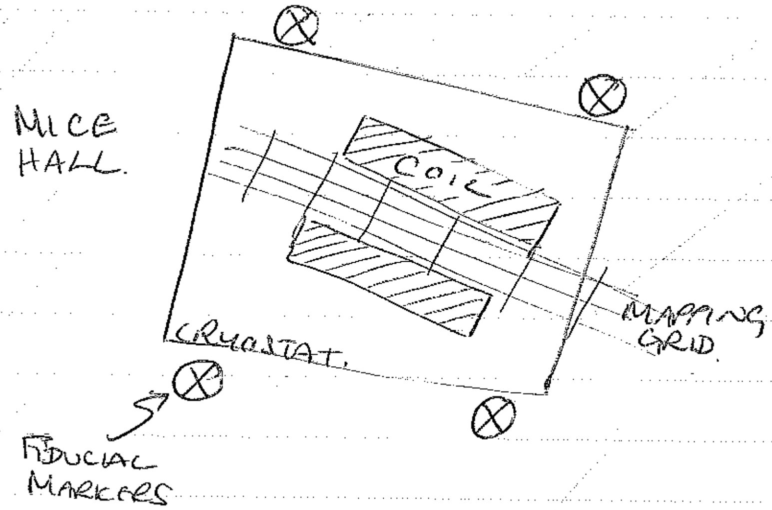
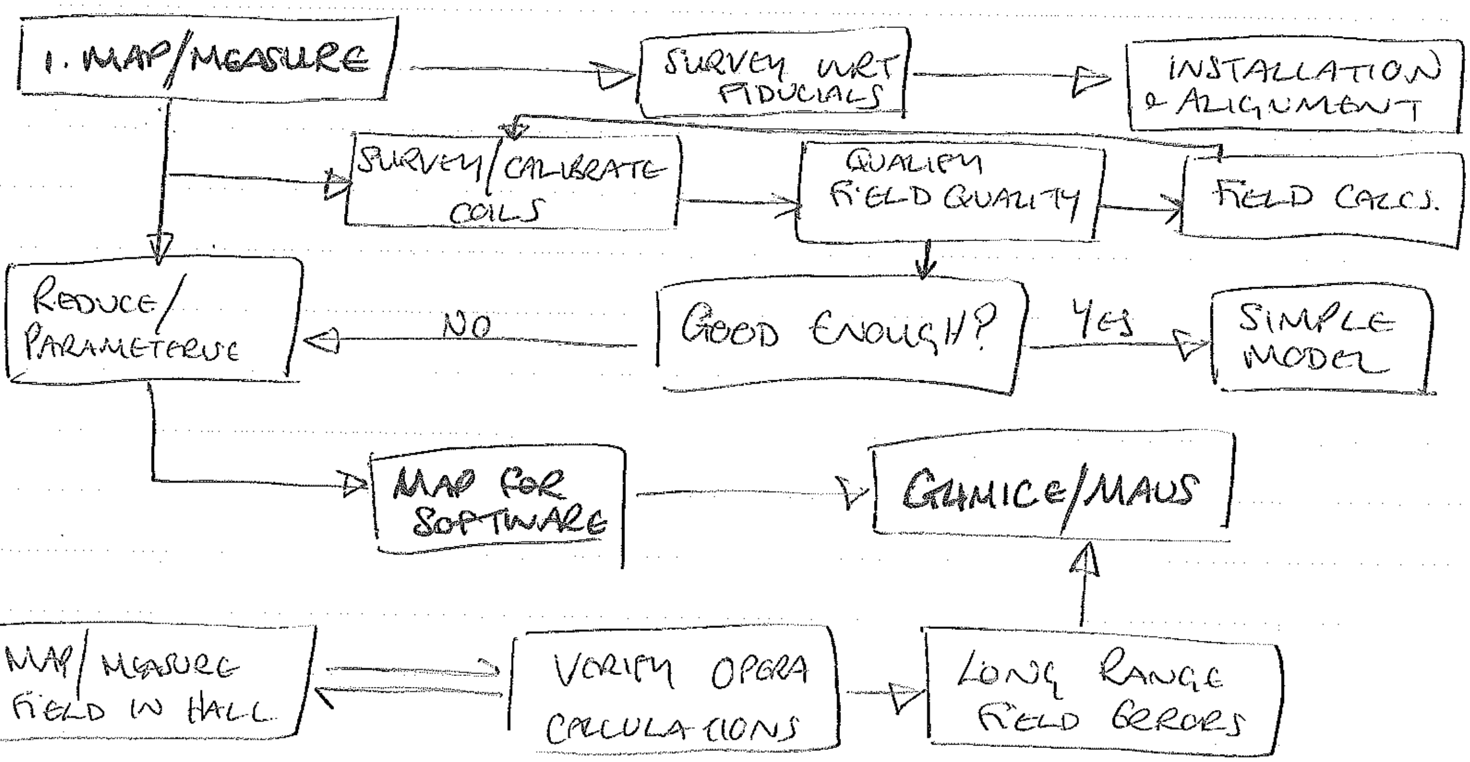
Notes on Field Mapping.

J. Cobb's flow diagram:

- Have a coil, somewhere inside a cryostat, which sits somewhere inside the hall, which is somehow related to some fiducial markers.



- Then there is a (potentially!) complicated set of ~~surveys that~~ procedures that we could follow:



- From what I can tell, this process is a solved problem. It has been done *many* times for many solenoid magnets (e.g. ATLAS's Inner Detector magnet most recently). Our magnets/configuration may through up some peculiarities, however...

→ Subject of quite a few papers and thesis! I don't *think* we need to entirely reinvent the wheel.

Standard Procedure

1. Develop a magnetic field model
2. Compare with measurements
3. Include sources of systematic error.

Magnetic Field Models

- We could derive a very simple field map by interpolating between the measured data points, but there are a few problems with this:

1. We will need to correct for systematic effects acquired during the measurement process (will come back to this)

2. We have a *lot* of different conditions and currents (and configurations, e.g. step 4, step 5) we can run our coils and magnets in. Ideally, we'd like to save ourselves some time and measure *some* of these to confirm our models are correct. Then we have more faith in our models for simulating close to the experimental reality.

- There are two approaches for deriving this field model, and they're generally combined for a better result and/or some way of quantifying the field map quality.

1. Coil Model / Geometry Model
2. Maxwell's Equations.

Coil Model

- Basically, represent the coil as a series of closed current loops. Then evaluate the magnetic field as a superposition of all of those loops. We use the known/surveyed geometry of each coil we're interested in and integrate the Biot-Savart law using either the expected or measured conductor currents (depending on if we're evaluating or simulating our field map).

The simplest approach is just to assume X closed current loops, where X = number of turns, and assume that the fact that there is some pitch between the coils is negligable. However, it can be made into a much more detailed model by considering survey information (from construction), wire pitch, welds joining coils and distortion due to magnetic forces.

Considering only the simple approach, the magnetic field of a closed current loop in cylindrical co-ordinates is [4]:

$$B_z(r,\phi,z) = \frac{\mu_0 I}{\pi} \frac{1}{2\alpha^2\beta}\left[(R^2-\rho^2)E(k) + \alpha^2 K(k)\right]$$

$$B_r(r,\phi,z) = \frac{\mu_0 I}{\pi} \frac{z}{2\alpha^2\beta r}\left[(R^2+\rho^2)E(k) - \alpha^2 K(k)\right]$$

$$B_\phi(r,\phi,z) = 0$$

where R = radius of the loop
I = current in loop
K = complete elliptic integral of the 1st kind
E = complete elliptic integral of the 2nd kind
$\rho^2 = r^2 + z^2$
$\alpha^2 = R^2 + \rho^2 - 2Rr$
$\beta^2 = R^2 + \rho^2 + 2Rr$
$k^2 = 1 - \alpha^2/\beta^2$

→ Then sum over the number of loops (i.e. use the principle of superposition).

- Complications
  1. Elliptic integrals are evil.
  2. Takes a lot of computation to calculate the magnetic field at any point (can speed this up if you have a look-up table for K and E).

3. Maxwell's Equations

- This is described very nicely in both [3] and [4], but I'll repeat it here for completeness sake. This is the more general way of modelling the field based on Maxwell's equations, which is more often known as the Fourier-Bessel (FB) model.

In the region where we measure the field ~~in~~ (i.e. in the bore of the solenoid) we have:

a) no currents
b) no magnetic material

so,

$$\nabla \cdot \underline{B} = 0$$
$$\nabla \times \underline{B} = 0$$

We then express $\underline{B}$ in terms of the magnetic scalar potential Φ:

$$B = -\nabla\Phi$$

where Φ satisfies Laplace's equation: $\nabla^2\Phi = 0$. A solution to this is found by separation of variables (in cylindrical polar co-ordinates):

$$\Phi(r, \varphi, z) = R(r)P(\varphi)Z(z)$$

and the axial and radial factors take 3 different forms according to the separation constants:

$$Z(z) = A\sin(\lambda z) + B\cos(\lambda z)$$
$$R(r) = CI_n(\lambda r)$$

$$Z(z) = A\sinh(\lambda z) + B\cosh(\lambda z)$$
$$R(r) = CJ_n(\lambda r)$$

$$Z(z) = Az + B$$
$$R(r) = Cr^n$$

J_n = Bessel function
I_n = Modified Bessel function } Hence the name of this model (or part of it!)
$\lambda > 0$
$n = 0, 1, 2 \ldots$
A, B, C = Arbitrary constants

The azimuthal factor is always of the form:

$$P(\varphi) = A\sin(n\varphi) + B\cos(n\varphi)$$

The general form of ϕ is an infinite sum over these terms (will come back to this).

The components of $\underline{B}$ can be found from $\underline{B} = -\nabla\phi$ and are rather longwinded:

$$B_z(r,\phi,z) = \sum_{n=0}^{\infty}\sum_{l=1}^{\infty} A_{nl} I_n\left(\frac{l\pi}{z_{max}} r\right)\cos(n\phi + \alpha_{nl})\cos\left(\frac{l\pi}{z_{max}} z\right)$$
$$- \sum_{n=0}^{\infty}\sum_{l=1}^{\infty} B_{nl} I_n\left(\frac{l\pi}{z_{max}} r\right)\cos(n\phi + \beta_{nl})\sin\left(\frac{l\pi}{z_{max}} z\right)$$
$$+ \sum_{n=0}^{\infty} A_{n0} r^n \cos(n\phi + \alpha_{n0})$$
$$+ \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} C_{nm} J_n\left(\frac{\zeta_{nm}}{r_{max}} r\right)\cos(n\phi + \gamma_{nm})\cosh\left(\frac{\zeta_{nm}}{r_{max}} z\right)$$
$$+ \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} D_{nm} J_n\left(\frac{\zeta_{nm}}{r_{max}} r\right)\cos(n\phi + \delta_{nm})\sinh\left(\frac{\zeta_{nm}}{r_{max}} z\right)$$

$$B_r(r,\phi,z) = \sum_{n=0}^{\infty}\sum_{l=1}^{\infty} A_{nl} I_n'\left(\frac{l\pi}{z_{max}} r\right)\cos(n\phi + \alpha_{nl})\sin\left(\frac{l\pi}{z_{max}} z\right)$$
$$+ \sum_{n=0}^{\infty}\sum_{l=1}^{\infty} B_{nl} I_n'\left(\frac{l\pi}{z_{max}} r\right)\cos(n\phi + \beta_{nl})\cos\left(\frac{l\pi}{z_{max}} z\right)$$
$$+ \sum_{n=0}^{\infty} A_{n0}\, n r^{n-1}\cos(n\phi + \alpha_{n0}) z$$
$$+ \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} C_{nm} J_n'\left(\frac{\zeta_{nm}}{r_{max}} r\right)\cos(n\phi + \gamma_{nm})\sinh\left(\frac{\zeta_{nm}}{r_{max}} z\right)$$
$$+ \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} D_{nm} J_n'\left(\frac{\zeta_{nm}}{r_{max}} r\right)\cos(n\phi + \delta_{nm})\cosh\left(\frac{\zeta_{nm}}{r_{max}} z\right)$$
$$+ \sum_{n=0}^{\infty} E_n\, n r^{n-1}\cos(n\phi + \varepsilon_n)$$

$$
\begin{aligned}
B_\phi(r,\phi,z) = &-\sum_{n=0}^{\infty}\sum_{l=1}^{\infty} A_{nl}\,\frac{z_{max}}{l\pi r}\, I_n\!\left(\frac{l\pi}{z_{max}}r\right)\sin(n\phi+\alpha_{nl})\sin\!\left(\frac{l\pi}{z_{max}}z\right) \\
&-\sum_{n=0}^{\infty}\sum_{l=1}^{\infty} B_{nl}\,\frac{z_{max}}{l\pi r}\, I_n\!\left(\frac{l\pi}{z_{max}}r\right)\sin(n\phi+\beta_{nl})\cos\!\left(\frac{l\pi}{z_{max}}z\right) \\
&-\sum_{n=0}^{\infty} A_{n0}\, n r^{n-1}\sin(n\phi+\alpha_{n0})\, z \\
&-\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} C_{nm}\left(\frac{z_{max}}{\zeta_{nm} r}\right) J_n\!\left(\frac{\zeta_{nm}}{z_{max}}r\right)\sin(n\phi+\gamma_{nm})\sinh\!\left(\frac{\zeta_{nm}}{r_{max}}z\right) \\
&-\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} D_{nm}\,\frac{z_{max}}{\zeta_{nm} r}\, J_n\!\left(\frac{\zeta_{nm}}{z_{max}}r\right)\sin(n\phi+\delta_{nm})\cosh\!\left(\frac{\zeta_{nm}}{r_{max}}z\right) \\
&-\sum_{n=0}^{\infty} E_n\, n r^{n-1}\sin(n\phi-\varepsilon_n)
\end{aligned}
$$

$A_{nl}, B_{nl}, C_{nm}, D_{nm}, E_n$ = arbitrary constants
$\alpha_{nl}, \beta_{nl}, \gamma_{nm}, \delta_{nm}, \varepsilon_n$ = arbitrary phases
ζ_{nm} = zeros of the J_n Bessel function.
I_n', J_n' = 1st derivatives of I_n and J_n.

→ Note: All of these series must be truncated for the fitting procedure. The number of terms must <u>also</u> be less than the number of points measured in each direction to avoid any oscillating behaviour of the model.

→ A_{nl}, B_{nl} can be determined from the measured field on the curved surface of the cylinder.
→ C_{nm} and D_{nm} can be found from the fields at the end of the cylinder
→ E_n = multipole terms, independent of z and making no contribution to B_z.

- Then reconstruct the field throughout a volume of interest based on measurements over the <u>surface of the volume.</u> So although we measure at many different radii, this approach only utilises the points on a cylindrical surface and at its ends.

Advantages

1. Don't need to know anything about the geometric distribution of current that gives rise to the magnetic field.

2. All information comes from constraints on Maxwell's equations and measured fields on the surface of a cylindrical volume.

Disadvantages

1. There is no direct physical meaning we can derive from this method, so we have no way of extrapolating beyond the measured volume. A shame, since we'd be quite interested in this!

2. Have to find all the parameters using a minimisation method. This procedure is detailed in Sections 2.1 to 2.3 of [3].

3. The "Compromise" Model.

1. The measured field ~~[illegible]~~ obeys Maxwell's equations.

2. The closed current loops model obeys Maxwell's equations.

3. Therefore the difference must also obey Maxwell's equations. Then use the Fourier-Bessel model to describe this difference. The overall shape of the field is given by the coil model, with corrections by Fourier-Bessel.

Fitting & sources of systematic uncertainty

- The aim of fitting is to minimise the difference between the measured data and our models, providing us with a (hopefully!) well-understood field map. We minimise:

$$\chi^2 = \sum_{i,c} \left(\frac{B_{i,c}^{data} - B_c^{model}(r_i)}{x} \right)^2$$

where x is our error estimation in Gauss (used to normalise the function to some reasonable level for the fitting program), $c = r, \phi$ or z, and $B^{model}(r_i)$ is the predicted field of the model at the co-ordinates of $B_{i,c}^{data}$, i runs over all measured points.

- Before fitting, we will probably have to make corrections to our measured points to account for:

a) Hall probe normalisation

b) Probe alignments, positions (3 measurements are <u>not</u> at exactly the same points)

c) Probe tilts

d) Measurement carriage tilts

e) Survey marks.

Once this has all been accounted for, we can consider our residual field to come from measurements and models based in the "same" co-ordinate system.

- ~~Minimise~~ Next, find the residual from the coil/geometric model and the data. This residual field obeys Maxwell's equations and can be described by the Fourier-Bessel model. This gets divided into 3 parts that have to be fitted consecutively (corresponding to the different constants), then add together all of the calculated fields. We then have a residual field derived from the sum of the coil model and Fourier-Bessel model:

$$\underline{B}^{model} = B^{coil} + B^{FB1} + B^{FB2} + B^{FB3} \rightarrow \text{into } \chi^2 \text{ equation.}$$

- The final field has to obey Maxwell's equations, so all residual fields are due to truncation effects or uncertainties in the probe positions.

- Estimate the error on the field map by the RMS of the residuals! We can also double-check our model by comparing an NMR probe and the value at $B_z(0,0,0)$.

- For speed, it's been found that interpolating a pre-generated map of this type is more than accurate enough.

References

[1] "Results of the ATLAS Solenoid Field Map", M. Aleksa et. al. J. Phys. Conf. Series 110 (2008) 092018

[2] ATLAS Note (ATL-MAGNET-PUB-2007-001) "Measurement of the solenoid magnetic field". M Aleksa et. al.

[3] ATLAS Note (ATL-MAGNET-PUB-2007-002) "Corrections to the solenoid field measurement", J. C. Hart.

[4] Thesis: "Magnetic Field Maps for a Large TPC Prototype", C. Grefe, 2008, University of Hamburg.