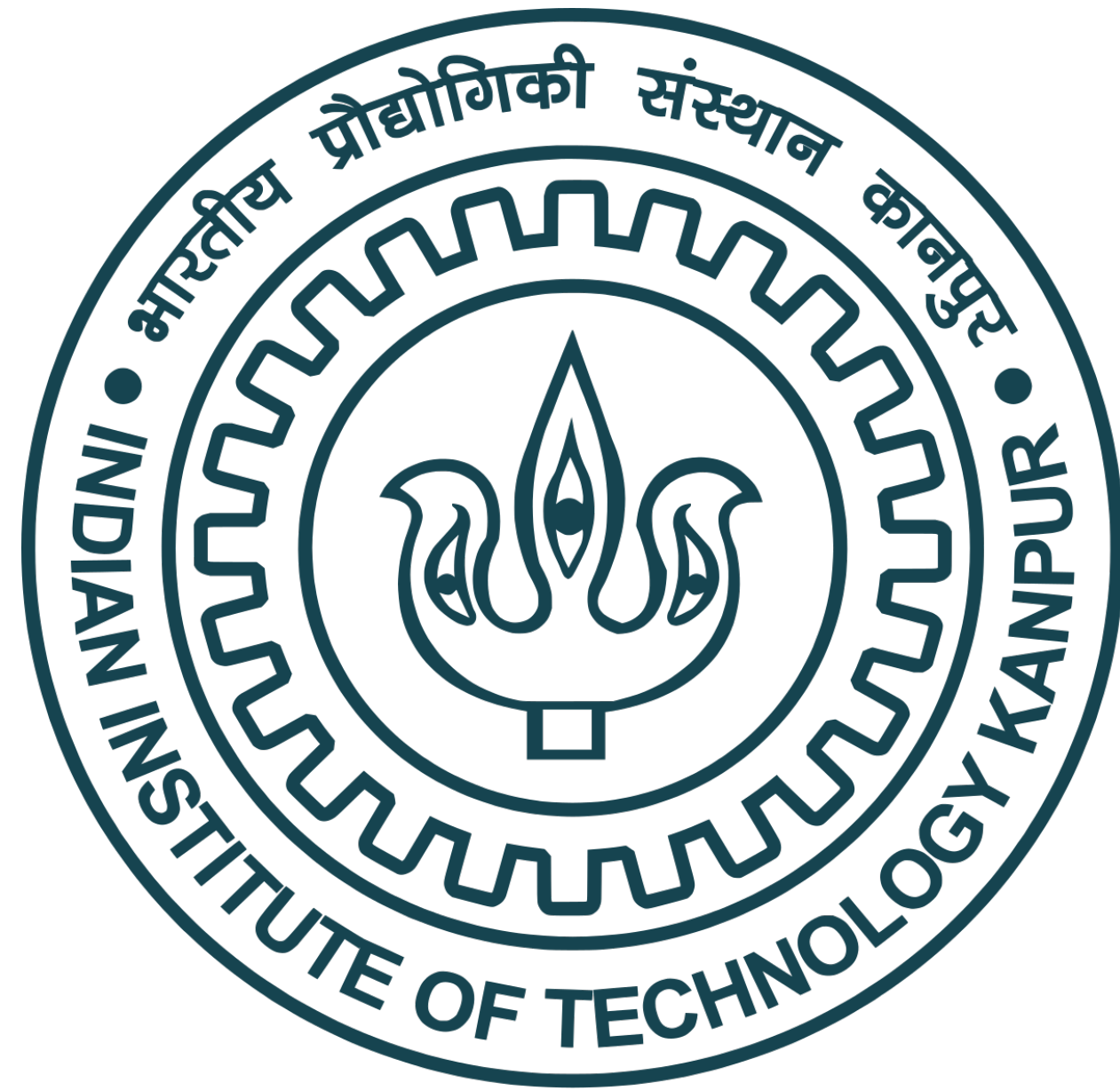
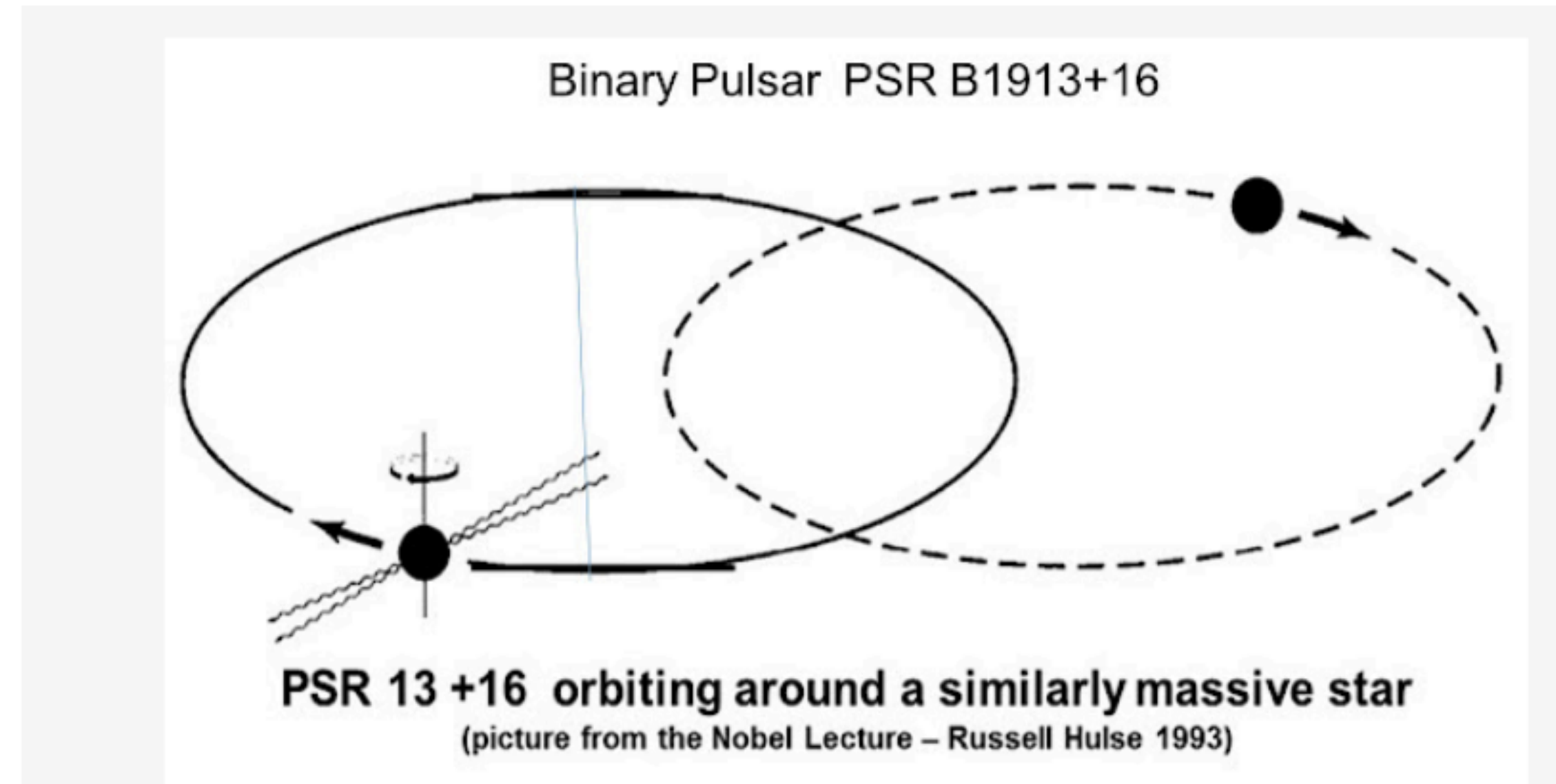


Looking for gravitons in Gravitational Waves



Subhendra Mohanty
IIT Kanpur

Hulse Taylor binary

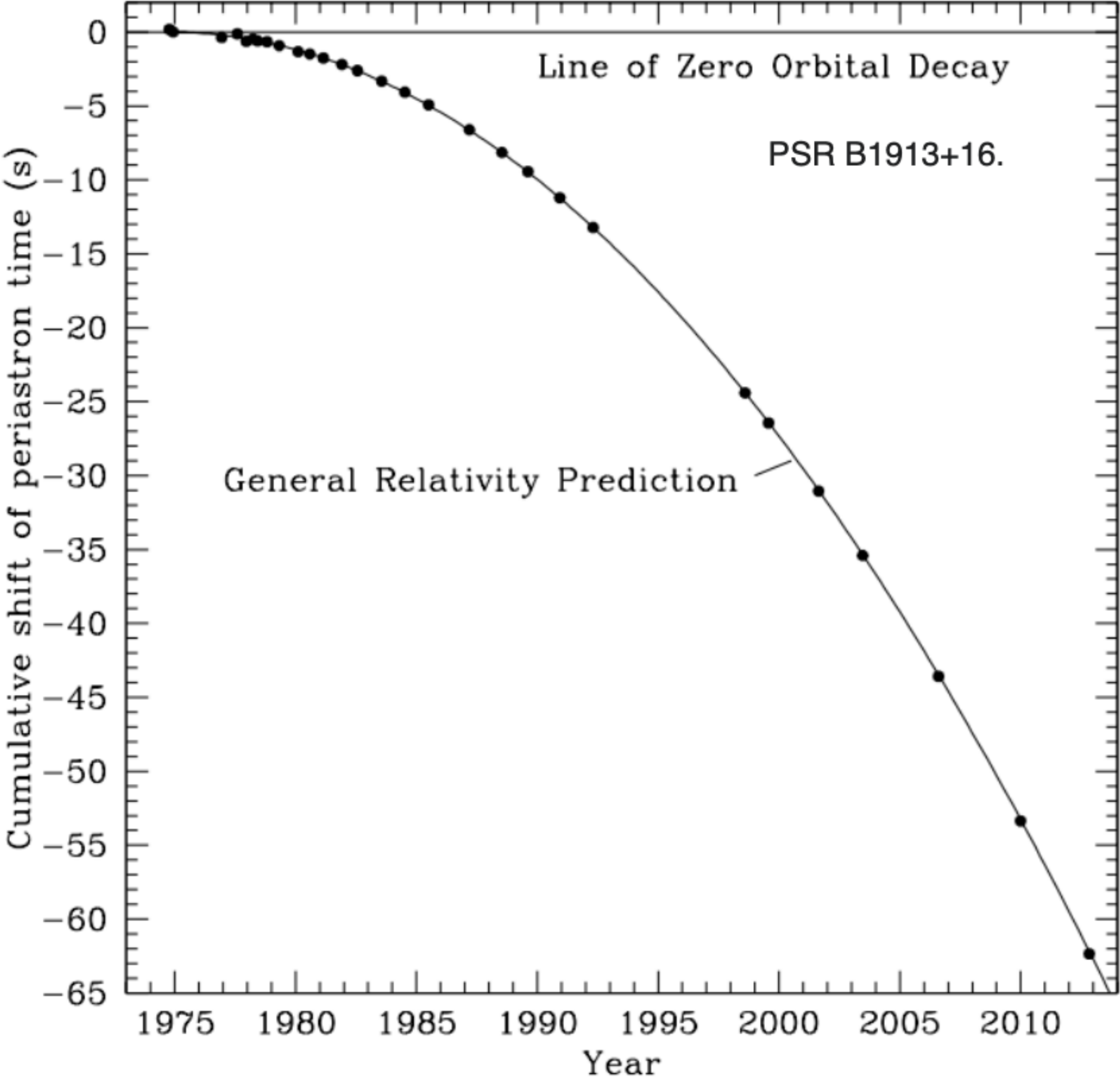


Energy loss by gravitational wave radiation in compact binaries

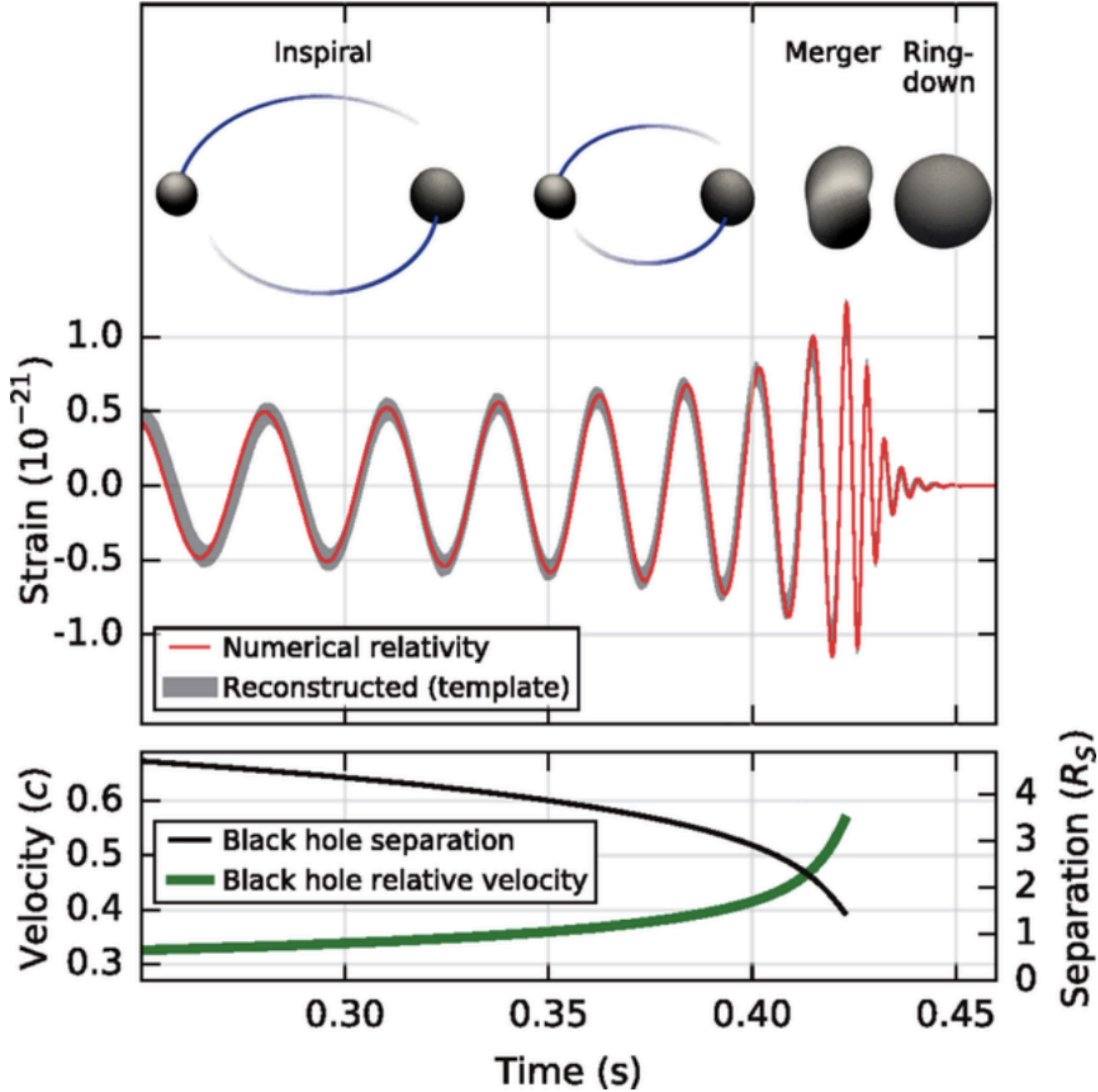
$$\left(\frac{dE}{dT}\right)^{\text{GW}} = \frac{32G}{5} \Omega^6 \left(\frac{m_1 m_2}{m_1 + m_2}\right)^2 a^4 (1 - e^2)^{-7/2} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4\right)$$

P.C Peters and J. Mathews (1963)

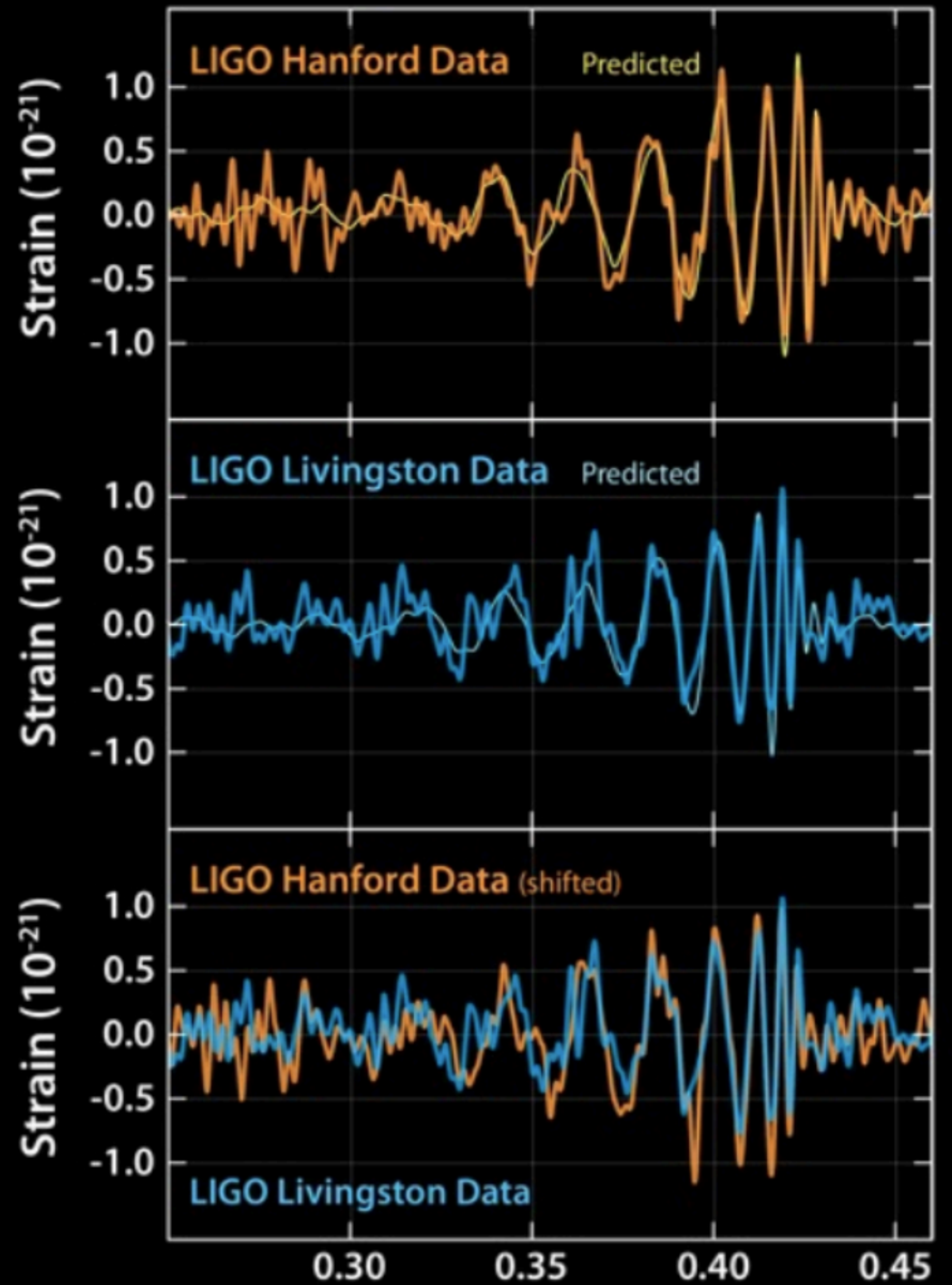
Time period loss of binary pulsars by gravitational wave radiation

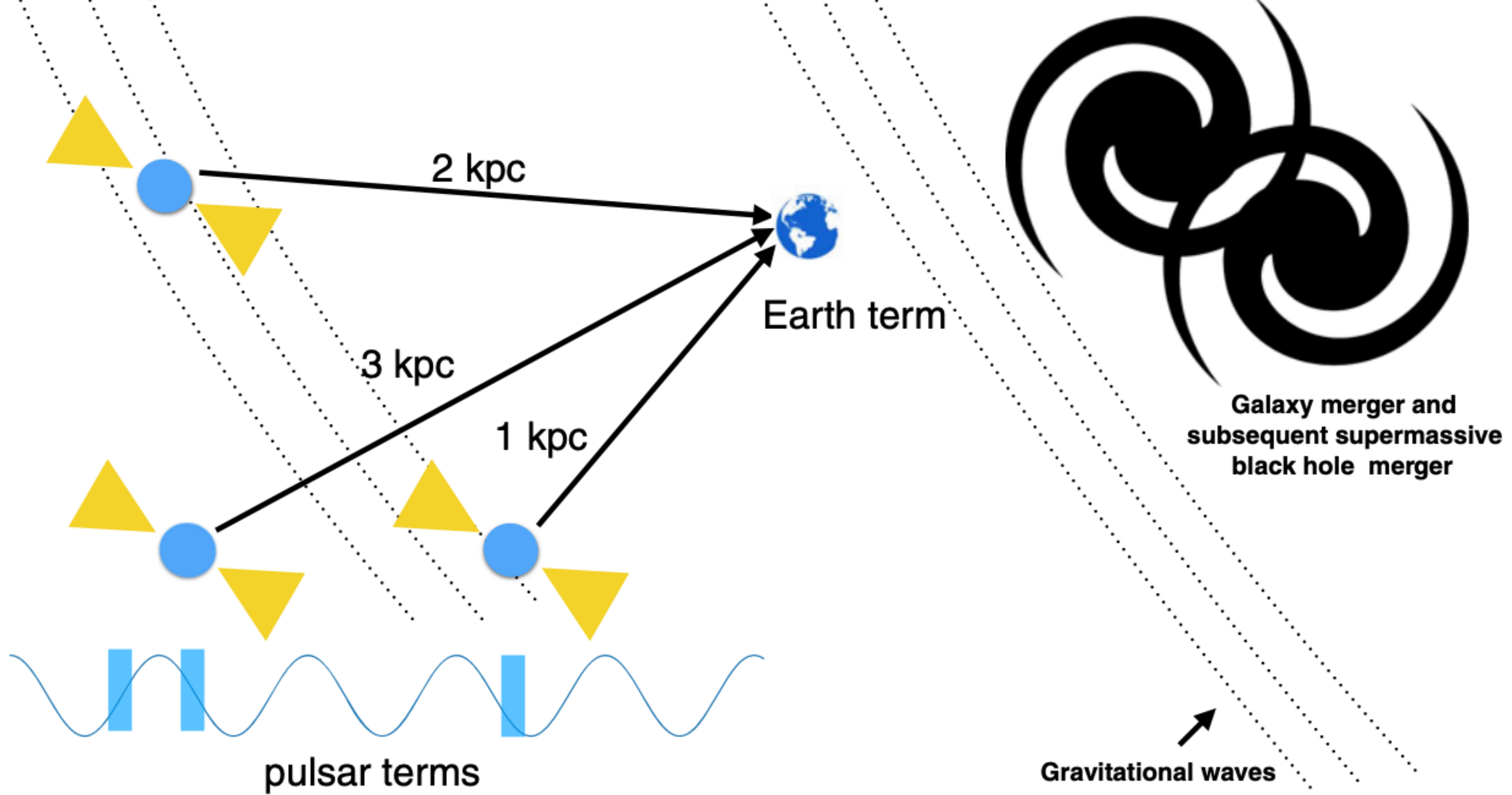


Black-hole mergers



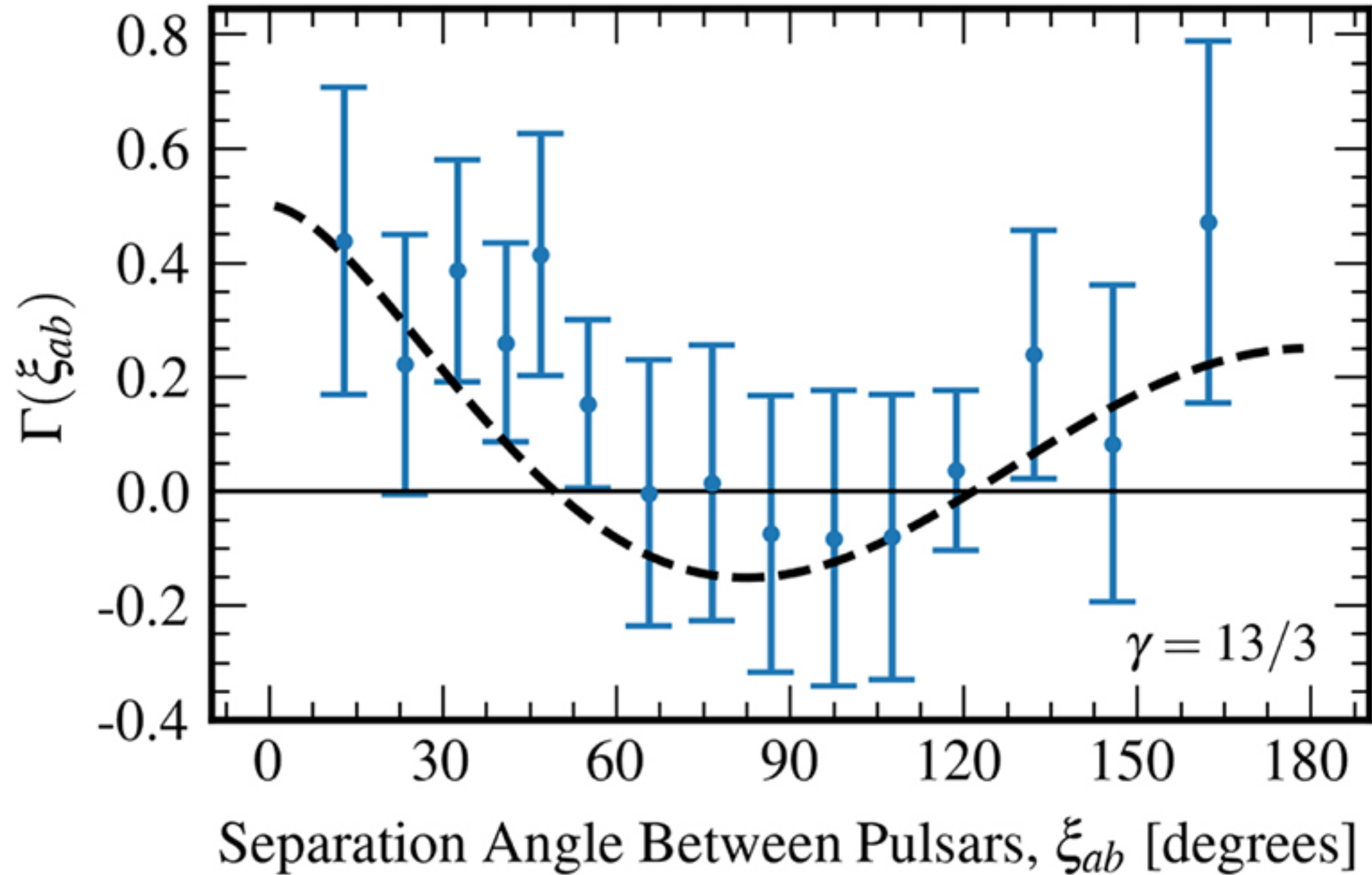
LIGO Event

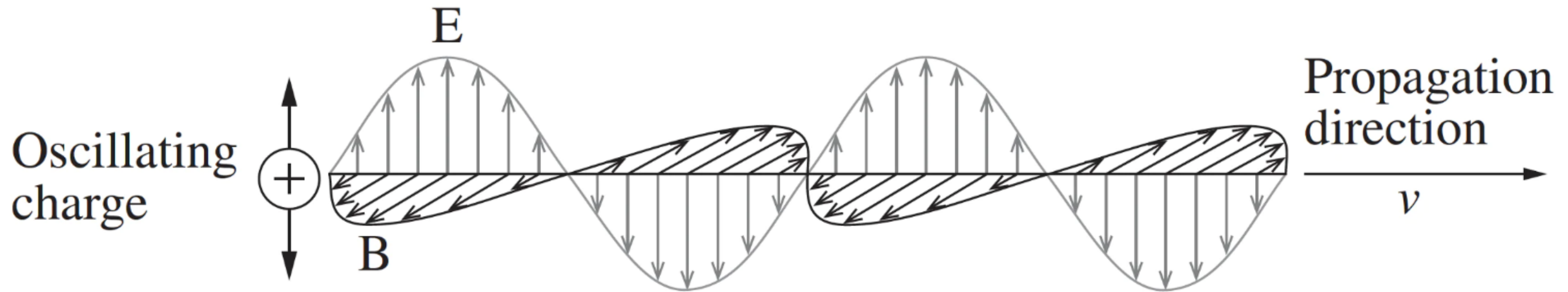




s. The astrophysics of stochastic gravitational waves, Burke-Spolaor et al.

NANOGrav observations- Hellings Downs curve

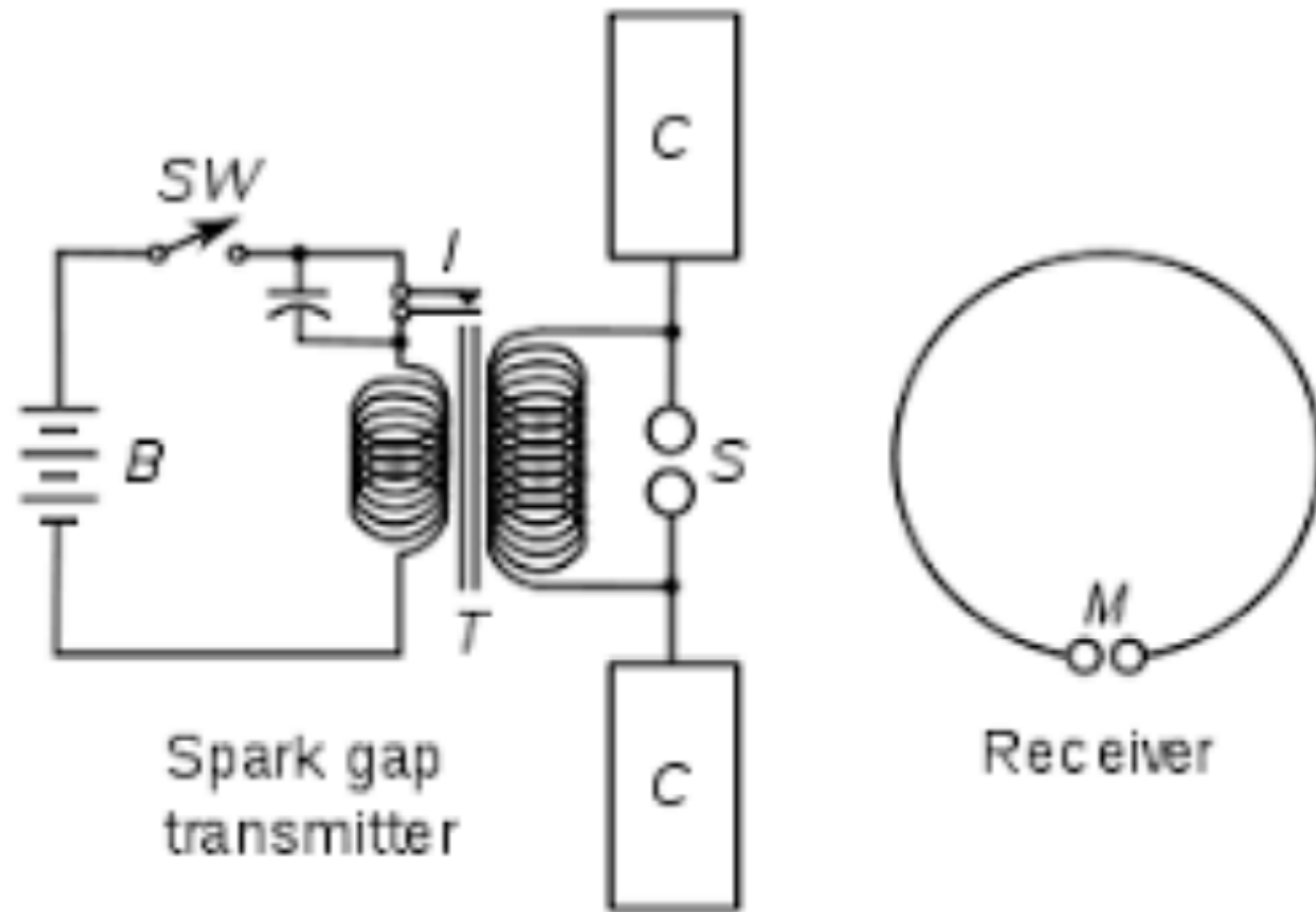




$$v = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

"We can scarcely avoid the conclusion that light consists in the transverse undulations of the same medium which is the cause of electric and magnetic phenomena."

Maxwell 1865



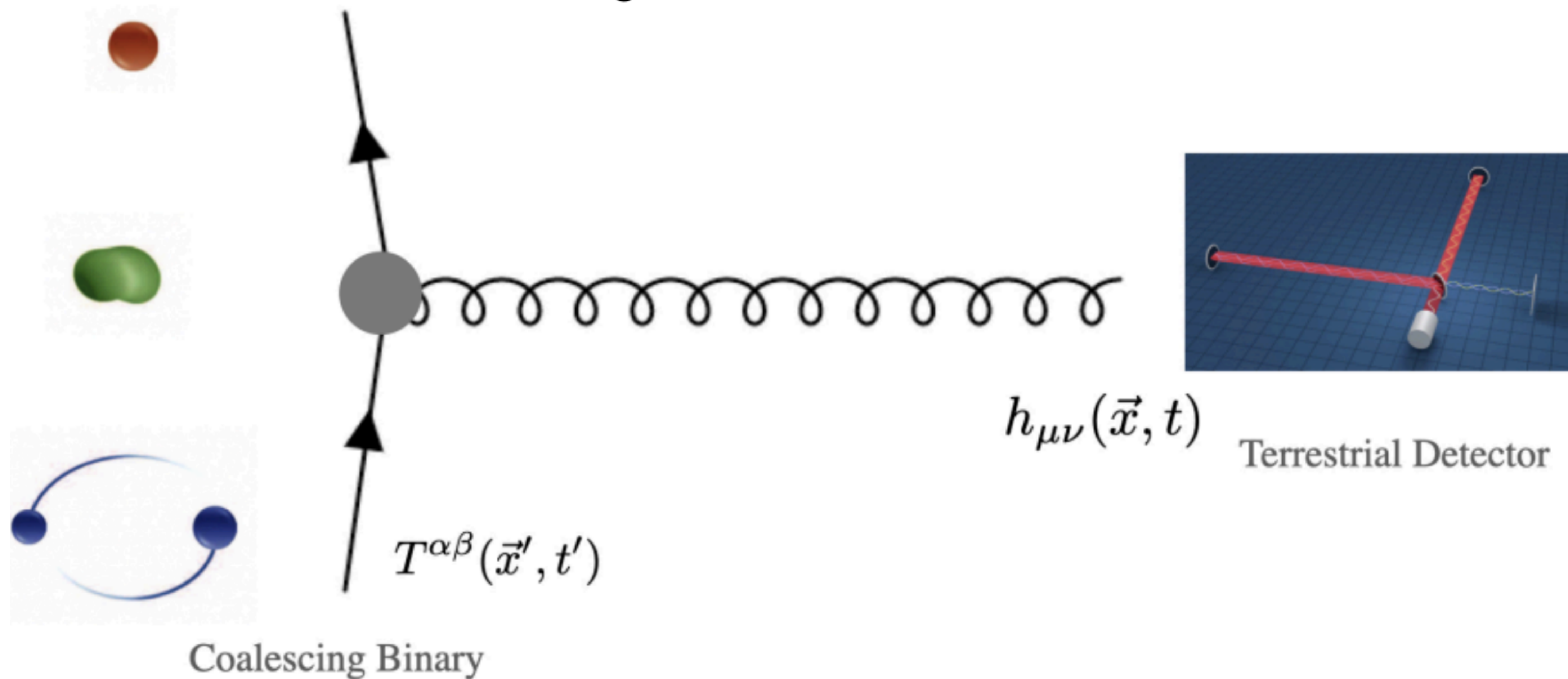
"It's of no use whatsoever... this is just an experiment that proves Maestro Maxwell was right—we just have these mysterious electromagnetic waves that we cannot see with the naked eye. But they are there."

Heinrich Hertz, 1887

Einstein's quadrupole formula for energy radiated in gravitational wave (1918)

$$\left(\frac{dE}{dT}\right)^{\text{GW}} = \frac{G}{c^5} \left\{ \frac{1}{5} \frac{d^3 Q_{ab}}{dT^3} \frac{d^3 Q_{ab}}{dT^3} + \mathcal{O}\left(\frac{1}{c^2}\right) \right\}$$

Diagrammatic calculation of GW emission



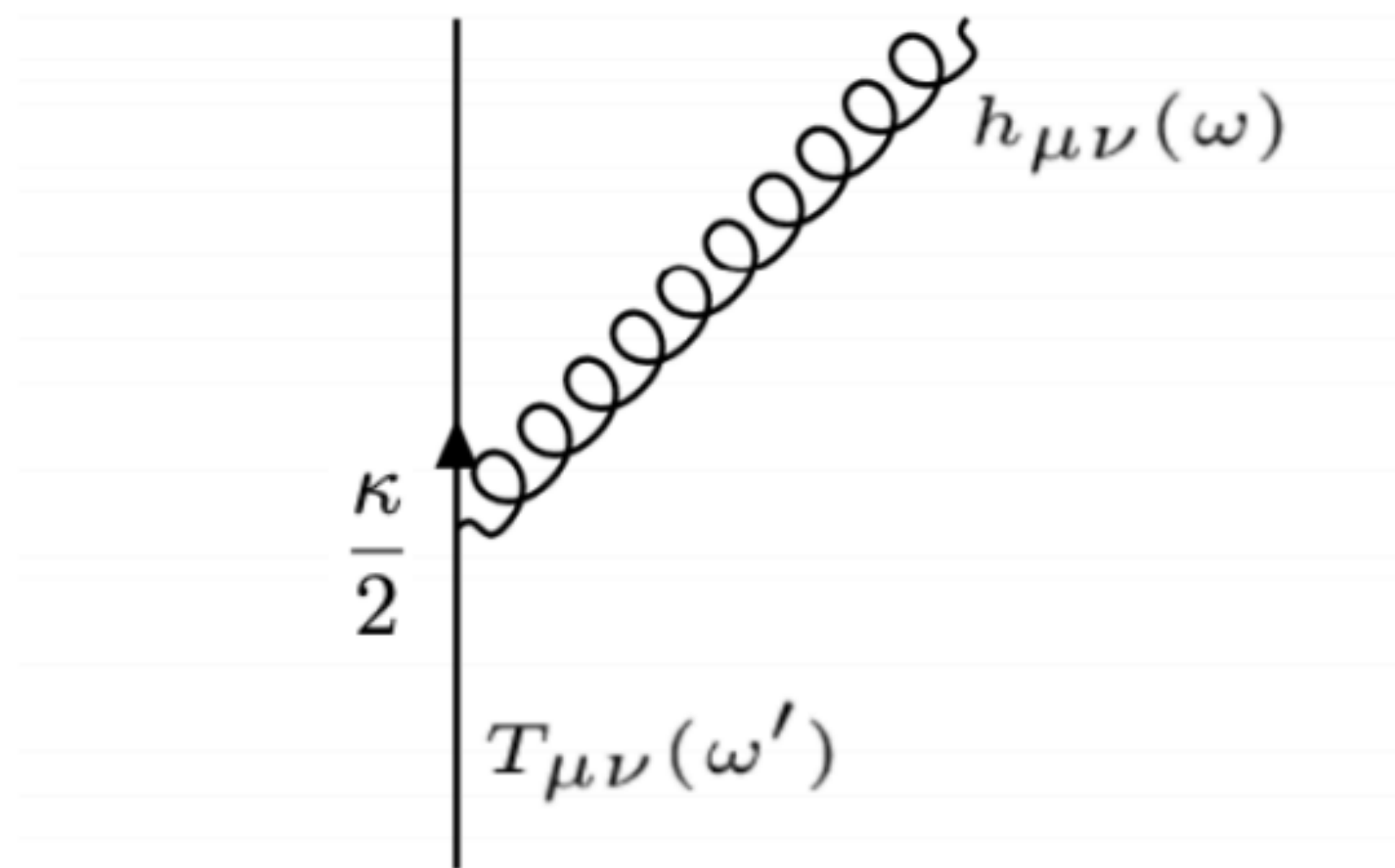
Gravitational radiation from elliptical orbits

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

$$\kappa = \sqrt{32\pi G}$$

Interaction vertex:

$$\frac{\kappa}{2} T_{\mu\nu}(k') \epsilon_{\lambda}^{\mu\nu}(k)$$



$$T_{\mu\nu}(x') = \mu \delta^3(\mathbf{x}' - \mathbf{x}(t)) U_{\mu} U_{\nu}$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$U_{\mu} = (1, \dot{x}, \dot{y}, 0)$$

Keplerian orbit $x = a(\cos \xi - e), \quad y = a\sqrt{(1 - e^2)} \sin \xi, \quad \Omega t = \xi - e \sin \xi, \quad \Omega = \left[G \frac{(m_1 + m_2)}{a^3} \right]^{\frac{1}{2}}$

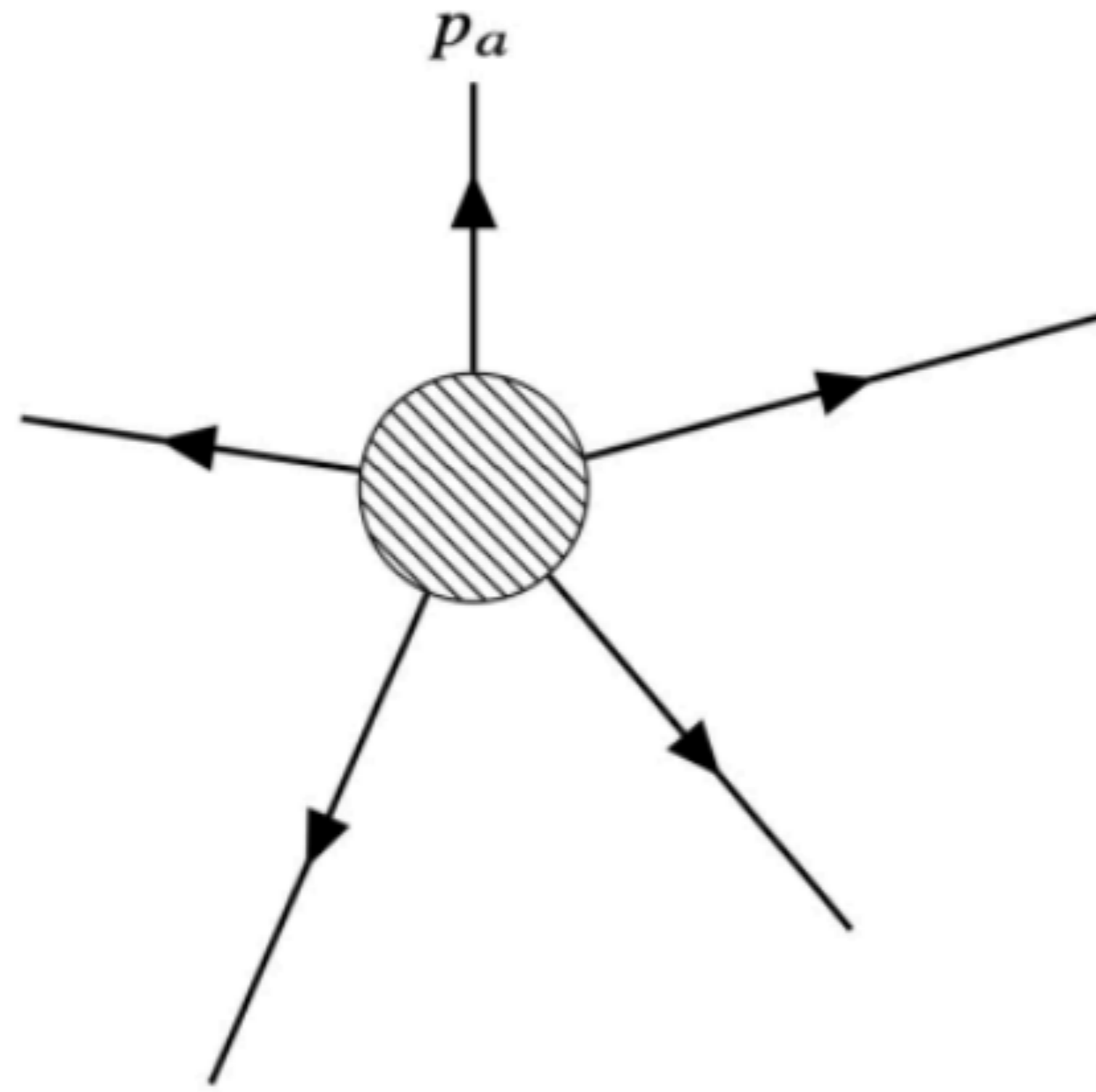
Energy loss from elliptical binaries in gravitational wave radiation

$$\begin{aligned}\frac{dE}{dt} &= \frac{\kappa^2}{8(2\pi)^2} \int \frac{8\pi}{5} \left[T_{ij}(\omega') T_{ji}^*(\omega') - \frac{1}{3} |T^i_i(\omega')|^2 \right] \delta(\omega - \omega') \omega^2 d\omega \\ &= \frac{32G}{5} \Omega^6 \mu^2 a^4 \sum_{n=1}^{\infty} n^6 f(n, e) \\ &= \frac{32G}{5} \Omega^6 \left(\frac{m_1 m_2}{m_1 + m_2} \right)^2 a^4 \frac{1}{(1 - e^2)^{7/2}} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right)\end{aligned}$$

SM & P.K.Panda Phys Rev D (1996)

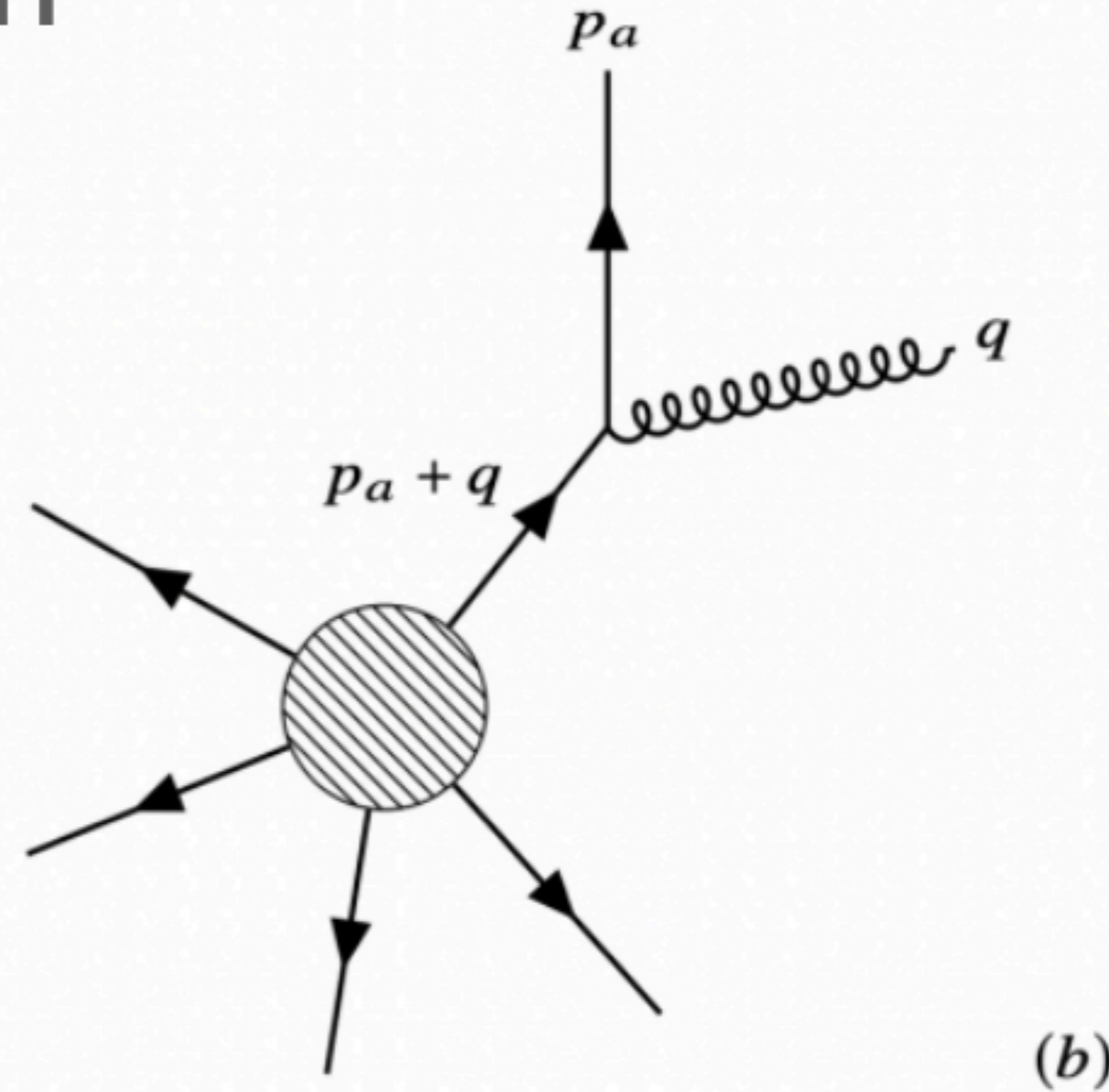
Matches Peters-Mathews formula (1963) from classical general relativity

Soft graviton theorem



$\mathcal{A}_n(p_a)$

(a)



\mathcal{A}_{n+1}

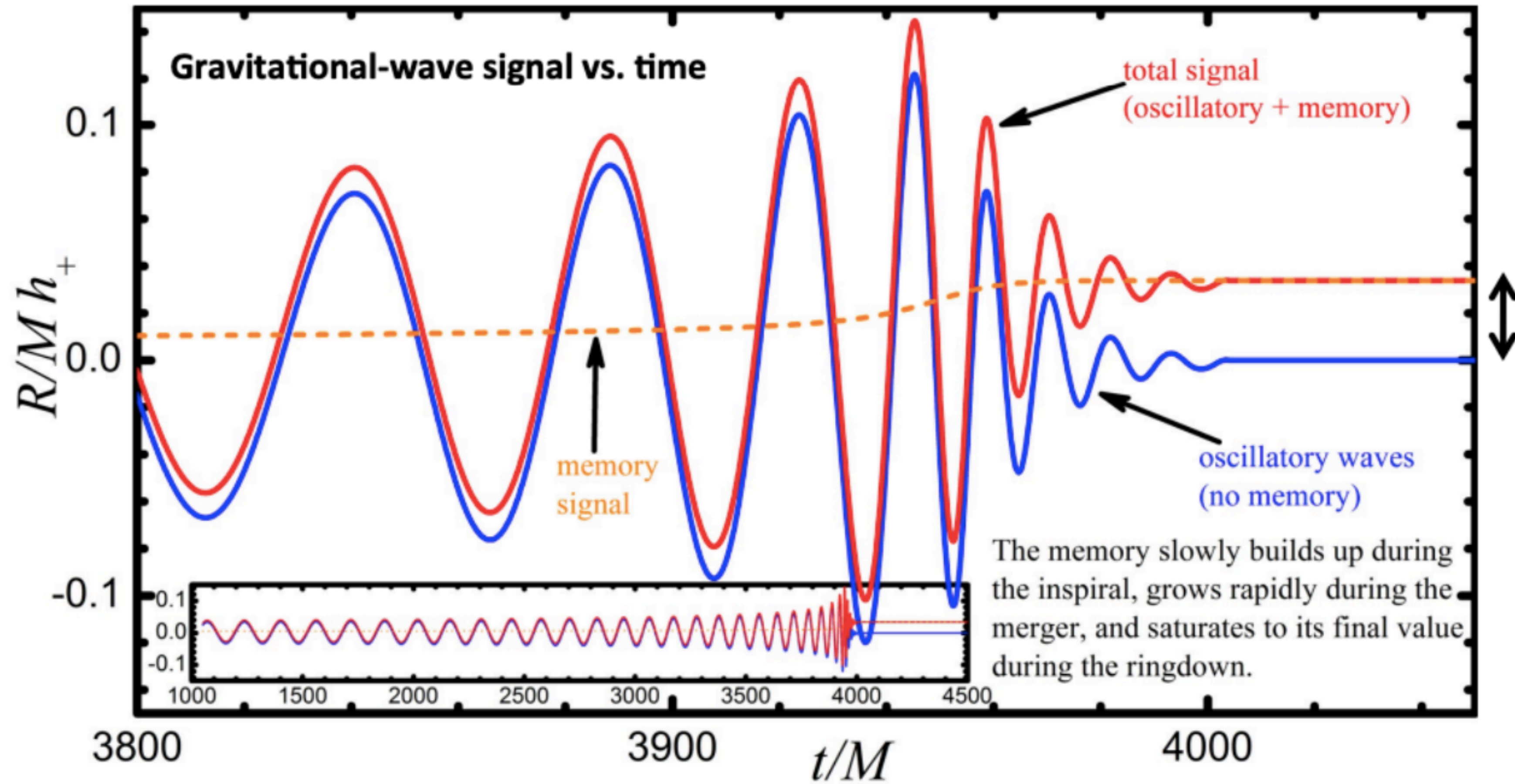
(b)

$$\mathcal{A}_{n+1}(p_a, q) = \frac{\kappa}{2} \epsilon^{*\mu\nu}(q) \sum_{a=1}^n \frac{p_{a\mu} p_{a\nu}}{p_a \cdot q} \mathcal{A}_n(p_a)$$

$$\int \frac{dk_0}{2\pi i k_0} e^{ik_0(t-r)} = \Theta(t-r)$$

The step function gives the DC signal in gravitational waves and this is called the memory effect.

Gravitational memory



Marc Favata (ICTS talk)

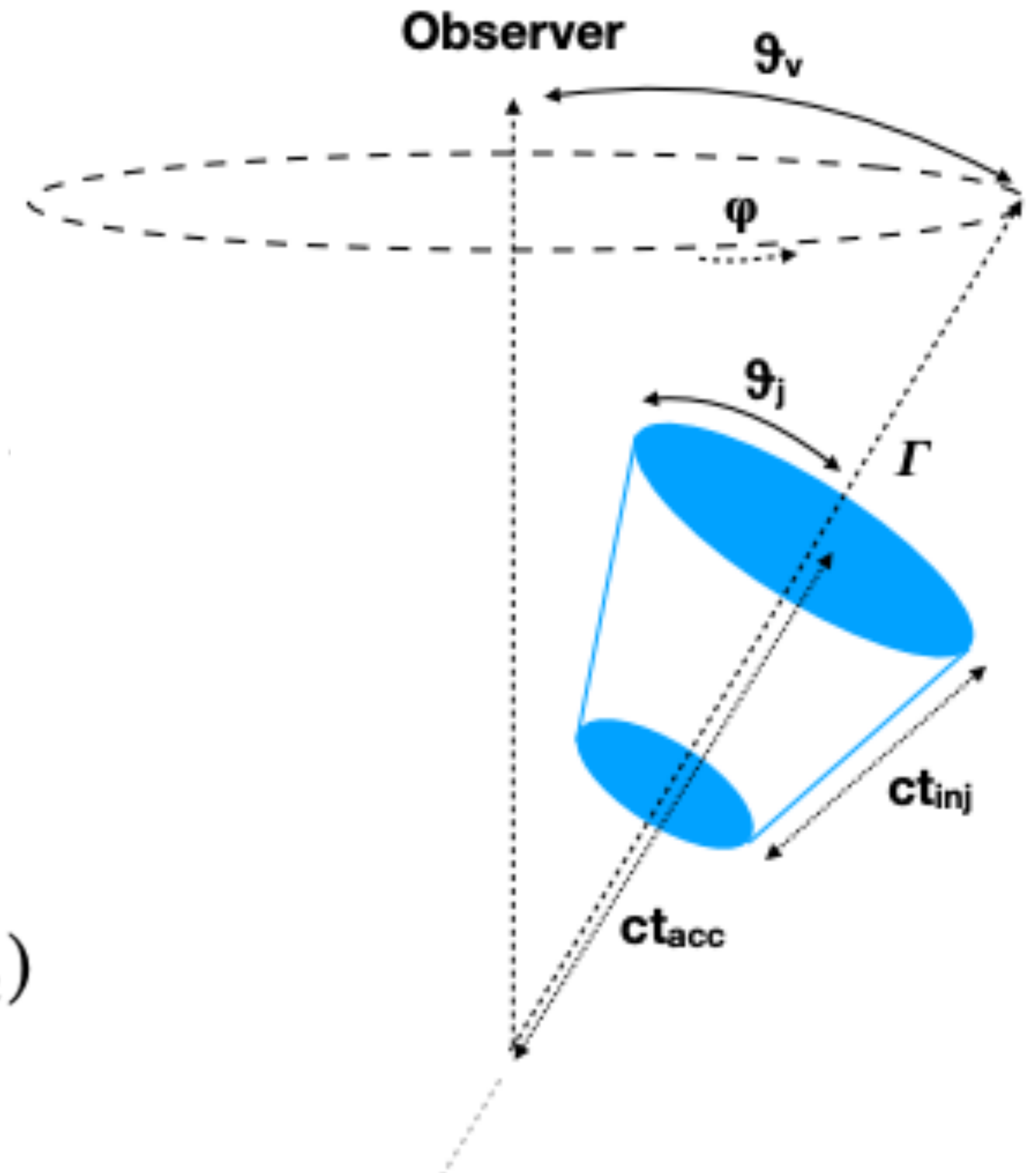
$$h(t, \theta_v) = h(\theta_v) \mathcal{H}(t)$$

$$\tilde{h}(f, \theta_v) = h(\theta_v) / f$$

$$h^{TT}(\theta_v) = h_+ + ih_x = \frac{2\mathcal{E}\beta^2}{r} \frac{\sin^2 \theta_v}{1 - \beta \cos \theta_v}$$

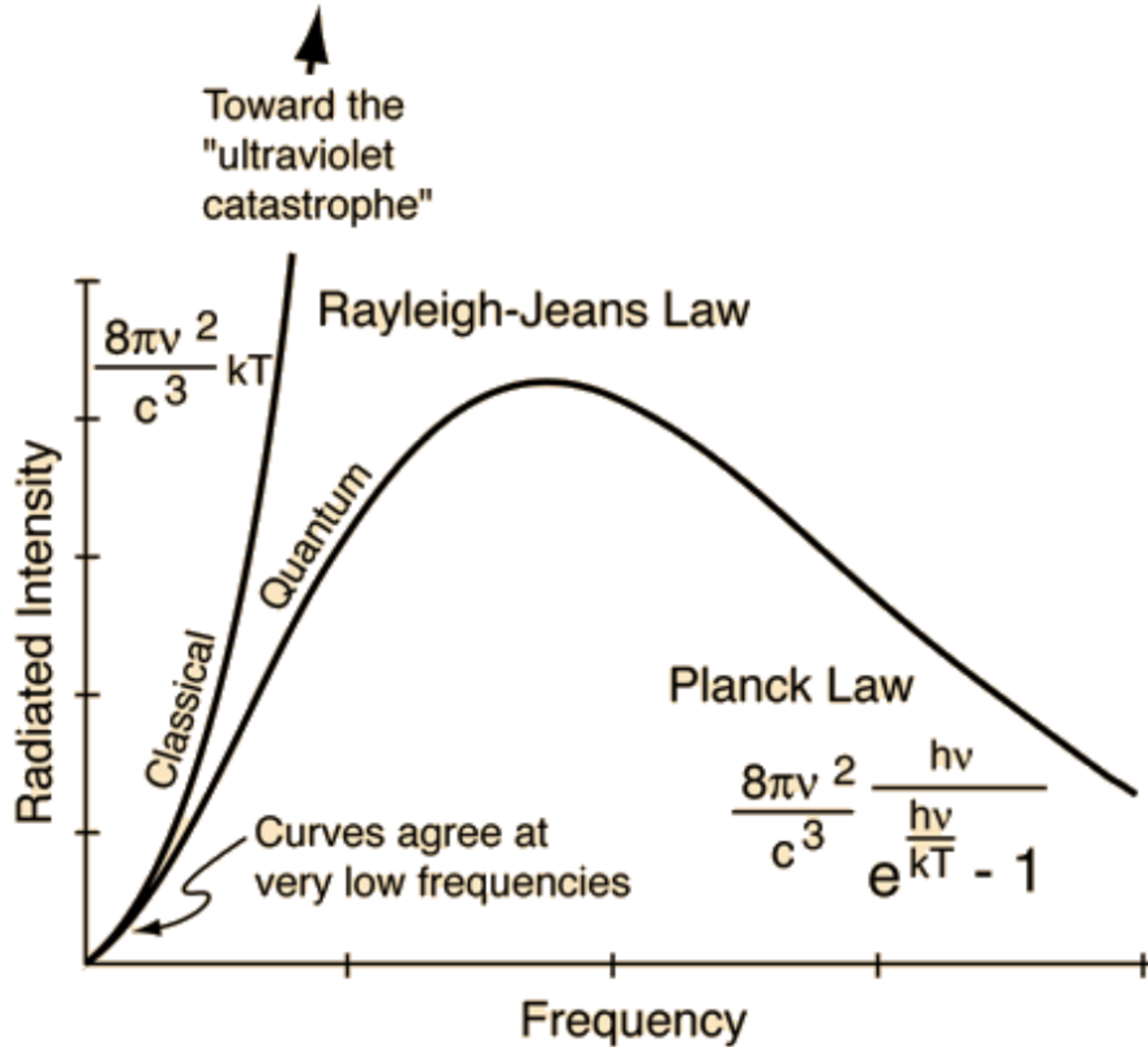
Can be derived from soft-theorem

$$\mathcal{A}_{n+1}(p_a, q) = \frac{\kappa}{2} \epsilon^{*\mu\nu}(q) \sum_{a=1}^n \frac{p_{a\mu} p_{a\nu}}{p_a \cdot q} \mathcal{A}_n(p_a)$$



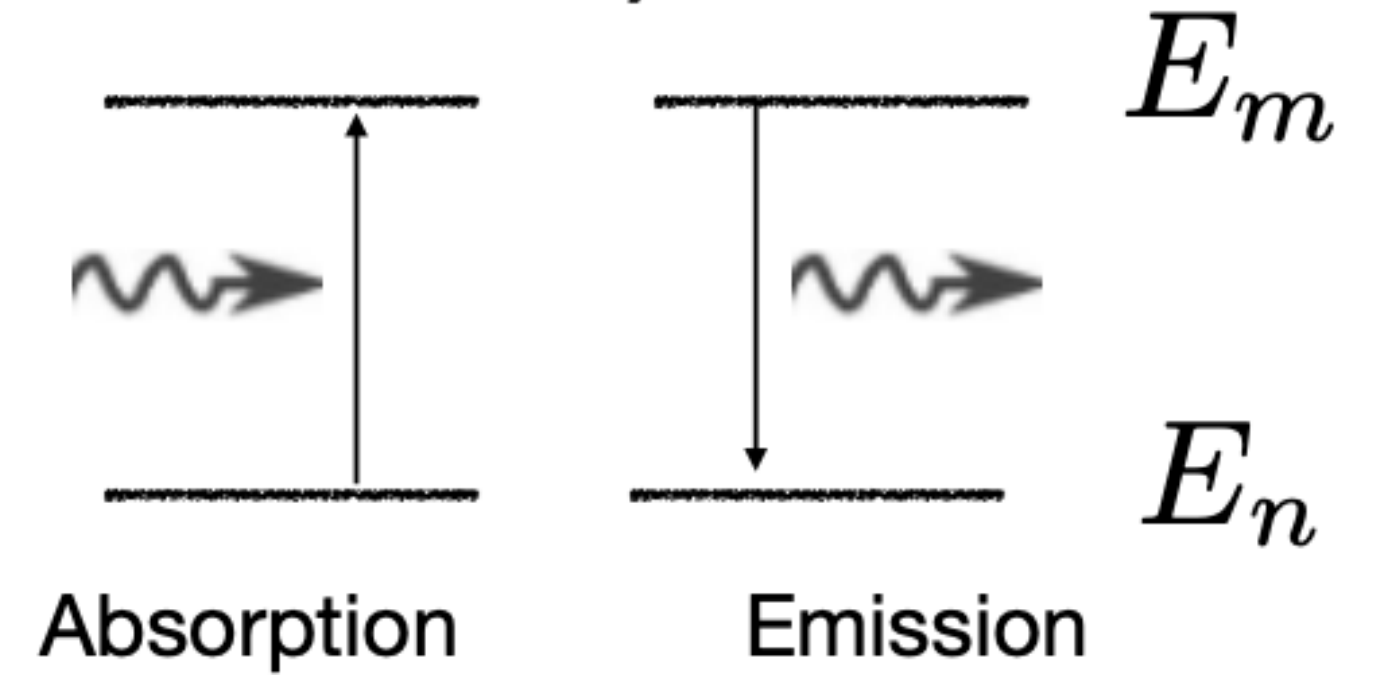
Can we look for the signature of gravitons in gravitational wave observations?

Planck's Radiation Law, 1900



Spontaneous and stimulated emissions (Einstein 1916)

$$\frac{dN_n}{dt} = \underbrace{A_{m \rightarrow n} N_m}_{\text{Emission}} + \underbrace{B_{m \rightarrow n} N_m u(\omega)}_{\text{Absorption}} - \underbrace{B_{n \rightarrow m} N_n u(\omega)}_{\text{Absorption}}$$



Thermal Equilibrium

$$\frac{N_n}{N_m} = e^{-(E_n - E_m)/T} = e^{-\omega/T} \quad \text{and} \quad \frac{dN_n}{dt} = 0$$

$$A_{m \rightarrow n} = B_{m \rightarrow n} \left(e^{\omega/T} - 1 \right) u(\omega)$$

$$A_{m \rightarrow n} = 8\pi\omega^3 B_{m \rightarrow n} \iff u(\omega) = \frac{8\pi\omega^3}{e^{\omega/T} - 1}$$

Net emission rate

$$\begin{aligned}\Gamma_{m \rightarrow n}^{emi} &= A_{m \rightarrow n} + B_{m \rightarrow n} u(\omega) \\ &= 8\pi\omega^3 B_{m \rightarrow n} (1 + n(\omega))\end{aligned}$$

Net absorption rate

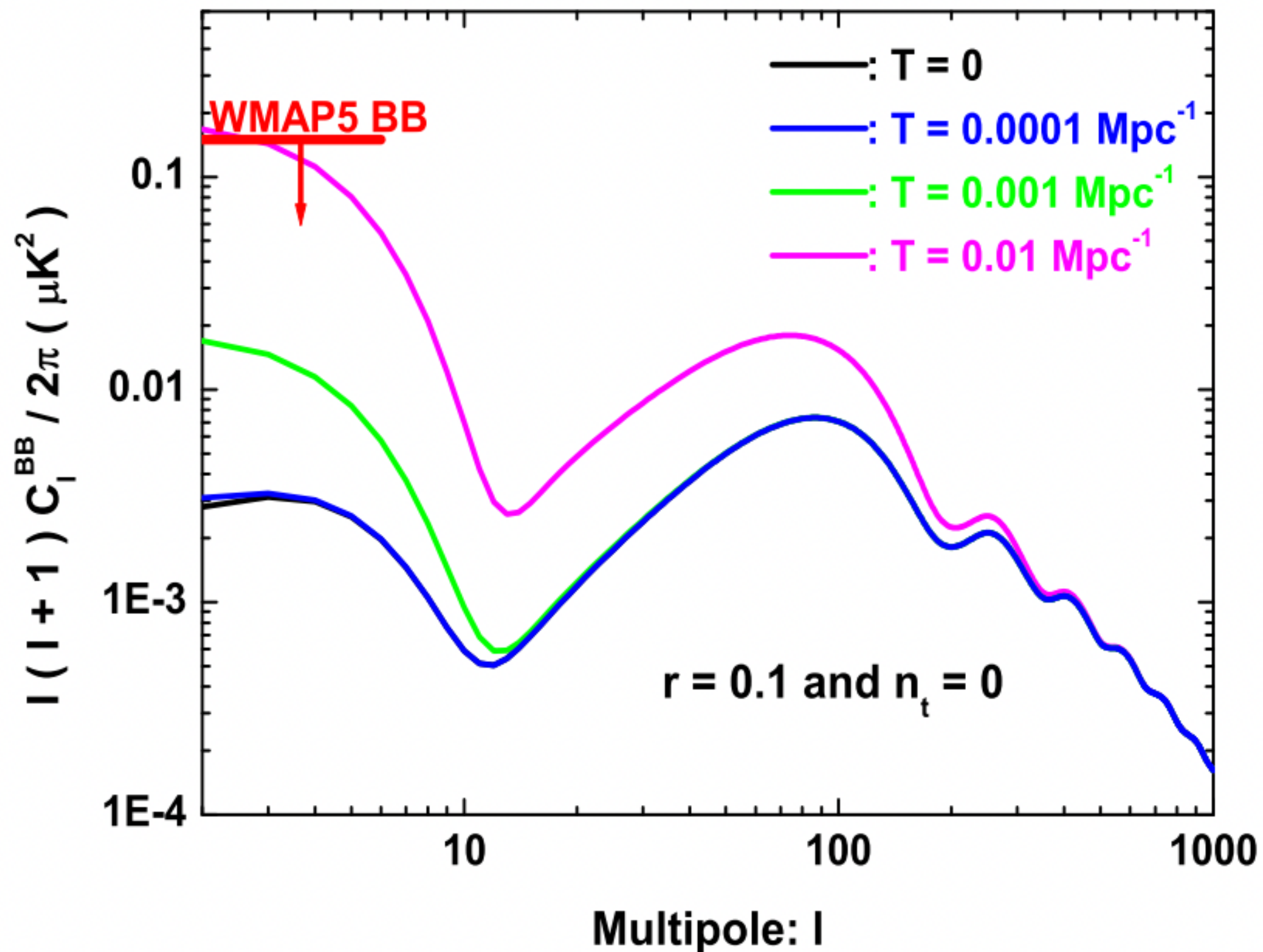
$$\begin{aligned}\Gamma_{n \rightarrow m}^{abs} &= B_{m \rightarrow n} u(\omega) \\ &= 8\pi\omega^3 B_{m \rightarrow n} n(\omega).\end{aligned}$$

Bose-Einstein distribution

$$n(\omega) = \frac{1}{e^{\omega/T} - 1}$$

Inflation power spectrum in a thermal background

Bhattacharya, Mohanty, Nautiyal , *Phys.Rev.Lett.* 97 (2006) 251301



Zhao, Baskaran, Coles 2009

During inflation zero-point fluctuations become classical

Zero-point fluctuations in thermal background of gravitons

$$P_t(k) = A_t(k_0) \left(\frac{k}{k_0} \right)^{n_t} \left(1 + \frac{2}{e^{k/T} - 1} \right)$$

Bose-Einstein distribution

$$n(k) = \frac{1}{e^{k/T} - 1}$$

Creation-annihilation commutation

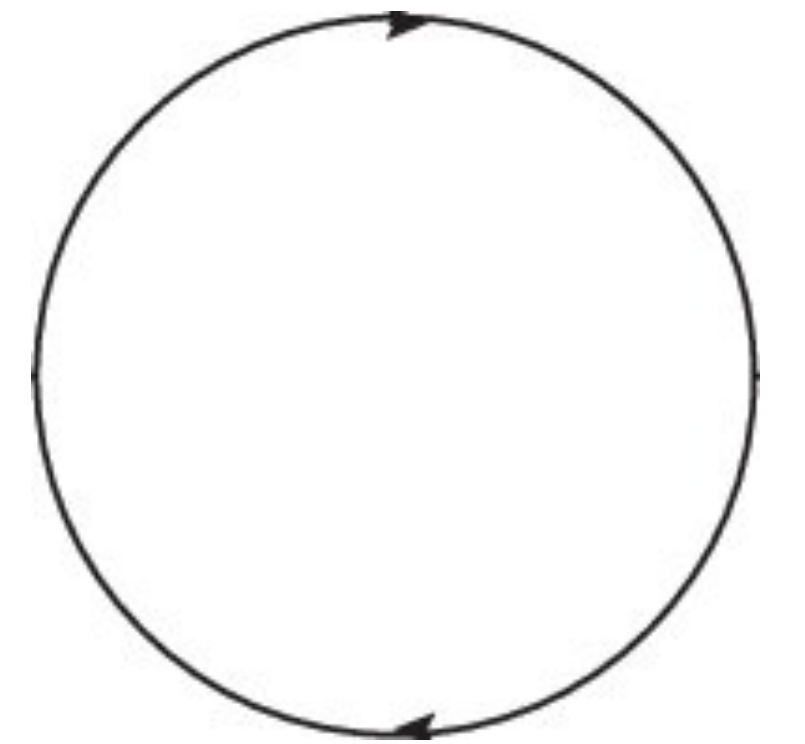
$$[a_k, a_{k'}^\dagger] = \delta_{kk'}$$

Zero-point fluctuation in vacuum

$$\langle 0 | a_k a_k^\dagger + a_k^\dagger a_k | 0 \rangle = \langle 0 | 1 + 2a_k^\dagger a_k | 0 \rangle = 1$$

Zero-point fluctuation in a thermal bath

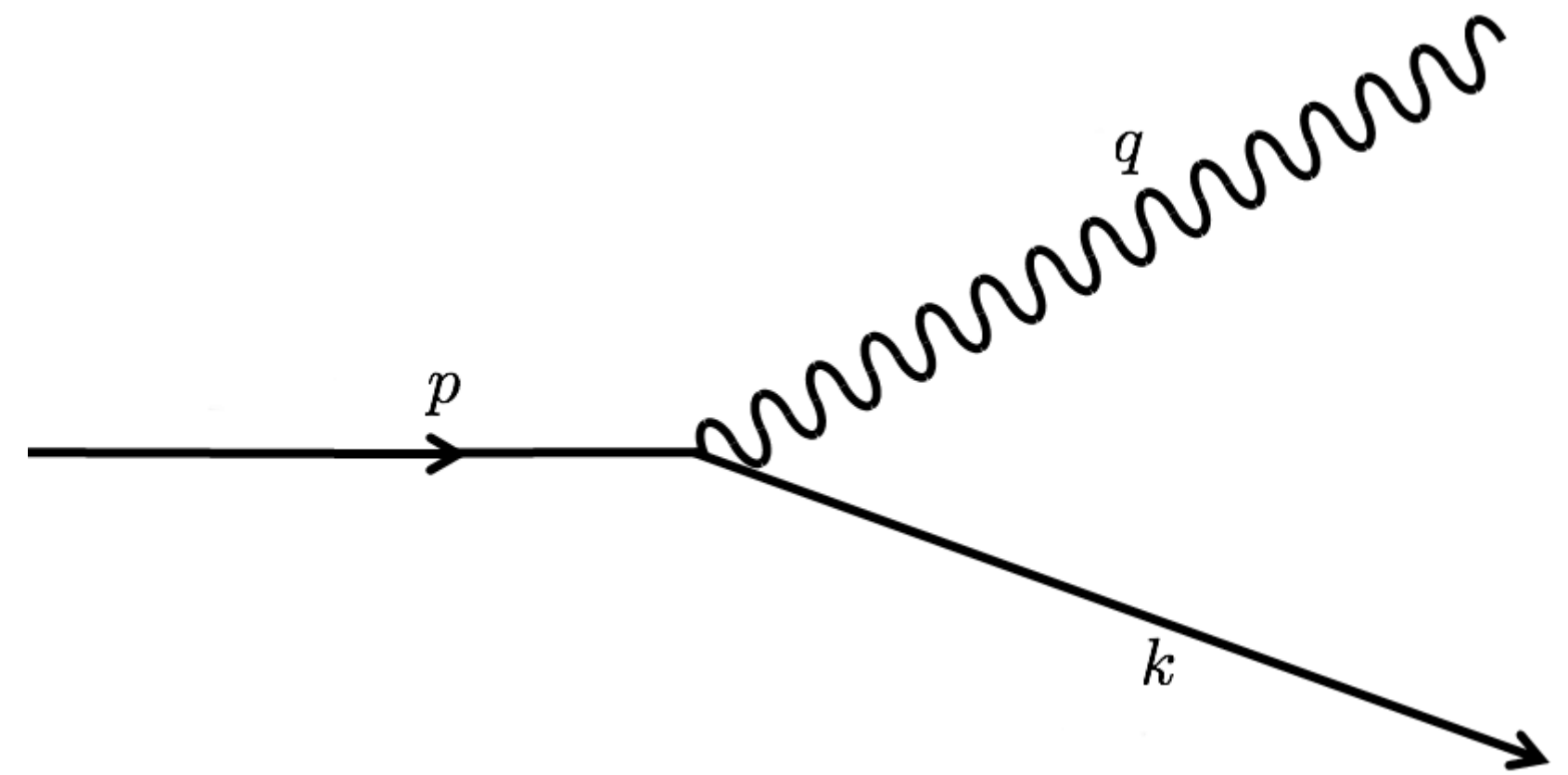
$$\langle n | a_k a_k^\dagger + a_k^\dagger a_k | n \rangle = \langle n | 1 + 2a_k^\dagger a_k | n \rangle = 1 + 2n$$



Is there a stimulated emission of gravitational waves in a thermal graviton thermal background?

Graviton emission in vacuum

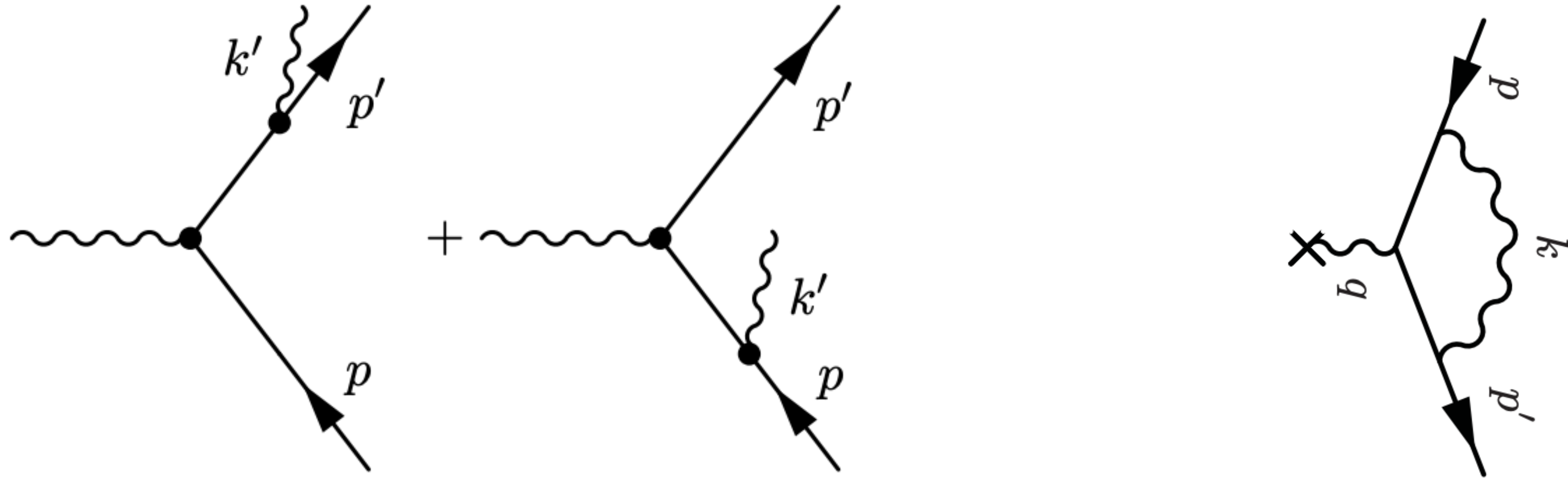
$$\langle 1 | a_k a_k^\dagger | 0 \rangle = 1$$



Graviton radiation in a thermal bath of gravitons- stimulated emission

$$\langle n | a_k a_k^\dagger | n \rangle = \langle n | 1 + a_k^\dagger a_k | n \rangle = 1 + n$$

Bloch-Nordsieck counterpart for soft gravitons?



F. Bloch and A. Nordsieck, "Note on the Radiation Field of the electron," *Phys. Rev.*, vol. 52, pp. 54–59, 1937.

Graviton loop corrections may be needed to correct infrared divergences in gravitational radiation calculations

Thank You