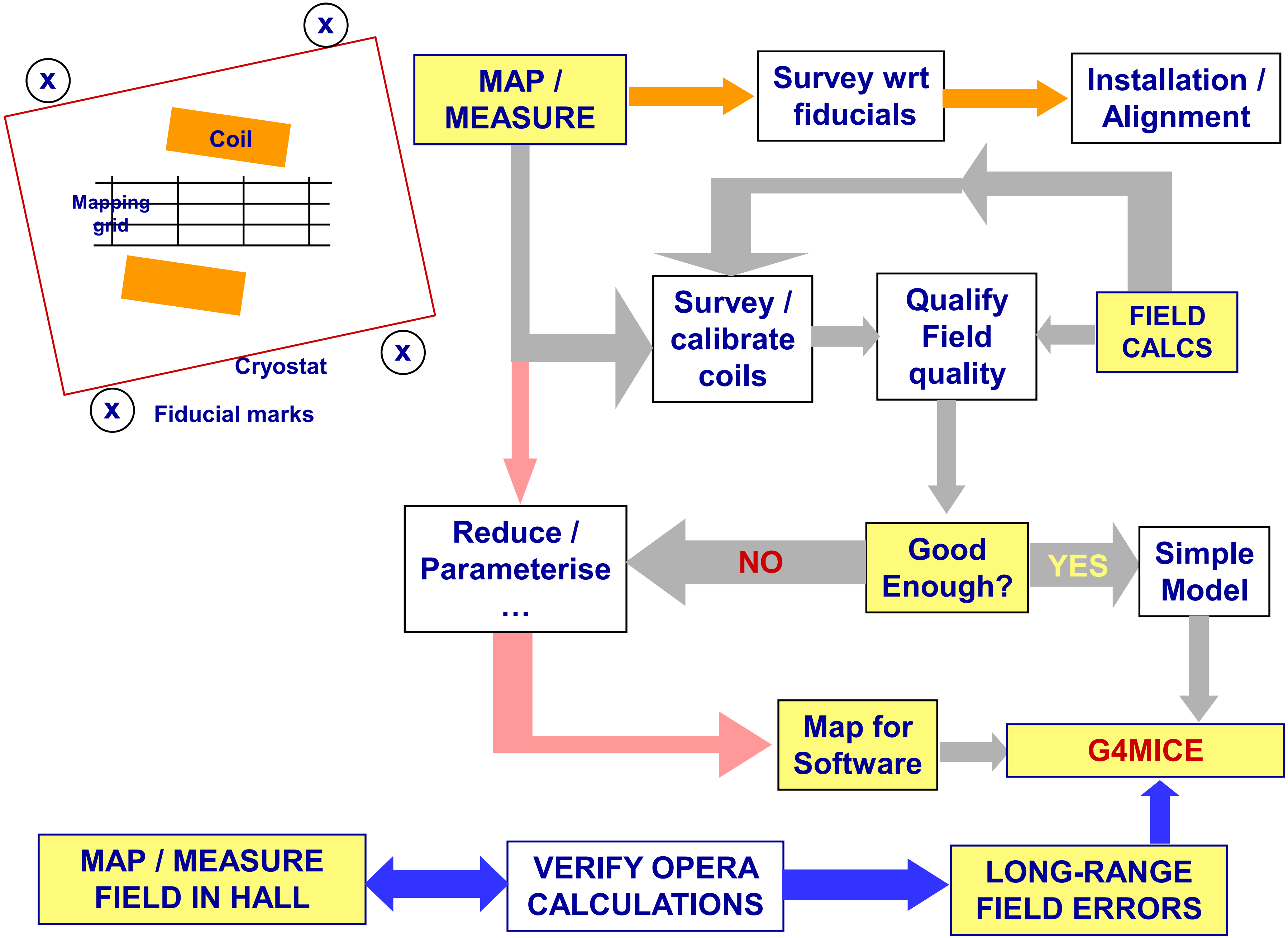
FIELD MAPPING

and what do we do with it?

Shown once before at an Analysis Meeting

- **CERN measurement system will produce a lot of field measurements**
- **What do we want to do with them?**
  - **One slide**
- **Are they good enough?**
  - ***Have looked at how well we can find axes of FCs***
- ***Don’t underestimate the work involved***


x
x
Coil
Mapping grid
Cryostat
x
x
Fiducial marks
MAP / MEASURE
Survey wrt fiducials
Installation / Alignment
Survey / calibrate coils
Qualify Field quality
FIELD CALCS
Reduce / Parameterise …
NO
Good Enough?
YES
Simple Model
Map for Software
G4MICE
MAP / MEASURE FIELD IN HALL
VERIFY OPERA CALCULATIONS
LONG-RANGE FIELD ERRORS

Magnet mapping – Finding the axis

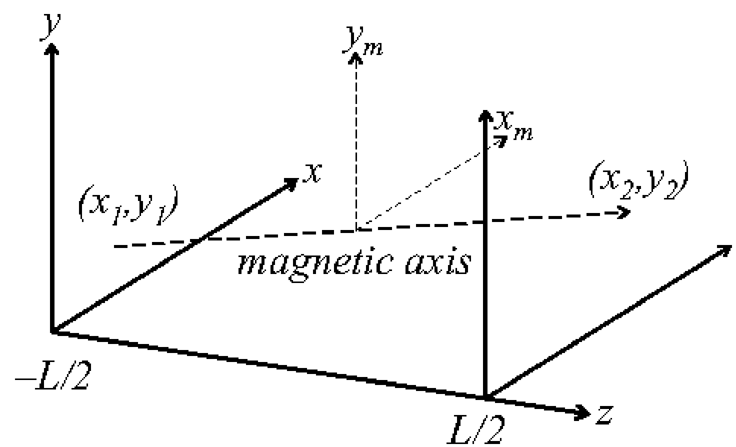


Figure 1: Magnetic axis of coil in the external coordinate system.

Attempting to answer question *'How well can we determine magnetic axis?'*.

Want to know (find by mapping) the axis of magnet in some external system.

Axis may be offset and tilted wrt external system.

Assume any tilt of magnetic axis wrt external system is small (as it will be), $z_m = z$.

Axis passes through (x_1, y_1) and (x_2, y_2) at $z = \pm L/2$.

We want to find (x_1, y_1) and (x_2, y_2).

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Equation of magnetic axis in external system is

$$x_a = \frac{(x_1 + x_2)}{2} + \frac{(x_2 - x_1)}{L} z$$

and likewise for y_a.

If measurement made at (x, y) in external coordinates then in magnet system

$$\begin{aligned} x_m &= x - x_a(z) \\ y_m &= y - y_a(z) \,. \end{aligned}$$

Close to the magnetic axis the Maxwell – Gauss equation $\nabla \cdot \mathbf{B} = 0$ gives

$$B_x \sim -\frac{x_m}{2} \frac{\partial B_z}{\partial z}(0, 0, z)$$

Transverse fields linear in distance from axis and proportional to field gradient

and likewise for B_y.

The axis (i.e. x_1, y_1, x_2, y_2) can be found by measuring B_x, B_y on a grid of N points in (x, y, z).

A χ^2 can be constructed from the measurements and minimised wrt the four unknowns.

Treat x and y independently.

$$\begin{aligned}\chi^2 &= \sum_N \frac{(B_x - k(z)(x - x_a(z)))^2}{\sigma_B^2} \\ &= \frac{1}{\sigma_B^2} \sum_N \left(B_x - k(z) \left[x - x_1 (\frac{1}{2} - \frac{z}{L}) - x_2 (\frac{1}{2} + \frac{z}{L}) \right] \right)^2\end{aligned}$$

where σ_B^2 is the field measurement error, assumed to be the same at each point, and $k(z) = \frac{1}{2} \frac{\partial B_z}{\partial z}(0, 0, z)$.

(In the summation, B_x, x and z should all have index i; summation is over i.)

This is a straightforward linear minimisation problem, though $k(z)$ must be known. $k(z)$ could either be calculated (fields on axis are easy to calculate) or obtained from measurements. It doesn't have to be known that well.

First partial derivatives will give x_1 and x_2; second derivatives will give errors on those quantities.

$$\begin{aligned}\frac{\partial \chi^2}{\partial x_1} &= \frac{1}{\sigma_B^2} \sum_N 2k(z)(\frac{1}{2} - \frac{z}{L}) \left(B_x - k(z) \left[x - x_1 (\frac{1}{2} - \frac{z}{L}) - x_2 (\frac{1}{2} + \frac{z}{L}) \right] \right) \\ \frac{\partial \chi^2}{\partial x_2} &= \frac{1}{\sigma_B^2} \sum_N 2k(z)(\frac{1}{2} + \frac{z}{L}) \left(B_x - k(z) \left[x - x_1 (\frac{1}{2} - \frac{z}{L}) - x_2 (\frac{1}{2} + \frac{z}{L}) \right] \right) .\end{aligned}$$

Can write above in terms of sums and solve pair of simultaneous equations for x_1 and x_2.

Errors are more interesting.

$$\begin{aligned}
\frac{\partial^2\chi^2}{\partial x_1^2} &= \frac{1}{\sigma_B^2}\sum_N 2k^2(z)(\frac{1}{2}-\frac{z}{L})^2 \\
\frac{\partial^2\chi^2}{\partial x_2^2} &= \frac{1}{\sigma_B^2}\sum_N 2k^2(z)(\frac{1}{2}+\frac{z}{L})^2 \\
\frac{\partial^2\chi^2}{\partial x_1\partial x_2} &= \frac{1}{\sigma_B^2}\sum_N 2k^2(z)(\frac{1}{2}-\frac{z}{L})(\frac{1}{2}+\frac{z}{L})\,.
\end{aligned} \tag{1}$$

These expressions give the inverse of the x_1, x_2 error matrix; the variance of x_1 is given by $\sigma_{x_1}^2 = 2(\frac{\partial^2\chi^2}{\partial x_1^2})^{-1}$.

Numerical estimate

Consider focus coils.

Current in both coils in same direction, $J = 100\text{A}/\text{mm}^2$.

21 equally spaced measurements of B_x along z on (or near – doesn't matter) axis.

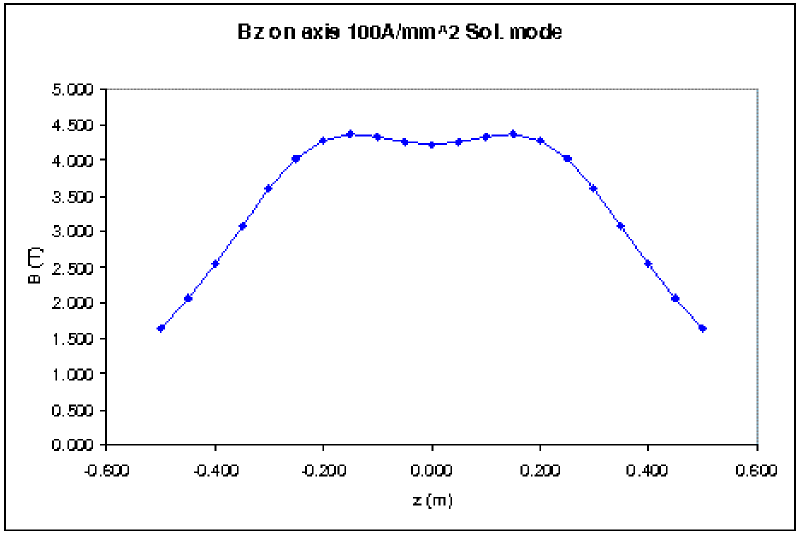

Figure 2: B_z on axis of Focus Coils, $J = 100\text{A}/\text{mm}^2$, solenoid mode.

$k(z) = \frac{1}{2}\frac{\partial B_z}{\partial z}(0, 0, z)$ calculated.

Sum in Equation 1 = 205.2.

Assume $\sigma_B = 2\text{mT}$, then $\sigma_{x_1} = 0.000197\text{m} = 0.2\text{mm}$.

With (say) 9 times more measurements close to axis, e.g., 3 points on each side of 60mm × 60mm square around axis, or more points in z, or both $\sigma_{x_1} \to\sim 0.07\text{mm}$

Surely good enough – other errors (to be thought about) may dominate.