Searching for Light New Scalars with Atomic and Nuclear Clocks

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based on work with

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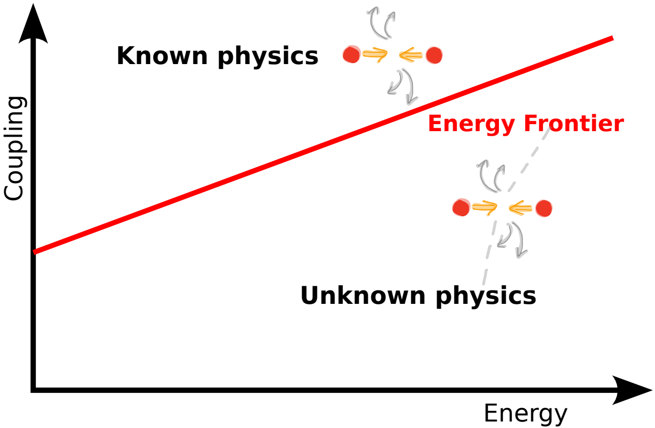
E. Fuchs, Eric Madge, Chaitanya Paranjape, Ekkehard Peik, Gilad Perez, Wolfram Ratzinger, Johannes Tiedau

Extended Scalar Sectors from All Angles, CERN, 25^{th} October 2024

Outline

Outline

Where is the New Physics?



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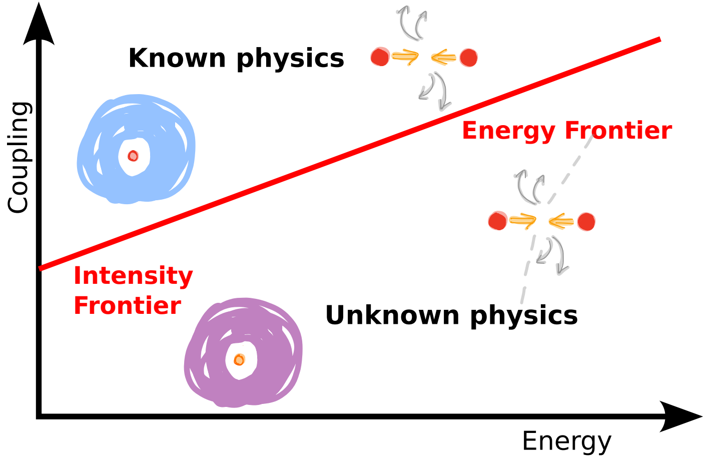

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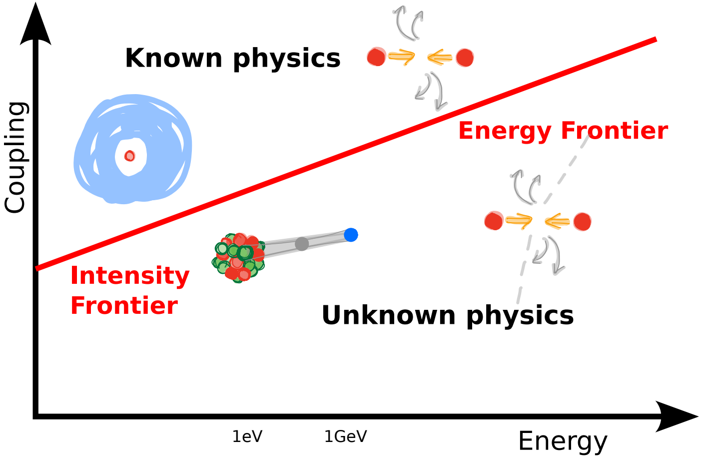

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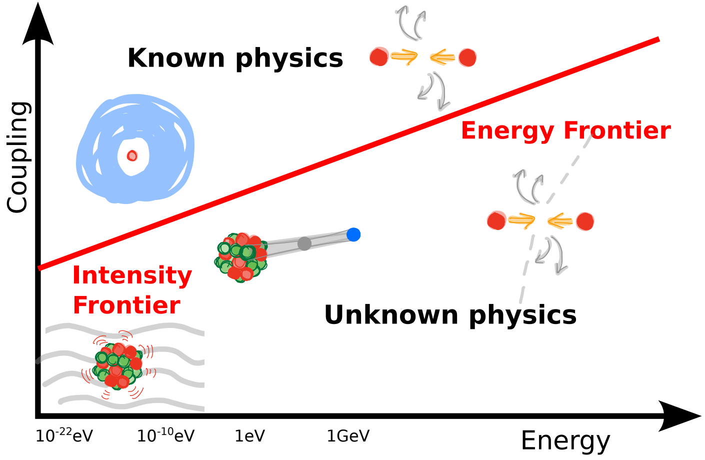


Outline

What Are We Looking For?

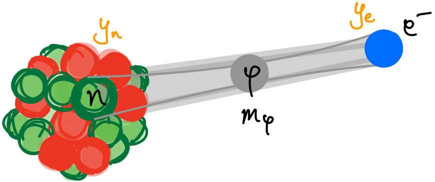
Search With Atomic Clocks

Search With Nuclear Clock

Search With Atomic Clocks:

Light Scalar

Coupling to Neutrons And Electrons



Why Atomic Clocks?

The most accurately measured numbers in physics are ratios of atomic clock transition frequencies:

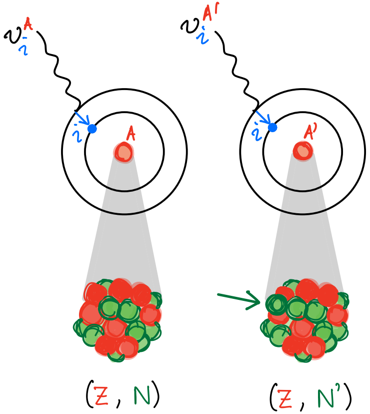
- $\nu_{\mathrm{Al}^+}/\nu_{\mathrm{Hg}^+} = 1.052871833148990438(55)$ (NIST; $\sigma_\nu/\nu \sim 5.2 \times 10^{-17}$)
  [Rosenband et al. Science 319, 1808 (2008)]
- $\nu_{\mathrm{Yb}}/\nu_{\mathrm{Sr}} = 1.207507039343337749(55)$ (RIKEN; $\sigma_\nu/\nu \sim 4.6 \times 10^{-17}$)
  [Nemitz et al. Nat. Photonics 10, 258 (2016)]
- $\nu_{\mathrm{E3}}/\nu_{\mathrm{E2}} = 0.932829404530965376(32)$ (PTB; $\sigma_\nu/\nu \sim 3.4 \times 10^{-17}$)
  [Lange et al. PRL 126 011102 (2021)]
- $\nu_{\mathrm{In}^+}/\nu_{\mathrm{Yb}^+} = 1.9737735915557215789(9)$ (PTB; $\sigma_\nu/\nu \sim 4.4 \times 10^{-18}$)
  [Hausser et al. arXiv: 2402.16807 (2024)]

$\Rightarrow$ These are sensitive to "everything", but we cannot calculate the spectrum below around 1% accuracy.

So what can we do with these?

[based on slide by Julian Berengut]

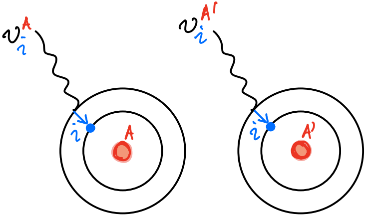
The King-Plot: Fit to Isotope Shift Data



Isotope shift:

$$\nu_i^{AA'} \equiv \nu_i^A - \nu_i^{A'}$$

The King-Plot: Fit to Isotope Shift Data

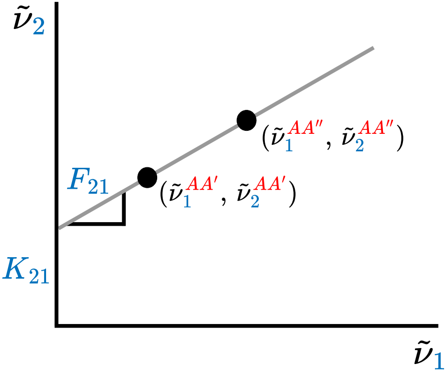


[W. King, J. Opt. Soc. Am. 53, 638 (1963)]

$$\nu_i^{AA'} \equiv \nu_i^A - \nu_i^{A'}$$

$\tilde{\nu}_i^A$: data, K_{21}, F_{21}: linear fit

$$\tilde{\nu}_2^{AA'} = K_{21} + F_{21}\tilde{\nu}_1^{AA'}$$
$$\tilde{\nu}_2^{AA''} = K_{21} + F_{21}\tilde{\nu}_1^{AA''}$$



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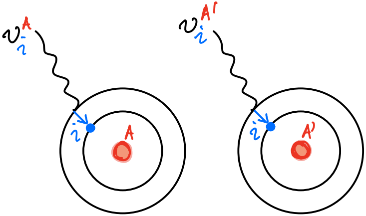
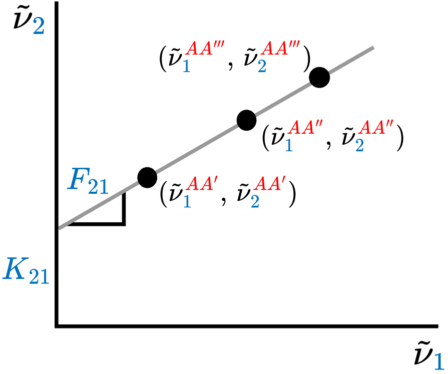


$$\nu_i^{AA'} \equiv \nu_i^A - \nu_i^{A'}$$

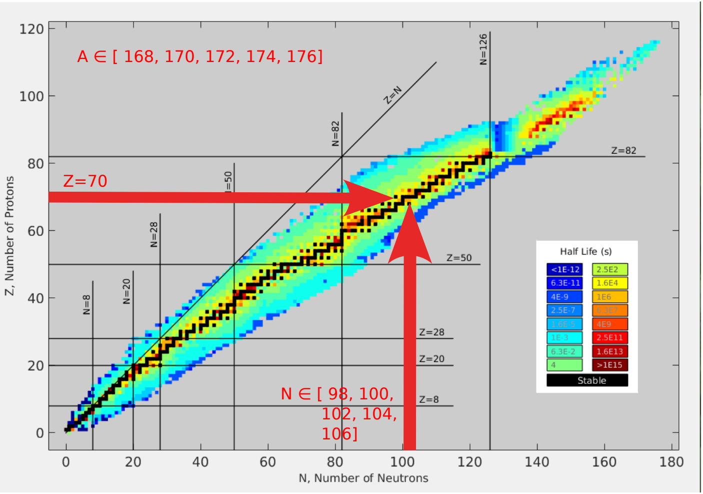
$\tilde{\nu}_i^A$: data, K_{21}, F_{21}: linear fit

$$\begin{aligned}\tilde{\nu}_2^{AA'} &= K_{21} + F_{21}\tilde{\nu}_1^{AA'}\\ \tilde{\nu}_2^{AA''} &= K_{21} + F_{21}\tilde{\nu}_1^{AA''}\\ \tilde{\nu}_2^{AA'''} &= K_{21} + F_{21}\tilde{\nu}_1^{AA'''}\end{aligned}$$

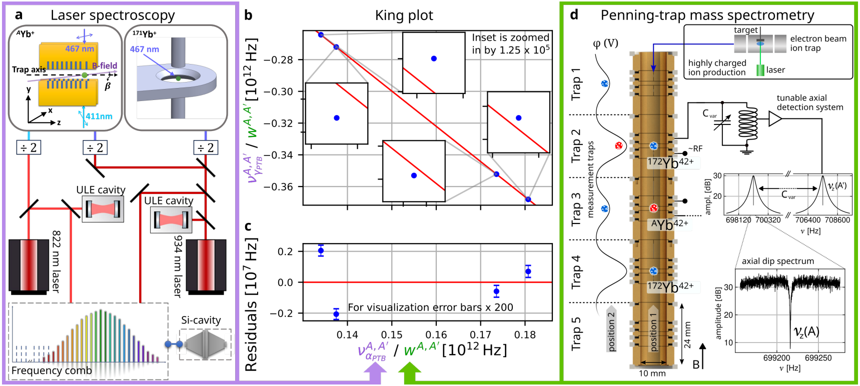
[W. King, J. Opt. Soc. Am. 53, 638 (1963)]



Ytterbium's Stable Isotopes

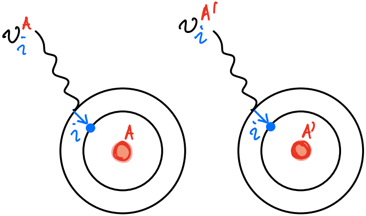


PTB + MPIK = New Yb King Plot



Observed King plot nonlinearity: $\sim 20.17(2)$ kHz

The King-Plot: Fit to Isotope Shift Data



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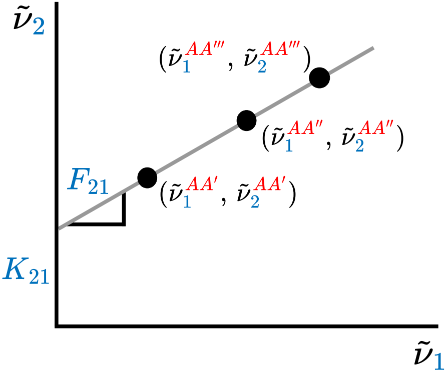
$$\nu_i^{AA'} \equiv \nu_i^A - \nu_i^{A'}$$



$\tilde{\nu}_i^A$: data, K_{21}, F_{21}: linear fit

$$\begin{aligned}
\tilde{\nu}_2^{AA'} &= K_{21} + F_{21}\tilde{\nu}_1^{AA'} \\
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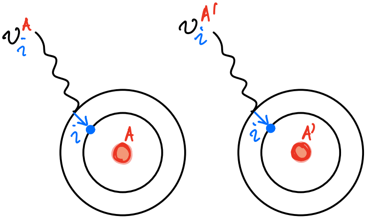
\end{aligned}$$

The King-Plot: Fit to Isotope Shift Data



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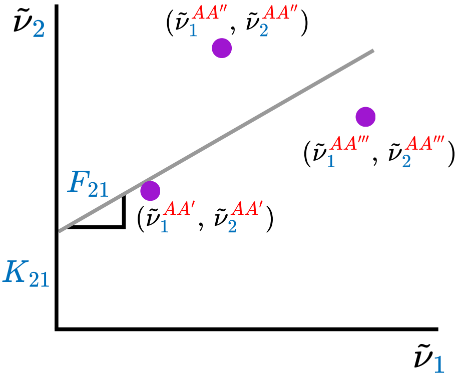

$$\nu_i^{AA'} \equiv \nu_i^A - \nu_i^{A'}$$

$\tilde{\nu}_i^A$: data, K_{21}, F_{21}: linear fit

$$\tilde{\nu}_2^{AA'} = K_{21} + F_{21}\tilde{\nu}_1^{AA'} + ?$$
$$\tilde{\nu}_2^{AA''} = K_{21} + F_{21}\tilde{\nu}_1^{AA''} + ?$$
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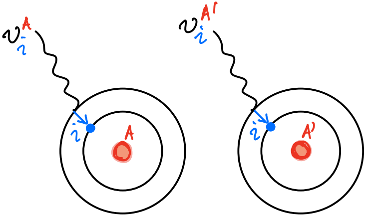
The King-Plot: Fit to Isotope Shift Data



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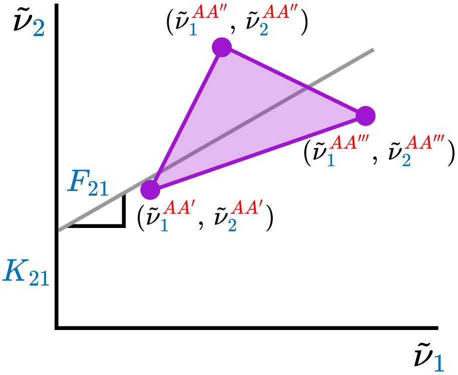

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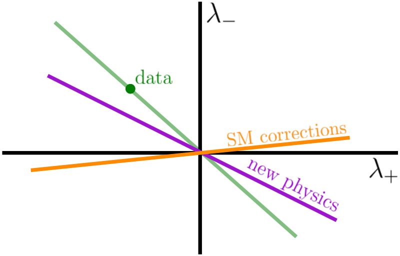
The Nonlinearity Decomposition Plot



- Plane of King linearity: $\mathbf{1} = (1, 1, 1, 1)$,
  $\tilde{\boldsymbol{\nu}}_{\boldsymbol{i}} = (\tilde{\nu}_i^{AA'}, \tilde{\nu}_i^{AA''}, \tilde{\nu}_i^{AA'''}, \tilde{\nu}_i^{AA''''})$, i=1,2,...

$$\tilde{\boldsymbol{\nu}}_j \approx F_{j1}\tilde{\boldsymbol{\nu}}_1 + K_{j1}\mathbf{1}\,, \qquad j > 1.$$

- Project isotope-shift data onto $\tilde{\boldsymbol{\nu}}_1$, $\mathbf{1}$, $\boldsymbol{\Lambda}_+$, $\boldsymbol{\Lambda}_-$ with $\boldsymbol{\Lambda}_\pm \perp (\tilde{\boldsymbol{\nu}}_1, \mathbf{1})$:

$$\tilde{\boldsymbol{\nu}}_j = (\tilde{\boldsymbol{\nu}}_1, \mathbf{1}, \boldsymbol{\Lambda}_+, \boldsymbol{\Lambda}_-)\,(F_{j1}, K_{j1}, \lambda_+, \lambda_-)^T$$

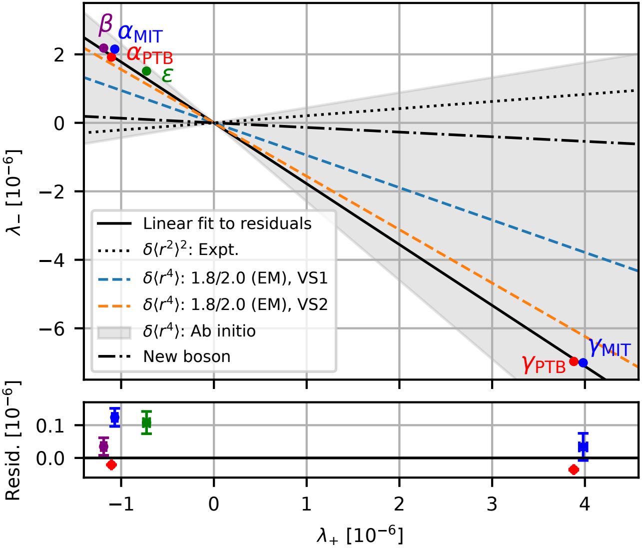
In presence of just one nonlinearity,

$$\tilde{\boldsymbol{\nu}}_j \approx F_{j1}\tilde{\boldsymbol{\nu}}_1 + K_{j1}\mathbf{1} + G_{j1}^{(4)}\boldsymbol{\delta}\widetilde{\langle \boldsymbol{r}^4 \rangle}\,, \qquad j > 1.$$

slope: $\frac{\lambda_-}{\lambda_+} \equiv \frac{G_{j1}^{(4)}\boldsymbol{\delta}\widetilde{\langle \boldsymbol{r}^4 \rangle}_-}{G_{j1}^{(4)}\boldsymbol{\delta}\widetilde{\langle \boldsymbol{r}^4 \rangle}_+} = \frac{\boldsymbol{\delta}\widetilde{\langle \boldsymbol{r}^4 \rangle}_-}{\boldsymbol{\delta}\widetilde{\langle \boldsymbol{r}^4 \rangle}_+} \Rightarrow$ transition-universal

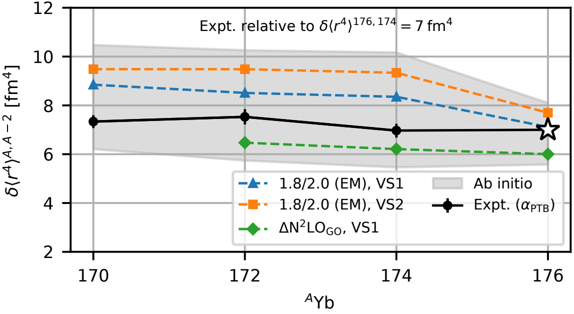
[arXiv:2004.11383, arXiv:2201.03578]

The Nonlinearity Decomposition Plot



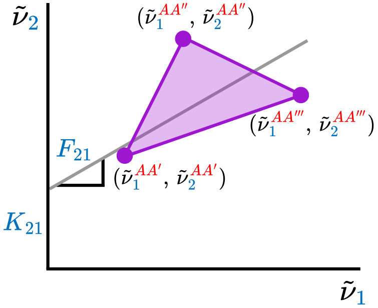
Extracting Nuclear Physics from Isotope-Shift Measurements

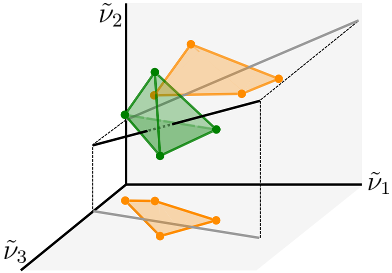
- **Assuming $\delta\langle r^4\rangle$ dominates,** what does the isotope-shift data tell us about the evolution of $\delta\langle r^4\rangle$ along the isotope chain?



blue, **orange**, **green**: Calculations by group of Prof. Achim Schwenk
black: new spectroscopic method, fixed at ⋆

King-Plot Method in Presence of Nuclear Effects: The Generalised King Plot [arXiv:1704.05068,2005.06144]





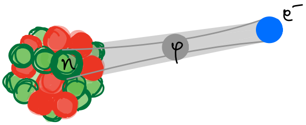
⇒ test King linearity

⇒ account for one King nonlinearity

⇒ put bound on 2nd

⇒ **King-plot method also works in presence of nuclear effects.**

King-Plot Bounds on New Bosons [arXiv:1704.05068,2005.06144]



New effective Yukawa-potential

$$V_\phi(r) = -\alpha_{\mathrm{NP}}(A - Z)\frac{e^{-m_\phi r}}{r}$$

Induces new term in the isotope shift:

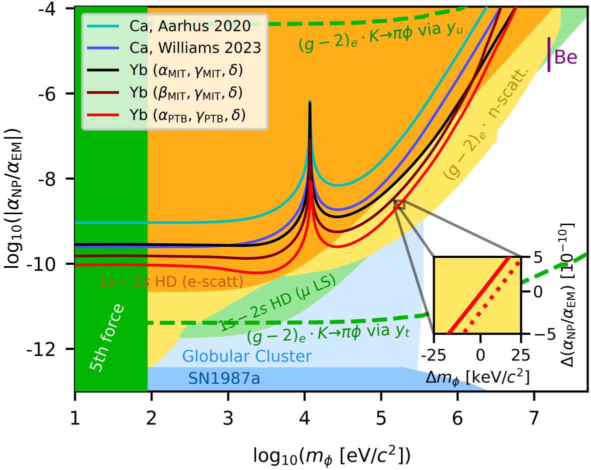
$$\tilde{\nu}_i^{AA'} = K_i \tilde{\mu}^{AA'} + F_i \delta\langle \widetilde{r^2}\rangle^{AA'} + G_i^{(4)} \delta\langle \widetilde{r^4}\rangle^{AA'} + \alpha_{\mathrm{NP}} X_i \tilde{\gamma}^{AA'}$$

$\Rightarrow$ Extract α_{NP} from fraction of volumes spanned by frequency vectors:

$$\alpha_{\mathrm{NP}} = \frac{Vol.}{Vol.|_{th,\alpha_{\mathrm{NP}}=1}} = \frac{2!\,\det\left(\vec{\tilde{\nu}}_1, \vec{\tilde{\nu}}_2, \vec{\tilde{\nu}}_3, \vec{\tilde{\mu}}\right)}{\varepsilon_{ijkl}\,\det\left(X_i \vec{\tilde{\gamma}}, \vec{\nu}_j, \vec{\nu}_k, \vec{\nu}_l\right)}$$

$\{\vec{\nu}_i\}$: data vectors in isotope-pair space, $\vec{\tilde{\mu}} \equiv (1,1,1,1)$, X_i, $\vec{\tilde{\gamma}}$: th. input

New Spectroscopy Bounds on New Physics



- $m_\phi \to 0$: $>$ size atom
- $m_\phi \to \infty$: not sensitive to contact interactions
- "Peaks" due to cancellations among electronic coefficients

Outline

What Are We Looking For?

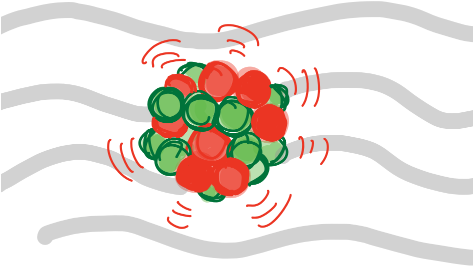
Search With Atomic Clocks

Search With Nuclear Clock

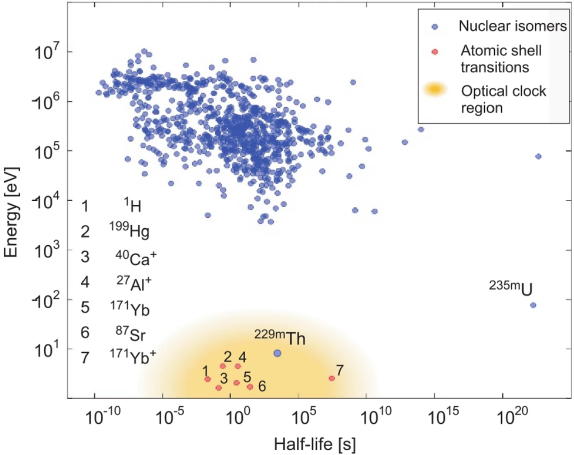
Search With Nuclear Clock:

Ultralight Scalar

Coupling to QCD



Fine-Tuning in Thorium-229?



Nuclear transition energy in ^{229}Th:

$$\Delta E = \Delta E_{\mathrm{EM}} + \Delta E_{\mathrm{nuc}}$$

$$\Delta E \ll |\Delta E_{\mathrm{EM}}| \sim |\Delta E_{\mathrm{nuc}}|$$

$$8\ \mathrm{eV} \ll 0.1\ \mathrm{MeV}$$

$\Rightarrow$ **Fine tuning?**

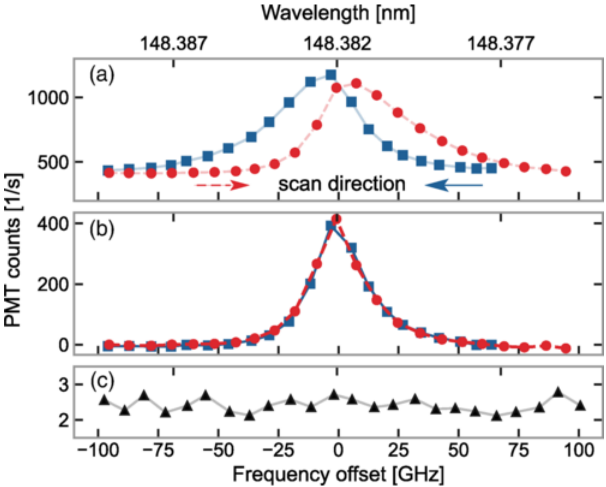
$\Rightarrow$ **Exceptionally sensitive probe of QCD at low energy?**

Progression of precision $\delta\nu/\nu$:

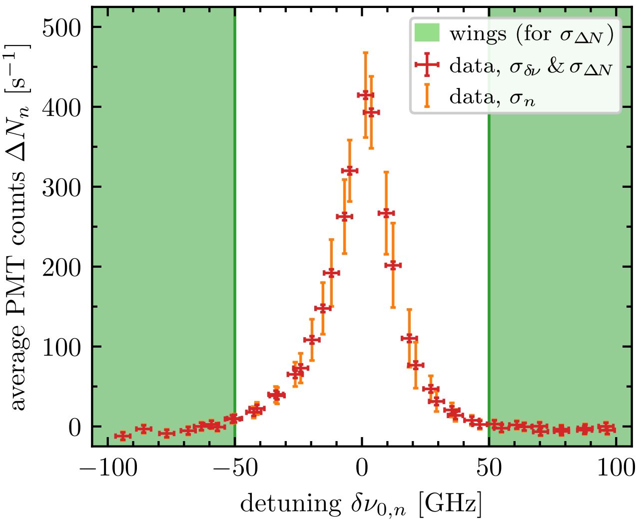
10^{-1} (2020), 10^{-3} (2022, ISOLDE), 10^{-6} (March 2024, PTB), 10^{-11} (June 2024, JILA)

Nuclear Lineshape Analysis

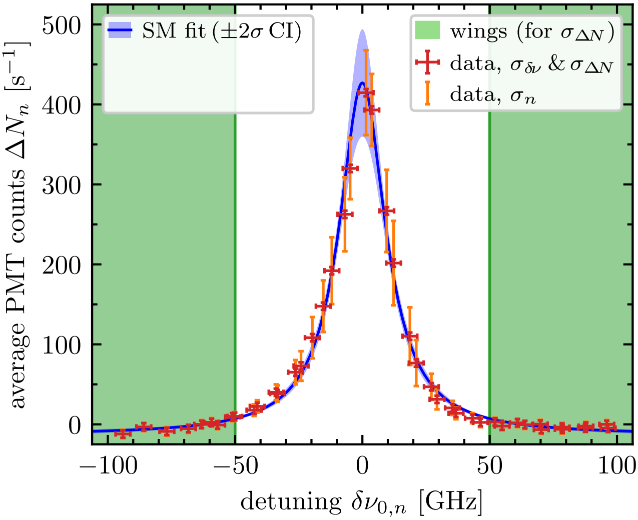
First laser-excitation of a nuclear transition: PTB 2024 [PRL 132, 182501]



Nuclear Lineshape Analysis



Nuclear Lineshape Analysis



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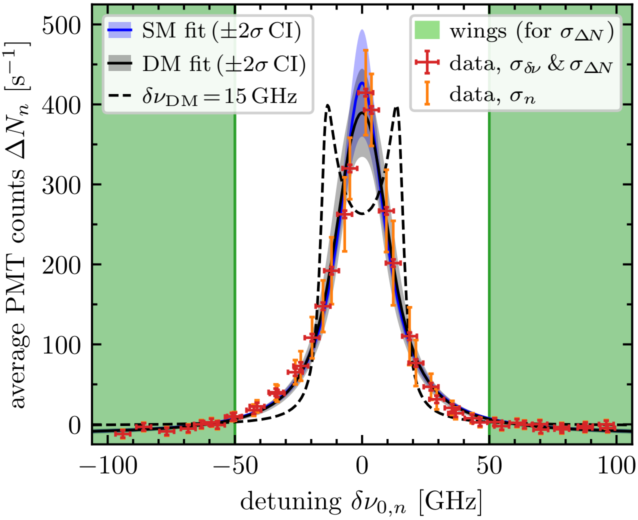
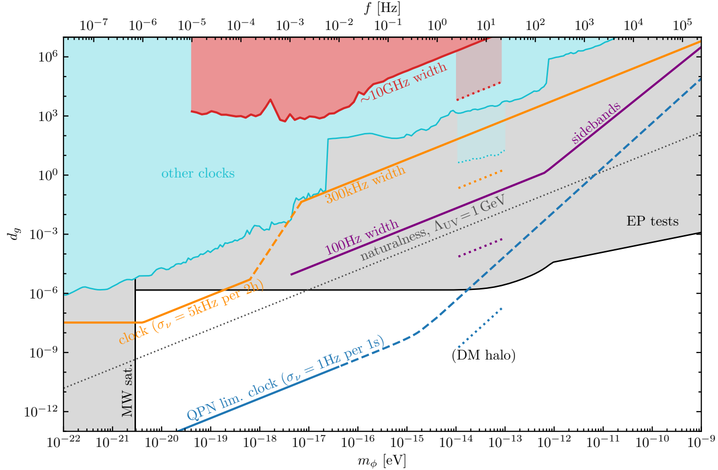


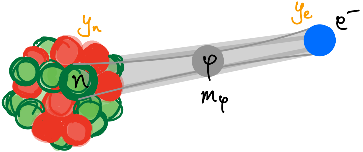
Bounds on Ultralight Scalar Coupling to QCD

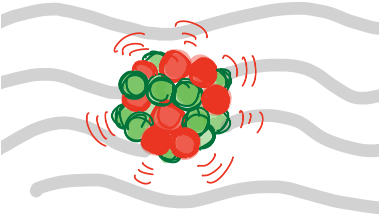


$$\mathcal{L}_\phi \supset -d_g \frac{\phi}{M_{\mathrm{Pl}}} \frac{\sqrt{\pi}\beta_s}{g_s} G^a_{\mu\nu} G^{a\mu\nu}$$

Conclusions

Atomic and nuclear clocks are sensitive probes of light new scalars:





Light scalar coupling to e^- & n

Isotope shift spectroscopy

Ultralight scalar coupling to QCD

Nuclear line-shape analysis

...And on the way we can learn about

Nuclear deformation

Fine-tuning in thorium

Check out our papers on the arXiv:

- Yb King plot: `https://arxiv.org/abs/2403.07792`
- Th-229 & ULDM: `https://arxiv.org/abs/2407.15924`

Stay tuned for:

- `Kifit`: Global King-plot analysis
- King-plot analysis of highly-charged Ca ions

Thank you for your attention.

Backup slides

Why Light New Physics

How?

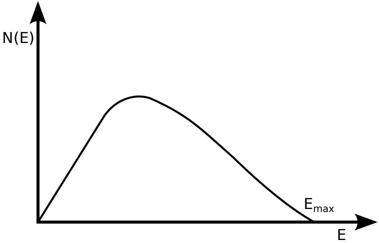
- Spontaneous breaking of exact symmetries $\rightarrow$ massless particles
  - Approximate symmetries broken $\rightarrow$ low-mass particles
- Traditional example:
  pion $\sim$ Goldstone boson of spontaneously broken chiral symmetry
  $\Rightarrow$ much lighter than other mesons

Historic Example for Light New Physics: β Decay

- β decay was assumed to involve only nucleons and electrons: $n \to p + e$

$\Rightarrow$ Expect discrete energy spectrum for e **but** continuous spectrum observed

- Pauli proposed a radical solution involving a neutrino: $n \to p + e + \bar{\nu}_e$
- Example of a **"hidden sector"** involving light new physics:
  - New light particle
  - Electrically neutral, weakly interacting
  - Manifests itself through a **"portal"**: weak interaction



[Slide inspired by Philippe Mermod, Flavour 2015, Munich, 3 June 2015]

Why Light New Physics

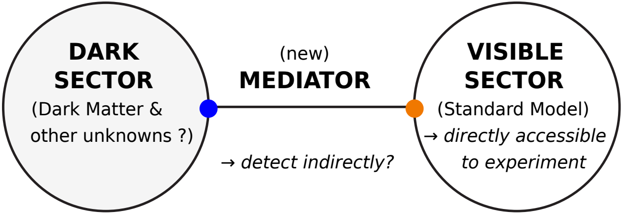
Examples of well-motivated light new particles:

- Sterile neutrino (neutrino masses, Dark Matter, matter-antimatter asymmetry)
- Axion (strong CP problem)
- Dark photon (mediator to Dark Matter)

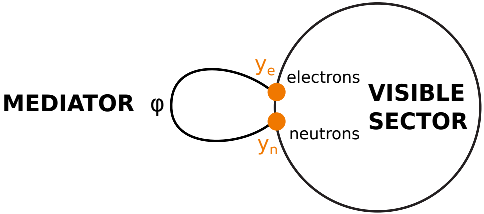
Dark Portals / Portals to Dark Matter?

Portal	**Coupling**
Scalar (Dark Higgs) S	$(\mu S + \lambda S^2) H^\dagger H$
Vector (Dark Photon) A'_μ	$-\frac{\varepsilon}{2} F'_{\mu\nu} F^{\mu\nu}$
Spinor (Sterile Neutrino) N	$y_N LHN$
Pseudoscalar (Axion) a	$\frac{a}{f_a} F_{\mu\nu} \tilde{F}^{\mu\nu}$, $\frac{a}{f_a} G_{\mu\nu} \tilde{G}^{\mu\nu}$, $\frac{\partial_\mu a}{f_a} \bar{\psi} \gamma^\mu \gamma^5 \psi$

Dark Portals



Dark Portals and Isotope Shift Measurements



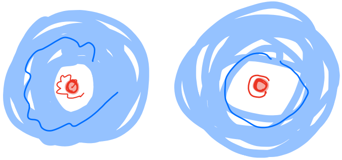
Isotope Shifts: Mass Shift & Field Shift

$$\nu_i^{AA'} = K_i \mu^{AA'} + F_i \delta\langle r^2\rangle^{AA'} + \dots$$

Mass Shift

Different motion of the nuclei
$\Rightarrow$ Correction to e^- kin. energy

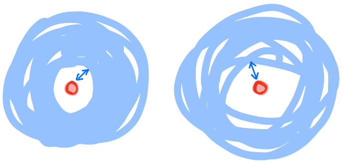
$$\mu^{AA'} = \frac{1}{m^A} - \frac{1}{m^{A'}}$$



Field Shift

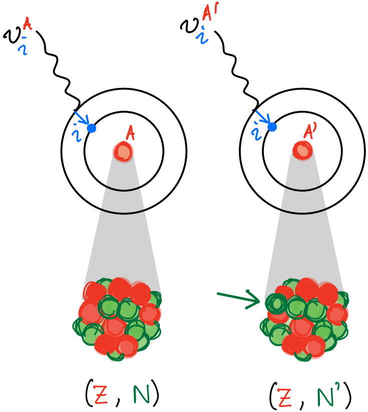
Different nucl. charge distrib.
$\Rightarrow$ Different contact interactions betw. e^- & nuclei

$$\delta\langle r^2\rangle^{AA'} = \langle r^2\rangle^A - \langle r^2\rangle^{A'}$$



Isotope Shifts

Factorisation of electronic and nuclear contributions.



Isotope shifts:

$$\nu_i^{AA'} = K_i \mu^{AA'} + F_i \delta\langle r^2\rangle^{AA'} + \dots$$

$\nu_i^{AA'} \equiv \nu_i^A - \nu_i^{A'}$

i: transition index

AA': isotope pair index

$K_i, F_i, \dots$: electronic coeffs.

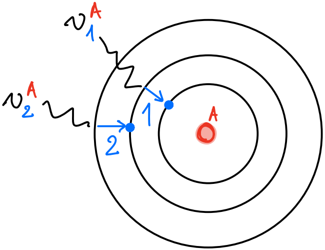
$\mu^{AA'}, \delta\langle r^2\rangle^{AA'}, \dots$: nuclear coeffs.

Z: number of protons

N, N': number of neutrons in A, A'

The King-Plot: Trade Data for Nuclear Physics

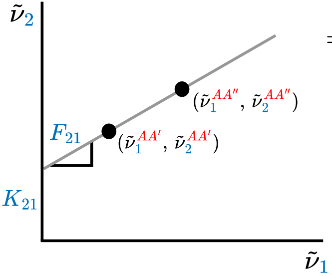
[W. King, J. Opt. Soc. Am. 53, 638 (1963)]



Issue: Large uncertainty on charge radius variance $\delta\langle r^2\rangle^{AA'}$

$\Rightarrow$ Measure isotope shifts for 2 transitions

$$\nu_1^{AA'} = K_1\mu^{AA'} + F_1\delta\langle r^2\rangle^{AA'}$$
$$\nu_2^{AA'} = K_2\mu^{AA'} + F_2\delta\langle r^2\rangle^{AA'}$$



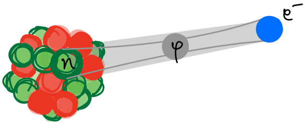
$\Rightarrow$ Eliminate charge radius variance $\delta\langle r^2\rangle^{AA'}$

$$\tilde{\nu}_2^{AA'} = K_{21} + F_{21}\tilde{\nu}_1^{AA'}$$

$\tilde{\nu}_i^{AA'} \equiv \nu_i^{AA'}/\mu^{AA'}$ $\Rightarrow$ data

$F_{21} \equiv F_2/F_1$ $K_{21} \equiv K_2 - F_{21}K_1$ $\Rightarrow$ fit

King-Plot Bounds on New Bosons [arXiv:1704.05068,2005.06144]



New effective Yukawa-potential

$$V_\phi(r) = -\alpha_{\mathrm{NP}}(A-Z)\frac{e^{-m_\phi r}}{r}$$

with $\alpha_{\mathrm{NP}} = (-1)^s \frac{y_e y_n}{4\pi}$, $s = 0, 1, 2$ (spin)

Induces new term in the King-relation:

$$\tilde{\nu}_2^{AA'} = K_{21}\tilde{\mu}^{AA'} + F_{21}\tilde{\nu}_1^{AA'} + \alpha_{\mathrm{NP}} X_{21}\tilde{\gamma}^{AA'}$$

$X_{21} = X_2 - F_{21}X_1$: NP electronic coefficient

$\tilde{\gamma}^{AA'} \equiv (A - A')/\mu^{AA'}$: NP nucl. coeff.

$\Rightarrow$ Extract α_{NP} from fraction of volumes spanned by frequency vectors:

$$\alpha_{\mathrm{NP}} = \frac{Vol.}{Vol.|_{th,\alpha_{\mathrm{NP}}=1}} = \frac{\det\left(\vec{\tilde{\nu}}_1, \vec{\tilde{\nu}}_2, \vec{\tilde{\mu}}\right)}{\varepsilon_{ijk}\det\left(X_i\vec{\tilde{\gamma}}, \vec{\nu}_j, \vec{\nu}_k\right)}$$

$\{\vec{\nu}_i\}$: data vectors in isotope-pair space, $\vec{\tilde{\mu}} \equiv (1, 1, 1)$, X_i, $\vec{\tilde{\gamma}}$: theory input

α_{NP} from Determinants

(No-Mass King-Plot:)

$$\begin{aligned}
\vec{\nu}_1 =& K_1\vec{\mu} + F_1\overrightarrow{\delta\langle r^2\rangle} + \alpha_{\mathrm{NP}} X_1 \vec{\gamma} \\
\vec{\nu}_2 =& K_2\vec{\mu} + F_2\overrightarrow{\delta\langle r^2\rangle} + \alpha_{\mathrm{NP}} X_2 \vec{\gamma} \\
\vec{\nu}_3 =& K_3\vec{\mu} + F_3\overrightarrow{\delta\langle r^2\rangle} + \alpha_{\mathrm{NP}} X_3 \vec{\gamma} \\
\Rightarrow \det\left(\vec{\nu}_1, \vec{\nu}_2, \vec{\nu}_3\right) =& \alpha_{\mathrm{NP}} \det\left(\vec{K}, \vec{F}, \vec{X}\right) \det\left(\vec{\mu}, \overrightarrow{\delta\langle r^2\rangle}, \vec{\gamma}\right) \\
\Rightarrow \alpha_{\mathrm{NP}} =& \frac{Vol}{Vol|_{th,\alpha_{\mathrm{NP}}=1}} = \frac{\det\left(\vec{\nu}_1, \vec{\nu}_2, \vec{\nu}_3\right)}{\det\left(\vec{K}, \vec{F}, \vec{X}\right) \det\left(\vec{\mu}, \overrightarrow{\delta\langle r^2\rangle}, \vec{\gamma}\right)} \\
=& \frac{\det\left(\vec{\nu}_1, \vec{\nu}_2, \vec{\nu}_3\right)}{\frac{1}{2}\varepsilon_{ijk} \det\left(X_i\vec{\gamma}, \vec{\nu}_j, \vec{\nu}_k\right)}
\end{aligned}$$

Choose your King-Plot

Extraction of α_{NP} using the "determinant method" requires

Type of King-Plot	Isotope-Pairs	Transitions	
Generalised King-Plot:	n	$n-1$	[PRR 2, 043444 (2020)]
No-Mass King-Plot:	n	n	[PRR 2, 043444 (2020)]

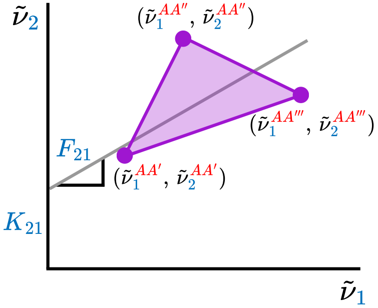
$n \geq 3$ (else cannot search for nonlinearities)

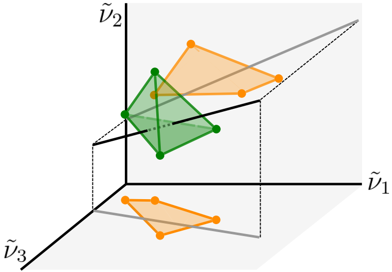
$$\alpha_{\mathrm{NP}} = \frac{V}{V|_{\mathrm{th},\alpha_{\mathrm{NP}}=1}} = \frac{(n-2)!\,\det\left(\vec{\tilde{\nu}}_1, \ldots, \vec{\tilde{\nu}}_{n-1}, \vec{\tilde{\mu}}\right)}{\varepsilon_{i_1,\ldots,i_{n-1}} \det\left(X_{i_1}\vec{\tilde{\gamma}}, \vec{\tilde{\nu}}_{i_2}, \ldots, \vec{\tilde{\nu}}_{i_{n-1}}, \vec{\tilde{\mu}}_{i_n}\right)}$$

$$\alpha_{\mathrm{NP}} = \frac{v}{v|_{\mathrm{th},\alpha_{\mathrm{NP}}=1}} = \frac{(n-1)!\,\det\left(\vec{\nu}_1, \vec{\nu}_2, \ldots, \vec{\nu}_n\right)}{\varepsilon_{i_1,i_2,\ldots,i_n} \det\left(X_{i_1}\vec{\gamma}, \vec{\nu}_{i_2}, \ldots, \vec{\nu}_{i_n}\right)}$$

King-Plot Method in Presence of Nuclear Effects: The Generalised King Plot

[arXiv:1704.05068,2005.06144]





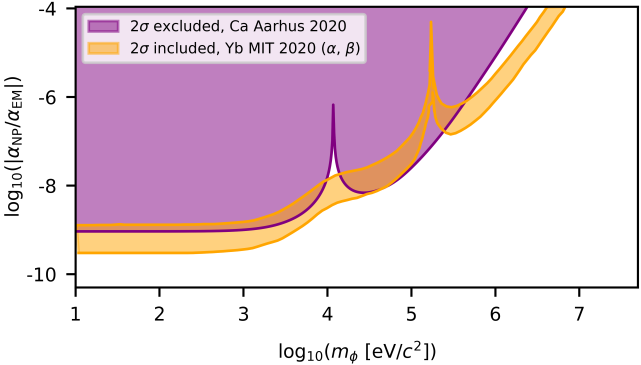
⇒ test King linearity

⇒ account for one King nonlinearity

⇒ put bound on 2^{nd}

⇒ **King-plot method also works in presence of nuclear effects.**

Upper Bounds on $|\alpha_{\mathrm{NP}}|$ vs. New Mediator Mass m_ϕ



Nonlinear King plot relation:

$$\tilde{\nu}_2^{AA'} = K_{21}\tilde{\mu}^{AA'} + F_{21}\tilde{\nu}_1^{AA'} + G_{21}^{(2)}\delta\langle r^2\rangle^2 + G_{21}^{(4)}\delta\langle r^4\rangle + \ldots ?$$

X Coefficients

Overlap of new physics potential and electronic wavefunction

$$X_i = \int \mathrm{d}^3 r \frac{e^{-m_\phi r}}{r} \left[|\psi_b(r)|^2 - |\psi_a(r)|^2 \right]$$

$|\psi(r)|^2$: electron density in absence of new physics,

a, b initial, final states

Requirement for searches for new light bosons:

- At least one of ψ_a or ψ_b should have good overlap with new potential.
- For tight bounds on α_{NP}, one X_i needs to be large.

Recipe for the Nonlinearity Decomposition Plot

[arXiv:2004.11383, arXiv:2201.03578]

1. Arrange the isotope-shift data for all transitions $\tau \in \{\alpha, \beta, \gamma, \delta, \epsilon\}$ in n-vectors $\tilde{\boldsymbol{\nu}}_\tau$, where n is the number of isotope pairs (here 4):

$$\tilde{\boldsymbol{\nu}}_\tau = \left(\tilde{\nu}_\tau^{168,170}, \tilde{\nu}_\tau^{170,172}, \tilde{\nu}_\tau^{172,174}, \tilde{\nu}_\tau^{174,176}\right)$$

2. Choose a reference transition, say δ.
3. Plane of King linearity is defined by the relations ($\mathbf{1} = (1, 1, 1, 1)$)

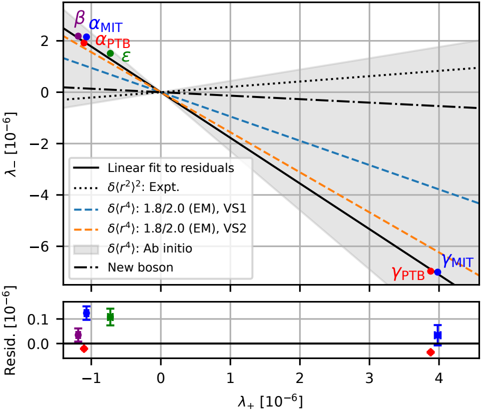
$$\tilde{\boldsymbol{\nu}}_\tau \approx F_{\tau\delta} \tilde{\boldsymbol{\nu}}_\delta + K_{\tau\delta} \mathbf{1}\,.$$

4. Define two ($n = 4$)$-$vectors $\Lambda_\pm$ that are orthogonal to $\tilde{\boldsymbol{\nu}}_\delta$, $\mathbf{1}$.
5. Project all isotope-shift data onto the four vectors $\tilde{\boldsymbol{\nu}}_\delta$, $\mathbf{1}$, Λ_+, Λ_-:

$$\tilde{\boldsymbol{\nu}}_\tau = \begin{pmatrix} \tilde{\boldsymbol{\nu}}_\delta & \mathbf{1} & \boldsymbol{\Lambda}_+ & \boldsymbol{\Lambda}_- \end{pmatrix} \begin{pmatrix} F_{\tau\delta} & K_{\tau\delta} & \lambda_+^{(\tau)} & \lambda_-^{(\tau)} \end{pmatrix}^T$$

6. Plot all points $(\lambda_+^{(\tau)}, \lambda_-^{(\tau)})$ in the same plane.

The Nonlinearity Decomposition Plot



Notation	Transition	Refs.
$\alpha_{\rm MIT,PTB}$	${}^2S_{1/2} \to {}^2D_{5/2}$ E2 in Yb^+	MIT, t.w.
β	${}^2S_{1/2} \to {}^2D_{3/2}$ E2 in Yb^+	MIT
$\gamma_{\rm MIT,PTB}$	${}^2S_{1/2} \to {}^2F_{7/2}$ E3 in Yb^+	MIT, t.w.
δ	${}^1S_0 \to {}^3P_0$ in Yb	Kyoto
ϵ	${}^1S_0 \to {}^1D_2$ in Yb	Mainz

- $\delta\langle r^2\rangle^2$ estimated using Angeli & Marinova Tables of experimental nuclear ground state charge radii
- $\delta\langle r^4\rangle$: Calculations by group of Prof. Achim Schwenk, TU Darmstadt

In presence of just one nonlinearity,

$$\tilde{\boldsymbol{\nu}}_j \approx F_{j1}\tilde{\boldsymbol{\nu}}_1 + K_{j1}\mathbf{1} + G_{j1}^{(4)}\boldsymbol{\delta}\widetilde{\langle \boldsymbol{r}^4\rangle}\,, \qquad j>1.$$

slope: $\dfrac{\lambda_-}{\lambda_+} \equiv \dfrac{G_{j1}^{(4)}\boldsymbol{\delta}\langle\widetilde{\boldsymbol{r}^4}\rangle_-}{G_{j1}^{(4)}\boldsymbol{\delta}\langle\widetilde{\boldsymbol{r}^4}\rangle_+} = \dfrac{\boldsymbol{\delta}\langle\widetilde{\boldsymbol{r}^4}\rangle_-}{\boldsymbol{\delta}\langle\widetilde{\boldsymbol{r}^4}\rangle_+} \Rightarrow$ transition-universal

Extracting Nuclear Physics from Isotope-Shift Measurements

- **Assuming $\delta\langle r^4\rangle$ dominates,** what does the isotope-shift data tell us about the evolution of $\delta\langle r^4\rangle$ along the isotope chain?

$\Rightarrow$ **"Put the King plot on it's head.":**

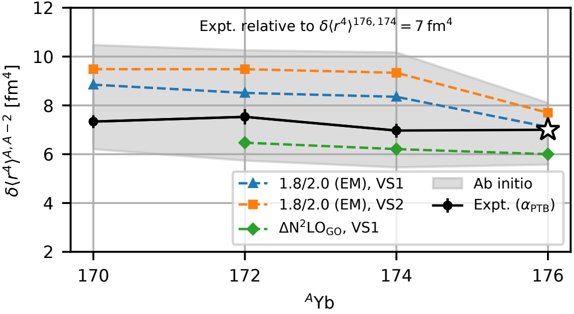
1. Instead of eliminating $\delta\langle r^2\rangle$ from the system of equations, we use experimental data (Angeli & Marinova) to determine it.
2. Perform a fit to determine the field shift coefficient F_τ from the data.
3. Use theoretical input for the electronic coefficient $G_\tau^{(4)}$ (J. Berengut)
4. Solve for object

$$Q^{AA',RR'} \equiv \delta\langle r^4\rangle^{AA'} - \frac{\mu^{AA'}}{\mu^{RR'}}\delta\langle r^4\rangle^{RR'} \,,$$

where RR': reference isotope pair, AA': any of remaining isotope pairs.

Extracting Nuclear Physics from Isotope-Shift Measurements

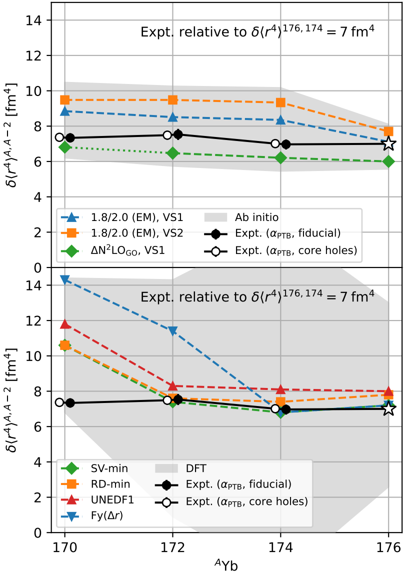
- **Assuming $\delta\langle r^4\rangle$ dominates,** what does the isotope-shift data tell us about the evolution of $\delta\langle r^4\rangle$ along the isotope chain?



blue, **orange**, **green**: Calculations by group of Prof. Achim Schwenk

black: new spectroscopic method, fixed at $\star$

$\delta\langle r^4\rangle$ Calculations: Ab initio vs. DFT



"Ab initio": Starting from chiral effective field theory interactions

DFT: Density Functional Theory

- Experimental $\delta\langle r^4\rangle^{AA'}$ values relative to $\delta\langle r^4\rangle^{176,174} = 7$ fm^4 extracted from isotope shifts from the α transition using atomic theory (fiducial, core holes)
- Above: ab initio calculations (t.w.)
- Below: density functional theory calculations (PRL.128.163201)
- Gray bands: estimated theory uncertainties

Advantages of Nuclear Clocks wrt. Atomic Clocks

+ Nucleus $\ll$ Atom $\Rightarrow$ Shielded from external fields
  $\Rightarrow$ **Higher accuracy**
+ Nucleus less polarisable than atom $\Rightarrow$ **Higher accuracy**
+ Use solids? $\Rightarrow$ Higher statistics $\Rightarrow$ **Higher stability**
+ Higher frequency $\Rightarrow$ **Higher stability**
+ Probes QCD $\Rightarrow$ **Sensitive to NP coupling to QCD**
+ Low transition frequency due to accidental cancellation (?)

$$\Delta E = \Delta E_{\mathsf{EM}} + \Delta E_{\mathsf{nuc}} \qquad \Delta E \ll |\Delta E_{\mathsf{EM}}| \sim |\Delta E_{\mathsf{nuc}}|$$

$$8 \text{ eV} \ll 0.1 \text{ MeV}$$

$\Rightarrow$ **Extraordinary sensitivity to new physics?**

[arXiv:2012.09304,2407.17526]

Sensitivity of Nuclear Clocks to New Physics

[arXiv:2012.09304,2407.17526]

$$\Delta E = \Delta E_{\mathsf{EM}} + \Delta E_{\mathsf{nuc}} \qquad \Delta E \ll |\Delta E_{\mathsf{EM}}| \sim |\Delta E_{\mathsf{nuc}}|$$

$$8 \text{ eV} \ll 0.1 \text{ MeV}$$

$$\frac{\delta(\Delta E)}{\Delta E} = \frac{1}{\Delta E}\left(\frac{\partial \Delta E_{\mathsf{EM}}}{\partial \alpha_{\mathsf{EM}}}\delta\alpha_{\mathsf{EM}} + \frac{\partial \Delta E}{\partial \alpha_s}\delta\alpha_s\right)$$

$$K_{\mathsf{EM}} \equiv \frac{1}{\Delta E}\frac{\partial \Delta E_{\mathsf{EM}}}{\partial \log \alpha_{\mathsf{EM}}} \simeq \frac{\Delta E_{\mathsf{EM}}}{\Delta E} \sim 10^5$$

$$K_s^{\mathsf{EM}} \equiv \frac{1}{\Delta E_{\mathsf{EM}}}\frac{\partial \Delta E_{\mathsf{EM}}}{\partial \log \alpha_s} \sim \beta K_{\mathsf{EM}}\,, \qquad \beta \sim \mathcal{O}(1)?$$

$$K_s^{\mathsf{EM}} \sim K_s^{\mathsf{nuc}}?$$

Some Phenomenology of Ultralight New Physics

- ϕ oscillates around potential minimum (cold dark matter):

$$\phi(t,x) \sim \phi_0 \cos(m_\phi t)$$

- Interacts with the Standard Model:

$$\mathcal{L}_\phi \supset \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}m_\phi\phi^2 + \frac{\phi}{M_{\rm Pl}}\left[\frac{d_e}{4e^2}F_{\mu\nu}F^{\mu\nu} - \frac{d_g\beta_s}{2g_s}G^a_{\mu\nu}G^{a\mu\nu} - d_{m_e}m_e\bar{e}e - \sum_{q=u,d}\left(d_{m_q}+\gamma_{m_q}d_g\right)m_q\bar{q}q\right]$$

$\Rightarrow$ Oscillating fundamental constants $\alpha_{\rm EM}$, $\Lambda_{\rm QCD}$, m_f

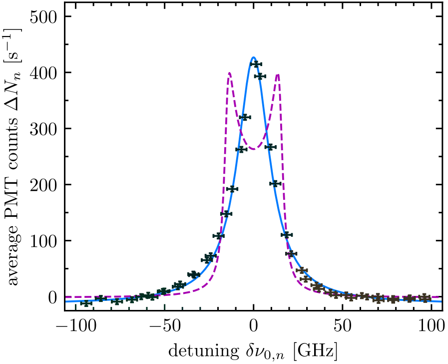
$\Rightarrow$ Oscillating transition frequencies

$$\nu \sim \nu_0\left(1 + \left(K_g d_g + K_e d_e + \ldots\right)\phi(t)/M_{\rm Pl}\right)$$

$$\Rightarrow\ \nu(t) \simeq \nu_0 + \delta\nu_{\rm DM}\cos\left(2\pi\nu_{\rm DM}t + \varphi_{\rm DM}\right)$$

Nuclear Lineshape Analysis in the Limit $\delta\nu_{\mathrm{DM}} \gg \nu_{\mathrm{DM}}$

$$\nu(t) \simeq \nu_0 + \delta\nu_{\mathrm{DM}} \cos\left(2\pi\nu_{\mathrm{DM}} t + \varphi_{\mathrm{DM}}\right)$$



- In absence of DM, $I(\nu) = \delta(\nu - \nu_0)$
- In presence of DM, average over $T_{\mathrm{DM}} = 1/\nu_{\mathrm{DM}}$:

$$\langle I(\nu)\rangle_{T_{\mathrm{DM}}} = \int_0^{T_{\mathrm{DM}}} \frac{\mathrm{d}t}{T_{\mathrm{DM}}}\delta(\nu - \nu(t))$$
$$= \frac{\theta\left(1 - \left|\frac{\nu-\nu_0}{\delta\nu_{\mathrm{DM}}}\right|\right)/\pi}{\sqrt{\delta\nu_{\mathrm{DM}}^2 - (\nu - \nu_0)^2}}$$

$\Rightarrow$ Convolve with resonance lineshape

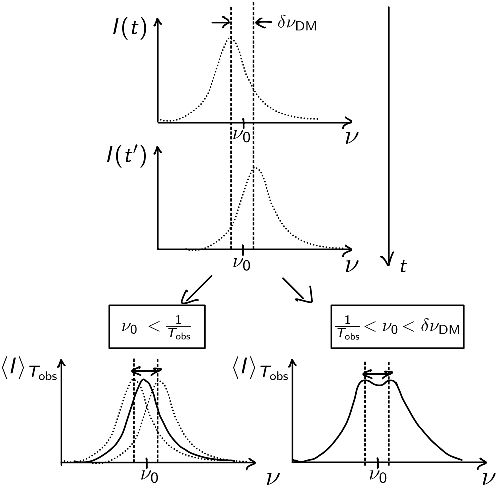
$\Rightarrow$ Take into account experimental procedure

$\Rightarrow$ ...

$\Rightarrow$ Curve fit (MCMC/ODR)

Nuclear Lineshape Analysis Regimes: Current

$$\boxed{\nu(t) = \nu_0 + \delta\nu_{\mathrm{DM}} \cos\left(2\pi\nu_{\mathrm{DM}} t + \varphi_{\mathrm{DM}}\right)}$$



Drift of resonance frequency

Line broadening

Nuclear Lineshape Analysis Regimes: Future