Higgs trilinears at NLO and implications for $gg \to hh$

predicting κ_λ and σ_{hh} in *any* model.
Based on works with Henning Bahl, Johannes Braathen, Kateryna Radchenko, Georg Weiglein.

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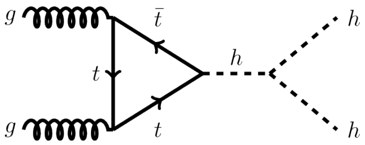
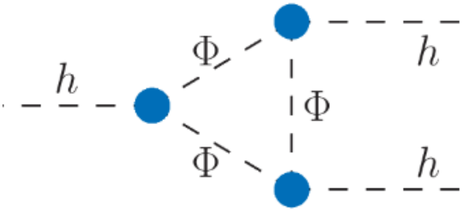
Why the trilinear self-coupling?

- probes electroweak symmetry breaking mechanism
- influences shape of the potential
- important for electroweak phase transition
- very sensitive to BSM loops (Part I)
- important input for di-Higgs production (Part II)

$$V_{\text{SM}} \supset \frac{m_h^2}{2} h^2 + \frac{1}{3!}\frac{3m_h^2}{v}\kappa_\lambda h^3 + \frac{1}{4!}\frac{3m_h^2}{v^2}\kappa_{2\lambda} h^4$$

$$\kappa_\lambda^{(n)} \equiv \frac{\lambda_{hhh}^{(n),\,\text{BSM}}}{\lambda_{hhh}^{(0),\,\text{SM}}},$$ n=loop−order in given BSM model





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- > probes electroweak symmetry breaking mechanism
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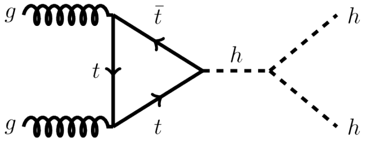
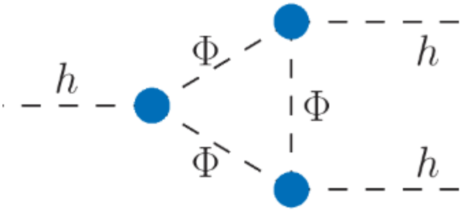
- > very sensitive to BSM loops (Part I)
  - two-loop also important (Part Ib) (if there is time left)
- > important input for di-Higgs production (Part II)





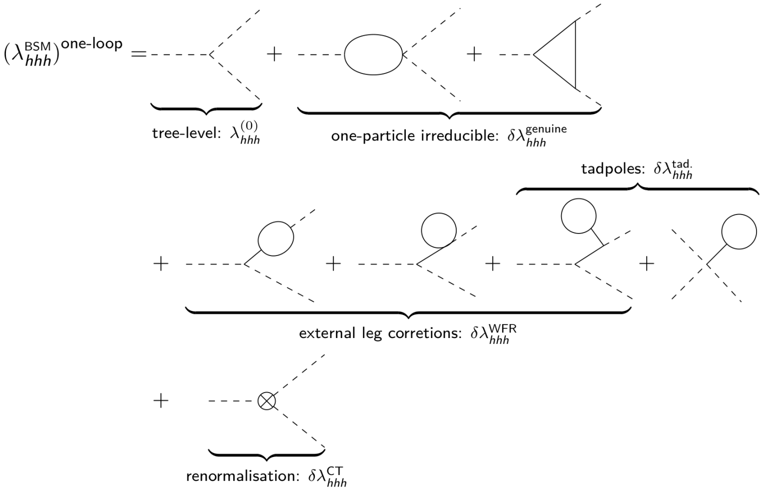
λ_{hhh} in and beyond the SM (Part I)

- Many studies for λ_{hhh} already exist
  - SM [Kanemura et al. '04][Senaha '18][Braathen et al. '19] ,
  - additional singlets [Kanemura et al. '16][Basler et al. '19],
  - doublets [Kanemura et al. '04][Basler et al. '17][Braathen et al. '19],
  - triplets [Aoki et al. '18][Chiang et al. '18],
  - SUSY: MSSM [Hollik et al. '02][Brucherseifer et al. '13] + NMSSM [Dao et al. '13][Dao et al. '15][Borschensky et al. '22]
- Higher-order corrections can be significant
- Many more details and models to explore!
  - suitable renormalisation schemes
  - estimate theoretical uncertainties
  - simple scanning / re-usability

$\rightarrow$ anyH3 [Bahl, Braathen, MG, Weiglein '23]:

automated tool to calculate λ_{hhh} (soon also $\lambda_{h_i h_j h_k}$ and $\sigma_{h_i h_j}$) in *any* model

Higher-order corrections to λ_{hhh} in any renormalisable theory



- Solid lines:
  - scalars,
  - fermions,
  - gauge/vector bosons,
  - ghosts
- possibility to exclude/restrict certain particles and/or topologies
- automatic non-trivial renormalisation
  - OS or $\overline{\text{MS}}$ masses
  - $\sim$ size of two-loop

Many more details to discuss!

How do you handle the

- treatment of tadpole corrections,
- treatment of external-leg corrections,
- renormalisation of electroweak VEV,
- renormalisation of mixing angles

...in a flexible way, that is applicable to:

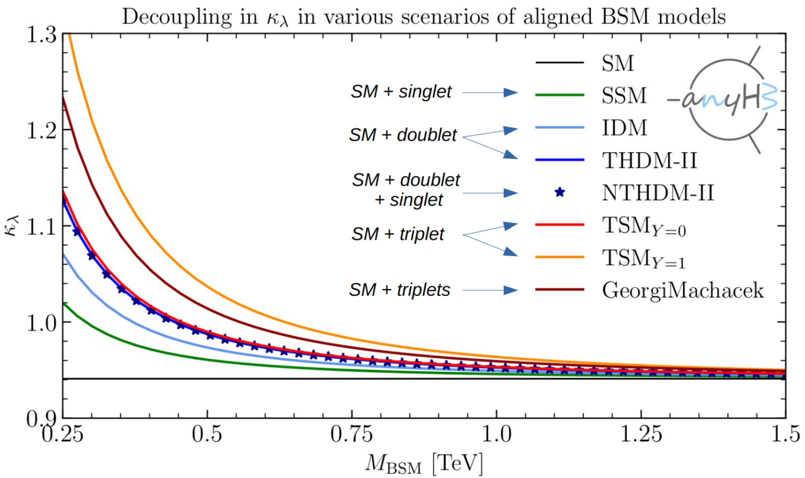
- a broad class of BSM models,
- a broad class of renormalisation schemes?

Answer: we do! Backup slides: feel free to ask questions!

Numerical results for κ_λ

note: also analytic results (`Mathematica`/`SymPy`) easily available!

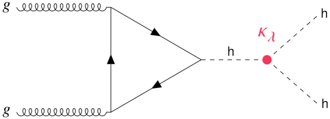
Decoupling → alignment

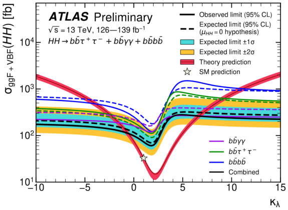


- ensure *appropriate* decoupling behaviour
- recover SM result for $M_{\mathrm{BSM}} \to \infty$
- further checks
  - literature (if available, e.g. MSSM)
  - UV-finiteness
  - FeynArts/FormCalc
- $\mathcal{O}(20)$ models built-in and cross-checked
- easy to implement new models (`UFO`)

Constraining BSM parameter space using κ_λ

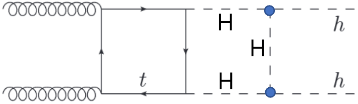
Leading-order parametrization used by experiment:



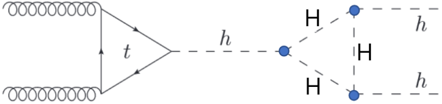


$pp \to hh$: $-1.2 < \kappa_\lambda < 7.2$ [ATLAS-CONF-2024-006]

Which BSM effects can this approach actually capture?



$\propto \mathcal{O}(y_t^2 g_{hhHH}^2)$ (not included)



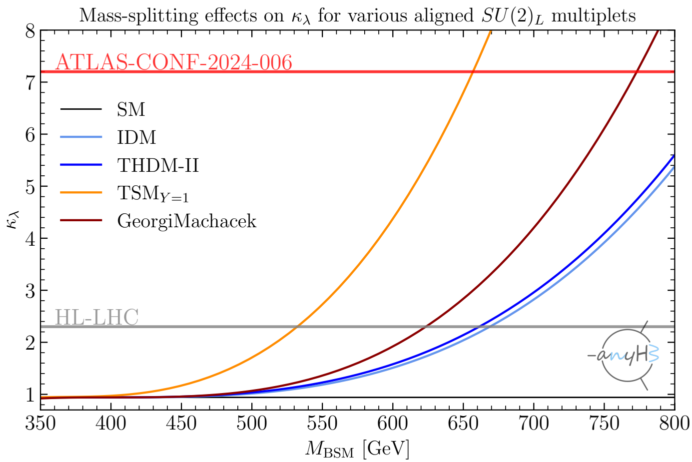
$\propto \mathcal{O}(y_t g_{hhHH}^3)$ (included)

When to apply the κ_λ-constraint to BSM models?

- no additional resonance in s-channel
- only κ_λ is *significantly* modified by BSM physics
- all other couplings SM-like

$\to$ a scenario often enforced by experimental constraints

Alignment w/o decoupling



- alignment: choose parameters such that $\kappa^{\text{tree-level}}_{hX_{\text{SM}}X_{\text{SM}}} = 1$
- introduce hierarchy within multiplet: $M_{\text{BSM}} > M^{\text{L}}_{\text{BSM}}$ (=400 GeV)
- induces large couplings for $\text{M}_{\text{BSM}} \to \infty : g_{hhHH} \gg y_t$
- corrections large-enough to exclude parameter space
- see [Bahl, Braathen, Weiglein '22] for in-depth discussion (THDM-I)

Simplest case: $V(\Phi_{\text{SM}}, H) \supset g_{hhHH}|\Phi_{\text{SM}}|^2 H^2 + \mu_H^2 H^2 \quad \Rightarrow \quad g_{hhHH} \propto (M_H^2 - \mu_H^2)/v_{\text{SM}}^2$

More results in the backup

- investigation of momentum dependence
- estimate missing higher-order BSM corrections
- dependence on m_t-scheme
- relative sign of κ_λ and κ_t

$\lambda_{h_i h_j h_k}$ and $\sigma_{gg\to hh}$ (Part II)

WIP [Bahl, Braathen, MG, Radchenko, Weiglein]

New update coming soon (`anyHH` / `anyBSM v2.0`):

- ability to compute arbitrary trilinear couplings
- even more flexible renormalisation
- double Higgs production cross-sections
  - few BSM predictions exist (SM+singlet(s),doublet,SUSY,...)
  - mostly NLO-QCD (K-factor≈ 2, HTL[Dawson et al. '98] or full m_t-dependence [Baglio et al. '21 and '23])
  - higher-order BSM corrections?

Double Higgs production

Independent calculation:

- at leading-order
  - hh, hH and AA production (later: Ah as well)
  - triangle and box form factors for generic theory [Plehn et al. '96]
  - pre-integrated luminosities and/or LHAPDF [Buckley et al.]
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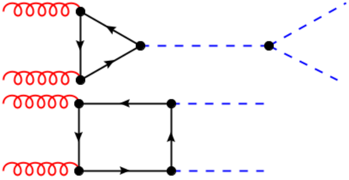

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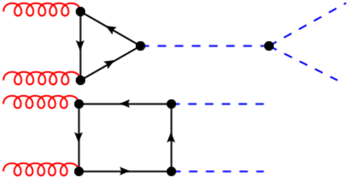
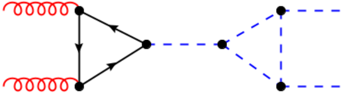


- at NLO
  - BSM: capture corrections to triangle formfactor; propagator corrections
  - QCD: flat K-factor ≈ 2 (HTL [Dawson, Dittmaier, Spira '98] : WIP)



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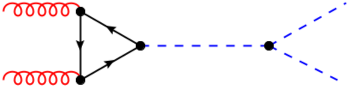

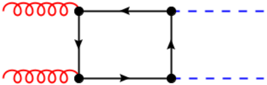

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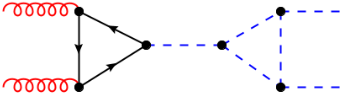

> flexible setup

- total cross-section + differential distributions
- automatically makes use of loop-induced couplings
- turn on/off individual resonances, couplings etc. pp....

$\rightarrow$ individual definitions of resonant/non-resonant contributions (exp. constraints!)

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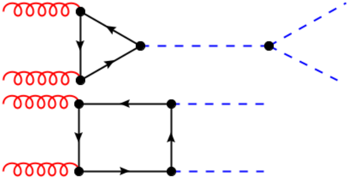

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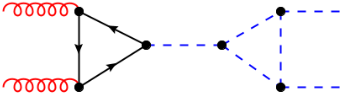

- flexible setup
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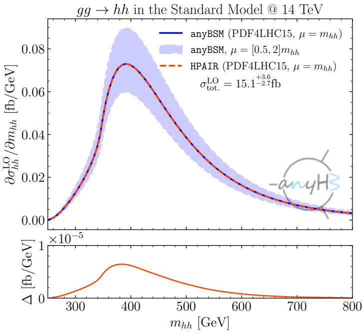
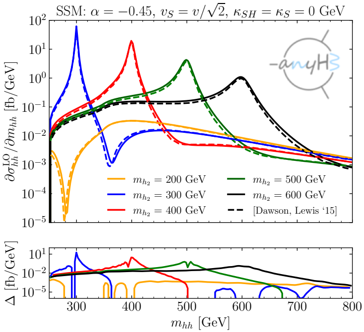
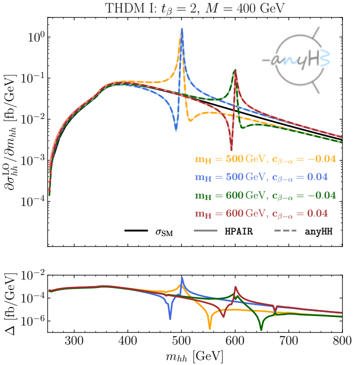
- WIP: MG5-integration for arbitrary processes

Cross-checks: SM,SSM,THDM perfect agreement with HPAIR







Double Higgs production: inert complex triplet Δ $(Y=1)$

$$
\begin{aligned}
V(\Phi,\Delta) = {} & m^2\Phi^\dagger\Phi + M^2\mathrm{Tr}(\Delta^\dagger\Delta) \\
& + \lambda_1(\Phi^\dagger\Phi)^2 + \lambda_2\left[\mathrm{Tr}(\Delta^\dagger\Delta)\right]^2 + \lambda_3\left[\mathrm{Tr}(\Delta^\dagger\Delta)^2\right] \\
& + \lambda_4(\Phi^\dagger\Phi)\mathrm{Tr}(\Delta^\dagger\Delta) + \lambda_5\Phi^\dagger\Delta\,\Delta^\dagger\Phi\,.
\end{aligned}
$$

> $\Delta = ((H^+/\sqrt{2}, -H^{++})^T, (H^0, -H^+/\sqrt{2})^T)$.

> invariant under $\mathbb{Z}_2 : \Delta \to -\Delta$ (forbid triplet VEV)

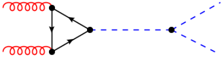
> most-relevant parameters: M_{H^+}, $M_{H^{++}}$, λ_4

> SM-like Higgs: in exact alignment with SM Higgs (protected by $\mathbb{Z}_2$): $\kappa_\lambda^{(0),\,\mathrm{TSM}} = 1$

However: $\delta^{(1)}\lambda_{hhh} \propto \frac{1}{(4\pi)^2}\frac{\left(M^2_{H^+}-M^2_{H^{++}}\right)^2}{v^3}$!

Double Higgs production: inert complex triplet Δ $(Y=1)$

> LO$^{\rm BSM}$ (tree-level $\lambda^{(0)}_{hhh}$): $\sigma^{\rm TSM}_{hh}(\lambda^{(0)}_{hhh}) = \sigma^{\rm SM}_{hh}$



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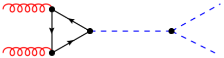
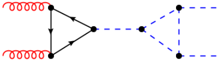
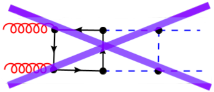
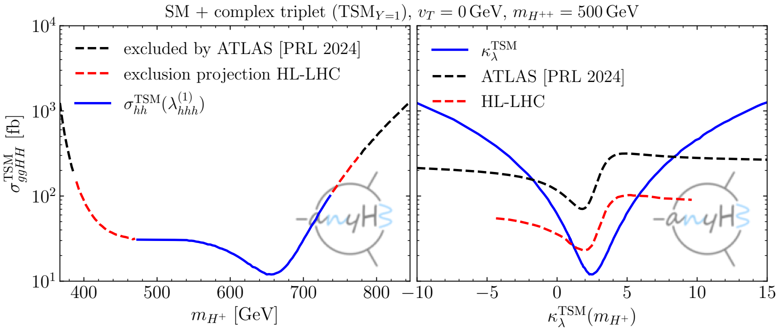


- NLO$^{\mathrm{BSM}}$ (one-loop $\lambda^{(1)}_{hhh}$): $\sigma^{\mathrm{TSM}}_{hh}(\lambda^{(1)}_{hhh}) \approx \sigma^{\mathrm{SM}}_{hh}(\kappa_\lambda = \kappa_\lambda^{(1),\,\mathrm{TSM}})$



BSM double-box diagram doesn't exist! $\rightarrow$ full NLO$^{\mathrm{BSM}}$ prediction





attention: only works if *hH*-mixing / *hhH*-coupling is protected by a symmetry!

Impact of loop-induced couplings: additional resonances

What if alignment is only *accidental*?

Impact of loop-induced couplings: additional resonances

What if alignment is only *accidental*?

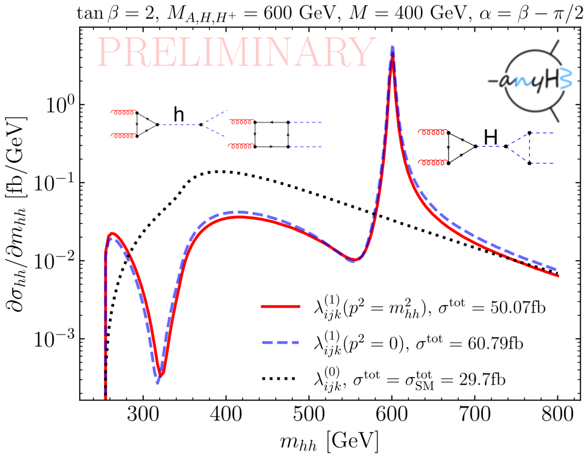
Example:

> THDM-II with $\alpha = \tan\beta + \pi/2$

- two CP-even Higgs bosons h, H ($+$ A, $H^{\pm}$)
- tree-level: h same couplings as in the SM
- tree-level: hhH-coupling vanishes
  $\rightarrow$ heavy resonance not contributing
  $\rightarrow$ indistinguishable from SM-prediction

> leading NLO$^{\mathrm{BSM}}$ corrections:

- OS renormalisation of α, $\tan\beta$, m_h, m_H
- non-zero hhH-coupling
  $\rightarrow$ peak appears
- slight distortion due to momentum dependence



w/o alignment: see Kateryna's talk [Heinemeyer, Mühlleitner, Radchenko, Weiglein '24]

Estimating missing higher-order (BSM) corrections

Notation: $h=1,\ H=2$

...via scheme conversion

- simple scheme: all masses and mixing angles OS (KOSY-like) but $M^2 = -\frac{m_{12}^2}{\sin\beta\cos\beta}$ is $\overline{\text{MS}}$
- non-minimal ren. of M?
- considering alignment limit $(\alpha = \beta - \pi/2)$:

$$\lambda_{111}^{(0)} = \frac{3m_h^2}{v}, \quad \lambda_{112}^{(0)} = 0, \quad \lambda_{122}^{(0)} = \frac{(m_h^2 + 2m_H^2 - 2M^2)}{2\,v}, \quad \lambda_{222}^{(0)} = \frac{M^2 - M_H^2}{v}\frac{6}{t_{2\beta}}$$

- 222^{OS}-scheme:
  - ren. condition:

$$\lambda_{222}^{(1),\,\text{ren.}} = \lambda_{222}^{(0)}(M^{\text{OS}}) + \underbrace{\delta^{(1)}\lambda_{222}(M^{\text{OS}})}_{\text{diagrams+vertex CTs}} + (\delta^{\text{CT}}M^{\text{OS}})\frac{\partial}{\partial M^{\text{OS}}}\lambda_{222}^{(0)} \stackrel{!}{=} \lambda_{222}^{(0)}$$

  - solve for

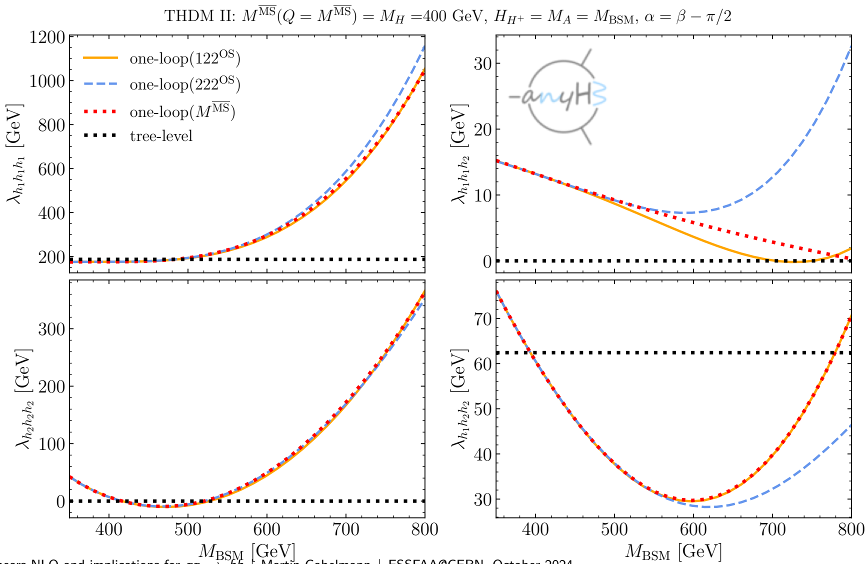
$$\delta^{\text{CT}}M = \frac{\lambda_{222}^{(0)} - \delta^{(1)}\lambda_{222}}{\partial\lambda_{222}^{(0)}/\partial M}$$

←anyH3 needs only this equation

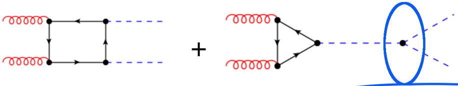
  - starting with $M^{\overline{\text{MS}}}$ and converting to M^{OS} we generate higher-orders

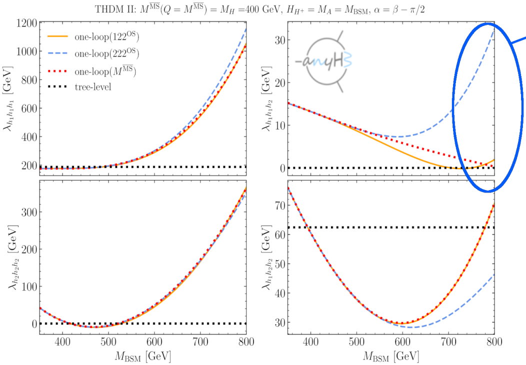
$$\lambda_{ijk}(M^{\text{OS}}) = \lambda_{ijk}(M^{\overline{\text{MS}}} - \delta^{\text{CT}}M^{\text{OS,fin}}) = \lambda_{ijk}(M^{\overline{\text{MS}}}) - \lambda'_{ijk}\delta^{\text{CT}}M^{\text{OS, fin}} + \lambda''_{ijk}\left(\delta^{\text{CT}}M^{\text{OS, fin}}\right)^2/2 + \dots$$

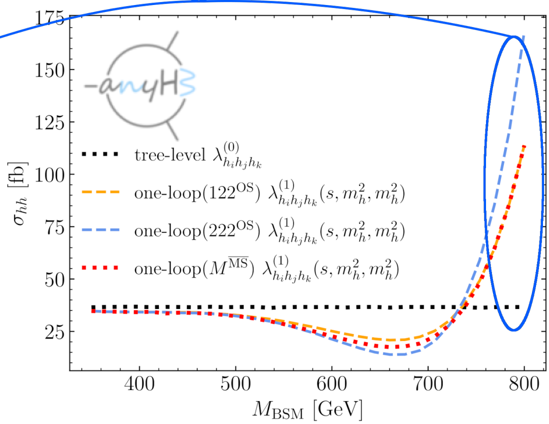
Estimating missing higher-order (BSM) corrections: λ_{ijk}



Estimating missing higher-order (BSM) corrections: σ_{hh}





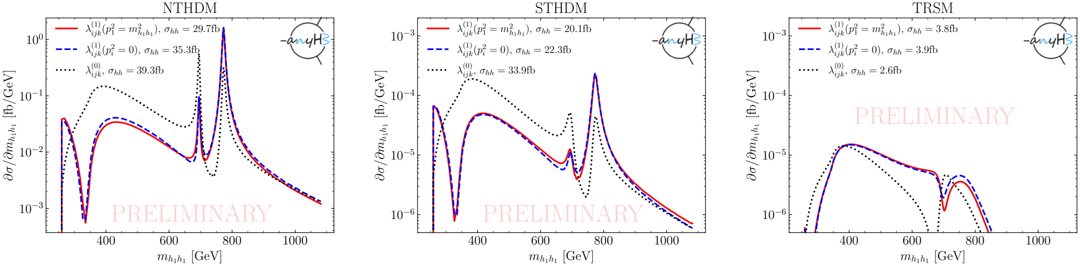


Double Higgs production: multiple resonances

- THDM + real singlet (NTHDM)
- THDM + complex singlet (STHDM)
- SM + two real singlets (TRSM)

} Three CP-even Higgs bosons $h_1,\ h_2,\ h_3$. Two possibly resonant!

With same masses and mixing angles:



very simple to generalise / run new models!

Two-loop corrections to scalar amplitudes (Part Ib)

WIP [Bahl, Braathen, MG, Paßehr]

- Large one-loop corrections to λ_{hhh}
  $\rightarrow$ strong motivation to study two-loop corrections
  $\rightarrow$ study new genuine two-loop effects (e.g. BSM self-couplings in inert scenarios)

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- Large one-loop corrections to λ_{hhh}
  → strong motivation to study two-loop corrections
  → study new genuine two-loop effects (e.g. BSM self-couplings in inert scenarios)
- generic setup at two-loops: (FeynArts + TwoCalc)
  - generic tadpoles + self-energies (two-loop counterterm) ✓
  - generic two-loop three-point function (e.g. di-Higgs) ✓
  - generic two-loop four-point function (e.g. tri-Higgs or EFT-UV matchings) ✓
  - no gauge-less limit applied!
- more details in backup

Example: two-loop corrections to λ_{hhh} in a singlet extension

- SM + real singlet S, $\langle S \rangle = v_S$:

$$V_{\text{SSM}} = \mu^2|\Phi|^2 + \frac{\lambda_H}{2}|\Phi|^4 + \frac{m_S^2}{2}S^2 + \frac{\kappa_S}{3}S^3 + \frac{\lambda_S}{2}S^4 + \kappa_{SH}S|\Phi|^2 + \frac{\lambda_{SH}}{2}S^2|\Phi|^2 \,.$$

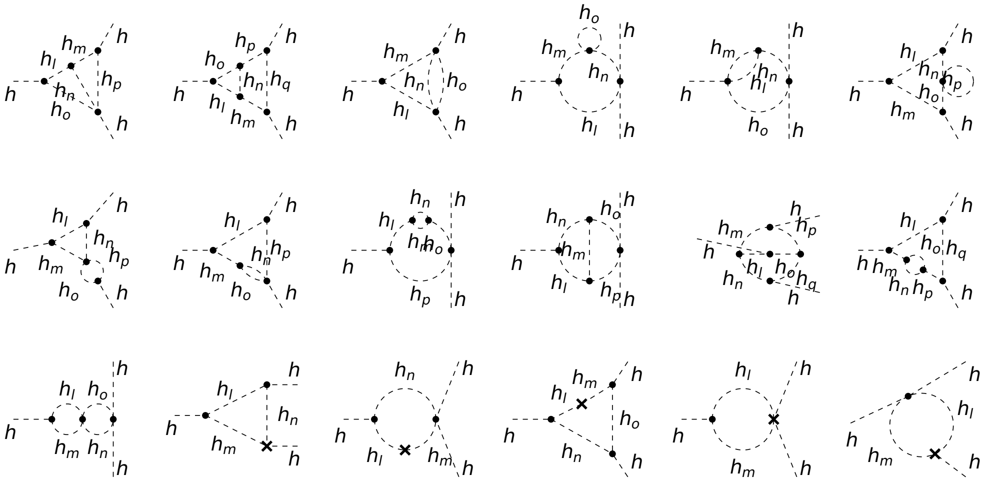
- consider heavy-singlet case $m_s \gg m_h$
- and no mixing $\alpha = 0$ (alignment)
  - the two-loop CT depends on $\left(\delta^{(1)}\alpha\right)^2$
    $\to$ proper OS/MS treatment

$$\delta^{(2)}\lambda_{hhh}^{\text{OS}} = \frac{3}{v^2}\left[\delta^{(2)}t_h - v\,\delta^{(2)}m_h^2 + (\delta^{(1)}\alpha)^2\frac{\kappa_{SH}v^4}{v_S} + \frac{3}{2}\left(\delta^{(1)}t_h - v\delta^{(1)}m_h^2 + \hat{\lambda}_{hhh}^{(1)}\right)\delta^{(1)}Z_{hh}\right]$$
$$- \frac{3\kappa_{SH}v}{4v_S}\left(\delta^{(1)}Z_{sh}\right)^2 + \frac{3}{2}\left[\left(2\frac{v}{v_S}\kappa_{SH} + \frac{m_s^2}{v}\right)\delta^{(1)}\alpha + \hat{\lambda}_{hhs}^{(1)}\right]\left(\delta^{(1)}Z_{sh}\right) + \delta^{(2)}\lambda_{hhh}^{\text{diag.}}$$
$$\approx -\frac{1}{(4\pi)^4}\frac{9\kappa_{SH}^3v^3}{2v_S^5} + \mathcal{O}\Big(\frac{m_h^2}{m_s^2}, \frac{\kappa_{SH}^2}{m_s^2}, \frac{\kappa_S^2}{m_s^2}\Big)$$

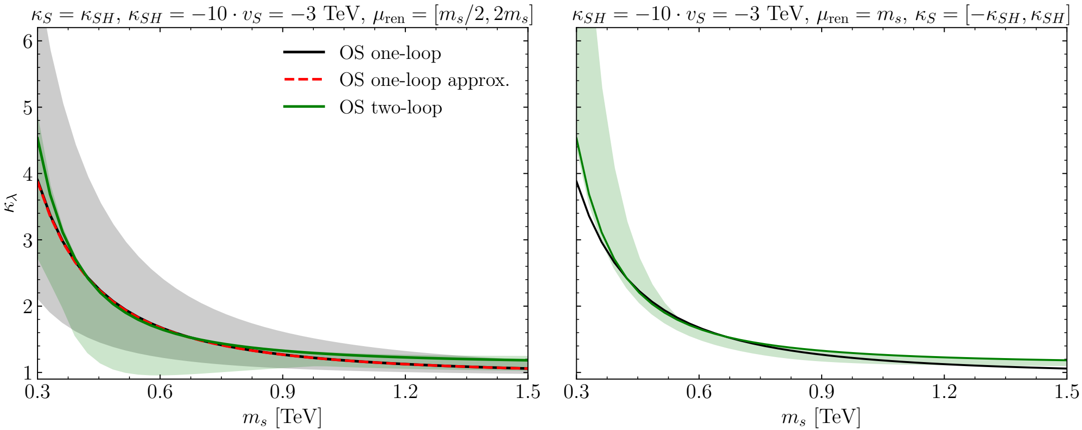
(full result in backup slides)

Example: two-loop corrections to λ_{hhh} in a singlet extension

reduction of diagrams using canonical edges: only a handful of diagrams left



Example: two-loop corrections to λ_{hhh} in a singlet extension



- left: reduction of theoretical uncertainty
- right: dependence on κ_S (singlet self-coupling) appears first at two-loop

Outlook / Summary

- λ_{hhh} in arbitrary ren. QFTs
  - at the full one-loop order
  - optional momentum dependence
  - flexible choice of renormalisation schemes
- analytical results; fast numerical results $\mathcal{O}$(ms)
- already studied many models:
  SM, SM+**singlets, doublets, triplets**, **SUSY**, ...
- found large mass-splitting effects
- $\lambda^{\text{(one-loop)}}_{h_i h_j h_k}$, $\lambda^{\text{(two-loop)}}_{hhh}$ **and** σ_{hh} **coming soon**

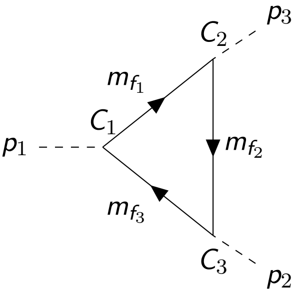
More info

- `pip install anyBSM`
- `anyBSM --help`
- documentation, tutorials and examples: anybsm.gitlab.io

Backup

Example: generic fermion triangle

Idea: compute *generic* diagrams i.e. assume most generic



> couplings $C_i = P_L C_i^L + P_R C_i^R$, $P_{R/L} = \frac{1 \pm \gamma_5}{2}$

> as well as loop-masses m_{f_i} and

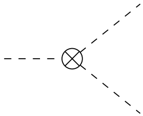
> external momenta p_i, $i = 1, 2, 3$.

$$= 2\mathbf{B0}(p_3^2, m_2^2, m_3^2)(C_1^L(C_2^L C_3^R m_{f_1} + C_2^R C_3^R m_{f_2} + C_2^R C_3^L m_{f_3}) + C_1^R(C_2^R C_3^L m_{f_1} + C_2^L C_3^L m_{f_2} + C_2^L C_3^R m_{f_3})) + m_{f_1}\mathbf{C0}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)(((C_1^L C_2^L C_3^R + C_1^R C_2^R C_3^L)(p_1^2 + p_2^2 - p_3^2) + 2(C_1^L C_2^L C_3^L + C_1^R C_2^R C_3^R) m_{f_2} m_{f_3} + 2m_{f_1}(C_1^L(C_2^L C_3^R m_{f_1} + C_2^R C_3^R m_{f_2} + C_2^R C_3^L m_{f_3}) + C_1^R(C_2^R C_3^L m_{f_1} + C_2^L C_3^L m_{f_2} + C_2^L C_3^R m_{f_3}))) + \mathbf{C1}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)(2p_2^2(C_1^L C_3^R(C_2^L m_{f_1} + C_2^R m_{f_2}) + C_1^R C_3^L(C_2^R m_{f_1} + C_2^L m_{f_2})) + (p_1^2 + p_2^2 - p_3^2)((C_1^L C_2^L C_3^R + C_1^R C_2^R C_3^L) m_{f_1} + (C_1^L C_2^R C_3^L + C_1^R C_2^L C_3^R) m_{f_3})) + \mathbf{C2}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)((p_1^2 + p_2^2 - p_3^2)(C_1^L C_3^R(C_2^L m_{f_1} + C_2^R m_{f_2}) + C_1^R C_3^L(C_2^R m_{f_1} + C_2^L m_{f_2})) + 2p_1^2((C_1^L C_2^L C_3^R + C_1^R C_2^R C_3^L) m_{f_1} + (C_1^L C_2^R C_3^L + C_1^R C_2^L C_3^R) m_{f_3}))$$

> insert concrete BSM model (UFO [Degrande et al. '11])

> evaluate with the help of (py)COLLIER [Denner et al. '16]

Generic renormalisation of λ_{hhh}

$\delta\lambda^{\text{CT}}_{hhh} =$  $= \; ?$

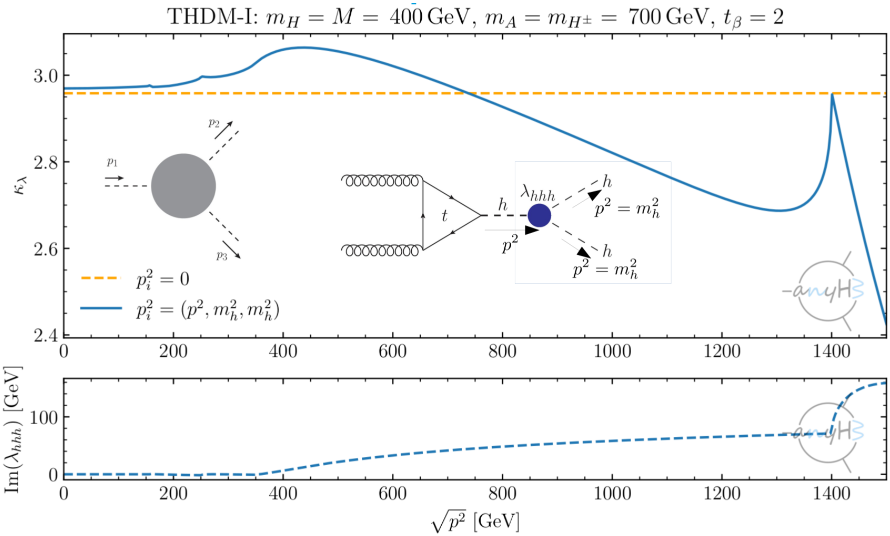
- one-loop $\rightarrow$ renormalisation of all parameters entering $\lambda^{(0),\text{BSM}}_{hhh}$ at tree-level
- In the SM $\lambda^{(0),\text{SM}}_{hhh} = \frac{3m_h^2}{v}$
- In general:

$$\lambda^{(0),\text{BSM}}_{hhh} \equiv \lambda^{(0),\text{BSM}}_{hhh}\Big(\underbrace{v^{\text{SM}},\, m_h^{\text{SM}}}_{\text{SM Higgs sector}},\ \underbrace{m_{X_i}}_{\text{further (OS) masses}},\ \underbrace{v_j}_{\text{BSM VEVs}},\ \underbrace{\alpha_k}_{\text{mixing angles}},\ \underbrace{\rho_l}_{\text{indep. parameters}}\Big)$$

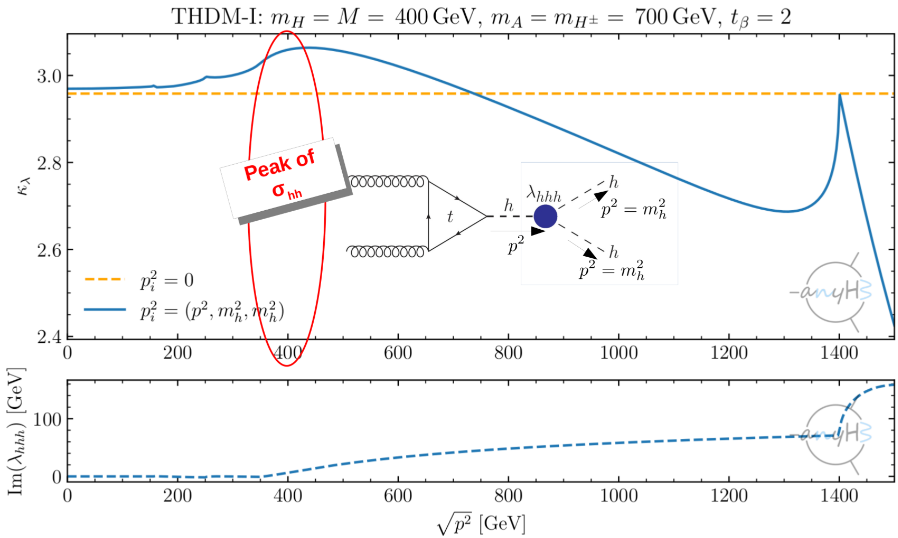
- user's choice:
  - **SM sector**: fully OS or $\overline{\text{MS}}/\overline{\text{DR}}$ (using $\alpha_{\text{QED}}(0)$, m_W, m_Z, m_h, see backup slides)
  - **BSM masses** (scalars/vectors/fermions): OS **or** $\overline{\text{MS}}/\overline{\text{DR}}$
  - **Additional couplings/vevs/mixings**: $\overline{\text{MS}}/\overline{\text{DR}}$ by default. **Custom ren. conditions possible!**

$$\delta^{(1)}_{\text{CT}}\lambda_{hhh} = \sum_p \left(\frac{\partial\lambda^{(0),\text{BSM}}_{hhh}}{\partial p}\right)\delta^{\text{CT}}p, \text{ with } p = \{m_h^{SM}, v^{\text{SM}}, m_{X_i}, \alpha_j, \dots\}^{\overline{\text{MS}}/\text{OS}/\text{custom}}$$

Momentum dependence in the THDM

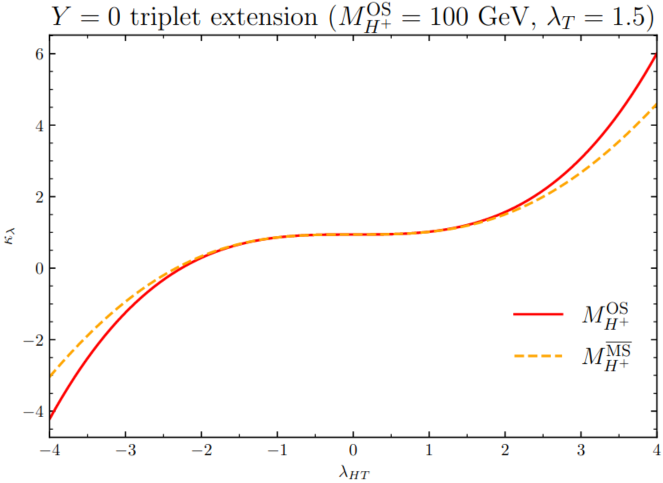


Momentum dependence in the THDM



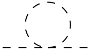
Uncertainty estimate: a real inert triplet $T = ((T^0/\sqrt{2}, T^+)^T, (T^-, T^0/\sqrt{2})^T)$

$$V(\Phi, T) = \mu^2|\Phi|^2 + \frac{\lambda}{2}|\Phi|^4 + \frac{M_T^2}{2}|T|^2 + \frac{\lambda_T}{2}|T|^4 + \frac{\lambda_{HT}}{2}|T|^2|\Phi|^2, \ \langle T^0 \rangle = 0, \ \langle \Phi^0 \rangle = v_{SM}$$



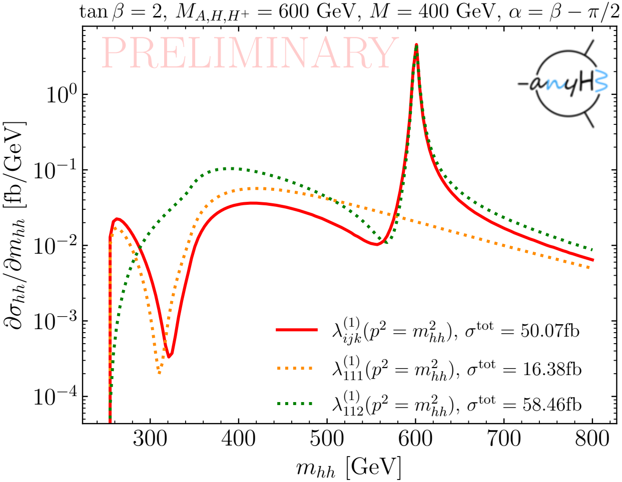
- at one-loop no explicit dependence of $\delta^{(1)}\lambda_{hhh}$ on λ_T
- but:

  $\delta^{(1),\text{OS}} M_{H^+} = \Sigma_T(p^2 = M_{H^+}^2)$

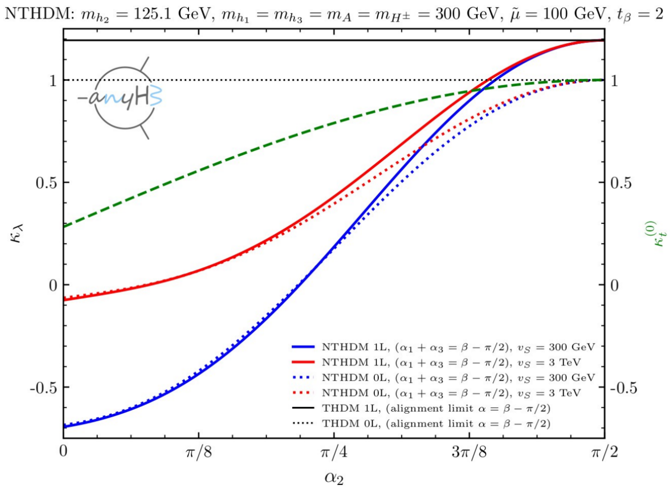
  $\supset$ 

  depends on λ_T
- $\lambda_{hhh}(M_{H^+}^{OS}) - \lambda_{hhh}(M_{H^+}^{\overline{MS}})$: estimates two-loop corrections generated by triplet self-coupling λ_T

Impact of loop-induced couplings: individual corrections

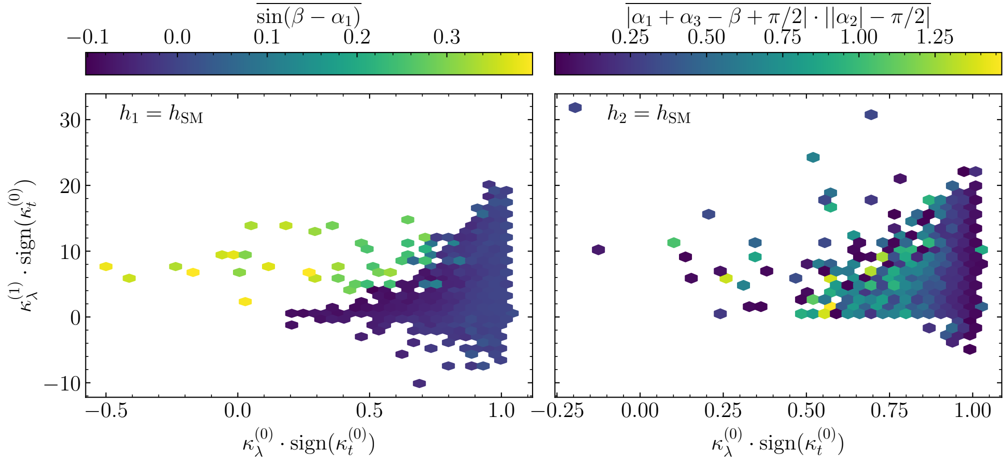


The sign of κ_λ in the NTHDM



- NTHDM=THDM+ real singlet
- 3 CP-even scalars $h_{1,2,3}$, 3 mixing angles $\alpha_{1,2,3}$
- $\alpha_2 \to \pi/2$: decoupling of singlet + alignment
- **attention**: from ggHH we only get $sgn(\kappa_t/\kappa_\lambda)$, the relative sign of top- and Higgs modifiers!
- $\kappa_t = \frac{y_t^{BSM}}{y_t^{SM}}$ strongly constrained

The sign of κ_λ in the NTHDM: full-fledged parameter scan



[WIP: Bosse, Braathen, MG, Hanning, Weiglein]

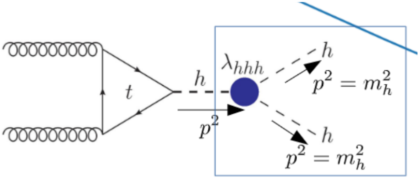
Treatment of external leg corrections

default treatment of external legs:

$$\delta^{(1)\,\text{,ext.-legs}}\lambda_{hhh} = -\sum_i \left[\frac{1}{2}\Sigma'_{hh}(p_i^2)\lambda^{(0)}_{hhh} + \underbrace{\sum_{j,h_j\neq h} \frac{\Sigma_{hh_j}(p_i^2)}{p_i^2 - m_{h_j}^2}\lambda^{(0)}_{h_j hh}}_{=0,\text{ for alignment}} \right]$$

> Attention: insert into di-Higgs production: need one off- and two on-shell Higgses:

$$\delta^{(1)\text{,ext.-legs}}\lambda_{hhh} = -\left(\tfrac{1}{2}+\tfrac{1}{2}\right)\Sigma'_{hh}(m_h^2)\lambda^{(0)}_{hhh}$$



> possible to turn-off default behaviour and specify ext.-leg contributions in terms of selfenergies

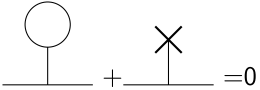
Treatment of tadpoles: many possibilities

At tree-level:

- define $t_h = \frac{\partial V}{\partial h}\Big|_{h=0}$ and $m_h^2 = \frac{\partial^2 V}{\partial h^2}\Big|_{h=0}$
- then $V_{\text{SM}} \supset t_h h + \frac{1}{2} m_h^2 h^2 + \frac{m_h^2 - t_h/v}{2v} h^3 + \frac{m_h^2 - t_h/v}{8v^2} h^4$
- popular choice $t_h = 0$ (but not the only choice!)

At one-loop: in general the renormalized tadpole consists of $\hat{t}_h = t_h + t_h^{(1)} + \delta t_h^{(1)}$

- "OS" tadpoles [Bohm '86, Denner '93]
  - demand $\hat{t}_h = t_h = 0$ at one-loop such that $t_h^{(1)} = -\delta t_h^{(1)}$
  - effectively no need to "attach" tadpoles to any diagrams



- "Fleischer-Jegerlehner (FJ)" tadpoles [Fleischer, Jegerlehner '01]
  - demand $t_h = 0$ at one-loop but let $\delta t_h^{(1)}$ cancel only divergent pieces
  - need to consider finite contributions of *all* 1PI diagrams
- "tadpole-free $\overline{\text{MS}}$ scheme" [Martin '01]
  - set $\delta t_h^{(1)} = 0$ and demand $\hat{t}_h = 0 \Rightarrow t_h = -t_h^{(1)}$
- Pinched scheme, GIVS [Dittmaier, Rzehak '22] ...(less relevant for this work)

Treatment of tadpole corrections for λ_{hhh}

w/o specifying a concrete scheme, nor the vacuum (in the alignment limit):

$$\lambda_{hhh}^{\text{tadpoles}} = -\underbrace{\frac{3t_h}{v^2}}_{\text{tree-level}} - \underbrace{\frac{6}{v^2}\delta^{(1)}_{\text{CT}}t_h}_{\text{CT-inserted diagrams}} + \underbrace{\delta^{(1)}_{\text{tadpoles}}\lambda_{hhh}}_{\text{tadpole diagrams}} + \underbrace{\frac{3}{v}\delta^{(1)}_{\text{CT, tadpoles}}m_h^2 - \frac{3m_h^2}{v^2}\delta^{(1)}_{\text{CT, tadpoles}}v}_{\text{tad. contr. to input parameters}}$$

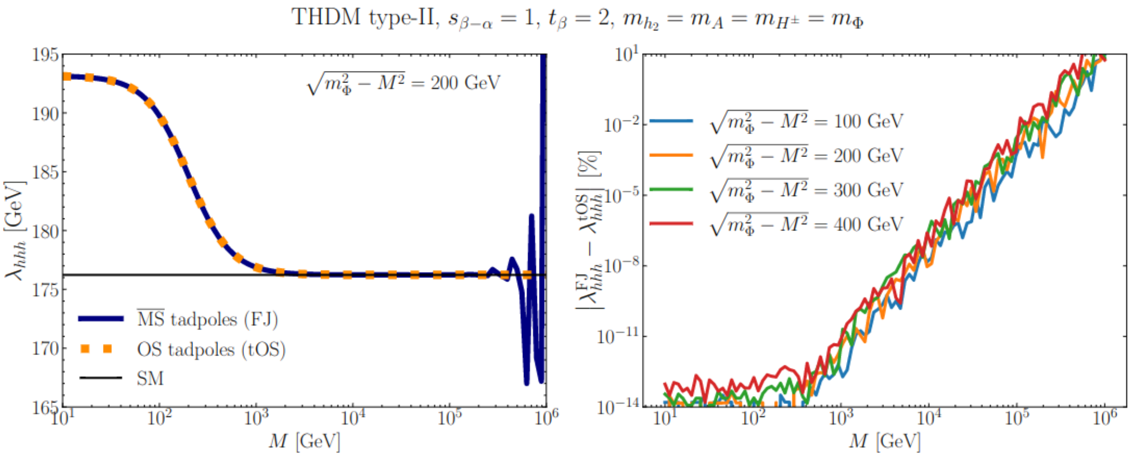
- In the SM (and BSM+alignment): once λ_{hhh} is expressed in terms of *physical* input parameters, its result is independent of the treatment (OS, FJ, ...) of the tadpoles (up to higher orders):

$$\delta^{(1)}\lambda_{hhh} \supset \frac{3}{v^2}\delta^{(1)}t_h|_{\text{finite}}$$

- However: `UFO` models do (often) **not** contain the explicit dependence on the tree-level/one-loop/one-loop-CT tadpoles.
- Thus: we choose the Fleischer-Jegerlehner treatment $t_h^{\text{tree-level}} = 0$ and renormalize $\delta^{(1)}t_h^{\text{CT}}|_{\text{finite}} = 0$ in the $\overline{\text{MS}}$ scheme per default (can also turn-off automatic tadpoles and implement own scheme).
- only need to take into account tadpole contributions

  to all two- and three-point functions:  and 

OS vs FJ tadpole treatment



Details on renormalisation of the SSM: OS scheme

OS conditions:

$$\delta^{(1)} m_s^2 = -\Sigma_s(p^2 = m_s^2)$$

$$\delta^{(1)} Z_{ij} = -\delta^{(1)} Z_{ij} = -2\Sigma_{ij}(p^2 = 0)/m_s^2, \; i \neq j,$$

$$\delta^{(1)} Z_{ii} = \frac{\partial}{\partial p^2} \Sigma_{ii}(p^2) \Big|_{p^2=0}, \; i, j = s, h,$$

$$\delta^{(2)} t_h = -t_h^{(2)} - \frac{1}{2} \delta^{(1)} Z_{hs} \delta^{(1)} t_s - \frac{1}{2} \delta^{(1)} Z_{hh} \delta^{(1)} t_h,$$

$$\delta^{(1)} m_{hs}^2 = (m_h^2 - m_s^2)\, \delta^{(1)} \alpha = -m_s^2\, \delta^{(1)} \alpha = \Sigma_{hs}(p^2 = 0)\, .$$

$\overline{\text{MS}}$ conditions:

$$(4\pi)^2 \delta^{(1)} \kappa_S^{\overline{\text{MS}}} = \frac{3}{\epsilon} \left(6\kappa_S \lambda_S + \kappa_{SH} \lambda_{SH}\right) ,$$

$$(4\pi)^2 \delta^{(1)} \kappa_{SH}^{\overline{\text{MS}}} = \frac{\lambda_{SH}}{\epsilon} \left(\kappa_S + 2\kappa_{SH}\right) .$$

Full result for λ_{hhh} in the SSM: OS scheme

$$(4\pi)^2 \lambda_{hhh}^{(1),\,\text{OS}} = -\frac{\kappa_{SH}^3 v^3}{2 v_S^3 m_s^2} + \mathcal{O}(m_h^2/m_s^2)\,,$$

$$(4\pi)^2 \lambda_{hhs}^{(1),\,\text{OS}} = -\frac{\kappa_{SH}^2 v^2}{4 v_S^4 m_s^2}\left(6 m_s^2 v_S - 2\kappa_S v_S^2 + 3\kappa_{SH} v^2\right) + \mathcal{O}(m_h^2/m_s^2)\,,$$

$$(4\pi)^4 \delta^{(2)} \lambda_{hhh}^{\text{OS}} = -\frac{9\kappa_{SH}^3 v^3}{2 v_S^5} - \frac{3\kappa_{SH}^3 v^3}{2 m_s^2 v_S^4}\left[(\kappa_S + 2\kappa_{SH})\overline{\ln} m_s^2 - 2(\kappa_S - \kappa_{SH}) - 3\kappa_{SH}\frac{v^2}{v_S^2}\right] \underbrace{- \frac{\kappa_{SH}^3 v^3}{8 m_s^4 v_S^3}\left[4\kappa_S^2 + \kappa_{SH}(5\kappa_{SH} - 12\kappa_S)\frac{v^2}{v_S^2} + 9\kappa_{SH}^2\frac{v^4}{v_S^4}\right]}_{\xrightarrow{m_s\to\infty} 0}\,.$$

behaves "nice" for $m_s \to \infty$

Full result for λ_{hhh} in the SSM: $\overline{\text{MS}}$ scheme

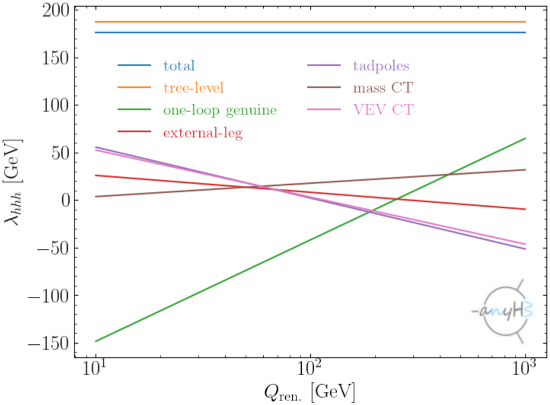
$$
\begin{aligned}
(4\pi)^4\delta^{(2)}\lambda_{hhh}^{\overline{\text{MS}}} = &- \frac{3}{8}\frac{\kappa_{SH}^2 v}{v_S^5}\Big[6\kappa_S v_S^2(3\overline{\ln} m_s^2 + \overline{\ln}^2 m_s^2 - 3) + 8\kappa_{SH} v_S^2(-2\overline{\ln} m_s^2 + \overline{\ln}^2 m_s^2 + 1) \\
&\qquad + \kappa_{SH} v^2(-23\overline{\ln} m_s^2 - 3\overline{\ln}^2 m_s^2 + 35)\Big] \\
&- \frac{1}{m_s^2}\frac{3\kappa_{SH}^2 v}{16 v_S^6}\Big[\kappa_{SH}^2 v^4(35 - 17\overline{\ln} m_s^2) - 4\kappa_S^2 v_S^4(\overline{\ln} m_s^2 - 1) \\
&\qquad + 4\kappa_{SH} v_S^2 v^2\left(\kappa_S(3\overline{\ln} m_s^2 - 8) - 6\kappa_{SH}(\overline{\ln} m_s^2 - 1)\right)\Big] \\
&+ \frac{1}{m_s^4}\frac{\kappa_{SH}^3 v^3(\overline{\ln} m_s^2 - 2)}{16 v_S^7}\Big[4\kappa_S^2 v_S^4 + 4\kappa_{SH} v_S^2 v^2(2\kappa_{SH} - 3\kappa_S) + 9\kappa_{SH}^2 v^4\Big] \\
&+ m_s^2\frac{9\kappa_{SH}^2 v}{4 v_S^4}\left(4\overline{\ln} m_s^2 + \overline{\ln}^2 m_s^2 - 4\right).
\end{aligned}
$$

Shows non-decoupling behaviour for $m_s \to \infty$!

Need to simultaneously scale $v_s \propto m_s$.

Simple cross-check: UV-finiteness in the SM

Numerically: scale independent result



Analytically: cancellaiton of $^1/_\epsilon$ poles

```
<< anyBSM`
LoadModel["SM"]
lam = lambdahhh[];
(lam["total"] - lam["treelevel"] //. UVparts // Simplify) == 0

True
```

(Default) Renormalization choice of $(v^{\text{SM}})^{\text{OS}}$ and $(m_i^2)^{\text{OS}}$

- $v^{\text{OS}} \equiv \frac{2M_W}{e}\sqrt{1-\frac{M_W^2}{M_Z^2}}$ with (remember: $\lambda_{hhh}^{(0)} \approx 3m_h^2/v$)
  - $\delta^{(1)} M_V^2 = \frac{\text{Re}\Pi_V^{(1),T}}{M_V^{2\ OS}}$, $V = W, Z$
  - $\delta^{(1)} e = \frac{1}{2}\Pi_\gamma + \text{sign}(\sin\theta_W)\frac{\sin\theta_W}{M_Z^2\cos\theta_W}\Pi_{\gamma Z}$
- attention *(i)*: $\rho^{\text{tree-level}} \neq 1 \rightarrow$ further CTs needed (depends on the model)

  $\rightarrow$ ability to define *custom* renormalisation conditions
- scalar masses: $m_i^{\text{OS}} = m_i^{\text{pole}}$
  - $\delta^{\text{OS}} m_i^2 = -\text{Re}\Sigma_{h_i}^{(1)}\big|_{p^2=m_i^2}$
- attention *(ii)*: scalar mixing may also require further CTs/tree-level relations

All bosonic one- & two-point functions and their derivatives for general QFTs are required for flexible OS renormalisation.

Feature list (so far) of `anyH3`

- import/convert arbitrary `UFO` models
- (semi)automatic renormalisation
  - OS or MS mass renormalization
  - OS or MS electroweak VEV
  - provide custom renormalization *conditions* (no need to compute diagrams) $\rightarrow$ estimate size of missing higher-orders
- optional: full p^2 dependence
- numerical / analytical / $\LaTeX$ outputs
- `Python`-library with `command-line`- and `Mathematica`-interface
- ...

```
pip install anyBSM
```

```python
from anyBSM import anyH3
myfancymodel = anyH3('path/to/UFO/model')
result = myfancymodel.lambdahhh()
```

more examples at anybsm.gitlab.io

Example for OS scheme definition (THDM)

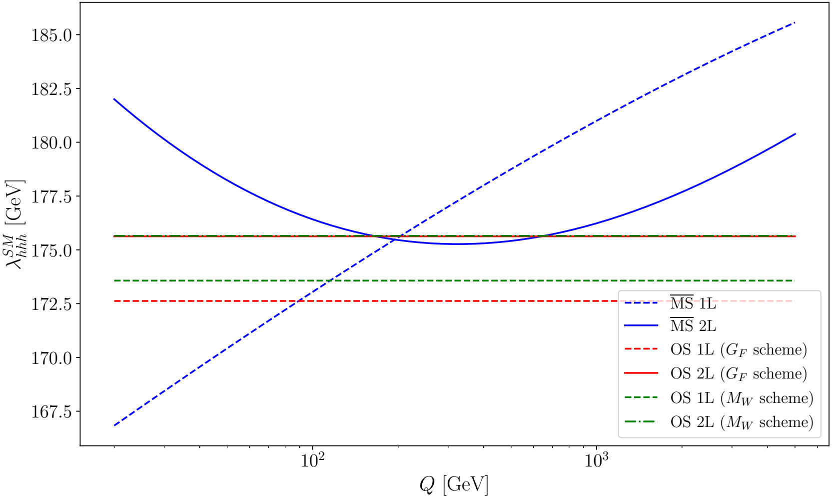
New simplified syntax in v2!

```
tadpoles: False
mass_counterterms:
  h1: OS
  h2: OS
parameter_counterterms:
  - parameter: TadH1
    counterterm: dTadH1
    condition: Tadpole('h2')*cos(alphaH) - Tadpole('h1')*sin(alphaH)
  - parameter: TadH2
    counterterm: dTadH2
    condition: Tadpole('h1')*cos(alphaH) + Tadpole('h2')*sin(alphaH)
  - parameter: betaH
    counterterm: dbetaH
    condition: (Re(Sigma('Hm1','Hm2',momentum='MHm1**2')) + Re(Sigma('Hm2','Hm1',momentum='MHm2**2')) + 2*(dTadH2*c
#     condition: (Re(Sigma('Ah1','Ah2',momentum='MAh1**2')) + Re(Sigma('Ah2','Ah1',momentum='MAh2**2')) + 2*(dTadH2*
    warn: False # turns-off warning that betaH is not an UFO input
  - parameter: TanBeta # this is the actual UFO input
    counterterm: dTanBeta
    condition: dbetaH/cos(betaH)**2 # depends on CT defined above
  - parameter: alphaH
    counterterm: dalphaH
    condition: (Re(Sigma('h1','h2',momentum='Mh1**2')) + Re(Sigma('h2','h1',momentum='Mh2**2')))/(2*(Mh1**2-Mh2**2)
    # countererm of M: takes into account running of M from Q=M to Q=Qren
  - parameter: M
```

Beyond anyH3

Two-loop effects in the SM

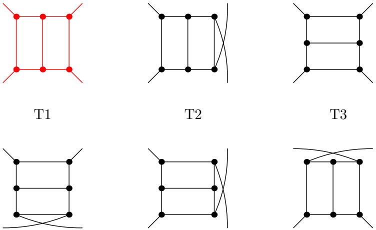
...and estimate of missing 3L effects



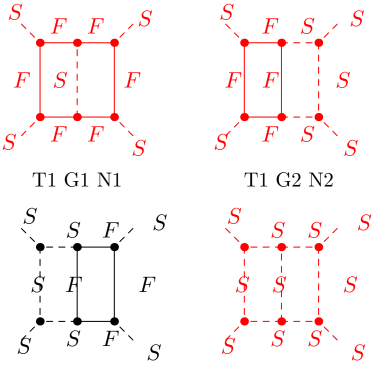
see [Braathen, Kaneura '19] for earlier works.

Geneneric Two-loop: Symmetries $\rightarrow$ reducing the number of diagrams

- external states identical $(h \rightarrow h;\ h, h \rightarrow h;\ h, h \rightarrow h, h)$
- external momenta to zero $p^2_{\text{ext.}} = 0$
- many diagrams identical
- example: double-box with fermion-scalar insertion



T1 T2 T3 T4 T5 T6



T1 G1 N1 T1 G2 N2 T1 G3 N3 T1 G4 N4

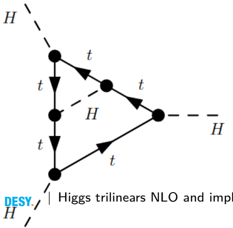
$\rightarrow$ only 3 (instead of 24) unique generic diagrams!

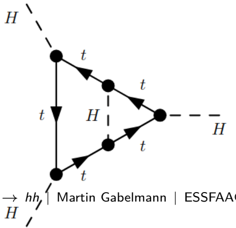
Canonical form of diagrams

"canonical edge" = unique representation of diagram:

- list of "edges" (=lines)
- identical diagrams $\leftrightarrow$ permutations
- canonical form = special ordering

```
{edge[v[1], v[4],  S[1]], edge[v[2], v[5],  S[1]],
 edge[v[3], v[6],  S[1]], edge[v[4], v[7], -F[3]],
 edge[v[4], v[8],  F[3]], edge[v[5], v[6],  F[3]],
 edge[v[5], v[8], -F[3]], edge[v[6], v[7],  F[3]],
 edge[v[7], v[8],  S[1]]}
```





- canonical-edges algorithm in pseudo code:
  - identify internal indices
  - identify external indices
  - generate permutations of external indices
  - generate permutations of internal indices
  - combine permutations of internal and external indices
  - permute edge list following the combined list of permutations
  - sort list of permuted edge lists
  - return first edge list after sorting

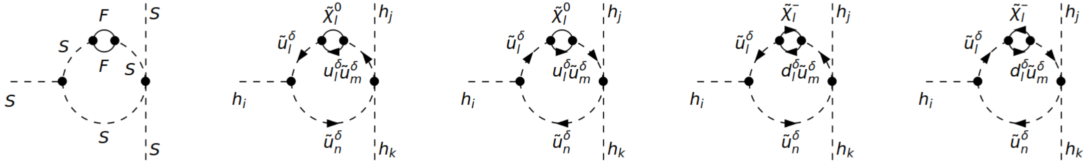
Symmetries: reducing the number of diagrams

- $n = 0, 1, 2, 3, 4$-point function with identical external fields
- count number of two-loop diagrams before$\rightarrow$after reduction of diagrams using canonical edges
  - at the topology-level
  - and field-level
- reduction of up to one order of magnitude!
- not counted: model-specific particle-insertions and summation over generation indices

n	topology-level	field-level
0	$2 \rightarrow 2$	$11 \rightarrow 11$
1	$3 \rightarrow 3$	$25 \rightarrow 25$
2	$9 \rightarrow 8$	$121 \rightarrow 92\,(102)$
3	$40 \rightarrow 13$	$936 \rightarrow 229\,(291)$
4	$265 \rightarrow 29$	$10496 \rightarrow 698\,(928)$

Cross-check: CP-violating NMSSM

- $\lambda_{hhh}^{\mathcal{O}(\alpha_t^2)}$ first computed in [Borschensky et al. '22] (see talk by MG@KUTS23)
- w/o symmetry-reduction: check on diagram-by-diagram level



1 2 3 4 5

(YES) Topology 12: my: $\left\{-1.71058 - \frac{0.098447}{\epsilon^2} + \frac{0.487837}{\epsilon}, -1.71058 - \frac{0.098447}{\epsilon^2} + \frac{0.487837}{\epsilon}, -0.710459 - \frac{0.0492235}{\epsilon^2} + \frac{0.213257}{\epsilon}, -0.710459 - \frac{0.0492235}{\epsilon^2} + \frac{0.213257}{\epsilon}\right\}$

new: $\left\{-1.71058 - \frac{0.098447}{\epsilon^2} + \frac{0.487837}{\epsilon}, -1.71058 - \frac{0.098447}{\epsilon^2} + \frac{0.487837}{\epsilon}, -0.710459 - \frac{0.0492235}{\epsilon^2} + \frac{0.213257}{\epsilon}, -0.710459 - \frac{0.0492235}{\epsilon^2} + \frac{0.213257}{\epsilon}\right\}$ my-new: {0, 0, 0, 0}

- full numerical agreement for all genuine 2L diagrams
- w/ symmetry reduction:

diagrams	topology-level	field-level
genuine two-loop	$39 \to 12$	$213 \to 67(32)$
sub-loop	$15 \to 5$	$36 \to 12(7)$

W mass prediction

- start with HO corrections to muon decay: $M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)=\frac{\pi\alpha_{\text{em}}}{\sqrt{2}G_F}\left[1+\Delta r\right]$
- and solve for: $M_W^2=M_Z^2\left[\frac{1}{2}+\sqrt{\frac{1}{4}-\frac{\pi\alpha_{\text{em}}}{\sqrt{2}G_F M_Z^2}(1+\Delta r)}\right]$
- with: $\Delta r^{(1)}=2\delta^{(1)}e+\frac{\Pi_W^{(1),T}(0)-\delta^{(1)}M_W^2}{M_W^2}-\frac{\delta^{(1)}\sin^2\theta_W}{\sin^2\theta_W}+\delta_{\text{vertex+box}}$
- and: $\frac{\delta^{(1)}\sin^2\theta_W}{\sin^2\theta_W}=\frac{\cos^2\theta_W}{\sin^2\theta_W}\left(\frac{\Pi_W^{(1),T}(M_W^2)}{M_W^2}-\frac{\Pi_W^{(1),T}(M_W^2)}{M_W^2}\right)$

It's all there but:

- $\delta^{\text{SM}}_{\text{vertex+box}}=-\frac{2\,\text{sign}(\sin\theta_W)}{\cos\theta_W\sin\theta_W M_Z^2}\Pi_{Z\gamma}(p^2=0)+\frac{\alpha_{QED}}{4\pi\sin^2\theta_W}\left(6+\frac{7-4\sin^2\theta_W}{2\sin^2\theta_W}\right)\log(\cos^2\theta_W)$
- $\delta^{\text{BSM}}_{\text{vertex+box}}=$ **needs to be implemented**

However:

- in many models $\Delta r\supset\frac{\delta\sin^2\theta_W}{\sin\theta_W}\approx\delta\rho$ is the dominant effect!

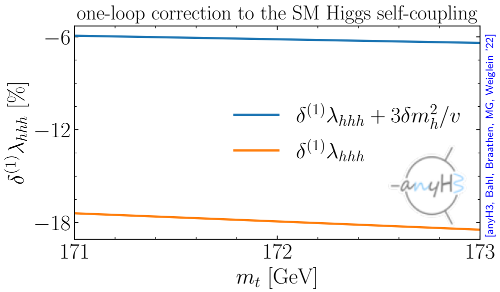
λ_{hhh} in the SM and in SUSY

In the SM at tree-level:

$$V(h) \supset \frac{m_h^2}{2}h^2 + \frac{m_h^2}{2v}h^3 + \ldots \quad \Rightarrow \quad \lambda_{hhh}^{\text{SM}} = \frac{\partial^3 V(h)}{\partial^3 h} = \frac{3m_h^2}{v}$$

Thus $\lambda_{hhh}^{\text{SM}}$ can be predicted perturbatively as a function of the SM parameters.

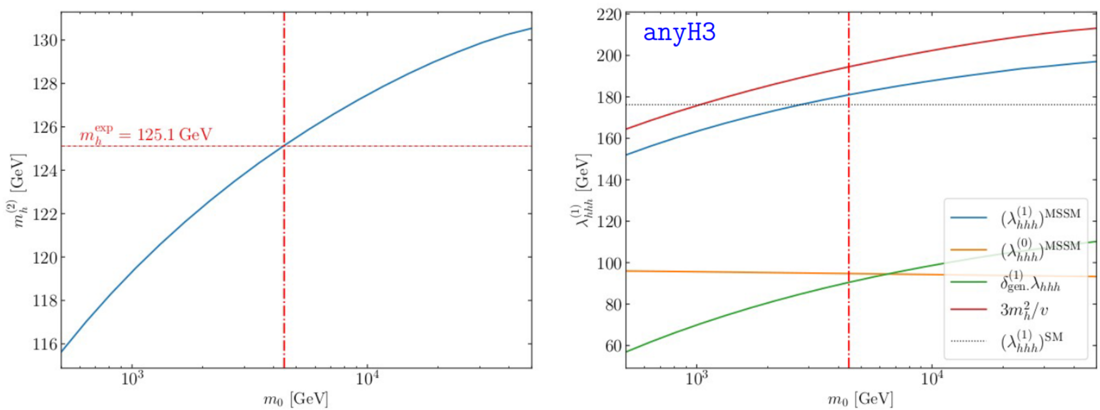
- corrections to λ_{hhh} are expected to behave similar to those of the Higgs boson mass
- OS scheme for m_h allows to "absorb" large part of corrections
- in SUSY:
  - $\lambda_{hhh} = 3m_h^2/v$ approximate[Dobado, Herrero, Hollik, Penaranda '02]
  - but m_h not free and $m_h \lesssim m_Z$ at tree-level!
    → requires loop corrections of about 40 GeV (15-30%)
    → can't stop at one-loop; need higher orders (→KUTS)



→ the precision of λ_{hhh} (order in perturbation theory) should match those of m_h!

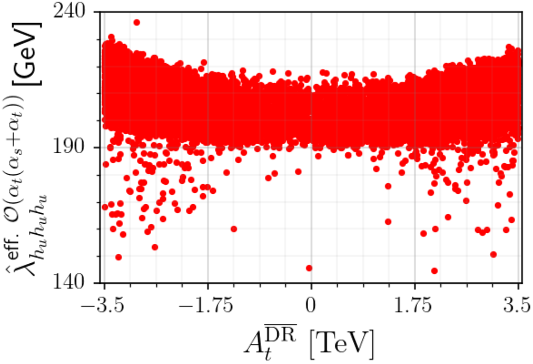
Full MSSM result: interface anyH3 to SPheno

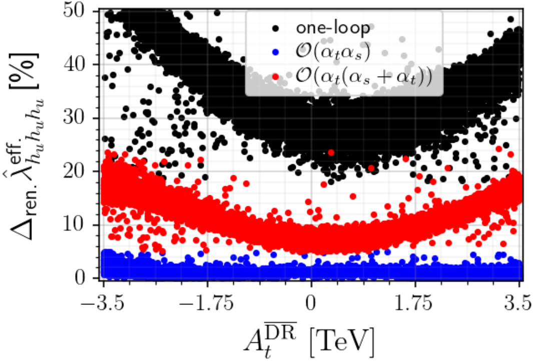
CMSSM, $m_0 = m_{1/2} = -A_0$, $\tan\beta = 10$, $\mathrm{sgn}(\mu) = 1$, with m_h computed at 2L in SPheno



- Example for a very simple version of the constrained MSSM → BSM parameters m_0, $m_{1/2}$, A_0, sgn(μ), tanβ
- For each point, M_h computed at 2L with SPheno, and SLHA output of SPheno used as input of anyH3

λ_{hhh} in the NMSSM at two-loops



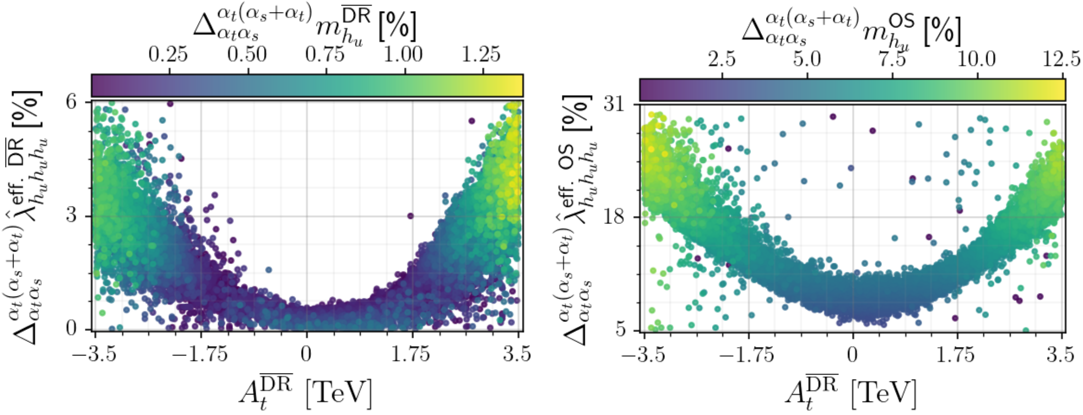


$$\Delta_{\text{ren.}}\lambda_{hhh} = \frac{\lambda_{hhh}(m_t^{\overline{\text{DR}}}, A_t^{\overline{\text{DR}}}) - \lambda_{hhh}(m_t^{\text{OS}}, A_t^{\text{OS}})}{\lambda_{hhh}(m_t^{\overline{\text{DR}}}, A_t^{\overline{\text{DR}}})}$$ $\sim$ higher-orders $\rightarrow$ estimates theory uncertainty

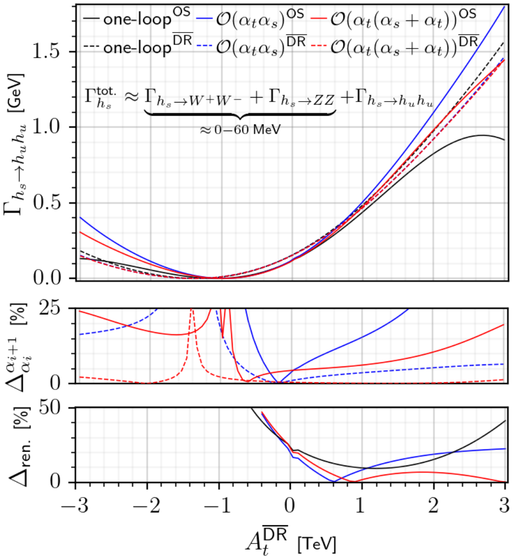
(Points checked against `HiggsSignals 2.6.2` and `HiggsBounds 5.10.2` as well as model-independent constraints on SUSY masses.)

Size of the $\mathcal{O}(\alpha_t^2)$-corrections to λ_{hhh}

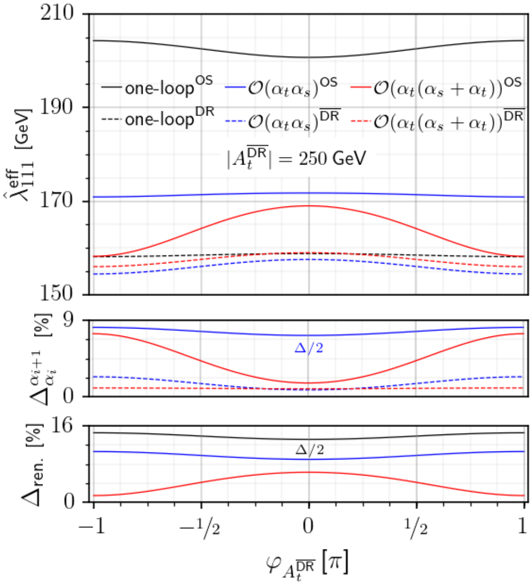
...and correlation to $\mathcal{O}(\alpha_t^2)$ m_h-corrections



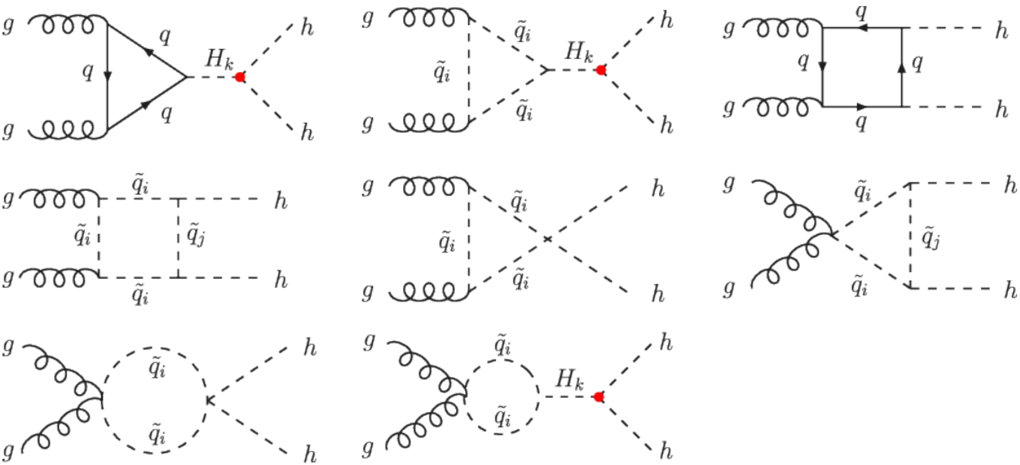
Effective couplings and Higgs to Higgs decays



Dependence on CP-violating phases



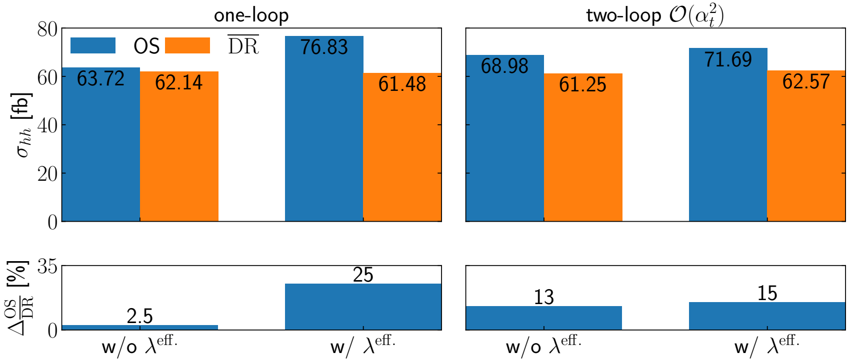
Double Higgs production in the NMSSM



Use $\lambda_{hhh}^{\alpha_t^2}$ as input in HPAIR [Spira] to estimate higher-order effects in σ_{hh}.

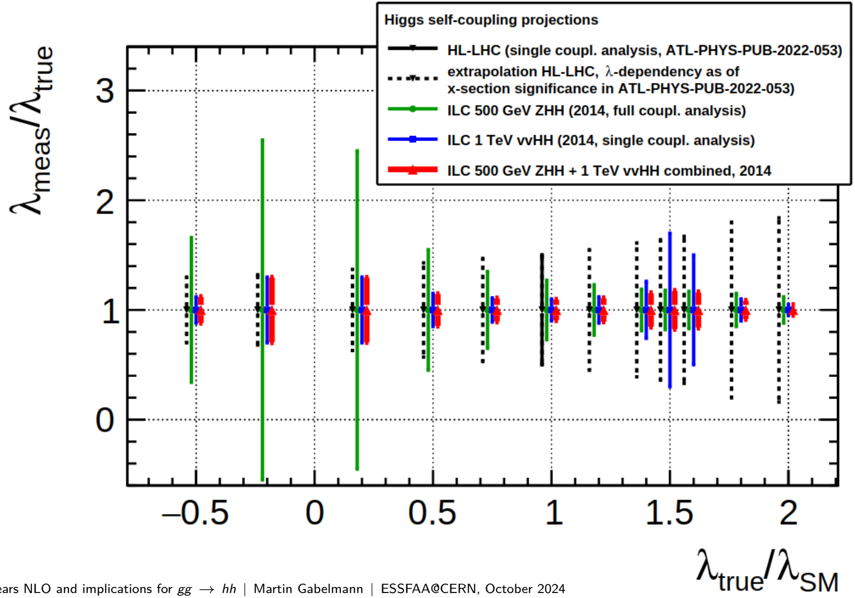
Double Higgs production in the NMSSM

Parameter point with resonant contribution from intermediate BSM Higgs:



- w/o $\lambda^{\text{eff.}}$: loop corrections to masses/mixing angles (and according LSZ-factors)
  $\rightarrow$ corrections to the input parameters
- w/ $\lambda^{\text{eff.}}$: additionally use effective coupling at respective order
  $\rightarrow$ corrections to the di-Higgs process

Projections for κ_λ "measurements"



[J.List et al. '23]