Renormalisation of extended scalar sectors

Overview and selected recent results

Johannes Braathen (DESY)

Extended Scalar Sectors From All Angles 2024,
CERN | 25 October 2024

HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE

Emmy Noether-Programm
DFG Deutsche Forschungsgemeinschaft

DESY.

Disclaimer

- The renormalisation theory of BSM models is an extremely broad and active topic, and it is impossible to make justice to it in a single talk…

→ I have tried to find a balance between overview and interesting recent results

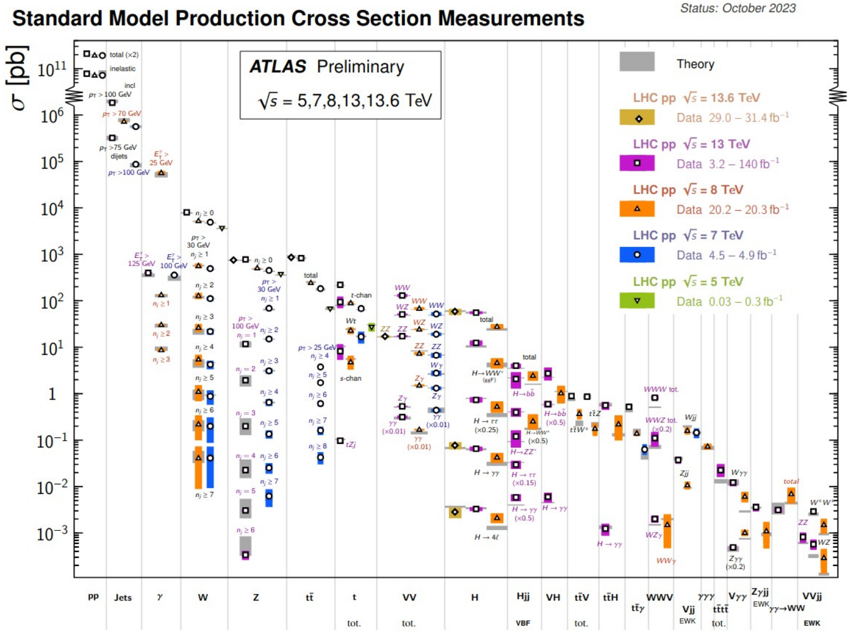
→ I apologise if I don't have time to cover your work!

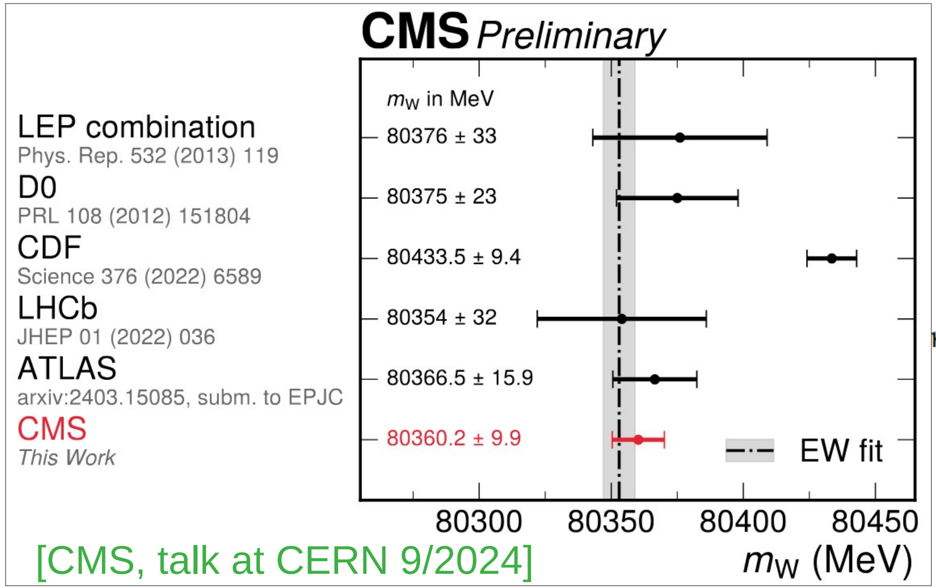
→ Also, I won't cover the renormalisation of the electroweak sector

(→ for that, see your favourite QFT book or e.g. [Böhm, Denner, Joos])

Introduction:
Renormalisation basics

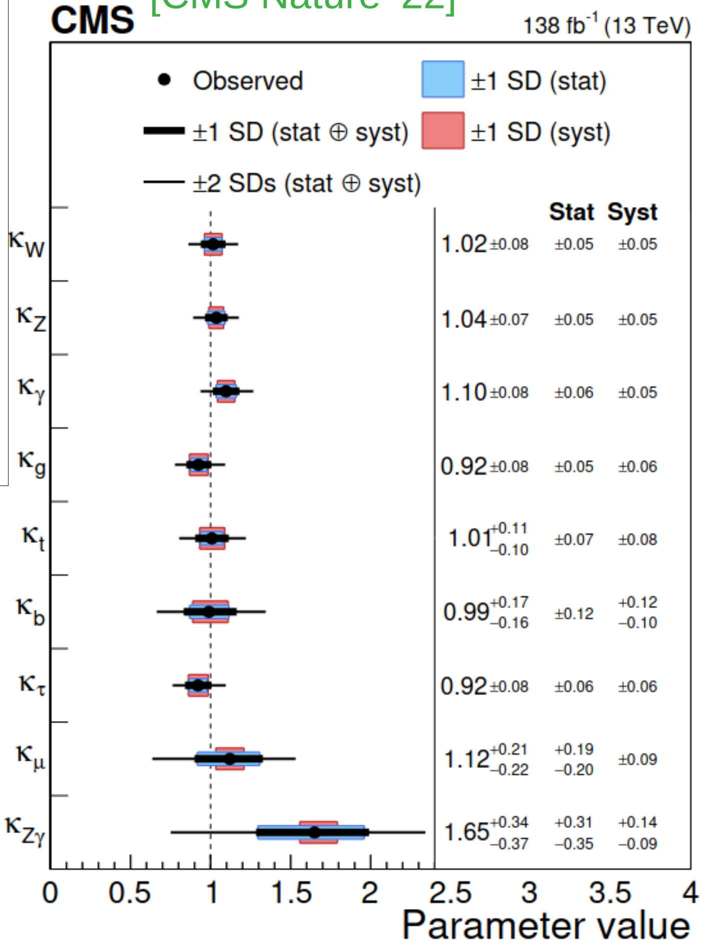
Precision calculations for precision measurements

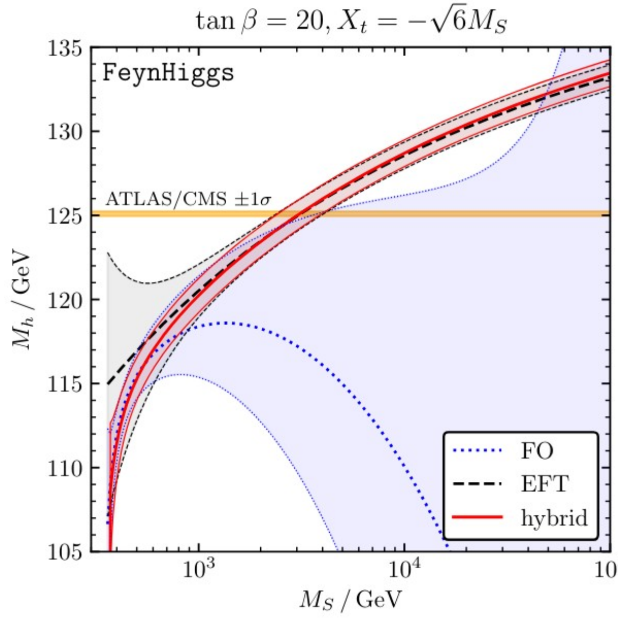




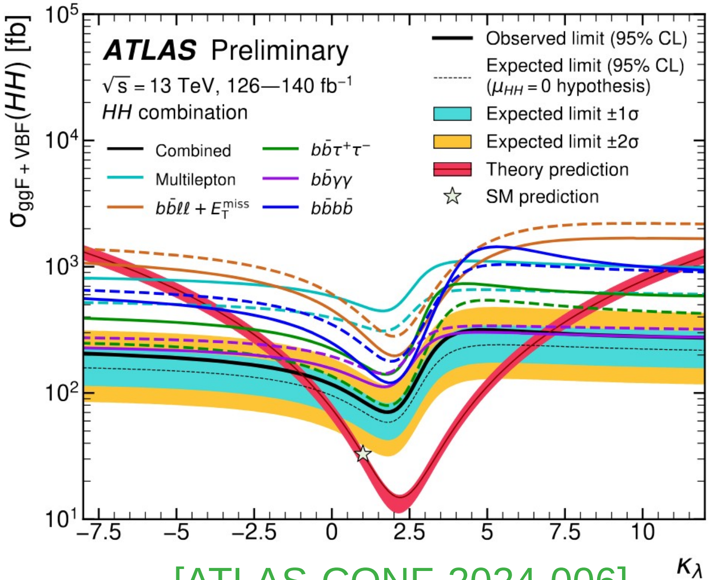
[CMS, talk at CERN 9/2024]

[CMS Nature '22]





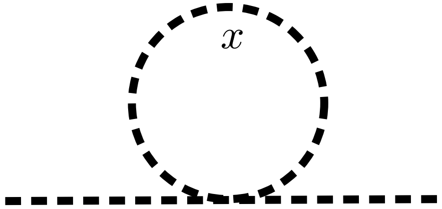
[Slavich, Heinemeyer et al. '20]



[ATLAS-CONF-2024-006]

Infinities in loop calculations and regularisation

- Calculation of quantum corrections, i.e. loop corrections, **contain divergences!**



$$\Rightarrow \mathbf{A_0}(x) = -(16\pi^2)\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2+x} = -2\int_{k=0}^{\infty}\frac{k^3dk}{k^2+x} \rightarrow -\infty$$

- *First step*: **regularisation**, i.e. modify theory to make loop integrals mathematically well-defined,

  Various options are possible, e.g.:

- **Cut-off Λ**

$$\mathbf{A_0}(x) \rightarrow -2\int_{k=0}^{\Lambda}\frac{k^3dk}{k^2+x} = -\Lambda^2 + x\log\left(1+\frac{\Lambda^2}{x}\right)$$

… but breaks Lorentz invariance!

- **Dimensional regularisation** (DREG), i.e. work in d = 4 - 2ε dimensions [*NB: for SUSY models, DRED*]

$$\mathbf{A_0}(x) \rightarrow -(16\pi^2)\mu^{2\epsilon}\int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2+x} = x\left[\frac{1}{\epsilon} - \gamma_E + \log 4\pi + 1 - \log\frac{x}{\mu^2}\right]$$

… and many more (e.g. Pauli-Villars, etc.)

μ : regularisation scale

$Q \equiv (4\pi e^{-\gamma_E})^{1/2}\mu$: renormalisation scale

Renormalisation

- *Second step*: **renormalisation, replace "bare" parameter by renormalised parameter + counterterm**

$$g \xrightarrow{\text{renormalisation transf.}} \underbrace{g_{\text{bare}}}_{\text{bare param.}} = \underbrace{g}_{\text{ren. param.}} + \underbrace{\delta^{\text{CT}} g}_{\text{counterterm}}$$

- ***Mathematical interpretation***: absorb divergences into parameter counterterms

- ***Physical interpretation***: determine the physical meaning of Lagrangian parameters, which are not physical observables, order by order in perturbation theory

  2 main choices:
  - ***relate parameter to some measured/measurable observable***
    → on-shell-like conditions; common e.g. for masses of particles
  - ***choose a simple/convenient definition of parameter***
    → $\overline{\text{MS}}$/$\overline{\text{DR}}$-like conditions or specific schemes; useful when the parameter is not easily related to an observable (e.g. hidden sector coupling, BSM VEVs, etc.) or obtained from UV theory (e.g. via matching and/or RG running)

- "Renormalisability" of a theory: all divergences compensated by a *finite number of counterterms*

Renormalisation of models with extended scalar sectors

The Two-Higgs-Doublet Model

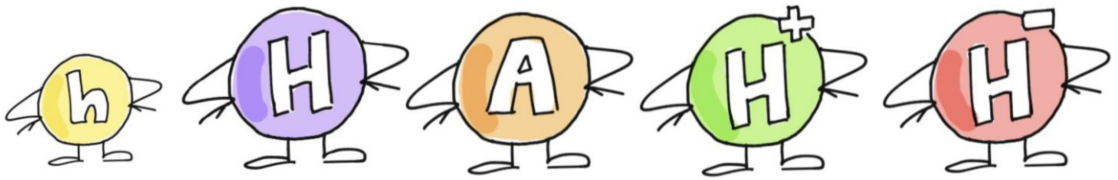


Figure by [K. Radchenko Serdula ‘24]

- 2 SU(2)$_L$ doublets $\Phi_{1,2}$ of hypercharge ½
- CP-conserving 2HDM, with softly-broken Z_2 symmetry ($\Phi_1 \to \Phi_1$, $\Phi_2 \to -\Phi_2$) to avoid tree-level FCNCs

$$V^{(0)}_{\text{2HDM}} = m_1^2|\Phi_1|^2 + m_2^2|\Phi_2|^2 - m_3^2(\Phi_2^\dagger\Phi_1 + \Phi_1^\dagger\Phi_2) + \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_2^\dagger\Phi_1|^2 + \frac{\lambda_5}{2}\Big((\Phi_2^\dagger\Phi_1)^2 + \text{h.c.}\Big)$$

- **Mass eigenstates**:
  h, H: CP-even Higgs bosons *(h = h$_{125}$); *A: CP-odd Higgs boson; H±: charged Higgs boson

$$\Phi_i = \begin{pmatrix} w_i^+ \\ \frac{1}{\sqrt{2}}(v_i + h_i + iz_i) \end{pmatrix} \quad \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix} \quad \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} G^0 \\ A \end{pmatrix} \quad \begin{pmatrix} w_1^+ \\ w_2^+ \end{pmatrix} = \begin{pmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} G^+ \\ H^+ \end{pmatrix}$$

$$v_1^2 + v_2^2 = v^2 = (246\text{ GeV})^2$$

- **Tadpole equations** *(minimisation of the scalar potential)*

$$t_1^{(0)} = 0 = m_1^2 - m_3^2 t_\beta + \frac{1}{2}\big[\lambda_1 c_\beta^2 + (\lambda_3 + \lambda_4 + \lambda_5)s_\beta^2\big]v^2$$

$$t_2^{(0)} = 0 = m_2^2 - \frac{m_3^2}{t_\beta} + \frac{1}{2}\big[\lambda_2 s_\beta^2 + (\lambda_3 + \lambda_4 + \lambda_5)c_\beta^2\big]v^2$$

- **BSM parameters**: 3 BSM masses m_H, m_A, $m_{H\pm}$, BSM mass scale M (defined by $M^2 \equiv 2m_3^2/s_{2\beta}$), angles α (CP-even Higgs mixing angle) and β (defined by $\tan\beta = v_2/v_1$), 2 tadpole parameters t_1, t_2 (or t_h, t_H)

Renormalising the 2HDM

- **Parameter renormalisation**:

  Tadpoles: $t_i \to t_i + \delta^{\text{CT}} t_i, \qquad i = h, H$

  Physical masses: $m_i^2 \to m_i^2 + \delta^{\text{CT}} m_i^2, \qquad i = h, H, A, H^\pm$

  BSM mass parameter: $M^2 \to M^2 + \delta^{\text{CT}} M^2$ *EW VEV*: $v \to v + \delta^{\text{CT}} v$

  Mixing angles: $\alpha \to \alpha + \delta^{\text{CT}} \alpha, \qquad \beta \to \beta + \delta^{\text{CT}} \beta$

- **Field renormalisation**:

$$\begin{pmatrix} H \\ h \end{pmatrix} \to \begin{pmatrix} 1 + \frac{1}{2}\delta^{\text{CT}} Z_{HH} & \delta^{\text{CT}} Z_{Hh} \\ \delta^{\text{CT}} Z_{hH} & 1 + \frac{1}{2}\delta^{\text{CT}} Z_{hh} \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}$$

$$\begin{pmatrix} G \\ A \end{pmatrix} \to \begin{pmatrix} 1 + \frac{1}{2}\delta^{\text{CT}} Z_{GG} & \delta^{\text{CT}} Z_{GA} \\ \delta^{\text{CT}} Z_{AG} & 1 + \frac{1}{2}\delta^{\text{CT}} Z_{AA} \end{pmatrix} \begin{pmatrix} G \\ A \end{pmatrix}$$

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} \to \begin{pmatrix} 1 + \frac{1}{2}\delta^{\text{CT}} Z_{G^\pm G^\mp} & \delta^{\text{CT}} Z_{G^\pm H^\mp} \\ \delta^{\text{CT}} Z_{H^\pm G^\mp} & 1 + \frac{1}{2}\delta^{\text{CT}} Z_{H^\pm H^\mp} \end{pmatrix} \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix}$$

→ **22 counterterms**

Renormalising the 2HDM

- **Parameter renormalisation**:

  Tadpoles: $t_i \to t_i + \delta^{\text{CT}} t_i, \qquad i = h, H$

  Physical masses: $m_i^2 \to m_i^2 + \delta^{\text{CT}} m_i^2, \qquad i = h, H, A, H^\pm$

  BSM mass parameter: $M^2 \to M^2 + \delta^{\text{CT}} M^2$ *EW VEV*: $v \to v + \delta^{\text{CT}} v$

  Mixing angles: $\alpha \to \alpha + \delta^{\text{CT}} \alpha, \qquad \beta$

- **Field renormalisation**:

$$\begin{pmatrix} H \\ h \end{pmatrix} \to \Big(\begin{matrix} 1 + \frac{1}{2}\delta^{\text{CT}} Z_{H} \\ \delta^{\text{CT}} Z_{hH} \end{matrix}$$

$$\begin{pmatrix} G \\ A \end{pmatrix} \to \Big(\begin{matrix} 1 + \frac{1}{2}\delta^{\text{CT}} Z_{GG} \\ \delta^{\text{CT}} Z_{AG} \end{matrix}$$

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} \to \Big(\begin{matrix} 1 + \frac{1}{2}\delta^{\text{CT}} Z \\ \delta^{\text{CT}} Z_{H^\pm} \end{matrix}$$

→ **22 counterterms**

Aparté: renormalisation of the EW VEV (and BSM VEVs)

- Divergent part:

$$\frac{\delta^{\text{CT}} v}{v} = \frac{\delta^{\text{CT}} M_W^2}{M_W^2} + \frac{\cos^2\theta_w}{2\sin^2\theta_w}\left(\frac{\delta^{\text{CT}} M_Z^2}{M_Z^2} - \frac{\delta^{\text{CT}} M_W^2}{M_W^2}\right) - \frac{\delta^{\text{CT}} e}{e}$$

- Finite part:
  depends on EW input scheme
  $\{G_F, \alpha_{em}, M_Z\}$ vs $\{M_W, \alpha_{em}, M_Z\}$, etc.

- Renormalisation of BSM VEVs in general,
  see e.g. [Sperling, Stöckinger, Voigt '13]

Desirable properties for renormalisation schemes

See e.g. [Freitas,Stöckinger ’02]
[Denner, Dittmaier, Lang ’18]

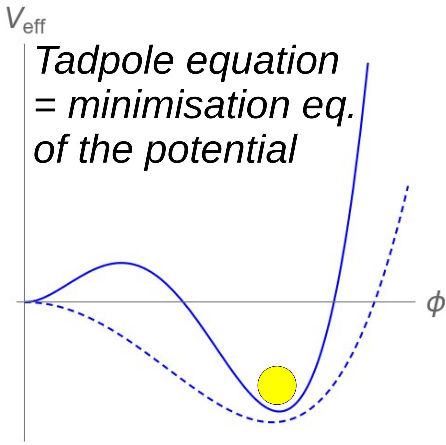
Always a matter of choice, but some properties that one can care about:

- **Simplicity**/applicability to new or generic models
- **Numerical** (or perturbative) **stability**
  → avoid artificial enhancements of higher-order corrections
  → avoid breakdown of calculations in regions of BSM parameter space (e.g. degenerate masses, special mixing angle like alignment limit, etc.)
- **Gauge independence**
- Preserve symmetry(ies) and/or structure of the theory
- Process independence

Tadpole schemes

Tadpole schemes down the hole….

Tadpoles and VEVs at (one-)loop level



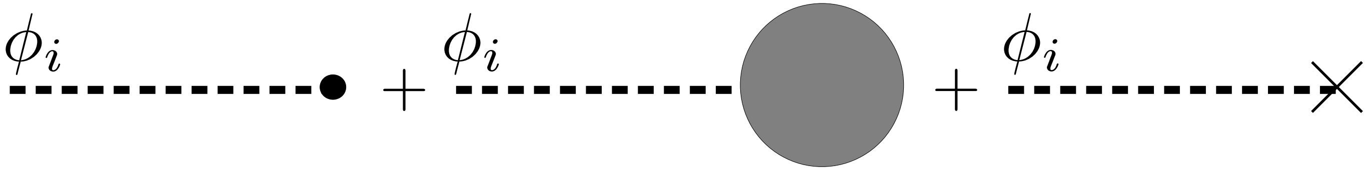
Tadpole equation = minimisation eq. of the potential

- Tree-level tadpole eqs.: $\left.\frac{\partial V^{(0)}}{\partial \phi_i}\right|_{\min} \equiv t_i^{(0)} = 0 \quad \Leftrightarrow$

$= 0$

e.g. in SM: $t_h^{(0)} \equiv \mu^2 v + \lambda v^3 = 0$

- Loop-level tadpole eqs.: $0 = \left.\frac{\partial V_{\text{eff}}}{\partial \phi_i}\right|_{\min} = \left.\frac{\partial V^{(0)}}{\partial \phi_i}\right|_{\min} + \left.\frac{\partial \Delta V}{\partial \phi_i}\right|_{\min}$



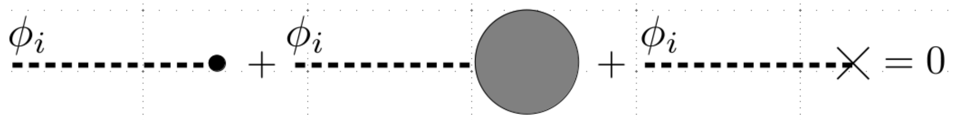
$= 0$

$$T_i^{(1)} \equiv t_i^{(0)} + t_i^{(1)} + \delta^{\text{CT}} t_i = 0$$

- Divergent part of $\delta^{\text{CT}} t_i$ fixed to $-t_i^{(1)}|_{\text{div}}$, but **what choice for finite part?**

Tadpoles and VEVs at (one-)loop level

Approaches to consider loop corrections to tadpole equations



$$T_i^{(1)} \equiv t_i^{(0)} + t_i^{(1)} + \delta^{\text{CT}} t_i = 0$$

- **Parameter-renormalised tadpole scheme (PRTS)** (see e.g. [Böhm, Hollik, Spiesberger '86], [Denner '93]):
  Absorb corrections to tadpole equation into finite $\delta^{\text{CT}} t_i$, but at cost of this appearing in other CTs

$$\delta^{\text{CT}} t_i = -t_i^{(1)} \qquad\qquad t_i^{(0)}(\{p_i\}) = 0$$

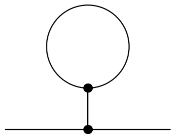
- **Tadpole-less scheme** (see e.g. [Martin '01, '03]):
  Fix the VEV as minimum of *loop corrected* potential, and solve tadpole eq. including loop corrections

$$\delta^{\text{CT}} t_i = -t_i^{(1)}\Big|_{\text{div.}} \qquad\qquad t_i^{(0)}(\{p_i\}) + t_i^{(1)}(\{p_i\})\Big|_{\text{fin.}} = 0$$

- **Fleischer-Jegerlehner tadpole scheme (FJTS)** [Fleischer, Jegerlehner '81]:
  Take the VEV as minimum of *tree-level* potential, solve the tree-level tadpole equation for one of the parameters p_i in the model, and include finite tadpole contributions $t_i^{(1)}|_{\text{fin}}$ in loop calculations

$$\delta^{\text{CT}} t_i = -t_i^{(1)}\Big|_{\text{div.}} \qquad\qquad t_i^{(0)}(\{p_i\}) = 0 \qquad\qquad t_i^{(1)}(\{p_i\}) \neq 0$$

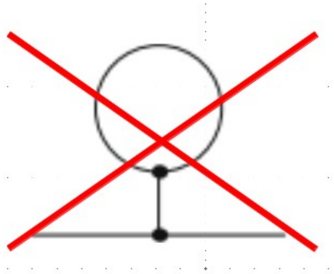
e.g.:



- **Gauge-Invariant Vacuum expectation value Scheme (GIVS)** [Dittmaier, Rzehak '22]
  → combine advantages of PRTS and FJTS, *more in backup* + *in talk by R. Feser this afternoon*

See also [Löschner, Grimus '18]

PRTS and Tadpole-less Scheme: Example in the SM



- In both schemes:
  1) EW VEV is minimum of loop-corrected potential
  2) No explicit tadpole diagrams in loop calcultations

- ***Parameter Renormalised Tadpole Scheme (PRTS)***:
  - Solution of tadpole eq.: $t_h^{(0)} = 0 \quad \Rightarrow \quad \mu^2 = -\lambda v^2$
  - Tree-level Higgs mass: $(m_h^2)^{(0)} = \mu^2 + 3\lambda v^2 = 2\lambda v^2 + t_h^{(0)}/v = 2\lambda v^2$
  - One-loop Higgs mass: $M_h^2 = 2\lambda v^2 + \frac{1}{v}\delta^{\text{CT}} t_h|_{\text{fin.}} - \Sigma_{hh}^{(1)}(M_h^2) = 2\lambda v^2 - \frac{1}{v} t_h^{(1)}|_{\text{fin.}} - \Sigma_{hh}^{(1)}(M_h^2)$

- ***Tadpole-less scheme***:
  - Solution of tadpole eq.: $t_h^{(0)} + t_h^{(1)}|_{\text{fin.}} = 0 \quad \Rightarrow \quad \mu^2 = -\lambda v^2 - \frac{1}{v} t_h^{(1)}|_{\text{fin.}}$
  (μ² becomes formally 1L)
  - Tree-level Higgs mass: $(m_h^2)^{(0)} = \mu^2 + 3\lambda v^2 = 2\lambda v^2 - \frac{1}{v} t_h^{(1)}|_{\text{fin.}}$
  - One-loop Higgs mass: $M_h^2 = \mu^2 + 3\lambda v^2 - \Sigma_{hh}^{(1)}(M_h^2) = 2\lambda v^2 - \frac{1}{v} t_h^{(1)} - \Sigma_{hh}^{(1)}(M_h^2)$

Fleischer-Jegerlehner Tadpole Scheme

- Take the VEV as minimum of *tree-level* potential

- Solve the tree-level tadpole equation for one of the scalar parameters,
  e.g. in the SM
$$\mu^2 = -\lambda v^2$$

- As *we aren't working at the minimum of the loop corrected potential*, we must include **finite contributions from tadpole diagrams** in all processes, e.g.

$$M_h^2 = 2\lambda v^2 - \frac{6\lambda v}{m_h^2} t_h^{(1)}|_{\text{fin.}} - \Sigma_{hh}^{(1)}(M_h^2)$$

Not same v as in PRTS/tadpole-free scheme!

- This can also be seen as a finite shift of the VEV v

$$\Delta^{(1)} v = -\frac{1}{m_h^2} t_h^{(1)}$$

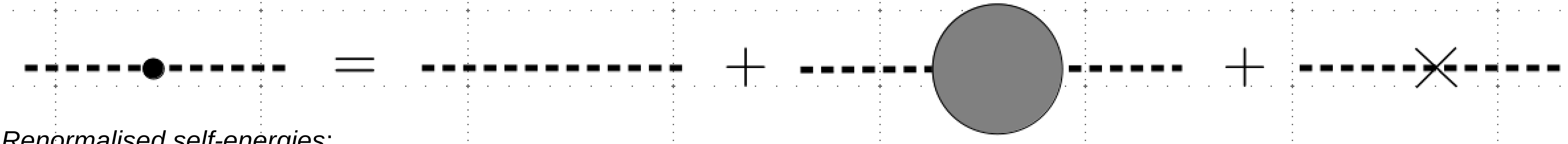
→ in a BSM model, this means we let New Physics disrupt the EW hierarchy
(c.f. also [Farina, Pappadopulo, Strumia '13])

Tadpole Schemes: Advantages and disadvantages

	PRTS	Tadpole-less	FJTS
Explicit tadpoles in loop calculations	No (but $\delta^{CT}t_i\vert_{fin.}$)	No	Yes
Numerical stability *(or where is it lost?)*	Stable in SM and many BSM theories … but problems if small BSM VEVs (e.g. singlet or triplet VEV) $-\frac{1}{v_T}t_T^{(1)} \sim \frac{1}{4\text{ GeV}}\frac{(1\text{ TeV})^3}{16\pi^2} \sim 10^6\text{GeV}^2$ with $v_T = 4\text{ GeV},\ m_T = 1\text{ TeV}$	Same as PRTS + mixes order in perturbation theory (→ can aggravate issues like Goldstone Boson Catastrophe, see [Martin '14], [Elias-Miro, Espinosa, Konstandin '14], [Pilaftsis, Teresi '15], [Kumar, Martin '16], [JB, Goodsell '16], [JB, Goodsell, Staub '17])	Tadpole diagrams can be numerically very large and spoil numerical accuracy (especially if light scalar masses) but better for scenarios with small BSM VEVs (see e.g. [JB, Goodsell, Paßehr, Pinsard '21])
Gauge dependence in calculations	Yes	Yes	No
Can it be used for generic theories / automated codes	Not easily (but ~doable in `anyH3`, see later)	Yes e.g. in SARAH [Staub '07-'15]	Yes

Renormalising masses

Renormalising masses and wave functions



Renormalised self-energies:

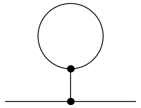
$$\widehat{\Sigma}_{ij}(p^2) = \Sigma_{ij}(p^2) - \delta^{\mathrm{CT}} m^2_{ij} + \frac{1}{2}(\delta^{\mathrm{CT}} Z^\dagger)_{ik}(p^2 - m_k^2)\delta_{kj} + \frac{1}{2}(p^2 - m_i^2)\delta_{ik}\delta^{\mathrm{CT}} Z_{kj}$$

- **Mass renormalisation**:

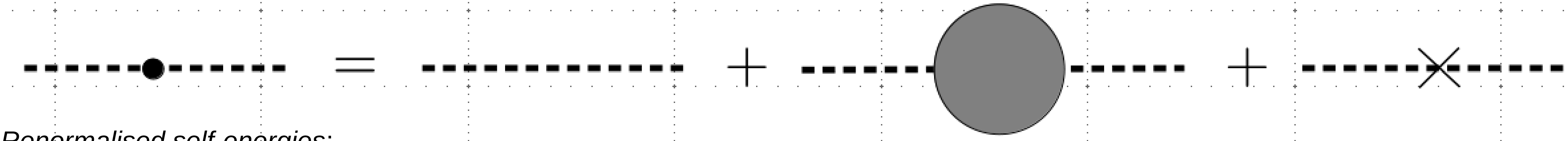
OS condition: $\mathrm{Re}\widehat{\Sigma}_{ii}(p^2 = M_i^2) \overset{!}{=} 0$ **$\overline{\text{MS}}$ condition**: $\mathrm{Re}\widehat{\Sigma}_{ii}(p^2 = M_i^2)|_{\mathrm{div.}} \overset{!}{=} 0$

Tadpoles enter depending on choice of scheme:
- PRTS: tadpole CT can enter mass CT matrix (depending on how tadpole eq. is solved)

- Tadpole-less scheme: no tadpoles in self-energies (but typically in tree-level mass matrix)

- FJTS: $\Sigma_{ij} \longrightarrow \Sigma_{ij}^{\mathrm{tad.}}$ (includes self-energies with tadpole-diagram insertions, i.e. )

Renormalising masses and wave functions



Renormalised self-energies:

$$\widehat{\Sigma}_{ij}(p^2) = \Sigma_{ij}(p^2) - \delta^{\text{CT}} m^2_{ij} + \frac{1}{2}(\delta^{\text{CT}} Z^\dagger)_{ik}(p^2\delta_{kj} - m^2_{kj}) + \frac{1}{2}(p^2\delta_{ik} - m^2_{ik})\delta^{\text{CT}} Z_{kj}$$

- **Mass renormalisation**:

  OS condition: $\text{Re}\widehat{\Sigma}_{ii}(p^2 = M_i^2) \overset{!}{=} 0$ **$\overline{\text{MS}}$ condition**: $\text{Re}\widehat{\Sigma}_{ii}(p^2 = M_i^2)|_{\text{div.}} \overset{!}{=} 0$

- **Diagonal WFR**:

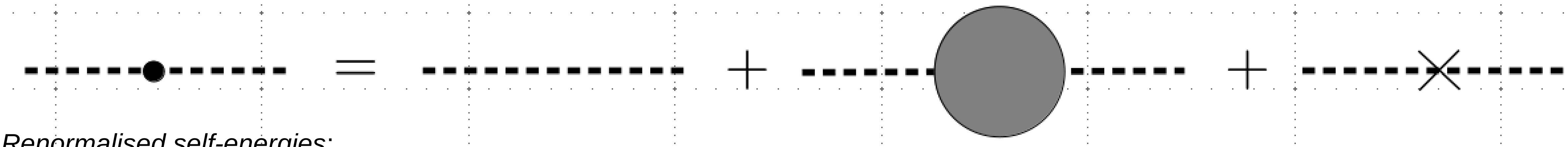
  OS condition: $\text{Re}\left[\frac{\partial}{\partial p^2}\widehat{\Sigma}_{ii}\right]_{p^2=M_i^2} \overset{!}{=} 0$ **$\overline{\text{MS}}$ condition**: *same with div. part*

- **Off-diagonal WFR**:

  OS conditions: $\text{Re}\widehat{\Sigma}_{ij}(p^2 = M_i^2) = \text{Re}\widehat{\Sigma}_{ij}(p^2 = M_j^2) \overset{!}{=} 0$

  $\overline{\text{MS}}$ conditions: *same with div. part*

Renormalising masses and wave functions



Renormalised self-energies:

$$\widehat{\Sigma}_{ij}(p^2) = \Sigma_{ij}(p^2) - \delta^{\mathrm{CT}} m^2_{ij} + \frac{1}{2}(\delta^{\mathrm{CT}} Z^\dagger)_{ik}(p^2\delta_{kj} - m^2_{kj}) + \frac{1}{2}(p^2\delta_{ik} - m^2_{ik})\delta^{\mathrm{CT}} Z_{kj}$$

$$\boxed{\mathrm{Re}\widehat{\Sigma}_{ii}(p^2 = M_i^2) \overset{!}{=} 0,\ \mathrm{Re}\left[\frac{\partial}{\partial p^2}\widehat{\Sigma}_{ii}\right]_{p^2=M_i^2} \overset{!}{=} 0,\ \ \mathrm{Re}\widehat{\Sigma}_{ij}(p^2 = M_i^2) = \mathrm{Re}\widehat{\Sigma}_{ij}(p^2 = M_j^2) \overset{!}{=} 0}$$

$$\delta^{\mathrm{CT}} m_i^2 = \mathrm{Re}\big[\Sigma_{ii}(p^2 = m_i^2) - \delta^{\mathrm{CT}} T_{ii}\big]\ (\mathrm{PRTS})\ \text{or}\ \mathrm{Re}\big[\Sigma^{\mathrm{tad.}}_{ii}(p^2 = m_i^2)\big]\ (\mathrm{FJTS})$$

$$\delta^{\mathrm{CT}} Z_{ii} = -\mathrm{Re}\left[\frac{\partial}{\partial p^2}\Sigma_{ii}\right]_{p^2=m_i^2}$$

$$\delta^{\mathrm{CT}} Z_{ij} = \frac{2}{m_i^2 - m_j^2}\mathrm{Re}\big[\Sigma_{ij}(m_j^2) - \delta^{\mathrm{CT}} T_{ij}\big]\ (\mathrm{PRTS})\ \text{or}\ \frac{2}{m_i^2 - m_j^2}\mathrm{Re}\big[\Sigma^{\mathrm{tad.}}_{ij}(p^2 = m_j^2)\big]\ (\mathrm{FJTS})$$

Renormalising mixing angles

Mixing angle renormalisation: overview

- **$\overline{\text{MS}}$ scheme**
  + Simplicity
  + Process independence
  – Scale dependence, but this can be used to estimate th. uncertainty and/or check perturbative stability
  – Tadpole scheme dependent: FJTS/PRTS/etc.
  – Possible numerical instabilities (especially with FJTS)

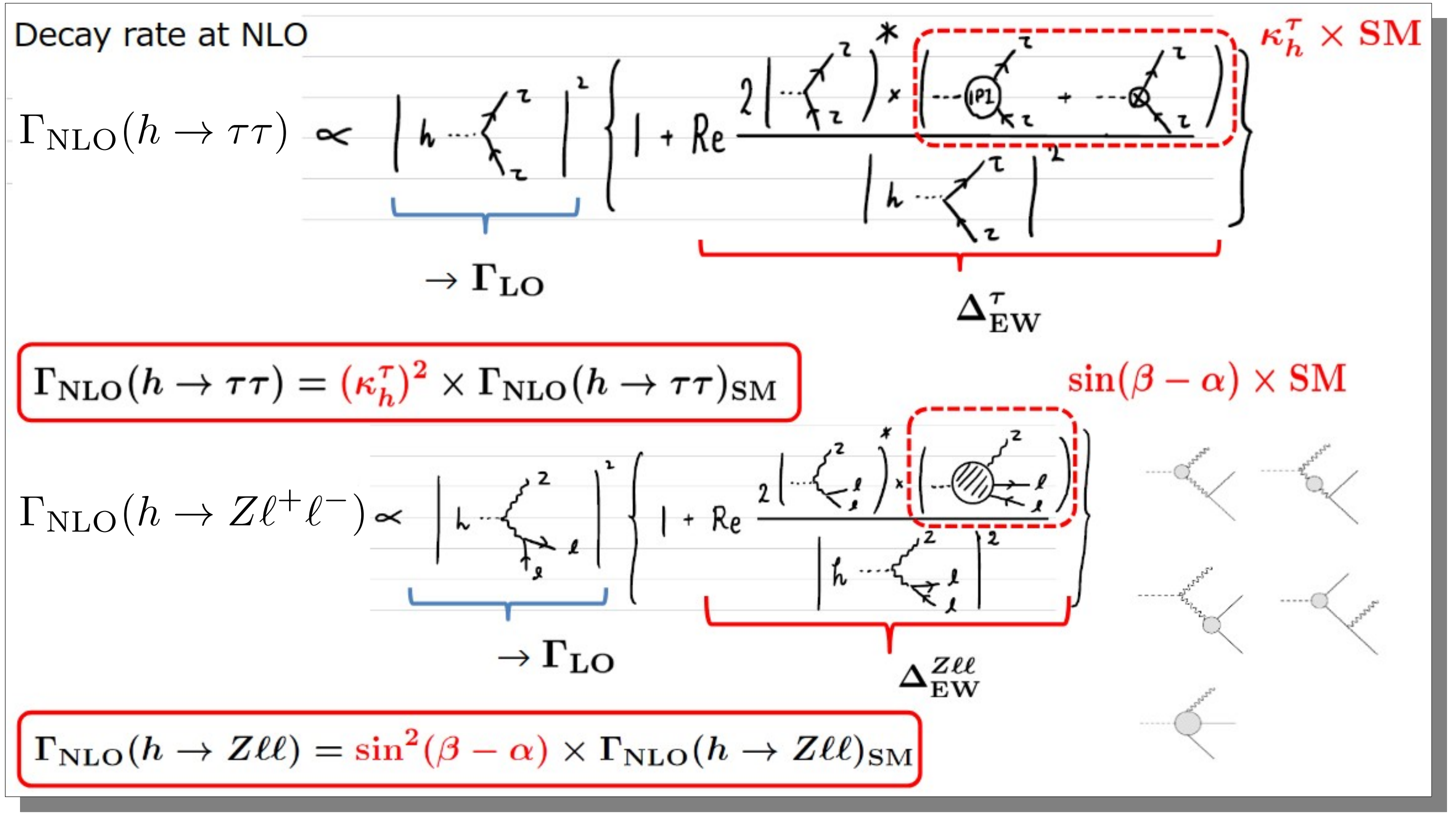
- **Momentum-subtraction schemes (Process-independent OS)**
  Ren. condition based on $\Sigma_{ij}(p^2)$ at some momentum p^2
  [Kanemura, Okada, Senaha, Yuan '04], [Krause et al. '16], and many more
  + Process independence
  + Stable coverage of BSM parameter space
  – Possible gauge dependence (often removed ad hoc, which are difficult to interpret or adapt – *see next slide*)
  • Can be adapted/extended using symmetries of theory or of UV divergences

- **Process-specific OS**
  e.g. $\Gamma(h \to XY) \overset{!}{=} \Gamma^{\text{LO}}(h \to XY)$
  + Gauge independence
  – Process dependence
  – Possible perturbative/numerical instabilities in parts of BSM parameter space
  – Difficult beyond 1L

- **OS conditions on ratios of amplitudes**
  e.g. $\frac{\mathcal{M}(h \to XY)}{\mathcal{M}(H \to XY)} \overset{!}{=} \frac{\mathcal{M}^{\text{LO}}(h \to XY)}{\mathcal{M}^{\text{LO}}(H \to XY)}$
  [Denner, Dittmaier, Lang '18]
  + Gauge independence
  + Tadpole contributions drop out (scheme choice irrelevant)
  • Process independence (by adding auxilary "dummy" fields which only serve for renormalisation condition)
  + Stable coverage of BSM parameter space
  – Difficult beyond 1L

Mixing angle renormalisation and “alignment-ness”

- sin(β-α) → 1 controls the *alignment limit* at tree level, but this picture is lost at loop level
  → *can a scheme be devised to recover this?*



Decay rate at NLO

$$\Gamma_{\rm NLO}(h\to\tau\tau) \propto \underbrace{\left|\;\cdots\;\right|^2}_{\to\,\Gamma_{\rm LO}} \underbrace{\left\{1+\mathrm{Re}\,\frac{2\left(\cdots\right)^*\times\left(\cdots\right)}{\left|\;\cdots\;\right|^2}\right\}}_{\Delta^{\tau}_{\rm EW}}$$

$\kappa_h^\tau \times$ SM

$$\boxed{\Gamma_{\rm NLO}(h\to\tau\tau) = (\kappa_h^\tau)^2 \times \Gamma_{\rm NLO}(h\to\tau\tau)_{\rm SM}}$$

$\sin(\beta-\alpha)\times$ SM

$$\Gamma_{\rm NLO}(h\to Z\ell^+\ell^-) \propto \underbrace{\left|\;\cdots\;\right|^2}_{\to\,\Gamma_{\rm LO}} \underbrace{\left\{1+\mathrm{Re}\,\frac{2\left(\cdots\right)^*\times\left(\cdots\right)}{\left|\;\cdots\;\right|^2}\right\}}_{\Delta^{Z\ell\ell}_{\rm EW}}$$

$$\boxed{\Gamma_{\rm NLO}(h\to Z\ell\ell) = \sin^2(\beta-\alpha) \times \Gamma_{\rm NLO}(h\to Z\ell\ell)_{\rm SM}}$$

- **Newly-proposed renormalisation conditions**:

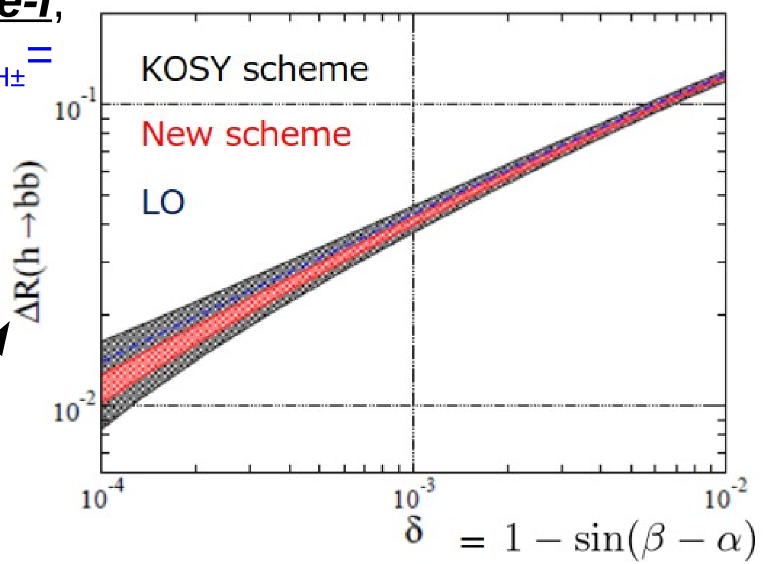
$$\Delta^{\tau}_{\rm EW} \overset{!}{=} \Delta^{\tau}_{\rm EW}|_{\rm SM}, \quad \Delta^{Z\ell\ell}_{\rm EW} \overset{!}{=} \Delta^{Z\ell\ell}_{\rm EW}|_{\rm SM} \Rightarrow \delta^{\rm CT}\alpha,\ \delta^{\rm CT}\beta$$

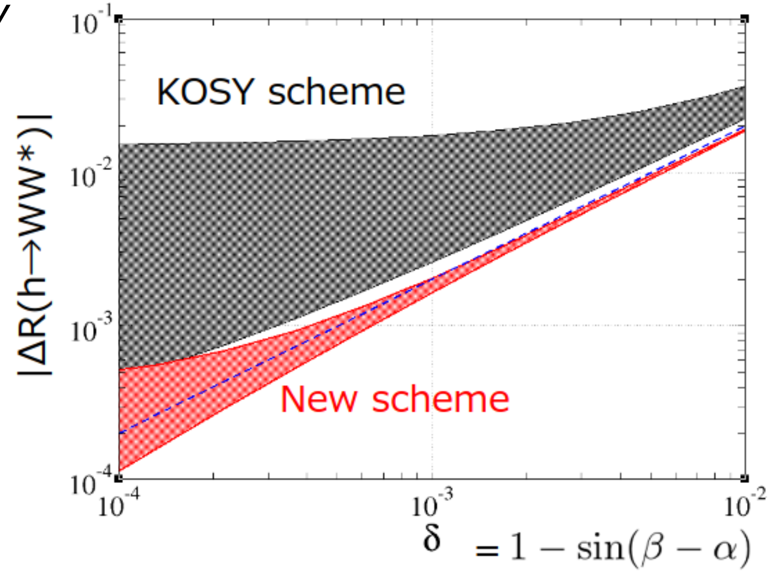
Slide elements by K. Yagyu

[Kanemura, Kikuchi, Yagyu ‘24]

2HDM type-I,
$m_H=m_A=m_{H\pm}=$ 300 GeV,
tanβ=2
Vary M

BSM deviation in Higgs decay





Renormalising other BSM parameters

Here: focus on ***renormalisation of BSM mass parameters****, like M in 2HDM*

In backup: ***renormalisation of Lagrangian trilinear couplings*** *(see also talk by A. Verduras Schaeidt yesterday!)*

Renormalisation of BSM mass scales

- **BSM parameters in 2HDM**:

$$M,\ m_H,\ m_A,\ m_{H^\pm},\ \alpha,\ \beta$$

- **BSM parameters in IDM**:

$$\mu_2,\ m_H,\ m_A,\ m_{H^\pm},\ \lambda_2$$

quartic self-coupling of inert doublet

- **Masses of BSM scalars in (many) extended sectors**

$$m_\Phi^2 = \mathcal{M}^2 + \tilde{\lambda}_\Phi v^2$$

$$\mathcal{M} = M, \mu_2, \cdots \text{ depending on model}$$

- ***How to renormalise BSM mass parameters like M or μ_2?***

- *Easiest choice*: $\overline{\text{MS}}$
… but residual renormalisation scale dependence

- *Process-dependent OS scheme*
(see e.g. [Abe, Sato '15], [Banerjee, Boudjema, Chakrabarty, Sun '21] for μ_2 in IDM)

Fix some renormalised amplitude, dependent on parameter at tree level, to its tree-level value, e.g.

$$\Gamma^{(1)}_{hHH}(m_h^2, m_H^2, m_H^2) \overset{!}{=} \Gamma^{(0)}_{hHH}$$

where $\Gamma^{(0)}_{hHH} \propto \dfrac{2(m_H^2 - \mu_2^2)}{v}$

… but may not be suited for all of parameter space of model + difficult beyond 1L (same as for mixing angles…)

Decoupling-inspired renormalisation of BSM mass scales

- Can we find a prescription related to the role of M, μ_2 in controlling the ***decoupling of BSM states***?

- Taking here the example of calculations of higher-order corrections to the trilinear Higgs coupling λ_{hhh}

At one loop:

$$\delta^{(1)}\lambda_{hhh} = \frac{16m_\Phi^4}{v^3}\left(1-\frac{M^2}{m_\Phi^2}\right)^3 \qquad [m_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2]$$

- Decoupling of BSM contributions:

$$(m_\Phi^2)^{n-1}\left(1-\frac{M^2}{m_\Phi^2}\right)^n \underset{m_\Phi^2=M^2+\tilde{\lambda}_\Phi v^2}{=} \frac{(\tilde{\lambda}_\Phi v^2)^n}{M^2+\tilde{\lambda}_\Phi v^2} \xrightarrow[\tilde{\lambda}_\Phi v^2 \text{ fixed}]{M\to\infty} 0$$

What about two loops?

If we express the two-loop corrections to λ_{hhh} in terms of **OS BSM scalar masses but M in $\overline{\text{MS}}$ scheme**

$$\delta^{(2)}\lambda_{hhh} \supset \frac{9M_\Phi^6\cot^2 2\beta}{4\pi^4 v_{\text{OS}}^5}\left(1-\frac{M^2}{M_\Phi^2}\right)^3\left[1-\frac{M^2}{M_\Phi^2}\log\frac{M_\Phi^2}{Q^2}\right]$$

Written out here for $M_H=M_A=M_{H\pm}=M_\Phi$ for brievity

→ ***doesn't seem to show decoupling!***

(in fact, one-loop relation between M_Φ and M is required!)

Decoupling-inspired renormalisation of BSM mass scales

- **Idea:** define CT for mass parameter to make decoupling of BSM states apparent with relation of the form: $M_\Phi^2 = \tilde{M}^2 + \tilde{\lambda}_\Phi v^2$

- **2HDM**: $\tilde{M}$
  [JB, Kanemura ‘19], [Degrassi, Slavich ‘23]

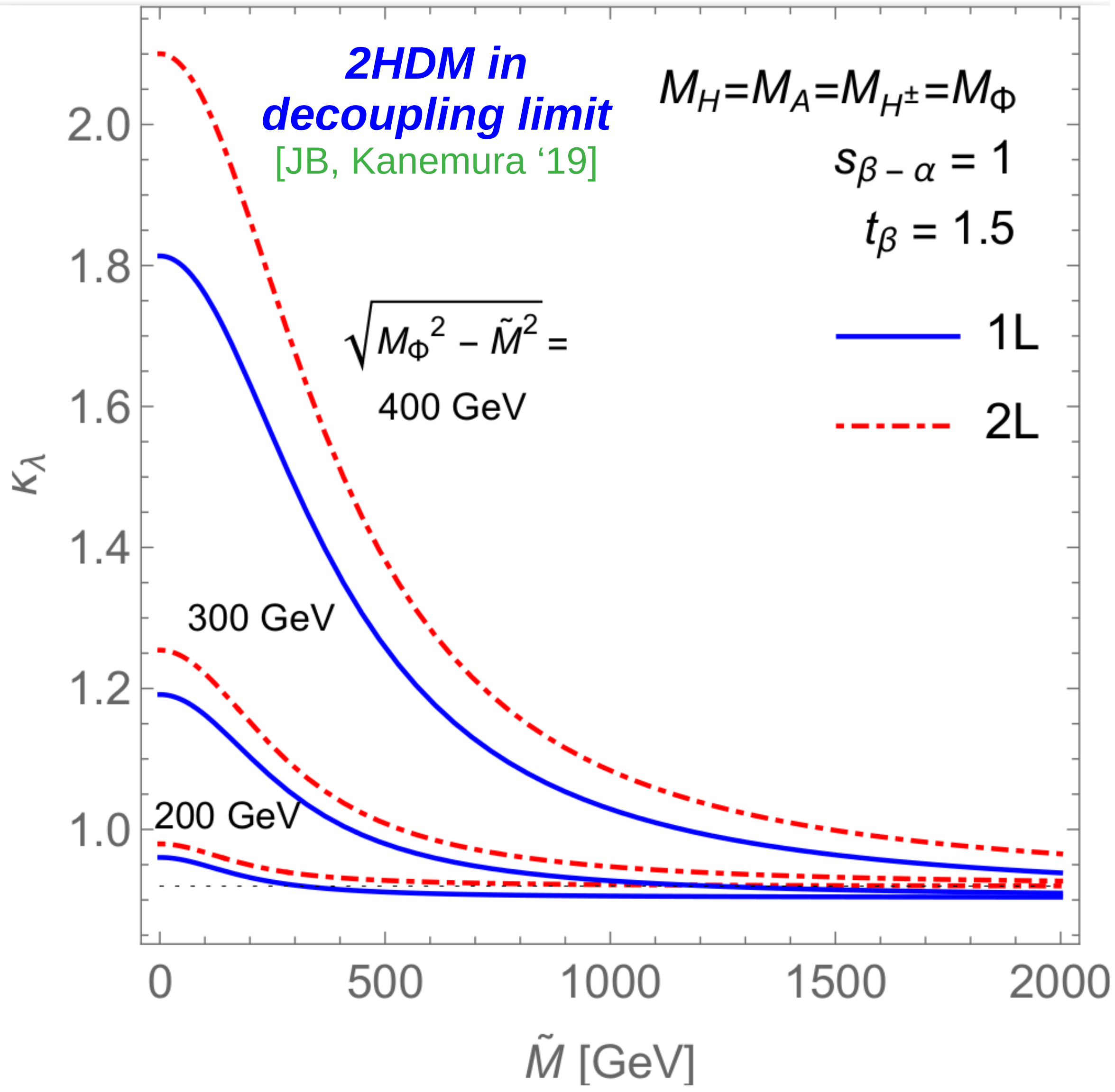
$$\delta^{\mathrm{CT,DI}} M^2 = \frac{M^2}{16\pi^2}\left[(\lambda_3 + 2\lambda_4 + 3\lambda_5)\left(\Delta_{\mathrm{UV}} + 1 - \log\frac{M^2}{Q^2}\right) + 3y_t^2 c_\beta^2\left(\Delta_{\mathrm{UV}} + 2 - \log\frac{M^2}{Q^2}\right)\right]$$

- **IDM**: μ_2^{DI} [Aiko, JB, Kanemura ‘23]

$$\delta^{\mathrm{CT,DI}} \mu_2^2 = \frac{\lambda_2 \mu_2^2}{16\pi^2}\left[3\Delta_{\mathrm{UV}} + 6 - \frac{1}{2}\left(\log\frac{m_H^2}{Q^2} + \log\frac{m_A^2}{Q^2} + 4\log\frac{m_{H^\pm}^2}{Q^2}\right)\right]$$

$$\left[\Delta_{\mathrm{UV}} \equiv \frac{1}{\epsilon} - \gamma_E + \log 4\pi\right]$$

- **Scheme defined for calculations of λ_{hhh}, but found to work also (out of the box) for Γ(h→γγ)**



Aparté: OS-like renormalisation à la BSMPT

[Basler, Mühlleitner ‘18]
[Basler, Mühlleitner, Müller ‘20]
[Basler et al ‘24]

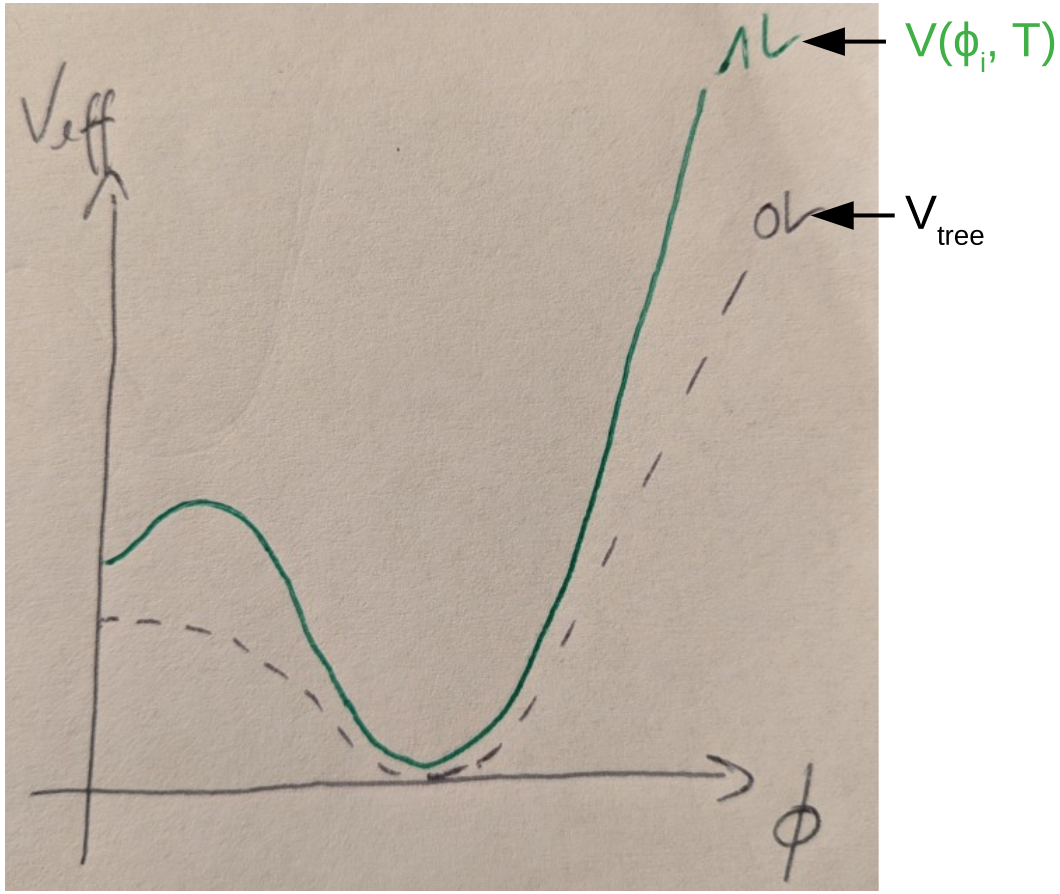
- Impact of loop and thermal corrections to electroweak phase transition dynamics can be obfuscated if these corrections modify the EW minimum
  → **“OS-like” renormalisation conditions**

$$0 = \partial_{\phi_i}\left(V^{\text{CW}} + V^{\text{CT}}\right)\Big|_{\phi_k=\langle\phi_k\rangle(T=0)},$$
$$0 = \partial_{\phi_i}\partial_{\phi_j}\left(V^{\text{CW}} + V^{\text{CT}}\right)\Big|_{\phi_k=\langle\phi_k\rangle(T=0)},$$

- Fixes finite CTs entering in effective potential:

$$V(\phi_i, T) = V_{\text{tree}} + V_{\text{CW}} + V_{\text{T}} + V_{\text{CT}} + V_{\text{daisy}}$$

V_{tree}: tree-level potential
V_{CW}: Coleman-Weinberg (T=0) one-loop corrections
V_T: thermal corrections
V_{daisy}: resummation of thermal Daisy diagrams



Renormalisation scheme conversions and uncertainty estimates

Renormalisation scheme conversions

$$g_{\text{bare}} = g_{\text{scheme A}} + \delta^{\text{CT}}_{\text{scheme A}} g = g_{\text{scheme B}} + \delta^{\text{CT}}_{\text{scheme B}} g$$

$$\Rightarrow \quad g_{\text{scheme B}} = g_{\text{scheme A}} + \underbrace{\delta^{\text{CT}}_{\text{scheme A}} g - \delta^{\text{CT}}_{\text{scheme B}} g}_{\text{finite because } \delta^{\text{CT}}_{\text{scheme A}} g\big|_{\text{div.}} = \delta^{\text{CT}}_{\text{scheme B}} g\big|_{\text{div.}}}$$

- Scheme conversion via difference of CTs

- Suppose one takes an expression for one's favourite observable, O_{ABC}, in terms of an $\overline{\text{MS}}$ parameter $x^{\overline{\text{MS}}}$

$$\mathcal{O}_{ABC} = f^{(0)}(x^{\overline{\text{MS}}}) + \frac{1}{16\pi^2} f^{(1)}(x^{\overline{\text{MS}}}) + \frac{1}{(16\pi^2)^2} f^{(2)}(x^{\overline{\text{MS}}})$$

and want to convert it in terms of the OS-renormalised parameter X^{OS}

$$x^{\overline{\text{MS}}} = X^{\text{OS}} + \frac{1}{16\pi} \delta^{(1)} x + \frac{1}{(16\pi^2)^2} \delta^{(2)} x$$

then $\mathcal{O}_{ABC} = f^{(0)}(X^{\text{OS}}) + \frac{1}{16\pi^2}\left[f^{(1)}(X^{\text{OS}}) + \frac{\partial f^{(0)}}{\partial x}(X^{\text{OS}})\delta^{(1)}x\right]$

$$+\frac{1}{(16\pi^2)^2}\left[f^{(2)}(X^{\text{OS}}) + \frac{\partial f^{(1)}}{\partial x}(X^{\text{OS}})\delta^{(1)}x + \frac{\partial f^{(0)}}{\partial x}(X^{\text{OS}})\delta^{(2)}x + \frac{1}{2}\frac{\partial^2 f^{(0)}}{\partial x^2}(X^{\text{OS}})(\delta^{(1)}x)^2\right] + 3\text{ loops}$$

→ scheme conversion **generates higher-order** (here 3L) **terms**

→ **this can serve as an estimate of unknown higher-order corrections** ***– provided both scheme are stable!***

Scheme conversions and uncertainty estimates

[Bahl, JB, Gabelmann, Weiglein '23]

$$g_{\text{scheme B}} = g_{\text{scheme A}} + \delta^{\text{CT}}_{\text{scheme A}} g - \delta^{\text{CT}}_{\text{scheme B}} g$$

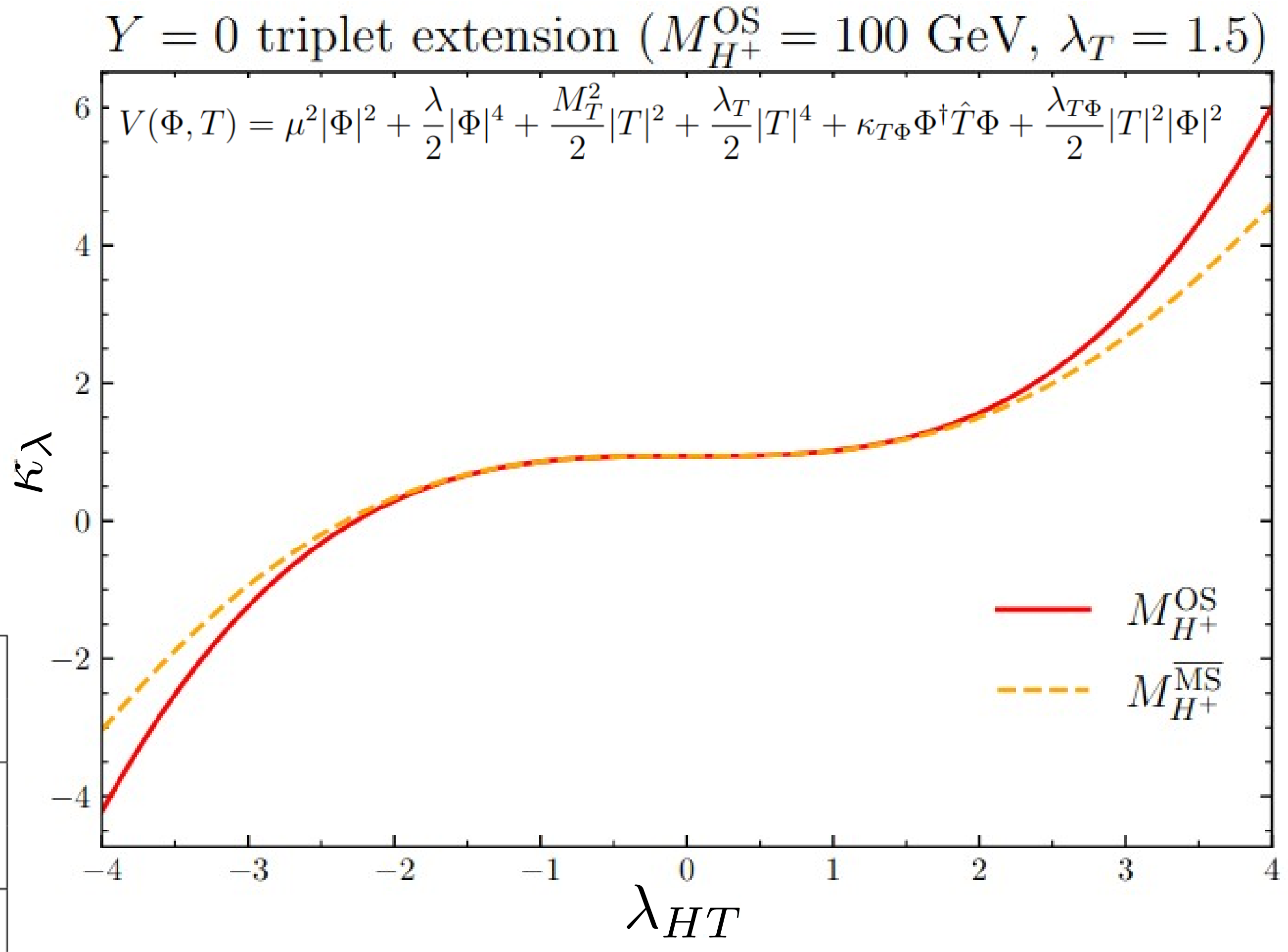
Examples with 1L conversion of BSM scalar masses entering in λ_{hhh} from 1L (but not at 0L)

→ **estimate of 2L corrections**

IDM benchmarks: change $\mu_2,\ \lambda_2$ and fix

$M_H = 400$ GeV, $M_A = 410$ GeV, $M_{H^\pm} = 415$ GeV

	Inputs		$\overline{\text{MS}}$ masses (at $Q = 300$ GeV)			anyH3 results		
BP	μ_2 [GeV]	λ_2 –	$m_H^{\overline{\text{MS}}}$ [GeV]	$m_A^{\overline{\text{MS}}}$ [GeV]	$m_{H^\pm}^{\overline{\text{MS}}}$ [GeV]	$(\lambda_{hhh}^{(1)})^{\text{OS}}$ [GeV]	$(\lambda_{hhh}^{(1)})^{\overline{\text{MS}}}$ [GeV]	Δ [%]
1	250	0	403.7	413.8	418.6	220.6	223.7	1.4
2	250	2	406.7	416.7	421.4	220.6	226.2	2.5
3	0	0	409.9	419.9	424.6	356.1	373.9	4.8
4	0	2	412.9	422.7	427.4	356.1	379.4	6.1


$Y = 0$ triplet extension $(M_{H^+}^{\text{OS}} = 100$ GeV, $\lambda_T = 1.5)$

$$V(\Phi, T) = \mu^2|\Phi|^2 + \frac{\lambda}{2}|\Phi|^4 + \frac{M_T^2}{2}|T|^2 + \frac{\lambda_T}{2}|T|^4 + \kappa_{T\Phi}\Phi^\dagger\hat{T}\Phi + \frac{\lambda_{T\Phi}}{2}|T|^2|\Phi|^2$$

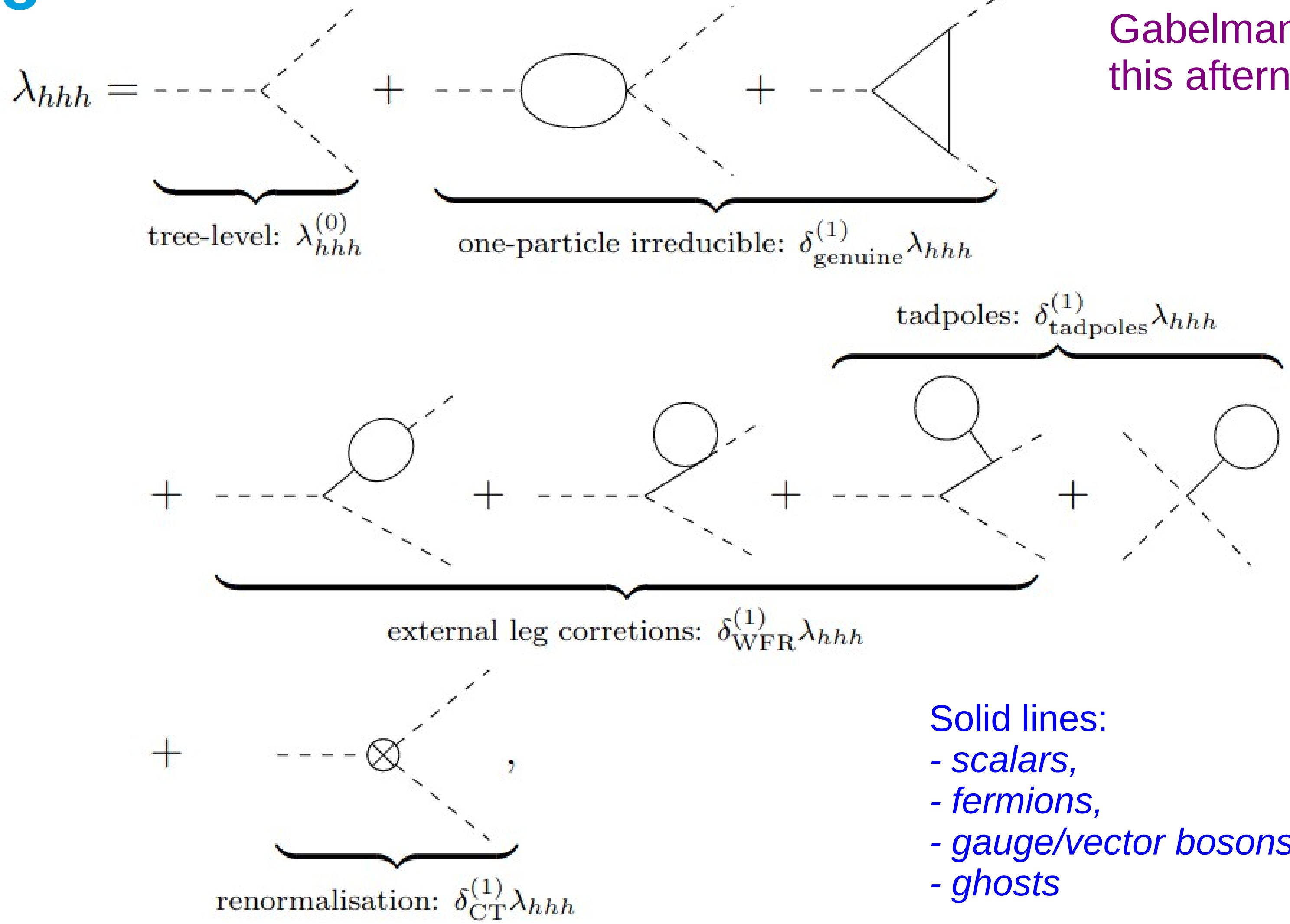
Here estimates in agreement with explicit 2L calculations for IDM [JB, Kanemura '19], and real triplet model [JB, Egle, Verduras Schaiedt WIP] (c.f. backup)

Towards automated renormalisation

Predictions for λ_{hhh} in general renormalisable theories

See talk by M. Gabelmann this afternoon!

- **Full one-loop generic results** applied to concrete (B)SM model, using inputs in `UFO` format [Degrande et al., ‘11], [Darmé et al. ‘23]
- Loop functions evaluated via `COLLIER` [Denner et al ‘16] interface, `pyCollier`
- Restrictions on **particles** and/or **topologies** possible
- **Renormalisation performed automatically**

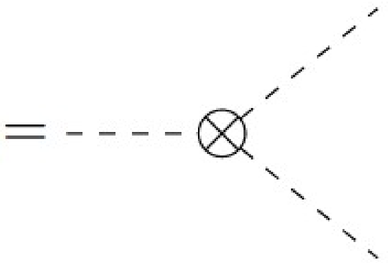


Solid lines:
- *scalars,*
- *fermions,*
- *gauge/vector bosons,*
- *ghosts*

v1 (public): λ_{hhh} [Bahl, JB, Gabelmann, Weiglein ‘23]

v2 (under dev.): λ_{ijk} *(+* `anyHH` *for di-Higgs prod.)* [Bahl, JB, Gabelmann, Radchenko Serdula, Weiglein *WIP*]

Flexible choice of renormalisation schemes

$\delta^{(1)}_{\rm CT}\lambda_{ijk} =$  $=$?

- **1L calculation** → renormalisation of all parameters entering λ_{hhh} at tree-level
- In general:

$$(\lambda^{(0)}_{ijk})^{\rm BSM} = (\lambda^{(0)}_{ijk})^{\rm BSM}(\underbrace{m_h \simeq 125\ {\rm GeV}, v \simeq 246\ {\rm GeV}}_{\text{SM sector}}, \underbrace{m_{\Phi_i}}_{\substack{\text{BSM}\\ \text{masses}}}, \underbrace{\alpha_i}_{\substack{\text{BSM}\\ \text{mixing angles}}}, \underbrace{v_i}_{\substack{\text{BSM}\\ \text{VEVs}}}, \underbrace{g_i}_{\substack{\text{indep.}\\ \text{BSM coups.}}})$$

- Most automated codes: $\overline{\rm MS}/\overline{\rm DR}$ only
- `anyH3`: much more flexibility, following **user choice**:
  - **SM sector** (m_h, v): fully OS or $\overline{\rm MS}/\overline{\rm DR}$
  - **BSM masses**: OS or $\overline{\rm MS}/\overline{\rm DR}$
  - **Additional couplings/vevs/mixings**: by default $\overline{\rm MS}$, but **user-defined ren. conditions** also possible!

$$\delta^{(1)}_{\rm CT}\lambda_{ijk} = \sum_x \left(\frac{\partial}{\partial x}(\lambda^{(0)}_{ijk})^{\rm BSM}\right)\delta^{\rm CT}x\,, \qquad \text{with } x \in \{m_h, v, m_{\Phi_i}, v_i, \alpha_i, g_i, \text{etc.}\}$$

Renormalised in $\overline{\rm MS}$, OS, in custom schemes, etc.

Flexible choice of renormalisation schemes

`schemes.yml`, here on-shell scheme for 2HDM as an example

```
OS:
  description: OS conditions for all input parameters and tadpoles
  SM_names:
    Higgs-Boson: h1
  VEV_counterterm: OS
  wfrs: 'OS' # set momenta in WFR topologies OS
  tadpoles: False
  mass_counterterms:
    h1: OS
    h2: OS
  parameter_counterterms:
    - parameter: TadH1
      counterterm: dTadH1
      condition: Tadpole('h2')*cos(alphaH) - Tadpole('h1')*sin(alphaH)
    - parameter: TadH2
      counterterm: dTadH2
      condition: Tadpole('h1')*cos(alphaH) + Tadpole('h2')*sin(alphaH)
    - parameter: betaH
      counterterm: dbetaH
      condition: (Re(Sigma('Hm1','Hm2',momentum='MHm1**2')) +
Re(Sigma('Hm2','Hm1',momentum='MHm2**2')) + 2*(dTadH2*cos(betaH) -
dTadH1*sin(betaH))/vSM)/(2*(MHm2**2+MHm1**2))
#         condition: (Re(Sigma('Ah1','Ah2',momentum='MAh1**2')) +
Re(Sigma('Ah2','Ah1',momentum='MAh2**2')) + 2*(dTadH2*cos(betaH) -
dTadH1*sin(betaH))/vSM)/(2*(MAh2**2+MAh1**2))
      warn: False # turns-off warning that betaH is not an UFO input
    - parameter: TanBeta # this is the actual UFO input
      counterterm: dTanBeta
      condition: dbetaH/cos(betaH)**2 # depends on CT defined above
    - parameter: alphaH
      counterterm: dalphaH
      condition: (Re(Sigma('h1','h2',momentum='Mh1**2')) +
Re(Sigma('h2','h1',momentum='Mh2**2')))/(2*(Mh1**2-Mh2**2))
```

Define which state is h_{125}

Turn off explicit tadpole diagrams (i.e. don't use FJTS)

OS renormalisation of scalar masses (can be `OS` or `MS`)

Define counterterms for parameters (here tadpoles in PRTS) *using 1-, 2-, 3-point functions*

Define counterterms for parameters (here for δβ) *using 1-, 2-, 3-point functions. Can use either charged or CP-odd sector for δβ*

Define counterterm δM by fixing λ_{122} to its tree-level value

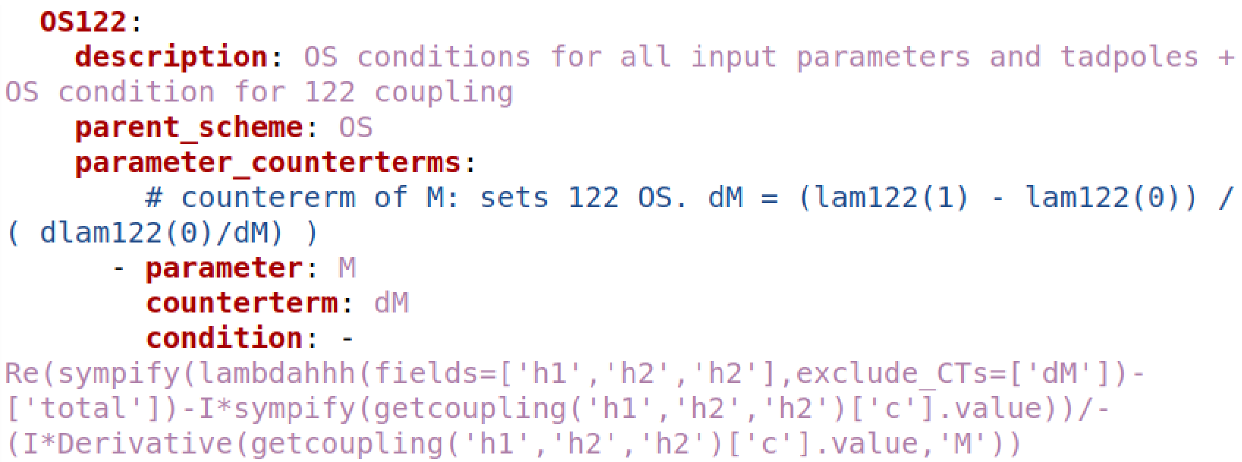
```
OS122:
  description: OS conditions for all input parameters and tadpoles +
OS condition for 122 coupling
  parent_scheme: OS
  parameter_counterterms:
      # countererm of M: sets 122 OS. dM = (lam122(1) - lam122(0)) /
( dlam122(0)/dM) )
    - parameter: M
      counterterm: dM
      condition: -
Re(sympify(lambdahhh(fields=['h1','h2','h2'],exclude_CTs=['dM'])-
['total'])-I*sympify(getcoupling('h1','h2','h2')['c'].value))/-
(I*Derivative(getcoupling('h1','h2','h2')['c'].value,'M'))
```

Flexible choice of renormalisation schemes – details

`schemes.yml`, here on-shell scheme for 2HDM as an example

$[h_1=h,\ h_2=H]$

Define counterterm δM by fixing λ_{122} to its tree-level value



```
 OS122:
   description: OS conditions for all input parameters and tadpoles +
OS condition for 122 coupling
   parent_scheme: OS
   parameter_counterterms:
       # countererm of M: sets 122 OS. dM = (lam122(1) - lam122(0)) /
( dlam122(0)/dM) )
     - parameter: M
       counterterm: dM
       condition: -
Re(sympify(lambdahhh(fields=['h1','h2','h2'],exclude_CTs=['dM'])-
['total'])-I*sympify(getcoupling('h1','h2','h2')['c'].value))/-
(I*Derivative(getcoupling('h1','h2','h2')['c'].value,'M'))
```

OS condition:

$$\lambda^{(1),\,\text{ren.}}_{h_1h_2h_2}=\lambda^{(0)}_{h_1h_2h_2}(M^{\text{OS}})+\delta^{(1)}\lambda_{h_1h_2h_2}(M^{\text{OS}})+\sum_{p_X\neq M}(\delta^{\text{CT}}p_X)\frac{\partial}{\partial p_X}\lambda^{(0)}_{h_1h_2h_2}+(\delta^{\text{CT}}M^{\text{OS}})\frac{\partial}{\partial M}\lambda^{(0)}_{h_1h_2h_2}\overset{!}{=}\lambda^{(0)}_{h_1h_2h_2}$$

$$\Rightarrow(\delta^{\text{CT}}M^{\text{OS}})=\frac{\lambda^{(1),\,\text{ren.}}_{h_1h_2h_2}|_{\delta^{\text{CT}}M\to0}-\lambda^{(0)}_{h_1h_2h_2}}{\frac{\partial}{\partial M}\lambda^{(0)}_{h_1h_2h_2}}=\frac{\sum_{p_X\neq M}(\delta^{\text{CT}}p_X)\frac{\partial}{\partial p_X}\lambda^{(0)}_{h_1h_2h_2}}{\frac{\partial}{\partial M}\lambda^{(0)}_{h_1h_2h_2}}$$

$\lambda^{(1),\,\text{ren.}}_{h_1h_2h_2}|_{\delta^{\text{CT}}M\to0}$: `lambdahhh(fields=['h1','h2','h2'],exclude_CTs=['dM'])['total'])`

$\lambda^{(0)}_{h_1h_2h_2}$: `I*sympify(getcoupling('h1','h2','h2')['c'].value))`

$\frac{\partial}{\partial M}\lambda^{(0)}_{h_1h_2h_2}$: `I*Derivative(getcoupling('h1','h2','h2')['c'].value,'M')`

Flexible choice of renormalisation schemes

`schemes.yml`, here <u>on-shell scheme for 2HDM</u> as an example

```
 OS:
  description: OS conditions for all input parameters and tadpoles
  SM_names:
   Higgs-Boson: h1
  VEV_counterterm: OS
  wfrs: 'OS' # set momenta in WFR topologies OS
  tadpoles: False
  mass_counterterms:
   h1: OS
   h2: OS
  parameter_counterterms:
   - parameter: TadH1
     counterterm: dTadH1
     condition: Tadpole('h2')*cos(alphaH) - Tadpole('h1')*sin(alphaH)
   - parameter: TadH2
     counterterm: dTadH2
     condition: Tadpole('h1')*cos(alphaH) + Tadpole('h2')*sin(alphaH)
   - parameter: betaH
     counterterm: dbetaH
     condition: (Re(Sigma('Hm1','Hm2',momentum='MHm1**2')) +
Re(Sigma('Hm2','Hm1',momentum='MHm2**2')) + 2*(dTadH2*cos(betaH) -
dTadH1*sin(betaH))/vSM)/(2*(MHm2**2+MHm1**2))
#         condition: (Re(Sigma('Ah1','Ah2',momentum='MAh1**2')) +
Re(Sigma('Ah2','Ah1',momentum='MAh2**2')) + 2*(dTadH2*cos(betaH) -
dTadH1*sin(betaH))/vSM)/(2*(MAh2**2+MAh1**2))
     warn: False # turns-off warning that betaH is not an UFO input
   - parameter: TanBeta # this is the actual UFO input
     counterterm: dTanBeta
     condition: dbetaH/cos(betaH)**2 # depends on CT defined above
   - parameter: alphaH
     counterterm: dalphaH
     condition: (Re(Sigma('h1','h2',momentum='Mh1**2')) +
Re(Sigma('h2','h1',momentum='Mh2**2')))/(2*(Mh1**2-Mh2**2))
```

Define which state is h_{125}

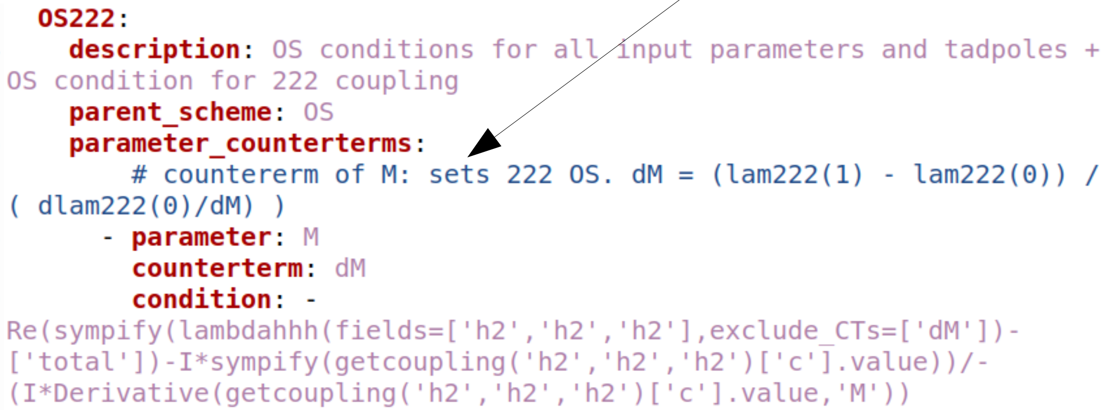
Turn off explicit tadpole diagrams (i.e. don't use FJTS)

OS renormalisation of scalar masses (can be `OS` or `MS`)

Define counterterms for parameters (here tadpoles in PRTS) *using 1-, 2-, 3-point functions*

Define counterterms for parameters (here for $\delta\beta$) *using 1-, 2-, 3-point functions. Can use either charged or CP-odd sector for $\delta\beta$*

Define counterterm δM by fixing λ_{222} to its tree-level value



```
  OS222:
    description: OS conditions for all input parameters and tadpoles +
OS condition for 222 coupling
    parent_scheme: OS
    parameter_counterterms:
        # countererm of M: sets 222 OS. dM = (lam222(1) - lam222(0)) /
( dlam222(0)/dM) )
      - parameter: M
        counterterm: dM
        condition: -
Re(sympify(lambdahhh(fields=['h2','h2','h2'],exclude_CTs=['dM'])-
['total'])-I*sympify(getcoupling('h2','h2','h2')['c'].value))/-
(I*Derivative(getcoupling('h2','h2','h2')['c'].value,'M'))
```

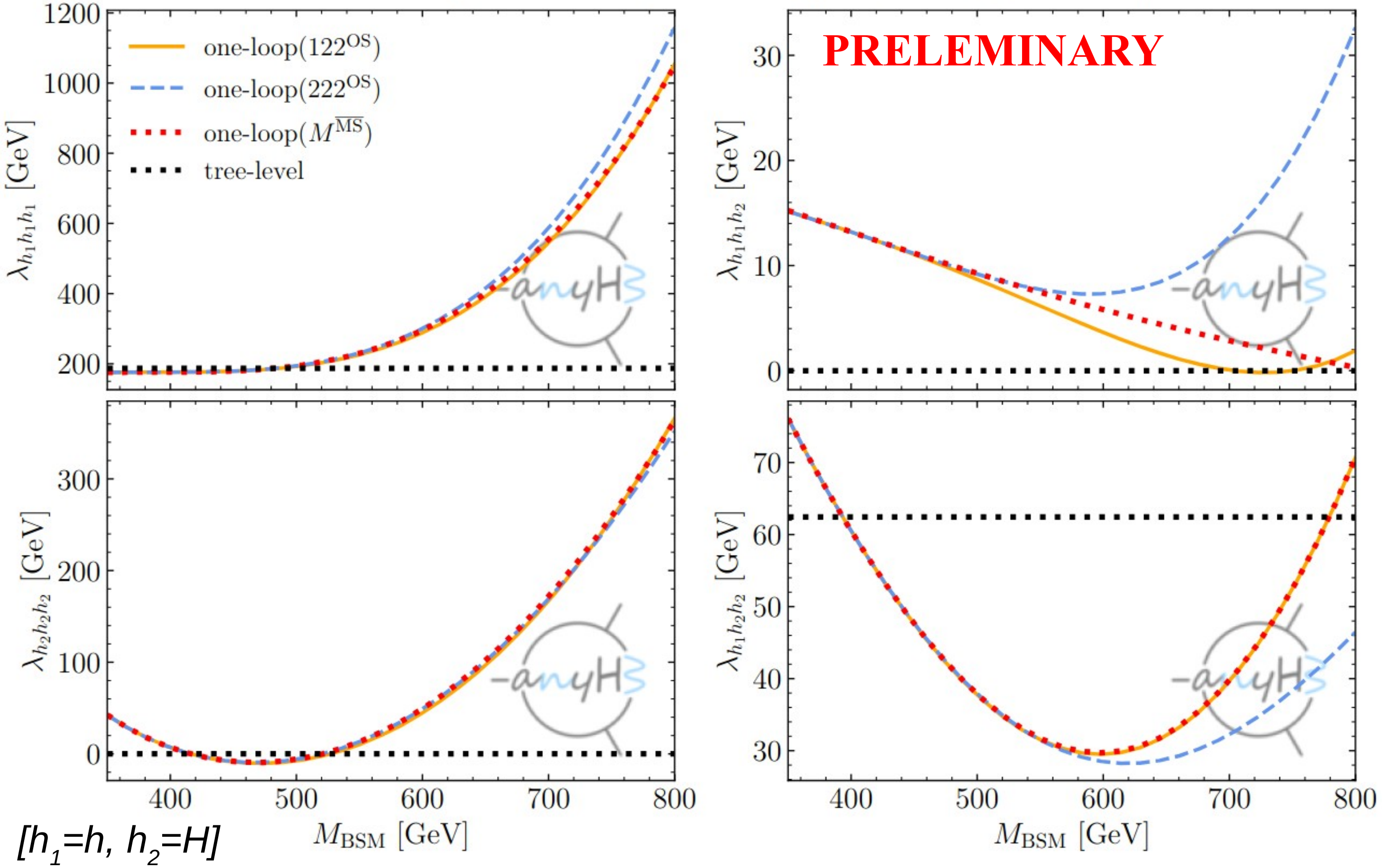
Scheme comparisons in the 2HDM

[Bahl, JB, Gabelmann, Radchenko
Serdula, Weiglein *WIP*]

2HDM type-II: $M^{\overline{\text{MS}}}(Q = M^{\overline{\text{MS}}}) = M_H = 400\text{ GeV},\ M_A = M_{H^\pm} \equiv M_{\text{BSM}},\ \alpha = \beta - \pi/2$

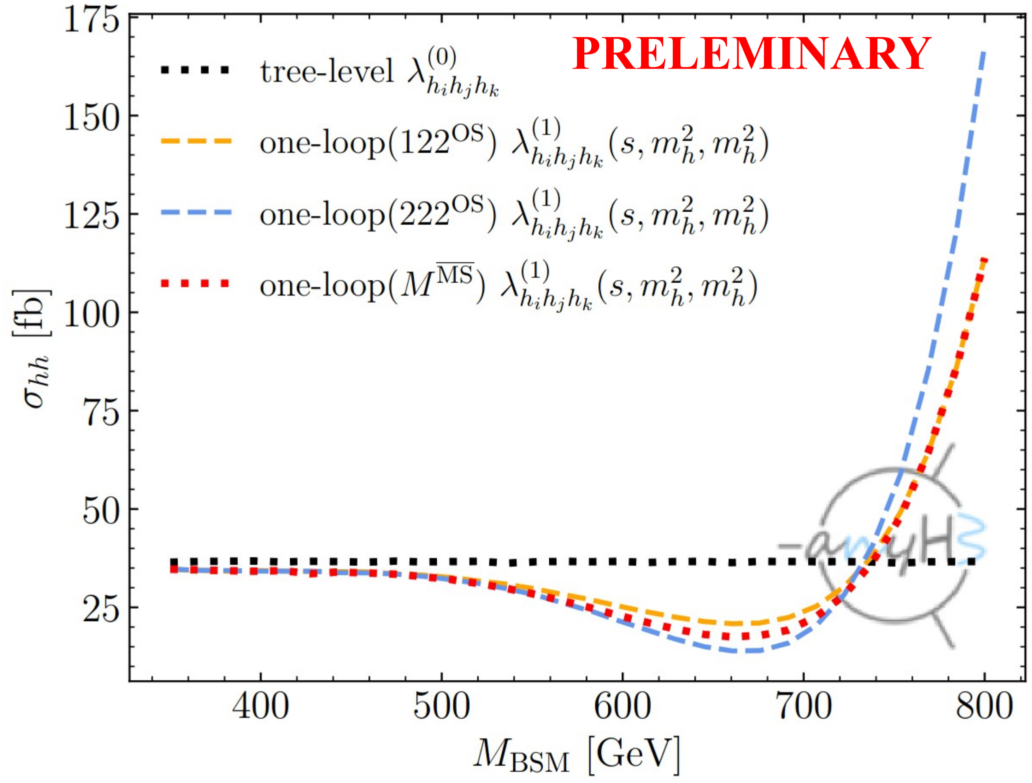
3 schemes for M: $\overline{\text{MS}}$, 122^{OS} (i.e. fix $\delta^{\text{CT}}M$ from $\lambda^{(1)}_{hHH} \overset{!}{=} \lambda^{(0)}_{hHH}$), 222^{OS} (i.e. fix $\delta^{\text{CT}}M$ from $\lambda^{(1)}_{HHH} \overset{!}{=} \lambda^{(0)}_{HHH}$)

Trilinear scalar couplings $\lambda^{(1)}_{ijk}$



[h_1=h, h_2=H]

Di-Higgs prod. cross-section σ_{hh}



See talk by M. Gabelmann this afternoon!

Summary

- **Precision calculations** are unavoidable to make use of the vast amount of data coming from various experimental directions to **test BSM theories**

- **Renormalisation in extended scalar sector is a crucial and very active topic of current research**
  → *devise schemes with desirable theoretical/phenomenological properties, without paying too much of a price in complexity or computational cost!*

- In general, there is **no scheme that fits for any model or any observable/quantity**

- Ongoing progress towards **automation of renormalisation** procedure in public code(s)
  (also automation of choice of renormalisation scheme, c.f backup)

Thank you very much for your attention!

Contact

DESY. Deutsches
Elektronen-Synchrotron

www.desy.de

Johannes Braathen
DESY Theory group
Building 2a, Room 208a
johannes.braathen@desy.de

Backup

A simple toy model

- Abelian Goldstone model + singlet scalar S

$$V^{(0)} = \mu^2\,|H|^2 + \frac{1}{4}\,\lambda\,|H|^4 + \frac{1}{2}\,m_S^2\,S^2 + a_{SH}\,S\,|H|^2 + \lambda_{SH}\,S^2\,|H|^2 + a_S\,S^3 + \lambda_S\,S^4$$

with $H \equiv \frac{1}{\sqrt{2}}(v + h + i\,G)\,, \quad S \equiv v_S + \hat{S}$

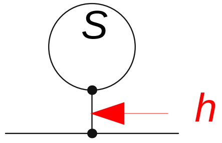
- 2 tadpole eqs. → solve for μ and v_s (or m_s)

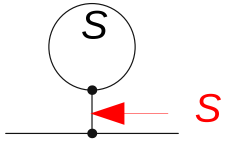
$$\text{For } 0 < v \ll m_S, \qquad v_S \sim -\frac{a_{SH} v^2}{2 m_S^2}$$

- Corrections to singlet mass:

$$\text{Option 1:} \quad \Delta M_S^2 = -\frac{1}{v_S} t_S + \Pi_{SS} \supset \frac{3 a_S m_S^2}{16\pi^2 v_S}\left(1 - \log\frac{m_S^2}{Q^2}\right) \simeq \frac{6 a_S m_S^4}{16\pi^2 a_{SH} v^2}\left(1 - \log\frac{m_S^2}{Q^2}\right)$$

$$\text{Option 2:} \quad \Delta M_S^2 = -\frac{a_{SH}^2}{32\pi^2 m_h^2} A(m_S^2) - \frac{3 a_S^2}{16\pi^2 m_S^2} A(m_S^2) + \Pi_{SS} \supset \frac{m_S^2}{32\pi^2}\left(\frac{a_{SH}^2}{m_h^2} - 24\lambda_S\right)\left(1 - \log\frac{m_S^2}{Q^2}\right)$$





Toy model scenarios

$$V^{(0)} = \mu^2\,|H|^2 + \frac{1}{4}\,\lambda\,|H|^4 + \frac{1}{2}\,m_S^2\,S^2 + a_{SH}\,S\,|H|^2 + \lambda_{SH}\,S^2\,|H|^2 + a_S\,S^3 + \lambda_S\,S^4$$

$$H \equiv \frac{1}{\sqrt{2}}\,(v + h + i\,G)\,, \quad S \equiv v_S + \hat{S}$$

$$\text{Option 1:} \quad \Delta M_S^2 = -\frac{1}{v_S} t_S + \Pi_{SS} \supset \frac{3 a_S m_S^2}{16\pi^2 v_S}\left(1 - \log\frac{m_S^2}{Q^2}\right) \simeq \frac{6 a_S m_S^4}{16\pi^2 a_{SH} v^2}\left(1 - \log\frac{m_S^2}{Q^2}\right)$$

$$\text{Option 2:} \quad \Delta M_S^2 = -\frac{a_{SH}^2}{32\pi^2 m_h^2} A(m_S^2) - \frac{3 a_S^2}{16\pi^2 m_S^2} A(m_S^2) + \Pi_{SS} \supset \frac{m_S^2}{32\pi^2}\left(\frac{a_{SH}^2}{m_h^2} - 24\lambda_S\right)\left(1 - \log\frac{m_S^2}{Q^2}\right)$$

- In the following: compare results from the 2 approaches, when taking the **same numerical inputs for the BSM VEV v_s** (with different interpretations)

- Compare different parameter points, to highlight the difficulty arising from the **choice of definition of inputs**

- Consistency check (with appropriate conversion of VEVs) → backup

$$V^{(0)} = \mu^2\,|H|^2 + \frac{1}{4}\,\lambda\,|H|^4 + \frac{1}{2}\,m_S^2\,S^2 + a_{SH}\,S\,|H|^2 + \lambda_{SH}\,S^2\,|H|^2 + a_S\,S^3 + \lambda_S\,S^4$$

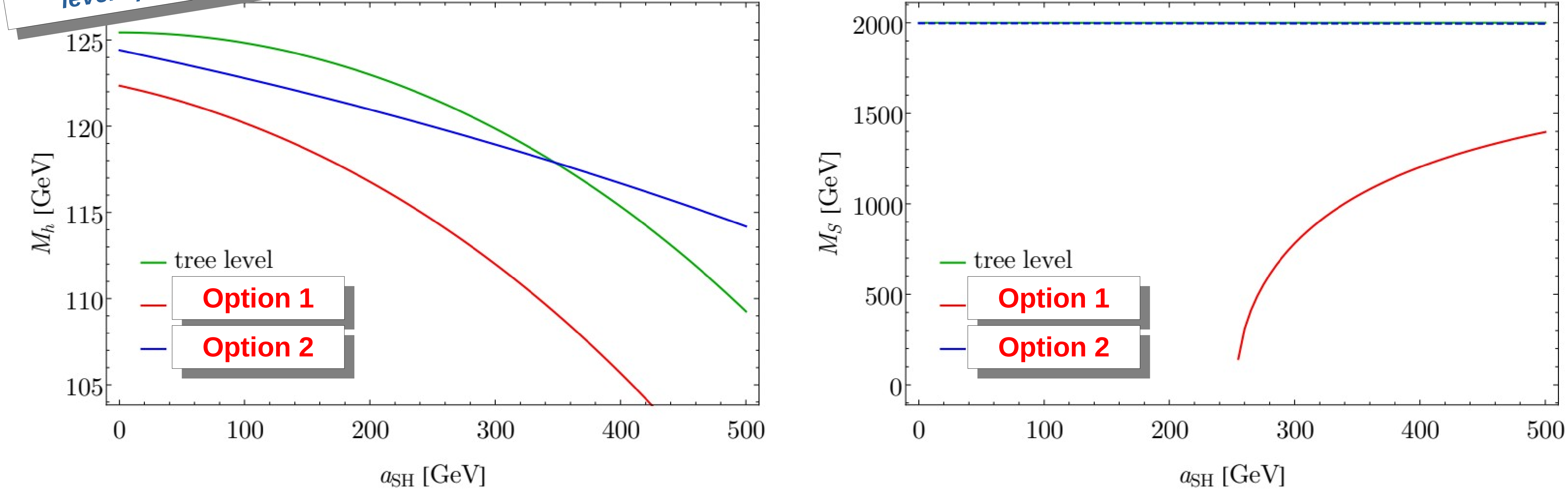
$$H \equiv \frac{1}{\sqrt{2}}\,(v + h + i\,G)\,, \quad S \equiv v_S + \hat{S}$$

Toy model scenario 1

***NB**: we compare points with same tree-level spectra, but different loop level spectra!*

$$\text{Option 1:} \quad \Delta M_S^2 = -\frac{1}{v_S} t_S + \Pi_{SS} \supset \frac{3 a_S m_S^2}{16\pi^2 v_S}\left(1 - \log\frac{m_S^2}{Q^2}\right) \simeq \frac{6 a_S m_S^4}{16\pi^2 a_{SH} v^2}\left(1 - \log\frac{m_S^2}{Q^2}\right)$$

$$\text{Option 2:} \quad \Delta M_S^2 = -\frac{a_{SH}^2}{32\pi^2 m_h^2} A(m_S^2) - \frac{3 a_S^2}{16\pi^2 m_S^2} A(m_S^2) + \Pi_{SS} \supset \frac{m_S^2}{32\pi^2}\left(\frac{a_{SH}^2}{m_h^2} - 24\lambda_S\right)\left(1 - \log\frac{m_S^2}{Q^2}\right)$$



- $(m_S)^{\rm tree}$=Q=2 TeV, a_S=100 GeV, λ=0.52 (for m_h), λ_{SH}=0, λ_S=1/24, vary a_{SH} → compute v_S (& μ) with *tree-level tad. eq.*
- Interpret *this value of v_S* as the minimum of one loop potential (*option 1*) vs tree level potential (*option 2*)

Toy model scenario 1

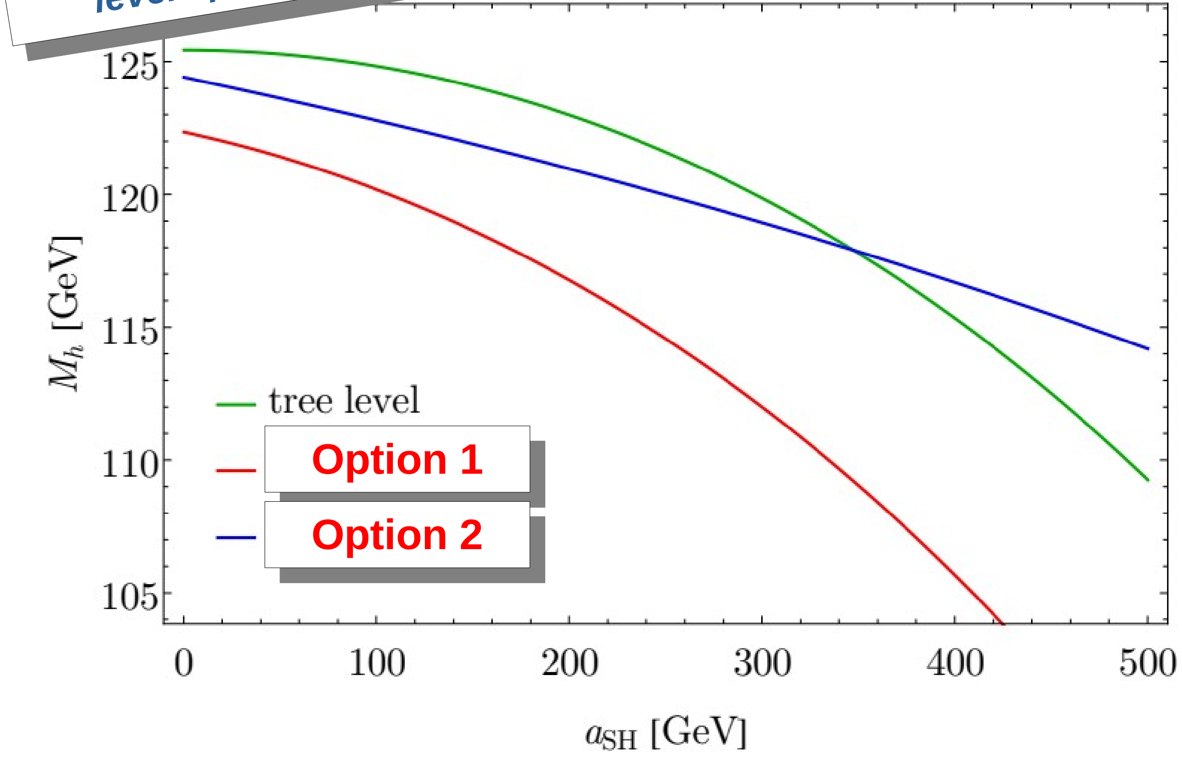
$$V^{(0)} = \mu^2 |H|^2 + \frac{1}{4}\lambda |H|^4 + \frac{1}{2} m_S^2 S^2 + a_{SH} S |H|^2 + \lambda_{SH} S^2 |H|^2 + a_S S^3 + \lambda_S S^4$$

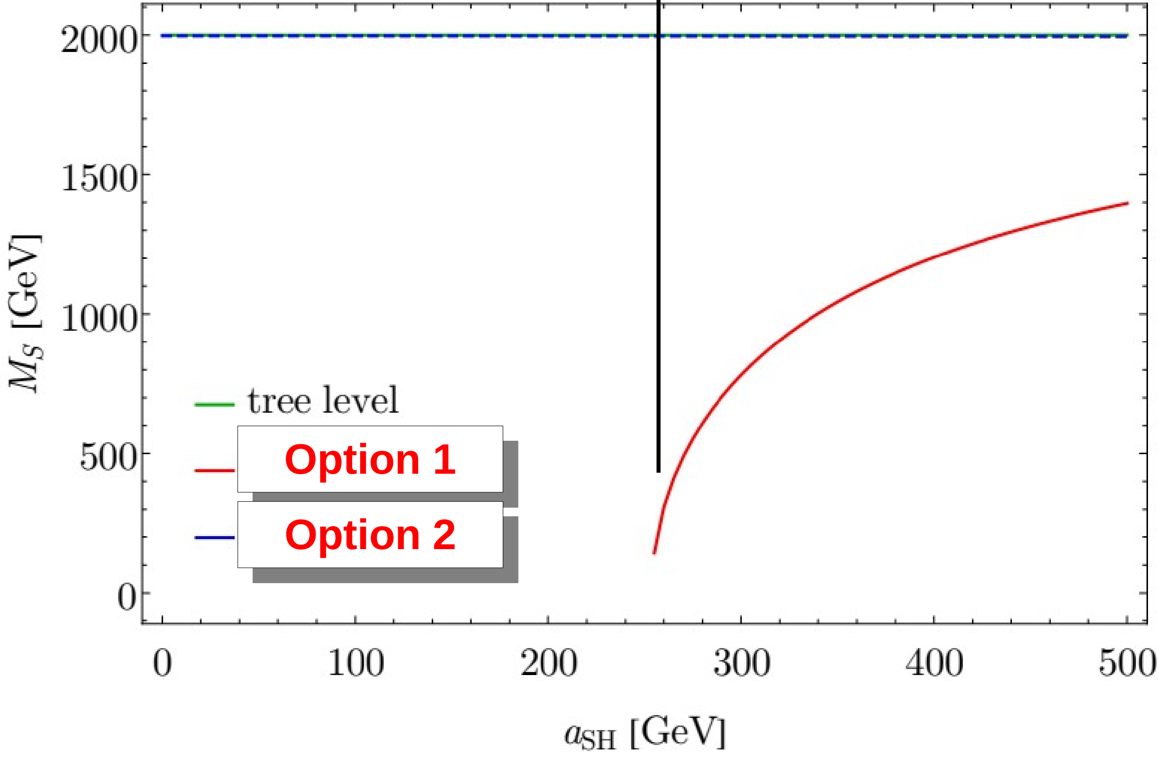
$$H \equiv \frac{1}{\sqrt{2}}(v+h+iG), \quad S \equiv v_S + \hat{S}$$

NB: we compare points with same tree-level spectra, but different loop level spectra!

$$\text{Option 1:} \quad \Delta M_S^2 = -\frac{1}{v_S} t_S + \Pi_{SS} \supset \frac{3 a_S m_S^2}{16\pi^2 v_S}\left(1-\log\frac{m_S^2}{Q^2}\right) \simeq \frac{6 a_S m_S^4}{16\pi^2 a_{SH} v^2}\left(1-\log\frac{m_S^2}{Q^2}\right)$$

$$\text{Option 2:} \quad \Delta M_S^2 = -\frac{a_{SH}^2}{32\pi^2 m_h^2} A(m_S^2) - \frac{3a_S^2}{16\pi^2 m_S^2} A(m_S^2) + \Pi_{SS} \supset \frac{m_S^2}{32\pi^2}\left(\frac{a_{SH}^2}{m_h^2} - 24\lambda_S\right)\left(1-\log\frac{m_S^2}{Q^2}\right)$$





- $(m_S)^{tree}$=Q=2 TeV, a_S=100 GeV, λ=0.52 (for m_h), λ_{SH}=0, λ_S=1/24, vary a_{SH} → compute v_S (& μ) with *tree-level tad. eq.*
- Interpret *this value of v_S* as the minimum of one loop potential (*option 1*) vs tree level potential (*option 2*)

Toy model scenario 2

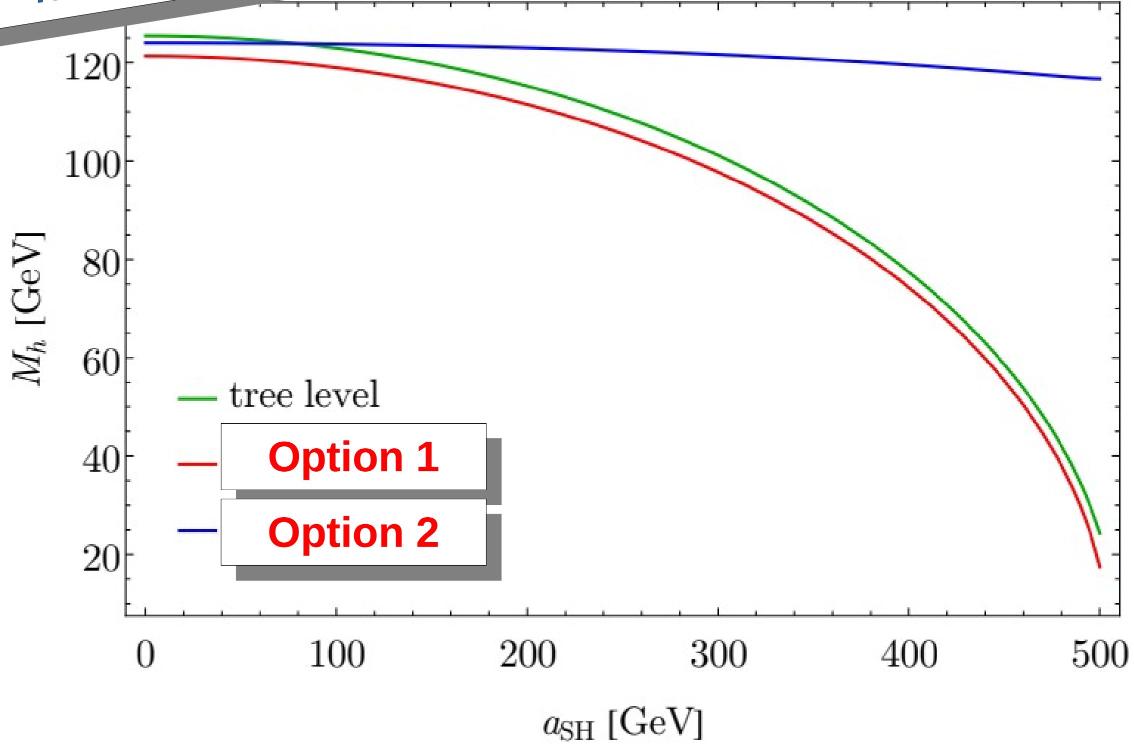
$$V^{(0)} = \mu^2 |H|^2 + \frac{1}{4}\lambda |H|^4 + \frac{1}{2} m_S^2 S^2 + a_{SH} S |H|^2 + \lambda_{SH} S^2 |H|^2 + a_S S^3 + \lambda_S S^4$$

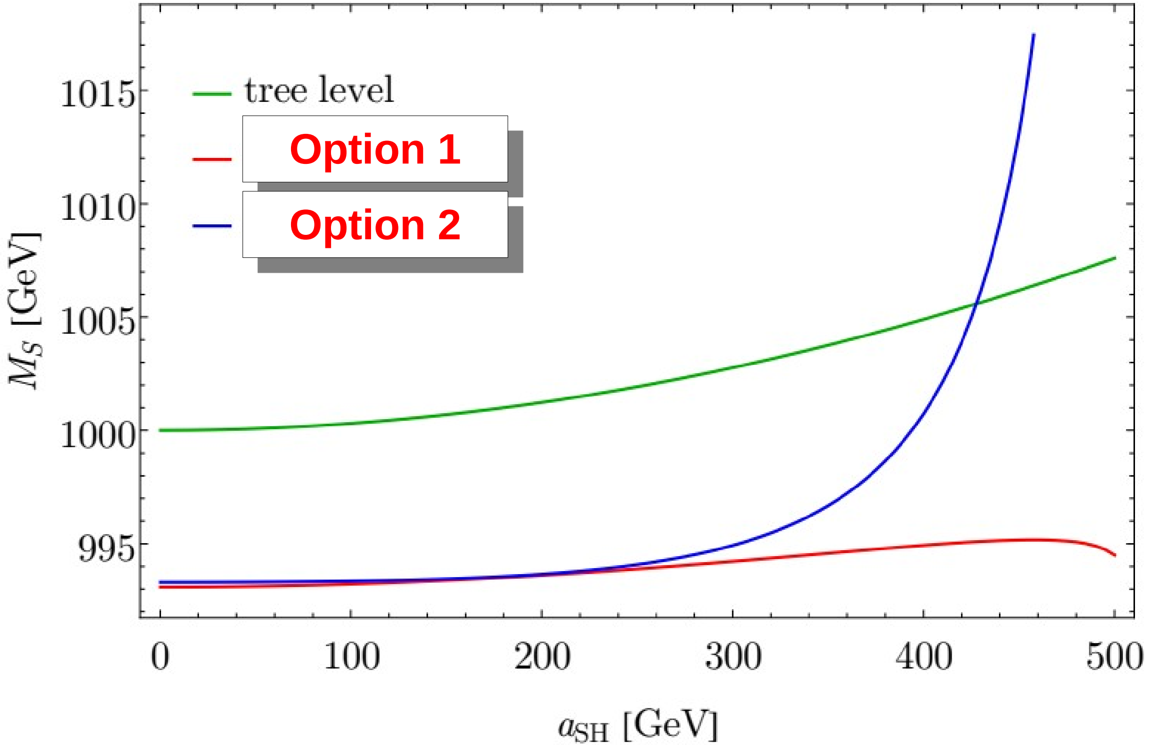
$$H \equiv \frac{1}{\sqrt{2}}(v + h + i G), \quad S \equiv v_S + \hat{S}$$

NB: we compare points with same tree-level spectra, but different loop level spectra!

$$\text{Option 1:} \quad \Delta M_S^2 = -\frac{1}{v_S} t_S + \Pi_{SS} \supset \frac{3 a_S m_S^2}{16\pi^2 v_S}\left(1 - \log\frac{m_S^2}{Q^2}\right) \simeq \frac{6 a_S m_S^4}{16\pi^2 a_{SH} v^2}\left(1 - \log\frac{m_S^2}{Q^2}\right)$$

$$\text{Option 2:} \quad \Delta M_S^2 = -\frac{a_{SH}^2}{32\pi^2 m_h^2} A(m_S^2) - \frac{3 a_S^2}{16\pi^2 m_S^2} A(m_S^2) + \Pi_{SS} \supset \frac{m_S^2}{32\pi^2}\left(\frac{a_{SH}^2}{m_h^2} - 24\lambda_S\right)\left(1 - \log\frac{m_S^2}{Q^2}\right)$$





- $(m_S)^{tree}$=1 TeV, Q=5 TeV, a_S=0 GeV, λ=0.52 (for m_h), λ_{SH}=0, λ_S=1/24, vary a_{SH} → compute v_S (& μ) with *tree-level tad. eq.*
- Interpret *this value of v_S* as the minimum of one loop potential (*option 1*) vs tree level potential (*option 2*)

Toy model scenario 2

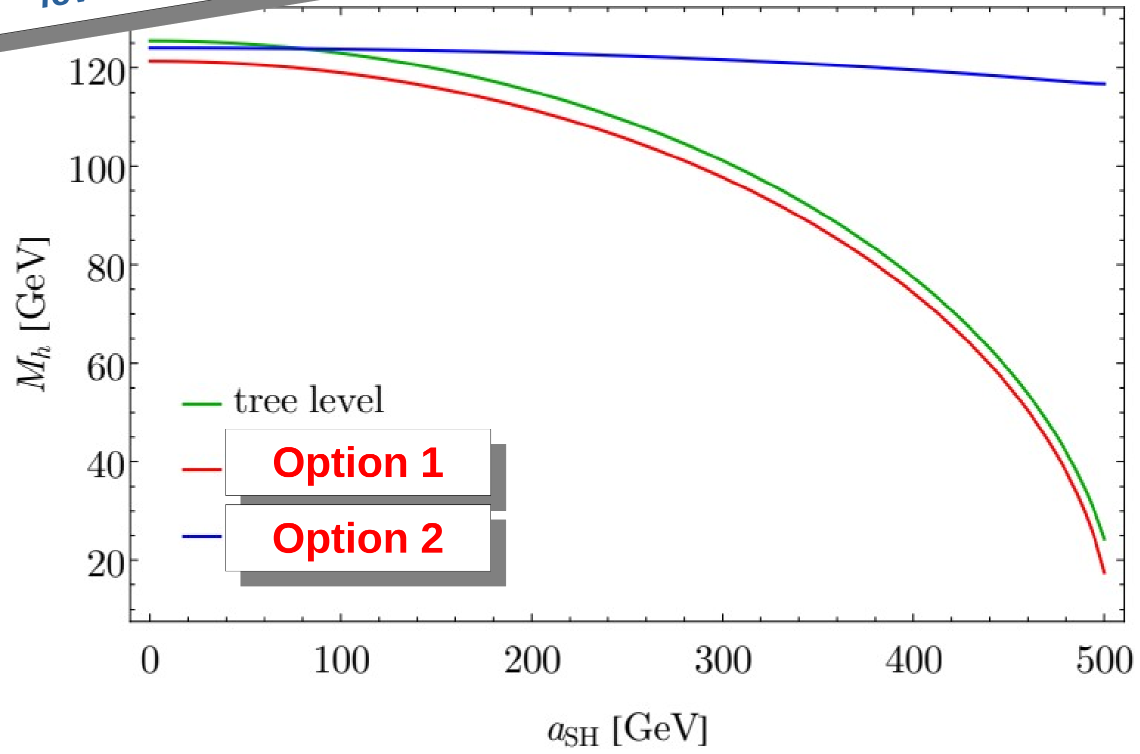
$$V^{(0)} = \mu^2\,|H|^2 + \frac{1}{4}\,\lambda\,|H|^4 + \frac{1}{2}\,m_S^2\,S^2 + a_{SH}\,S\,|H|^2 + \lambda_{SH}\,S^2\,|H|^2 + a_S\,S^3 + \lambda_S\,S^4$$

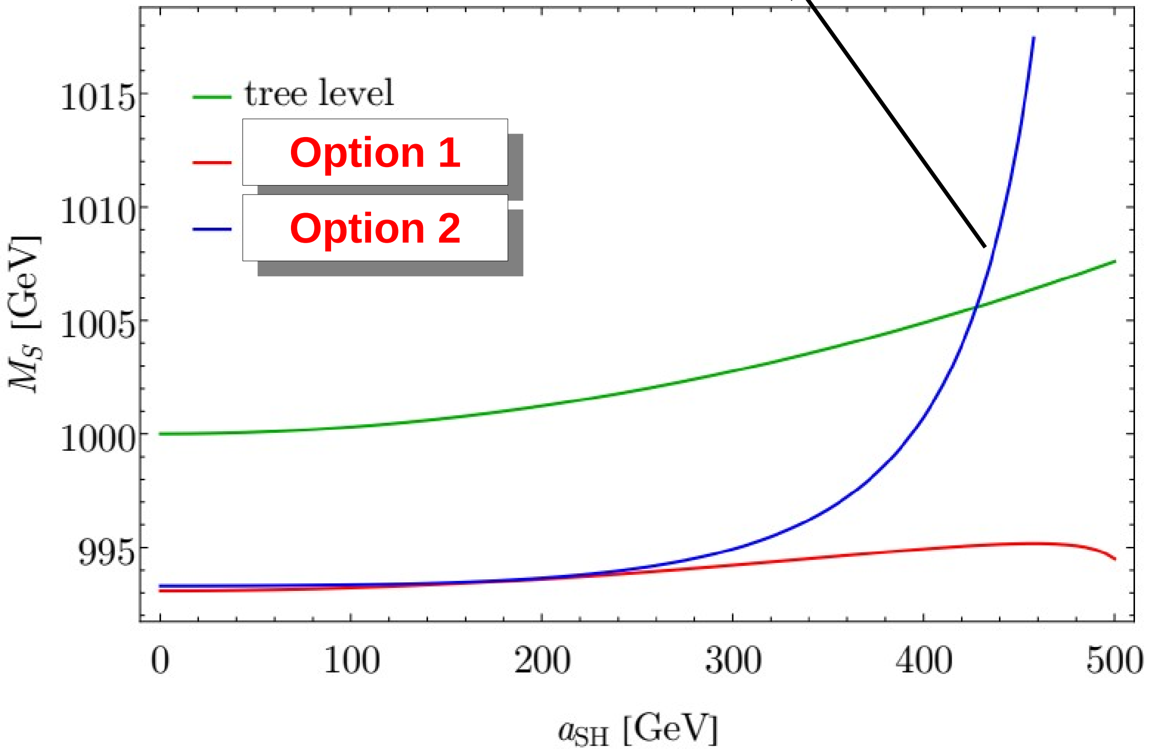
$$H \equiv \frac{1}{\sqrt{2}}\,(v + h + i\,G)\,,\quad S \equiv v_S + \hat{S}$$

***NB**: we compare points with same tree-level spectra, but different loop level spectra!*

$$\text{Option 1:}\quad \Delta M_S^2 = -\frac{1}{v_S}t_S + \Pi_{SS} \supset \frac{3a_S m_S^2}{16\pi^2 v_S}\left(1 - \log\frac{m_S^2}{Q^2}\right) \simeq \frac{6a_S m_S^4}{16\pi^2 a_{SH} v^2}\left(1 - \log\frac{m_S^2}{Q^2}\right)$$

$$\text{Option 2:}\quad \Delta M_S^2 = -\frac{a_{SH}^2}{32\pi^2 m_h^2}A(m_S^2) - \frac{3a_S^2}{16\pi^2 m_S^2}A(m_S^2) + \Pi_{SS} \supset \frac{m_S^2}{32\pi^2}\left(\frac{a_{SH}^2}{m_h^2} - 24\lambda_S\right)\left(1 - \log\frac{m_S^2}{Q^2}\right)$$





- $(m_S)^{\rm tree}$=1 TeV, Q=5 TeV, a_S=0 GeV, λ=0.52 (for m_h), λ_{SH}=0, λ_S=1/24, vary a_{SH} → compute v_S (& μ) with *tree-level tad. eq.*
- Interpret *this value of v_S* as the minimum of one loop potential (*option 1*) vs tree level potential (*option 2*)

Gauge-Invariant VEV Scheme

[Dittmaier, Rzehak '22]

See talk by R. Feser this afternoon!

- ➢ Combine advantages of PRTS (numerical stability) and FJTS (gauge invariance)

- ➢ Go to non-linear (NL) representation of SM (again, as example) Higgs sector

$$\Phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v+h+iG^0) \end{pmatrix} \longrightarrow \mathbf{\Phi} = \frac{1}{\sqrt{2}}(v+h)\exp[i\zeta_i\sigma_i/v]$$

Pauli matrices

Goldstone bosons

→ in this NL representation, (v+h) is gauge invariant

- ➢ Define tadpole CT in NL rep. same as PRTS: $(\delta^{\rm CT}t_h|^{\rm GIVS})_{\rm NL} = (\delta^{\rm CT}t_h|^{\rm PRTS})_{\rm NL} = -(t_h^{(1)})_{\rm NL}$
… but gauge independent thanks to NL rep.

- ➢ Convert back to linear rep.: $(\delta^{\rm CT}t_h|^{\rm GIVS}) = (\delta^{\rm CT}t_h|^{\rm GIVS})_1 + (\delta^{\rm CT}t_h|^{\rm GIVS})_2 \overset{!}{=} -t_h^{(1)}$
with $(\delta^{\rm CT}t_h|^{\rm GIVS})_1 \equiv -(t_h^{(1)})_{\rm NL}$

$$(\delta^{\rm CT}t_h|^{\rm GIVS})_2 \equiv (t_h^{(1)})_{\rm NL} - (t_h^{(1)}) = -m_h^2\Delta v^{\rm FJTS}|_{\rm gauge\text{-}dep.}$$

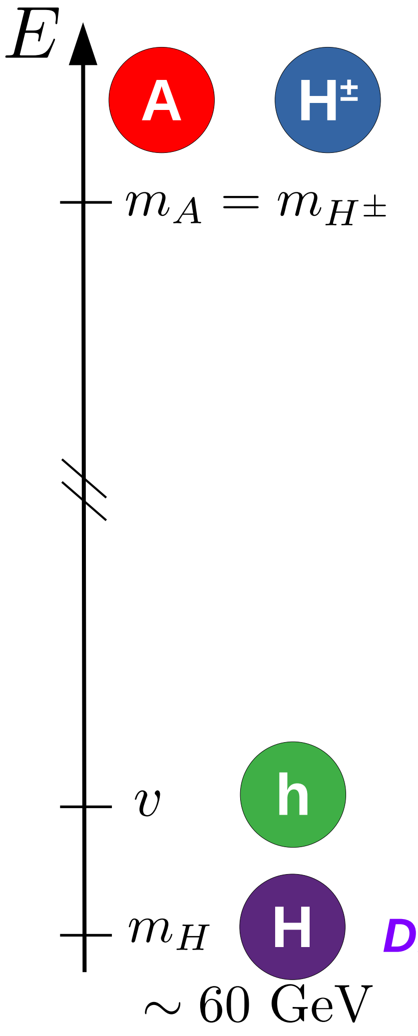
$$\Delta v^{\rm GIVS} = \Delta v^{\rm FJTS}|_{\rm gauge\ dep.}$$

Part 1: gauge independent, enters CTs in loop calculations (as with PRTS)
Part 2: gauge dependent, but drops out of any computed observable

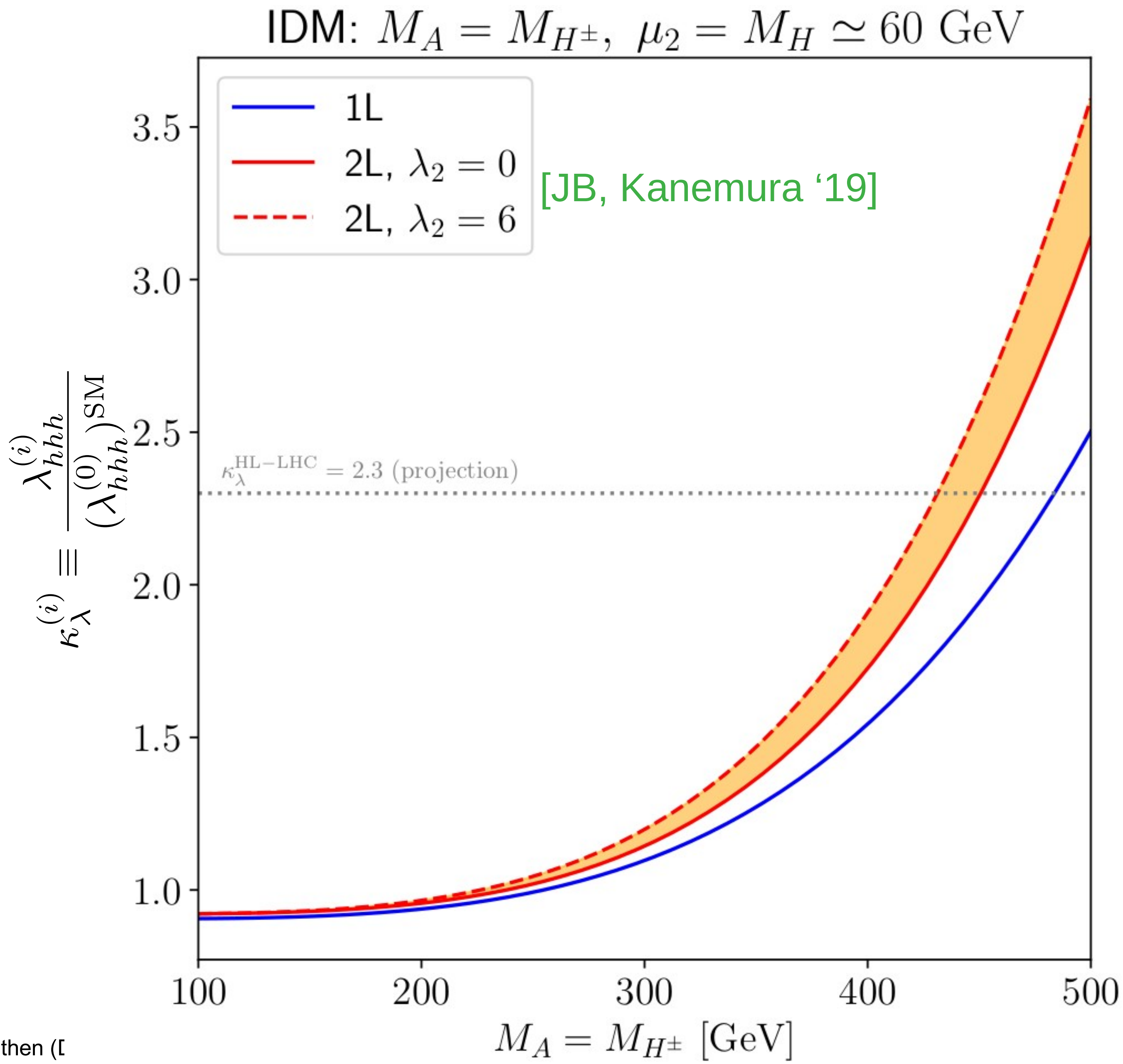
- ➢ Extended to 2HDM and Z2SSM
- ➢ Significant efforts needed to go beyond 1L …

Calculating κ_λ in the inert doublet model

Here: Inert Doublet Model in DM-inspired "Higgs resonance" scenario



DM candidate

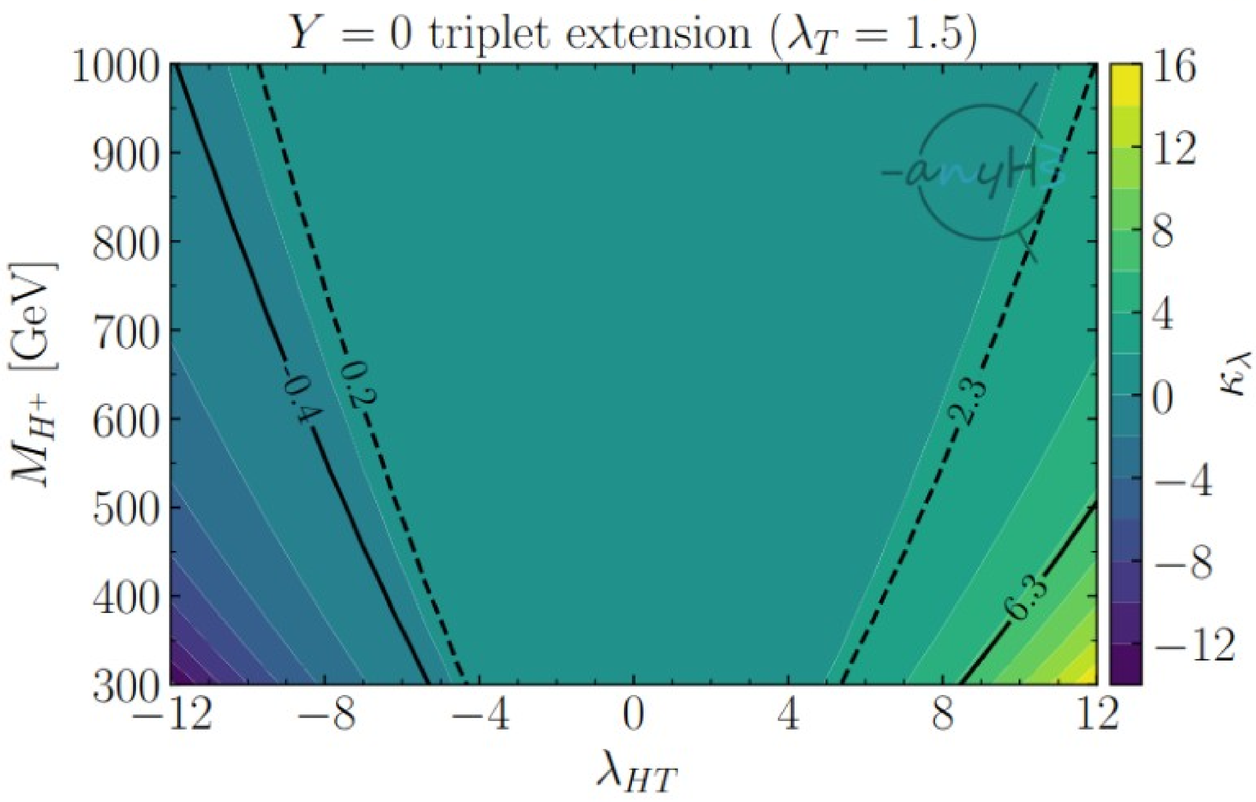


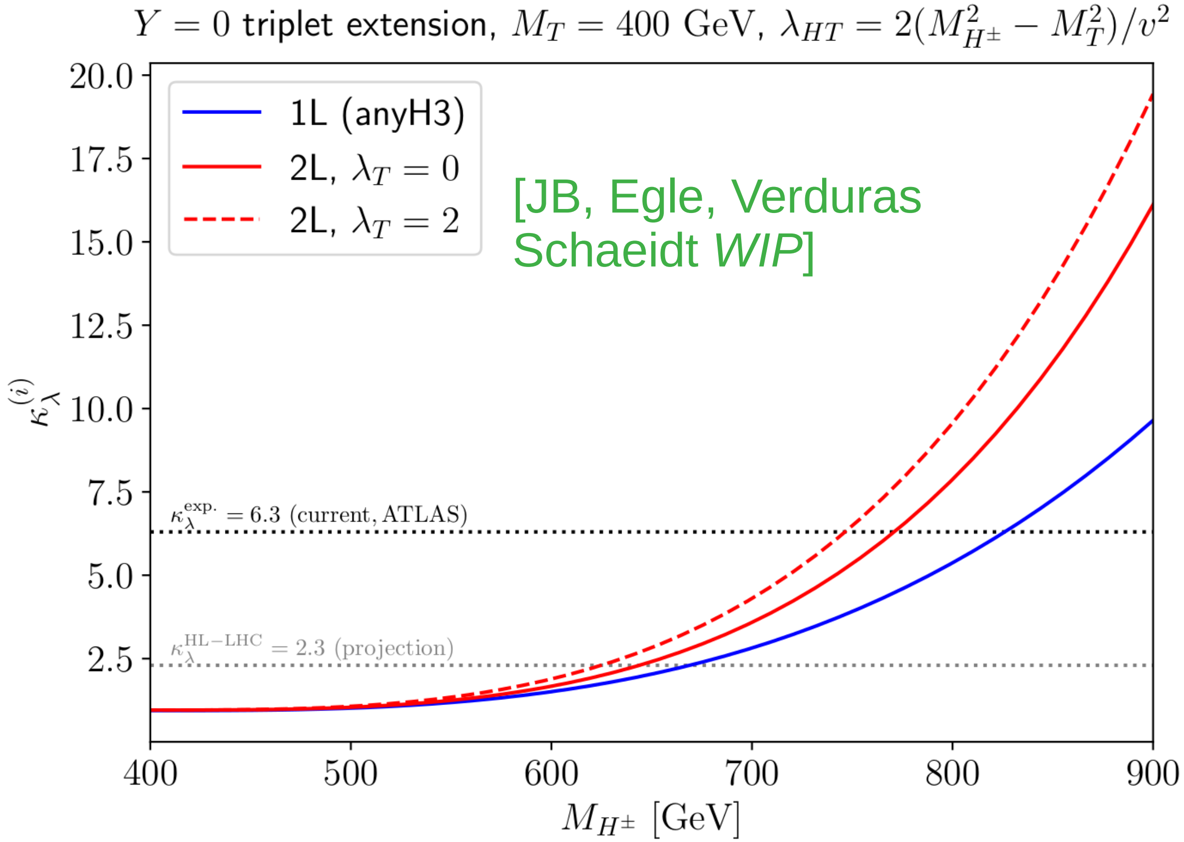
[JB, Kanemura '19]

Calculating κ_λ in the real triplet model

Real VEV-less triplet model:

$$V(\Phi,T)=\mu^2|\Phi|^2+\frac{\lambda}{2}|\Phi|^4+\frac{M_T^2}{2}|T|^2+\frac{\lambda_T}{2}|T|^4+\frac{\lambda_{HT}}{2}|T|^2|\Phi|^2,\ \ \langle T\rangle=0,\ \langle\Phi\rangle=v_{\text{SM}}$$





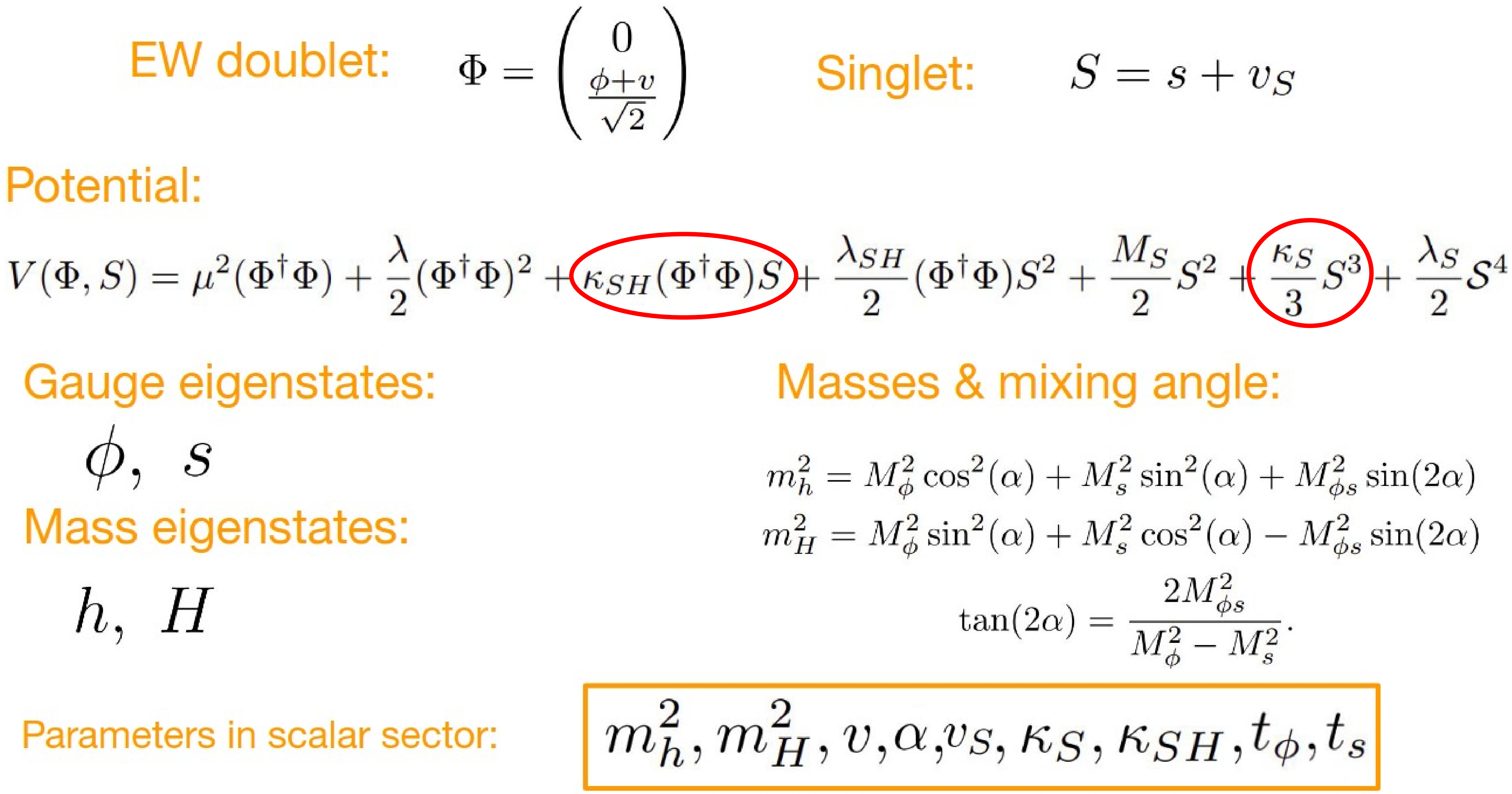
[JB, Egle, Verduras Schaeidt *WIP*]

- *Left*: κ_λ @ 1L in plane of $M_{H\pm}$ and λ_{HT} (portal coupling) with `anyH3`
- *Right*: κ_λ @ 2L, with results from [JB, Egle, Verduras Schaeidt *WIP*]

Renormalisation of Lagrangian trilinear couplings

Taking RxSM (singlet extension of SM) as example

Slide by A. Verduras Schaeidt



EW doublet: $\Phi = \begin{pmatrix} 0 \\ \frac{\phi+v}{\sqrt{2}} \end{pmatrix}$

Singlet: $S = s + v_S$

Potential:

$$V(\Phi, S) = \mu^2(\Phi^\dagger\Phi) + \frac{\lambda}{2}(\Phi^\dagger\Phi)^2 + \boxed{\kappa_{SH}(\Phi^\dagger\Phi)S} + \frac{\lambda_{SH}}{2}(\Phi^\dagger\Phi)S^2 + \frac{M_S}{2}S^2 + \boxed{\frac{\kappa_S}{3}S^3} + \frac{\lambda_S}{2}S^4$$

Gauge eigenstates:

$\phi, \ s$

Mass eigenstates:

$h, \ H$

Masses & mixing angle:

$$m_h^2 = M_\phi^2\cos^2(\alpha) + M_s^2\sin^2(\alpha) + M_{\phi s}^2\sin(2\alpha)$$

$$m_H^2 = M_\phi^2\sin^2(\alpha) + M_s^2\cos^2(\alpha) - M_{\phi s}^2\sin(2\alpha)$$

$$\tan(2\alpha) = \frac{2M_{\phi s}^2}{M_\phi^2 - M_s^2}.$$

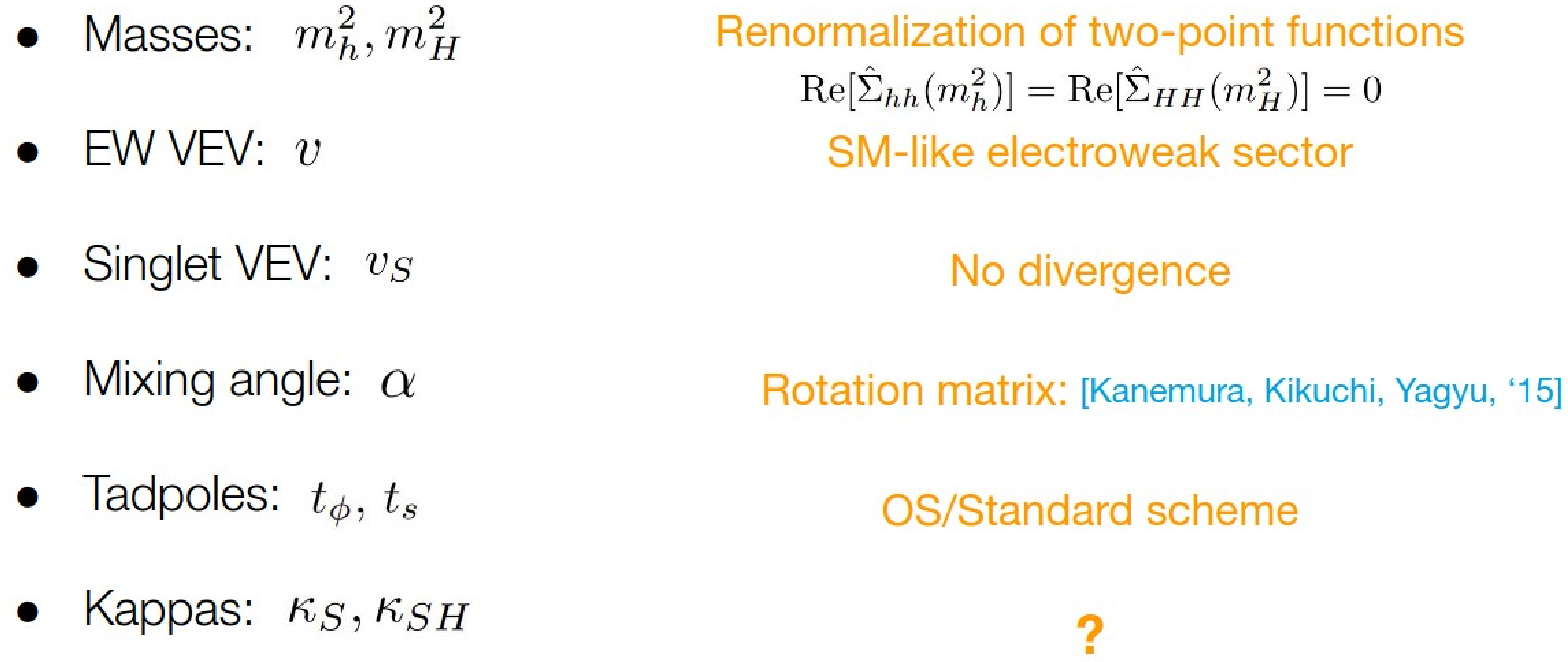
Parameters in scalar sector: $\boxed{m_h^2, m_H^2, v, \alpha, v_S, \kappa_S, \kappa_{SH}, t_\phi, t_s}$

Renormalisation of Lagrangian trilinear couplings

Slide by A. Verduras Schaeidt

On-shell renormalisation of RxSM

[JB, Heinemeyer, Verduras Schaeidt *WIP*]
+ talk by A. Verduras Schaeidt yesterday



Masses: m_h^2, m_H^2	Renormalization of two-point functions $\mathrm{Re}[\hat{\Sigma}_{hh}(m_h^2)] = \mathrm{Re}[\hat{\Sigma}_{HH}(m_H^2)] = 0$
EW VEV: v	SM-like electroweak sector
Singlet VEV: v_S	No divergence
Mixing angle: α	Rotation matrix: [Kanemura, Kikuchi, Yagyu, ‘15]
Tadpoles: t_ϕ, t_s	OS/Standard scheme
Kappas: κ_S, κ_{SH}	?

Renormalisation of Lagrangian trilinear couplings

Slide by A. Verduras Schaeidt

[JB, Heinemeyer, Verduras Schaeidt *WIP*]

On-shell renormalisation of RxSM

Our choice of renormalization conditions:

$$\boxed{\hat{\lambda}^{(1)}_{hHH} \overset{!}{=} \lambda^{(0)}_{hHH} \qquad \hat{\lambda}^{(1)}_{HHH} \overset{!}{=} \lambda^{(0)}_{HHH}}$$

$$\underbrace{\lambda^{(0)}_{hHH}} + \underbrace{\delta\lambda^{(1)}_{hHH}} + \underbrace{\delta\lambda^{m^2}_{hHH} + \delta\lambda^{v}_{hHH} + \delta\lambda^{tad}_{hHH} + \delta\lambda^{wfr}_{hHH} + \delta\kappa_S^{\text{CT}}\frac{\partial\lambda^{(0)}_{hHH}}{\partial\kappa_S} + \delta\kappa_{SH}^{\text{CT}}\frac{\partial\lambda^{(0)}_{hHH}}{\partial\kappa_S H}} = \lambda^{(0)}_{hHH}$$

Tree level | **Genuine one-loop contribution** | **Contribution from renormalization of different parameters and WFR**

$$\overbrace{\lambda^{(0)}_{HHH}} + \overbrace{\delta\lambda^{(1)}_{HHH}} + \overbrace{\delta\lambda^{m^2}_{HHH} + \delta\lambda^{v}_{HHH} + \delta\lambda^{tad}_{HHH} + \delta\lambda^{wfr}_{HHH} + \delta\kappa_S^{\text{CT}}\frac{\partial\lambda^{(0)}_{HHH}}{\partial\kappa_S} + \delta\kappa_{SH}^{\text{CT}}\frac{\partial\lambda^{(0)}_{HHH}}{\partial\kappa_S H}} = \lambda^{(0)}_{HHH}$$

Equations can be inverted, to obtain CTs for κ_S and κ_{SH}:

$$\delta\kappa_S^{\text{CT}} = \frac{\frac{\partial\lambda^{(0)}_{HHH}}{\partial\kappa_{SH}}\left(\delta\lambda^{(1)}_{hHH} + \sum\delta\lambda^{i}_{hHH}\right) - \frac{\partial\lambda^{(0)}_{hHH}}{\partial\kappa_{SH}}\left(\delta\lambda^{(1)}_{HHH} + \sum\delta\lambda^{i}_{HHH}\right)}{\frac{\partial\lambda^{(0)}_{hHH}}{\partial\kappa_{SH}}\frac{\partial\lambda^{(0)}_{HHH}}{\partial\kappa_S} - \frac{\partial\lambda^{(0)}_{hHH}}{\partial\kappa_S}\frac{\partial\lambda^{(0)}_{HHH}}{\partial\kappa_{SH}}}$$

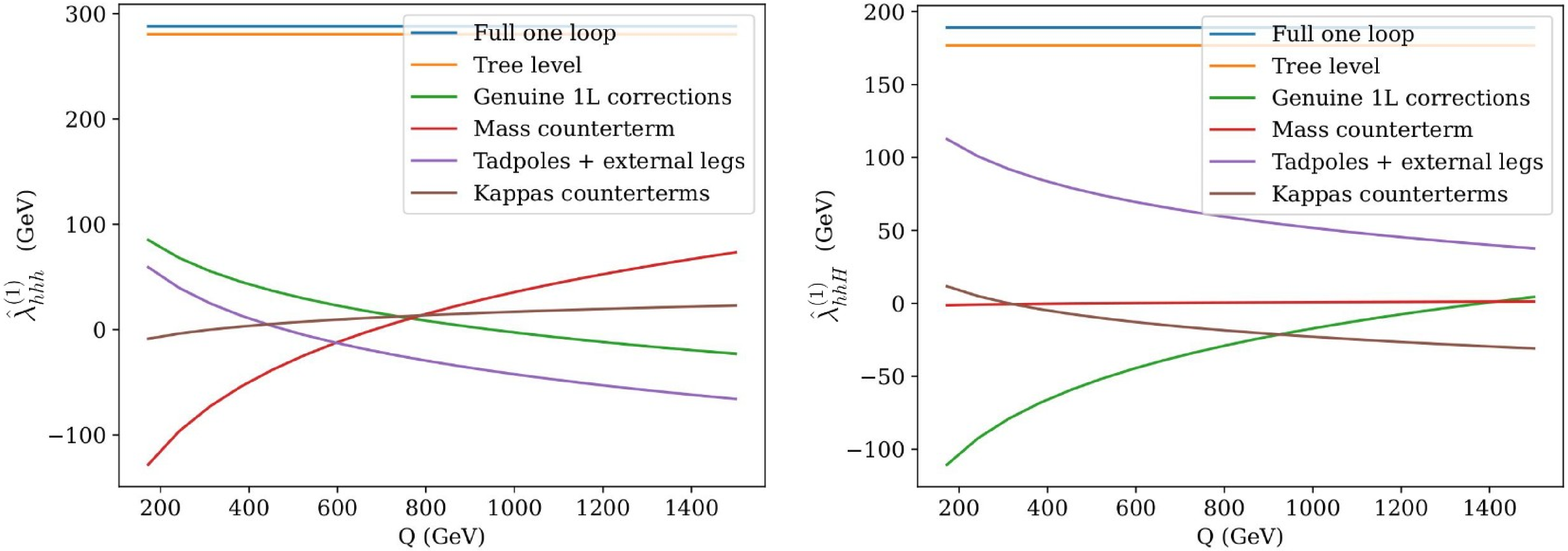
$$\delta\kappa_{SH}^{\text{CT}} = \frac{\frac{\partial\lambda^{(0)}_{hHH}}{\partial\kappa_S}\left(\delta\lambda^{(1)}_{HHH} + \sum\delta\lambda^{i}_{HHH}\right) - \frac{\partial\lambda^{(0)}_{HHH}}{\partial\kappa_S}\left(\delta\lambda^{(1)}_{hHH} + \sum\delta\lambda^{i}_{hHH}\right)}{\frac{\partial\lambda^{(0)}_{hHH}}{\partial\kappa_{SH}}\frac{\partial\lambda^{(0)}_{HHH}}{\partial\kappa_S} - \frac{\partial\lambda^{(0)}_{hHH}}{\partial\kappa_S}\frac{\partial\lambda^{(0)}_{HHH}}{\partial\kappa_{SH}}}$$

Renormalisation of Lagrangian trilinear couplings

Slide by A. Verduras Schaeidt

[JB, Heinemeyer,
Verduras Schaeidt *WIP*]

On-shell renormalisation of RxSM



- Predictions for trilinear scalar couplings λ_{hhh} and λ_{hhH}, *independent of renormalisation scale*, in this full OS scheme

→ in turn used for computing di-Higgs production cross-section (c.f. talk by A. Verduras Schaeidt yesterday)

- Calculations of λ_{ijk} (and CTs) performed with `anyH3` [Bahl, JB, Gabelmann, Weiglein ‘23], [Bahl, JB, Gabelmann, Radchenko, Weiglein *WIP*] + talk by M. Gabelmann this afternoon

What renormalisation scheme to choose?

[Heinemeyer, von der Pahlen '23]

- **Chargino/neutralino sector of MSSM**
  → 6 particles $\tilde{\chi}^{\pm}_{1,2}$, $\tilde{\chi}^0_{1,2,3,4}$
  → but only 3 masses can be renormalised OS at the same time

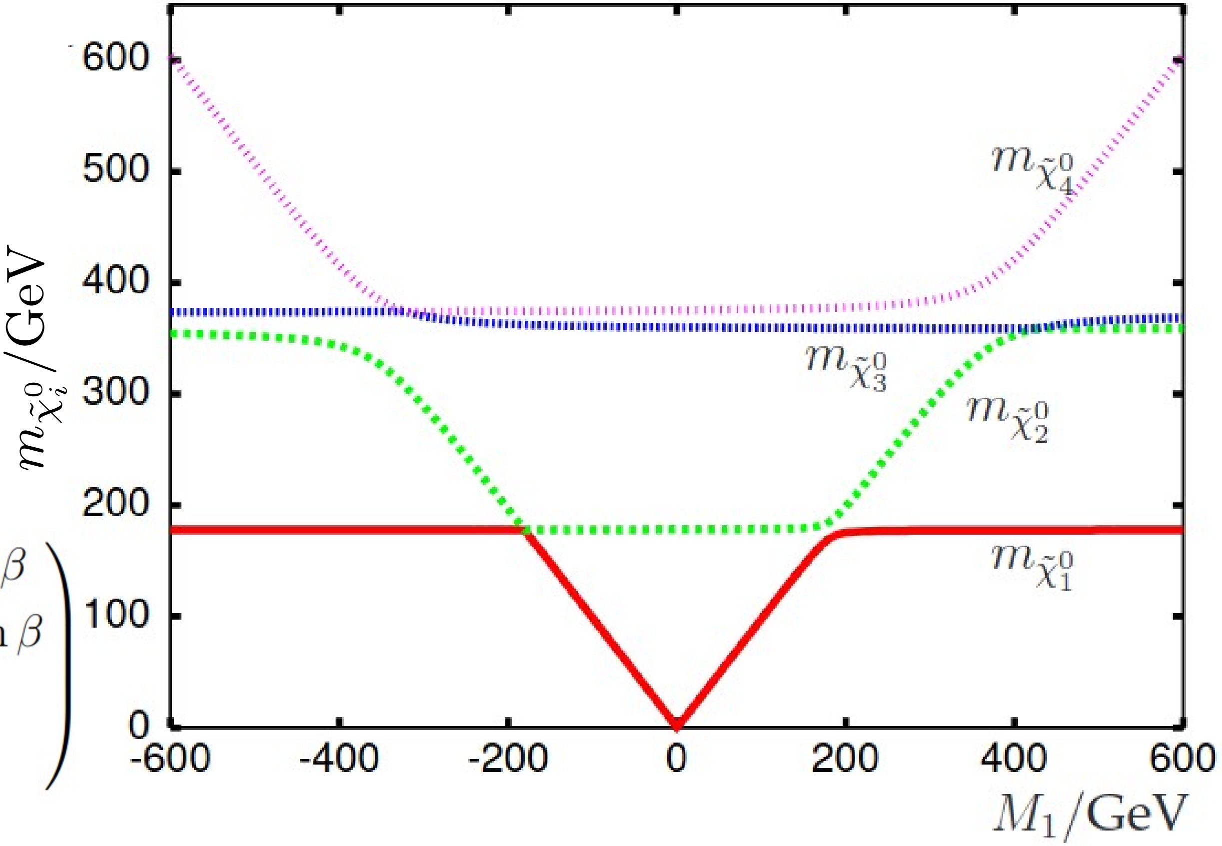
- Mass matrices

$$\mathcal{M}_C = \begin{pmatrix} M_2 & \sqrt{2}\sin\beta\, M_W \\ \sqrt{2}\cos\beta\, M_W & \mu \end{pmatrix}$$

$$\mathcal{M}_N = \begin{pmatrix} M_1 & 0 & -M_Z\, s_w\cos\beta & M_Z\, s_w \sin\beta \\ 0 & M_2 & M_Z\, c_w \cos\beta & -M_Z\, c_w\sin\beta \\ -M_Z\, s_w\cos\beta & M_Z\, c_w\cos\beta & 0 & -\mu \\ M_Z\, s_w\sin\beta & -M_Z\, c_w\sin\beta & -\mu & 0 \end{pmatrix}$$

μ: Higgsino mass term

M_1, M_2: gaugino (bino and wino) mass terms

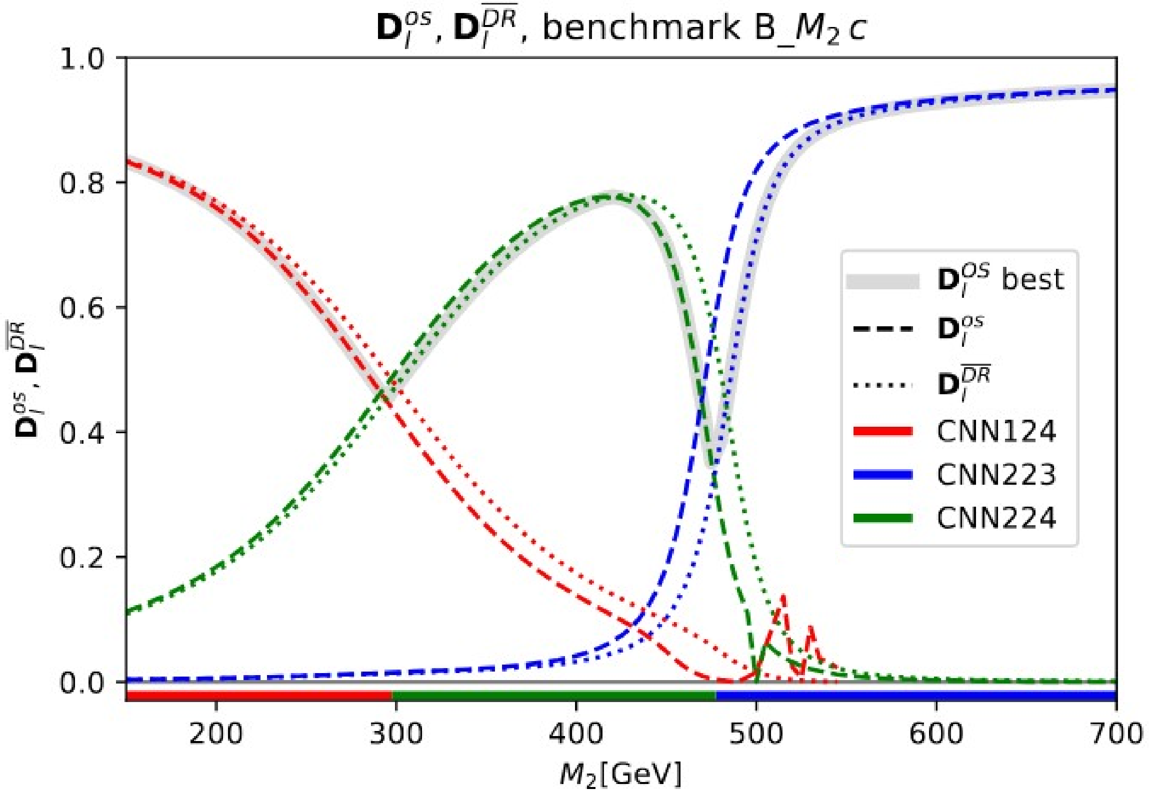


Plot from [Desch, Kalinowski, Moortgat-Pick, Nojiri, Polesello '03]

What renormalisation scheme to choose?

[Heinemeyer, von der Pahlen ‘23]

- **Metric to decide “most appropriate” scheme**: scheme that maximises dependence on underlying parameters of the theory
- CNNijk ≡ $m_{\tilde{\chi}_i^\pm},\ m_{\tilde{\chi}_j^0},\ m_{\tilde{\chi}_k^0}$ are renormalised OS



$$M_1 = 500\ \text{GeV},\ M_2 \in [150, 700]\ \text{GeV},\ \mu = 300\ \text{GeV},\ \tan\beta = 10$$