Dynamical phases in non-equilibrium Conformal Field Theories.

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IIT-Kanpur

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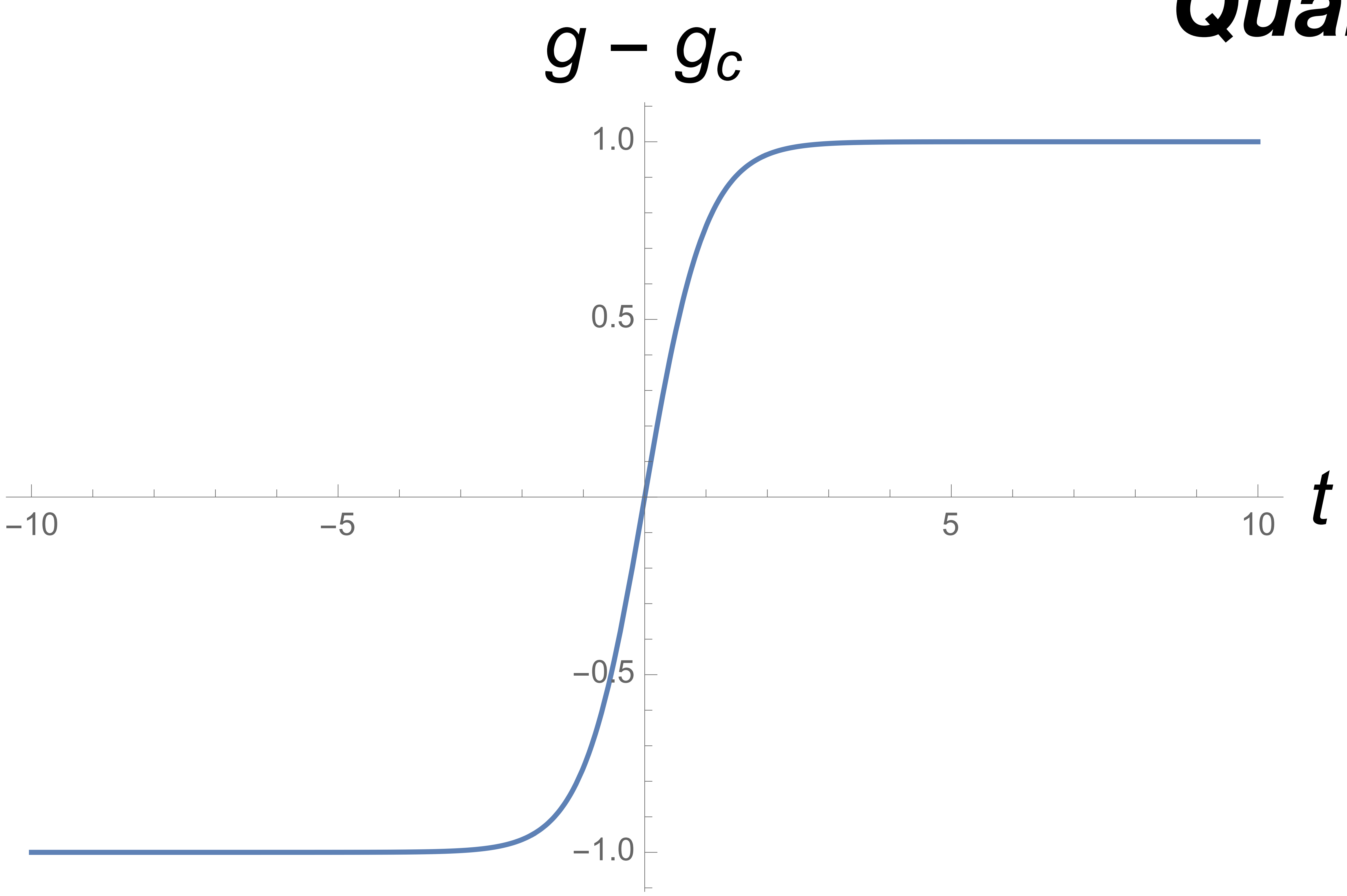
Quantum Quench



$$|\psi(t)\rangle = e^{-i\int^t dt' H(g(t'))}|\psi_0\rangle$$

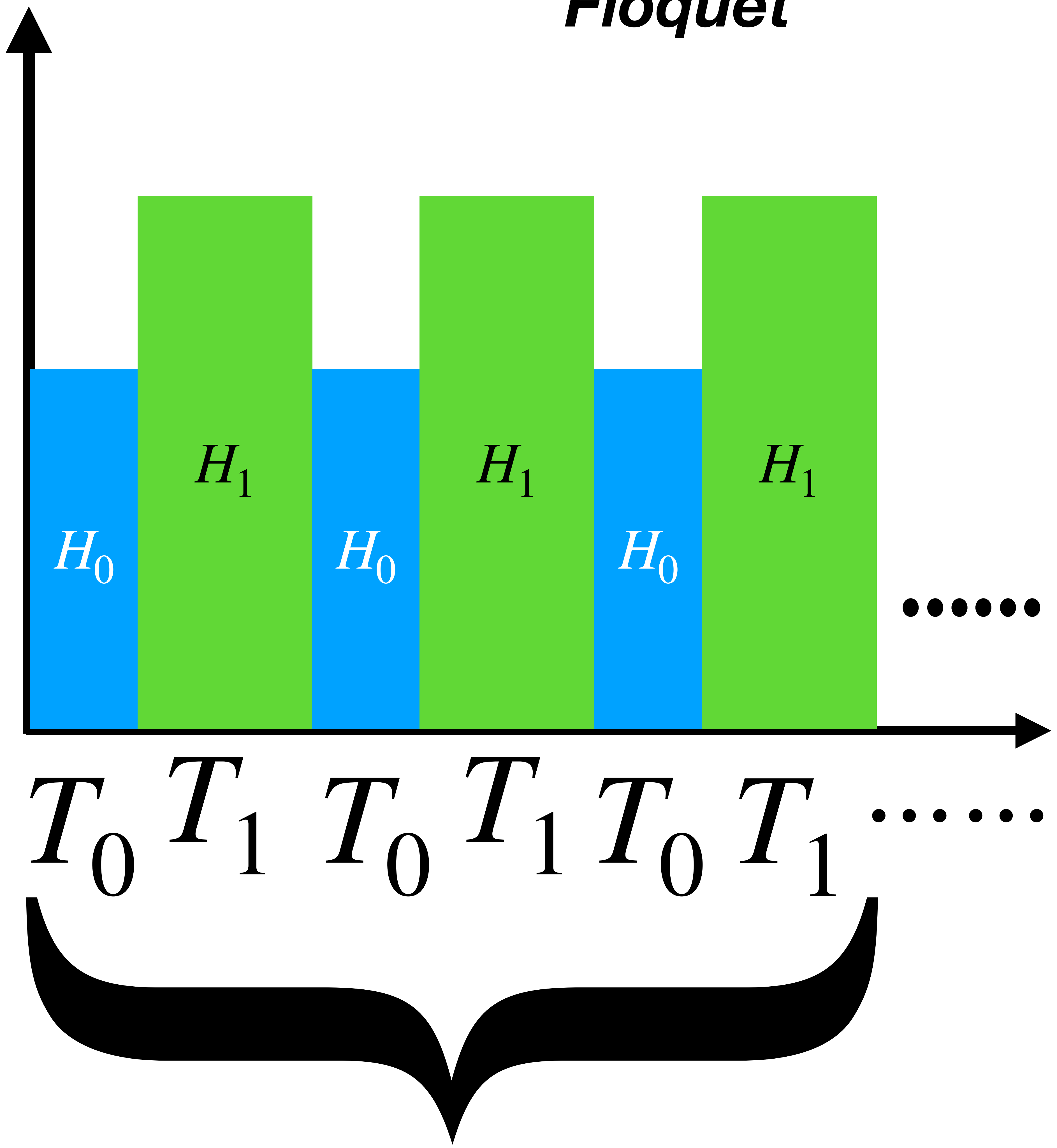
What is the time development of observables ?

$$\langle\psi(t)|\mathcal{O}_1\ldots\mathcal{O}_n|\psi(t)\rangle$$

What is the nature of the final state?
Is there an effective simple description?

$$\lim_{t\to\infty}|\psi(t)\rangle\langle\psi(t)| \sim e^{-\beta H_\infty}$$

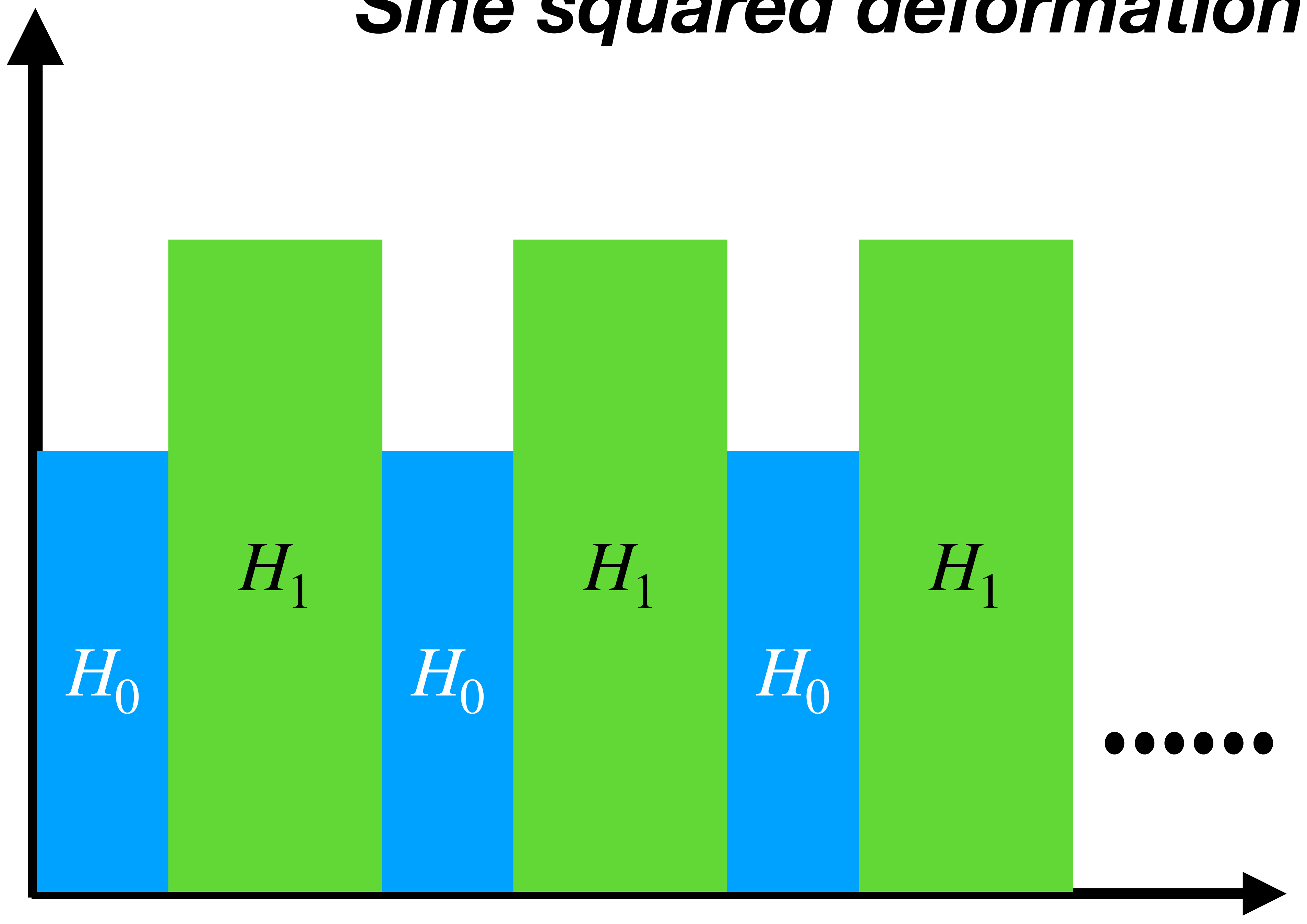
Floquet



n cycles → $U = \left(e^{-iH_1T_1}e^{-iH_0T_0}\right)^n$

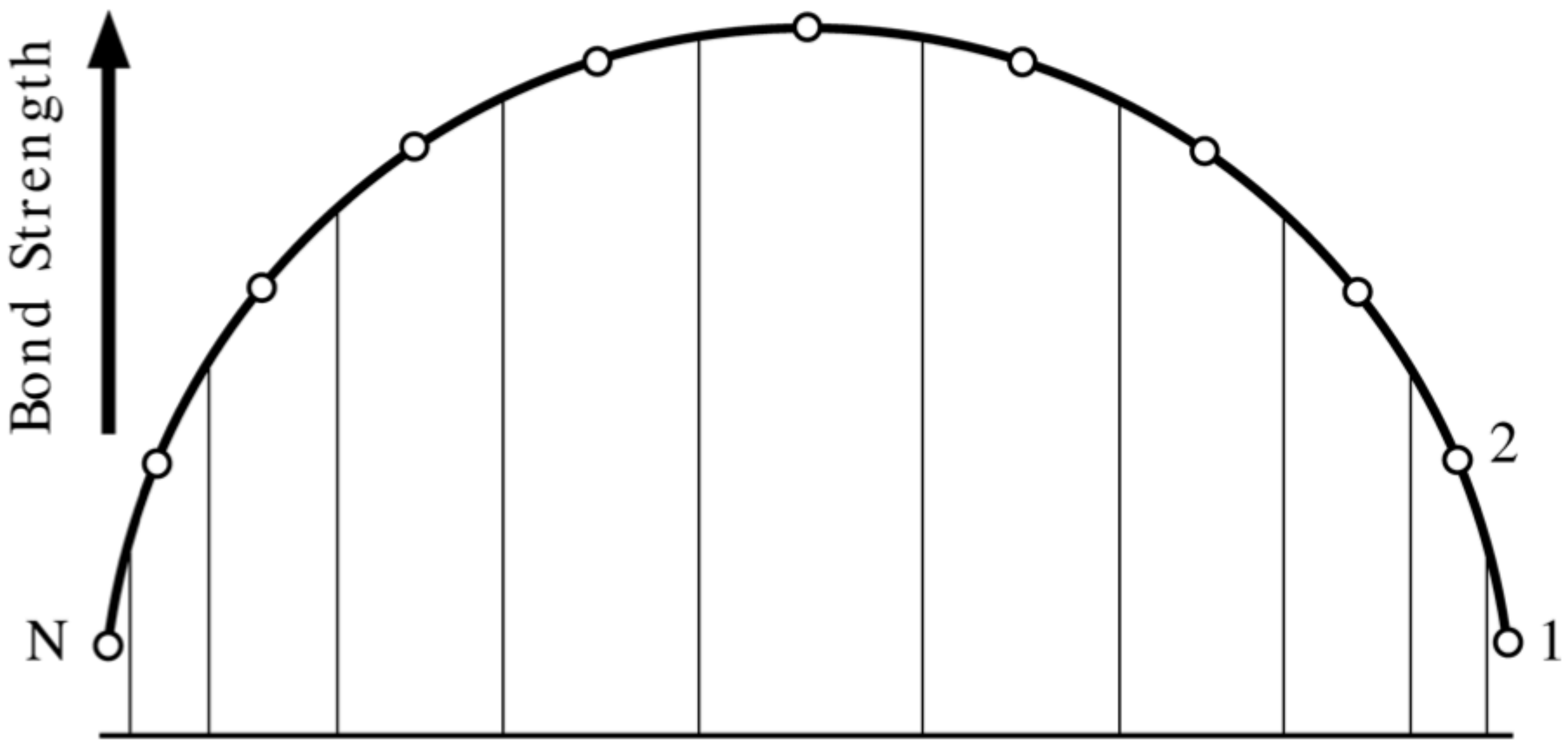
Sine squared deformation

$$H_0 = H_{CFT} = L_0 + \bar{L}_0$$



$$H_{CFT} = \int dx \; h(x)$$

$$H_1 = \int dx \; \sin^2(x/2)\, h(x)$$



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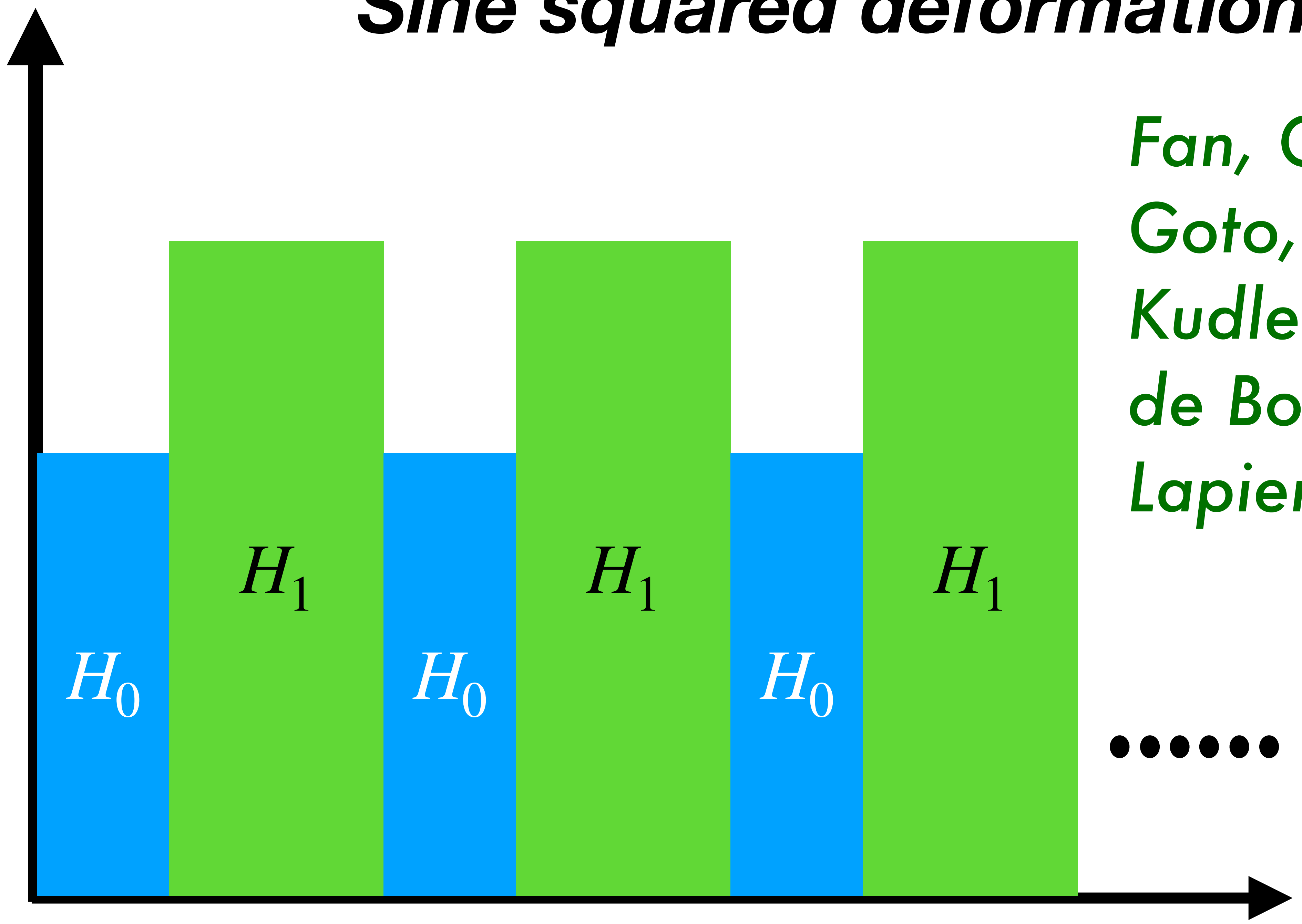

Fan, Gu, Vishwanath, Wen, Ryu, Ludwig, Wu, Goto, Nozaki, Tamaoka, Katsura, Nishino, Kudler-Flam, Numasawa, Ishibashi, Okunishi, de Boer, Godet, Keski-Vakkuri, Guo, Datta, Lapierre, Moosavi ......

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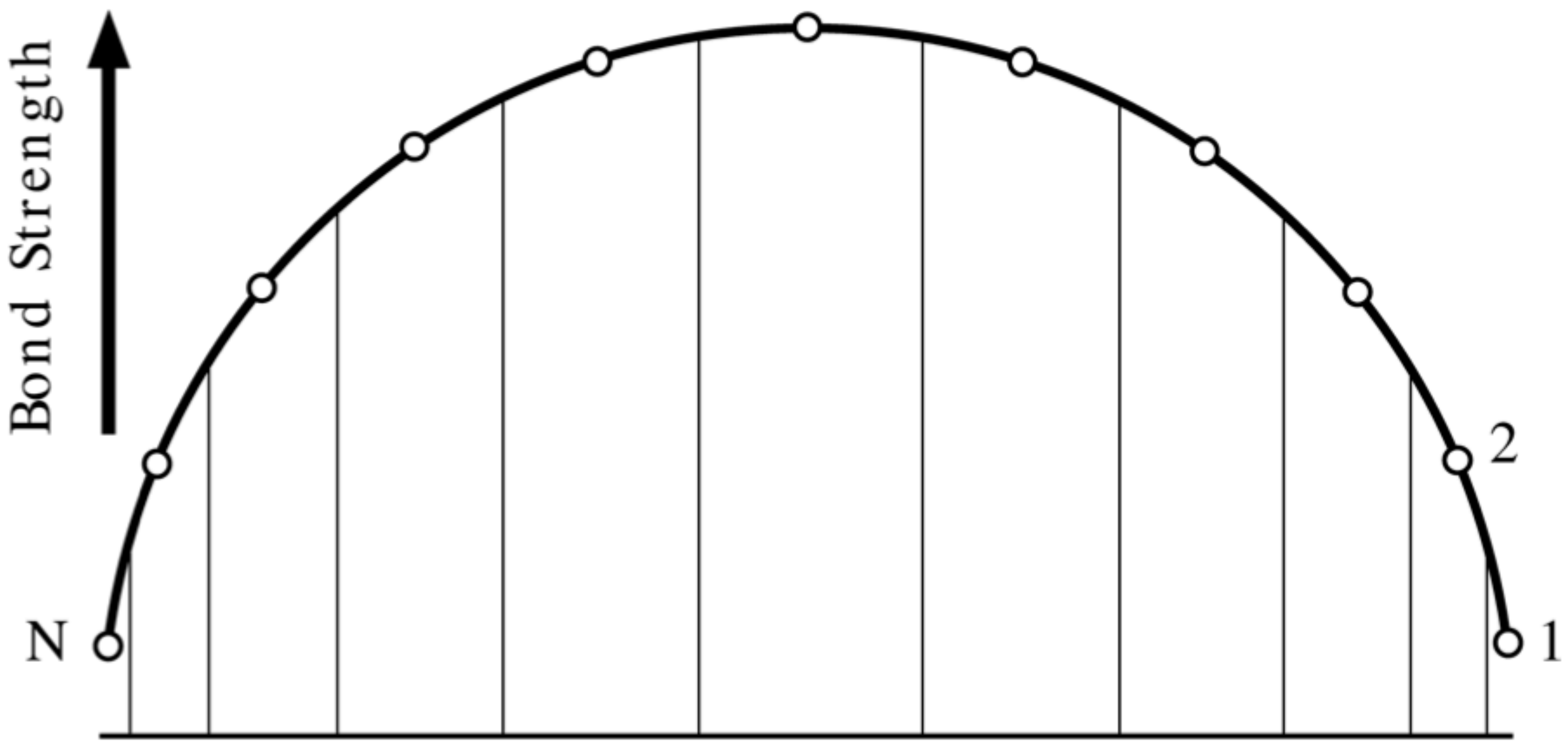
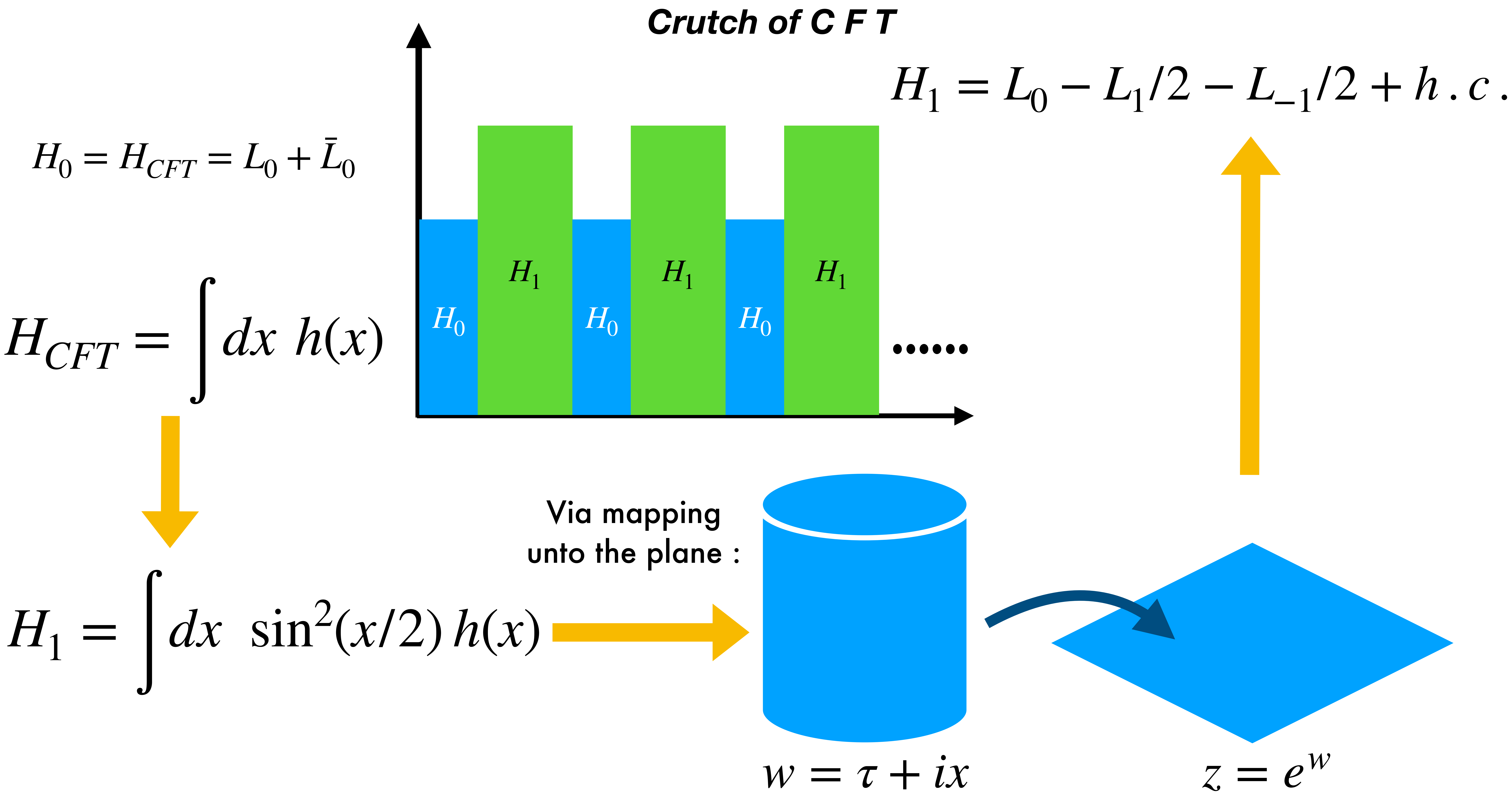


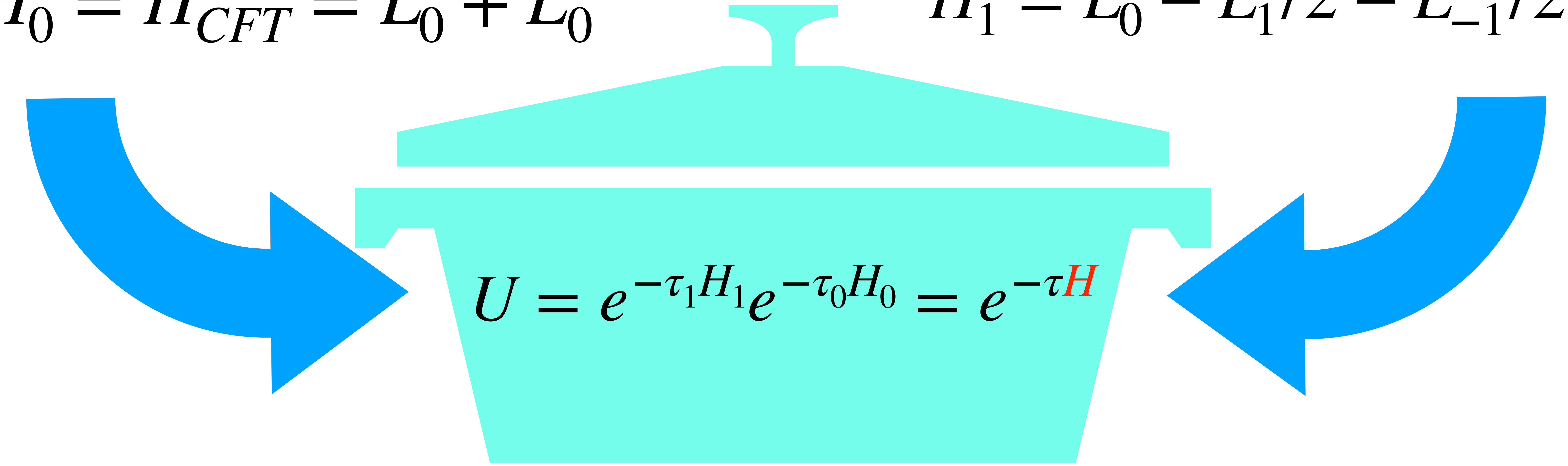


Crutch of C F T
$H_0 = H_{CFT} = L_0 + \bar{L}_0$
$H_1 = L_0 - L_1/2 - L_{-1}/2 + h.c.$
H_0
H_1
H_0
H_1
H_0
H_1
......
$H_{CFT} = \int dx \; h(x)$
$H_1 = \int dx \; \sin^2(x/2)\, h(x)$
Via mapping
unto the plane :
$w = \tau + ix$
$z = e^w$

Putting things together using algebra

$$H_0 = H_{CFT} = L_0 + \bar{L}_0$$

$$H_1 = L_0 - L_1/2 - L_{-1}/2 + h.c.$$



$$U = e^{-\tau_1 H_1} e^{-\tau_0 H_0} = e^{-\tau H}$$

Using the SL2 algebra we can BCH-ize: $[L_0, L_1] = -L_1,\ [L_0, L_{-1}] = L_{-1},\ [L_1, L_{-1}] = 2L_0$

$$\boxed{H = c_0 L_0 + c_1 L_1 + c_2 L_{-1} + h.c.}$$

Möbius shows the way

$$U = e^{-\tau H}$$

$$H = c_0 L_0 + c_1 L_1 + c_2 L_{-1} + h.c.$$

Geometrically SL_2 generates Möbius $\Longrightarrow$ we can find

$$z \to \tilde{z} = \frac{az + b}{cz + d}$$

such that $H = L_0^{(\tilde{z})}$.

And finally : a, b, c & d, are all functions of T_0 & T_1.

Möbius shows the way

$$U = e^{-\tau H}$$

$$H = c_0 L_0 + c_1 L_1 + c_2 L_{-1} + h.c.$$

Geometrically SL_2 generates Möbius $\implies$ we can find $z \to \tilde{z} = \dfrac{az+b}{cz+d}$, such that

$$H = L_0^{(\tilde{z})}.$$

And now the spacetime dependence of observables can be determined using behaviour under conformal transformations.

$$\langle \psi(t) | O_h(w, \bar{w}) | \psi(t) \rangle = \left(\frac{\partial z}{\partial w} \right)^h \left(\frac{\partial \tilde{z}}{\partial z} \right)^h \cdots \langle \psi(0) | O_h(\tilde{z}(w)) | \psi(0) \rangle$$

Iterating the map ...

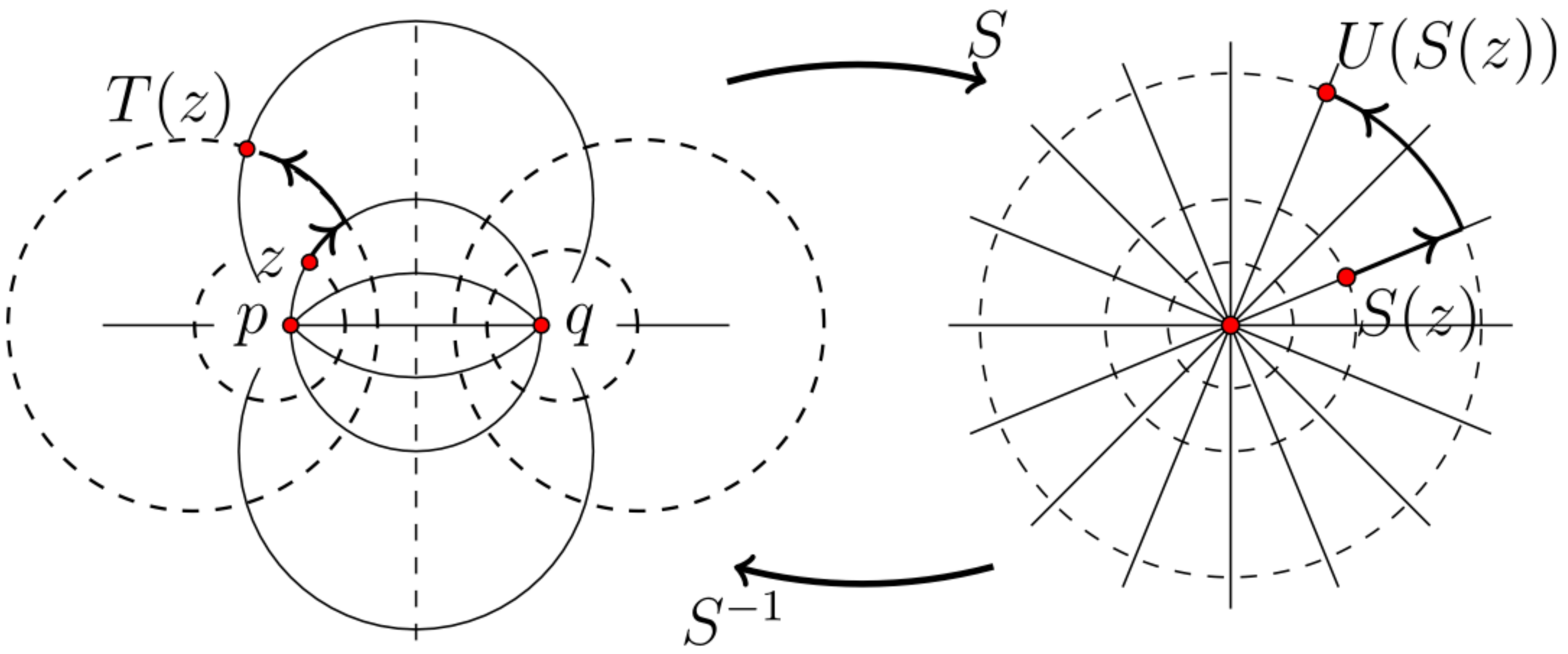
For n cycles we can find $z \to z_n = T\left(T\left(\cdots T \cdots T(z)\cdots\right)\right)$

n-times

$$\langle \psi(t_n) \,|\, O_h(w, \bar{w}) \,|\, \psi(t_n) \rangle = \left(\frac{\partial z}{\partial w}\right)^h \left(\frac{\partial z_n}{\partial z}\right)^h \cdots \langle \psi(0) \,|\, O_h(\tilde{z}(w)) \,|\, \psi(0) \rangle$$

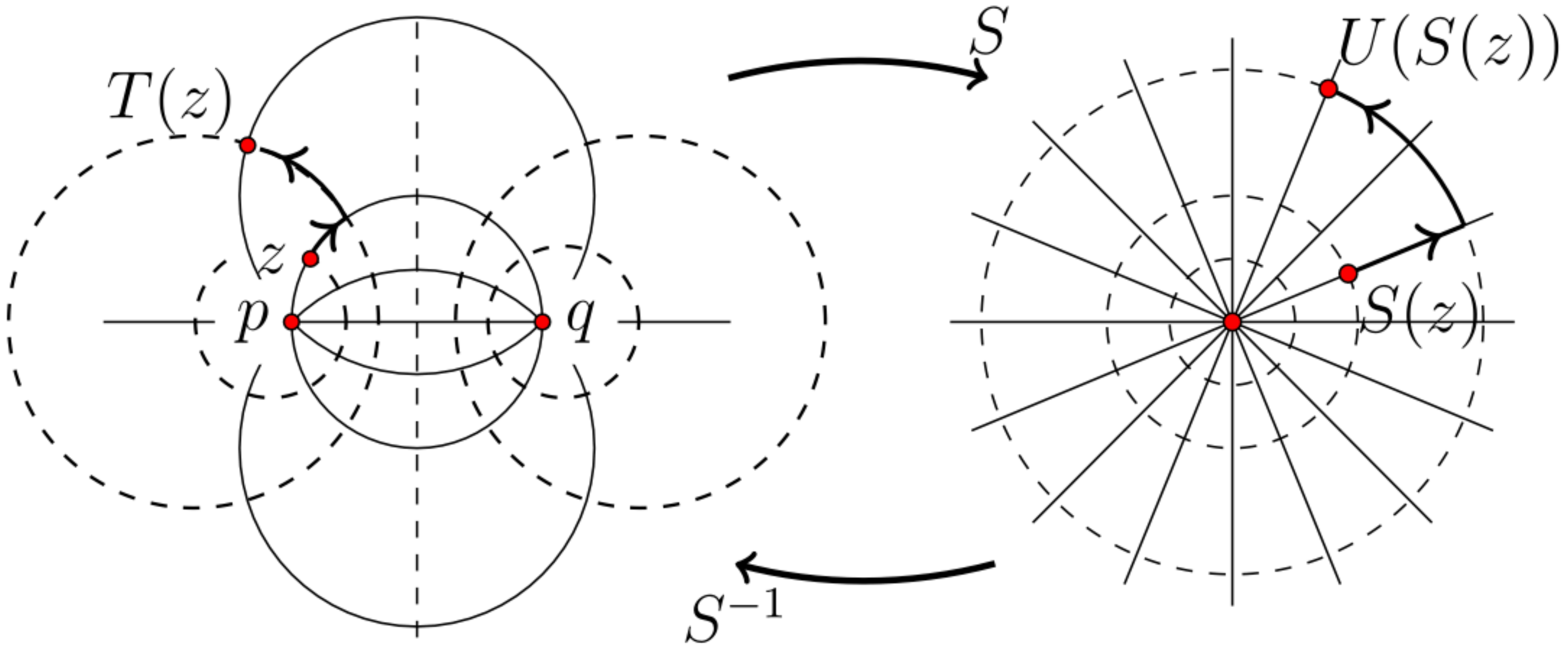
Möbius transformation

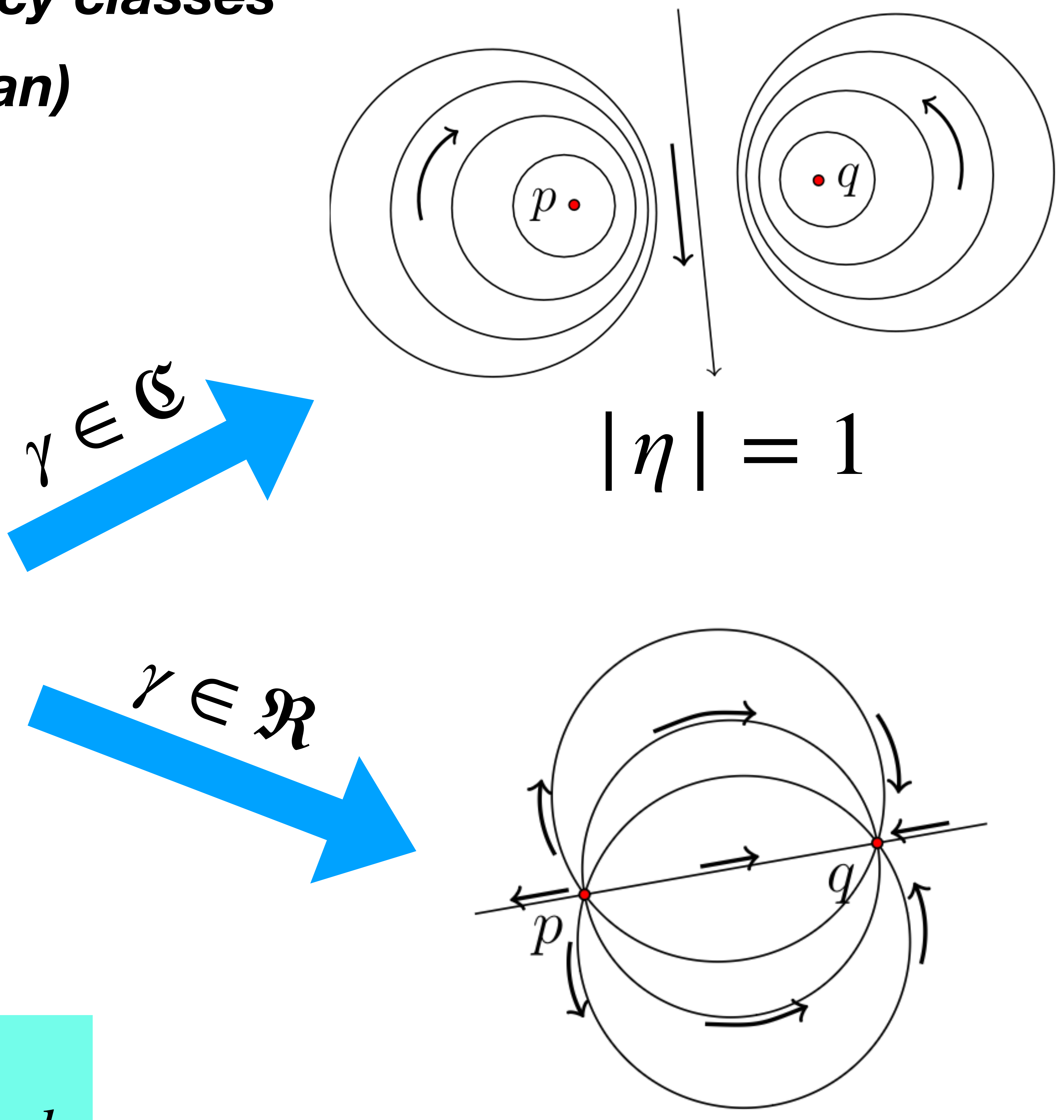
$$\frac{T(z)-\gamma_1}{T(z)-\gamma_2} = \eta \frac{z-\gamma_1}{z-\gamma_2}$$



$$f(\gamma) = \gamma = \frac{a-d \pm \sqrt{(a-d)^2 + 4bc}}{2c}, \ \eta = \frac{c\gamma_2 + d}{c\gamma_1 + d}$$

Möbius conjugacy classes
(Euclidean)





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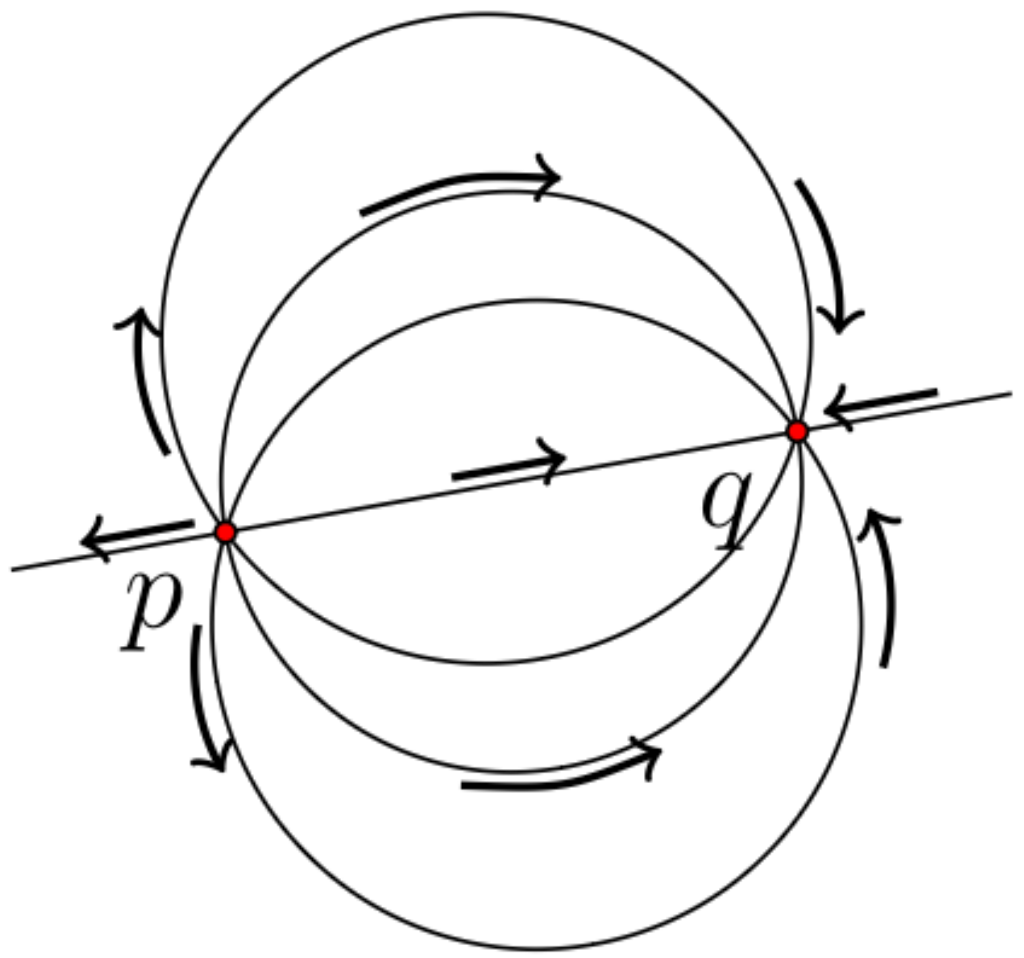
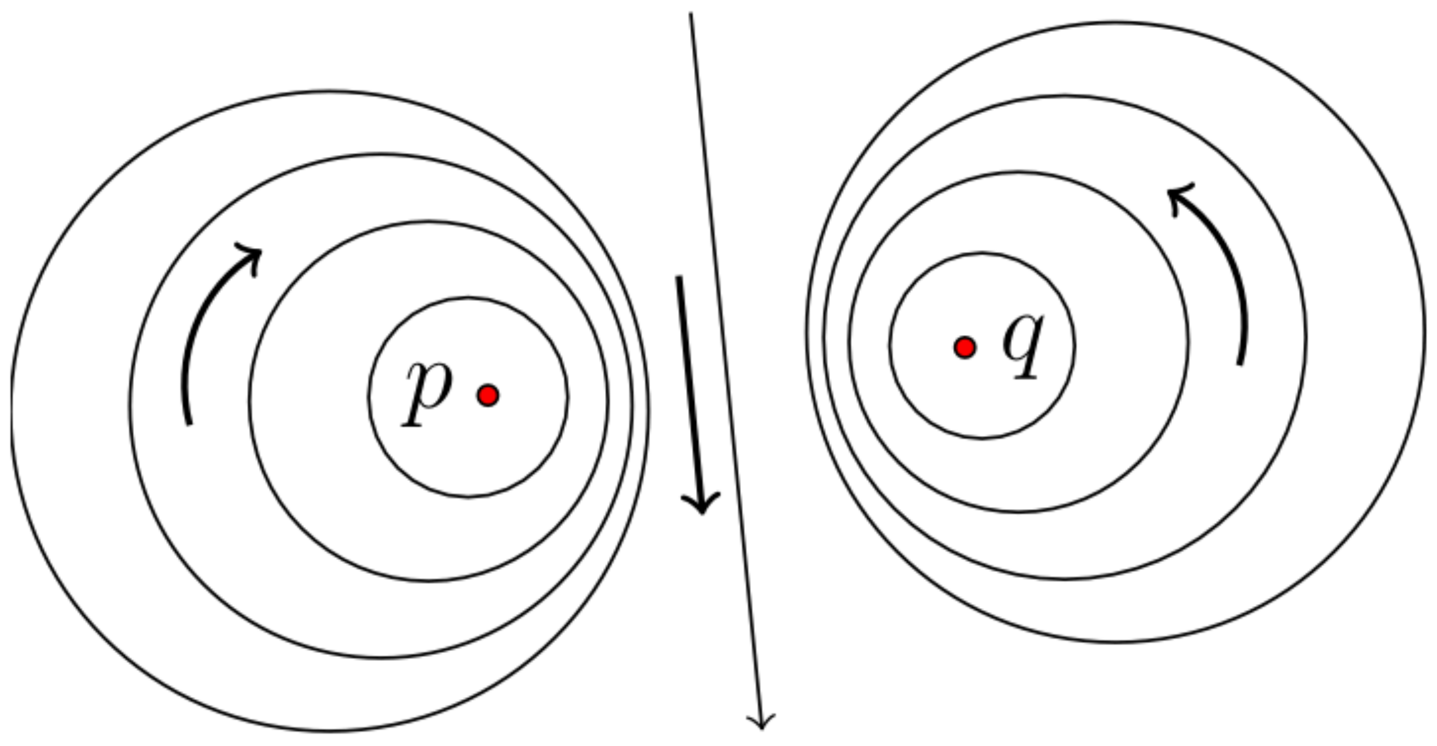
$\mathbf{Im}\,(\eta) = 0$

Spacetime asymmetries



$\mathrm{Im}\left(\eta\right) = 0$



$|\eta| = 1$

$$\langle \psi(t_n) \,|\, O_h(w,\bar{w}) \,|\, \psi(t_n) \rangle = \left(\frac{\partial z}{\partial w}\right)^h \left(\frac{\partial z_n}{\partial z}\right)^h \cdots \langle \psi(0) \,|\, O_h(\tilde{z}(w)) \,|\, \psi(0) \rangle$$

$\frac{\partial z_n}{\partial z} \sim \eta^n$, **when,** $z \to \gamma_1$, **and,** $\frac{\partial z_n}{\partial z} \sim \eta^{-n}$, **when,** $z \to \gamma_2$

Hence in unitary theories where $h > 0$, factors grow / shrink when $\eta \in \Re$.

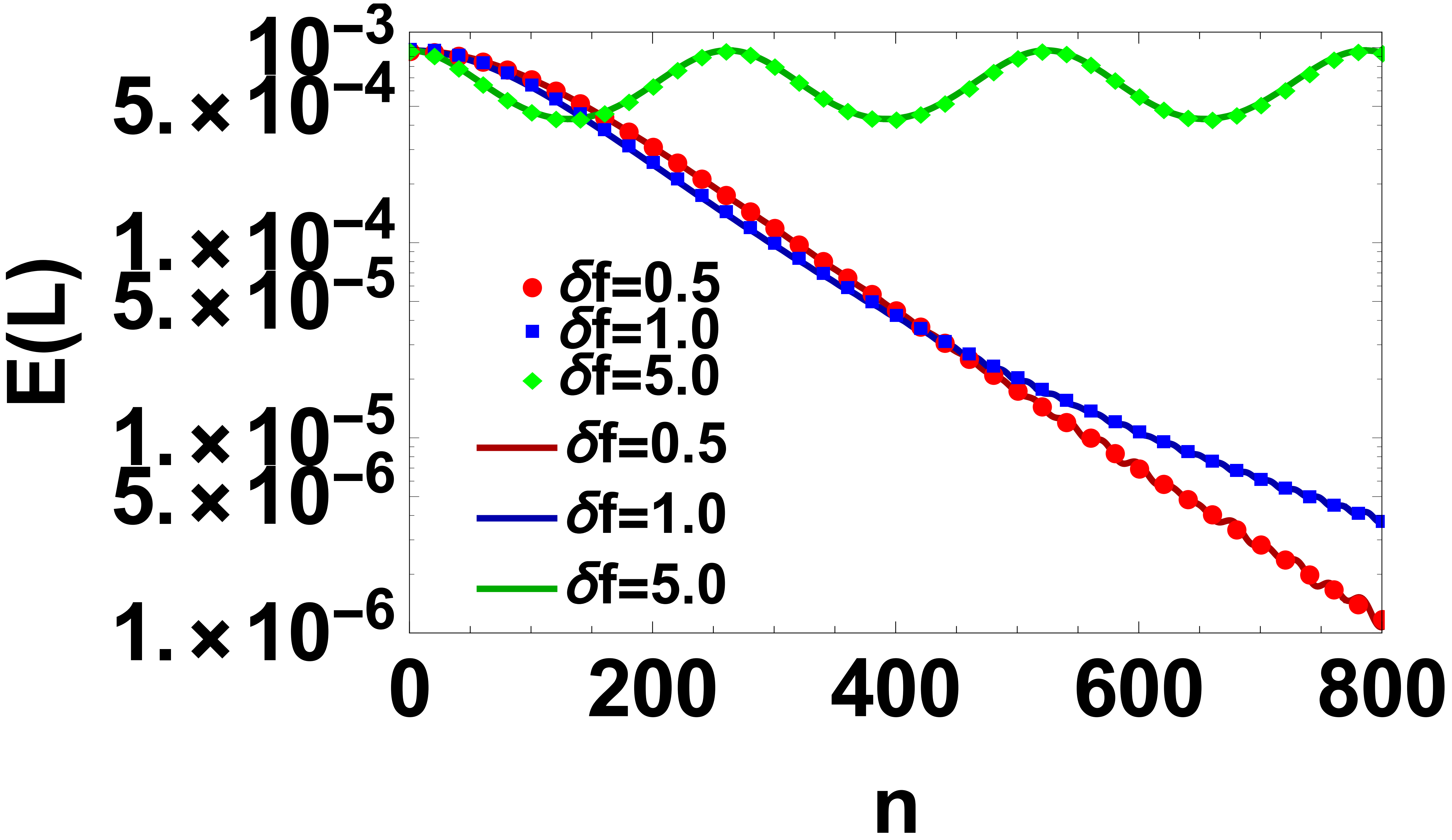
Hence observables show exponential growth / decay in time depending on their proximity to γ_1 / γ_2.

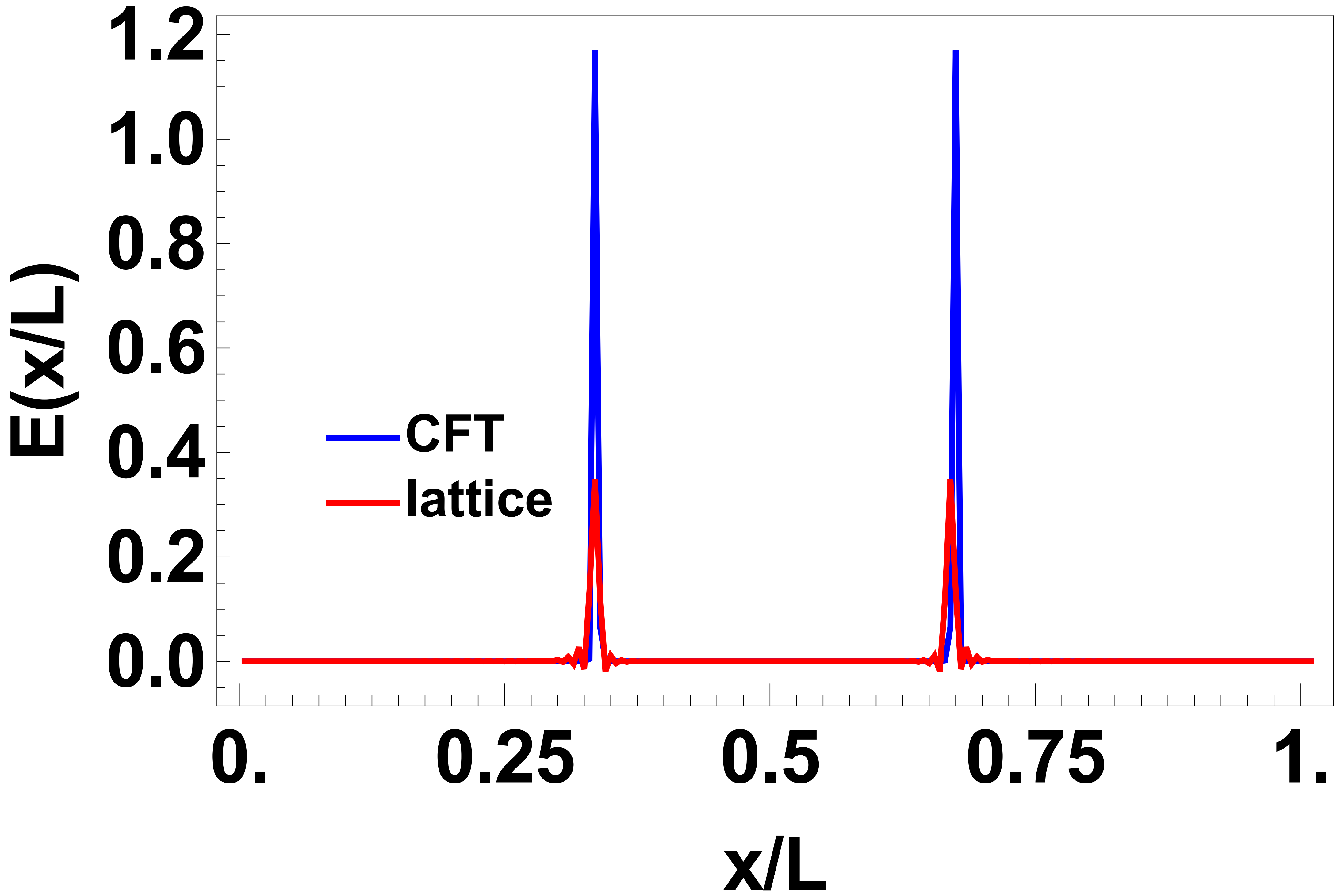
Example on the lattice

$$H = \sum_j \left(\delta f + J_0 \cos\left(\omega_D t\right)\right) c_j^\dagger c_{j+1} + 2J_1 \cos\left(\frac{2\pi}{L}(j - \frac{1}{2})\right) c_j^\dagger c_{j+1} + h.c.$$

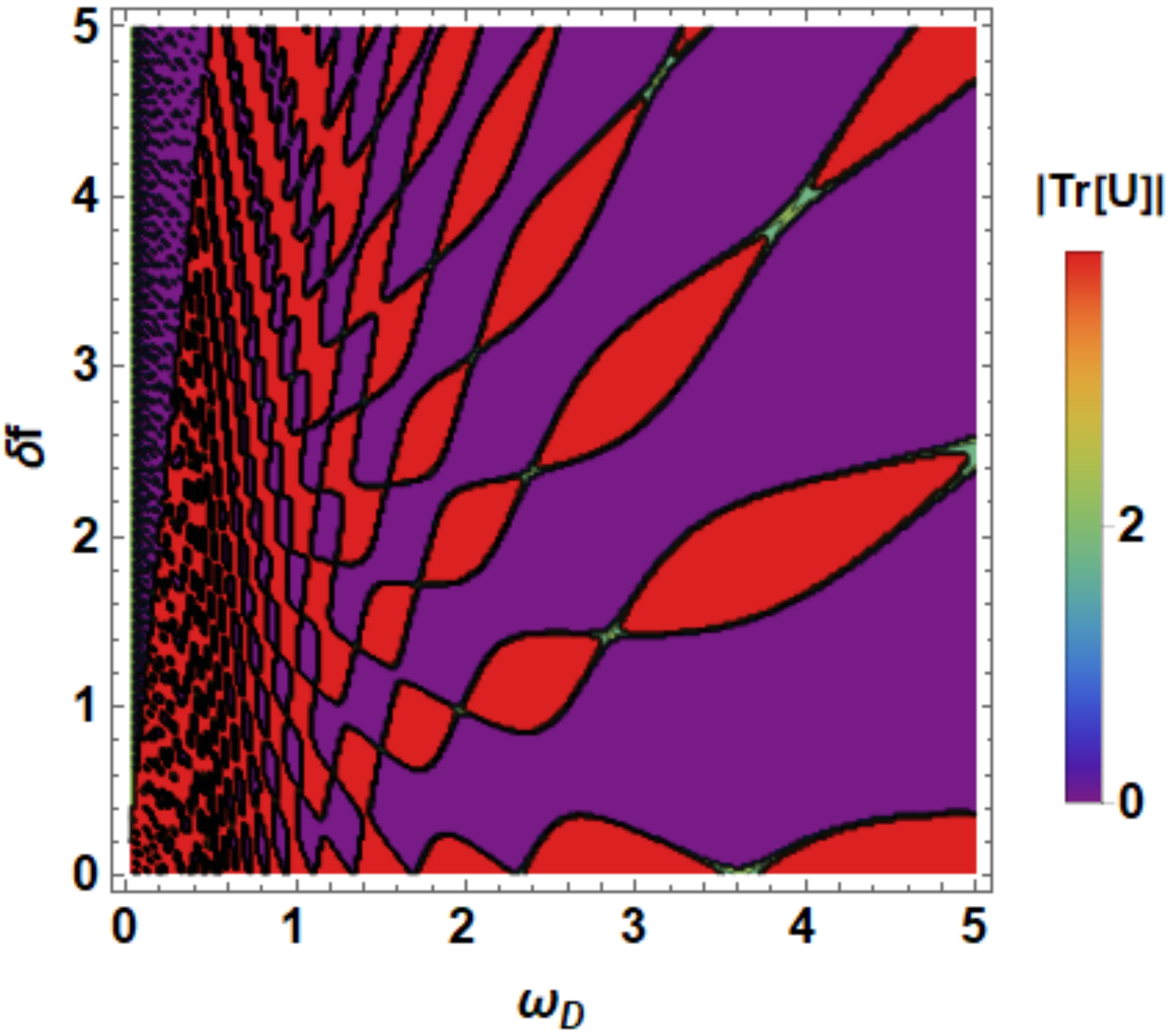
Look at the energy density : $\langle \psi(t) | \mathscr{E} | \psi(t) \rangle = \langle \psi(0) | \left(U^\dagger\right)^n T_{\tau\tau}(w)(U)^n | \psi(0) \rangle$

$$|\psi(0)\rangle = |\tfrac{1}{2}, \tfrac{1}{2}\rangle = c^\dagger_{k_F+\pi/L} c^\dagger_{-k_F-\pi/L} |FS\rangle$$





A phase diagram of heating and non-heating steady states.



Onto general dimensions

We only used $SL(2,R)$ subgroup in a CFT, this luxury is also present in a $d+1$ dimensional CFT.

$$[D, K_\mu] = -iK_\mu,\ [D, P_\mu] = iP_\mu,\ [K_\mu, P_\mu] = 2iD$$

Now for a CFT on $S^d \times \mathbb{R}$, we consider the deformation caused by $H = 2iD + i\beta\left(K_\mu + P_\mu\right)$, and once again many things go through : including the Möbius.

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In 4 dimensions, the representation turns out to be

QUATERNIONS

Transformations

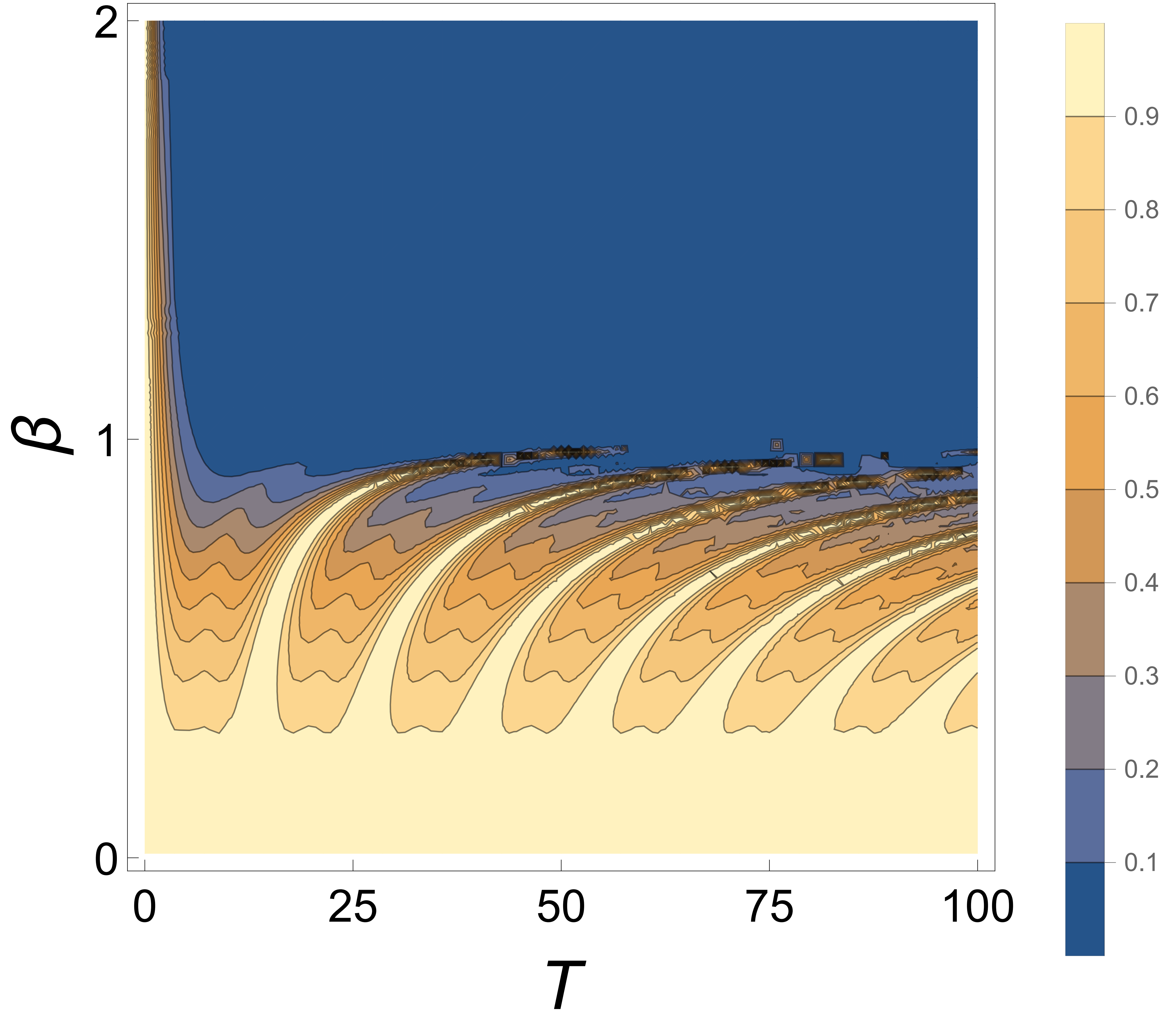
$$Q = \begin{pmatrix} x - iz & -i(\tau - iy) \\ -i(\tau + iy) & x + iz \end{pmatrix} \qquad U = e^{-w\left(2iD + i\beta(P_x + K_x)\right)} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$Q \to Q' = \left(aQ + b\mathbb{1}\right) \cdot \left(cQ + d\mathbb{1}\right)^{-1}$$

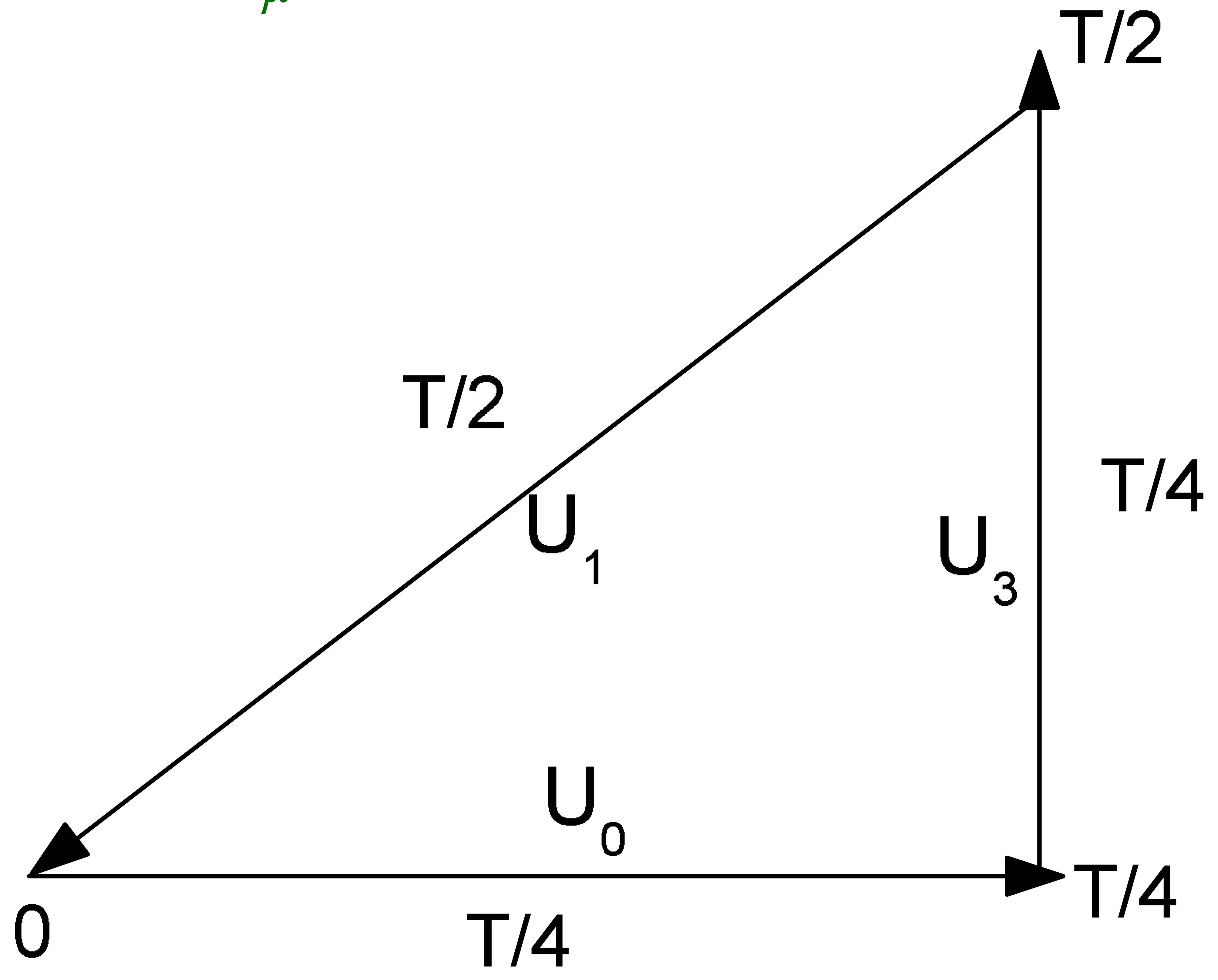
$$O(w) \xrightarrow{\text{Weyl}} e^{w\Delta} O(x^\mu) \xrightarrow{U[a,b;c,d]} \left| \frac{\partial x'}{\partial x} \right|^{\Delta/(d+1)} O(x^{\mu\prime}(w))$$

Fidelity

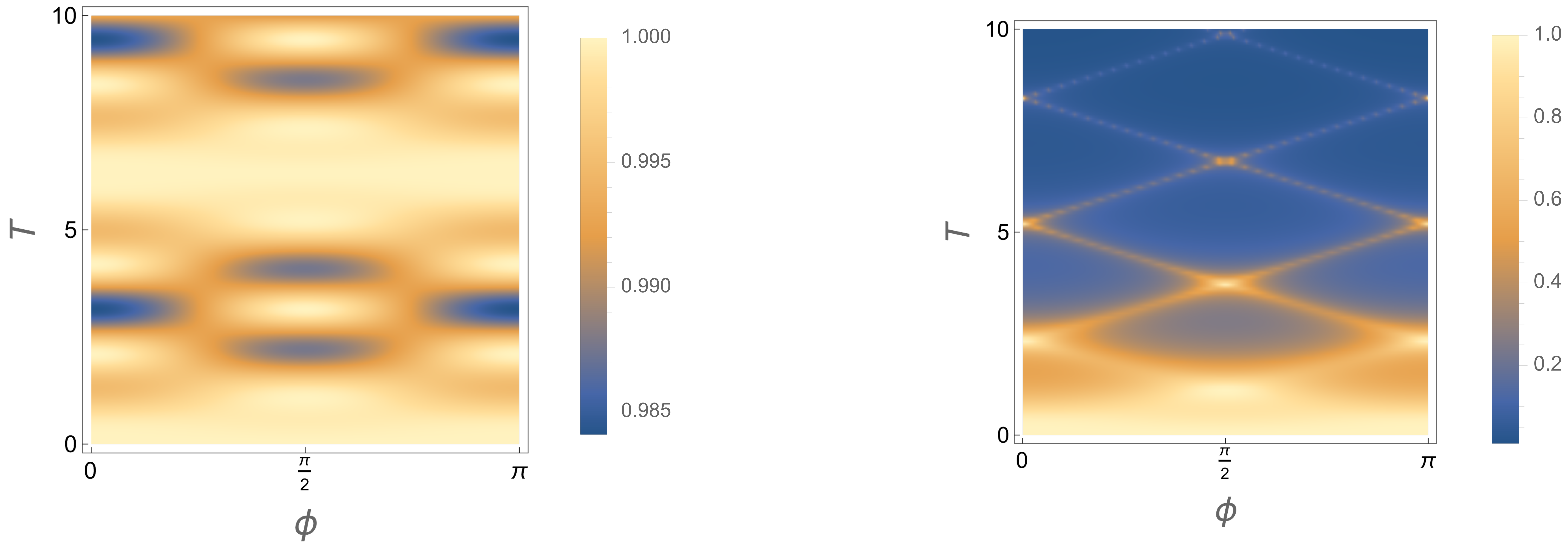
$$F(T) = \frac{1}{\langle \Delta | \Delta \rangle} \langle \Delta | U_1^\dagger \left(\frac{T}{2}\right) U_3 \left(\frac{T}{4}\right) U_0 \left(\frac{T}{4}\right) | \Delta \rangle$$



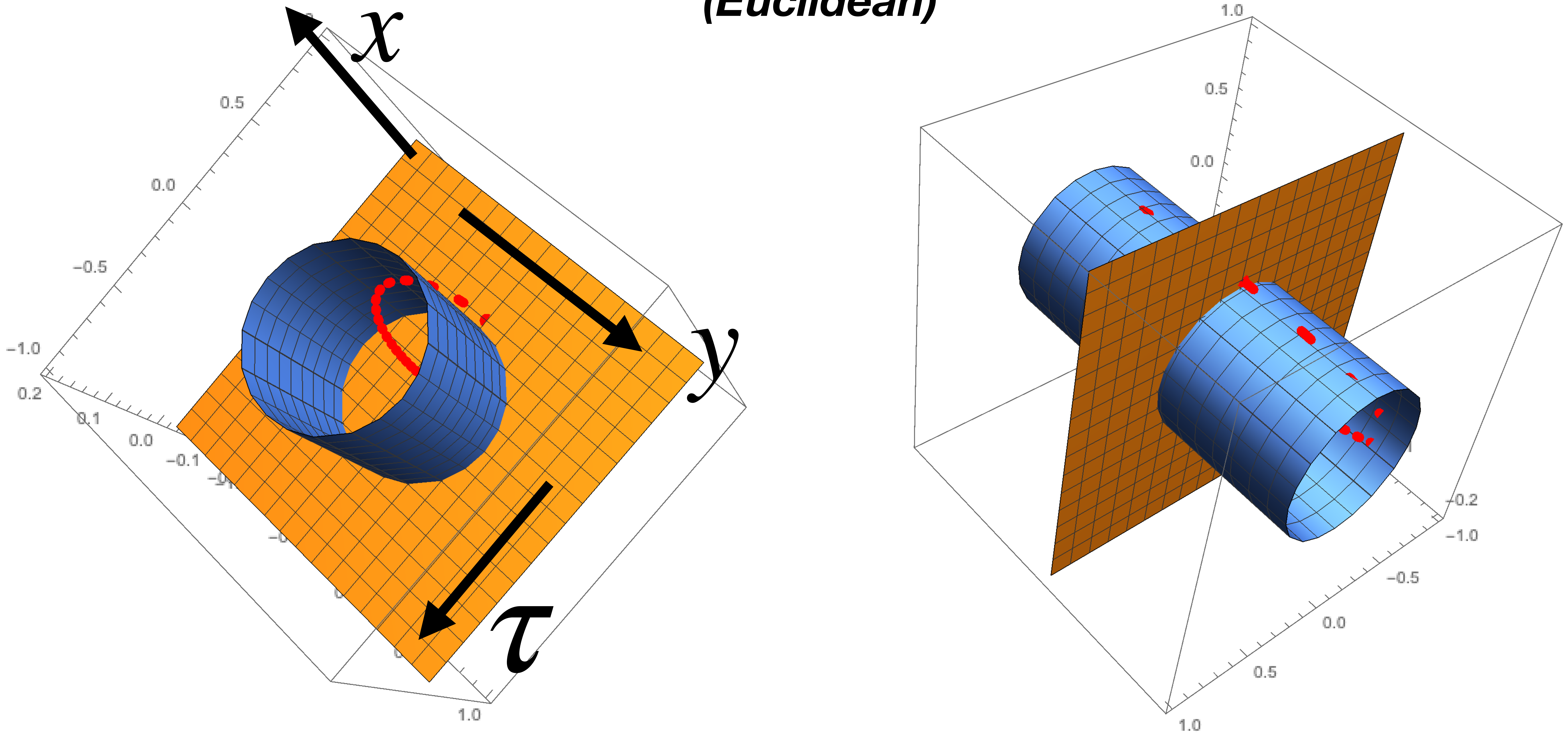
$$U_\mu = e^{-w\left(2iD + i\beta(P_\mu + K_\mu)\right)}$$



Oscillations & Localizations of $\mathscr{E}$



Movements around fixed surfaces
(Euclidean)



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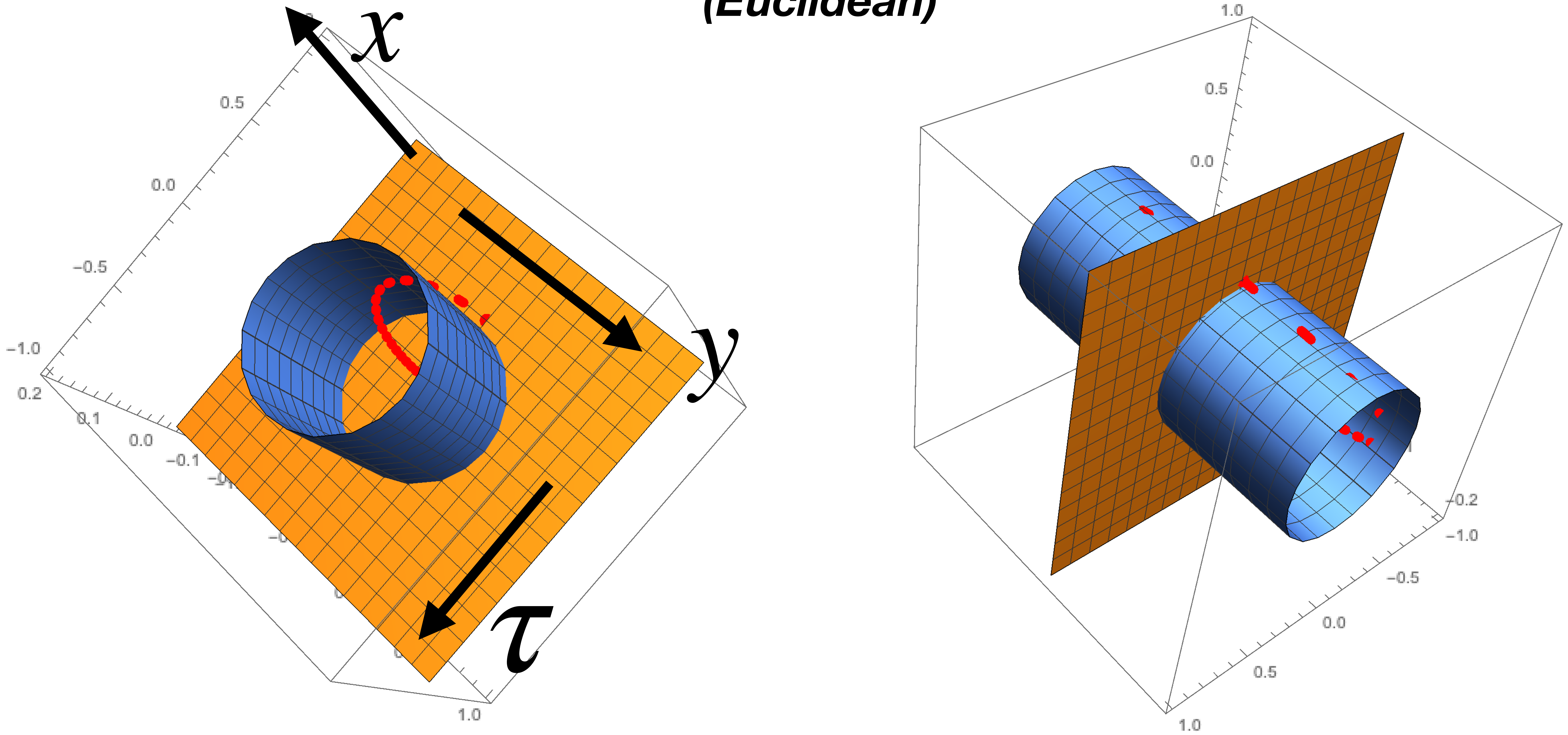

Disclaimer: *Verifying the n-cycle classification via conjugacy classes in the real time case remains open as of now.*

Other members of the band

Roopayan Ghosh,
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Arnab Kundu,
SINP Kolkata

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IACS Kolkata

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UKY Lexington

Open to Closed

In 2D one can also start from the CFT on a strip :

Open to Closed

In 2D one can also start from the CFT on a strip :

Note, that the Möbius deformation $H_1 = c_1 L_0 + c_2 L_q + c_3 L_{-q}$ *moves points (even on boundaries) into the*

BULK!

Open to Closed

This in fact was the observation in the first paper to have studied the sine-squared deformation in the first place! Gendiar, Krcmar, Nishino; 2010

$$H_1 = \sum_{\ell=1}^{N-1} \sin \frac{\ell\pi}{N} \left(\hat{h}_{\ell,\ell+1} + \frac{\hat{g}_\ell + \hat{g}_{\ell+1}}{2} \right)$$

Behaves as a periodic system, of size $2N$.

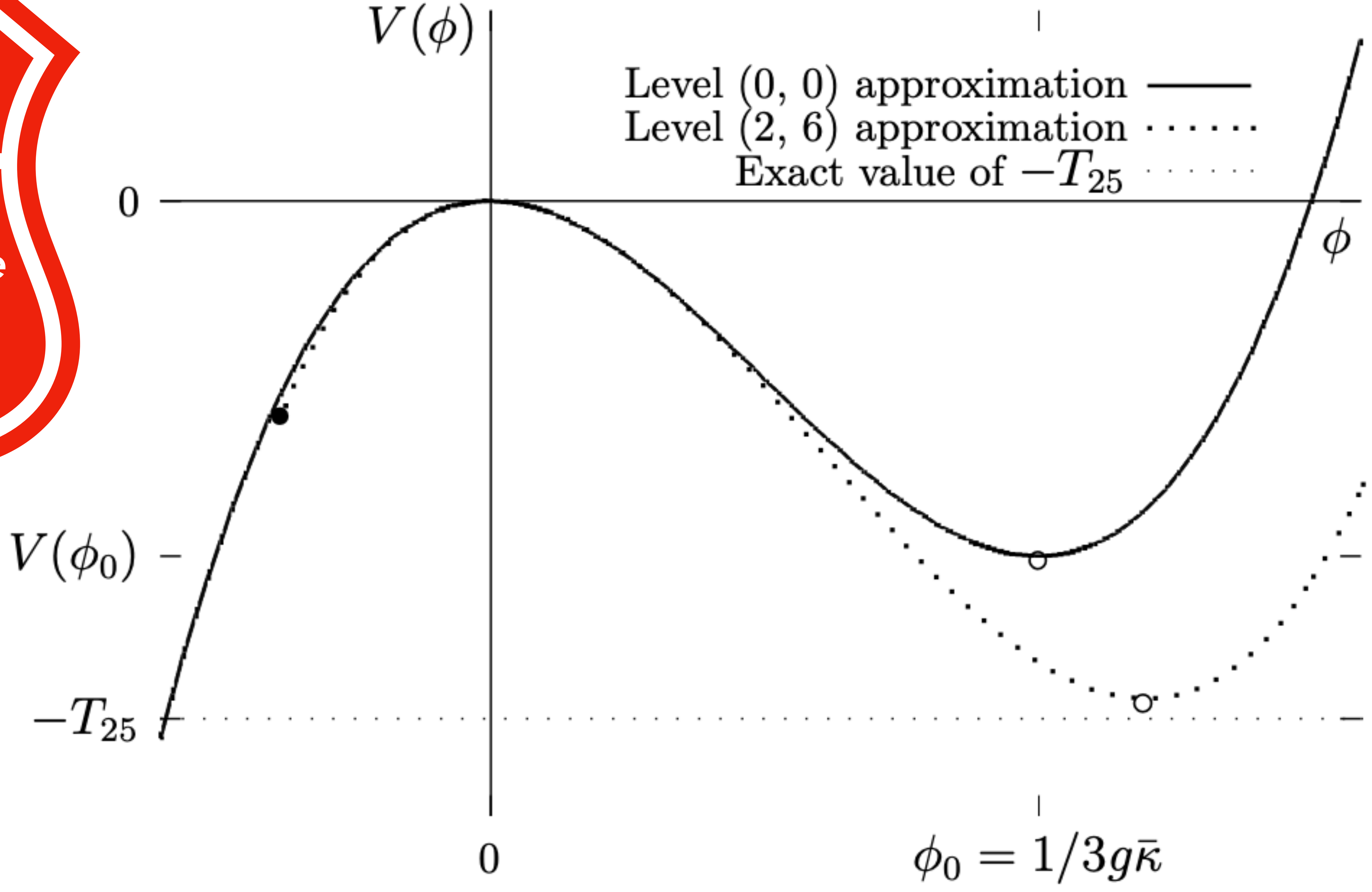
$$\frac{E_N^{(gs)}}{N} - \lim_{N\to\infty} \frac{E_N^{(gs)}}{N} = \begin{cases} 1/N & \textit{Open} \\ 1/N^2 & \textit{Closed} \end{cases}$$

A version of this observation has inspired a construction in the context of String Field Theory.

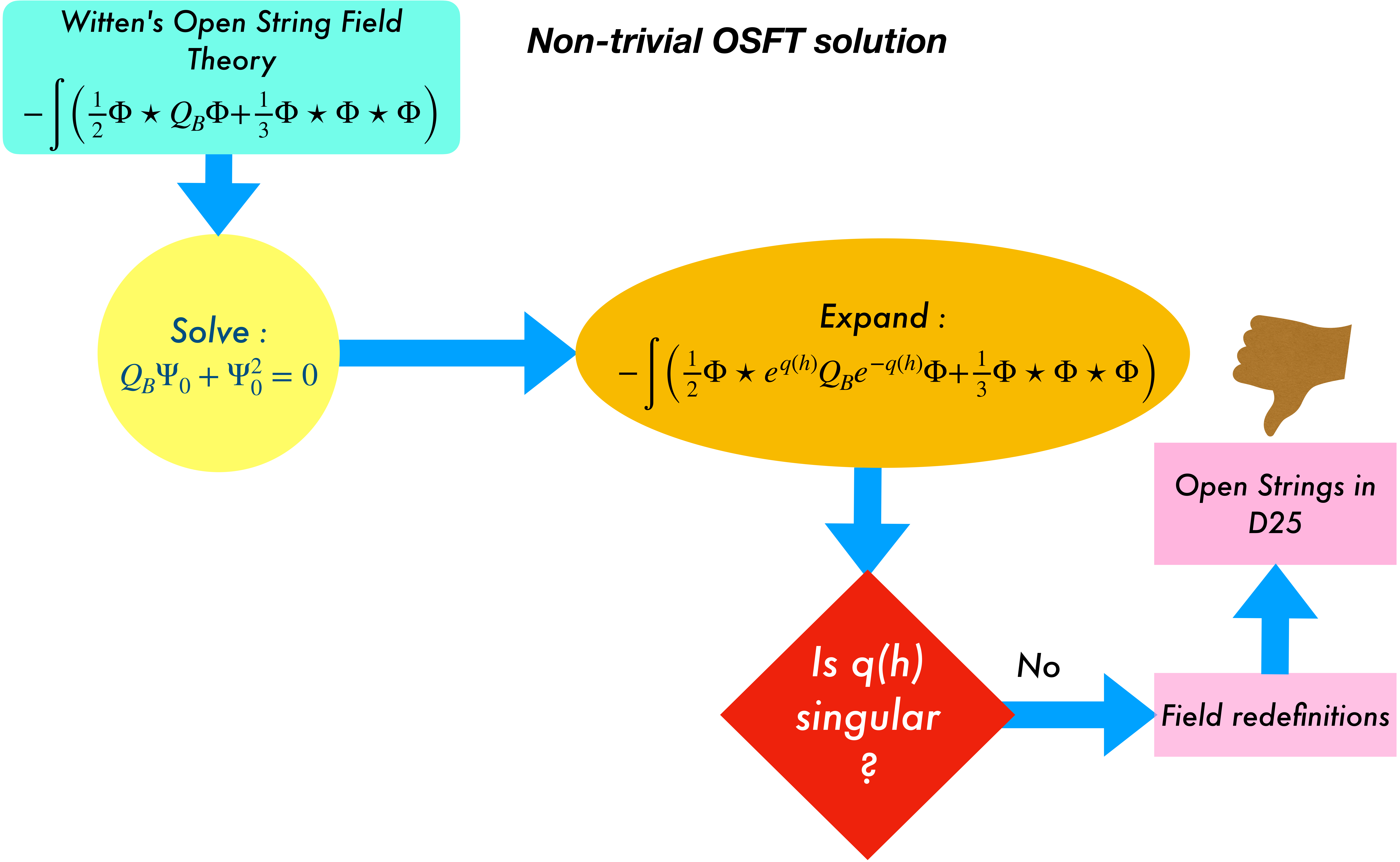
Tachyon condensation

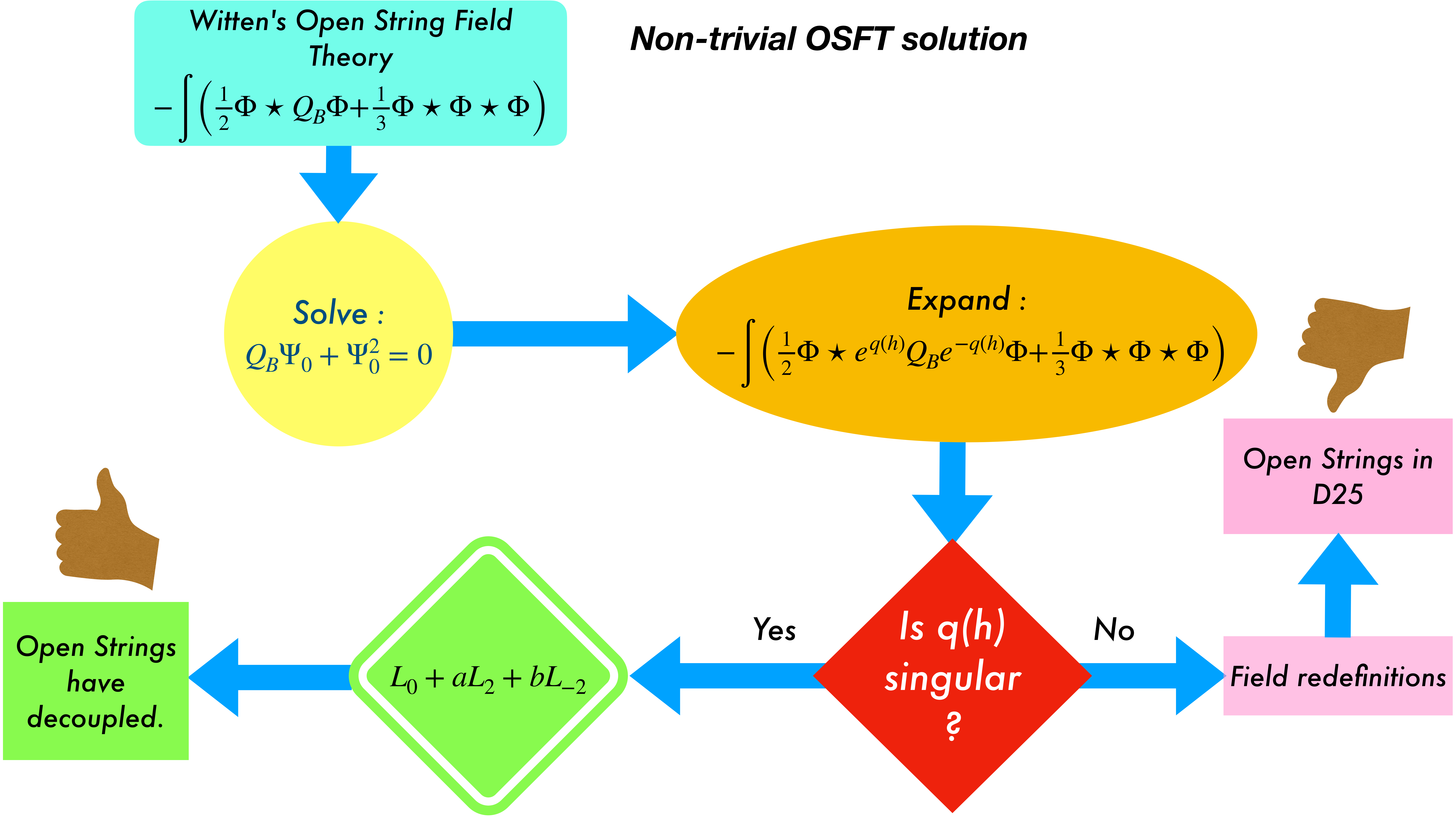
Sen, 1999.

Open strings should decouple from the true vacuum.

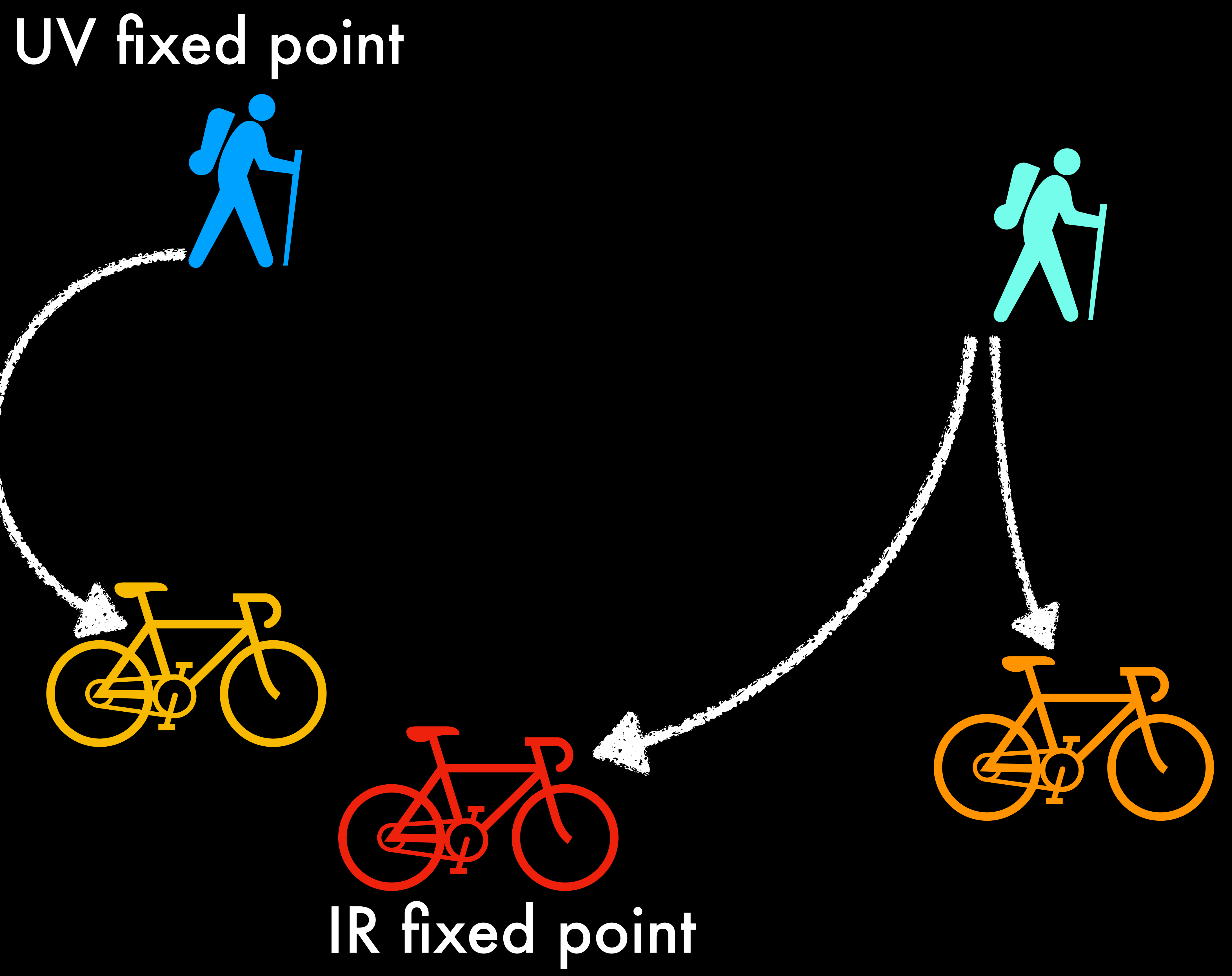


Non-trivial OSFT solution

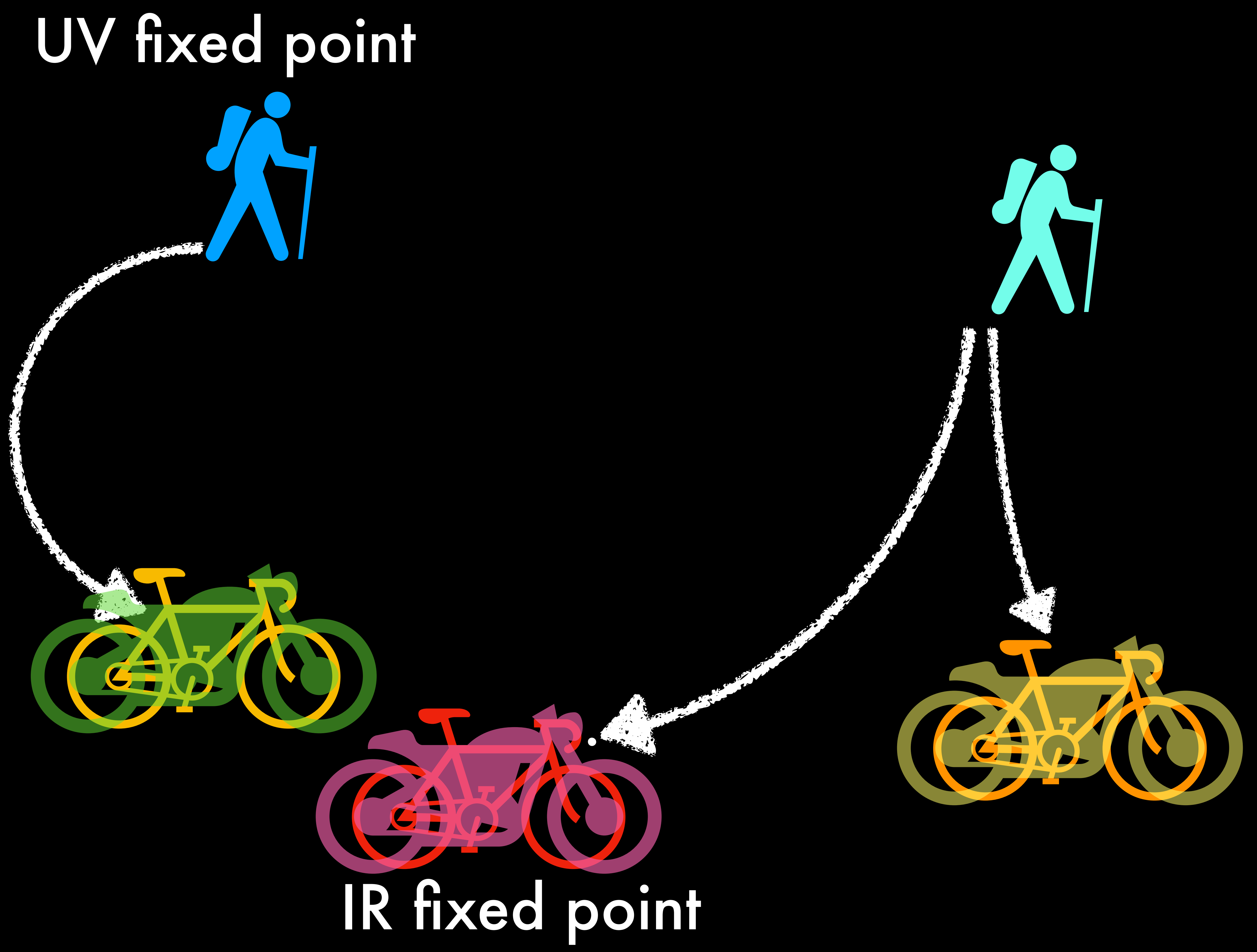




Witten's Open String Field Theory
$-\int\left(\frac{1}{2}\Phi\star Q_B\Phi+\frac{1}{3}\Phi\star\Phi\star\Phi\right)$
Non-trivial OSFT solution
Solve :
$Q_B\Psi_0+\Psi_0^2=0$
Expand :
$-\int\left(\frac{1}{2}\Phi\star e^{q(h)}Q_Be^{-q(h)}\Phi+\frac{1}{3}\Phi\star\Phi\star\Phi\right)$
Open Strings in D25
Is q(h) singular ?
Yes
No
Field redefinitions
$L_0+aL_2+bL_{-2}$
Open Strings have decoupled.



Space of Quantum Field Theories



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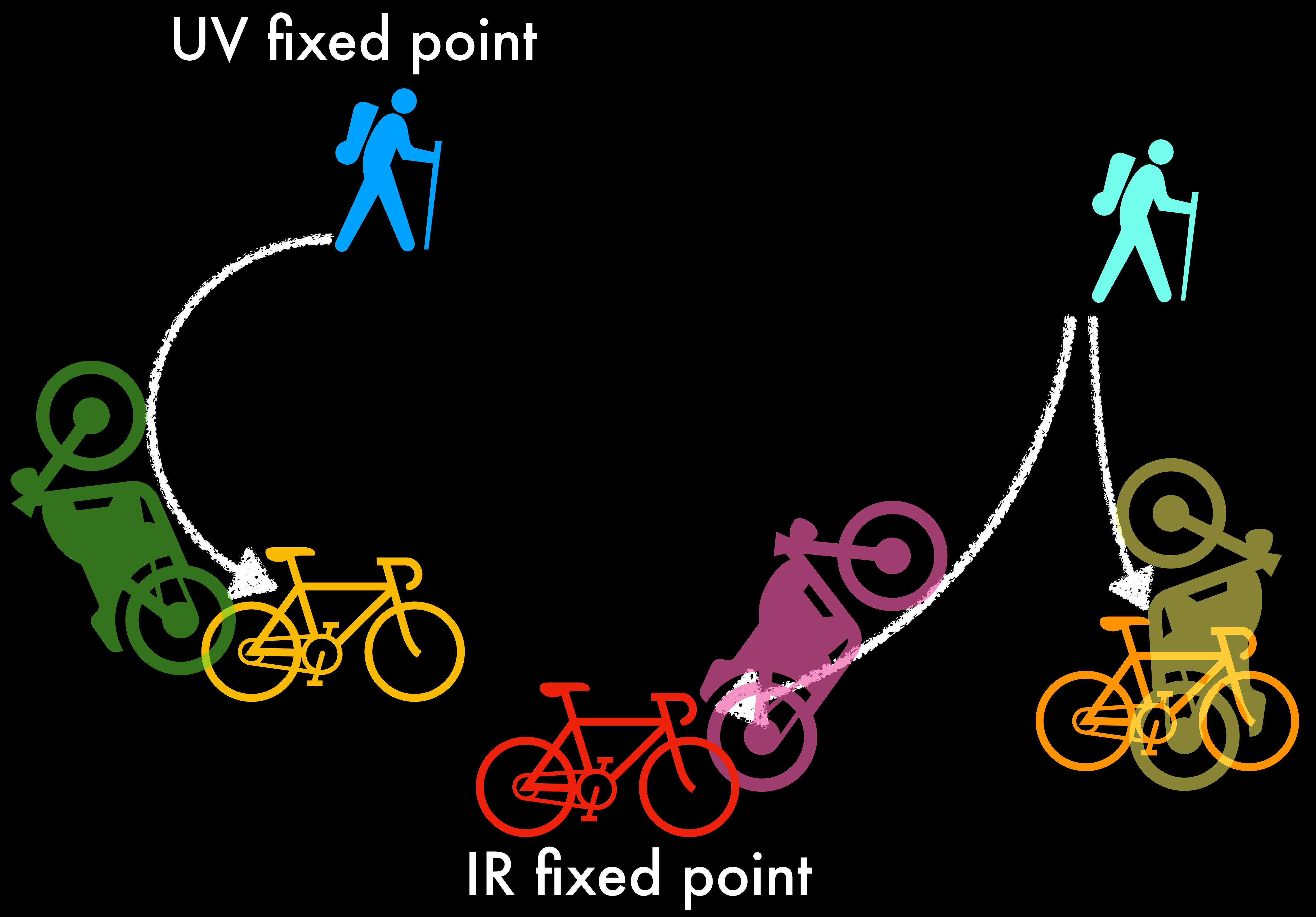

Thank you for your kind attention.