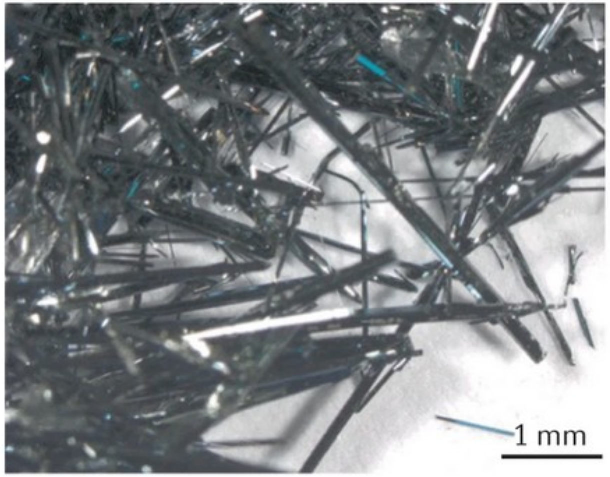

Axions in topological condensed matter

Arijit Kundu

Figure: Nature 575, 315 (2019)

$(TaSe4)_2I$ nanowires!

Outline

- Topological insulators. Character.
- How these lead to the θ term? What are possible θ?
- More outcomes of this term: axion ED and Farady rotations.
- Dynamical axion field. Axion polaritons.
- Weyl semimetals and chiral anomaly.

Nature Reviews Physics, 2, 682–696 (2020)
J. Appl. Phys. 129, 141 101 (2021)

Warning: I am not an expert of the underlying theories and probably my answers will be incomplete, in the best case!

Topological systems in cond-mat

Conventionally, metals and insulators have been distinguished by the existence of band gaps.

In 2005, a novel phase of matter that does not belong to either conventional metals or insulators, called the *topological insulator*, was discovered.

Topological insulators have gaps in their bulk (thus insulating) but also have gapless boundary (edge or surface) states (thus metallic). A topological insulator phase and a trivial insulator phase cannot be connected adiabatically to each other.

Topological systems in cond-mat

Different phases of matter had usually been distinguished from each other by the order parameters which indicate spontaneous symmetry breaking.

However, from the viewpoint of symmetry analysis, time-reversal invariant topological insulators and time-reversal invariant band insulators cannot be distinguished.

Topological systems in cond-mat

The ways to distinguish such topologically nontrivial and trivial insulator phases can be divided into two types (which of course give rise to equivalent results).

One way is introducing a "topological invariant" such as Z_2 invariant or Chern number (which are integers).

The other way is the "topological field theory", which describes the responses of topological phases to external fields.

The θ term

Electromagnetic response of topological insulators are best described by the action with a θ term.

$$S_\theta = \int dt d^3 r \, \frac{\theta e^2}{4\pi^2 \hbar c} \boldsymbol{E} \cdot \boldsymbol{B}$$

The phenomena described by the θ term is termed the axion electrodynamics

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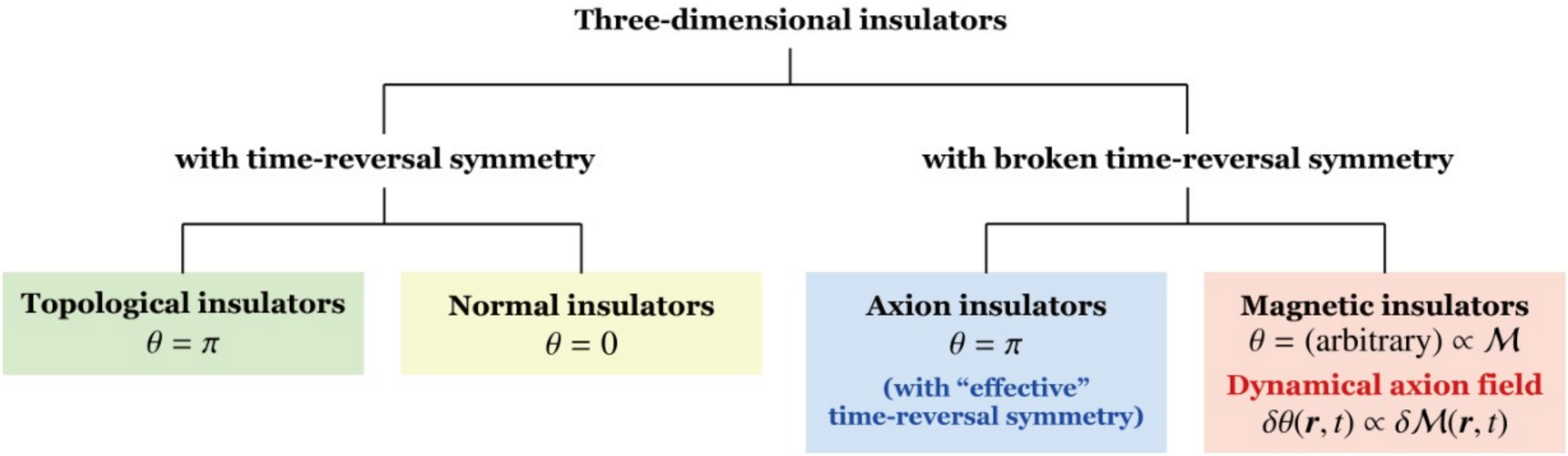

J. Appl. Phys. 129, 141 101 (2021)

The θ term

What is the matter with this term?

Take derivative with $\boldsymbol{E}$ and $\boldsymbol{B}$:

$$\boldsymbol{P} = \frac{\theta e^2}{4\pi^2\hbar c}\boldsymbol{B}, \qquad \boldsymbol{M} = \frac{\theta e^2}{4\pi^2\hbar c}\boldsymbol{E}$$

Linear magnetoelectric effect.

$$\boxed{S_\theta = \int dt d^3r \, \frac{\theta e^2}{4\pi^2\hbar c}\boldsymbol{E}\cdot\boldsymbol{B}}$$

Surface half-quantized anamalous Hall effect

The key (as we shall see) here is the fact that the surface of a TI host quasiparticles that follow a Dirac equation. A single Dirac node on each of the two surfaces.

$$\mathscr{H}_{\text{surface}}(\boldsymbol{k}) = \hbar v_{\mathrm{F}}(k_y \boldsymbol{\sigma}_x - k_x \boldsymbol{\sigma}_y)$$

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$$\mathscr{H}_{\text{surface}}(\boldsymbol{k}) = \hbar v_{\text{F}}(k_y \boldsymbol{\sigma}_x - k_x \boldsymbol{\sigma}_y)$$

In presence of magnetic impurities, it gets an additional $m\sigma_z$ term. Leading to a gapped spectrum:

$$E_{\text{surface}}(\boldsymbol{k}) = \pm\sqrt{(\hbar v_{\text{F}} k_x + m_y)^2 + (\hbar v_{\text{F}} k_y - m_x)^2 + m_z^2}$$

Additionally, the Hall conductivity becomes $\sigma_{xy} = -\text{sign}(m)\frac{e^2}{2h}$

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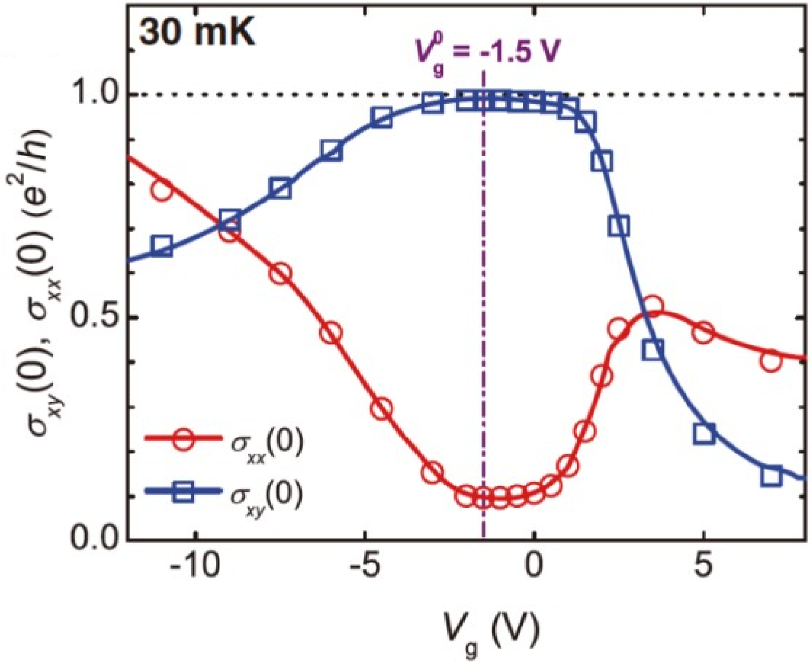


Cr dopped $(Bi,Sb)_2Te_3$

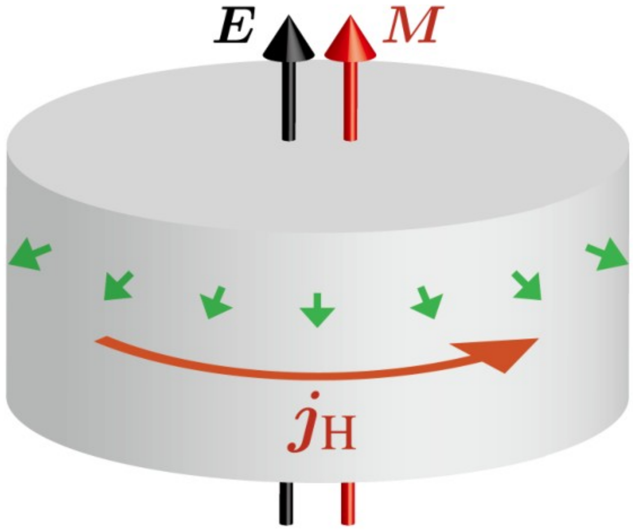
Net conductance is double, as there are two surfaces (fermion doubling!)

Science 340, 167 (2013)

How the θ term arises?

When an external electric field E is applied parallel to the cylinder, a surface anomalous Hall current induces

$$\boldsymbol{j}_{\mathrm{H}} = -\mathrm{sgn}(m)\frac{e^2}{2h}\hat{\boldsymbol{n}} \times \boldsymbol{E}$$



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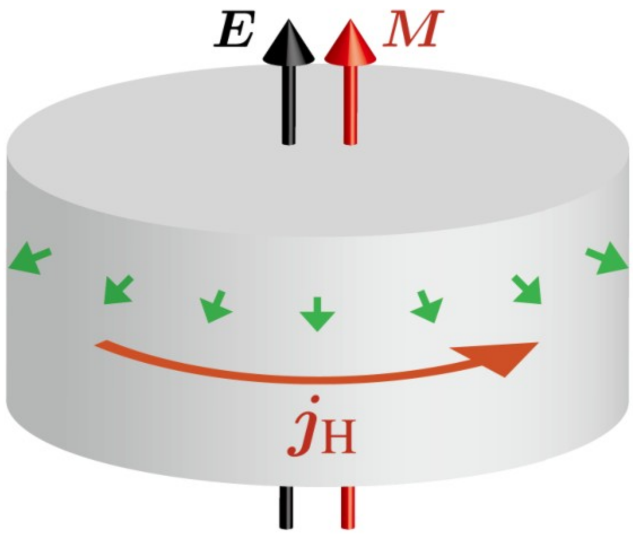
The magnetization is then (Ampere's law)

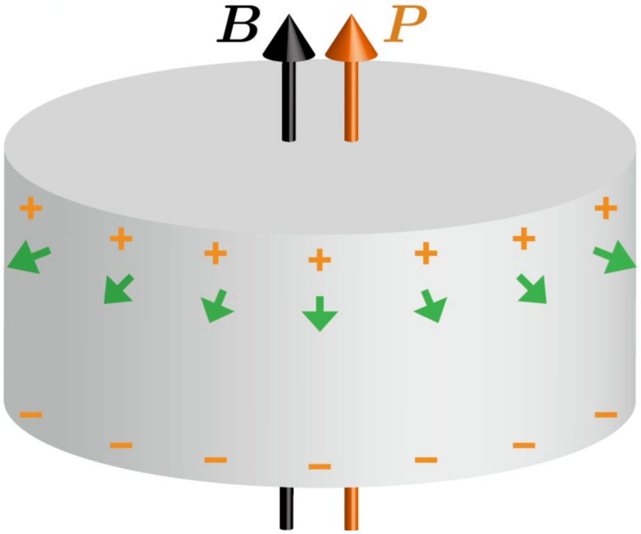
$$\boldsymbol{M} = \mathrm{sgn}(m)\frac{e^2}{2hc}\boldsymbol{E}$$



How the θ term arises?

Similarly, when an external magnetic field B is applied parallel to the cylinder, the circulating electric field normal to the magnetic field is induced as

$$\nabla \times \boldsymbol{E}^{\mathrm{ind}} = -\partial \boldsymbol{B}/\partial t$$



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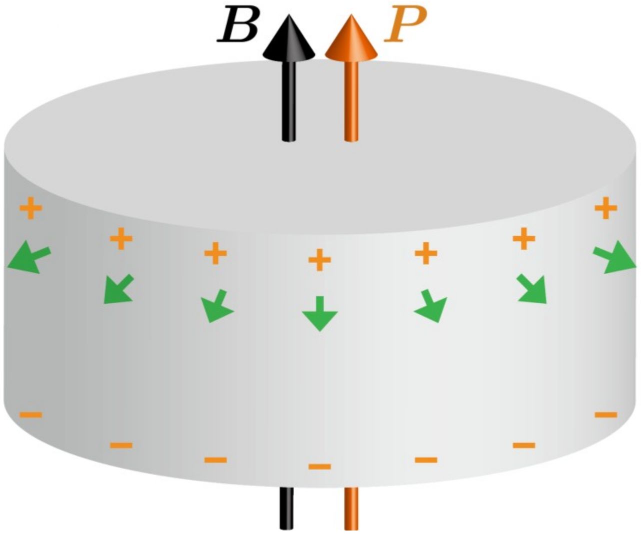
$$\boldsymbol{j}_{\text{H}} = \text{sgn}(m)\frac{e^2}{2h}\frac{\partial \boldsymbol{B}}{\partial t}$$



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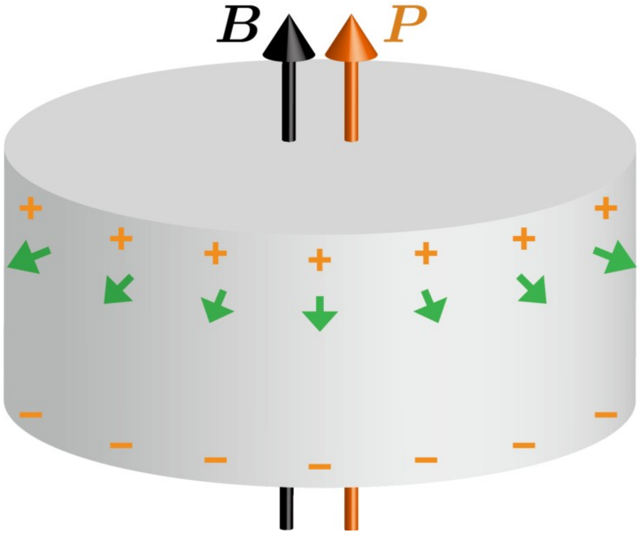
A polarization current is equivalent to the time derivative of the electric polarization

$$\boldsymbol{P} = \mathrm{sgn}(m)\frac{e^2}{2hc}\boldsymbol{B}$$



How the θ term arises?

The magnetization and polarization are simply derivatives of the free energy. So, we can write the free energy as:

$$F = -\int d^3r \, \frac{e^2}{2hc} \boldsymbol{E} \cdot \boldsymbol{B} = -\int d^3r \, \frac{\theta e^2}{4\pi^2 \hbar c} \boldsymbol{E} \cdot \boldsymbol{B}$$

In addition to this simple understanding, more rigorous microscopic derivations are also possible!

Axion electrodynamics

The θ term changes the Maxwell's equations.

$$\begin{aligned}
\nabla \cdot \mathbf{E} &= \rho - \kappa \nabla\theta \cdot \mathbf{B} \\
\nabla \times \mathbf{E} &= -\partial \mathbf{B}/\partial t \\
\nabla \cdot \mathbf{B} &= 0 \\
\nabla \times \mathbf{B} &= \partial \mathbf{E}/\partial t + \mathbf{j} + \kappa\,(\partial\theta/\partial t \mathbf{B} + \nabla\theta \times \mathbf{E})
\end{aligned}$$

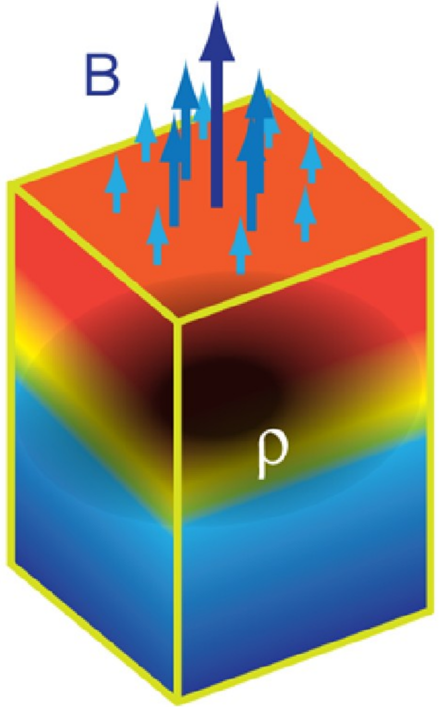
Note that, only the gradient of θ appears. Thus this has to be surface effect!

Axion electrodynamics

The first term $\nabla \cdot \mathbf{E} = \rho - \kappa \nabla \theta \cdot \mathbf{B}$ implies an applied magnetic field can give rise to the contribution to the net charge density!

In an applied field, one expects that there would be a charge build up on the surface of a TI.

More interestingly, this excess charge would be exactly half quantized!

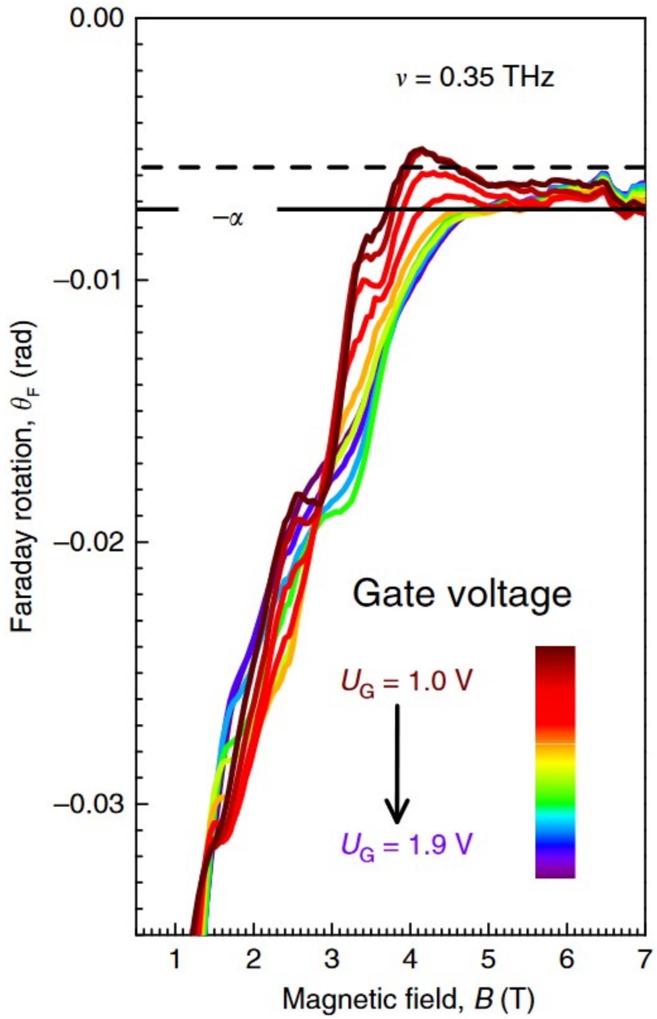


Physics 1, 36 (2008)

Faraday rotations

The modified ED also implies that, when an electromagnetic wave is irradiated on the surface of a TI, its polarization angle will be rotated.

This rotation was also observed!



Nat. Commun. 8, 15 197 (2017)

Dynamical axion field

This is when the θ is a field, as a function of space and time. This can happen in many cases in condensed matter, such as, in anti-ferromagnets (essentially as you need to break both time-reversal as well as inversion symmetry).

A derivation of this in the so called Fu-Kane-Mele-Hubbard is relatively straight-forward, but beyond the scope of this talk. See, Nat. Phys. 6, 284 (2010), for example.

Example materials are magnetically doped Tis, or layered antiferromagnets.

Axionic polariton

When θ is a field, many interesting phenomena follows. One of them is the formation of axionic polariton. This are essentially bound-states formed by the axion and photons.

This is relatively easy to see. The electromagnetic wave equations modify to become:

$$\frac{\partial^2 \boldsymbol{E}}{\partial t^2} - c'^2 \nabla^2 \boldsymbol{E} - \frac{\alpha}{\pi \varepsilon} \boldsymbol{B}_0 \frac{\partial^2 \delta\theta}{\partial t^2} = 0,$$

$$\frac{\partial^2 \delta\theta}{\partial t^2} - v^2 \nabla^2 \delta\theta + m^2 \delta\theta - \frac{\alpha}{8\pi^2 g^2 J} \boldsymbol{B}_0 \cdot \boldsymbol{E} = 0$$

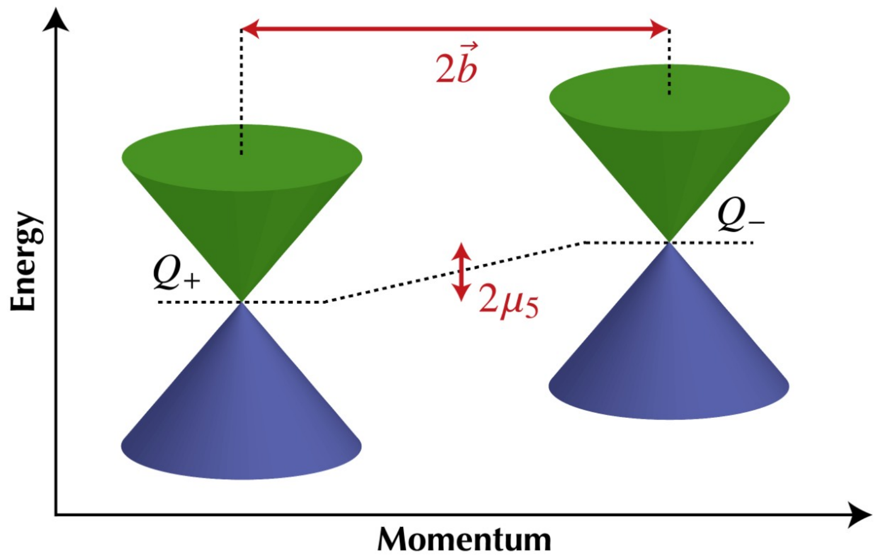
And the dispersion of the photon field becomes:

$$2\omega_{\pm}(k) = c'^2 k^2 + m^2 + b^2 \pm \sqrt{(c'^2 k^2 + m^2 + b^2)^2 - 4c'^2 k^2 m^2}$$

θ term in Weyl semimetals

Weyl semimetals are not insulators. They have an even number of Weyl nodes (places near which electrons follow Weyl equations).

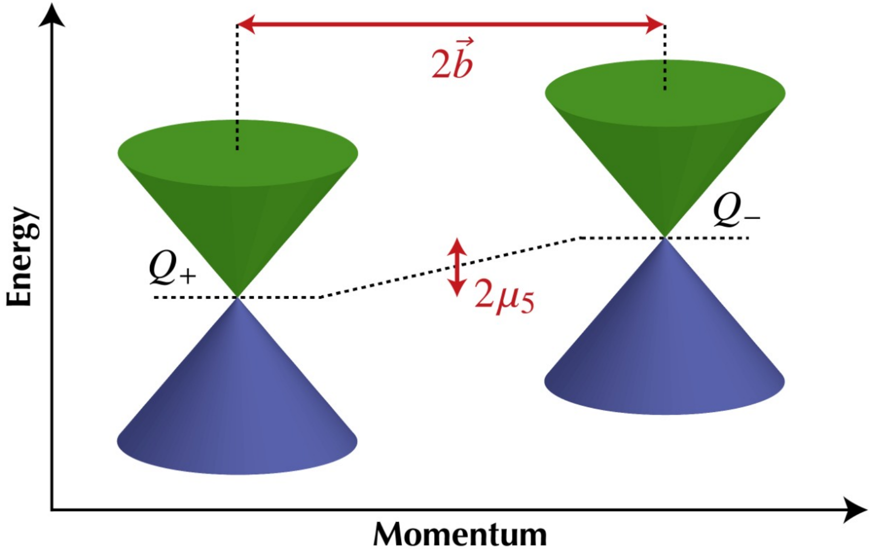
These nodes can be separated in the momentum or in energy.



θ term in Weyl semimetals

The effective action of these systems contains a theta term, which is a function of this momentum/energy separation.

$$S = \int dt d^3 r \, \bar{\psi} [i \gamma^\mu (\partial_\mu - ie A_\mu)] \psi + \frac{e^2}{2\pi^2 \hbar} \int dt d^3 r (\boldsymbol{b} \cdot \boldsymbol{r} - \mu_5 t) \boldsymbol{E} \cdot \boldsymbol{B}$$

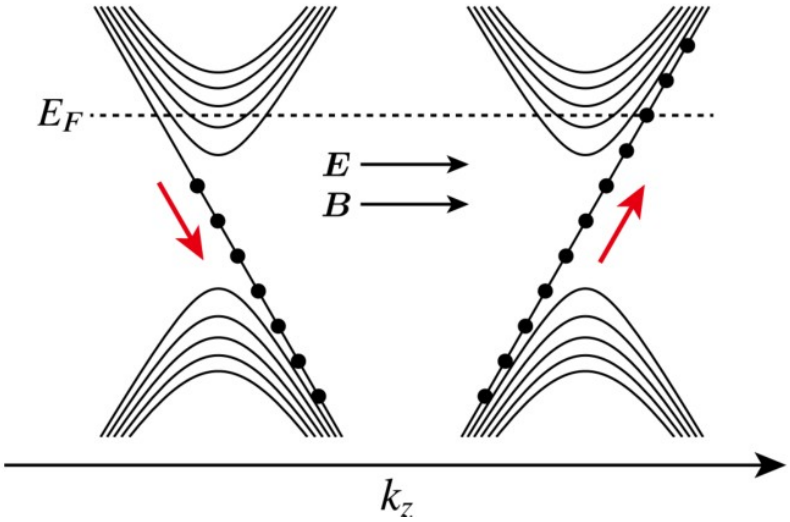


Chiral magnetic effect in Weyl semimetals

The axion term here can generate non-equal population difference between the two valleys!

$$\frac{\partial N_i}{\partial t} = Q_i \frac{e^2}{4\pi^2\hbar^2 c}\boldsymbol{E}\cdot\boldsymbol{B}$$

This is essentially a manifestation of chiral anomaly.



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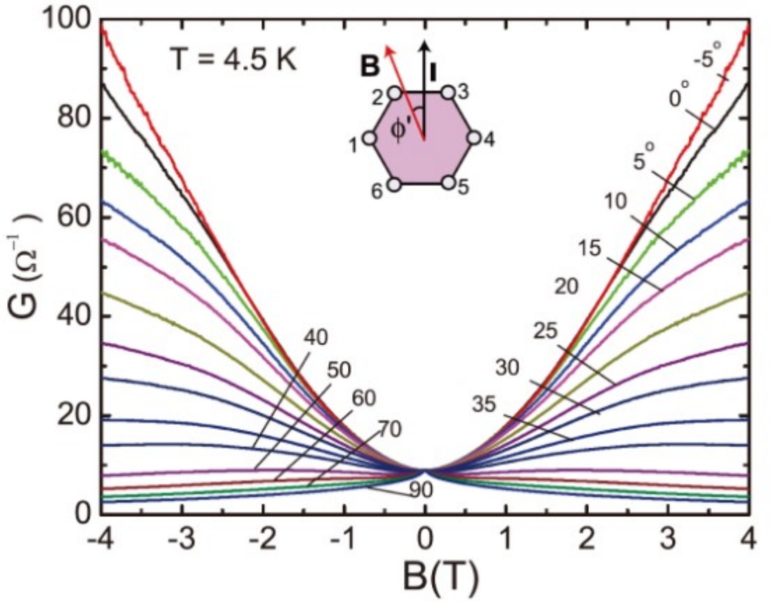
Little more calculations show that this gives rise to negative linear magneto-resistance!



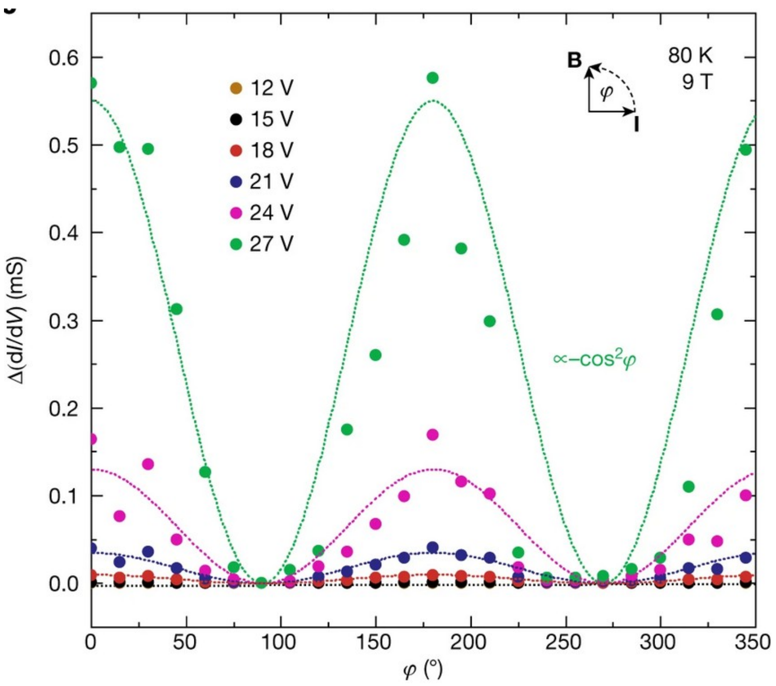
Science 350, 413 (2015)

Charge-density wave coupled to axions

The Weyl nodes, if interacting, can couple to each other in forming a CDW state. It is predicted that this is an axion insulator and the phase part of the CDW order parameter can behave like an axion field.

In this case the longitudinal magnetoresistance is positive, with a linear in B term, which has a coupling constant originating from chiral anomaly.

Recent experiment suggest that, but.. remains controversial to date.



Nature 575, 315 (2019)

Thank you!