\sim **Symmetry and Dark matter candidates M. N. Rebelo CFTP/IST, U. Lisboa Complex** *S***3-symmetric 3HDM A. Kunčinas,***^a* **O.M. Ogreid,***^b* **P. Osland***^c* **and M.N. Rebelo***^a*

Cooler Cootore from a **Extended Scalar Sectors from all Angles** *Instituto Superior Técnico, Universidade de Lisboa, Avenida Rovisco Pais nr. 1, 1049-001 Lisboa, Portugal*

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07 (2023) 013 e-Print: 2302.07210 Lhep-JHEP 07 (2023) 013 e-Print: 2302.07210 [hep-ph]

Work under way in collaboration with: *^cDepartment of Physics and Technology, University of Bergen,* D. Emmanuel-Costa, O. M. Ogreid, P. Osland Work done in collaboration with A. Kunčinas, O. M. Ogreid and P. Osland,

Work Partially supported by:

FCT Fundação para a Ciência e a Tecnologia MINISTÉRIO DA EDUCAÇÃO E CIÊNCIA

European Union

QUADRO DE REFERÊNCIA **ESTRATÉGICO** NACIONAL ORTUGAL 2007.2013

Motivation for three Higgs doublets

Why not more? Three fermion generations may suggest three doublets

New sources of CP violation in the scalar sector

-
- Possibility of having a discrete symmetry and still have CP violation, explicit or
	-
	-

spontaneous

Rich phenomenology, including DM candidates

- Motivation for imposing discrete symmetries
- Symmetries reduce the number of free parameters leading to (testable) predictions
- Symmetries help control HFCNC (e.g. NFC or MFV suppression in BGL models)

Symmetries are needed to stabilise DM

Our work

We discuss a three-Higgs-doublet model with an underlying $S_3\;$ symmetry allowing in principle for complex couplings

specifying whether it can be explicit or spontaneous

without soft symmetry breaking terms

- S_3 symmetry to have either spontaneous or explicit C violation in the scalar sector, depending on the scalar sector, depending on \mathcal{C}
- the regions of parameter space corresponding to the different possible vacua of the *S*³ We list all possible vacuum structures allowing for CP violation in the scalar sector ϵ and existence of a sufficient condition for ϵ to be explicit the best discussed. We discuss a three-Higgs-doublet model with an underlying model with an underlying ϵ
- This classification is based strictly on the exact S_3 -symmetric scalar potential S_3 -symmetric scalar potential -symmetric scalar potential
- vacua with inprivations $t_{\rm eff}$ spontaneous or explicit $C_{\rm eff}$ violation in the scalar sector, depending on the scalar sector, depending on \sim the regions of parameter space corresponding to the different possible vacua of the *S*³ Different regions of parameter space correspond to different vacua with implications
- Maskawa matrix is complex. In order to understand what are the possible sources of CP taneously for special couplings was studied. In that case violated spontaneously for special In a previous work the scalar potential with real couplings was studied. In that case CP was explicitly conserved and could only be violated spontaneously for special
	- Osland, M. N. R, 2016
. el-Costa, Ogreid, Osland, M. N. R, 2016
. Emmanuel-Costa, Ogreid, Osland, M. N. R, 2016

that are outlined in our work

vacua, which we identified

The Scalar potential

S₃ is the permutation group involving three objects, ϕ_1, ϕ_2, ϕ_3

here all fields appear on equal footing this representation is not irreducible, for instance, the combination $\phi_1 + \phi_2 + \phi_3$ real. In the complete of multi-Higgs extensions of the Standard Model in the Standard Model in the Standard Mo
Standard Model in positive of the Standard Model in positive of the Standard Model in the Standard Model in th t econtotion c

doublet and singlet:

remains invariant, it splits into two irreducible representations,

Derman, 1979

$$
+\frac{1}{2}E_3[(\phi_i^{\dagger}\phi_i)(\phi_k^{\dagger}\phi_j)+\text{hc}]+\frac{1}{2}E_4[(\phi_i^{\dagger})
$$

$$
\left\langle\begin{array}{c} h_1 \\ h_2 \end{array}\right\rangle
$$

 $\{\phi_j\}(\phi_i^{\dagger}\phi_k) + \text{hc}]\}$

$$
\Big), \quad h_S \qquad \qquad \text{of} \ \ S_3
$$

 p_{e} p_{e} p_{e} Derman, 1979 to be explicitly conserved. We discuss a three-Higgs-doublet model with an underlying S_3

$$
V_2 = -\lambda \sum_{i} \phi_i^{\dagger} \phi_i + \frac{1}{2} \gamma \sum_{i < j} [\phi_i^{\dagger} \phi_j + \text{hc}]
$$
\n
$$
V_4 = A \sum_{i} (\phi_i^{\dagger} \phi_i)^2 + \sum_{i < j} \{ C(\phi_i^{\dagger} \phi_i) (\phi_j^{\dagger} \phi_j) + \bar{C}(\phi_i^{\dagger} \phi_j) (\phi_j^{\dagger} \phi_i) + \frac{1}{2} D[(\phi_i^{\dagger} \phi_j)^2 + \text{hc}] \}
$$
\n
$$
+ \frac{1}{2} E_1 \sum_{i \neq j} [(\phi_i^{\dagger} \phi_i) (\phi_i^{\dagger} \phi_j) + \text{hc}] + \sum_{i \neq j \neq k \neq i, j < k} \{ \frac{1}{2} E_2[(\phi_i^{\dagger} \phi_j) (\phi_k^{\dagger} \phi_i) + \text{hc}]
$$

1 2 1 2 Xİl $\begin{array}{c} \hline \end{array}$ $\Delta=$ $\sqrt{2}$ \blacksquare 111 111 \setminus A $\frac{7}{7}$ $\overline{}$ $1, 1, 1$ uuni 111 \setminus $\overline{}$ Harrison, Perkins and Scott, 1999 and Scott 1999 S_3

$$
\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}
$$

Decomposition into these two irreducible representations **Decomposition into these two irreducible representations** *E-mail:* Anton.Kuncinas@tecnico.ulisboa.pt, omo@hvl.no, Per.Osland@uib.no, rebelo@tecnico.ulisboa.pt

This definition does not treat equally ϕ_1, ϕ_2, ϕ_3 they could be interchanged e interchanged occurs in the framework of the electroweak symmetry breaking whenever the Lagrangian

explicitly **SCalar, potential in the single** $V_{\rm F}$ = 24. (*h*₁ + 22). Online of 200 2110 Chenged 1 $\frac{1}{10}$ $\tilde{\mathbf{f}}$ *λ*4 $\sqrt{\frac{1}{2}}$ $\frac{\partial}{\partial x} \left(\frac{\partial}{\partial y} \right) \left(\frac{\partial}{\partial z} \right) \left(\frac{\partial}{\partial z} \right) \left(\frac{\partial}{\partial x} \right) \left(\frac{\partial}{\partial z} \right) \$ $\{A_{5}^{2}A_{6}^{3}A_{8}^{5}h_{8}^{7}h_{8}^{7}\}\{h_{1}^{7}h_{1}^{+}\}_{h_{2}^{1}h_{2}^{3}\}\}^{27}$ $+\frac{1}{\lambda}$ \int λ \int $\frac{1}{2}$ *PERMITTED SCALAR PRATCARED** *S*3-symmetric potential can be written as [62–64]: Sh^T ch^T h^T h^T h^T h^T h^T h^T h^T $\frac{1}{2} \sum_{k=1}^{n} \left(\frac{\lambda_{5}}{h_{5}^{2}} \right) \left(\frac{h_{1}^{2}}{h_{1}^{2}} \right) \left(\frac{h_{1}^{2}}{h_{1}^{2}} \right) + \left(\frac{h_{2}^{2}}{h_{2}^{2}} \right) \left(\frac{h_{2}^{2}}{h_{1}^{2}} \right)$ $\lambda_{R}^{(n)}(h_{1}^{n}) + \lambda_{R}^{(n)}(h_{1}^{n})$ $\overline{\lambda}$ $\bigg\{$ λ_S^7 \mathbf{z} 1) (bⁱn₁)+ n₅ (b₁) + + (b₁⁰, b₂)(b₁) + h₂) = h₂)² + − h22) + > $\begin{bmatrix} \text{arg} & \text{arg$ w ($\sin g$ $\left[e f \right/ \sin d (\lambda \cot h) \sinh^2 \lambda + v^2$ μ is an are the μ and μ the set μ μ and μ the set of μ so the μ of μ and σ classified σ previously for the real potential by allowing for complex coefficients. en defined **Progress his attended to the choice of the choice of the contract * $\lim_{\Delta z \to 0} \lim_{\Delta z \to 0}$ $\int \sin^2\theta \sin^2\theta$ $XM_S^2X^T$ $\qquad \qquad \underline{\Phi}_1, \qquad \underline{\Phi}_2, \ \underline{\Phi}_3$ S_3 w($\sin g$ let $/$) and λ doublet the potential can be written cas $1 \begin{bmatrix} \text{L IIETC} & \text{dge} & \text{U4O} & \text{C99} & \text{DIII1188} \\ \text{(1)} & \text{I2} & \text{I31} & \text{C2} & \text{I4} \end{bmatrix}$ 2 _a(gg ?hoted₂In2)[36], (18] grven⁷ry1) ¹*h*¹ *h†* ²*h*2) + h.c.] + 5(*h† AhA*)(*h†* ¹*h*¹ + *h†* ²*h*2) ²*hA*)] + 7[(*h† Ah*1)(*h† Ah*1)+(*h† Ah*2)(*h† ^Ah*2) + h.c.] $A_{\text{A}} + A_{\text{B}} + A_{\text{B}}$ of the \mathcal{B}_3 singless and detailed in the representation of written as (278) , $4, 5, 6, 7$ he *p*otential in terms of the S₃ singlet and doublet If the \mathcal{B}_3 singlessmet.doublet fields, the potential can be written as $(2, 3)$, $4, 5, 6, 7$ $+\frac{\cos \beta}{4}h^{\dagger}\hbar \frac{\sin \beta}{4}h^{\dagger}\hat{u}^2$ $\frac{1}{2}h^{\dagger}\hbar \frac{\sin \beta}{4}h^{\dagger}\frac{\sin \beta}{2}$ $\pm 2\lambda 6\sqrt{h^{\dagger}h^{\dagger$ $\frac{1}{2} \frac{\lambda_7}{\lambda_8} \left[(h_S^{\dagger} h_1^{\dagger}) (h_S^{\dagger} h_1^{\dagger}) + (h_S^{\dagger} h_2^{\dagger}) (h_S^{\dagger} h_2^{\dagger}) + \text{h.c.} \right]$ $+\lambda_4[(\Lambda\bar{\zeta}_H\Lambda\gamma_0\bar{s}_1^{\gamma}\eta_4\chi_1^{\gamma}\eta_2^{\gamma}\eta_4\chi_1+(\lambda\gamma_5\bar{s}_2^{\gamma}\eta_4\chi_1^{\gamma}\eta_2^{\alpha}\eta_1^{\gamma}\eta_2^{\gamma}\eta_2^{\gamma}\eta_3^{\gamma}+\gamma^2\lambda_8\lambda_2^{\alpha}\eta_3^{\gamma}\eta_2\chi_3^{\gamma})$ $\overline{1}$ $2\widetilde{v}_{\bf s}$ ⇥ 28*v*˜³ *^S* (⁵ ⁺ ⁶ + 27)(˜*v*² ¹ + ˜*v*² ²)˜*v^S* + 4(˜*v*² ² 3˜*v*² ¹)˜*v*² $\lim_{\tilde{U}_{\infty}}$ **plays a special tole** \tilde{w}^2 $+\tilde{w}^2$ \tilde{p} \tilde{x} $+\tilde{\chi}^2$ $(\tilde{v}^2 - 3\tilde{v}^2) \tilde{v}$ ₂, \tilde{v} doublet representation, 1 $\Omega_{\!\scriptscriptstyle\beta}$ $\frac{1}{2}$ $\int_{\mathcal{B}} \frac{1}{\sqrt{2}} \int_{\mathcal{B}} \frac{1}{\sqrt{2}} \int_{\mathcal{C}} \frac{1}{\sqrt{2}} \$ \vec{d} $2\tilde{\psi}$ $\overline{\mathbf{b}}$ (⁵ ⁺ ⁶ + 27)˜*v*2*v*˜² *^S* 2(¹ ⁺ 3)˜(˜*v*² ¹ + ˜*v*² ²)˜*v*² + 34(˜*v*² ² *^v*˜² ¹)˜*v^S* Stab following $\mathscr{D}(62)$ awdure \mathscr{D}^{max} (2.21b) at ically consistent. $\lim\!\!\!\!\mathrm{if}\ \sum\limits_{\ell=1}^{\infty}$: χ_1 λ a $\mathcal{L}_S^{\mathcal{L}}$ two couplings, λ_A and λ_7 , that could be complex_{ed} <u>.</u>
रेने W nitten h as h^3 at 351 : *MAI 14194* **B** 1 *†* ¹¹ + *†* ²² $\sum_{n=1}^{\infty} \frac{1}{3}$ **hare moted in [37], as given by Party COUDHILES ARE TO DE FOALIQUOP BY Now the there is supply the symmetry for the symmet** $\mathbf{F}(\mathbf{u}) = \mathbf{F}(\mathbf{u})$ NH STANHYCH \mathcal{A} \mathcal{A} \mathcal{B} $\mathcal{$ $\text{HC}(A)$ With magnetic burned A_{A} carded algregis λ A_3 singless and download along the vacuum in the asymptomy initial A . m^2 ± 2 $(\lambda_2 + \lambda_3)$ sin $\beta + \lambda_7$ cbs² β ¹ v^2 which one poethold becomes $(17b)$ $\{\langle \lambda_2 + \lambda_3 \rangle \sin^2 \beta + \lambda_7 \cos^2 \beta \rangle \}$ SS Goldsteiner Gunus the standard as follows of the master *A*² ◆ = ✓ $COS \left(\right) \right)$ $\sin 4 \sqrt{4} \cos \beta$ **入场名** *z*3 \mathbf{Q}^{\prime} w_1 w_2 w_3 w_4 w_2 w_3 w_2 w_3 w_4 w_2 w_3 w_4 w_5 w_6 w_7 $\frac{a_{2}}{A_{2}} = \frac{\pm_{2} \lambda_{5} (n_{1} \lambda_{5} n_{1}) (n_{1} \lambda_{5} n_{1} + (n_{2} \lambda_{2}) (n_{2} \lambda_{5}))}{\pm_{2} \lambda_{5} (n_{2} \lambda_{5} n_{2}) (n_{1} \lambda_{1} n_{2} + n_{2} \lambda_{2} n_{3})} + \lambda_{6} (n_{2} \lambda_{1}) (n_{1} \lambda_{2} n_{2} + n_{2} \lambda_{3} n_{3})$ art we have : \int $(05^{\prime\prime}_{1}\frac{\textit{p}}{\text{p}}\frac{\text{p}}{\text{p}})+\frac{\textit{p}}{\text{p}}\frac{\textit{p}}{\text{p}}$ \mathcal{P}_S $-B'_S$ **X** -*B*⁰ **S 2 SDE** $\frac{1}{2}$ *v*3 \mathbf{q} $\frac{1}{2}$ $\frac{2}{\sqrt{2}}$ $\frac{1}{\sqrt{2}}$ \frac **ht 2 Frien Messing Certify Nd 2 Day Xor Hot Hins Set 381 (85 HHM) Risht** he appearance of a massless scalar is hot surprising. One can easily vehity that Stth fph potential $\mathcal{G}(162)$ symmetry $\mathcal{G}(12)$ fph plays a special rôle

The state wand take a potential in the second them to by year so that CP symmetry is representation α of ential in terms of the S_2 singlet and doublet represented the singlet representation. In this case the case the case of the single $V_{2}^{n_{A+1}}$ suffit $\frac{1}{2}$ (λ ₂ + λ) sin² β + λ $\frac{1}{2}$ ($\frac{1}{2}$) $\frac{1}{2}$ the $\frac{1}{2}$ $\frac{1}{2$ $\frac{1}{2}$ *d* $\frac{1}{2}$ ^h $\frac{1}{2}$ hb $\frac{1}{2}$ 2 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ hbilitty h_2 h_1 e) $2\overline{s}$ $\left.\right\}$ + h*.*c*.* $\overline{1}$ $\big]_\mathsf{L}$ + h*.*c*.* \int_0^1 $\rightarrow \lambda_8 (h \frac{1}{5} h \frac{1}{25})$ 2 *.* acN or \mathbb{R}^5 and \mathbb{R}^5 and \mathbb{R}^5 in the sum of \mathbb{R}^5 and \mathbb $\begin{bmatrix} S_3 \text{w} \text{ (sine)} \text{ (s)}' \text{ and } \text{ (dothable)} \text{ (}4_1^2 + v_2^2 \text{)}, & \lambda_1 + \lambda_3 + \lambda_5 + \lambda_6 + 2\lambda_7 + \lambda_8 \end{bmatrix} \times \begin{bmatrix} 192 |\lambda_4| \text{ (4g)} \end{bmatrix}$ \mathcal{A} There are two couplings, λ_4 and λ_7 , that could be complex_{ed} Hence, CP symmetry can be L'RONS Das and Dey, 20^(4d) $V_2^{\hat{n}^2} = \frac{1}{2} \left(\lambda_2 + 2 \lambda_3 + \lambda_4 \lambda_5 \right)$ $\sin^2 \beta_1 + \lambda_7 \cos^2 \beta_1 \cos^2 \beta_2 \cos \theta$ is the context of two Higgs-doublet models (2HDMs) [32]. For the defined potograpor singlet and a doublet and country seasonal perception of the season of the second section of the second $\frac{1}{2}$ $V = \int_{-\infty}^{\infty} \frac{\sin^2(\sqrt{2}}{\sin(\sqrt{2})} \int_0^1 \frac{1}{2} \int_0^$ $\sum_{i=1}^{n} \sum_{j=1}^{n} \binom{n}{j} \binom{n}{j} \binom{n}{j}$ $\overline{\lambda}$ $\frac{1}{5}$ (2) **3** *.* There are two $\zeta_8 \bar{\zeta}_8$ plings, λ_4 and λ_7 , that could be complex_s. Hence, CP symmetry can be $b^{\pm}_{\rm eff}$ is \pm explicitly. The settles coupling \mathbb{R} \mathbb{R} of \mathbb{R} and \mathbb{R} the hermiticity of the potential. Andel and Algem (weither the Uniteratury of the graps at the street Identals to the graphic repart of the seates potenti the whitteh as $b^3 + 35 + 35$ is a potential by allowing for complex coefficients. en defined in the context of the second of the context of two rings abused models (211DMs) [02]. To the
en defined in the second of starting to the enarged part energies in stige stise difpsered of call the second coupled idst<u>onder on the standard and the top</u> of the thellowing *(the send conditions for hb*e should be hold it will be asymptotic $\overline{\mathcal{L}}$.
h 1212260 ac**Nonsymmitetry sor the interinter change of the avating post is 2|** λ_7 **|,** (4f) The general property of the complex, but we assume them to be real so that CP symmetry is not broken explicitly. For the stability of the vacuum in the asymptotic limit we under the requirement that there should be no direction in the field space along which the potential becomes infinitely negative. The necessary and $\overline{\lambda}_1^{\text{L}}$ $\overline{\lambda}_2^{\text{L}}$ $\overline{\lambda}_3^{\text{L}}$ $\overline{\lambda}_4^{\text{L}}$ $\overline{\lambda}_5^{\text{L}}$ $\overline{\lambda}_6^{\text{L}}$ $\overline{\lambda}_7^{\text{L}}$ $\overline{\lambda}_8^{\text{L}}$ $\overline{\lambda}_7^{\text{L}}$ $\overline{\lambda}_8^{\text{L}}$ $\overline{\lambda}_7^{\text{L}}$ $\overline{\lambda}_8^{\text{L}}$ $\overline{\lambda}_9^{\text{L}}$ $\overline{\lambda}_9^{\text{L$ $\lambda_{8\perp}$ λ_{4}^{180} , λ_{6}^{1} , λ_{7}^{1} , λ_{8}^{1} (4b) $\lambda_1 + \lambda_3 > 0, \lambda' > 0$ (4c) $\frac{1}{2} \left(\frac{1}{2} \frac{1}{2} \frac{1}{2} \left(\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \right) + \frac{1}{2} \left(\frac{1}{2} \frac{1$ $\lambda_5 + 2\sqrt{\lambda_8(\lambda_1 + \lambda_3)} \quad \text{(4e)}$ $\sqrt[3]{2}$ is $\frac{1}{2}$ and $\frac{1}{2}$ $,\phi$ _p $\left(\frac{2}{3}, \frac{21}{9}\right)$ city of the notantial (5) has also been westing the literature with terms of this men doublet, the mention part of the scalar potential is \mathbf{v}^{\intercal} GONSIS \mathbf{f} GA λ^2 $\hat{\beta}_1 + \beta_2(\gamma^{\dagger} \gamma_2)(\gamma^{\dagger} \gamma_1) + \frac{\beta_4}{\gamma_1}$ $^{24} (d_0^{\dagger} d_2)^2$ $\lambda_5 + 2\sqrt{\lambda_8(\lambda_1 + \lambda_3)} \geq (198)$ λ_2)[,] $\lambda_1 + \lambda_3 + \lambda_5 + \lambda_6 + 2\lambda_7 + \lambda_8$ ≤ 19

Choice of a suitable basis for the analysis of the complex scalar potential + *λ*5(*h† ShS*)(*h†* ¹*h*¹ + *h†* ²*h*2) + *λ*6[(*h† Sh*1)(*h†* 1*hS*)+(*h† Sh*2)(*h†* ²*hS*)] 1919 <mark>|</mark> (*h† Sh*1)(*h† Sh*1)+(*h† Sh*2)(*h†* \blacksquare + h*.*c*.* $\ddot{}$ + *λ*8(*h† ^ShS*) $\overline{}$ Choice of a suitable basis for the analysis of the complex scalar potenti + *|λ*4*|* $\overline{}$ *^Sh*¹ ²*h*¹ "& (*i*didi + *ei*(*θ*2+*α*4) *h† ^Sh*¹ *h†* ¹*h*² + *h† ^Sh*² *h†* ¹*h*¹ − *h†* λ [†] ¹*h*² + *h† ^Sh*² " !*h† hacis* for 1 **M**

- The most general approach of allowing for λ_4 and λ_7 to be complex together with two vacuum phases would yield redundant solutions words the potential. most general approach of allowing for λ_4 and λ_7 to be complex together with two vacuum phases would yield redundant solutions st gerieral approach of allowing for λ_4 $\frac{1}{2}$ + $\frac{1}{2}$ **.** *h† ^Sh*¹ d vield red :
: **ndant solution** $\frac{1}{2}$ and $\frac{1}{2}$ to be complex together with $\frac{1}{2}$ and $\frac{1}{2}$ to be complex together with *e*^{*i*}(2*θ*₁₊*αλ* **Here, we see that the scalar potential is sensitive to different** *b* **phases of eq. (2.3). In part** *b**is a* **phase software** θ **and** θ **an** *h† ^Sh*² " !*h†* ¹*h*¹ − *h†* ²*h*² ould $\begin{bmatrix} 74 & 011 \\ 1 & 11 \end{bmatrix}$
- Another option with real veve and complex couplings through: $i\theta_{i,1}$ is not unitary transformation into the defining representation of θ In principle we could consider a basis with real vevs and complex couplings through: rotation of two of
The SU(2) doublets, two of $h_i = e^{i\theta_i}h'_i$. i , $i = \{1, 2\}$ *.* Here, we could consider a pasis with real vevs and complex couplings in philophogy $h_i = e^{i v_i} h_i, \quad i = \{1, 2\}.$ der a basis with real vevs and complex couplings through: $a_i = e^{i\theta_i}h'$, $i = \{1, 2\}$ Scal

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ve Hig Here, we see that the scalar potential is sensitive to different *θⁱ* phases of eq. (2.3). In principle we could consider a basis with real vevs and complex couplings of the couplings of the present complex couplings of the present complex coupling of the present coupling of the present complex coupling of the pres $a_{ij} = e^{-\lambda} h_i, \quad i = \{1, 2\}.$ principle can consider a basis with real vers and complex consider a basis with relativished and complex consider above. We note that the sum (*λ*² + *λ*3) would then get a phase, while the form of the $n_i = e^{i\omega_i} n_i, \quad i = \{1, 2\}.$ $\begin{array}{c} \text{com} \ \text{be co} \ \text{com} \ \text{hasir} \ \text{or} \ \text{and} \ \text{es in} \ \text{secl} \end{array}$
- Due to the global U(1) symmetry the phase of the *S*³ singlet, *hS*, can always be rotated however, in this case $(\lambda_2 + \lambda_3)$ would get a phase and the potential would change form $\begin{array}{c} \sqrt{2} \end{array}$ such that the coupling should be real. This is due to a possible to a possi $\frac{1}{2}$ nowever, in this case $(2\pi/3)$ would get a phase and the potential would change form correta troute origing to simplify
- \overline{a} This can be avoided by choosing $\,\theta_1=\theta_2\equiv \theta\, \,\,$ in any rephasing of the Higgs double scalar potential (2.1) suggests that the real α potential be real. This is due to a possible This can be avoided by choosing $v_1-v_2=0$. In any rephasing of the Higgs This can be avoided by choosing $\,\theta_1=\theta_2\equiv\theta\,\,\,$ in any rephasing of the Higgs doublets α asing of the Higgs doublets This can be avoided by choosing $\theta_1 = \theta_2 \equiv \theta$ in any rephasing of the Higgs doublets
- *This phase can be chosen in such a way that either* λ_4 *or* λ_7 *become real* This phase can be chosen in such a way that either λ_4 or λ_7 become real *in such a way that either* λ_4 or λ_7 become real This phase can be chosen in such a way that either This phase can be chosen in such a way that either λ_4 or λ This phase can be chosen in such a way that either λ_4 or λ_7 become real
- of that in depend we are left with two vacuum phases and one complex coupling so that, in general, we are left with two vacuum phases a *A* each that in general, we are left with two vacuum phases and one complex cos real. Explicitly, this choice gives for the phase-dependent part, *so that, in general, we are left with two vacuum phases and one complex coupling* so that, in general, we are left with two vacuum phases and one complex coupling
- *<i>i* \blacksquare *i* = *ei<i>P* **=** *ei* = *eiready* analysed real. Explicitly, this choice gives for the phase-dependent part, $\mathbf C$ ⁼ *[|]λ*4*|ei*(*θ*+*α*4) ' ! aneous CP VI $\overline{}$ \mathfrak{r} *h* 2011 Mere airea $\overline{ }$ **y** *h†* yse *h†* " (**S** with "
ב \mathbf{r} ⁼ *[|]λ*4*|ei*(*θ*+*α*4) ph:
ana *h† ^Sh*¹ in : *h†* ²*h*¹ \overline{C} U_ľ li
I *h† ^Sh*¹ sin. *h†* ¹*h*² We are only interested in cases with non-vanishing phases in the couplings since the :h *V contribute the contribution* **JU** ⁼ *[|]λ*4*|ei*(*θ*+*α*4) We are only interested in cases with non-vanishing phases in the couplings since the
cases with spontaneous CP violation were already analysed cases with spontaneous CP violation were already analysed scalar potential in calar potential (2.1) suggests that the set of a post of a post of a post of a post of a p
Me are only interested in cases with non-vanish
- 10000 01 1101011 11010 011 0007 011017000
1 #! \$ λ_4 the only co ...
= *h† ^Sh*¹ $\overline{2}$ $\overline{\mathsf{H}}$ $\frac{1}{2}$ $\frac{1}{2}$.
a
a
k er than λ_7 " !*h†* ²*h*¹ \overline{or} *h† ^Sh*¹ *to ch* hoc $\frac{1}{2}$!
!
! \overline{S} μ ith λ λ_4 the only ⁺ *[|]λ*7*|ei*(2*θ*+*α*7) It is convenient to choose a basis with It is convenient to choose a basis with $\,\lambda_4\,$ the the only complex coefficient rather than

$$
,\quad i=\{1,2\}.
$$

Results obtained previously for the real potential

 $\lambda_a = \lambda_b$

| Varuum | ρ_1, ρ_2, ρ_3 | w_1, w_2, w_S | Comment |
|---------|--------------------------|-----------------------|--|
| R-0 | 0,0,0 | 0,0,0 | Not interesting |
| R-1-1 | x, x, x | 0,0, w_S | $\mu_0^2 = -\lambda_8 w_S^2$ |
| R-1-2a | $x, -x, 0$ | $w, 0, 0$ | $\mu_1^2 = -(\lambda_1 + \lambda_3) w_1^2$ |
| R-1-2b | $x, 0, -x$ | $w, \sqrt{3}w, 0$ | $\mu_1^2 = -\frac{4}{3}(\lambda_1 + \lambda_3) w_2^2$ |
| R-1-2c | 0, x, -x | $w, -\sqrt{3}w, 0$ | $\mu_1^2 = -\frac{4}{3}(\lambda_1 + \lambda_3) w_2^2$ |
| R-II-1a | x, x, y | 0, w, w_S | $\mu_0^2 = \frac{1}{2}\lambda_4 \frac{w_2^2}{2} - \frac{1}{2}\lambda_0 w_2^2 - \lambda_8 w_5^2$ |
| R-II-1b | x, y, x | $w, -w/\sqrt{3}, w_S$ | $\mu_1^2 = -(\lambda_1 + \lambda_3) w_2^2 + \frac{3}{2}\lambda_4 w_2 w_S - \frac{1}{2}\lambda_4 w_5^2$ |
| R-II-1b | y, x, x | $w, w/\sqrt{3}, w_S$ | $\mu_1^2 = -4(\lambda_1 + \lambda_3) w_2^2 - \lambda_8 w_5^2$ |

$$
\lambda_a = \lambda_5 + \lambda_6 + 2\lambda_7,
$$

\n
$$
\lambda_b = \lambda_5 + \lambda_6 - 2\lambda_7.
$$

Complex vacua $\frac{1}{2}$ \sim mploy \sim 0.000 \sim

ectiv
And Contraint
Contraints on the product of the same
And the product of the produc $\xi = \sqrt{-3 \sin 2\rho_1/\sin 2\rho_2}, \psi = \sqrt{[3 + 3 \cos(\rho_2 - 2\rho_1)]/(2 \cos \rho_2)}$. With the constraints of $\begin{array}{c} \nabla \sin 2p_1 / \sin 2p_2, \ y \rightarrow \nabla \cos p_2 \cos 2p_1 \sin 2p_2. \n\end{array}$
the vacual abelled with an asterisk $(*)$ are in fact real Table 4 the vacua labelled with an asterisk (∗) are in fact real. Table 4 the vacua labelled with an asterisk (∗) are in fact real. Table 2: Complex vacua. Notation: $\epsilon = 1$ and -1 for C-III-d and C-III-e, respectively;

$\sum_{i=1}^n a_i = 1$ respectively. Where two possible signs (± or ∓) are given, they correspond to those of **Constraints** on constraints on complex vacual with an assemble with a continued with

the Lagrangian is invariant under CP and it at the same time the same time the same time the same the same the
The Eagrangian is no transformation of the spontance we can be violated spontaneously. For the can inspect the conded transformation with transformation and the set of the set engines, in verificity. this case the most general Calision is given by: $\limsup_{n\rightarrow\infty}\frac{1}{n}$ list of complex solutions presented in Table 2. CP can only be spontaneously violated if he Laprangian is indicated green at the same time to the same time time to same the same the same time the same that can be it that that the internal can be identified with the Lagrangian and the Lagrangian and the Lagrangian vacuum persposed temput he he spontaneous companies of the invariant proposed to the perspect of the invariant Leand Cure ls uldem star poatured transformation of the scalar doublet and the scalar doublet and the scalar doublet and and the sector sector can be sector cannot be seen to separate CP. In the models with several Higgs down to th
The sector complex of the sector complex with several Higgs down to the sector complex of the sector of the se $\mathrm{inc}\mathrm{e}$ sthis aransformation leaves the kinetic energy term of ithe Lagran star invariant. $\mathrm{d}\mathrm{n}$ his case the most general CP transformation is given by: $\overline{\cdot 2}$ CP vacuum invariant. The idea of spontaneous CP violation was first proposed by T. D. Let the complex for the context of the context of the context of the SM, the context of the Colluction of the c
In the complex for the context of the contract of the context of the context of the SM, with a technical of th doublet an eight and conjugation to the state of the strange to the state of the state of the state of the total
- doublet an eight amount complex complex complex complex complex complex complex complex doublet and the str and the scalar sector and the sector cannot with several Higgs particular the sector serveral Eugeneer of Frid
SCaral SCCLOT Cannot March MOLARE STREET The DOORES IN HILL SEX CLOL TIME COMPLEX COMPLEX C coupleur a virtual period with a unitary transformation with a unitary of the set of dominated transformation sulate the comparison leaves to this transformation in the coves the transformation of the Lagrangian in the L
In a memory term of the condition in the comparison in the comparison in the comparison of the condition and c the most case of the most general contract of the most general contract in the most general contract in the co this case the most general CP transformation is given by: $\overline{\text{11S}}$ C **+6 AI**
2. A **22**
22222 E ALAMA
A T *φ†* **1933**
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- 2017 **('**θ[†]΄
('d d)
) <u>A</u>Ω1 WERNE PT 0 11 F TOTT and the bit hit can consider the computation of the **4 E VIOLATI ACL. CHALLENE GEL. VACHZENHEZH CHALLEN** • C-III-a (0*, w* ˆ ²*eiσ*² *, w* ˆ $\widetilde{\mathrm{max}}$ $\mathrm{min}_{1} \widetilde{\mathrm{min}}$ $\widetilde{\mathrm{min}}$ $\widetilde{\mathrm{min}}$ $\widetilde{\mathrm{min}}$

 V olume 136B, number 5,6 V and V and V arbitrary matrix \sim 1 \boldsymbol{v} ording 150 \boldsymbol{v} , number \boldsymbol{v} , \boldsymbol{v} $V_0 = V_0$ and V_0 and V_0 and V_0 and V_0 and V_0 are V_0 are V_0 and V_0 are V_0 and V_0 are V_0 a Volume 136B, number 5,6 PHYSICS LETTERS 15 March 1984 **e** volume 150B, number 5,0

Φi −→ UijΦ[∗] $C \text{ IV } f\left(\begin{array}{cc} \sqrt{2\pi i} & \cos(\sigma_1 - 2\sigma_2) \\ 0 & \cos(\sigma_1 - 2\sigma_2) \end{array} \right)$ together with assumption that vacuum is invariant $\frac{\cos(\theta_1 - 2\theta_2)}{\cos \sigma_1}$ $\hat{w}_2 e^{i \sigma_1}$, $\hat{w}_2 e^{i \sigma_2}$, \hat{w}_S); • C-IV-f $\left(\sqrt{2+\frac{\cos(\sigma_1-2\sigma_2)}{\cos \sigma_1}}\hat{w}_2e^{i\sigma_1}, \hat{w}_2e^{i\sigma_2}, \hat{w}_S\right);$ $\overline{\cos \sigma _1}$ $\hat{w}_2e^{i\sigma_1}, \, \hat{w}_2e^{i\sigma_2}, \, \hat{w}_S$ \setminus :
?

number 5,6 PHYSICS LETTERS

the vacuum is CP invariant:

Complex vacua, Spontaneous CP Violation list of complex solutions presented in Table 2. CP can only be spontaneously violated if that can be identified with a CP transformation, leaving both the Lagrangian and the vacuum invariant. The idea of spontaneous CP violation was first proposed by T. D. Lee [3] in the context of two Higgs doublets. In the context of the SM, with a single Higgs doublet, a CP transformation of the scalar doublet amounts to its complex conjugation and the scalar sector cannot violate CP. In models with several Higgs doublets complex since this transformation leaves the kinetic energy term of the Lagrangian invariant. In CP **Next we present a few illustrative examples. Important tool:** We assumed, for simplicity, that all parameters of the potential are real. Therefore our −→ UijΦ[∗] with U an arbitrary unitary matrix⁴. This equation together with the assumption that ^j (8.1) that can be identified with a CP transformation, leaving both the Lagrangian and the CP −→ UijΦ[∗] Φi ^j (8.1) that can be identified with a CP transformation, leaving both the Lagrangian and the vacuum invariant. The idea of spontaneous CP violation was first proposed by T. D. Lee [3] in the context of two Higgs doublets. In the context of the SM, with a single Higgs doublet, a CP transformation of the scalar doublet amounts to its complex conjugation and the scalar sector cannot violate CP. In models with several Higgs doublets complex conjugation may be combined with a unitary transformation acting on the set of doublets, since this transformation leaves the kinetic energy term of the Lagrangian invariant. In this case the most general CP transformation is given by: Φi CP −→ UijΦ[∗] with U an arbitrary unitary matrix⁴. This equation together with the assumption that the vacuum is CP invariant: of whether or not CP can be violated spontaneously. For that purpose we can inspect the list of complex solutions presented in Table 2. CP can only be spontaneously violated if the Lagrangian is invariant under CP and if at the same time there is no transformation that can be identified with a CP transformation, leaving both the Lagrangian and the vacuum invariant. The idea of spontaneous CP violation was first proposed by T. D. Lee [3] in the context of two Higgs doublets. In the context of the SM, with a single Higgs doublet, a CP transformation of the scalar doublet amounts to its complex conjugation and the scalar sector cannot violate CP. In models with several Higgs doublets complex conjugation may be combined with a unitary transformation acting on the set of doublets, since this transformation leaves the kinetic energy term of the Lagrangian invariant. In CP −→ UijΦ[∗] with U an arbitrary unitary matrix⁴. This equation together with the assumption that • C-IV-c '[√]1 + 2 cos² *^σ*2*^w* Table 1: Spontaneous CP violation Vacuum ⁴ SCPV Vacuum ⁴ SCPV Vacuum ⁴ SCPV C-I-a X no C-III-f,g 0 no C-IV-c X yes C-III-a X yes C-III-h X yes C-IV-d 0 no C-III-b 0 no C-III-i X no C-IV-e 0 no C-III-c 0 no C-IV-a 0 no C-IV-f X yes C-III-d,e X no C-IV-b 0 no C-V 0 no − ' *φ†* ¹*φ*¹ (' *φ†* ²*φ*² − ' *φ†* ²*φ*² (' *φ†* ³*φ*³ − ' *φ†* ³*φ*³ (' *φ†* ²*φ*³ (⁺ [√] 2*λ*⁴ ' *φ†* ²*φ*¹ (' *φ†* ³*φ*¹ (− *i* √ 2*λ*⁴ ' *φ†* ³*φ*² (' ¹ was imposed as required by eq. (C.21), and where *λ*⁷ is real and *λ*⁴ is complex, see eq. (2.8). The potential has the structure of the ∆(54)-symmetric one, as We classify cases with complex scalar potential based on ref. [61]. We first list cases allowing for spontaneous CP violation when the scalar potential is real: *^S*); *, ±w* ˆ ²*eiσ*² *, w* ˆ *^S*); ˆ ²*, w* ˆ ²*eiσ*² *, w* ˆ *S* (;

Coming back to the complex potential

the scalar potential potential (2.1) in a more compact form, \blacksquare

Compact notation:

, (3.1a)

$$
V_2 = Y_{ab} \left(h_a^{\dagger} h_b \right),
$$

$$
V_4 = \frac{1}{2} Z_{abcd} \left(h_a^{\dagger} h_b \right)
$$

$$
Y_{11} = Y_{22} = \mu_1^2,
$$

\n
$$
Z_{1111} = Z_{2222} = 2\lambda_1 + 2\lambda_3,
$$

\n
$$
Z_{3333} = 2\lambda_8,
$$

\n
$$
Z_{1122} = Z_{2211} = 2\lambda_1 - 2\lambda_3,
$$

\n
$$
Z_{1133} = Z_{2233} = Z_{3311} = Z_{3322} = \lambda_5,
$$

\n
$$
Z_{1221} = Z_{2112} = -2\lambda_2 + 2\lambda_3,
$$

\n
$$
Z_{1331} = Z_{2332} = Z_{3113} = Z_{3223} = \lambda_6,
$$

\n
$$
Z_{1212} = Z_{2121} = 2\lambda_2 + 2\lambda_3,
$$

\n
$$
Z_{1313} = Z_{2323} = Z_{3131} = Z_{3232} = 2\lambda_7,
$$

\n
$$
Z_{1123} = Z_{1213} = Z_{1312} = Z_{1321} = Z_{2113} = Z_{2311} = -Z_{2223} = -Z_{2322} = .
$$

\n
$$
Z_{1132} = Z_{1231} = Z_{2131} = Z_{3112} = Z_{3121} = Z_{3211} = -Z_{2232} = -Z_{3222} = .
$$

Explicit CP violation possibilities of spontaneous CP violation or CP conservation. We shall proceed to write possibility of spontaneous CP conservation or CP conservation. We shall provide the conservation of \mathbb{R} \sim space compact form, we see that \sim secalar potential (2.1) in a more compact form, we see that \sim

$h_a^{\dagger} h_b$ $\left(h_c^{\dagger} h_d\right)$ " $\left[\begin{array}{c} n_b \end{array} \right] \left(\begin{array}{c} n_c' h_d \end{array} \right) \,,$

January
John Silva
John Silva
John Silva *,* (3.1b) Branco,Lavoura, Silva 1999 Propos Loveiro Cilvo 10

 $Z_{3333} = 2\lambda_8,$ $Z_{1133} = Z_{2233} = Z_{3311} = Z_{3322} = \lambda_5,$ $Z\lambda_3$, $Z_{1331} = Z_{2332} = Z_{3113} = Z_{3223} = \lambda_6$ $Z_{1123} = Z_{1213} = Z_{1312} = Z_{1321} = Z_{2113} = Z_{2311} = -Z_{2223} = -Z_{2322} = \lambda_4^R - i\lambda_4^I,$ $Z_{1132} = Z_{1231} = Z_{2131} = Z_{3112} = Z_{3121} = Z_{3211} = -Z_{2232} = -Z_{3222} = \lambda_4^R + i\lambda_4^I.$ $Y_{33} = \mu_0^2$

Powerful and elegant tool: CP odd Higgs basis invariants built from Y- and Z- tensors **Olation
Thermand Septem See references [65-71] in our paper**
See references [65-71] in our paper Powerful and elegant tool: GP-odd Higgs basis invariants by

$$
I_{5Z}^{(1)} = \text{Im} [Z_{aabc} Z_{dbef} Z_{cghe} Z_{idgh} Z_{fijj}],
$$

\n
$$
I_{5Z}^{(2)} = \text{Im} [Z_{abbc} Z_{daef} Z_{cghe} Z_{idgh} Z_{fjji}],
$$

\n
$$
I_{6Z}^{(1)} = \text{Im} [Z_{abcd} Z_{baef} Z_{gchi} Z_{djke} Z_{fkil} Z_{jglh}],
$$

\n
$$
I_{6Z}^{(2)} = \text{Im} [Z_{abcd} Z_{baef} Z_{gchi} Z_{dejk} Z_{fhkl} Z_{lgij}],
$$

\n
$$
I_{7Z} = \text{Im} [Z_{abcd} Z_{eafc} Z_{bgdh} Z_{iejk} Z_{gflm} Z_{hlkn} Z_{minj}],
$$

$I_{2Y3Z} = \text{Im} [Z_{abcd}Z_{befa}Z_{dchf}Y_{qa}Y_{eh}].$

–
Land Complex computation due to high number of contraction of indices requiring special simplification simplification techniques!

Explicit CP violation

The symmetric 3HDM potential, *The quadrilinear part of the symmetric 3HDM potential, and positive 3HDM potential,* $\frac{1}{2}$ *and* $\frac{1}{2}$ *and* $\frac{1}{2}$ *and* $\frac{1}{2}$ *and* $\frac{1}{2}$ *and* $\frac{1}{2}$ *and* $\frac{1}{2}$ *and \frac{1}{2}* Anton Kunčinas **Odd Magne Ogreid**

Next, we must prove that I $\overline{}$ \overline

Theorem 1. *The quadrilinear part of the S*3*-symmetric 3HDM potential, V*4*, explicitly conserves CP if and only if* I conserves CP if and only if $I_{5Z}^{(1)} = I_{5Z}^{(2)} = I_{6Z}^{(1)} = I_{6Z}^{(2)} = I_{7Z} = 0$. $\it city$ $\frac{1}{1-\epsilon}$ \mathcal{L}^{i} **Theorem 1.** The quadruinear part of the S_3 -symmetric 3HDM potential, V_4 $\begin{matrix}0&0\\0&0\end{matrix}$ ⁴*λ*7(*λ*¹ − *λ*² − *λ*8)

- **Solution 0:** $\lambda_4^I = 0$; • **Solution 0:** *λ*^I ${\bf u}$ (*x*) ${\bf u}$ ${\bf i}$ ${\bf j}$ ${\bf n}$ ${\bf j}$ ${\bf j}$
- $p_1 \cdot q_2 \cdot q_3 \cdot R_0$ α behavion **1.** $\alpha_4 = 0$, • **Solution 1:** $\lambda_4^{\text{R}} = 0$; λ \bullet Solution 1: $\lambda_4^{\prime\prime} = 0;$
- **Solution 2:** $\lambda_7 = 0$; • Solution 2: $\lambda_7 = 0$:
	- N_{max} Ω Ω Ω Ω Ω Ω • **Solution 3** $(\lambda_4^R \lambda_4^I \lambda_7 \neq 0)$: $\text{olution } 3 (\lambda_4^{\text{R}} \lambda_4^{\text{I}} \lambda_7 \neq 0)$: $\lambda_{23} \equiv \lambda_2 + \lambda_3$
 $\lambda_{23} \equiv \lambda_2 + \lambda_3$

$$
\lambda_{23} \equiv \lambda_2 + \lambda_3
$$

\n
$$
\left(\lambda_4^R\right)^2 = -\frac{(\lambda_{23} - \lambda_7)(2\lambda_{23} + \lambda_7)^2}{\lambda_7}, \qquad \lambda_5 = 2(\lambda_1 + \lambda_2),
$$

\n
$$
\left(\lambda_4^I\right)^2 = \frac{(\lambda_{23} + \lambda_7)(2\lambda_{23} - \lambda_7)^2}{\lambda_7}, \qquad \lambda_6 = 4\lambda_3,
$$

\n
$$
\lambda_8 = \lambda_1 - \lambda_2.
$$

$$
\left(\lambda_4^R\right)^2 = -\frac{(\lambda_{23} - \lambda_7)(2\lambda_{23} + \lambda_7)^2}{\lambda_7}, \qquad \lambda
$$

$$
\left(\lambda_4^I\right)^2 = \frac{(\lambda_{23} + \lambda_7)(2\lambda_{23} - \lambda_7)^2}{\lambda_7}, \qquad \lambda
$$

 α *s* and α **solution** α and α **solution** For each of these solutions we were able to show that there exists a real basis for V_4 eal b $\frac{1}{2}$ and the solutions we were able to show that there existed

\Box **Explicit CP violation** ⁶*^Z* = I7*^Z* = 0 imply a CP invariant *V*4. In Explicit CP violation \blacklozenge

Theorem 1 is that we now include the vanishing of I2*^Y* ³*Z*. We find that Solutions 0–2 make For each of the solutions we were able to snow that there exists a real basis for $\bf v$ No additional continuous sym
P For each of the solutions we were able to show that there exists a real basis for V No additional continuous symmetries for solution 3' and de Medeiros Varzielas, Ivan de Medeiros Varzielas, Ivanov 2019 The potential has the structure of the $\Delta(54)$ -symmetric **EXAMPLE CF VIOLATION**

Continuous CP of

and only if $I_{5Z}^{(1)} = I_{5Z}^{(2)} = I_{6Z}^{(1)} = I_{6Z}^{(2)} = I_{7X} = I_{2Y3Z} = 0$.

Solution $3' (\lambda_4^R \lambda_4^T \lambda_7 \neq 0)$:
 $\mu_1^2 = \mu_0^2$,
 $\left(\lambda_4^R\right)^2 = -\frac{(\lambda_{23} - \lambda_7)(2\lambda_{23} + \lambda_7)^2}{\lambda_7},$ ¹ **λ**
λ A A A A and the U strow that there exists a real basis for v
A *Medeire commetries for solution 3[/] λ*₂ = 2008, ²/₂ = 2008, 2019, le to show that there exists **1** No additional continuous symmetries for solution 3['] d **⁴***λ***⁷ != 0):** 112⁰ 112 ('*φ†* − *i* ()
() The potential has the structure of the $\Delta(54)$ -symmetric

r explicit given by eqs. (52) and (53) in ref. $\overline{5}$. The ref. $\overline{7}$

\Box out algebraically the solutions in terms of the potential parameters. The potential parameters. The difference from α Explicit CP violation

For Solution 3.2 we get the following expression: \mathbf{r} Ξχ \overline{a}

Theorem 2. *The* S_3 -symmetric 3HDM potential, $V = V_2 + V_4$, explicitly conserves CP if *and only if* I $\frac{(1)}{5Z} = \mathbf{I}_{5Z}^{(2)} = \mathbf{I}_{6Z}^{(1)} = \mathbf{I}_{6Z}^{(2)} = \mathbf{I}_{7Z} = \mathbf{I}_{2Y3Z} = 0.$ Theorem 1 is that we now include the vanishing of I2*^Y* ³*Z*. We find that Solutions 0–2 make **allieorein 2.** The 53-symmetric 3 in potential, $v = v_2 + v_4$, explicitly conserves \cup P and only if $I_{5Z}^{\geq} = I_{5Z}^{\geq} = I_{6Z}^{\geq} = I_{7Z} = I_{2Y3Z} = 0.$ $\bf{2.}$ The S_3 -symmetric 3HDM potential, $V = V_2 + V_4$, explicitly $2HDM$ potential, $V - V_2 + V_4$ explicitly conserves CP if *and only if* I \sum_{x} \sum_{y} \sum_{z} \sum_{z ⁵*^Z* = I(2) ⁵*^Z* = I(1) anon 9 The S_p commeters 6 and only if $I_{5Z}^{\left(1\right)} = I_{5Z}^{\left(2\right)} = I$

Production $2'(\lambda R)\lambda + 0$. First, we prove that if *V* is CP invariant, then I • **Solution 3'** $(\lambda_4^R \lambda_4^I \lambda_7 \neq 0)$: \overline{C} in \overline{C} *λ λ − λ − (C)*. (*C*) *−* (*C* • Solution 3' $(\lambda_4^R \lambda_4^I \lambda_7 \neq 0)$:

$$
\mu_1^2 = \mu_0^2,
$$

\n
$$
\left(\lambda_4^R\right)^2 = -\frac{(\lambda_{23} - \lambda_7)(2\lambda_{23} + \lambda_7)^2}{\lambda_7},
$$

\n
$$
\left(\lambda_4^I\right)^2 = \frac{(\lambda_{23} + \lambda_7)(2\lambda_{23} - \lambda_7)^2}{\lambda_7},
$$

\n
$$
\lambda_5 = 2(\lambda_1 + \lambda_2),
$$

\n
$$
\lambda_6 = 4\lambda_3,
$$

\n
$$
\lambda_8 = \lambda_1 - \lambda_2.
$$

For the general 3HDM, the necessary and sufficient set of CP-odd invariants needed. *For the general 3HDM, the necessary and sufficient set of CP-odd invariants need
for explicit CP conservation has not yet been identified λ*⁶ = 4*λ*3*,* (C.13e) I IIUIUI
. *λ*7 For the general 3HDM, the necessary and sufficient set of CP-odd invariants needed for explicit CP conservation has not yet been identified

 β C-IV-g results in at least two negative mass-squared eigenvalues. Introduction of soft symmetry breaking terms might solve the issue.

It is possible to have CP
wieletien without breaking violation without breaking S_3 (see R-I-1) S_3 (see R-I-1)

Thear
Thera
Thera
Instruction entries with "-" indicate that it is not possible to
can explicit is respected generate realistic masses and mixing symmetries with "-" indicate established that CP is violated that CP is violated in the flavour sector and the Cabibbo-Kobayashi-Kobayashi-

^α In C-IV-c and C-IV-f there is a massless scalar present. Soft symmetry breaking would remove the massless scalar.

Summary of different CP violating models conserves CP and the vacuum breaks it. This requires that not all vacuum breaks it. This requires that not all
This requires that not all vacuum expectations in the vacuum expectation of the vacuum expectations in the vac values be real. In the context of multi-Higgs extensions of the Standard Model imposing

- R-I-1 there is a pair of charged mass degenerate states and two pairs of neutral mass-degenerate states
- C-III-a realistic masses and mixing require the fermions to transform trivially under the symmetry and require complex Yukawa couplings. Has a viable DM candidate for a real potential
- C-III-h realistic masses and mixing require the fermions to transform trivially under the symmetry and require complex Yukawa couplings
- C-IV-c possible to fit both fermion masses and the CKM matrix however, there is an accidental massless scalar state in the model
- C -IV-f this vacuum is a generalisation of C-IV-c but a massless scalar state is also present
- C -IV-g possible to fit both fermion masses and mixing however, there are negative mass-squared scalars
- C-V possible to fit both fermion masses and the CKM matrix; can also yield a realistic scalar sector. Remarkable possibility of having light neutral scalars of order a few Mev escaping detection. More details in our paper.

Potentially realistic models with real Yukawa couplings C-IV-c C-IV-f C-IV-g C-V only C-V survives without the need for soft breaking terms due to unrealistic scalar spectrum were explored numerically. A complete systematic systematic study of the models presented here, $\frac{1}{2}$ Futchlially realistic hiddels with real fundwa couplings to draw a conclusion is a specific model can be regulated or not. In the regulation of α

A numerical study of C-V was performed fitting several parameters

- Masses of the up- and down-quarks;
- The absolute values, arguments of the unitarity triangle $(\alpha, \sin 2\beta, \gamma)$ and independent measure of CP violation (J) [89, 90] of the CKM matrix;
	- 15 • Interactions of the SM-like Higgs boson with fermions. We assume the Higgs boson signal strength in the *b*-quark channel [91–93] as a reference point and apply the corresponding limits to other channels;
	- Suppressed scalar mediated FCNC [94, 95];
	- CP properties of the SM-like Higgs boson [96, 97];
	- not kinematically suppressed [98, 99];

• Upper limit on the decay of the *t*-quark into lighter charged scalars when decays are

Figure 2. Scatter plots of masses that satisfy constraints in the C-V model. Top: the charged sector, H_i^{\pm} . Bottom: the active sector, H_i . In the neutral sector the red line indicates a 125 GeV state.

Many interesting aspects of the models presented here remain to be analysed

Conclusions

- Potential DM candidates exist as was shown in previous works of ours
- Khater, Kunčinas, Ogreid, Osland, MNR, 2021 Kunčinas, Ogreid, Osland, MNR, 2022
	- Many important studies of 3HDM have appeared in the literature, and several of them are cited in our paper.
		- Still many important questions remain open
		- Multi-Higgs models are at present a fertile ground of research
	- The LHC may bring important news for this field in the near future

Back-up slide

We have the following *S*³ doublets: $\sqrt{\bar{Q}}_1$ \bar{Q}_2 ◆ *L* We have th Q $\frac{2}{ }$ $^\prime$ I Singlets of *S*³ can be obtained from the multiplication of two singlets or two doublets, where one factor could allowing β doublets. $\langle \bar{O}_1 \rangle$ $\langle u_1 \rangle$ $\langle d_1 \rangle$

,

and singlets:

$$
\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}_R, \quad \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}_R, \quad (h_1 \ h_2)
$$

$\bar{Q}_{3L}, \quad u_{3R}, \quad d_{3R}, \quad h_S,$

indices 1,2,3 on quark fields \overline{Q} , u , d label the families. Mass terms arise from the following generic structures: $\bar{Q}_L \phi d_R$ or $\bar{Q}_L \tilde{\phi} u_R$, where ϕ and $\tilde{\phi} = -i[\phi^{\dagger} \sigma_2]^T$ are scalar SU(2) doublets. where indices 1,2,3 on quark fields \overline{Q} , *u*, *d* label the families. Mass terms arise from the following $\mathcal{L}_{\mathcal{L}}\varphi u_R$, where φ and $\varphi = -i[\varphi' \circ 2]$ are scalar $\mathcal{L}(\varphi)$ doublets.

As a result, the mass matrix will have the structure Λ_{α} cone factor contrivently done the atmostry in a roban, one made matrix with have one bet accure As a result, the mass matrix will have the structure

 $\mathcal{M} =$ $\overline{1}$ \overline{a}

 $\mathcal{M} = \begin{bmatrix} y_2^d w_1 & y_1^d w_S - y_2^d w_2 & y_4^d w_2 \end{bmatrix}$ $y_5^{\omega}w_1$ $y_5^{\omega}w_2$ $y_3^{\omega}w_5$ $\mathcal{M} = \begin{bmatrix} y_2^{\alpha}w_1 & y_1^{\alpha}w_5 - y_2^{\alpha}w_2 & y_4^{\alpha}w_2 \\ d_{\alpha}w_1 & d_{\alpha}w_2 & d_{\alpha}w_1 \end{bmatrix}$ $\frac{95 \omega_1}{800}$ $\frac{95 \omega_2}{800}$ $\frac{93 \omega_5}{800}$ $y_1^d w_S + y_2^d w_2$ *y*^{*d*}₂*w*₁ *y*^{*d*}₄*w*₁ $y_2^d w_1$ *y*^{*d*}₁*w*_S *y*^{*d*}₁*w*₂ *y*^{*d*}₁*w*₂ $y_5^d w_1$ *y*^{*d*}₅*w*₂ *y*^{*d*}₃*w*_S \setminus A *,* (1.4)

