

Natural Alignment and CP Violation Beyond the Standard Model

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Based on:

- P.S.B. Dev, AP, JHEP1412 (2014) 024
- AP, PRD93 (2016) 075012; [arXiv:2408.04511](#)
- N. Darvishi, AP, PRD99 (2019) 115014; PRD101 (2020) 095008 . . .
- N. Darvishi, AP, J.-H. Yu, JHEP05 (2024) 233

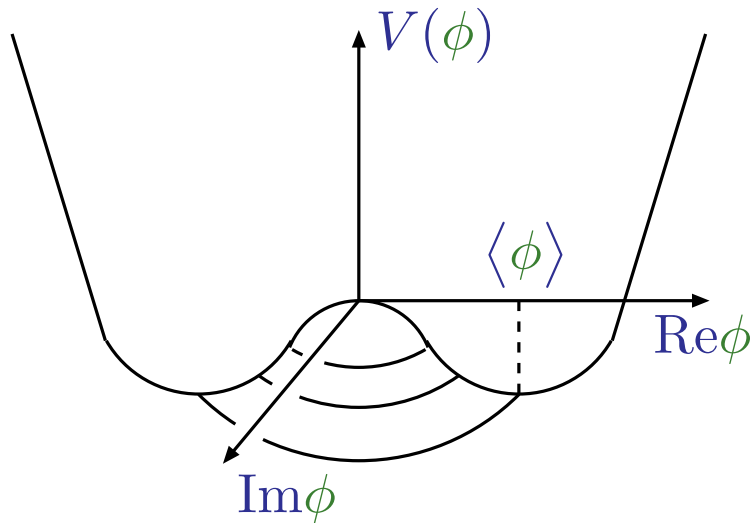
Outline:

- The Standard Theory of Electroweak **Symmetry Breaking**
- Brief history of **symmetries** for **natural SM alignment**
- SM **Alignment** in the 2HDM and **Beyond**
- **CP Violation** in 2HDM: **Explicit, Spontaneous and Mixed**
- Phenomenology at the **LHC**
- Summary

• The Standard Theory of Electroweak Symmetry Breaking

Higgs Mechanism in the SM: $SU(3)_{\text{colour}} \otimes SU(2)_L \otimes U(1)_Y$

[P. W. Higgs '64; F. Englert, R. Brout '64.]



Higgs potential $V(\phi)$

$$V(\phi) = -m^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 .$$

Ground state:

$$\langle \phi \rangle = \sqrt{\frac{m^2}{2\lambda}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

carries weak charge, but **no** electric charge and **colour**.

Custodial Symmetry of the SM with $g' = Y_f = 0$ and $V(\phi)$:

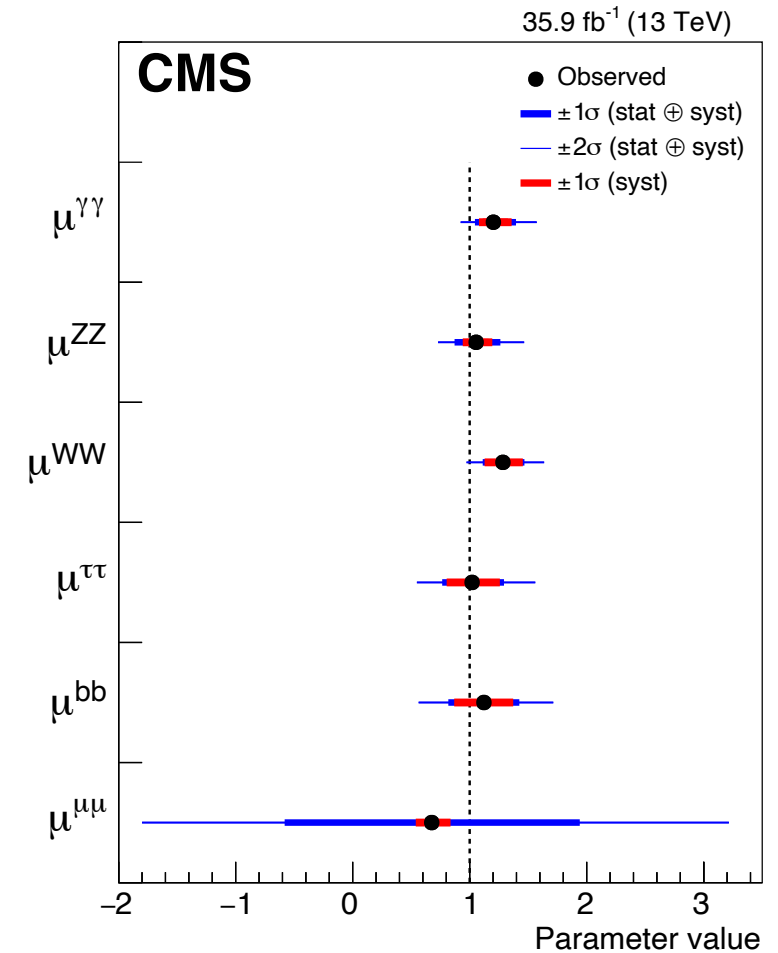
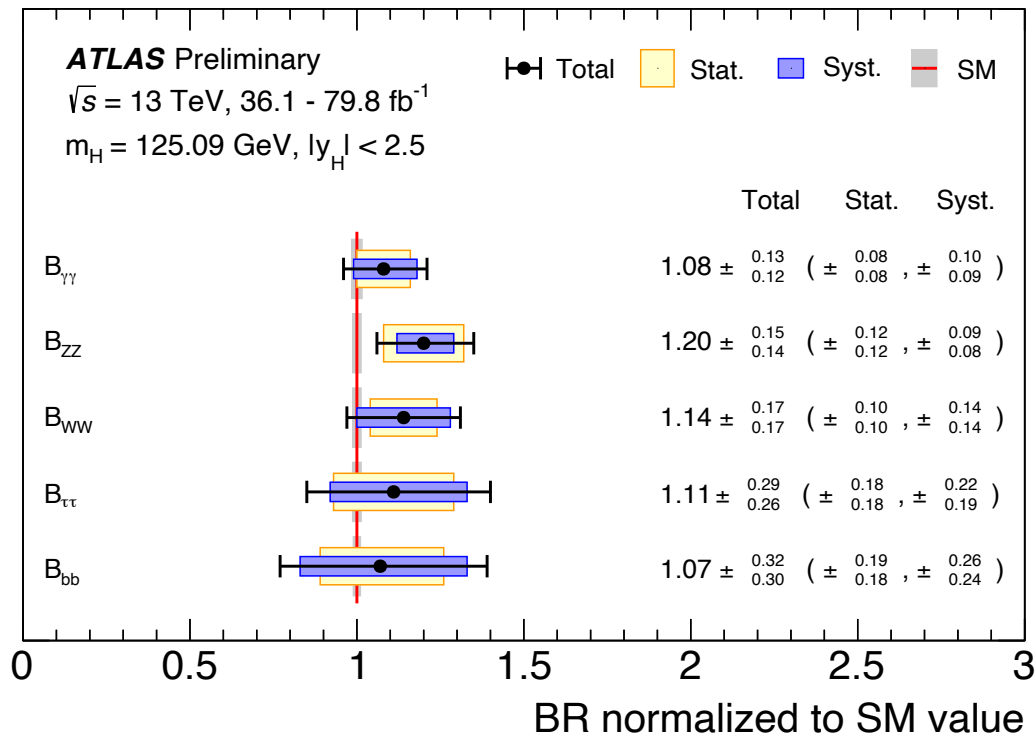
[P. Sikivie, L. Susskind, M. B. Voloshin, V. I. Zakharov '80.]

$$\Phi \equiv (\phi, i\sigma^2 \phi^*) \mapsto \Phi' \equiv U_L \Phi U_C ,$$

with $U_L \in SU(2)_L$ and $U_C \in SU(2)_C$, and $SU(2)_L \otimes SU(2)_C / \mathbb{Z}_2 \simeq SO(4)$.

On the SM Higgs-Boson at the LHC

ATLAS & CMS Results 2019:



• Brief history of **symmetries** for natural SM alignment

- Flavour unitarity of the CKM mixing matrix
[Gell-Mann, Levy '60; Cabbibo '63; Kobayashi, Maskawa '73]
- GIM mechanism to suppress s -changing interactions at the quantum level
(requires the existence of the c -quark) [Glashow, Iliopoulos, Maiani '70]
- Conditions for diagonal neutral currents in Z -boson interactions to quarks
[Paschos '77]
- **Natural** diagonal neutral currents in Z - & multi-Higgs-boson interactions to quarks
[Glashow, Weinberg '77; Peccei, Quinn '77]
- Renormalizable models with partial flavour **non-conservation** at tree level
(**GIM suppressed**). [Branco, Grimus, Lavoura '96]
- Yukawa **alignment** in the 2HDM broken by RG effects
(induced by non-diagonal PQ charges of Higgs doublets) [Pich, Tuzon '09]
- **Natural Alignment Beyond** the Standard Model [Dev, Pilaftsis '14, AP '16
(for all Higgs couplings to gauge bosons and fermions) & this talk]

• SM Alignment in the 2HDM and Beyond

• 2HDM potential

[TD Lee '73; AP, C Wagner '99;

Review: Branco, Ferreira, Lavoura, Rebelo, Sher, Silva '12.]

$$\begin{aligned}
 V = & -\mu_1^2(\phi_1^\dagger\phi_1) - \mu_2^2(\phi_2^\dagger\phi_2) - m_{12}^2(\phi_1^\dagger\phi_2) - m_{12}^{*2}(\phi_2^\dagger\phi_1) \\
 & + \lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\
 & + \frac{\lambda_5}{2}(\phi_1^\dagger\phi_2)^2 + \frac{\lambda_5^*}{2}(\phi_2^\dagger\phi_1)^2 + \lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + \lambda_6^*(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_1) \\
 & + \lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + \lambda_7^*(\phi_2^\dagger\phi_2)(\phi_2^\dagger\phi_1) .
 \end{aligned}$$

• Physical (CP-conserving) spectrum:

CP-even Higgs bosons H and h ; CP-odd scalar a ; charged scalars h^\pm .

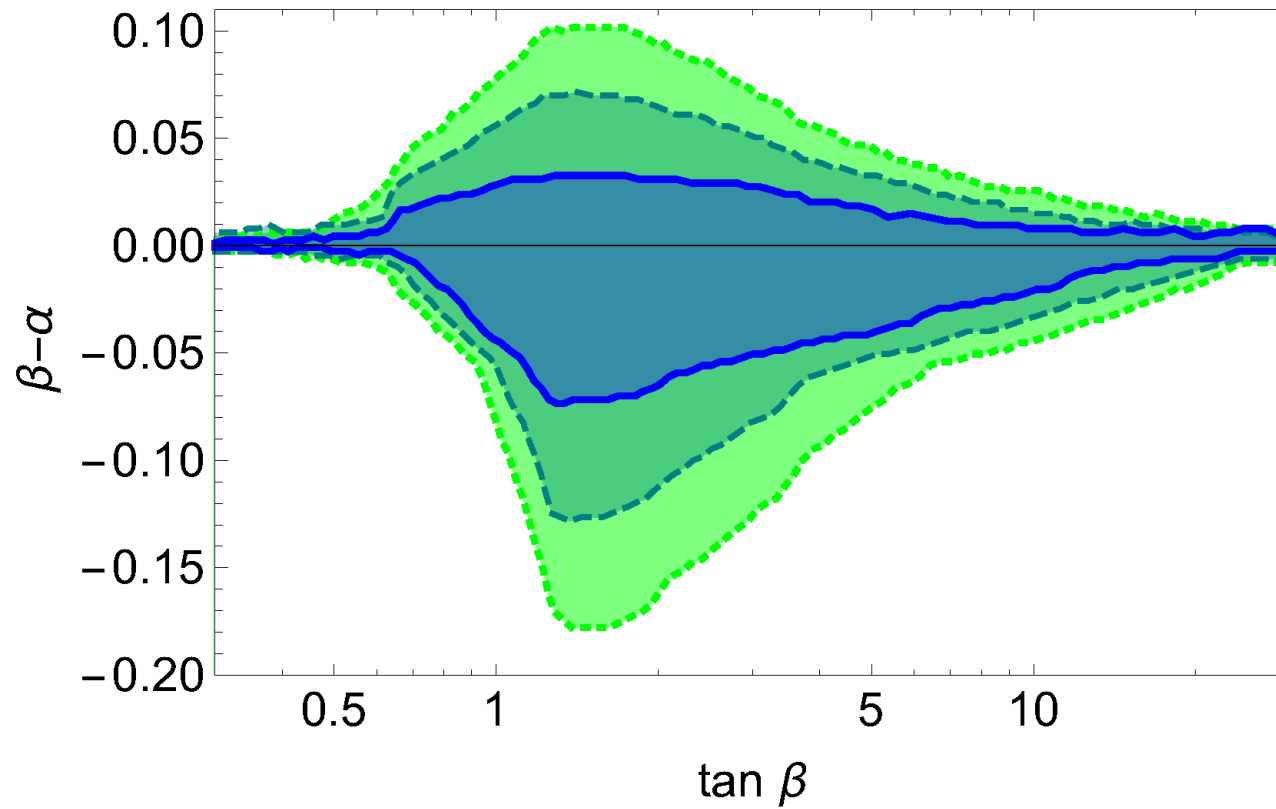
• Higgs coupling to gauge bosons $V = W, Z$:

$$g_{HVV} = \cos(\beta - \alpha) , \quad g_{hVV} = \sin(\beta - \alpha) ,$$

where $\tan \beta = \langle \phi_2 \rangle / \langle \phi_1 \rangle$ and α diagonalizes the CP-even mass matrix.

- **Global fit to SM mis-alignment**

[e.g. D. Chowdhury, O. Eberhardt, JHEP11 (2015) 052.]



Pheno favoured limit $\beta \rightarrow \alpha$: $g_{HVV} = \cos(\beta - \alpha) \rightarrow g_{H_{SM}VV} = 1$.

- **SM Alignment** $\beta \rightarrow \alpha$:

- (i) **(Non-)Decoupling:** [Appelquist, Carazzone '74; Georgi, Nanopoulos '79; Donoghue, Li '79; Gunion, Haber '03; Ginzburg, Krawczyk '05.]

$$M_h^2 \simeq M_a^2 + \lambda_5 v^2 \simeq M_{h^\pm}^2 \gg v_{\text{SM}}^2$$

$$M_H^2 \simeq 2\lambda_{\text{SM}} v^2 - \frac{v^4 s_\beta^2 c_\beta^2}{M_a^2 + \lambda_5 v^2} \left[s_\beta^2 (2\lambda_2 - \lambda_{345}) - c_\beta^2 (2\lambda_1 - \lambda_{345}) + \dots \right]^2$$

- (ii) **Fine-tuning:** [Krawczyk et al. '99; Carena, Low, Shah, Wagner '14; Dev, AP '14.]

$$\lambda_7 t_\beta^4 - (2\lambda_2 - \lambda_{345}) t_\beta^3 + 3(\lambda_6 - \lambda_7) t_\beta^2 + (2\lambda_1 - \lambda_{345}) t_\beta - \lambda_6 = 0$$

(iii) **Natural SM Alignment** (independent of M_{h^\pm} and t_β): [PSB Dev, AP '14
N Darvishi, AP, J-H Yu, arXiv:2312.00882]

$$\lambda_1 = \lambda_2 = \frac{\lambda_{345}}{2} \quad (\text{with } \lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5), \quad \lambda_6 = \lambda_7 = 0$$

Symmetries:

- | | |
|---|-----------------------------|
| 1. $Sp(4)$: | $\lambda_4 = \lambda_5 = 0$ |
| 2. $SU(2)_{HF} \times U(1)_Y$: | $\lambda_5 = 0$ |
| 3. $CP \times SO(2)_{HF} \times U(1)_Y$: | $\lambda_{3,4,5} \neq 0$ |
| 4* $CP \times Sp(2)_{\phi_1 \tilde{\phi}_2} \times Sp(2)_{\phi_2 \tilde{\phi}_1}$: | $\lambda_4 = -\lambda_5$ |
| 5. $SO(2)_{HF} \times Sp(2)_{\phi_1 + \phi_2}$: | $\lambda_4 = \lambda_5$ |

*Noted by Aiko, Kanemura '21

$Sp(4)$ Majorana field multiplet:

$$\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ i\sigma^2 \phi_1^* \\ i\sigma^2 \phi_2^* \end{pmatrix}, \quad \Phi = C\Phi^* \quad [\text{Battye, Brawn, AP, '11; AP, '12 AP '24}]$$

$\Rightarrow \Phi$ as a Dirac fermion: arXiv:2408.04511

References (*an incomplete list on SM Alignment in the 2HDM*)

- **On the SM Higgs basis (also Decoupling of FCNC Effects):**
H. Georgi and D. V. Nanopoulos, Phys. Lett. B82 (1979) 95.
- **Alignment via Decoupling:**
 - J. F. Gunion, H. E. Haber, Phys. Rev. D67 (2003) 075019.
 - I. F. Ginzburg, M. Krawczyk, Phys. Rev. D72 (2005) 115013.
- **Alignment via Fine-tuning:**
 - P. H. Chankowski, T. Farris, B. Grzadkowski, J. F. Gunion, J. Kalinowski, M. Krawczyk, Phys. Lett. B496 (2000) 195.
 - A. Delgado, G. Nardini, M. Quiros, JHEP1307 (2013) 054.
 - M. Carena, I. Low, N. R. Shah, C. E. M. Wagner, JHEP1404 (2014) 015.
- **Natural Alignment without Decoupling and without Fine-tuning:**
 - P.S.B. Dev, AP, JHEP1412 (2014) 024.
 - B. Grzadkowski, O. M. Ogreid, P. Osland, Phys. Rev. D94 (2016) 115002.

References (*an incomplete list on symmetries in the 2HDM*)

- **Spontaneous CP Violation:** T. D. Lee, Phys. Rev. D8 (1973) 1226;
G.C. Branco, Phys. Rev. D22 (1980) 2901;
G.C. Branco, M.N. Rebelo, Phys. Lett. B160 (1985) 117.
- **Z_2 symmetry:** S. L. Glashow, S. Weinberg, Phys. Rev. D15 (1977) 1958.
- **PQ U(1) symmetry:** R. D. Peccei, H. R. Quinn, Phys. Rev. Lett. 38 (1977) 1440.
- **Custodial $SU(2)_L$ -preserving symmetry:**
P. Sikivie, L. Susskind, M. B. Voloshin, V. I. Zakharov, Nucl. Phys. B173 (1980) 189.
- **Bilinear formalism:**
M. Maniatis, A. von Manteuffel, O. Nachtmann, F. Nagel, EPJC48 (2006) 805;
C. C. Nishi, Phys. Rev. D74 (2006) 036003.
- **$SU(2)_L \otimes U(1)_Y$ -preserving symmetries: 6**
I. P. Ivanov, Phys. Rev. D75 (2007) 035001;
P. M. Ferreira, H. E. Haber, J. P. Silva, Phys. Rev. D79 (2009) 116004.
- **Custodial $SU(2)_L$ -preserving symmetries: 7**
R. A. Battye, G. D. Brawn, AP, JHEP1108 (2011) 020.
- **On completeness and uniqueness of classification: 6 + 7 = 13**
AP, Phys. Lett. B706 (2012) 465.

• Maximally Symmetric Two Higgs Doublet Model

[P.S.B. Dev, AP '14]

$$G_{\Phi} = SU(2)_L \otimes Sp(4)/Z_2 \simeq SU(2)_L \otimes SO(5).$$

$$V = -\mu^2 \left(|\Phi_1|^2 + |\Phi_2|^2 \right) + \lambda \left(|\Phi_1|^2 + |\Phi_2|^2 \right)^2 = -\frac{\mu^2}{2} \Phi^\dagger \Phi + \frac{\lambda}{4} (\Phi^\dagger \Phi)^2,$$

where

[R. A. Battye, G. D. Brawn, AP, JHEP1108 (2011) 020.]

$$\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ i\sigma^2 \phi_1^* \\ i\sigma^2 \phi_2^* \end{pmatrix}, \quad \text{with } U_L \in SU(2)_L : \Phi \mapsto \Phi' = U_L \Phi,$$

so that under **global field transformations**, [AP, Phys. Lett. B706 (2012) 465 & '24]

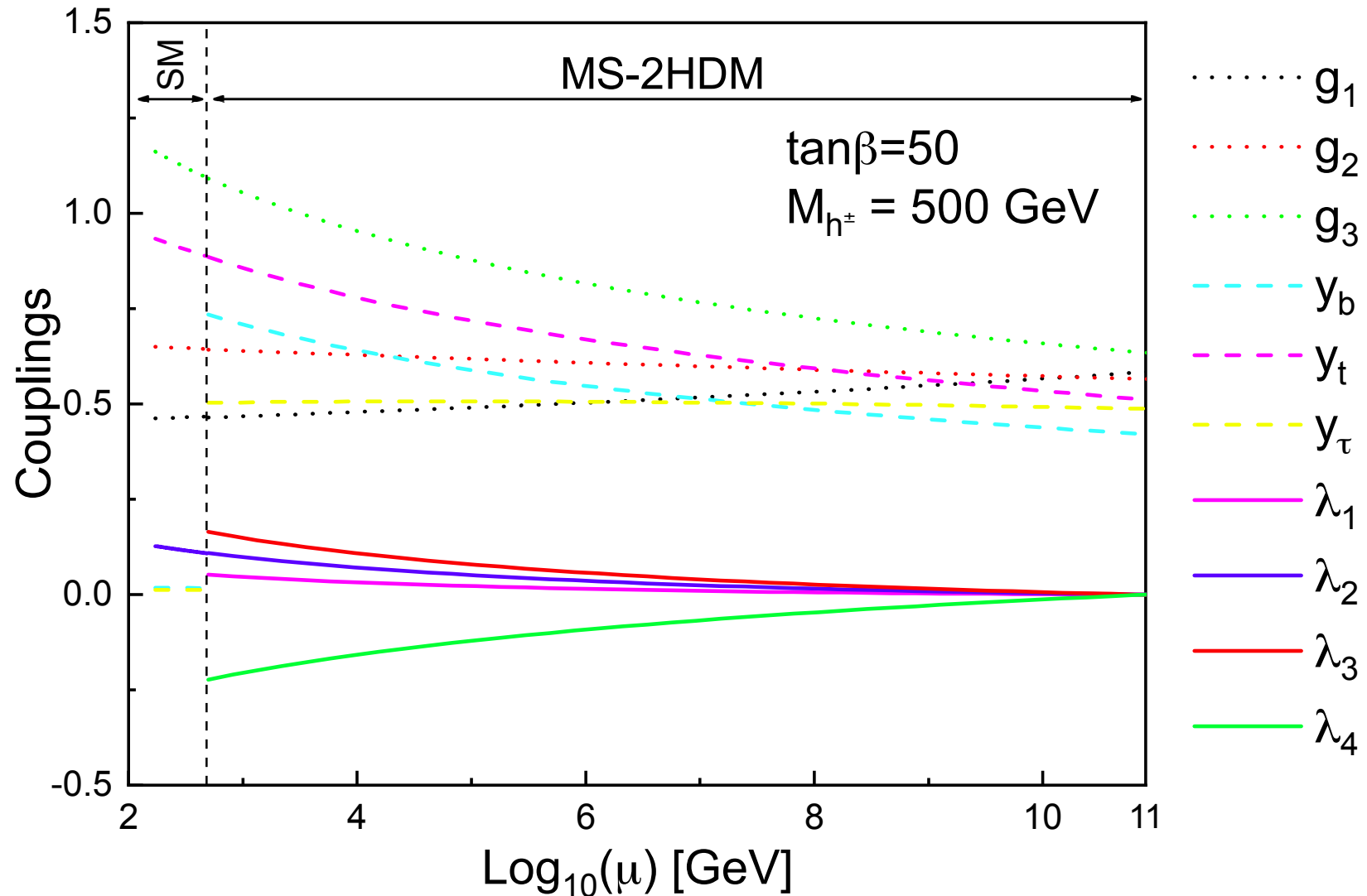
$$Sp(4) : \Phi \mapsto \Phi' = U \Phi, \quad \text{with } U \in U(4) \text{ \& } UCU^T = C \equiv i\sigma^2 \otimes \sigma^0$$

SU(2)_L gauge kinetic terms remain invariant.

Breaking Effects: $-m_{12}^2 \phi_1^\dagger \phi_2$, $U(1)_Y$ coupling g' , Yukawa couplings $\mathbf{Y}^{u,d}$.

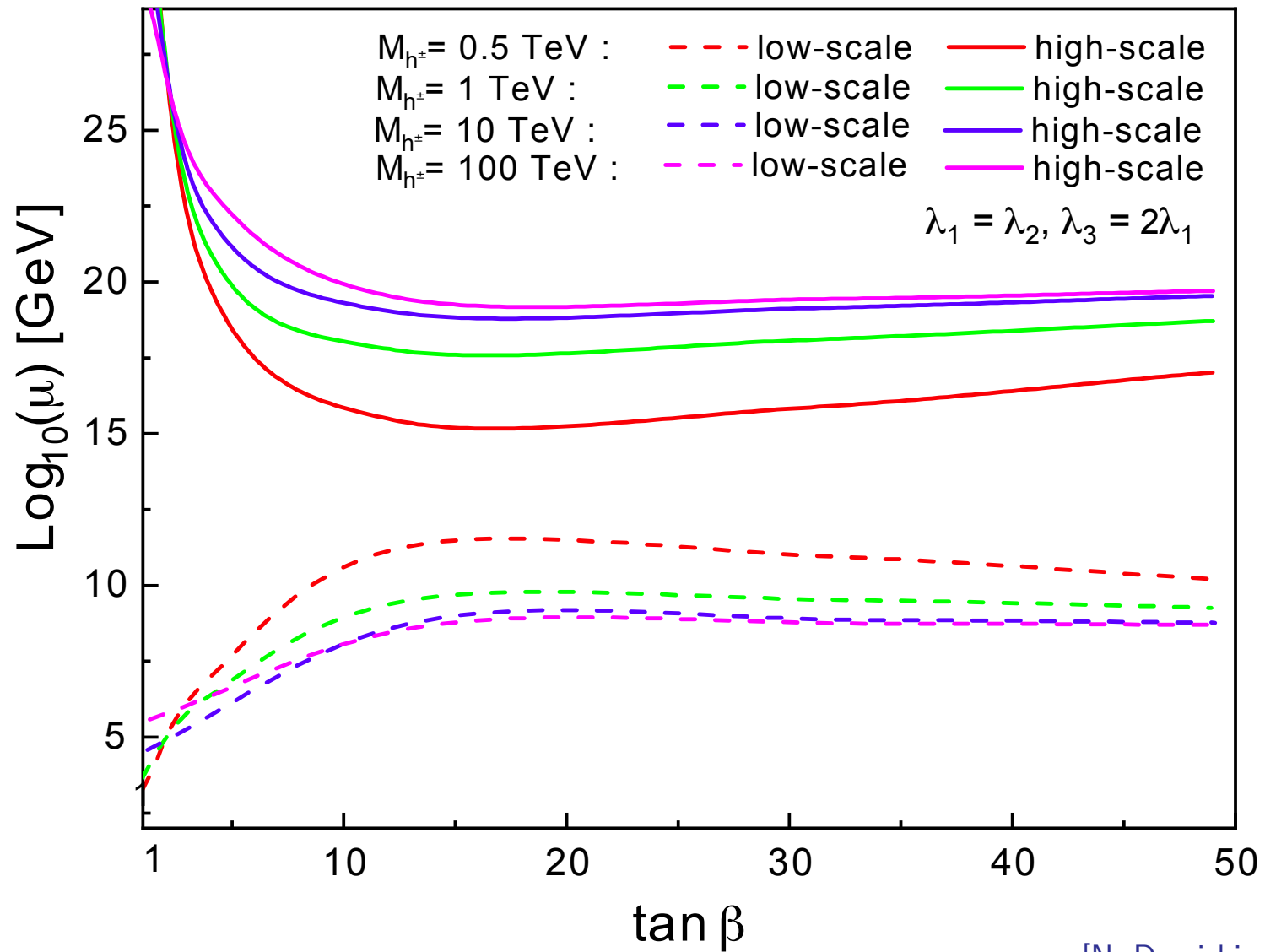
• **Quartic Coupling Unification (two loops)**

[N. Darvishi, AP '19]



First conformal unification point: $\mu_X^{(1)} \sim 10^{11}$ GeV (of order PQ scale)

– Low- and high-scale quartic coupling unification: $\tan \beta$ vs $\mu_X^{(1,2)}$



[N. Darvishi, AP '19]

• Natural Alignment Beyond the 2HDM

[AP '16]

For n HDM with $m < n$ inert scalar doublets, there are still **3** continuous alignment symmetries in the **field space of the non-inert sector**:

$$(i) \quad \text{Sp}(2N_H) \times \mathcal{D}; \quad (ii) \quad \text{SU}(N_H) \times \mathcal{D}; \quad (iii) \quad \text{SO}(N_H) \times \mathcal{CP} \times \mathcal{D},$$

where $N_H = n - m$, \mathcal{D} acts on the inert sector *only*, and \mathcal{CP} is the canonical CP: $\Phi_i(t, \mathbf{x}) \rightarrow \Phi_i^*(t, -\mathbf{x})$ (with $i = 1, 2, \dots, N_H$).

Symmetry **invariants**:

$$(i) \quad S = \Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 + \dots = \frac{1}{2} \Phi^\dagger \Phi$$

$$(ii) \quad D^a = \Phi_1^\dagger \sigma^a \Phi_1 + \Phi_2^\dagger \sigma^a \Phi_2 + \dots$$

$$(iii) \quad T = \Phi_1 \Phi_1^T + \Phi_2 \Phi_2^T + \dots$$

Symmetric part of the **scalar potential**:

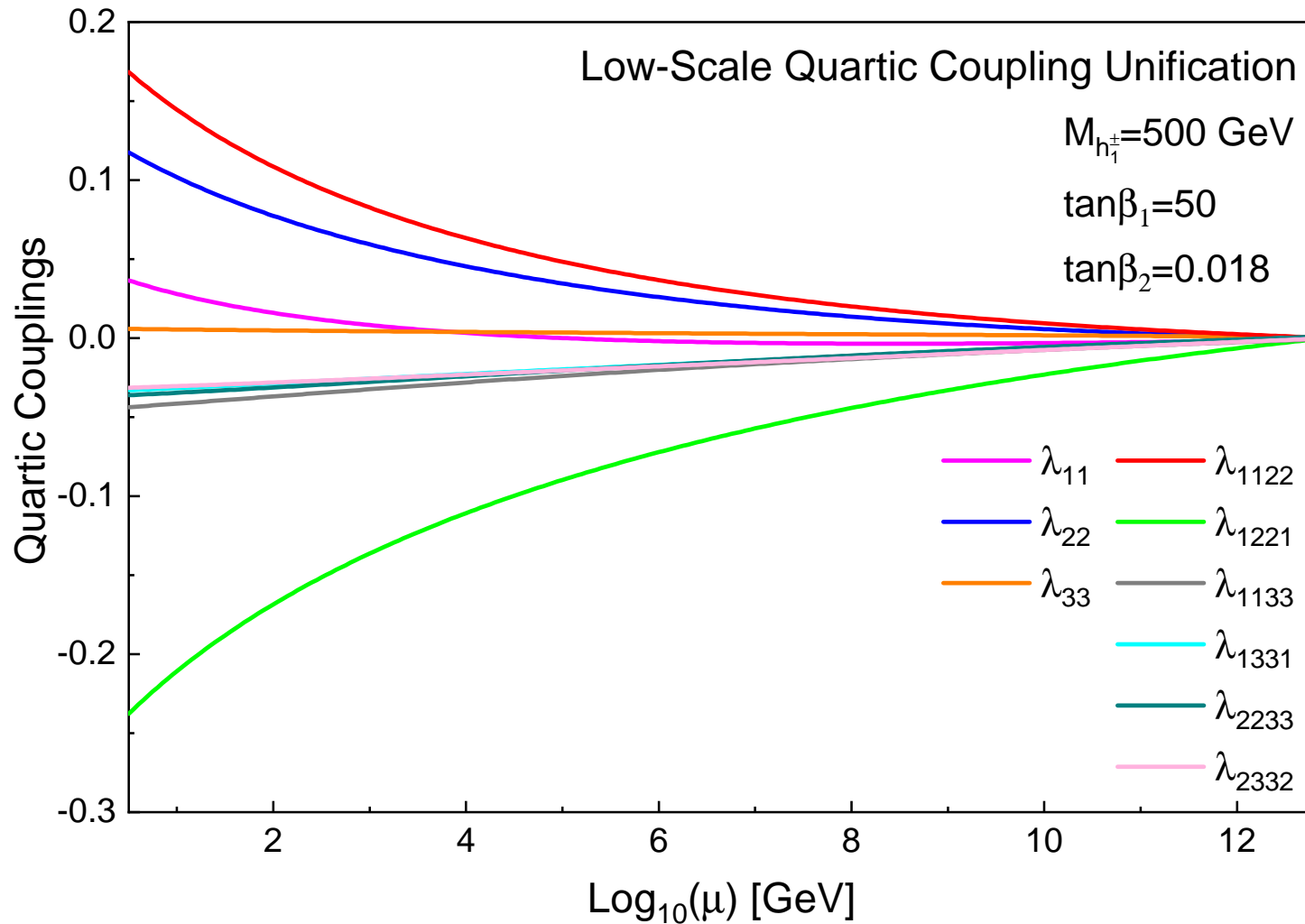
$$V_{\text{sym}} = -\mu^2 S + \lambda_S S^2 + \lambda_D D^a D^a + \lambda_T \text{Tr}(T T^*).$$

Minimal Symmetry of Alignment: $\mathbf{Z}_2^{\text{EW}} \times \mathbf{Z}_2^{\text{I}}$.

– Quartic Coupling Unification in the MS-3HDM

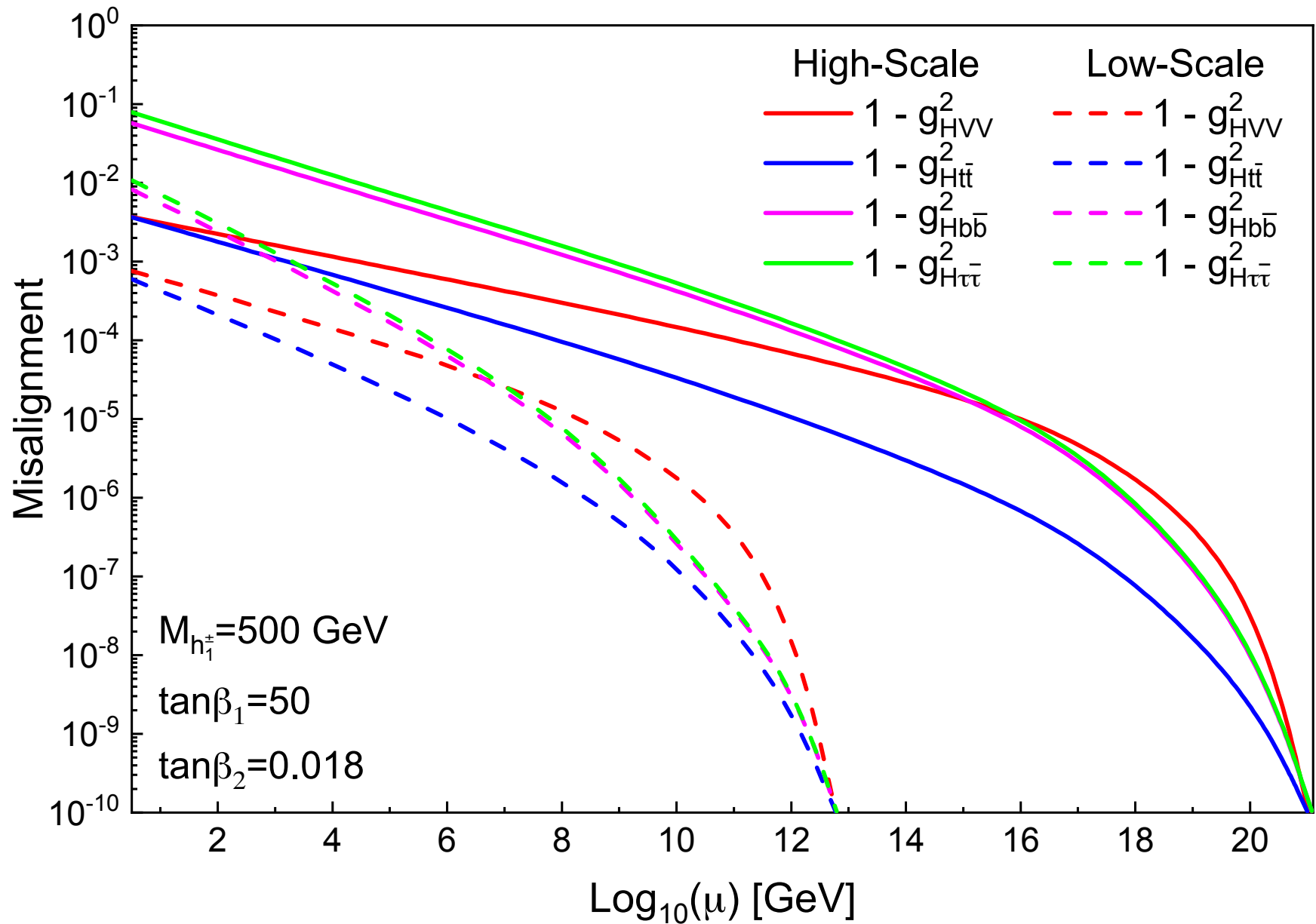
[N. Darvishi, M. Masouminia, AP '21]

Input parameters: $\tan \beta_1 = v_2/v_1$, $\tan \beta_2 = v_3/\sqrt{v_1^2 + v_2^2}$,
 $M_{h_{1,2}^\pm}$ and $h_1^\pm h_2^\mp$ -mixing angle: σ



Misalignment predictions in the MS-3HDM

[N. Darvishi, M. Masouminia, AP '21]



• CP Violation in 2HDM: Explicit, Spontaneous and Mixed

[N. Darvishi, AP, J.-H. Yu '2023]

$$R_\xi = \frac{\mathcal{V}_\xi(v_1, v_2 e^{i\xi})}{|\mathcal{V}_{\xi=0}|}$$

Weak basis:

$$\text{Im}\lambda_5 = 0, \lambda_6 = \lambda_7$$

$$|r_{\text{CP}}| > 1:$$

Explicit CPV, CPC

$$1/2 < |r_{\text{CP}}| < 1:$$

Explicit, Spontaneous,
& Mixed CPV.

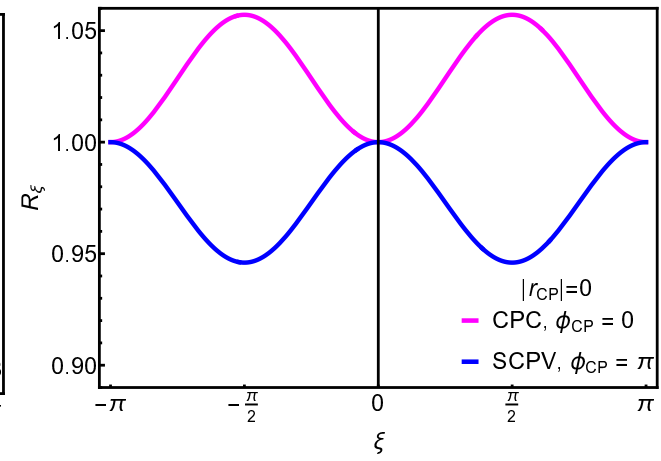
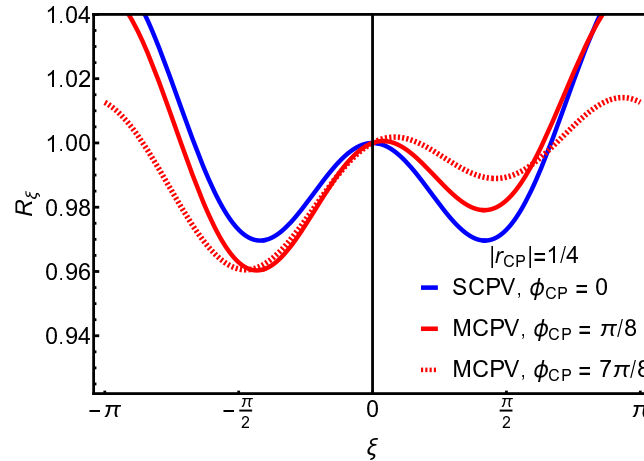
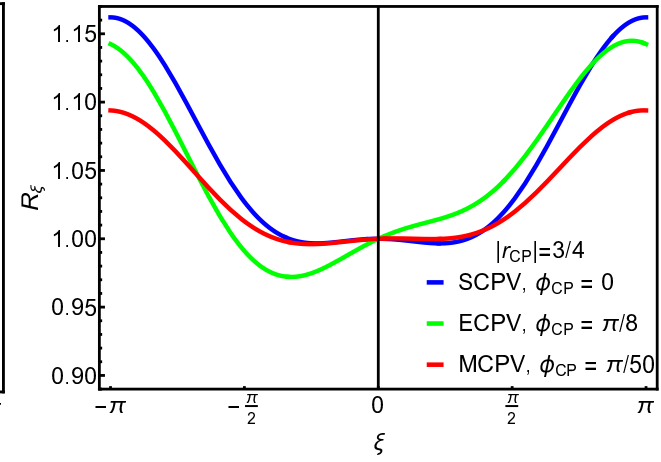
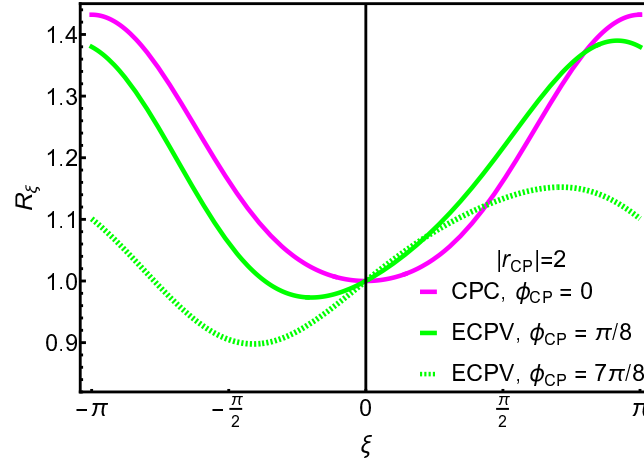
$$0 < |r_{\text{CP}}| < 1/2:$$

Spontaneous
& Mixed CPV.

**CP-odd
minimization**

condition:

$$\sin 2\xi = 2 |r_{\text{CP}}| \sin(\xi + \phi_{\text{CP}}), \quad r_{\text{CP}} = |r_{\text{CP}}| e^{i\phi_{\text{CP}}} = \frac{m_{12}^2 - \lambda_6 v^2}{\lambda_5 s_{2\beta} v^2}.$$



● CP Violation in Naturally Aligned (NA) 2HDM

[N. Darvishi, AP, J.-H. Yu '2023]

No.	Generators	Continuous Syms	Parameters
8	T^{0-9}	SO(5)	$m_{11}^2 = m_{22}^2, \lambda_1 = \lambda_2 = \lambda_3/2.$
7	$T^{0,2,5,7,8,9}$	CP1×O(4)	$m_{11}^2 = m_{22}^2, \lambda_1 = \lambda_2 = \lambda_3/2, \lambda_4 = -\text{Re}(\lambda_5).$
7'	$T^{0,1,4,6,8,9}$	CP2×O(4)'	$m_{11}^2 = m_{22}^2, \lambda_1 = \lambda_2 = \lambda_3/2, \lambda_4 = \text{Re}(\lambda_5).$
6	$T^{0,2,5,7,8,9}$	SO(4)	$m_{11}^2 = m_{22}^2, \text{Im}(m_{12}^2), \lambda_1 = \lambda_2 = \lambda_3/2,$ $\lambda_4 = -\text{Re}(\lambda_5), \text{Im}(\lambda_6) = \text{Im}(\lambda_7).$
6'	$T^{0,1,4,6,8,9}$	SO(4)'	$m_{11}^2 = m_{22}^2, \text{Re}(m_{12}^2), \lambda_1 = \lambda_2 = \lambda_3/2,$ $\lambda_4 = \text{Re}(\lambda_5), \text{Re}(\lambda_6) = \text{Re}(\lambda_7).$
5	$T^{0,2,4,6}$	O(2)×O(3)	$m_{11}^2 = m_{22}^2, \lambda_1 = \lambda_2 = \lambda_{345}/2, \lambda_4 = \text{Re}(\lambda_5).$
5'	$T^{0,1,5,7}$	O(2)'×O(3)	$m_{11}^2 = m_{22}^2, \lambda_1 = \lambda_2 = \bar{\lambda}_{345}/2, \lambda_4 = -\text{Re}(\lambda_5).$
4	T^{0-3}	O(3)×O(2) _Y	$m_{11}^2 = m_{22}^2, \lambda_1 = \lambda_2 = \lambda_{34}/2.$
3	$T^{0,5,7}$	SO(3)	$m_{11}^2, m_{22}^2, \text{Im}(m_{12}^2), \lambda_1, \lambda_2, \lambda_3, \lambda_4 = -\text{Re}(\lambda_5),$ $\text{Im}(\lambda_6), \text{Im}(\lambda_7).$
3'	$T^{0,4,6}$	SO(3)'	$m_{11}^2, m_{22}^2, \text{Re}(m_{12}^2), \lambda_1, \lambda_2, \lambda_3, \lambda_4 = \text{Re}(\lambda_5),$ $\text{Re}(\lambda_6), \text{Re}(\lambda_7).$
2	$T^{0,2}$	CP1×O(2)×O(2) _Y	$m_{11}^2 = m_{22}^2, \lambda_1 = \lambda_2 = \lambda_{345}/2.$
2'	$T^{0,1}$	CP2×O(2)'×O(2) _Y	$m_{11}^2 = m_{22}^2, \lambda_1 = \lambda_2 = \bar{\lambda}_{345}/2.$
1	$T^{0,2}$	O(2)×O(2) _Y	$m_{11}^2 = m_{22}^2, \text{Im}(m_{12}^2), \lambda_1 = \lambda_2 = \lambda_{345}/2,$ $\text{Im}(\lambda_6) = \text{Im}(\lambda_7).$
1'	$T^{0,1}$	O(2)'×O(2) _Y	$m_{11}^2 = m_{22}^2, \text{Re}(m_{12}^2), \lambda_1 = \lambda_2 = \bar{\lambda}_{345}/2,$ $\text{Re}(\lambda_6) = \text{Re}(\lambda_7).$

Abbreviations: $\lambda_{34} \equiv \lambda_3 + \lambda_4, \lambda_{345} (\bar{\lambda}_{345}) \equiv \lambda_3 + \lambda_4 + (-) \text{Re}(\lambda_5)$

– **CP-violating** breaking patterns in **NA-2HDM**

[N. Darvishi, AP, J.-H. Yu '2023]

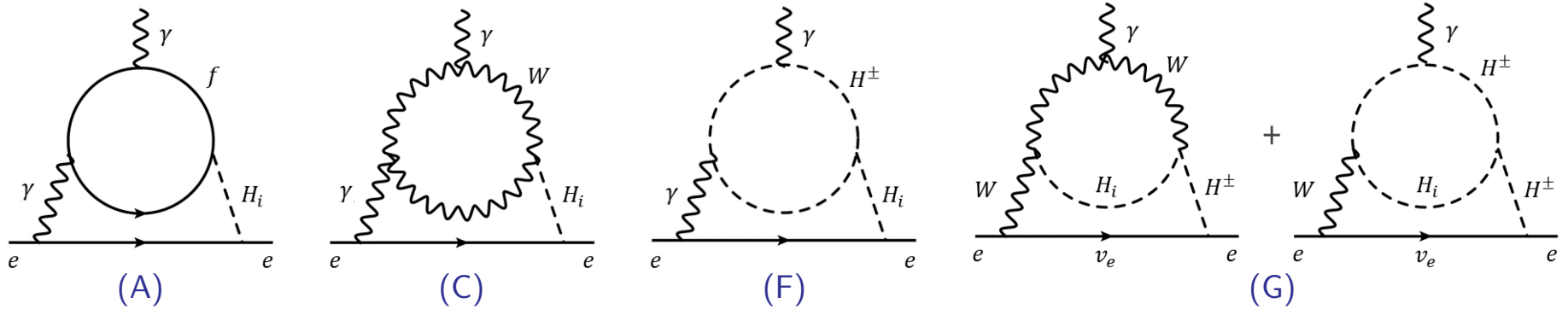
No.	Syms with NHAL	No.	Syms breaking of NHAL	Types of CPV after SB
8	SO(5)	—		
7	CP1×O(4)	6	SO(4)	ECPV/MCPV
7'	CP2×O(4)'	6'	SO(4)'	SCPV
5	O(2)×O(3)	6'	SO(4)'	SCPV
5'	O(2)'×O(3)	6	SO(4)	ECPV/MCPV
4	O(3)×O(2) _Y	—		
2	CP1×O(2)×O(2) _Y	1	O(2)×O(2) _Y	ECPV/MCPV
2'	CP2×O(2)'×O(2) _Y	1'	O(2)'×O(2) _Y	SCPV

SB: Soft symmetry-breaking **B**ilinear mass terms.

• EDM compatible scenarios with approximate natural alignment

[N. Darvishi, AP, J.-H. Yu '2023]

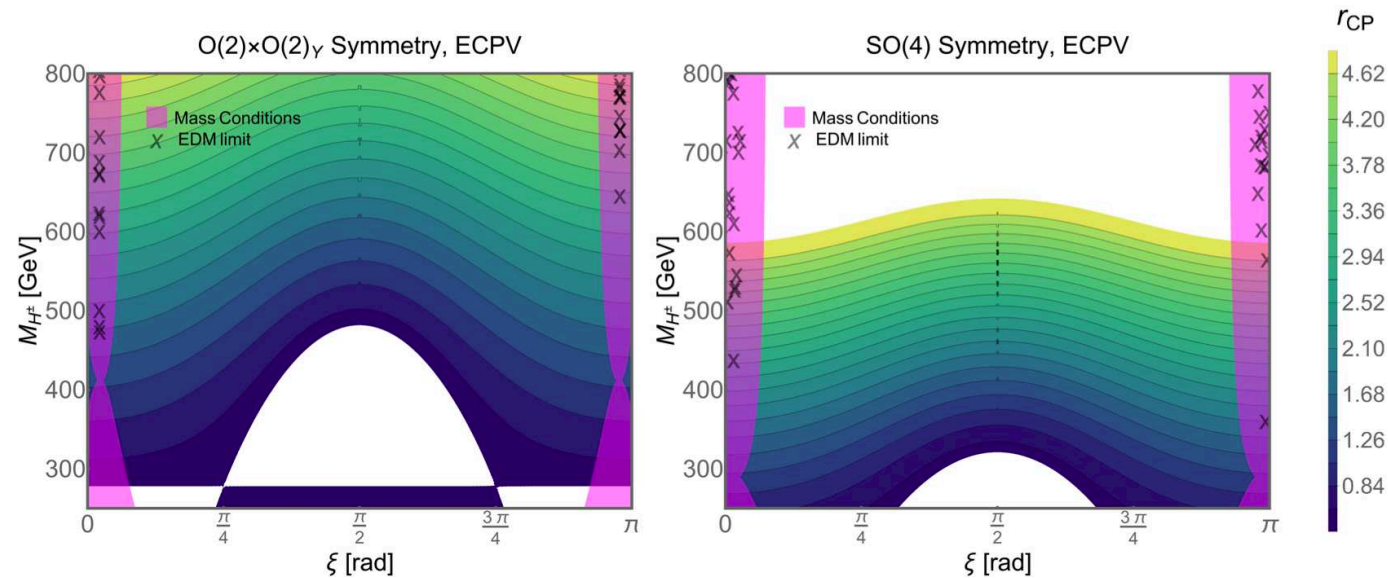
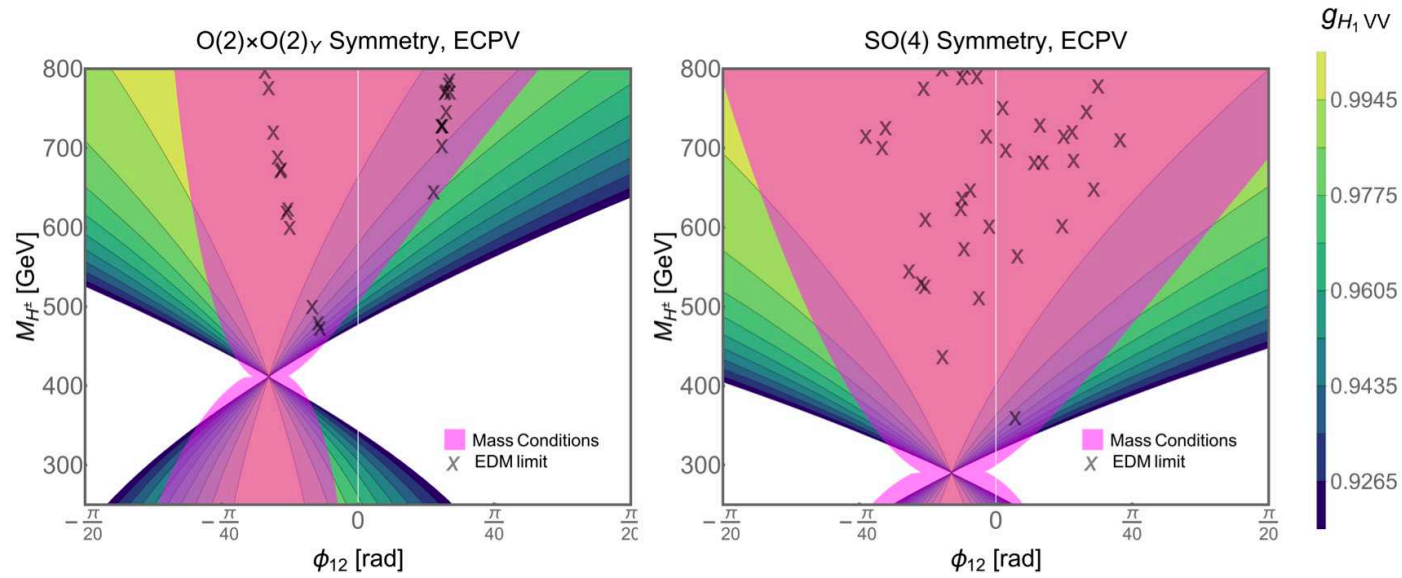
Symmetries	CPV types	$\tan \beta$	ξ	ϕ_{12}	M_{H_1}	M_{H_2}	M_{H_3}	M_{H^\pm}	$g_{H_1 VV}$
$O(2) \times O(2)_Y$	ECPV	1.608	0.002	-0.05	125.10	682.99	787.67	640.18	0.998
	MCPV	0.804	-1.93	0.27	125.21	225.11	270.50	380.54	0.989
$O(2)' \times O(2)_Y$	SCPV	0.71	0.67	0	125.52	275.23	448.01	551.38	0.957
SO(4)	ECPV	0.827	0.002	-0.02	125.28	506.93	570.58	570.59	0.995
	MCPV	0.740	-1.25	0.44	125.40	408.49	433.98	433.99	0.984
SO(4)'	SCPV	0.742	-2.42	0	125.68	216.63	265	442.21	0.948



Symmetries	CPV types	The ratio of EDM contributions over the total EDM							EDM [$e \cdot \text{cm}$]
		(A)	(B)	(C)	(D)	(E)	(F)	(G)	
$O(2) \times O(2)_Y$	ECPV	5.581	-0.072	-4.711	-0.308	0.0185	0.312	0.180	9.51×10^{-30}
	MCPV	2.008	-0.026	-0.942	-0.065	0.0015	0.028	-0.005	7.67×10^{-30}
$O(2)' \times O(2)_Y$	SCPV	2.003	-0.026	-0.866	-0.079	0.0002	0.004	-0.037	3.86×10^{-30}
SO(4)	ECPV	1.851	-0.021	-0.832	-0.055	0.0005	0.009	0.048	2.87×10^{-30}
	MCPV	1.995	-0.021	-0.887	-0.077	-0.0013	-0.023	0.016	1.07×10^{-30}
SO(4)'	SCPV	-1.211	0.014	1.875	0.172	0.0002	0.004	0.146	3.55×10^{-30}

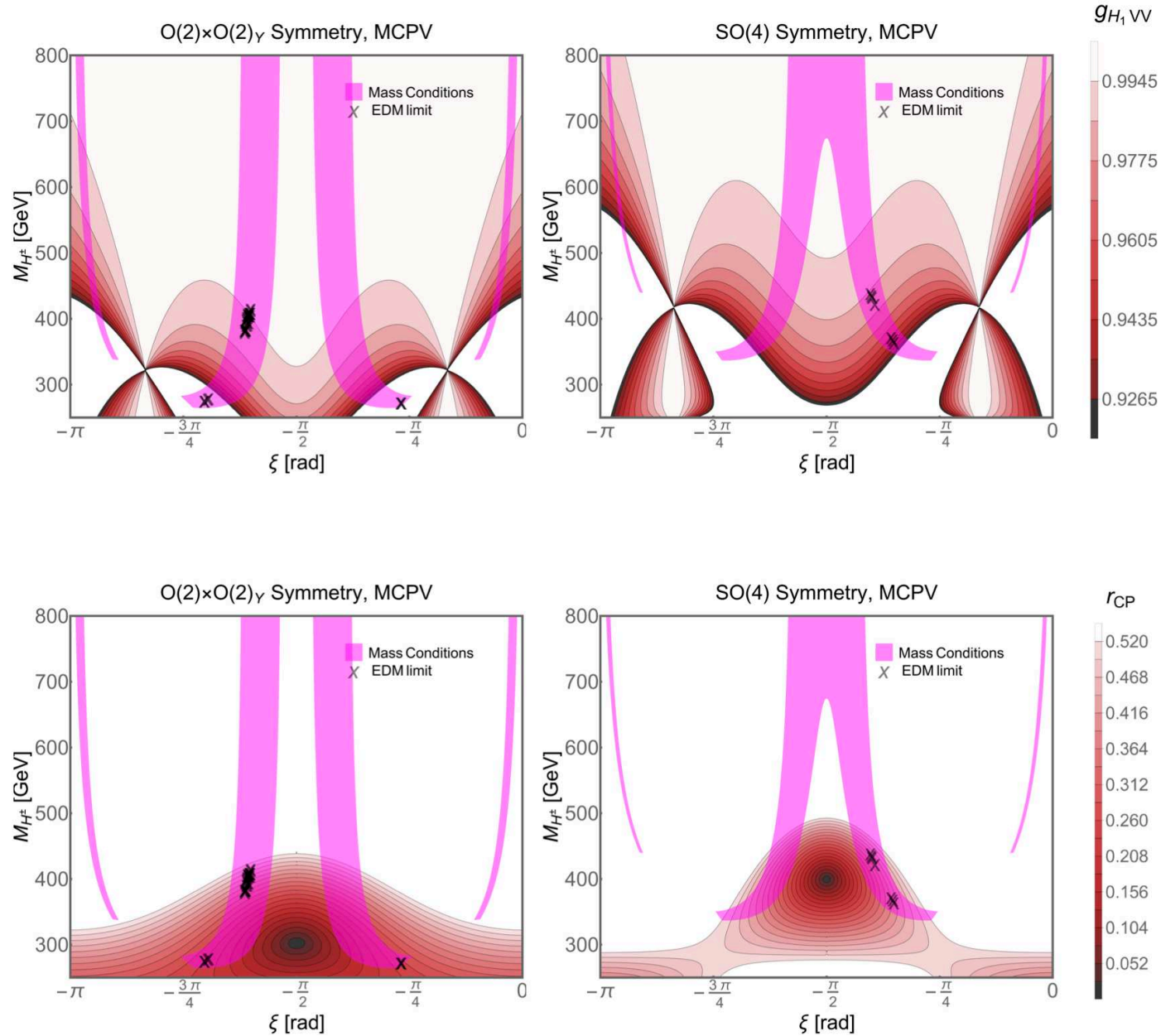
– Explicit CP Violation in NA-2HDM

[N. Darvishi, AP, J.-H. Yu '2023]



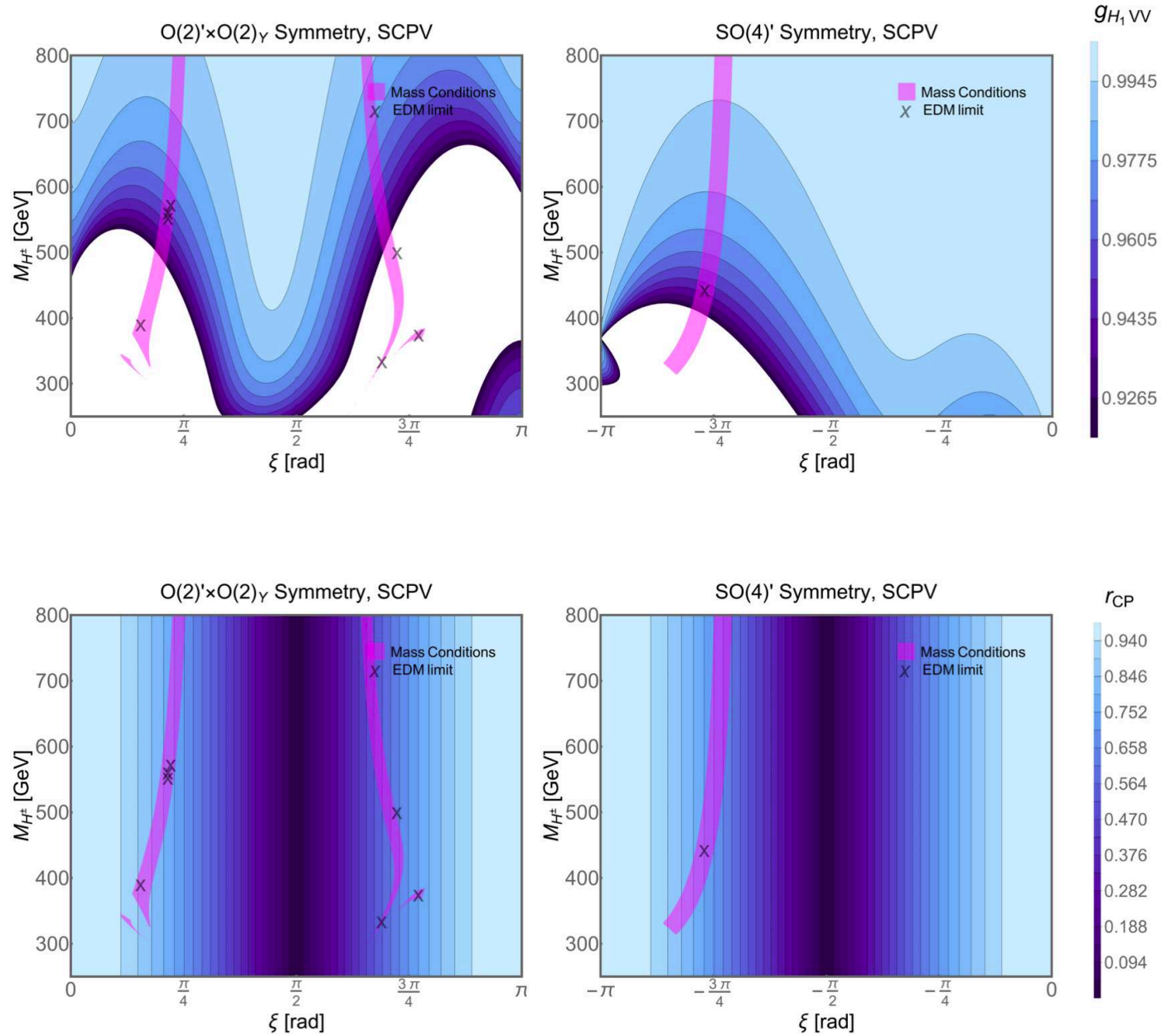
Mixed CP Violation in NA-2HDM

[N. Darvishi, AP, J.-H. Yu '2023]



– Spontaneous CP Violation in NA-2HDM

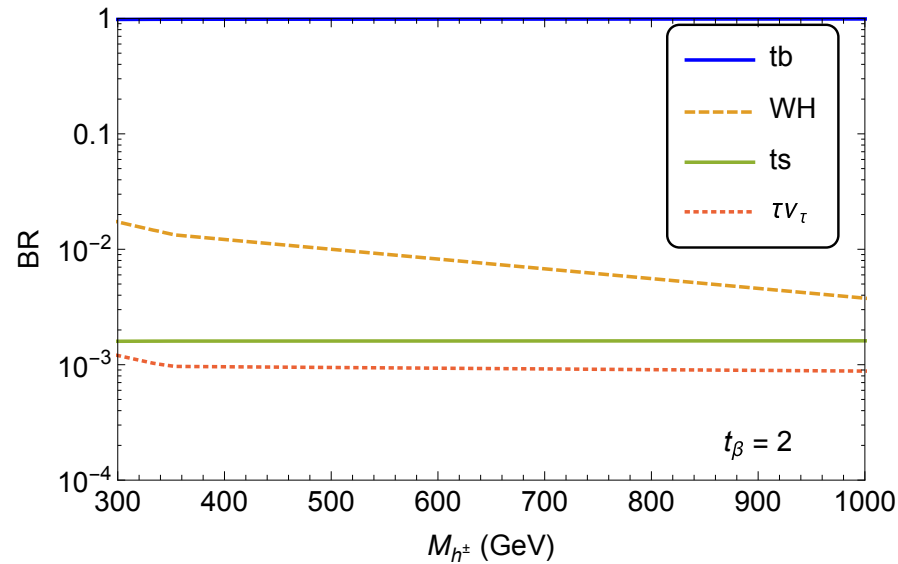
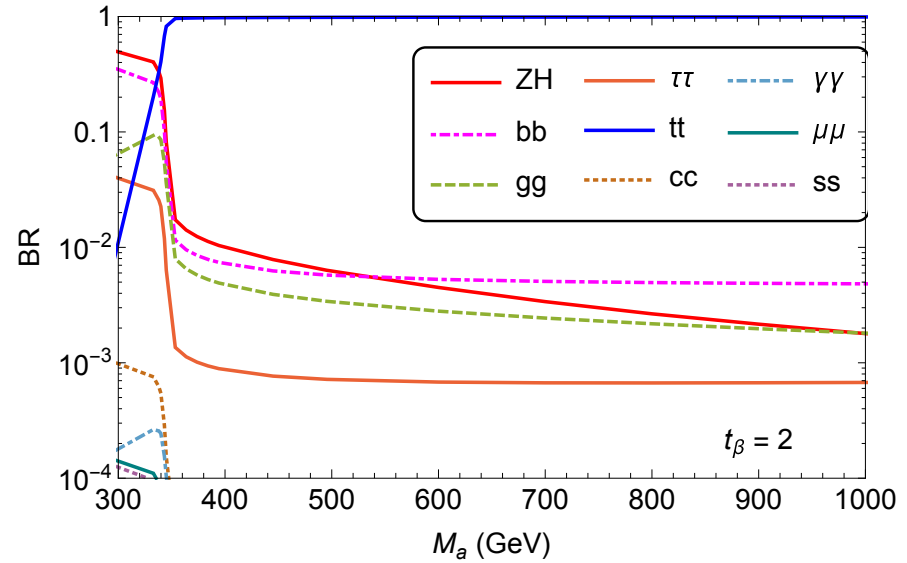
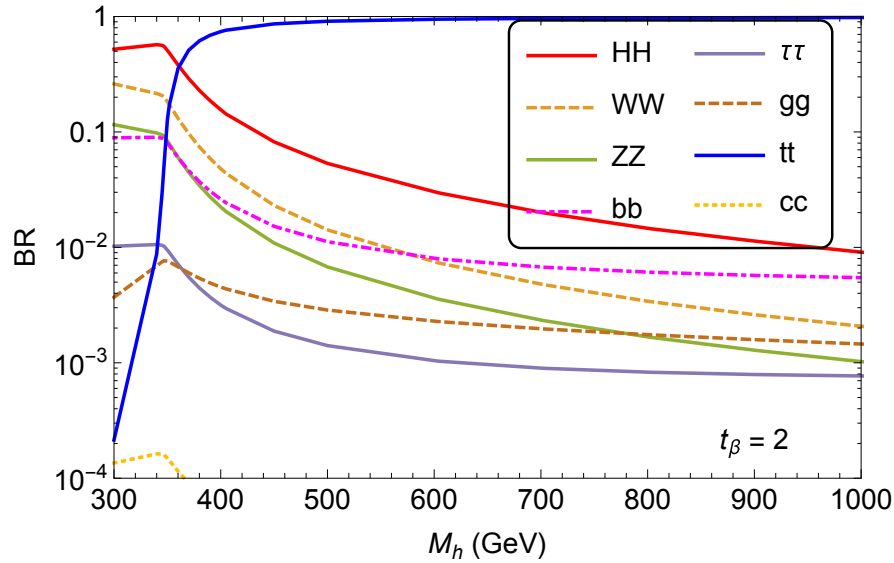
[N. Darvishi, AP, J.-H. Yu '2023]



- Phenomenology at the LHC

- Branching ratios in the MS-2HDM

[Dev, AP '14]

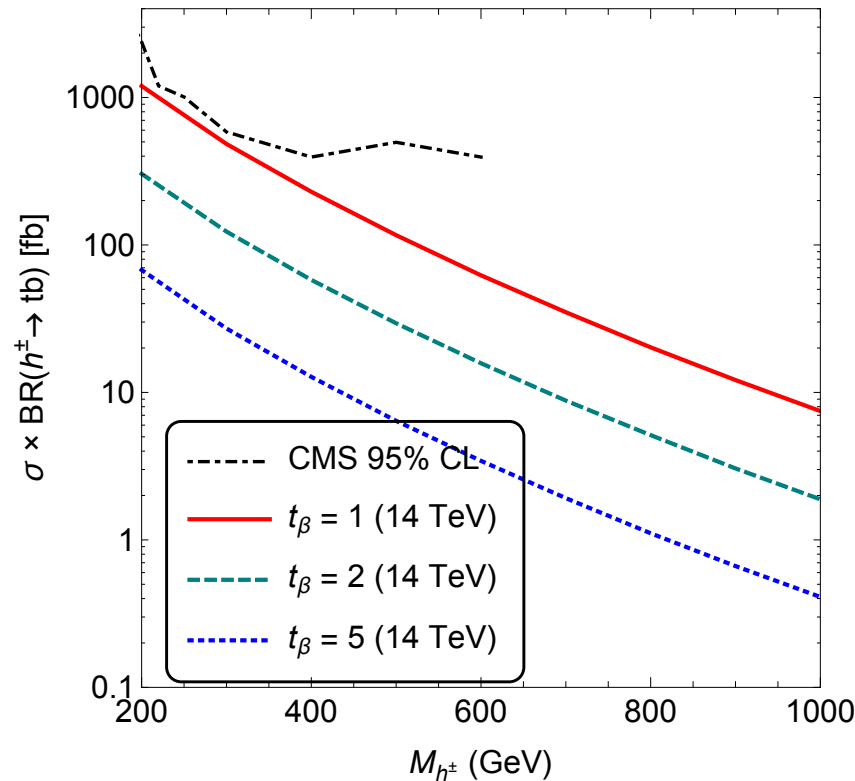
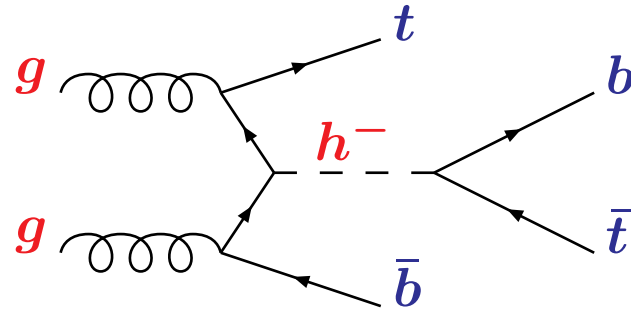


• Discovery channels for aligned Higgs doublets:

– $gg \rightarrow t\bar{b}h^- \rightarrow t\bar{b}t\bar{b}$

[Dev, AP '14;

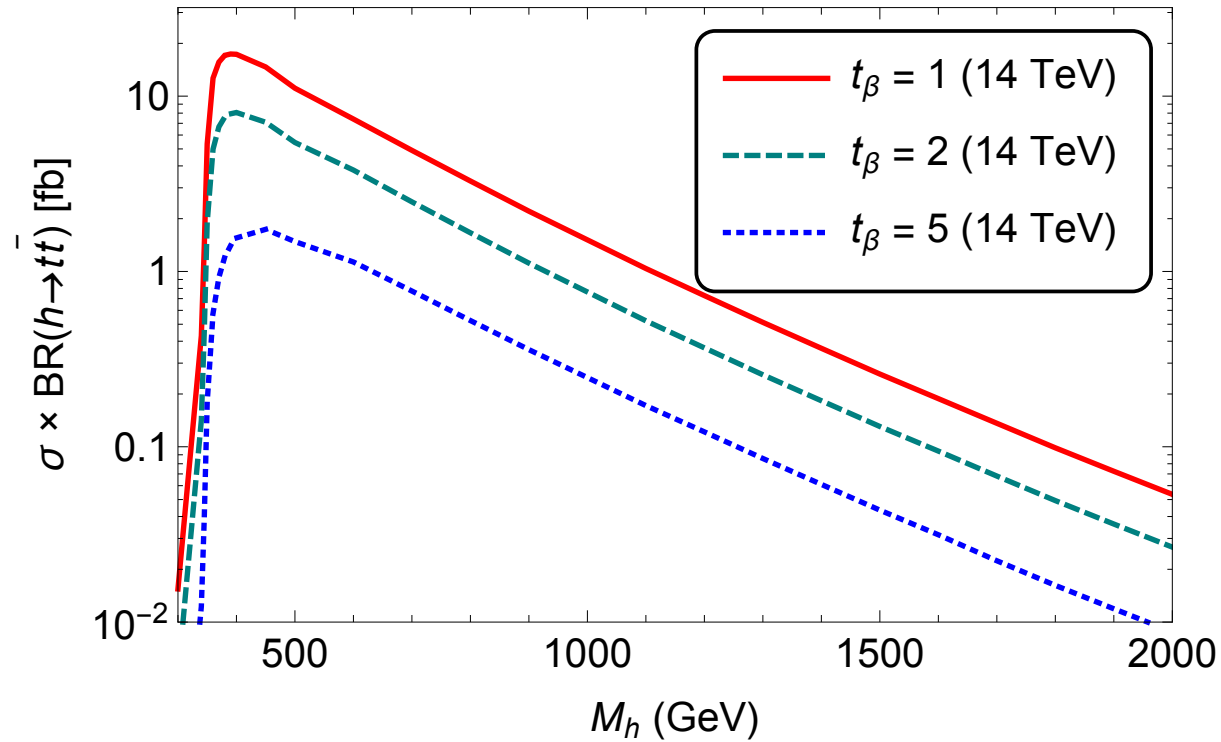
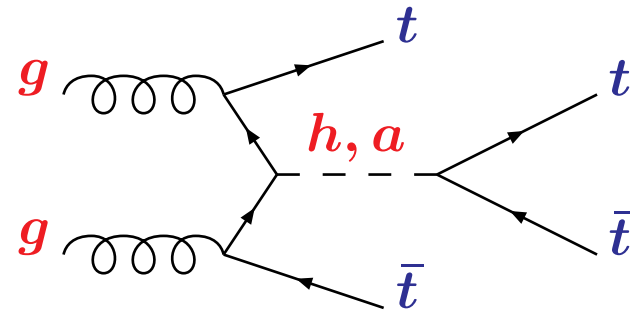
Yvonne Peters, AP et al, PRD100 (2019) 035026]

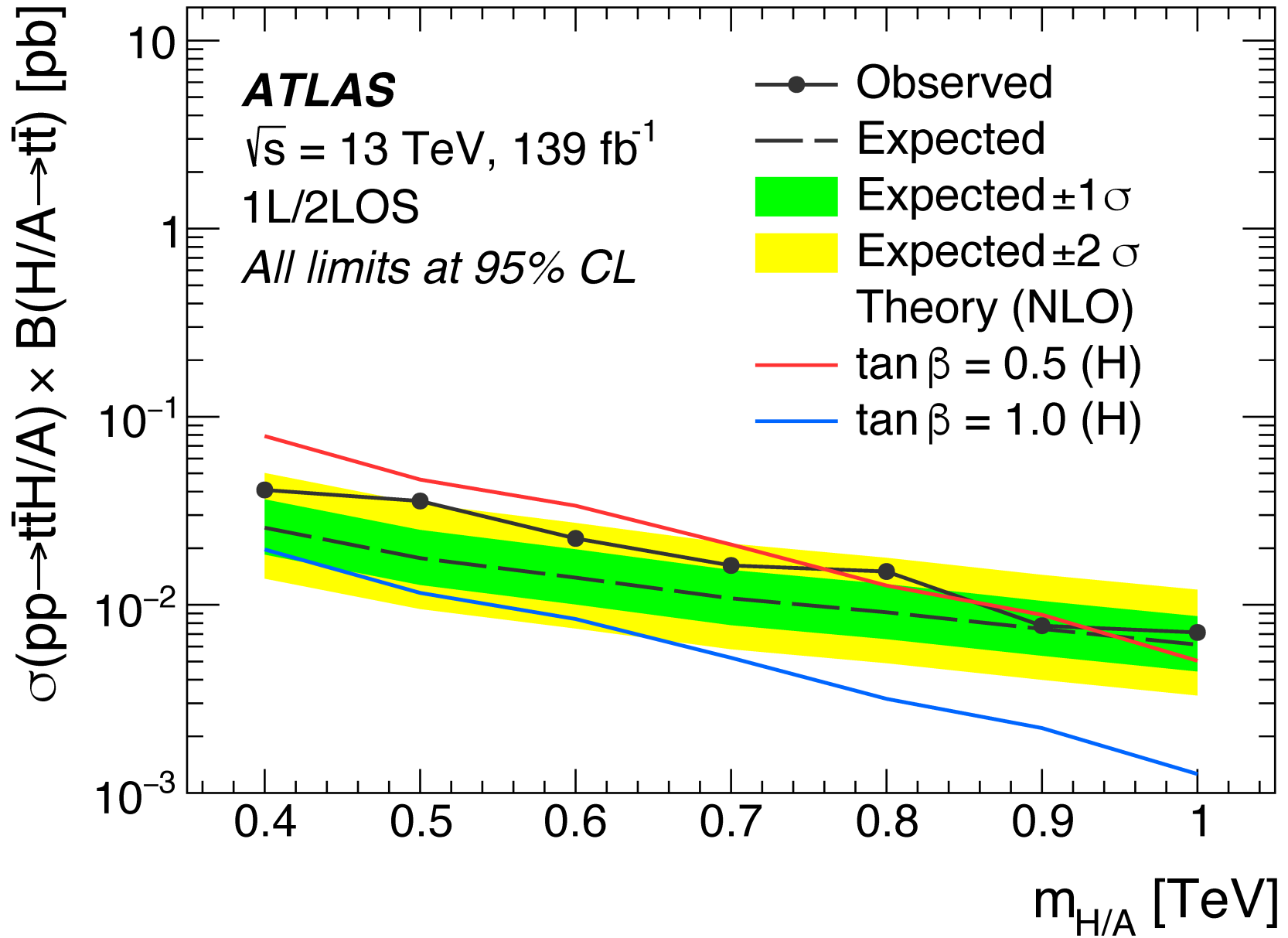


$p_T^\ell > 20 \text{ GeV},$
 $|\eta^\ell| < 2.5,$
 $\Delta R^{\ell\ell} > 0.4,$
 $M_{\ell\ell} > 12 \text{ GeV},$
 $|M_{\ell\ell} - M_Z| > 10 \text{ GeV},$
 $p_T^j > 30 \text{ GeV},$
 $|\eta^j| < 2.4,$
 $\cancel{E}_T > 40 \text{ GeV}.$

- $gg \rightarrow t\bar{t}(h, a) \rightarrow t\bar{t}t\bar{t}$

[Dev, AP '14]

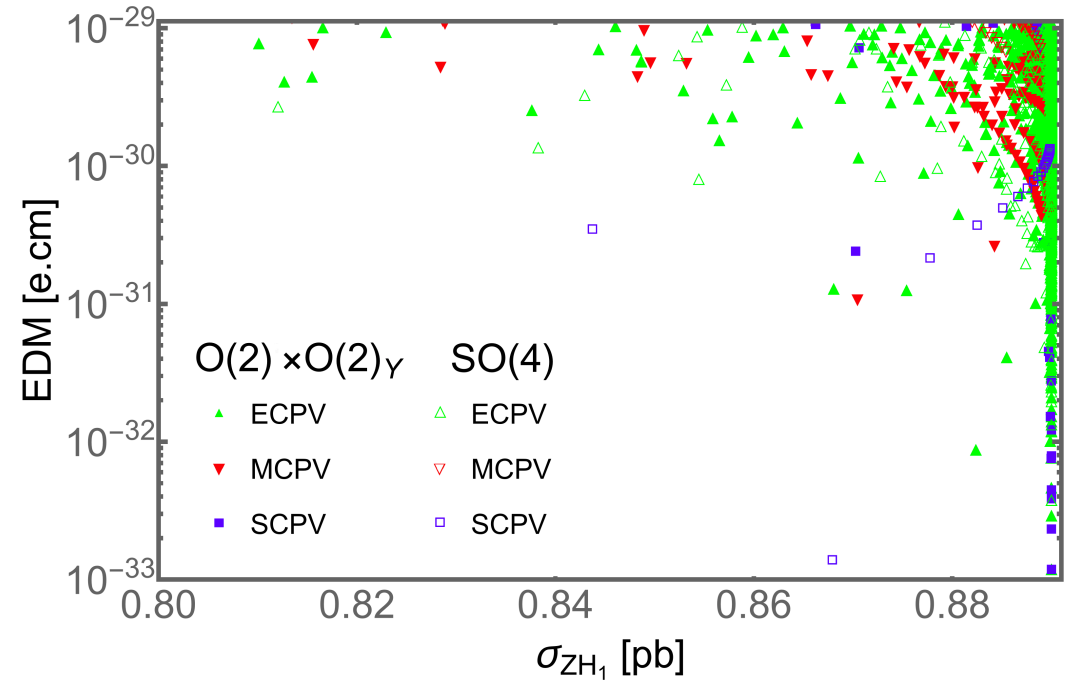
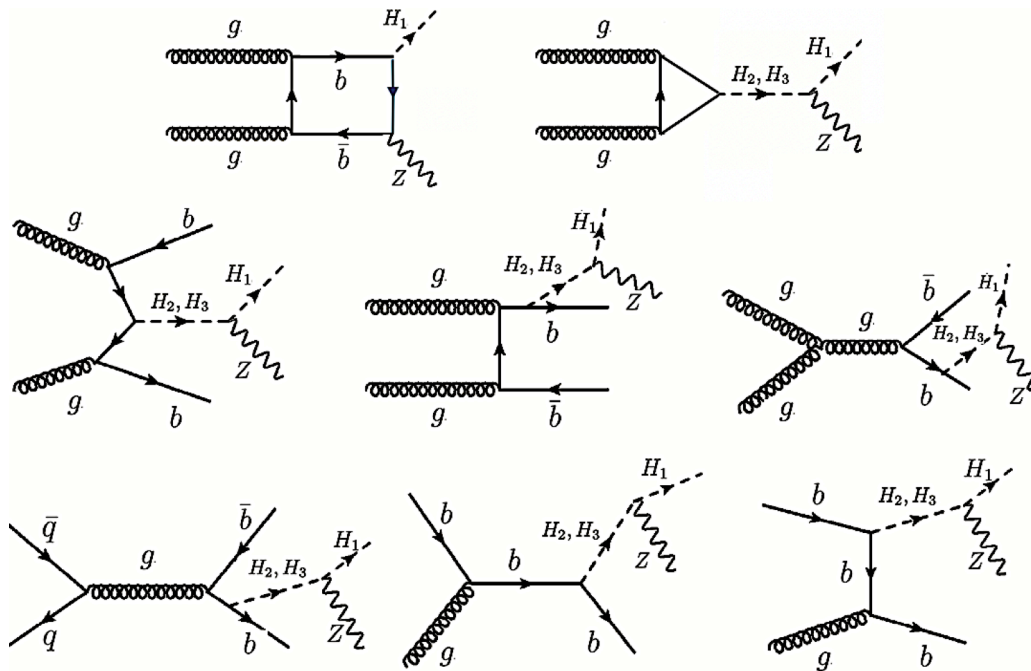




• Other production channels for **CP-violating NA Higgs bosons**

– $gg \rightarrow (Z^*, H_2, H_3) \rightarrow ZH_1(+b\bar{b})$

[N. Darvishi, AP, J.-H. Yu '2023]



Omitted are graphs with $H_{2,3} \rightarrow Z$ and $H_{2,3}$ radiation from \bar{b} quarks.

• SUMMARY

- 5 symmetries for **natural alignment** *without* decoupling in 2HDM:
 - (i) $\text{Sp}(4)$ (ii) $\text{SU}(2)_{\text{HF}}$ (iii) $\text{SO}(2)_{\text{HF}} \times \mathcal{CP}$
 - (iv) $\text{Sp}(2)_{\phi_1\tilde{\phi}_2} \times \text{Sp}(2)_{\phi_2\tilde{\phi}_1} \times \mathcal{CP}$ (v) $\text{SO}(2)_{\text{HF}} \times \text{Sp}(2)_{\phi_1+\phi_2}$
- **Soft breaking** \longrightarrow **minimal alignment** symmetry: $\mathbf{Z}_2^{\text{EW}} \times \mathbf{Z}_2^{\text{I}}$
 \longrightarrow **Naturally aligned (NA) heavy Higgs sector** is \mathbf{Z}_2^{EW} **odd**.
- Three forms of **CPV** in 2HDM: **Explicit**, **Spontaneous** and **Mixed**.
 Maximising **CPV** along **softly-broken** symmetry directions:
 - (i) $\text{SO}(2)_{\text{HF}}$ (ii) $\text{Sp}(2)_{\phi_1\tilde{\phi}_2} \times \text{Sp}(2)_{\phi_2\tilde{\phi}_1} \simeq \text{SO}(4)$,
 compatible with current **EDM** limits.
- $t\bar{t}t\bar{t}$ channel is a **powerful** probe for **NA-2HDMs**
- ZH_1 channel can test **CP-violating NA Higgs bosons** at the LHC.

Back-Up Slides

- **Quartic Coupling Unification in the MS-2HDM**

[Dev, AP '14; N. Darvishi, AP '19]

Symmetry-breaking of $\text{Sp}(4)/\mathbb{Z}_2 \sim \text{SO}(5)$:

- Soft breaking (e.g. through m_{12}^2):

$$M_H^2 = 2\lambda_2 v^2, \quad M_h^2 = M_a^2 = M_{h^\pm}^2 = \frac{\text{Re}(m_{12}^2)}{s_\beta c_\beta}$$

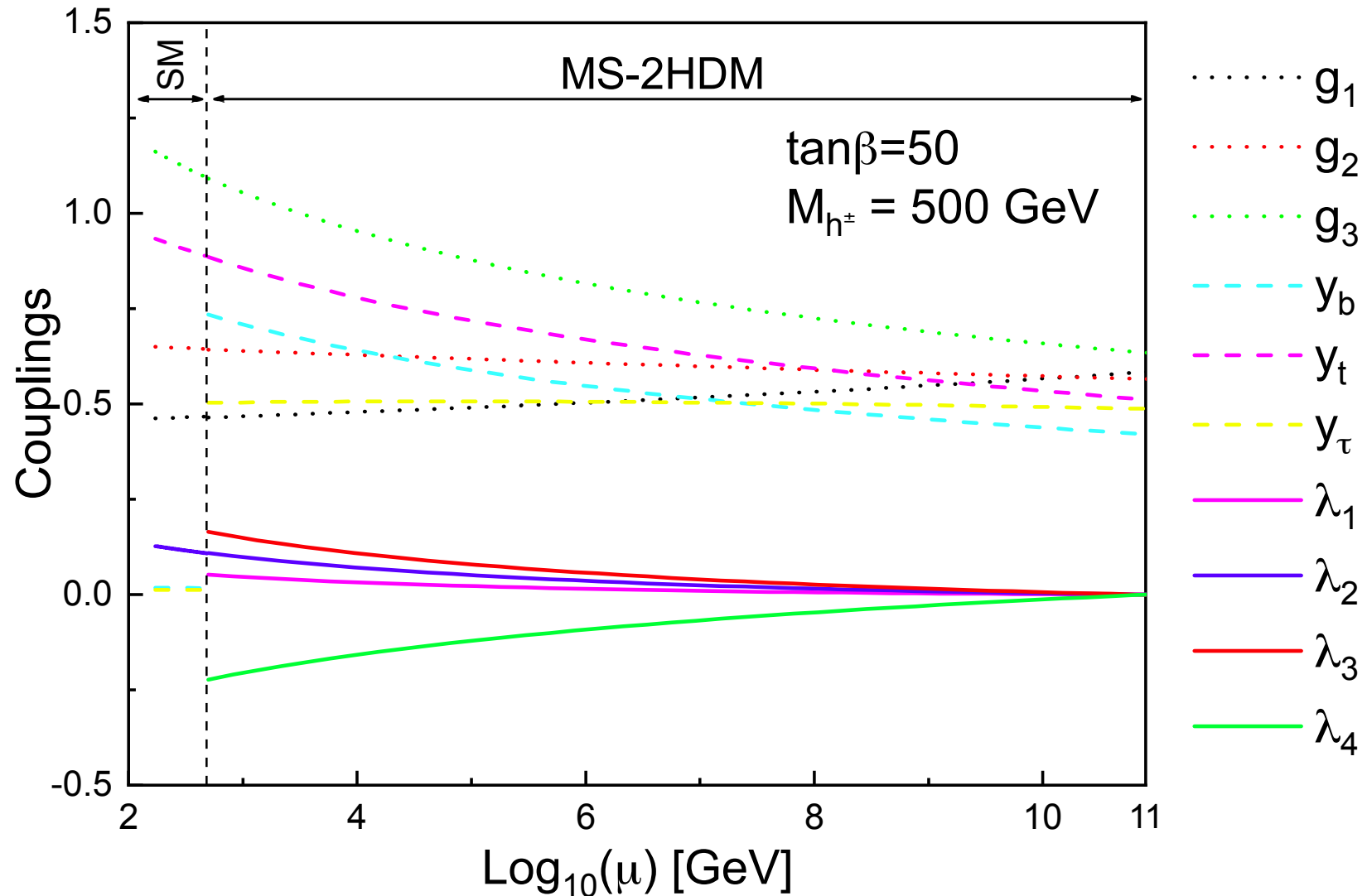
Heavy Higgs spectrum is **degenerate** at tree level.

- **Explicit breaking** through RG running (two loops):

$$\begin{aligned} \text{Sp}(4)/\mathbb{Z}_2 \otimes \text{SU}(2)_L &\xrightarrow{g' \neq 0} \text{SU}(2)_{\text{HF}} \otimes \text{U}(1)_Y \otimes \text{SU}(2)_L \\ &\xrightarrow{\mathbf{Y}^{u,d}} \text{U}(1)_{\text{PQ}} \otimes \text{U}(1)_Y \otimes \text{SU}(2)_L \\ &\xrightarrow[\langle \Phi_{1,2} \rangle]{m_{12}^2} \text{U}(1)_{\text{em}} \end{aligned}$$

• **Quartic Coupling Unification (two loops)**

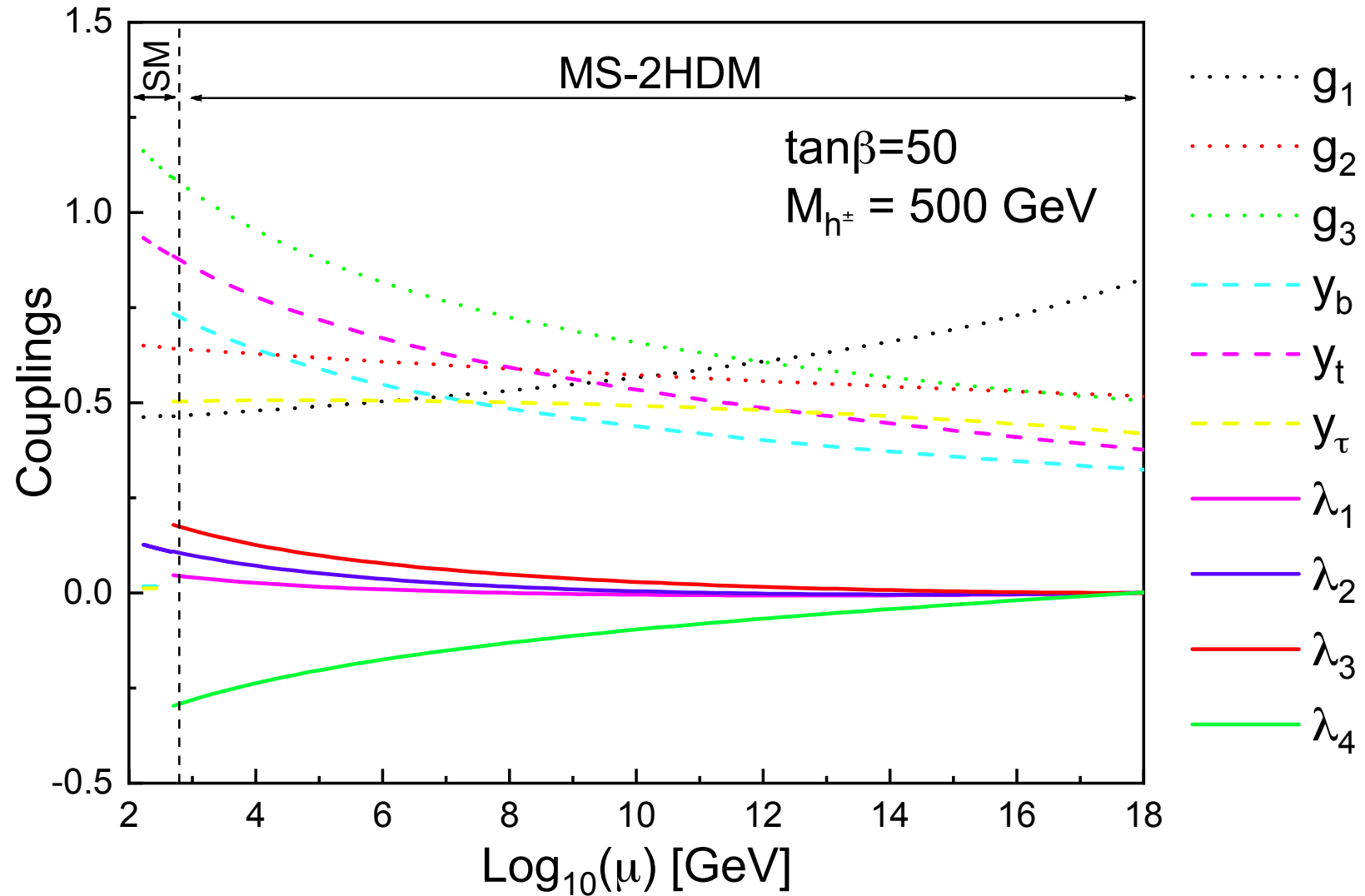
[N. Darvishi, AP '19]



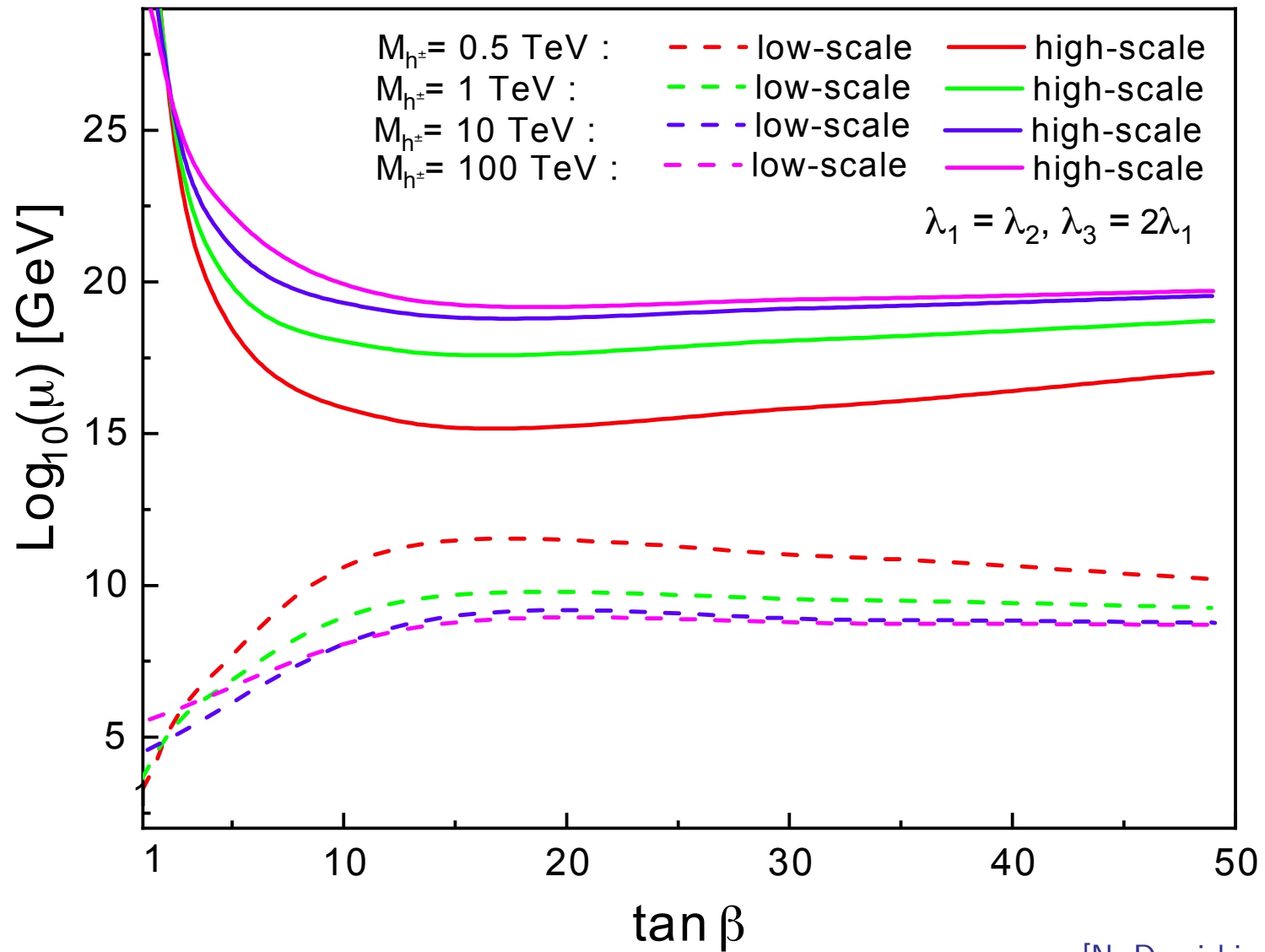
First conformal unification point: $\mu_X^{(1)} \sim 10^{11} \text{ GeV}$ (of order PQ scale)

Second conformal unification point: $\mu_X^{(2)} \sim 10^{18}$ GeV (of order m_{Pl})

[N. Darvishi, AP '19]



– Low- and high-scale quartic coupling unification: $\tan \beta$ vs $\mu_X^{(1,2)}$



[N. Darvishi, AP '19]

– Misalignment in the MS-2HDM

CP-even mass matrix in Higgs basis:

$$\mathcal{M}_S^2 = \begin{pmatrix} \hat{A} & \hat{C} \\ \hat{C} & \hat{B} \end{pmatrix} \xrightarrow[\text{approx.}]{\text{seesaw}} M_H^2 \simeq \hat{A} - \frac{\hat{C}^2}{\hat{B}} \quad \& \quad M_h^2 \simeq \hat{B} \gg \hat{A}, \hat{C}$$

Light-to-heavy scalar mixing:

$$\theta_S \equiv \frac{\hat{C}}{\hat{B}} = \frac{v^2 s_\beta c_\beta [s_\beta^2 (2\lambda_2 - \lambda_{34}) - c_\beta^2 (2\lambda_1 - \lambda_{34})]}{M_a^2 + 2v^2 s_\beta^2 c_\beta^2 (\lambda_1 + \lambda_2 - \lambda_{34})} \ll 1$$

Higgs couplings to $V = W, Z$:

$$g_{HVV} \simeq 1 - \frac{1}{2} \theta_S^2, \quad g_{hVV} \simeq -\theta_S$$

Higgs couplings to quarks:

$$\begin{aligned} g_{Huu} &\simeq 1 + t_\beta^{-1} \theta_S, & g_{Hdd} &\simeq 1 - t_\beta \theta_S, \\ g_{huu} &\simeq -\theta_S + t_\beta^{-1}, & g_{hdd} &\simeq -\theta_S - t_\beta. \end{aligned}$$

Misalignment predictions in the MS-2HDM with low- and high-scale quartic coupling unification, assuming $M_{h^\pm} = 500$ GeV.

[N. Darvishi, AP '19]

Couplings	ATLAS	CMS	$\tan \beta = 2$	$\tan \beta = 20$	$\tan \beta = 50$
$ g_{HZZ}^{\text{low-scale}} $	[0.86, 1.00]	[0.90, 1.00]	0.9999	0.9999	0.9999
$ g_{HZZ}^{\text{high-scale}} $			0.9981	0.9999	0.9999
$ g_{Htt}^{\text{low-scale}} $	$1.31^{+0.35}_{-0.33}$	$1.45^{+0.42}_{-0.32}$	1.0049	1.0001	1.0000
$ g_{Htt}^{\text{high-scale}} $			1.0987	1.0003	1.0001
$ g_{Hbb}^{\text{low-scale}} $	$0.49^{+0.26}_{-0.19}$	$0.57^{+0.16}_{-0.16}$	0.9803	0.9560	0.9590
$ g_{Hbb}^{\text{high-scale}} $			0.8810	0.9449	0.9427

→ Misalignment predictions consistent with experiment

- **Maximally Symmetric Three Higgs Doublet Model (MS-3HDM)**

[N. Darvishi, M. Masouminia, AP '21]

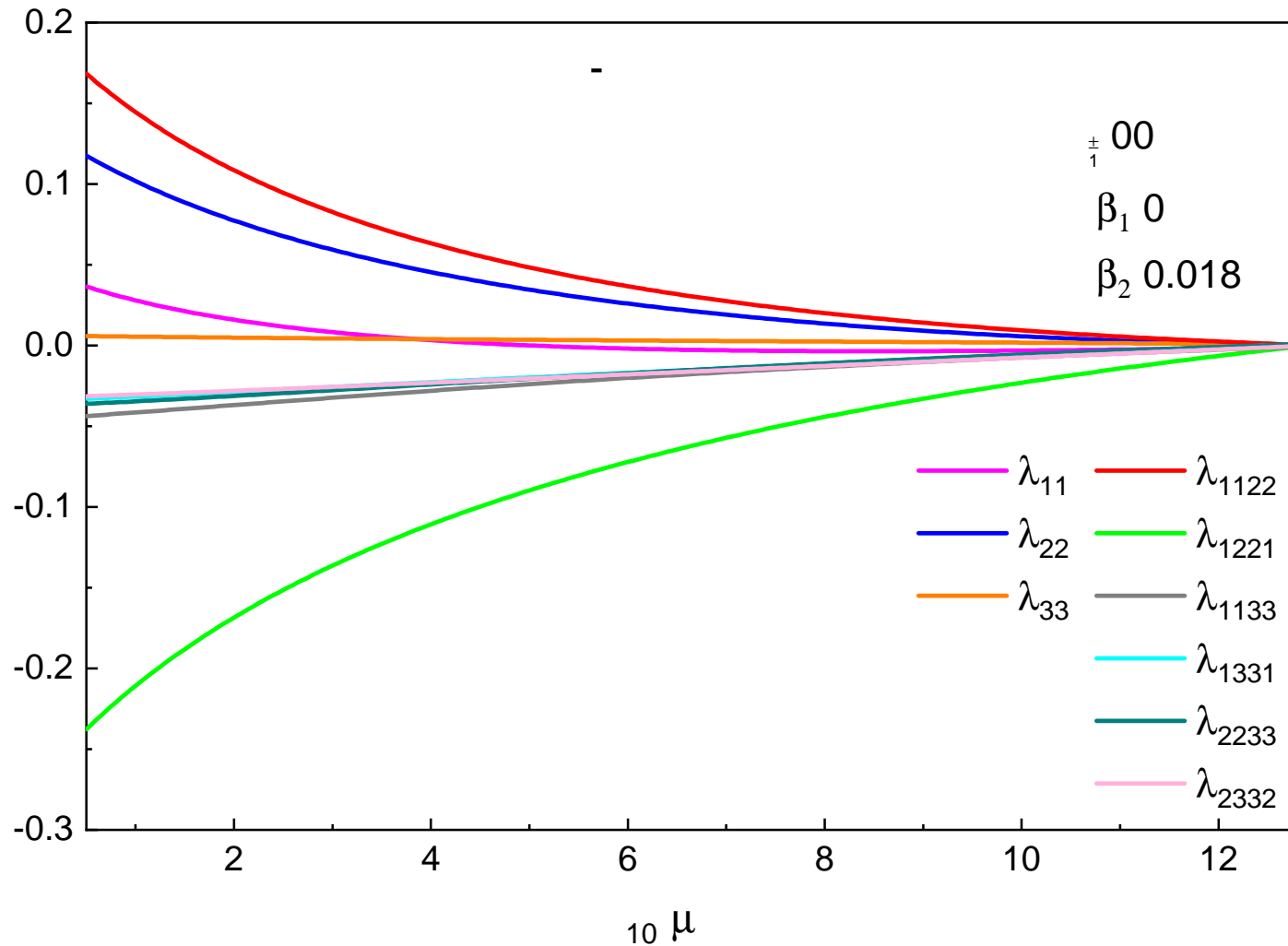
Breaking pattern:

$$\begin{array}{l}
 \text{Sp}(6)/\mathbb{Z}_2 \otimes \text{SU}(2)_L \xrightarrow{g' \neq 0} \text{SU}(3)_{\text{HF}} \otimes \text{U}(1)_Y \otimes \text{SU}(2)_L \\
 \xrightarrow[\text{Type V}]{\mathbf{Y}^{u,d,e}} \text{U}(1)_{\text{PQ}} \otimes \text{U}(1)'_{\text{PQ}} \otimes \text{U}(1)_Y \otimes \text{SU}(2)_L \\
 \xrightarrow[\text{soft } m_{ij}^2]{\langle \Phi_{1,2,3} \rangle} \text{U}(1)_{\text{em}}
 \end{array}$$

– Quartic Coupling Unification in the MS-3HDM

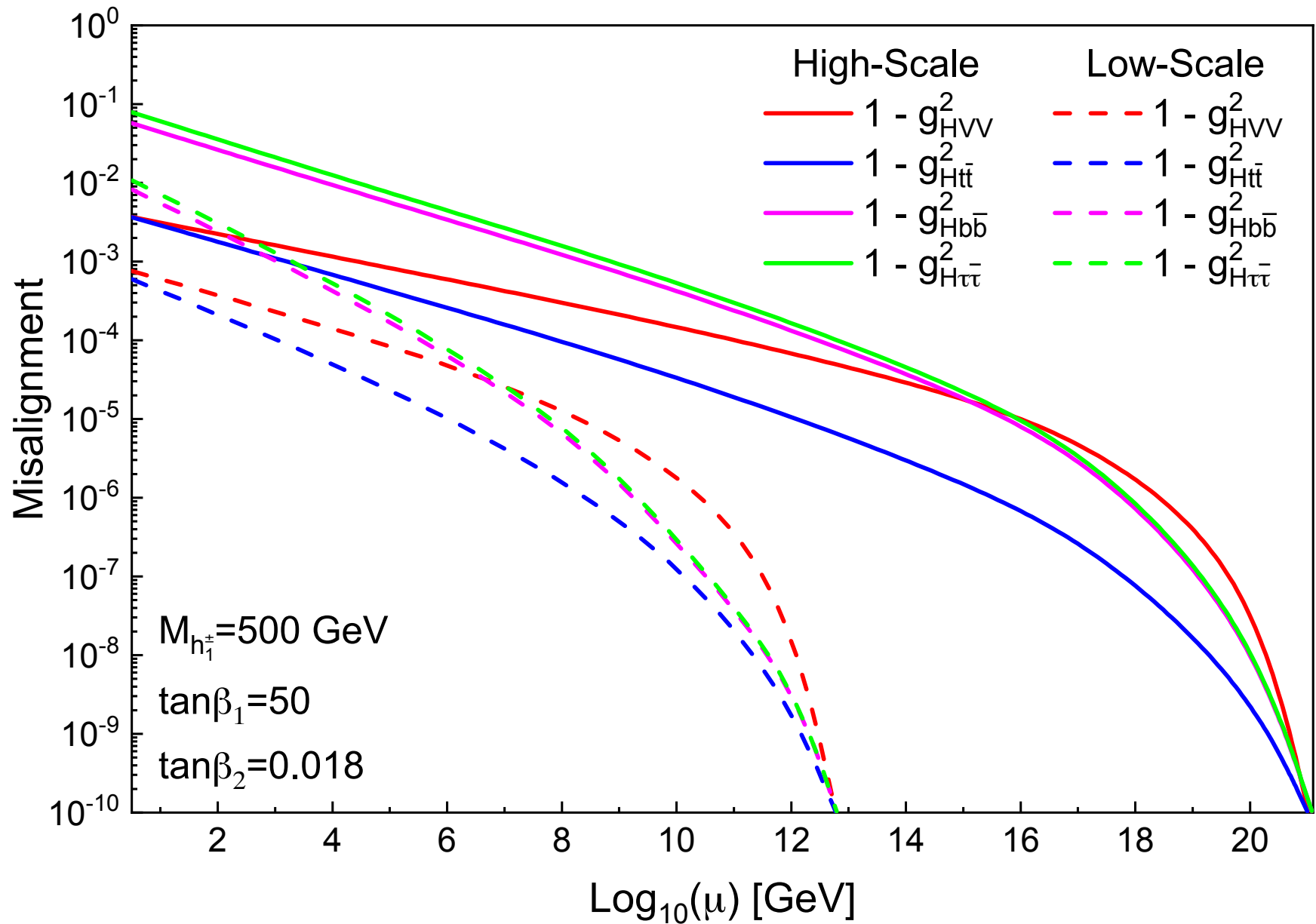
[N. Darvishi, M. Masouminia, AP '21]

Input parameters: $\tan \beta_1 = v_2/v_1$, $\tan \beta_2 = v_3/\sqrt{v_1^2 + v_2^2}$,
 $M_{h_{1,2}^\pm}$ and $h_1^\pm h_2^\mp$ -mixing angle: σ

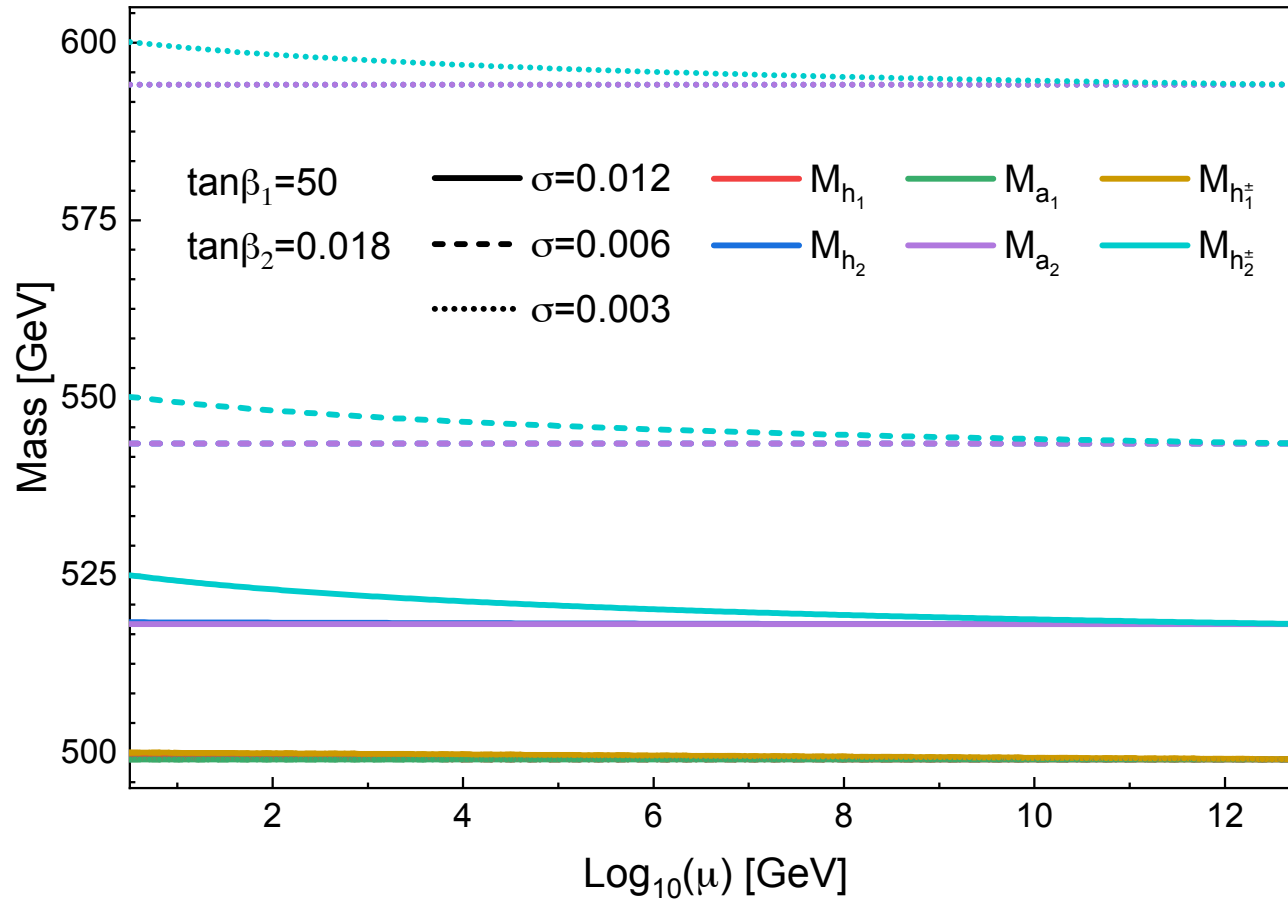


Misalignment predictions in the MS-3HDM

[N. Darvishi, M. Masouminia, AP '21]



– **Scalar Mass Spectrum in the MS-3HDM** [N. Darvishi, M. Masouminia, AP '21]



Predictions:

Alignment of masses: $M_{h_1} \sim M_{a_1} \sim M_{h_1^\pm}$ $M_{h_2} \sim M_{a_2} \sim M_{h_2^\pm}$

Alignment of **all heavy-sector** mixing angles in the Higgs basis: $\alpha \simeq \rho \simeq \sigma$

• Global Symmetries in 2HDM

• 2HDM potential

[TD Lee '73; AP, C Wagner '99;

Review: Branco, Ferreira, Lavoura, Rebelo, Sher, Silva '12.]

$$\begin{aligned}
 V = & -\mu_1^2(\phi_1^\dagger\phi_1) - \mu_2^2(\phi_2^\dagger\phi_2) - m_{12}^2(\phi_1^\dagger\phi_2) - m_{12}^{*2}(\phi_2^\dagger\phi_1) \\
 & + \lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\
 & + \frac{\lambda_5}{2}(\phi_1^\dagger\phi_2)^2 + \frac{\lambda_5^*}{2}(\phi_2^\dagger\phi_1)^2 + \lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + \lambda_6^*(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_1) \\
 & + \lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + \lambda_7^*(\phi_2^\dagger\phi_2)(\phi_2^\dagger\phi_1) .
 \end{aligned}$$

• Physical spectrum (CP-conserving limit):

CP-even Higgs bosons H and h ; CP-odd scalar a ; charged scalars h^\pm .

• Higgs coupling to gauge bosons $V = W, Z$:

$$g_{HVV} = \cos(\beta - \alpha), \quad g_{hVV} = \sin(\beta - \alpha),$$

where $\tan \beta = \langle \phi_2 \rangle / \langle \phi_1 \rangle$ and α diagonalizes the CP-even mass matrix.

- **Symmetries of the 2HDM Potential**

[R. A. Battye, G. D. Brawn, AP, JHEP1108 (2011) 020.]

Introduce the $SU(2)_L$ -covariant **8D** complex field multiplet

$$\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ i\sigma^2\phi_1^* \\ i\sigma^2\phi_2^* \end{pmatrix}, \quad \text{with } U_L \in SU(2)_L : \Phi \mapsto \Phi' = U_L \Phi .$$

Φ satisfies the **Majorana constraint**

$$\Phi = C \Phi^* ,$$

where C is the **charge conjugation 8D** matrix

$$C = \sigma^2 \otimes \sigma^0 \otimes \sigma^2 = \begin{pmatrix} \mathbf{0}_4 & \mathbf{1}_4 \\ -\mathbf{1}_4 & \mathbf{0}_4 \end{pmatrix} \otimes (-i\sigma_2) .$$

- The SO(1,5) Bilinear Formalism

Introduce the *null 6-Vector*

$$R^A = \Phi^\dagger \Sigma^A \Phi = \begin{pmatrix} \phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2 \\ \phi_1^\dagger \phi_2 + \phi_2^\dagger \phi_1 \\ -i \left[\phi_1^\dagger \phi_2 - \phi_2^\dagger \phi_1 \right] \\ \phi_1^\dagger \phi_1 - \phi_2^\dagger \phi_2 \\ \phi_1^\top i\sigma^2 \phi_2 - \phi_2^\dagger i\sigma^2 \phi_1^* \\ -i \left[\phi_1^\top i\sigma^2 \phi_2 + \phi_2^\dagger i\sigma^2 \phi_1^* \right] \end{pmatrix},$$

with $A = \mu, 4, 5$, and

$$\Sigma^\mu = \frac{1}{2} \begin{pmatrix} \sigma^\mu & \mathbf{0}_2 \\ \mathbf{0}_2 & (\sigma^\mu)^\top \end{pmatrix} \otimes \sigma^0,$$

$$\Sigma^4 = \frac{1}{2} \begin{pmatrix} \mathbf{0}_2 & i\sigma^2 \\ -i\sigma^2 & \mathbf{0}_2 \end{pmatrix} \otimes \sigma^0, \quad \Sigma^5 = \frac{1}{2} \begin{pmatrix} \mathbf{0}_2 & -\sigma^2 \\ -\sigma^2 & \mathbf{0}_2 \end{pmatrix} \otimes \sigma^0.$$

- The 2HDM Potential in the **SO(1,5) Formalism**

$$V_{2\text{HDM}} = -\frac{1}{2} M_A R^A + \frac{1}{4} L_{AB} R^A R^B ,$$

with

$$M_A = (\mu_1^2 + \mu_2^2, \quad 2\text{Re}(m_{12}^2), \quad -2\text{Im}(m_{12}^2), \quad \mu_1^2 - \mu_2^2, \quad 0, \quad 0) ,$$

$$L_{AB} = \begin{pmatrix} \lambda_1 + \lambda_2 + \lambda_3 & \text{Re}(\lambda_6 + \lambda_7) & -\text{Im}(\lambda_6 + \lambda_7) & \lambda_1 - \lambda_2 & 0 & 0 \\ \text{Re}(\lambda_6 + \lambda_7) & \lambda_4 + \text{Re}(\lambda_5) & -\text{Im}(\lambda_5) & \text{Re}(\lambda_6 - \lambda_7) & 0 & 0 \\ -\text{Im}(\lambda_6 + \lambda_7) & -\text{Im}(\lambda_5) & \lambda_4 - \text{Re}(\lambda_5) & -\text{Im}(\lambda_6 - \lambda_7) & 0 & 0 \\ \lambda_1 - \lambda_2 & \text{Re}(\lambda_6 - \lambda_7) & -\text{Im}(\lambda_6 - \lambda_7) & \lambda_1 + \lambda_2 - \lambda_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} .$$

- **Unitary Field Transformations:**

[AP, Phys. Lett. B706 (2012) 465.]

$$\text{Sp}(4) : \quad \Phi \mapsto \Phi' = U \Phi , \quad \text{with } U \in \text{U}(4) \quad \underline{\text{and}} \quad UCU^T = C$$

$$\text{SO}(5) : \quad R^I \mapsto R'^I = O^I{}_J R^J , \quad \text{with } O \in \text{SO}(5) \subset \text{SO}(1,5)$$

$$\implies \quad \text{SO}(5) \sim \text{Sp}(4)/\mathbf{Z}_2$$

• Symmetry Breaking Scenarios and pseudo-Goldstone Bosons

[AP, Phys. Lett. B706 (2012) 465.]

No	Symmetry	Generators $T^a \leftrightarrow K^a$	Discrete Group Elements	Maximally Broken SO(5) Generators	Number of Pseudo- Goldstone Bosons
1	$Z_2 \times O(2)$	T^0	D_{CP1}	–	0
2	$(Z_2)^2 \times SO(2)$	T^0	D_{Z_2}	–	0
3	$(Z_2)^3 \times O(2)$	T^0	D_{CP2}	–	0
4	$O(2) \times O(2)$	T^3, T^0	–	T^3	1 (a)
✓ 5	$Z_2 \times [O(2)]^2$	T^2, T^0	D_{CP1}	T^2	1 (h)
✓ 6	$O(3) \times O(2)$	$T^{1,2,3}, T^0$	–	$T^{1,2}$	2 (h, a)
7	$SO(3)$	$T^{0,4,6}$	–	$T^{4,6}$	2 (h^\pm)
8	$Z_2 \times O(3)$	$T^{0,4,6}$	$D_{Z_2} \cdot D_{CP2}$	$T^{4,6}$	2 (h^\pm)
9	$(Z_2)^2 \times SO(3)$	$T^{0,5,7}$	$D_{CP1} \cdot D_{CP2}$	$T^{5,7}$	2 (h^\pm)
10	$O(2) \times O(3)$	$T^3, T^{0,8,9}$	–	T^3	1 (a)
11	$SO(4)$	$T^{0,3,4,5,6,7}$	–	$T^{3,5,7}$	3 (a, h^\pm)
12	$Z_2 \times O(4)$	$T^{0,3,4,5,6,7}$	$D_{Z_2} \cdot D_{CP2}$	$T^{3,5,7}$	3 (a, h^\pm)
✓ 13	$SO(5)$	$T^{0,1,2,\dots,9}$	–	$T^{1,2,8,9}$	4 (h, a, h^\pm)

✓: Natural SM Alignment \mapsto

[Dev, AP, JHEP1412 (2014) 024.]

• Vacuum Topology of the 2HDM

[R Battye, G Brawn, AP, JHEP08 (2011) 020.]

$G_{\text{HF/CP}}$	$H_{\text{HF/CP}}$	$\mathcal{M}_{\Phi}^{\text{HF/CP}}$	Topological Defect
Z_2	\mathbf{I}	Z_2	Domain Wall
$U(1)_{\text{PQ}} \simeq S^1$	\mathbf{I}	S^1	Vortex
$SO(3)_{\text{HF}}$	$SO(2)_{\text{HF}}$	S^2	Global Monopole
$CP1 \simeq Z_2$	\mathbf{I}	Z_2	Domain Wall
$CP2 = Z_2 \otimes \Pi_2$	Π_2	Z_2	Domain Wall
$CP1 \otimes SO(2)$	$CP1$	S^1	Vortex

- Energy density of the topological defect $\phi_{1,2}(\mathbf{r})$:

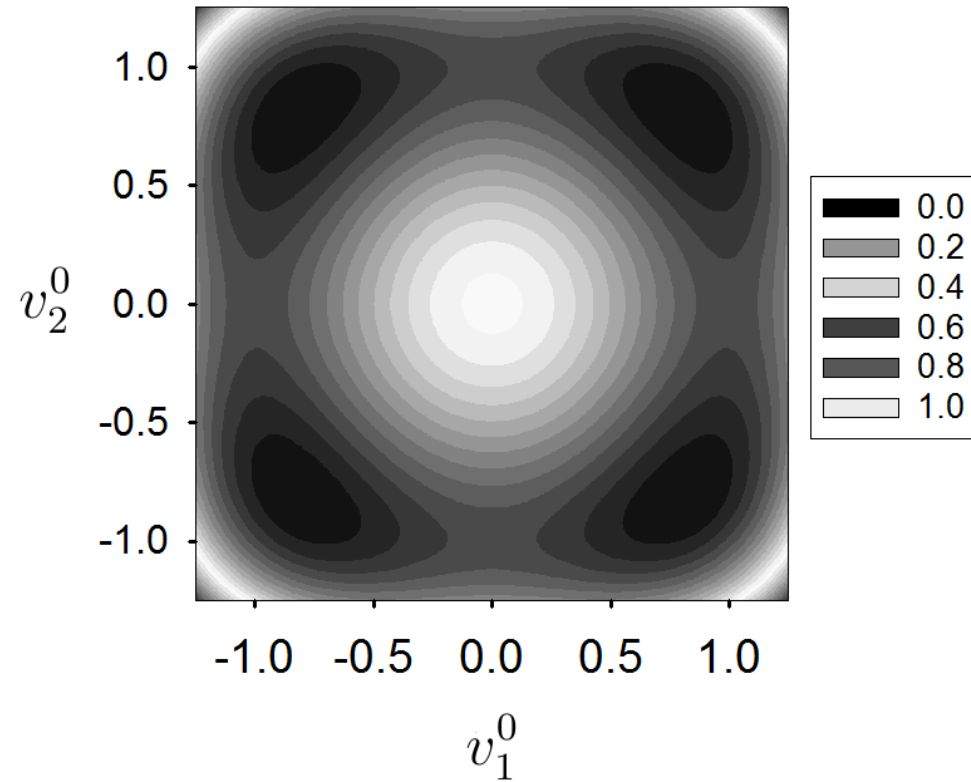
$$\mathcal{E}(\phi_1, \phi_2) = (\nabla\phi_1^\dagger) \cdot (\nabla\phi_1) + (\nabla\phi_2^\dagger) \cdot (\nabla\phi_2) + V(\phi_1, \phi_2) + V_0 .$$

- Gradient flow approach to numerically find $\phi_{1,2}(\mathbf{r})$

$$-\frac{\delta E[\phi_{1,2}]}{\delta\phi_{1,2}(\mathbf{r}, \tau)} = \frac{\partial\phi_{1,2}(\mathbf{r}, \tau)}{\partial\tau} \rightarrow 0 , \quad \text{for } \tau \gg 1 .$$

- Z₂ Domain Walls**

$$\begin{array}{ccc}
 \begin{pmatrix} v_1^0 \\ v_2^0 \end{pmatrix} & \xleftrightarrow{U(1)_Y} & \begin{pmatrix} -v_1^0 \\ -v_2^0 \end{pmatrix} \\
 \downarrow Z_2 & & \uparrow Z_2 \\
 \begin{pmatrix} v_1^0 \\ -v_2^0 \end{pmatrix} & \xleftrightarrow{U(1)_Y} & \begin{pmatrix} -v_1^0 \\ v_2^0 \end{pmatrix}
 \end{array}$$

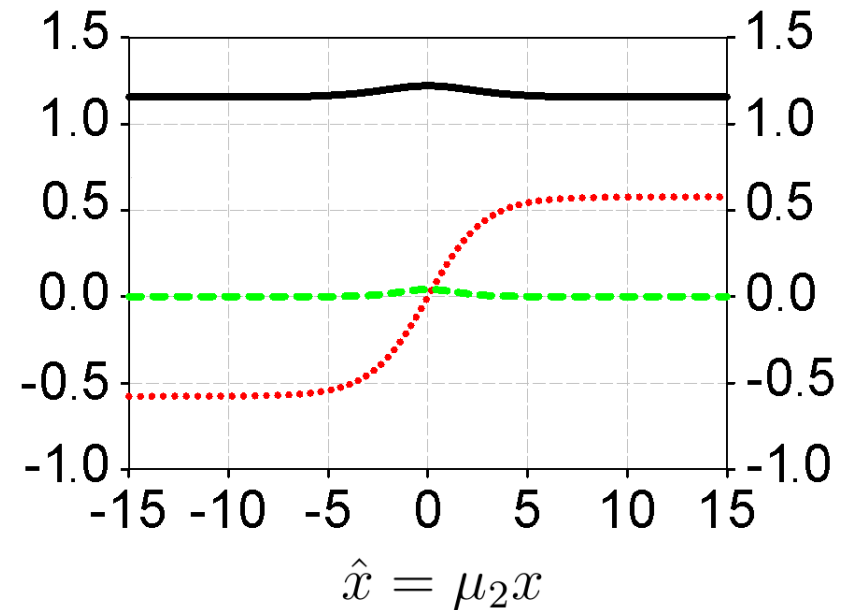
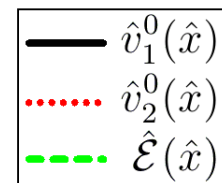
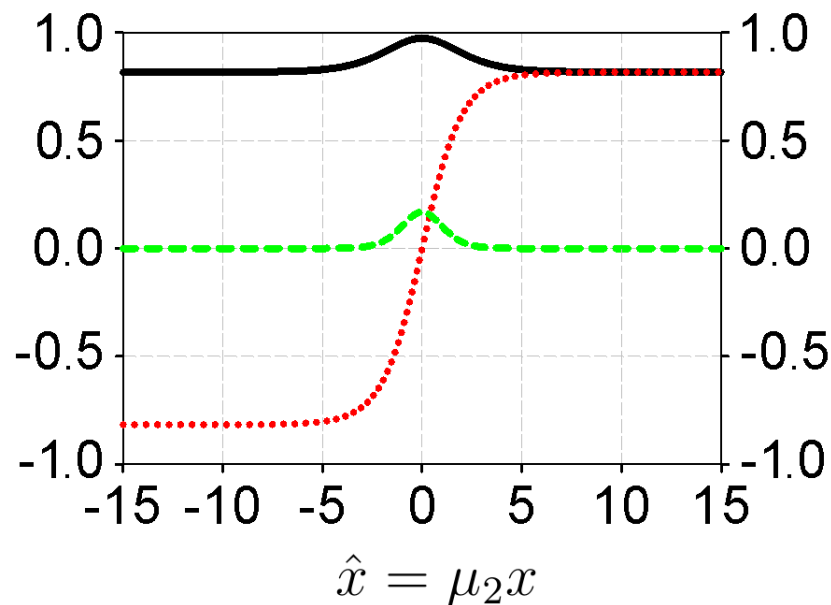


– Spatial profile of the Z_2 domain wall

[R Battye, G Brown, AP, JHEP08 (2011) 020.]

Introduce dimensionless quantities:

$$\hat{x} = \mu_2 x, \quad \hat{v}_{1,2}^0(\hat{x}) = \frac{v_{1,2}^0(x)}{\eta}, \quad \hat{E} = \frac{\lambda_2 E}{\mu_2^3}, \quad \text{with } \eta = \frac{\mu_2}{\sqrt{\lambda_2}}.$$



• Charge-Violating Domain Walls in the 2HDM

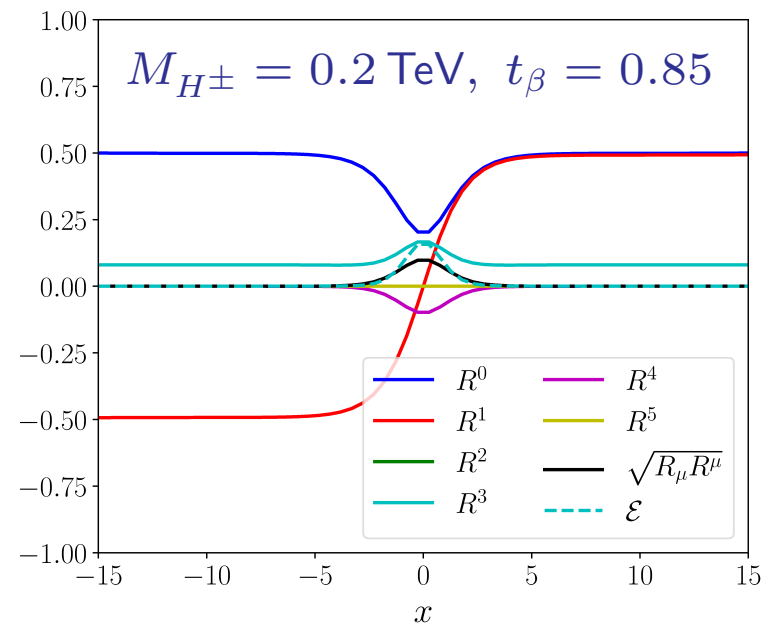
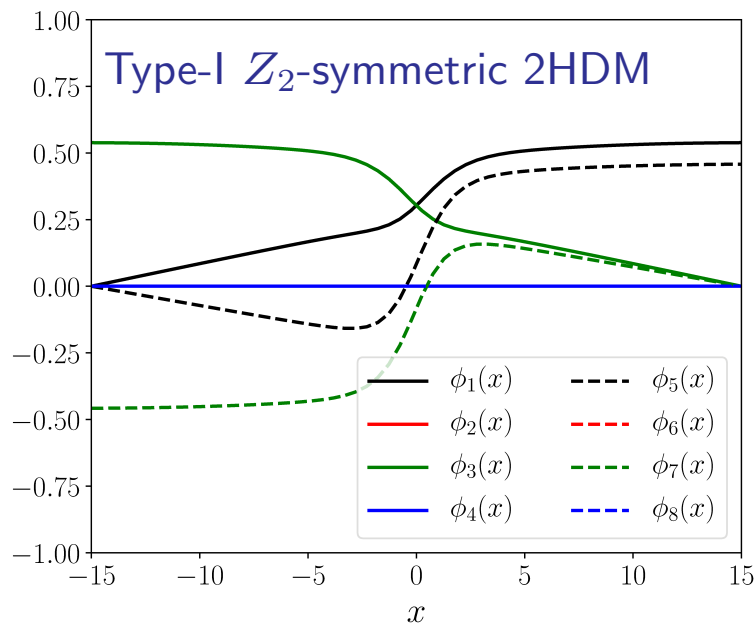
[R Battye, AP, D Viatic, JHEP2101 (2021) 105.
K.H. Law, AP, PRD105 (2022) 056007]

– Relatively **gauge-rotated** vacua at the boundaries:

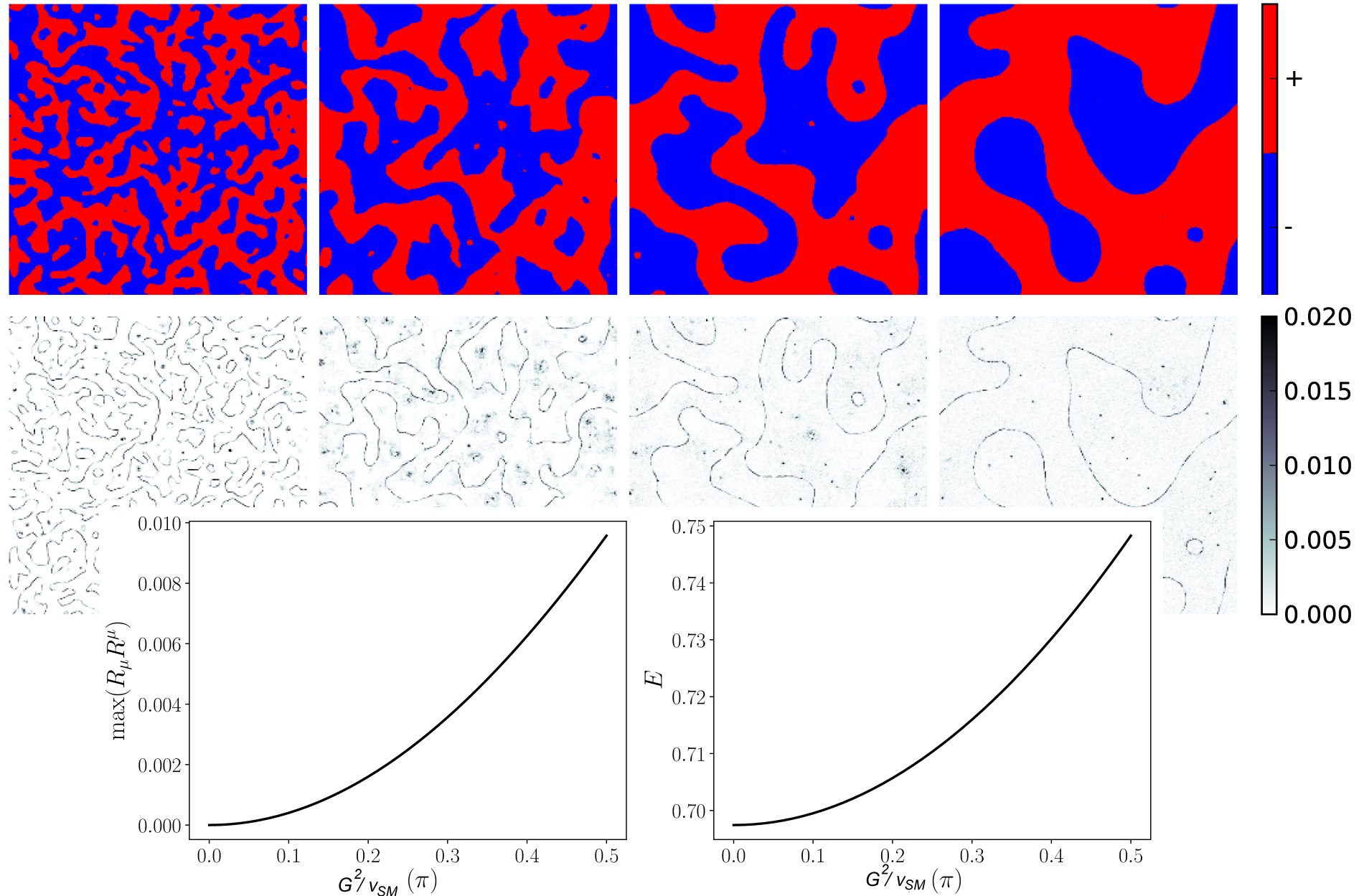
$$\Phi_1(-\infty) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \Phi_2(-\infty) = \frac{1}{\sqrt{2}} \begin{pmatrix} v_+ \rightarrow 0 \\ -v_2 e^{-i\xi} \end{pmatrix},$$

$$\Phi_1(+\infty) = U(+\infty) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \Phi_2(+\infty) = U(+\infty) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ +v_2 e^{+i\xi} \end{pmatrix},$$

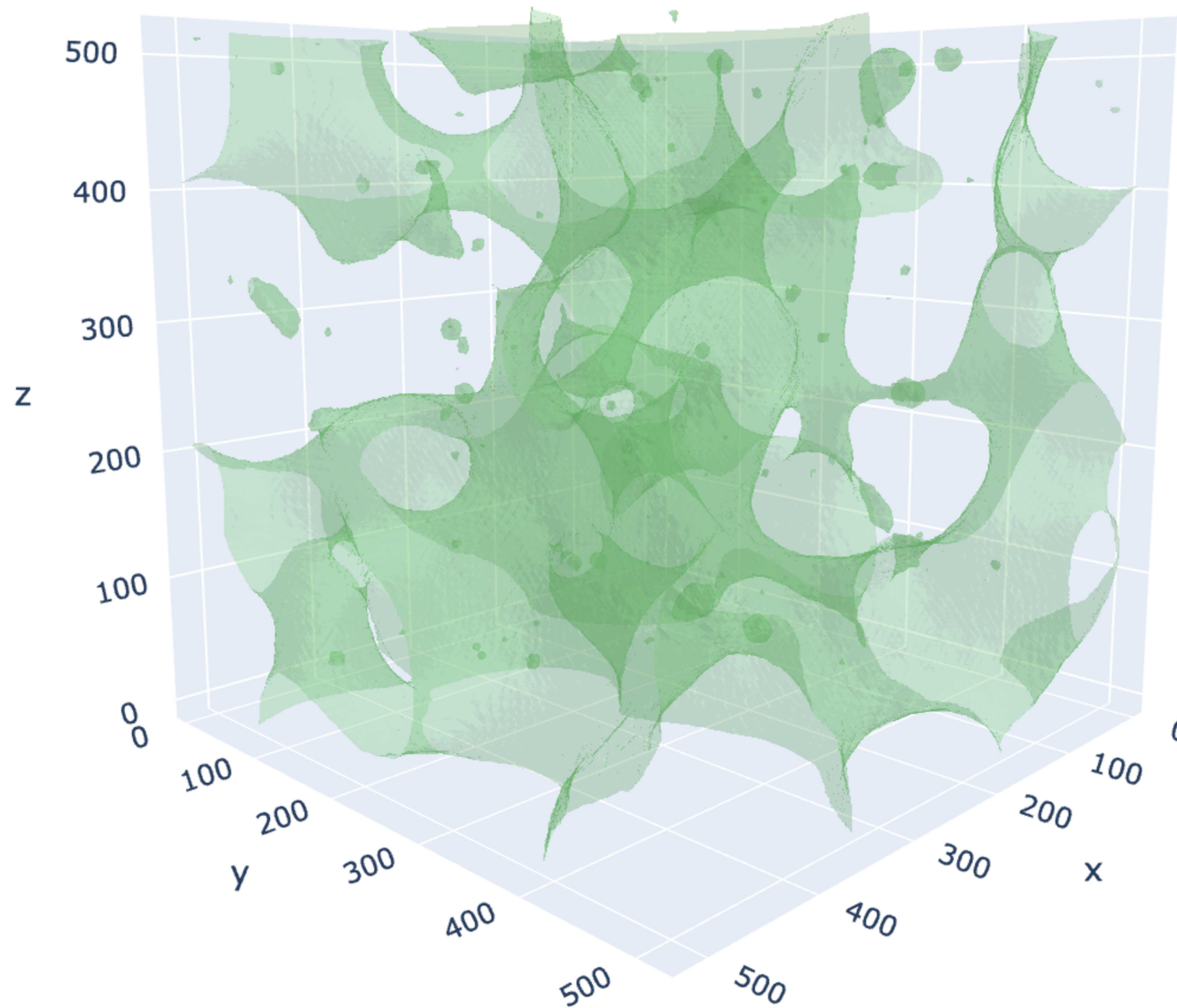
$$U(x) = e^{i\theta(x)} \exp\left(\frac{i G^i(x) \sigma^i}{v_{\text{SM}} 2}\right), \quad \text{with } U(-\infty) = \mathbf{1}_2.$$



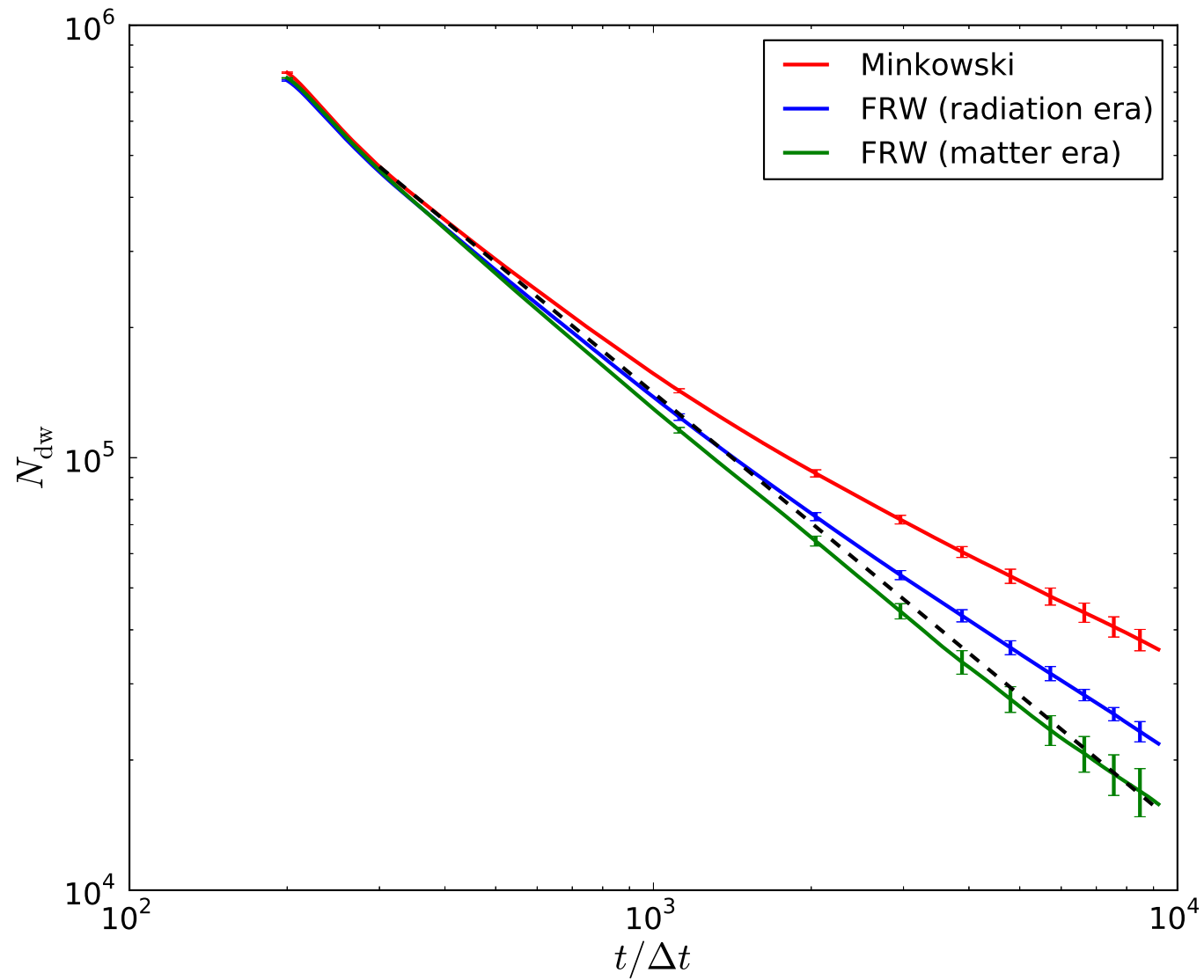
– 2D DW simulations in the Type-I Z_2 -symmetric 2HDM



– **3D DW network** in the Type-I Z_2 -symmetric 2HDM



– Evolution of DW number N_{dw} in the Type-I Z_2 -symmetric 2HDM



– QCD instantons in Type-II Z_2 -symmetric 2HDM

[R Battye, AP, D Viatc, PRD102 (2020) 123536;
RD Peccei, HR Quinn '77]

$$V_{\text{inst}} \sim \Lambda_{\text{QCD}}^4 \left[\left(\frac{\Phi_1^\dagger \Phi_2}{v_{\text{SM}}^2} \right)^{n_G} - \left(\frac{\Phi_1^\dagger \Phi_2 e^{i\theta_{\text{QCD}}}}{v_{\text{SM}}^2} \right)^{n_G} \right] + \text{H.c.}$$
$$\lesssim \frac{\Lambda_{\text{QCD}}^4}{v_{\text{SM}}^2} s_\beta^2 c_\beta^2 \left(1 - \cos(n_G \theta_{\text{QCD}}) \right) \Phi_1^\dagger \Phi_2 + \text{H.c.},$$

$$\implies \theta_{\text{QCD}} \gtrsim \frac{10^{-11}}{\sin \beta \cos \beta}$$

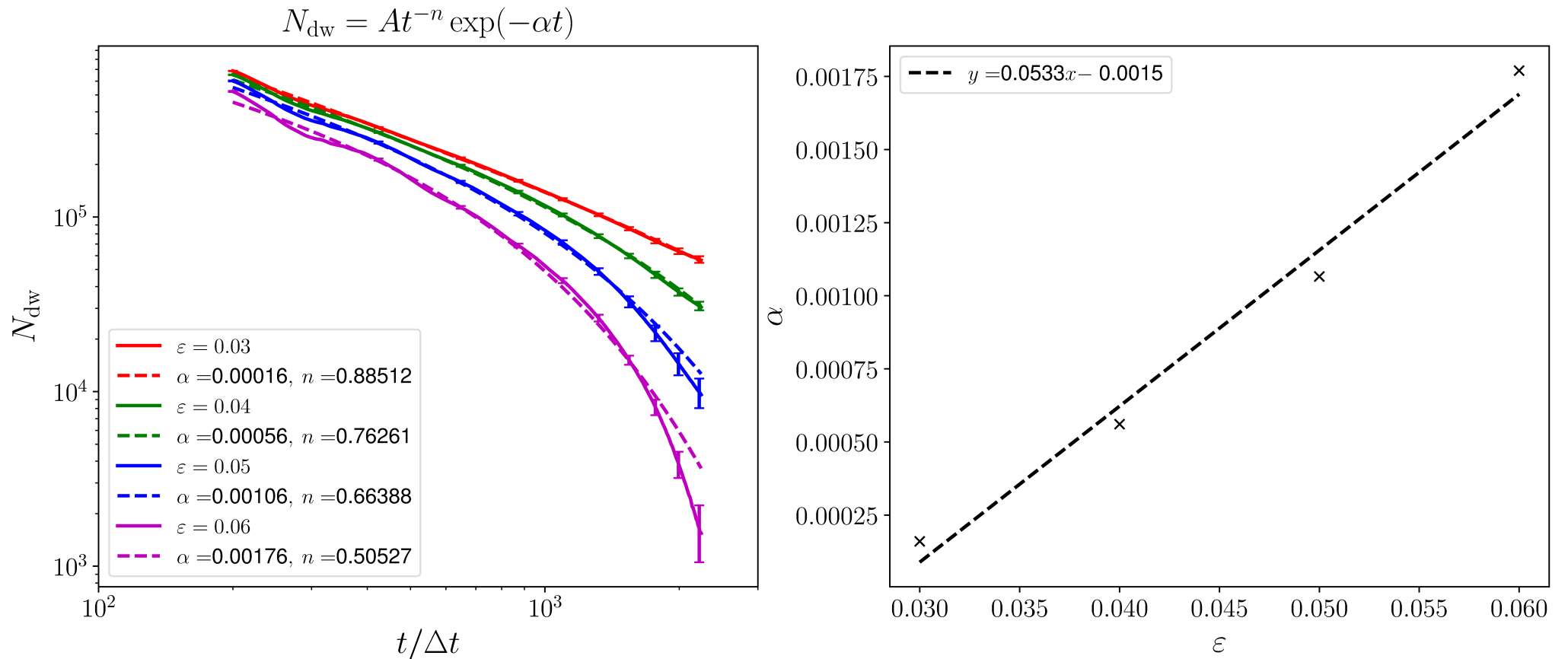
From neutron EDM limit: $\theta_{\text{QCD}} \lesssim 10^{-11} - 10^{-10}$

Loose constraint:

$$0.3 \lesssim \tan \beta \lesssim 3$$

– Biased initial conditions in Z_2 -symmetric 2HDMs

[R Battye, AP, D Viatic, PRD102 (2020) 123536]



Avoidance of DW domination in the Universe:

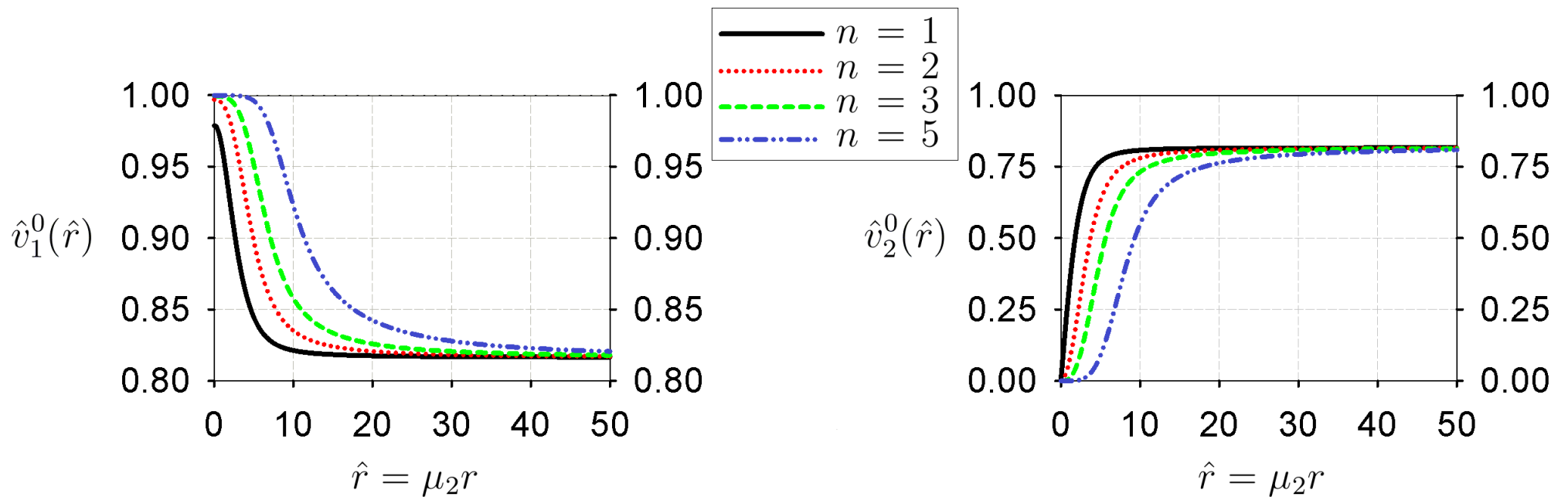
$$\epsilon > \frac{640\pi A \hat{E}}{3 e} \left(\frac{v_{\text{SM}}^{3/2}}{M_{\text{Pl}}} \right)^2 \simeq 2.5 \times 10^{-29} A \hat{E} \text{ GeV}, \quad \text{with } A, \hat{E} \sim 1.$$

• Other Topological Defects from the 2HDM Potential

• $U(1)_{PQ}$ Vortices

[R. A. Battye, G. D. Brawn, A.P., JHEP08 (2011) 020.]

$$\phi_1(r) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1^0(r) \end{pmatrix}, \quad \phi_2(r, \chi) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2^0(r) e^{in\chi} \end{pmatrix}.$$

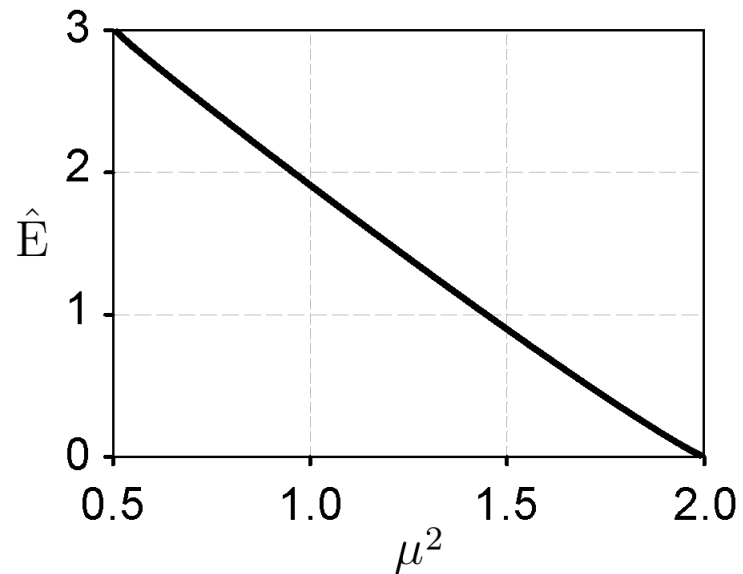


Energy dependence of the $U(1)_{PQ}$ Vortex

[R. A. Battye, G. D. Brawn, A.P., JHEP08 (2011) 020.]

Energy per unit length:

$$E = 2\pi \int_0^\infty r dr \mathcal{E}(\phi_1, \phi_2) ,$$



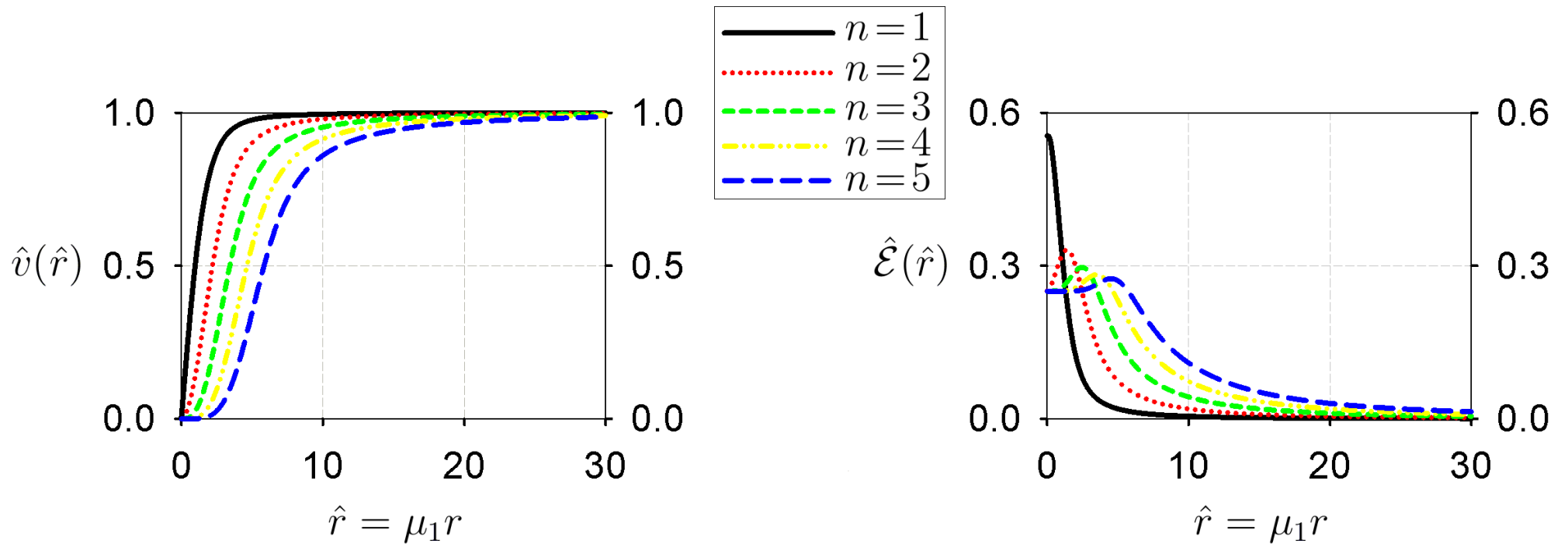
with

$$\mu^2 = \frac{\mu_1^2}{\mu_2^2} .$$

• **CP3 Vortices**

[R. A. Battye, G. D. Brawn, A.P., JHEP08 (2011) 020.]

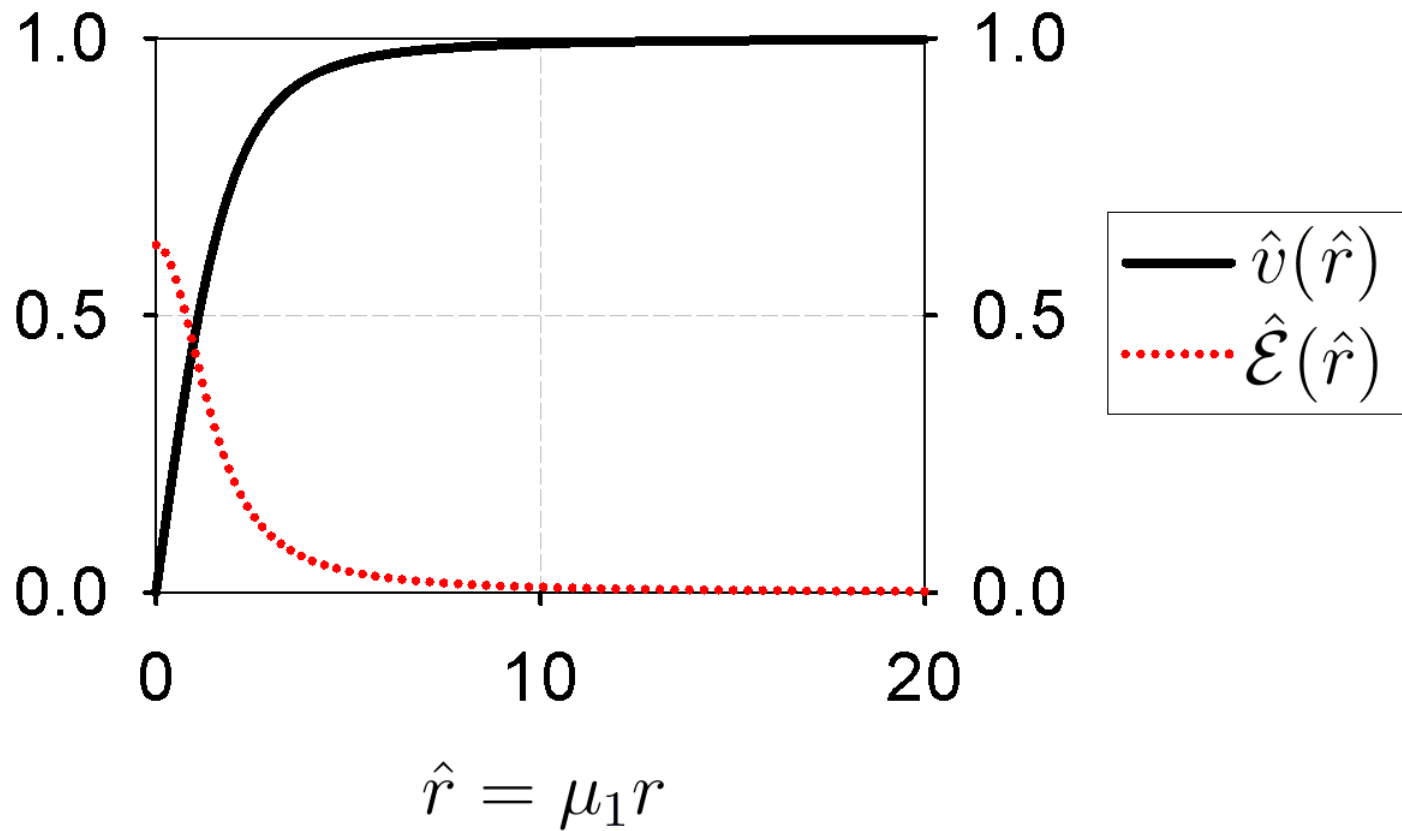
$$\phi_1(r, \chi) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v(r) \cos(n\chi) \end{pmatrix}, \quad \phi_2(r, \chi) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ -v(r) \sin(n\chi) \end{pmatrix}.$$



• **SO(3)_{HF} Global Monopole**

[R. A. Battye, G. D. Brawn, A.P., JHEP08 (2011) 020.]

$$\phi_1(r, \chi) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v(r) \sin \chi \end{pmatrix}, \quad \phi_2(r, \chi, \psi) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v(r) e^{i\psi} \cos \chi \end{pmatrix}.$$



• Natural Alignment Beyond the 2HDM

[AP '16]

– n HDM **potential** with m inert scalar doublets:

$$V_{n\text{HDM}} = V_{\text{sym}} + V_{\text{inert}} + \Delta V_{\text{soft}} ,$$

– **3** continuous **alignment symmetries** in the field space of the **active** EWSB sector ($N_H = n - m$):

$$(i) \text{ Sp}(2N_H) \times \mathcal{D} \quad (ii) \text{ SU}(N_H) \times \mathcal{D} \quad (iii) \text{ SO}(N_H) \times \mathcal{CP} \times \mathcal{D} ,$$

where \mathcal{D} acts on the inert sector *only*.

– **Symmetry invariants:**

$$(i) \quad S = \Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 + \dots = \frac{1}{2} \Phi^\dagger \Phi$$

$$(ii) \quad D^a = \Phi_1^\dagger \sigma^a \Phi_1 + \Phi_2^\dagger \sigma^a \Phi_2 + \dots$$

$$(iii) \quad T = \Phi_1 \Phi_1^\top + \Phi_2 \Phi_2^\top + \dots$$

– **Symmetric part of the scalar potential:**

$$V_{\text{sym}} = -\mu^2 S + \lambda_S S^2 + \lambda_D D^a D^a + \lambda_T \text{Tr}(T T^*) .$$

– Inert part of the scalar potential:

$$V_{\text{inert}} = \widehat{m}_{\widehat{a}\widehat{b}}^2 \widehat{\Phi}_{\widehat{a}}^\dagger \widehat{\Phi}_{\widehat{b}} + \lambda_{\widehat{a}\widehat{b}\widehat{c}\widehat{d}} (\widehat{\Phi}_{\widehat{a}}^\dagger \widehat{\Phi}_{\widehat{b}}) (\widehat{\Phi}_{\widehat{c}}^\dagger \widehat{\Phi}_{\widehat{d}}) + \lambda_{\widehat{a}\widehat{b}cd} (\widehat{\Phi}_{\widehat{a}}^\dagger \widehat{\Phi}_{\widehat{b}}) (\Phi_c^\dagger \Phi_d) \\ + \lambda_{a\widehat{b}\widehat{c}d} (\Phi_a^\dagger \widehat{\Phi}_{\widehat{b}}) (\widehat{\Phi}_{\widehat{c}}^\dagger \Phi_d) + \left[\lambda_{a\widehat{b}c\widehat{d}} (\Phi_a^\dagger \widehat{\Phi}_{\widehat{b}}) (\Phi_c^\dagger \widehat{\Phi}_{\widehat{d}}) + \text{H.c.} \right]$$

$$\mathbf{Z}_2^I : \quad \Phi_a \rightarrow \Phi_a \quad (a = 1, 2, \dots, N_H), \quad \widehat{\Phi}_{\widehat{b}} \rightarrow -\widehat{\Phi}_{\widehat{b}} \quad (\widehat{b} = \widehat{1}, \widehat{2}, \dots, \widehat{m})$$

– Soft-symmetry Breaking:

$$\Delta V_{\text{soft}} = m_{ab}^2 \Phi_a^\dagger \Phi_b$$

– Minimal Symmetry of Alignment in the Higgs basis:

$$\mathbf{Z}_2^{\text{EW}} : \quad \Phi'_1 \rightarrow \Phi'_1, \quad \Phi'_{a'} \rightarrow -\Phi'_{a'} \quad (a' = 2, 3, \dots, N_H)$$

where m_{ab}^2 becomes diagonal.

\implies

Minimal Alignment Symmetry: $\mathbf{Z}_2^{\text{EW}} \times \mathbf{Z}_2^I$

[AP '16]