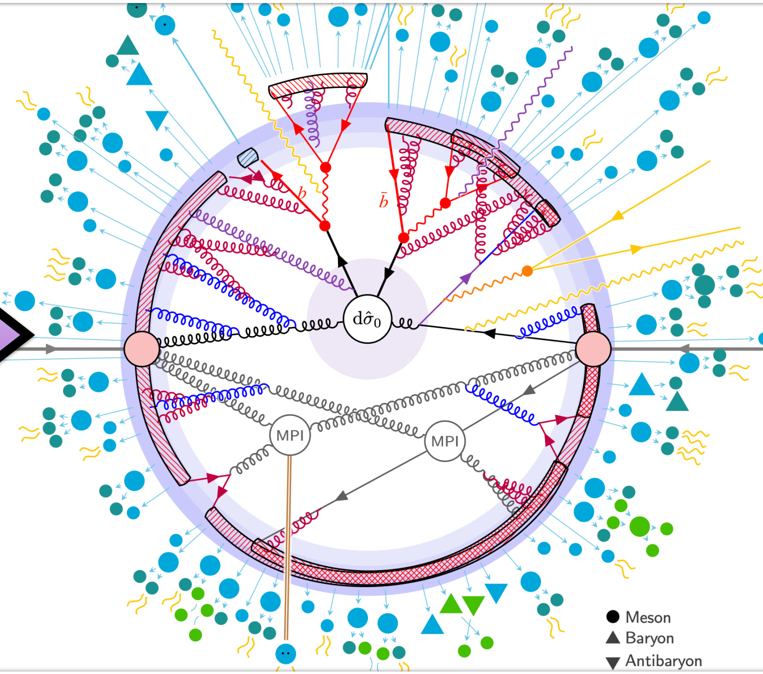
VINCIA NNLO

NNLO Matching in Vincia

Peter Skands — U of Oxford & Monash U.

Loops \ Legs	0	1	2
0	B	R	RR
1	V	RV	
2	VV		



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Introduction & Overview

Fixed-Order pQCD State of the Art: NNLO (→ N^3LO)

Resummation extends range of applicability: multi-scale problems

MCs: Showers, MPI, Hadronization → **Explicit collider studies**

Hadronization corrections, UE, IR sensitivity, tuning, measurement calibrations, detector response, …

1. Can use off-the-shelf (LL) showers, e.g. with MiNNLO$_{PS}$

Based on POWHEG-Box ⊕ Analytical Resummation ⊕ NNLO normalisation

Best you can do with LL showers but approximate; depends on some auxiliary scales & choices

2. This talk: VinciaNNLO

Based on nested shower-style phase-space generation with 2nd-order MECs

True NNLO matching (shower matches NNLO point by point) → Expect small matching systematics

So far only worked out for colour-singlet decays

Also developing extensions of the shower LL → **NLL** → NNLL (with L. Scyboz, B. El Menoufi)

Why go beyond **Fixed-Order** perturbation theory?

Simple example of a multi-scale observable:

For an arbitrary hard process ($Q_{\text{hard}} \gg 1\,\text{GeV}$)

Calculation of the **fraction of events** that pass a **jet veto**

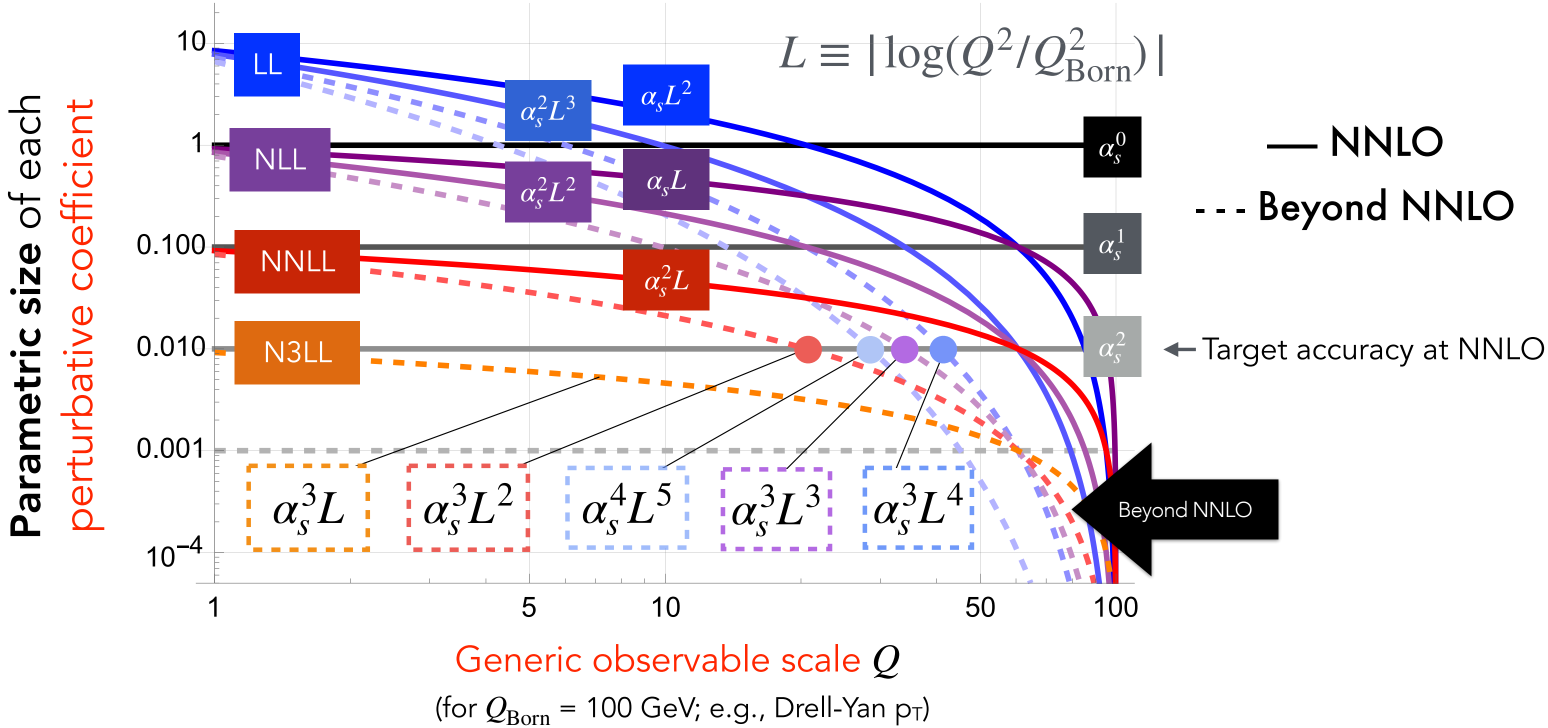
(i.e., **no additional jets** resolved above Q_{veto}):

$$\overbrace{1}^{\text{LO}} - \overbrace{\alpha_s(L^2 + L + F_1)}^{\text{NLO}} + \overbrace{\alpha_s^2(L^4 + L^3 + L^2 + L + F_2)}^{\text{NNLO}} + \ \ldots$$

$$L \propto \ln(Q_{\text{veto}}^2 / Q_{\text{hard}}^2)$$

$\left(\text{Logs arise from integrals over propagators} \propto \dfrac{1}{q^2}\right)$

The Case for Combining Fixed-Order Calculations with Resummations



Resummation extends domain of validity of perturbative calculations

Perturbation Theory as a Markov Chain

Stochastic differential evolution in "hardness" scale

dσ for generic observable "O", expressed as a Markov chain:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}O} = \int \mathrm{d}\Phi_0 \, \underbrace{|M_{\mathrm{Born}}^{\mathrm{LO}}|^2 \overbrace{\left(1 + F_{\mathrm{NLO}} + \ldots\right)}^{\text{Differential Born-level "k" factor}}}_{\substack{\text{Born-Level} \\ \text{Fixed-Order Matching Coefficients}}} \, \underbrace{\mathcal{S}(\Phi_0, O)}_{\text{Shower}}$$

(In general, the Fixed-Order matching coefficients M and F are **local** = functions of Φ_0)

A Simple FSR Shower

With only (iterated) $n \to n+1$ kernels

Sudakov expansions ➜ "virtuals"

$$\mathcal{S}_{+1}(\Phi_n, O) = \overbrace{\Delta\left(\Phi_n, Q_{\mathrm{IR}}\right)}^{\text{'Sudakov Factor'}} \overbrace{\delta\left(\hat{O}(\Phi_n) - O\right)}^{\text{Evaluate } O \text{ on } \Phi_n}$$

$$+ \int \mathrm{d}\Phi_{+1} \overbrace{\Delta\left(\Phi_n, Q_{n+1}\right)}^{\text{Sudakov Factor}} \overbrace{\frac{|\tilde{M}_{n+1}|^2}{|\tilde{M}_n|^2}}^{\text{Branching Kernel}} \mathcal{S}_{+1}(\Phi_{n+1}, O)$$

Branchings ➜ "reals"

MARKOV CHAIN

UNITARITY

Sudakov Factor $\Delta(\Phi_n, Q) = \exp\left(- \int_{Q^2}^{Q_n^2} \mathrm{d}\Phi_{+1} \frac{|\tilde{M}_{n+1}|^2}{|\tilde{M}_n|^2} \right)$

Branching Kernel

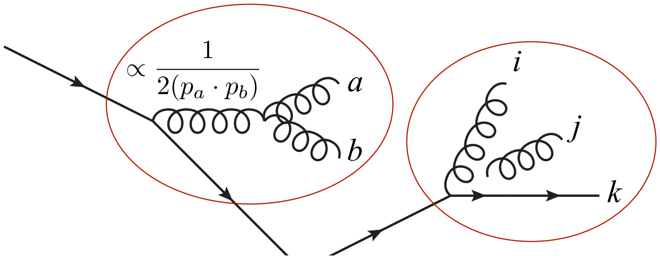
Soft-Collinear Approximations or tree-level MEs (MECs)

NB: partition of phase space and branching probabilities onto different terms not shown here

Branching Kernels (for single branchings)

Most bremsstrahlung is driven by **divergent propagators** → simple universal structure, independent of process details

Amplitudes *factorise* in singular limits



Collinear limits ➜ DGLAP splitting kernels:

$$|\mathcal{M}_{F+1}(\ldots,a,b,\ldots)|^2 \stackrel{a||b}{\rightarrow} g_s^2 \mathcal{C} \frac{P(z)}{2(p_a \cdot p_b)} |\mathcal{M}_F(\ldots,a+b,\ldots)|^2$$

Soft limits ($E_g/Q \to 0$) ➜ dipole factors (same as classical):

$$|\mathcal{M}_{F+1}(\ldots,i,j,k\ldots)|^2 \stackrel{j_g\to 0}{\rightarrow} g_s^2 \mathcal{C} \frac{(p_i \cdot p_k)}{(p_i \cdot p_j)(p_j \cdot p_k)} |\mathcal{M}_F(\ldots,i,k,\ldots)|^2$$

These limits are not independent; they overlap in phase space.

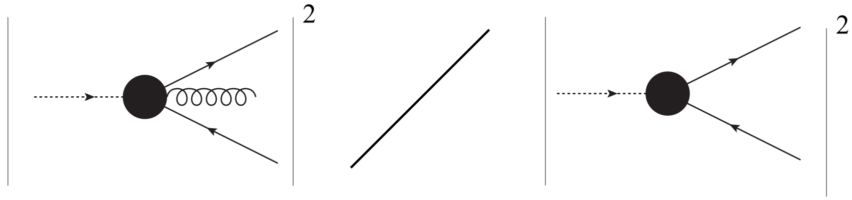
How to treat the two consistently has given rise to **many** individual approaches:

Angular ordering, **angular vetos**, **dipoles**, **global antennae**, **sector antennae**, …

DGLAP, Dipoles, Antennae

Factorisation of (squared) amplitudes in IR singular limits

(leading colour)



DGLAP

ij-collinear limit

jk-collinear limit

$$\frac{P_{q\to qg}(z_i)}{s_{qg}} + \frac{P_{q\to qg}(z_k)}{s_{g\bar{q}}}$$

One term for each **parton**

Requires angular ordering to get soft limits right

Antenna

Full ME (modulo nonsingular terms)

$$\frac{2s_{q\bar{q}}}{s_{qg}s_{g\bar{q}}} + \frac{1}{s}\left(\frac{s_{g\bar{q}}}{s_{qg}} + \frac{s_{qg}}{s_{g\bar{q}}}\right)$$

eikonal term

collinear terms

partitioning of eikonal

One term for each colour-connected pair of partons

Dipole (CS/Partitioned)

$$\frac{\mathscr{K}_{qg,\bar{q}}(z_q)}{s_{qg}} + \frac{\mathscr{K}_{\bar{q}g,q}(z_{\bar{q}})}{s_{g\bar{q}}}$$

Two terms for each colour-connected pair of partons

Note: this is (intentionally) oversimplified. Many subtleties (recoil strategies, gluon parents, global vs sector, colour factors, initial-state partons, mass terms) not shown.

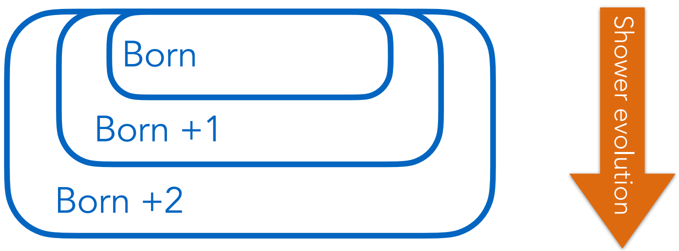
VinciaNNLO

VINCIA NNLO

Idea: Use (nested) Shower Markov Chain as NNLO Phase-Space Generator

Harnesses the power of showers as efficient phase-space generators for QCD

Pre-weighted with the (leading) QCD singular structures = soft/collinear poles



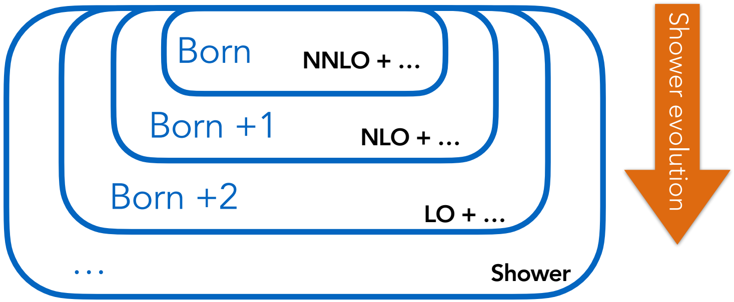
Different from conventional Fixed-Order phase-space generation (eg VEGAS)

VinciaNNLO

Continue shower afterwards …

No auxiliary / unphysical scales

⇒ expect small matching systematics



Proofs of concept for

$Z \to q\bar{q}$ @ NNLO

Hartgring, Laenen, **PZS** 2013
Li, **PZS** 2017
Campbell et al. 2023
Preuss, **PZS** 2024

Need:

1. Born-Local NNLO ($\mathcal{O}(\alpha_s^2)$) *K*-factors: $k^{\mathrm{NNLO}}(\Phi_0)$
2. NLO ($\mathcal{O}(\alpha_s^2)$) MECs in the first $2 \to 3$ shower emission: $k^{\mathrm{NLO}}_{2\to3}(\Phi_1)$
3. LO ($\mathcal{O}(\alpha_s^2)$) MECs for next (iterated) $2 \to 3$ shower emission: $k^{\mathrm{LO}}_{3\to4}(\Phi_2)$
4. Direct $2 \to 4$ branchings for unordered sector, with LO ($\mathcal{O}(\alpha_s^2)$) MECs: $k^{\mathrm{LO}}_{2\to4}(\Phi_2)$

Based on Sector Antenna Showers

Lopez-Villarejo & **PS** 1109.3608 Brooks, Preuss & **PS** 2003.00702

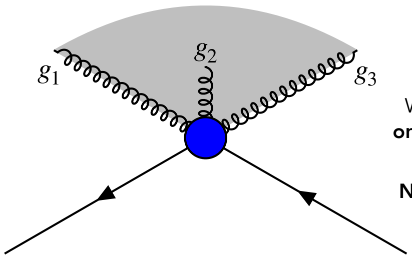
Sector antennae

Kosower, hep-ph/9710213 hep-ph/0311272 (+ Larkoski & Peskin 0908.2450, 1106.2182)

Divide the n**-gluon** phase space up into
n **non-overlapping sectors** →

Inside each of which **only the most singular** (~"classical") kernel is allowed to contribute.

Example: $Z \to q\bar{q}ggg$



Sectorization:
When 2 is "softest", the **only** contributing history is 2 emitted by 1 and 3
No "sum over histories"

Lorentz-invariant sector definitions based on "ARIADNE p_T":

Gustafson & Pettersson, NPB 306 (1988) 746

$$p_{\perp j}^2 = \frac{s_{ij}s_{jk}}{s_{ijk}}$$ with $s_{ij} \equiv 2(p_i \cdot p_j)$ *(+ generalisations for heavy-quark emitters)* Brooks, Preuss & PS 2003.00702

➜ Unique properties (which are useful for matching):

Clean scale definitions; shower operator is **bijective** & true **Markov chain**

NNLO Matching as a Markov chain

Campbell, Höche, Li, Preuss, **PZS**, 2108.07133

VINCIA NNLO

Focus on hadronic Z decays (for now)

"Two-loop MEC"

$$\langle O \rangle^{\text{NNLO+PS}}_{\text{Vincia}} = \int \mathrm{d}\Phi_0 \, B(\Phi_0) \, k_0^{\text{NNLO}}(\Phi_0) \, \mathcal{S}(\Phi_0, O)$$

Ingredients:

1. Born-Local NNLO ($\mathcal{O}(\alpha_s^2)$) K-factors: $k^{\text{NNLO}}(\Phi_0)$
2. NLO ($\mathcal{O}(\alpha_s^2)$) MECs in the first $2 \to 3$ shower emission: $k^{\text{NLO}}_{2\to3}(\Phi_1)$
3. LO ($\mathcal{O}(\alpha_s^2)$) MECs for next (iterated) $2 \to 3$ shower emission: $k^{\text{LO}}_{3\to4}(\Phi_2)$
4. Direct $2 \to 4$ branchings for "unordered sector", with LO ($\mathcal{O}(\alpha_s^2)$) MECs: $k^{\text{LO}}_{2\to4}(\Phi_2)$

NEW

$$\mathcal{S}(\Phi_n, O) = \mathcal{S}_{+1}(\Phi_n, O) + \mathcal{S}_{+2}(\Phi_n, O)$$

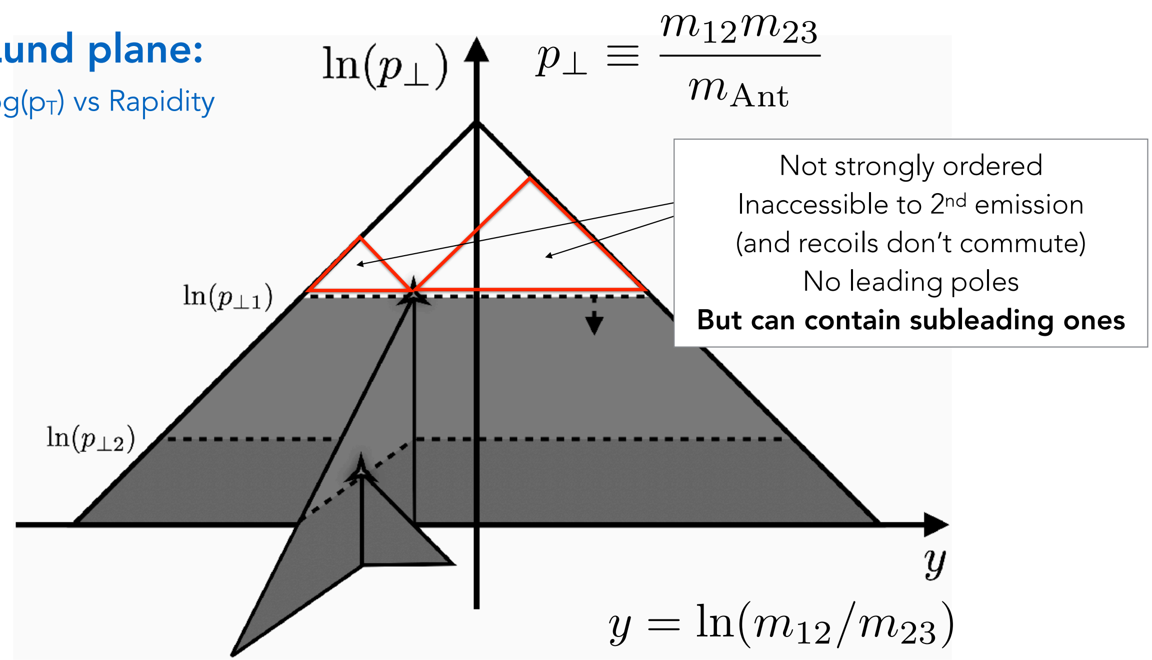
Why do we need direct 2→4 Branchings?

Iterated MECs not possible with off-the-shelf showers

E.g., strong $p_\perp$-ordering **cuts out** part of the second-order phase space

Lund plane:

Log(p_T) vs Rapidity



Example: $Z \to qg g\bar{q}$

Double-differential distribution in $\frac{p_{\perp 1}}{m_Z}$ & $\frac{p_{\perp 2}}{p_{\perp 1}}$

$$R_4 = \frac{\text{Sum(shower histories)}}{|M_{Z\to 4}^{(\text{LO,LC})}|^2}$$

Example phase-space point:

Q_0 = mZ = 91 GeV

p_{T1} = 5 GeV

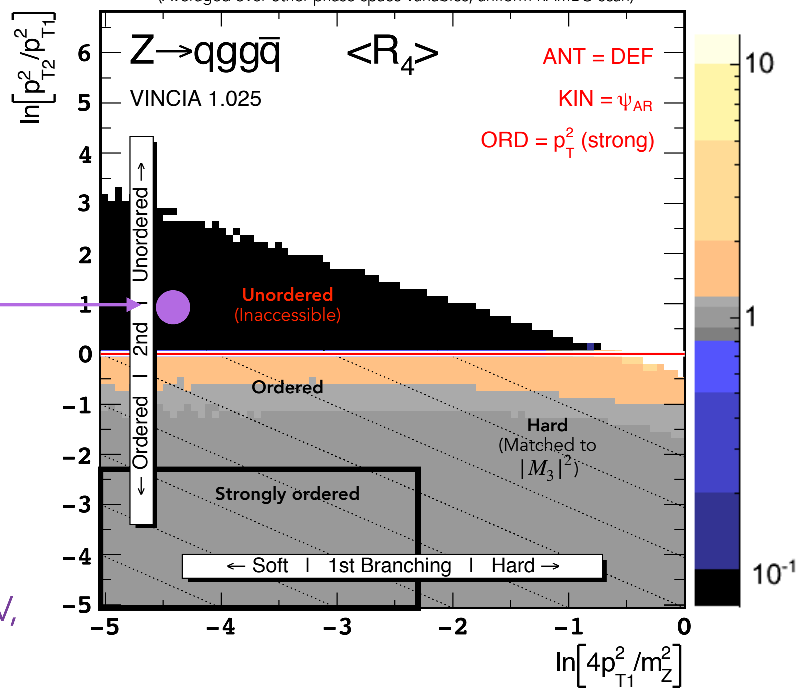
p_{T2} = 8 GeV

Unordered but has $p_{\perp 2} \ll Q_0$:
"Double Unresolved"

(Note: due to recoil effects, swapping the order of the two branchings does not simply give p_{T1} = 8 GeV, p_{T2} = 5 GeV but for this example just produces a different unordered set of scales.)

[Giele, Kosower, PS, 2011]

(Averaged over other phase-space variables, uniform RAMBO scan)



❶ Weight each Born-level event by **local** K-factor

$$k_{\mathrm{NNLO}}(\Phi_2) = 1 + \frac{\mathrm{V}(\Phi_2)}{\mathrm{B}(\Phi_2)} + \frac{\mathrm{I}_{\mathrm{S}}^{\mathrm{NLO}}(\Phi_2)}{\mathrm{B}(\Phi_2)} + \frac{\mathrm{VV}(\Phi_2)}{\mathrm{B}(\Phi_2)} + \frac{\mathrm{I}_{\mathrm{T}}(\Phi_2)}{\mathrm{B}(\Phi_2)} + \frac{\mathrm{I}_{\mathrm{S}}(\Phi_2)}{\mathrm{B}(\Phi_2)}$$
$$+ \int \mathrm{d}\Phi_{+1} \left[\frac{\mathrm{R}(\Phi_2, \Phi_{+1})}{\mathrm{B}(\Phi_2)} - \frac{\mathrm{S}^{\mathrm{NLO}}(\Phi_2, \Phi_{+1})}{\mathrm{B}(\Phi_2)} + \frac{\mathrm{RV}(\Phi_2, \Phi_{+1})}{\mathrm{B}(\Phi_2)} - \frac{\mathrm{T}(\Phi_2, \Phi_{+1})}{\mathrm{B}(\Phi_2)} \right]$$
$$+ \int \mathrm{d}\Phi_{+2} \left[\frac{\mathrm{RR}(\Phi_2, \Phi_{+2})}{\mathrm{B}(\Phi_2)} - \frac{\mathrm{S}(\Phi_2, \Phi_{+2})}{\mathrm{B}(\Phi_2)} \right]$$

← Iterated azimuthal averaging → 2 pairs

← Spin-averaged subtraction terms: Done with pairs of phase-space points at $\Delta\varphi$ = 90 degrees

Fixed-Order Coefficients:

Loops \ Legs	0	1	2
0	B	R	RR
1	V	RV	
2	VV		

Subtraction Terms:

Loops \ Legs	0	1	2
0	0	$\mathrm{S}^{\mathrm{NLO}}$	S
1	$\mathrm{I}_{\mathrm{S}}^{\mathrm{NLO}}$	T	
2	I_{S}, I_{T}		

(not **directly** tied to shower formalism — but must be fully local in Born kinematics Φ_2)

Note: requires "Born-local" NNLO subtraction terms

Not an immediate issue: trivial for decays; simple for colour-singlet production.

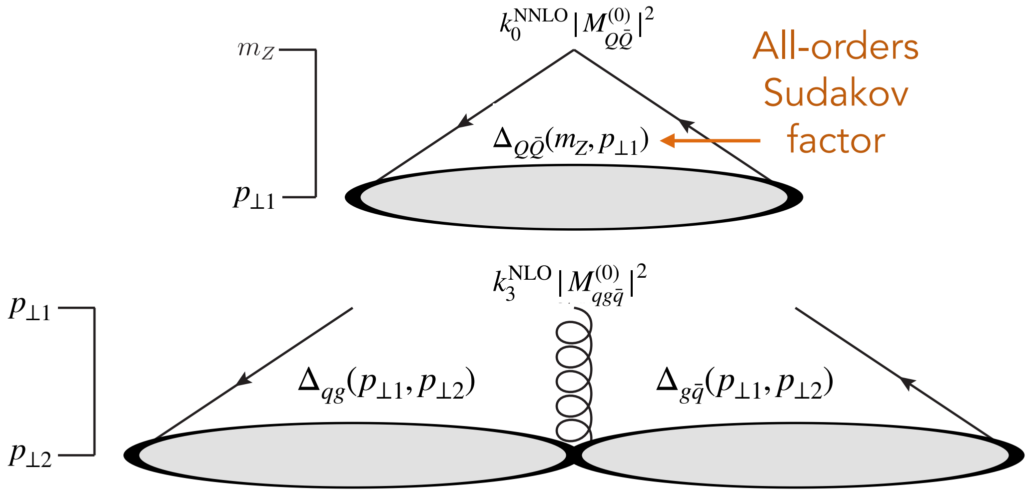
In general simple if shower kinematics preserve Φ_{Born} variables; otherwise compute "sector jet rates"

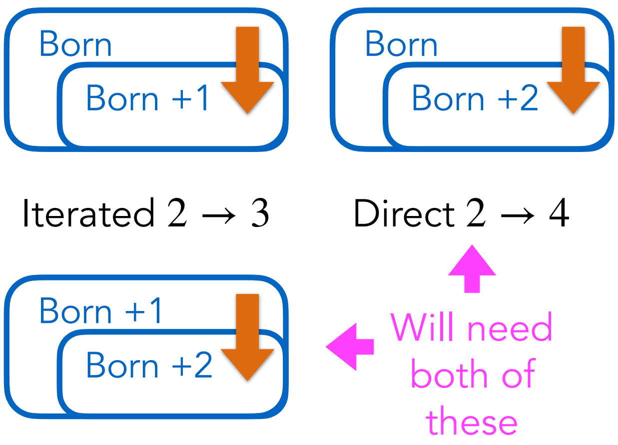
The Shower Operator (its 2nd-order expansion)

This is the part that differs most from standard fixed-order methods

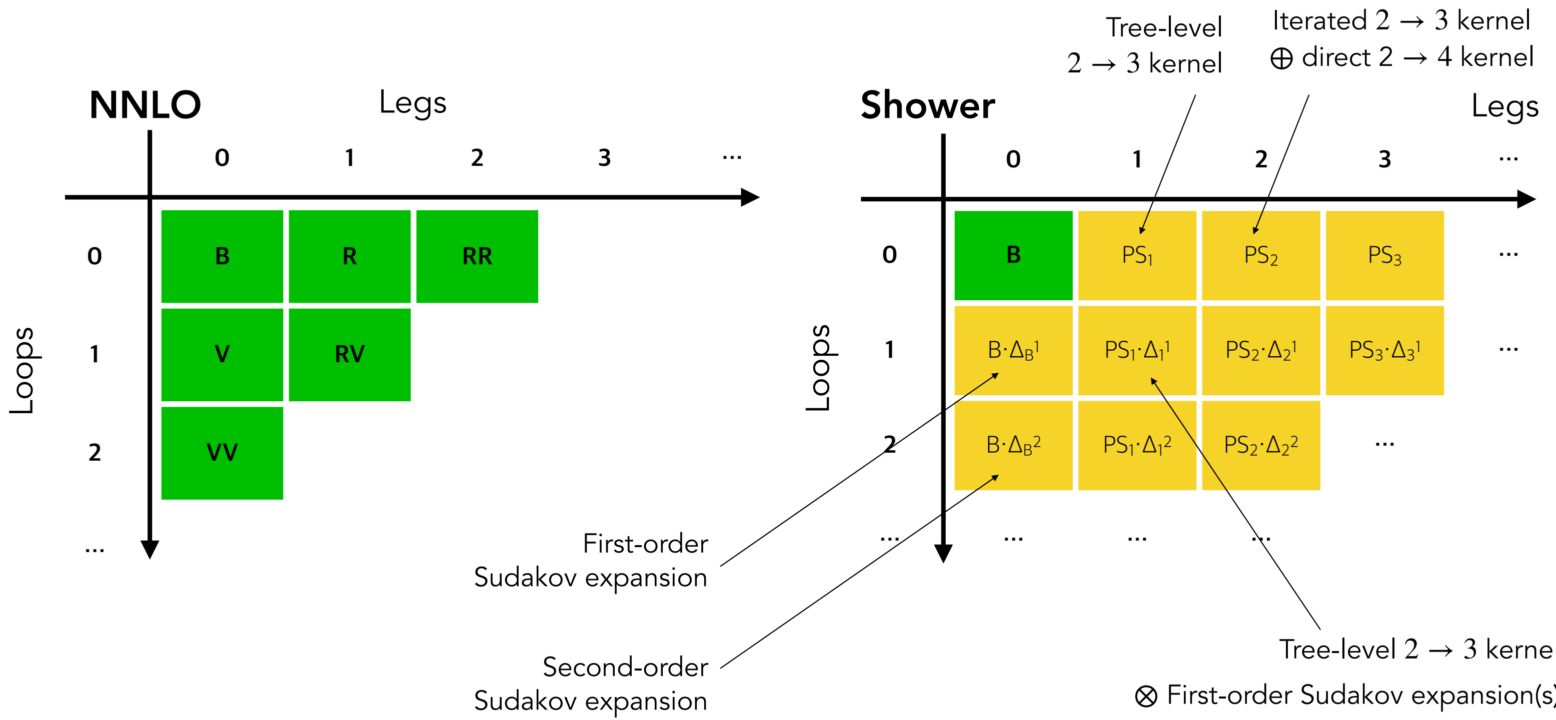
Recall: the +1 and +2 phase spaces are generated via nested sequences of shower-style branchings. Each of which produces an **all-orders** expansion!

We expand these to second order and correct them to NNLO





Coefficients of the Perturbative Expansions



Note: shower coefficients not independent — tied together by universality (→) and unitarity (↙)!

Also: shower "observable" ≡ fully differential rates in each of the (nested) phase spaces

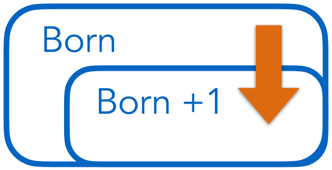
❷ & ❸ Iterated $2 \to 3$ Branchings with NNLO Corrections

Key Aspect:

Up to matched order, include **process-specific** $\mathcal{O}(\alpha_s^2)$ **corrections** into shower evolution

❷ Correct 1st branching to (fully differential) NLO 3-jet rate [Hartgring, Laenen, PZS (2013)]

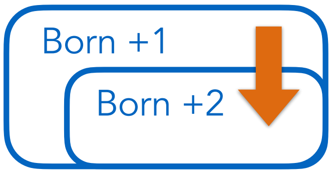
$$\Delta^{\mathrm{NLO}}_{2\to3}\left(\frac{m_Z}{2}, p_{\perp1}\right) = \exp\left\{-\int_{p_{\perp1}}^{\frac{m_Z}{2}} \mathrm{d}\Phi_{[0]+1} \frac{|M^{\mathrm{LO}}_{Z\to3}(\Phi_1)|^2}{|M^{\mathrm{LO}}_{Z\to2}(\Phi_0)|^2} k^{\mathrm{NLO}}_{Z\to3}(\Phi_0, \Phi_{+1})\right\}$$



Allowing for NLO correction factor $k^{\mathrm{NLO}}_{Z\to3}(\Phi_0, \Phi_{+1})$ **(will return to this)**

❸ Correct 2nd branching to LO ME [Giele, Kosower, PZS (2011); Lopez-Villarejo, PZS (2011)]

$$\Delta^{\mathrm{LO}}_{3\to4}(p_{\perp1}, p_{\perp2}) = \exp\left\{-\int_{p_{\perp2}}^{p_{\perp1}} \mathrm{d}\Phi_{[1]+1} \frac{|M^{\mathrm{LO}}_{Z\to4}(\Phi_2)|^2}{|M^{\mathrm{LO}}_{Z\to3}(\Phi_1)|^2}\right\}$$



Entirely based on sectorization and (iterated) Matrix-Element Corrections

(Sectorization defines $d\Phi_{[n]+1}$ and allows to use simple ME ratios instead of partial-fractionings)

Caveat: Double-Unresolved Phase-Space Points

Iterated shower branchings are strictly ordered in shower p_T

Not all 4-parton phase-space points can be reached this way!

In general, strong ordering cuts out part of the double-real phase space

~ double-unresolved regions; no leading logs here but can contain subleading ones

Vice to Virtue: [Li, PZS (2017)]

Divide double-emission phase space into **strongly-ordered** and **unordered** regions (according to the shower ordering variable)

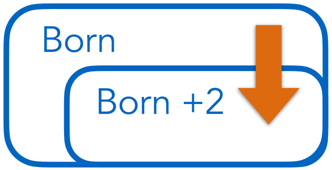
Unordered clusterings ⇔ new direct double branchings

Complementary phase-space regions:

$$d\Phi_{[0]+2} = \Theta(\hat{p}_{\perp 1} - p_{\perp 2}) d\Phi_{[0]+1} d\Phi_{[1]+1} + \Theta(\hat{p}_{\perp 1} + p_{\perp 2}) d\Phi_{[0]+2}$$

Generated by iterated, ordered branchings

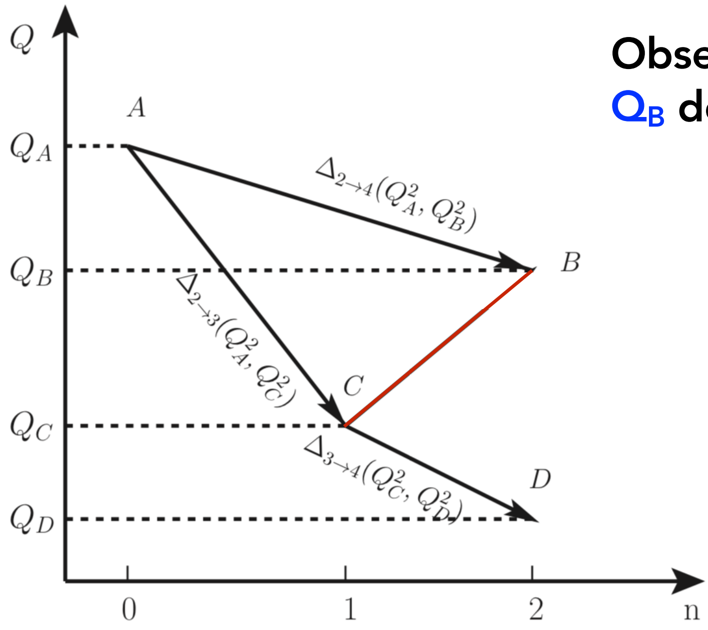
Generated by new direct $2 \to 4$ branchings



Vice to Virtue: Define Ordered and Unordered Phase-Space **Sectors**

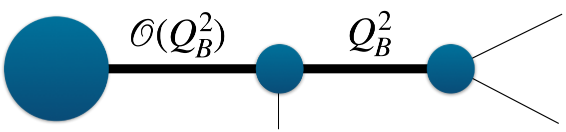
Ordered clusterings ⇔ **iterated single** branchings

Unordered clusterings ⇔ new **direct double** branchings



Observation: for direct double-branchings, Q_B defines the physical resolution scale

Corresponding Feynman diagram(s) have highly **off-shell** intermediate propagator



Intermediate "clustered" **on-shell** 3-parton state at (C) is merely a convenient stepping stone in phase space ⇨ integrate out

❹ (New: Direct $2 \to 4$ Double-Branching Generator)

Derived in: Li & PZS, *A Framework for Second-Order Showers*, PLB 771 (2017) 59

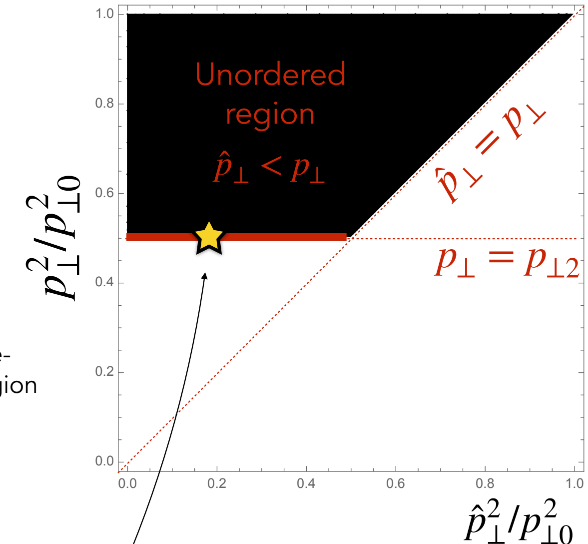
Sudakov trial integral for direct double branchings with $p_\perp \in [p_{\perp 0}, p_{\perp 2}]$:

Scale of intermediate 2→3 stepping stone

Unordered Sector: $\hat{p}_\perp < p_\perp$

$$-\ln \Delta(p_{\perp 0}^2, p_{\perp 2}^2) = \int_0^{p_{\perp 0}^2} \mathrm{d}\hat{p}_\perp^2 \int_{p_{\perp 2}^2}^{p_{\perp 0}^2} \mathrm{d}p_\perp^2 \, \Theta(p_\perp^2 - \hat{p}_\perp^2) \, \frac{N}{p_\perp^4}$$

Generic overestimate of double-branching kernel in unordered region



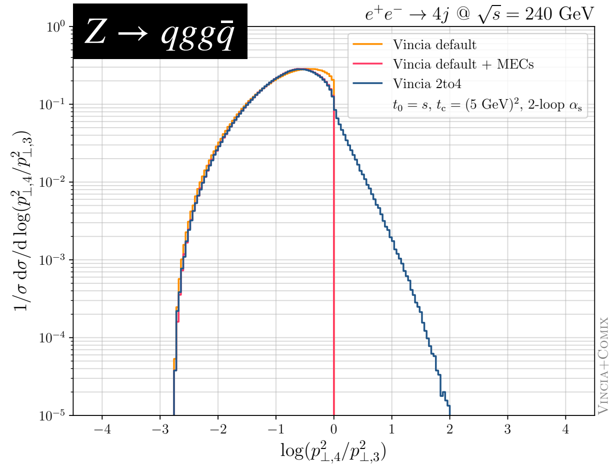
Trick: **swap integration order**

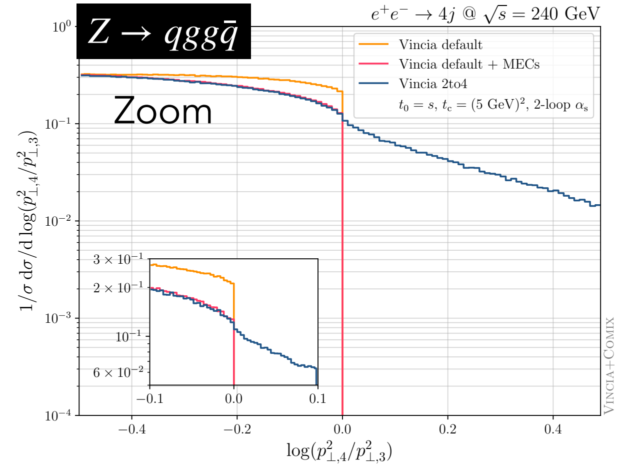
⇒ outer integral along $p_\perp$ instead of $\hat{p}_\perp$:

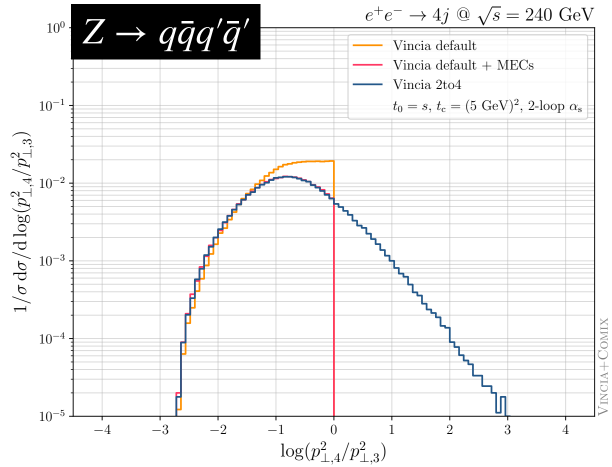
$$= \int_{p_{\perp 2}^2}^{p_{\perp 0}^2} \mathrm{d}p_\perp^2 \int_0^{p_\perp^2} \mathrm{d}\hat{p}_\perp^2 \, \frac{N}{p_\perp^4} \equiv \int_{p_{\perp 2}^2}^{p_{\perp 0}^2} \mathrm{d}p_\perp^2 \, F(p_\perp^2)$$

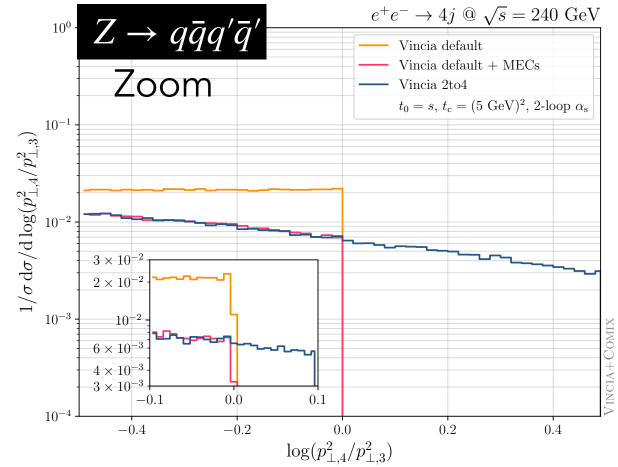
➜ **First** generate physical scale $p_{\perp 2}$, **then** generate $0 < \hat{p}_\perp < p_{\perp 2}$ + two z and φ choices

Validation: combining iterated $2 \to 3$ and direct $2 \to 4$ branchings









VINCIA NNLO

Summary: Shower Markov chain with NNLO Corrections

❷ **Correct 1st $(2 \to 3)$ branching to (fully differential) NLO 3-jet rate** [Hartgring, Laenen, PZS 2013]

$$\Delta^{\mathrm{NLO}}_{2\to3}\left(\frac{m_Z}{2}, p_{\perp1}\right) = \exp\left\{ - \int_{p_{\perp1}}^{\frac{m_Z}{2}} \mathrm{d}\Phi_{[0]+1} \frac{|M^{\mathrm{LO}}_{Z\to3}(\Phi_1)|^2}{|M^{\mathrm{LO}}_{Z\to2}(\Phi_0)|^2} k^{\mathrm{NLO}}_{Z\to3}(\Phi_0, \Phi_{+1}) \right\}$$

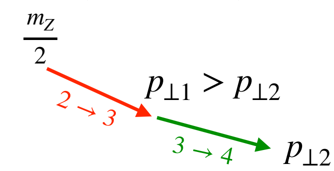
❸ **Correct 2nd $(3 \to 4)$ branching to LO ME** [Giele, Kosower, **PS** (2011); Lopez-Villarejo, PZS 2011]

$$\Delta^{\mathrm{LO}}_{3\to4}(p_{\perp1}, p_{\perp2}) = \exp\left\{ - \int_{p_{\perp2}}^{p_{\perp1}} \mathrm{d}\Phi^{\mathrm{Ord}}_{[1]+1} \frac{|M^{\mathrm{LO}}_{Z\to4}(\Phi_2)|^2}{|M^{(0)}_{Z\to3}(\Phi_1)|^2} \right\}$$

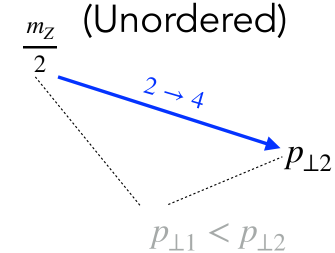
❹ **Add direct $2 \to 4$ branching and correct it to LO ME** [Li, PZS 2017]

$$\Delta^{\mathrm{LO}}_{2\to4}(p_{\perp1}, p_{\perp2}) = \exp\left\{ - \int_{p_{\perp2}}^{p_{\perp1}} \mathrm{d}\Phi^{\mathrm{Unord}}_{[2]+2} \frac{|M^{\mathrm{LO}}_{Z\to4}(\Phi_2)|^2}{|M^{\mathrm{LO}}_{Z\to2}(\Phi_0)|^2} \right\}$$

Iterated:
(Ordered)



Direct:
(Unordered)



Entirely based on MECs and Sectorization

By construction, expansion of extended shower **matches** NNLO singularity structure.

But shower kernels **do not** define NNLO subtraction terms* (!)

*This would be required for an MC@NNLO scheme (but difficult to realise in antenna showers)

Real-Virtual Corrections: NLO MECs (❷)

$$k^{\mathrm{NLO}}_{2\mapsto3} = (1 + w^{\mathrm{V}}_{2\mapsto3})$$

Hartgring, Laenen, PZS (2013)
Campbell, Höche, Li, Preuss, PZS, 2108.07133

Local correction given by **three terms:**

Spin-averaged subtraction terms: Done with pairs of phase-space points at $\Delta\varphi$ = 90 degrees

$$w^V_{2\mapsto3}(\Phi_0,\Phi_{+1}) = \Bigg(\mathrm{RV}(\Phi_0,\Phi_{+1}) + \mathrm{I}^{\mathrm{NLO}}(\Phi_0,\Phi_{+1};t_1)$$

NLO Born+1*j*

$$+ \int_0^{t_1} \mathrm{d}\Phi'_{+1} \left(\mathrm{RR}(\Phi_0,\Phi_{+1},\Phi'_{+1}) - \mathrm{S}^{\mathrm{NLO}}(\Phi_0,\Phi_{+1},\Phi'_{+1}) \right) \Bigg) \frac{1}{\mathrm{R}(\Phi_0,\Phi_{+1})}$$

NLO Born

$$- \left(\mathrm{V}(\Phi_0) + \mathrm{I}^{\mathrm{NLO}}(\Phi_0) + \int_0^{\mathrm{t}_0} \mathrm{d}\Phi'_{+1} \left(\mathrm{R}(\Phi_0,\Phi'_{+1}) - \mathrm{S}^{\mathrm{NLO}}(\Phi_0,\Phi'_{+1}) \right) \right) \frac{1}{\mathrm{B}(\Phi_0)}$$

Shower

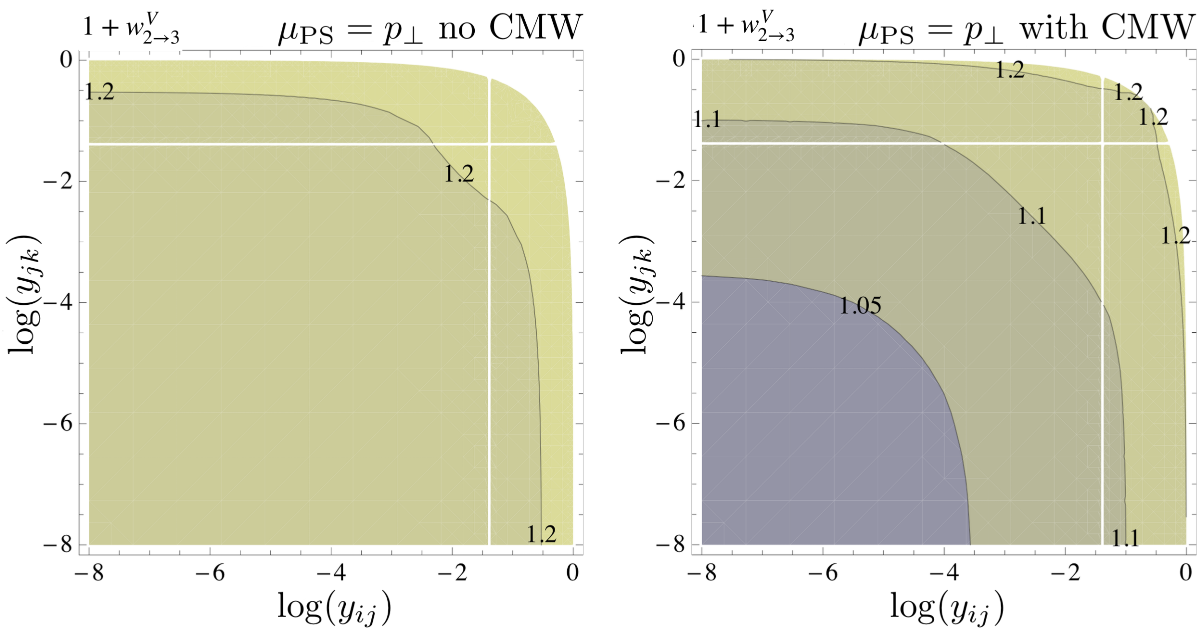
$$+ \left(\frac{\alpha_s}{2\pi} \log\left(\frac{\kappa^2_{\mathrm{CMW}}\mu^2_{\mathrm{PS}}}{\mu^2_{\mathrm{R}}} \right) + \int_{\mathrm{t}_1}^{\mathrm{t}_0} \mathrm{d}\Phi'_{+1}\, \mathrm{A}_{2\mapsto3}(\Phi'_{+1})\, k^{\mathrm{LO}}_{2\mapsto3}(\Phi_0,\Phi'_{+1}) \right)$$

Calculation can be **(semi-)automated**, given a suitable NLO subtraction scheme
(C. Preuss had a crucial realisation to separate this from the terms generated by the shower)

Size of the Real-Virtual Correction Factor (❷)

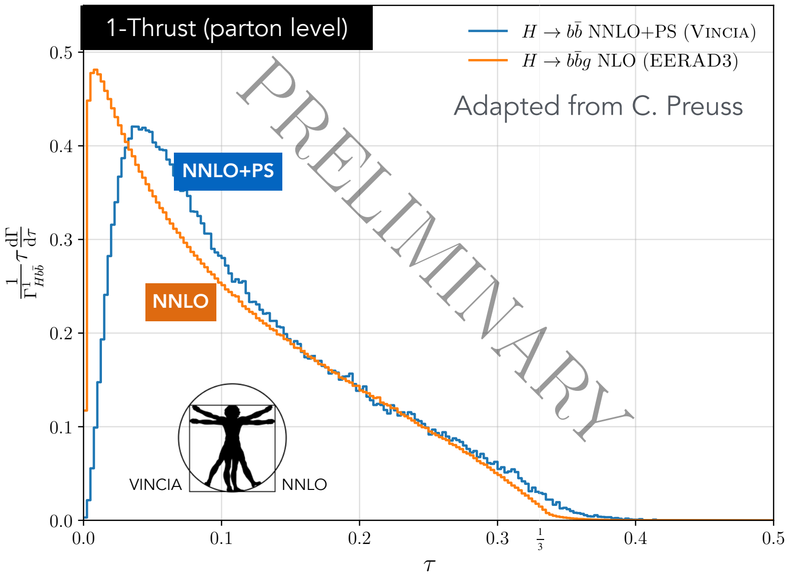
$$k^{\mathrm{NLO}}_{2\to3} = (1 + w^{\mathrm{V}}_{2\to3})$$

studied **analytically** in detail for $Z \to q\bar{q}$ in Hartgring, Laenen, PS JHEP 10 (2013) 127



⇒ now: **generalisation** & **(semi-)automation** in Vincia in form of NLO MECs

Preview: VinciaNNLO for $H \to b\bar{b}$

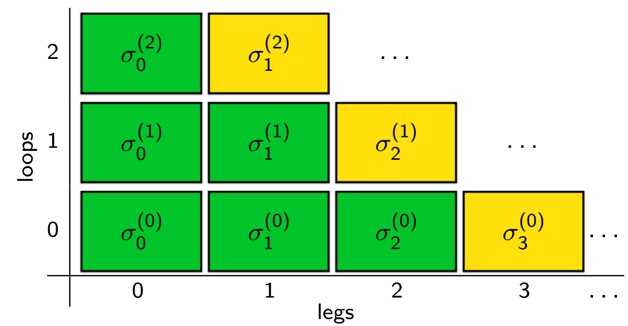


Note:

"NNLO Reference" = **EERAD3** NLO $H \to b\bar{b}g$

Coloretti, Gehrmann-de Ridder, Preuss, JHEP 06 (2022) 009

NNLO accuracy in $H \to 2j$ implies **NLO correction in first emission** and **LO correction in second emission.**

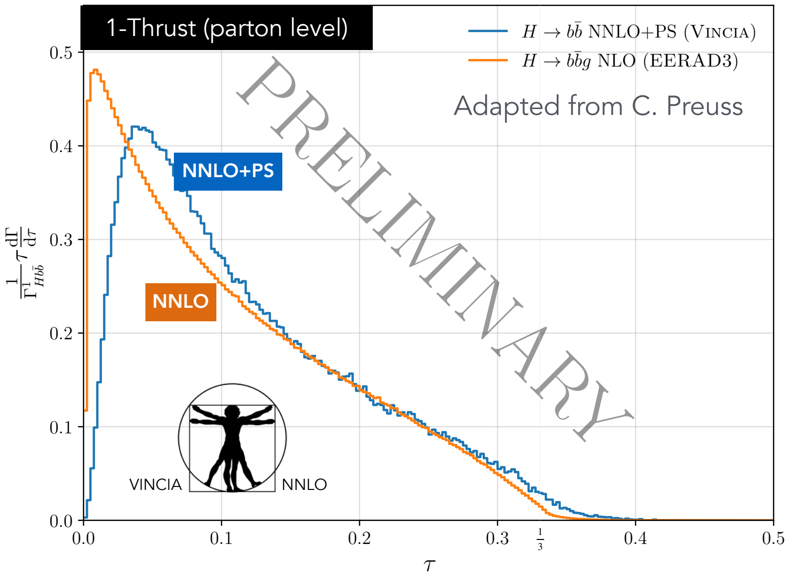


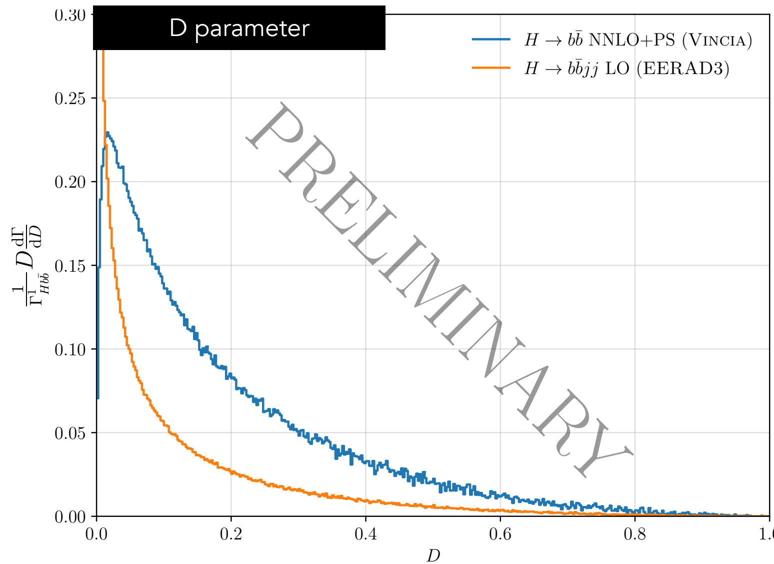
For Thrust, NNLO $H \to b\bar{b}$

NLO for $\tau < 1/3$

LO for $\tau > 1/3$

Preview: VinciaNNLO for $H \to b\bar{b}$





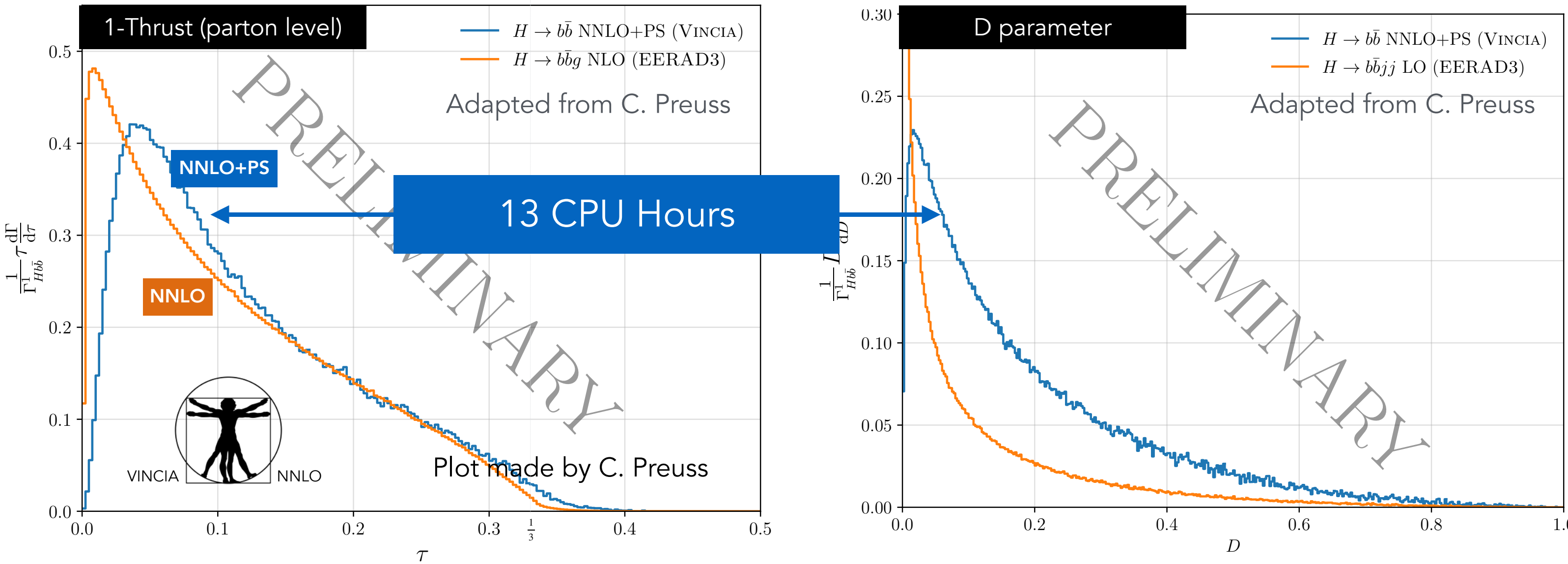
For Thrust, NNLO $H \to b\bar{b}$

- NLO for $\tau < 1/3$
- LO for $\tau > 1/3$

For D parameter, NNLO $H \to b\bar{b}$ = LO

Radiation from shower generates large corrections over entire range

Preview: VinciaNNLO for $H \to b\bar{b}$



VINCIA NNLO+PS: shower as phase-space generator: efficient & no negative weights!

- Looks ~ 5 x ***faster*** than **EERAD3*** (for equivalent unweighted stats)

  + is **matched to shower** + can be **hadronized**

Proof of concepts now done for $Z/H \to q\bar{q}$; work remains for pp (& for N^nLL accuracy)

* Already quite optimised: uses analytical MEs, "folds" phase space to cancel azimuthally antipodal points, and uses antenna subtraction (→ smaller # of NLO subtraction terms than Catani-Seymour or FKS).

Summary

VINCIA NNLO

Shower-style phase-space generation ⊗ 2nd-order MECs

Exploits **sectorization ➜** defines $d\Phi_{[n]+1}$, unique scales, and allows to use simple ME ratios (instead of sums over partial-fractionings)

Ingredients:

1. Born-Local NNLO ($\mathcal{O}(\alpha_s^2)$) *K*-factors: $k^{\mathrm{NNLO}}(\Phi_0)$
2. NLO ($\mathcal{O}(\alpha_s^2)$) MECs in the first $2 \to 3$ shower emission: $k^{\mathrm{NLO}}_{2\to3}(\Phi_1)$
3. LO ($\mathcal{O}(\alpha_s^2)$) MECs for next (iterated) $2 \to 3$ shower emission: $k^{\mathrm{LO}}_{3\to4}(\Phi_2)$
4. Direct $2 \to 4$ branchings for "unordered sector", with LO ($\mathcal{O}(\alpha_s^2)$) MECs: $k^{\mathrm{LO}}_{2\to4}(\Phi_2)$

Elaborate proofs of concept for $Z \to q\bar{q}$ and $H \to q\bar{q}$

Now working to make public in **Pythia 8** *(with J. Altmann, B. El Menoufi, C. Preuss, L Scyboz)*

Outlook: underlying shower ➜ NLL & NNLL; extend to pp, and matching ➜ N3LO

Extra Slides

MECs are **extremely** simple in **sector showers**

In global antenna subtraction & in conventional dipole/antenna showers:

Total **gluon-collinear DGLAP kernel** is partial-fractioned among neighbouring "sub-antenna functions" → factorially growing number of contributing terms in each phase-space point

Global Antenna

$$A^{\mathrm{gl}}_{qg\mapsto qgg}(i_q, j_g, k_g) \to \begin{cases} \frac{2s_{ik}}{s_{ij}s_{jk}} & \text{if } j_g \text{ soft} \\ \frac{1}{s_{ij}}\frac{1+z^2}{1-z} & \text{if } i_q \parallel j_g \\ \frac{1}{s_{jk}}\frac{1+z^3}{1-z} & \text{if } j_g \parallel k_g \end{cases}$$

= **partial-fractioned** $g \to gg$ DGLAP kernel

Sector Antenna

$$A^{\mathrm{sct}}_{qg\mapsto qgg}(i_q, j_g, k_g) \to \begin{cases} \frac{2s_{ik}}{s_{ij}s_{jk}} & \text{if } j_g \text{ soft} \\ \frac{1}{s_{ij}}\frac{1+z^2}{1-z} & \text{if } i_q \parallel j_g \\ \frac{1}{s_{jk}}\frac{2(1-z(1-z))^2}{z(1-z)} & \text{if } j_g \parallel k_g \end{cases}$$

= the **full** $g \to gg$ DGLAP kernel

⇒ Sector kernels can be replaced by direct ratios of (colour-ordered) tree-level MEs:

- **Global** shower: $A^{\mathrm{glb}}_{IK\to ijk}(i,j,k) \to A^{\mathrm{glb}}_{IK\to ijk}\frac{|M_{n+1}(\ldots,i,j,k,\ldots)|^2}{\sum_{\mathrm{h}\in\mathrm{histories}} A_h |M_n(\ldots I_h, K_h,\ldots)|^2}$ = **complicated**

  Fischer & Prestel EPJC77(2017)9

- **Sector** shower: $A^{\mathrm{sct}}_{IK\to ijk}(i,j,k) \to \frac{|M_{n+1}(\ldots,i,j,k,\ldots)|^2}{|M_n(\ldots I,K,\ldots)|^2}$ = **simple**

  Lopez-Villarejo & PZS JHEP 11 (2011) 150

Note: can just use ME also in denominator, not shower kernel, since we matched at previous order "already"

Colour MECs

Sector kernels can be replaced by ratios of (colour-ordered) **tree-level MEs:**

- **Global** shower: $A^{\text{glb}}_{IK\to ijk}(i,j,k) \to A^{\text{glb}}_{IK\to ijk} \dfrac{|M_{n+1}(\ldots,i,j,k,\ldots)|^2}{\sum_{\text{h}\in\text{histories}} A_h |M_n(\ldots I_h, K_h, \ldots)|^2}$ = **complicated** [Fischer & Prestel 1706.06218]

- **Sector** shower: $A^{\text{sct}}_{IK\to ijk}(i,j,k) \to \dfrac{|M_{n+1}(\ldots,i,j,k,\ldots)|^2}{|M_n(\ldots I, K, \ldots)|^2}$ = **simple** [Lopez-Villarejo & PS 1109.3608]

Can also incorporate (fixed-order) **sub-leading colour effects by "colour MECs":**

[Giele, Kosower, PS, 1102.2126]

$$w_{\text{col}} = \frac{\sum_{\alpha,\beta} \mathcal{M}_\alpha \mathcal{M}^*_\beta}{\sum_\alpha |\mathcal{M}_\alpha|^2}$$

Example: $Z \to q\bar{q} + 2g$

$$P_{\text{MEC}} = w_{\text{col}} \frac{A^0_4(1_q, 3_g, 4_g, 2_{\bar{q}})}{A^0_3(\widetilde{13}_q, \widetilde{34}_g, 2_{\bar{q}})} \theta(p^2_{\perp,134} < p^2_{\perp,243}) + w_{\text{col}} \frac{A^0_4(1_q, 3_g, 4_g, 2_{\bar{q}})}{A^0_3(1_q, \widetilde{34}_g, \widetilde{23}_{\bar{q}})} \theta(p^2_{\perp,243} < p^2_{\perp,134})$$

$$w_{\text{col}} = \frac{A^0_4(1,3,4,2) + A^0_4(1,4,3,2) - \frac{1}{N_C^2}\tilde{A}^0_4(1,3,4,2)}{A^0_4(1,3,4,2) + A^0_4(1,4,3,2)}$$

Colour-Ordered Projectors

Better: use smooth projectors [Frixione et al. 0709.2092]

$$\mathrm{RR}(\Phi_3, \Phi'_{+1}) = \sum_j \frac{C_{ijk}}{\sum_m C_{\ell mn}} \mathrm{RR}(\Phi_3, \Phi_{ijk}^{\mathrm{ant}})\,, \quad C_{ijk} = A_{IK\mapsto ijk} \mathrm{R}(\Phi_3)$$

- **But**: antenna-subtraction term **not positive-definite**!
- To render this well-defined, need to work on **colour-ordered** level

$$\mathrm{RR} = \mathcal{C} \sum_\alpha \mathrm{RR}^{(\alpha)} - \frac{\mathcal{C}}{N_\mathrm{C}^2} \sum_\beta \mathrm{RR}^{(\beta)} \pm \ldots$$

- Different colour factors enter with different sign, but **no sign changes** within one term

$$\mathcal{C} \left[\frac{C_{ijk}}{\sum_m C_{\ell mn}} \frac{\mathrm{RR}^{(\alpha)}(\Phi_3, \Phi_{ijk}^{\mathrm{ant}})}{\mathrm{R}(\Phi_3)} - A_{IK\mapsto ijk} \right]$$

$\Rightarrow$ Numerically **better behaved**, uses **standard antenna-subtraction** terms