Heating the Quantum Soup With Scalar & Fermion Noodles

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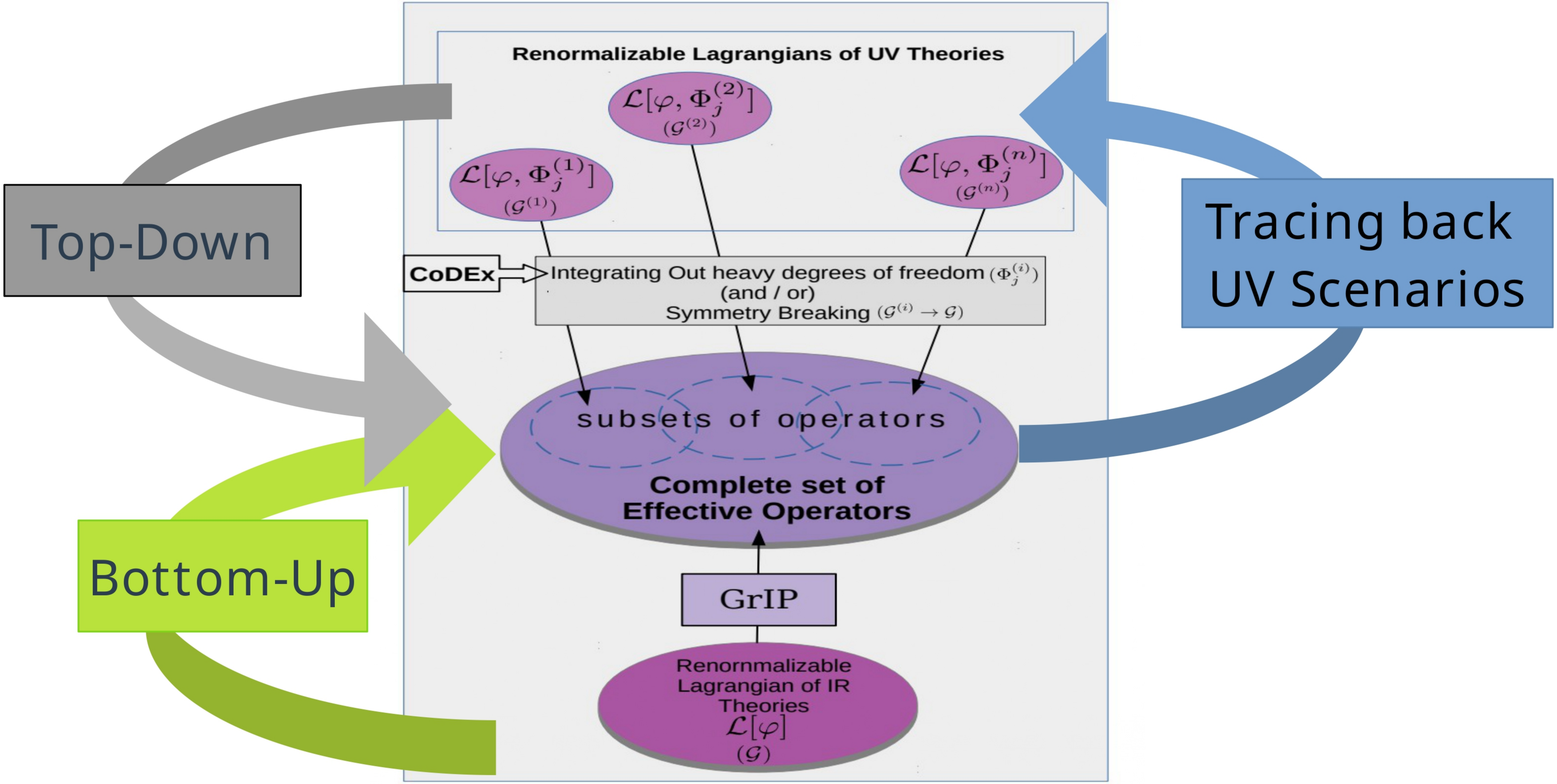
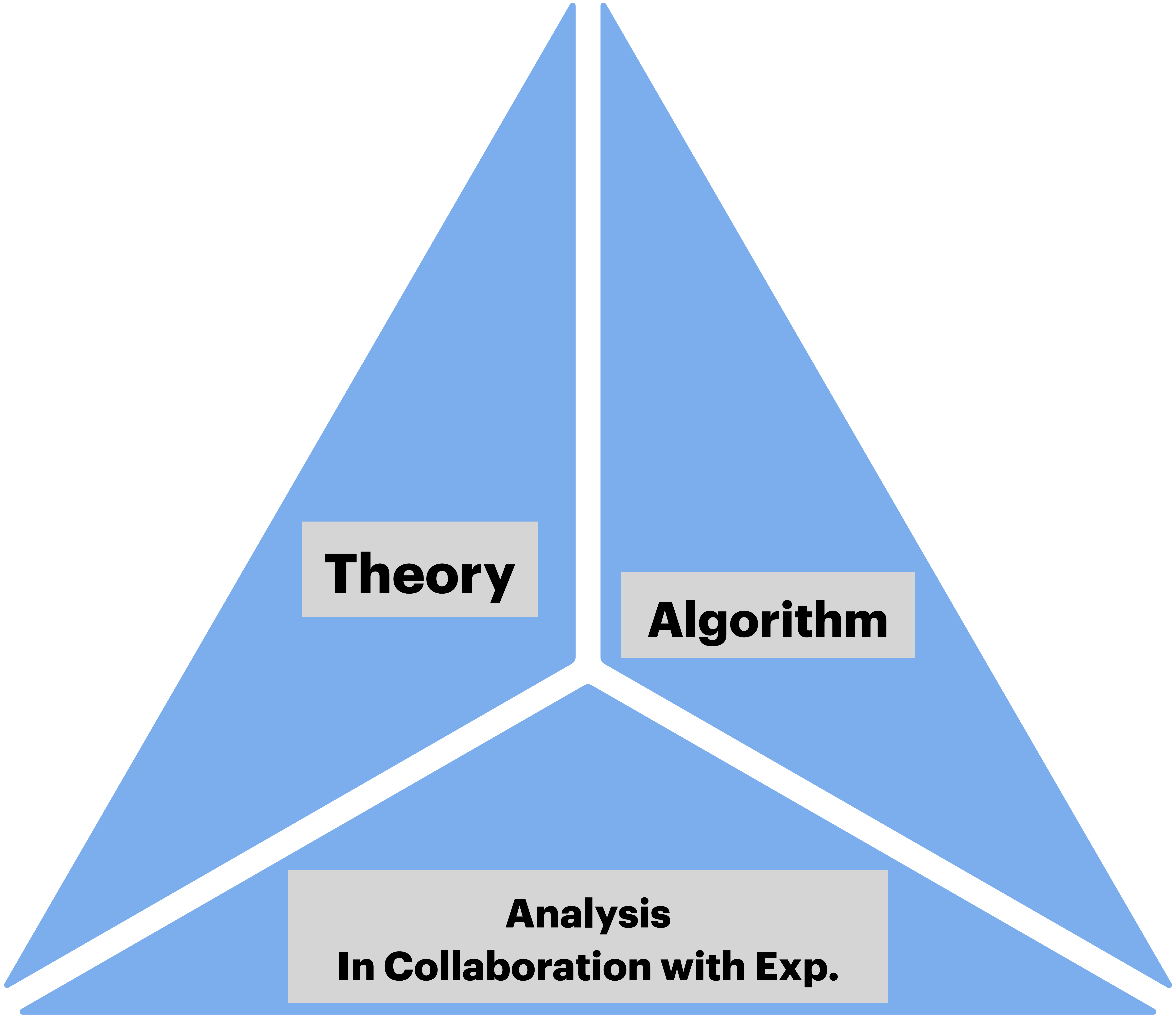
Mohamadreza Ahmadi Monfared – 2015 – Quantum Soup – Paintings and drawings – Mohsen Gallery in Tehran, Iran

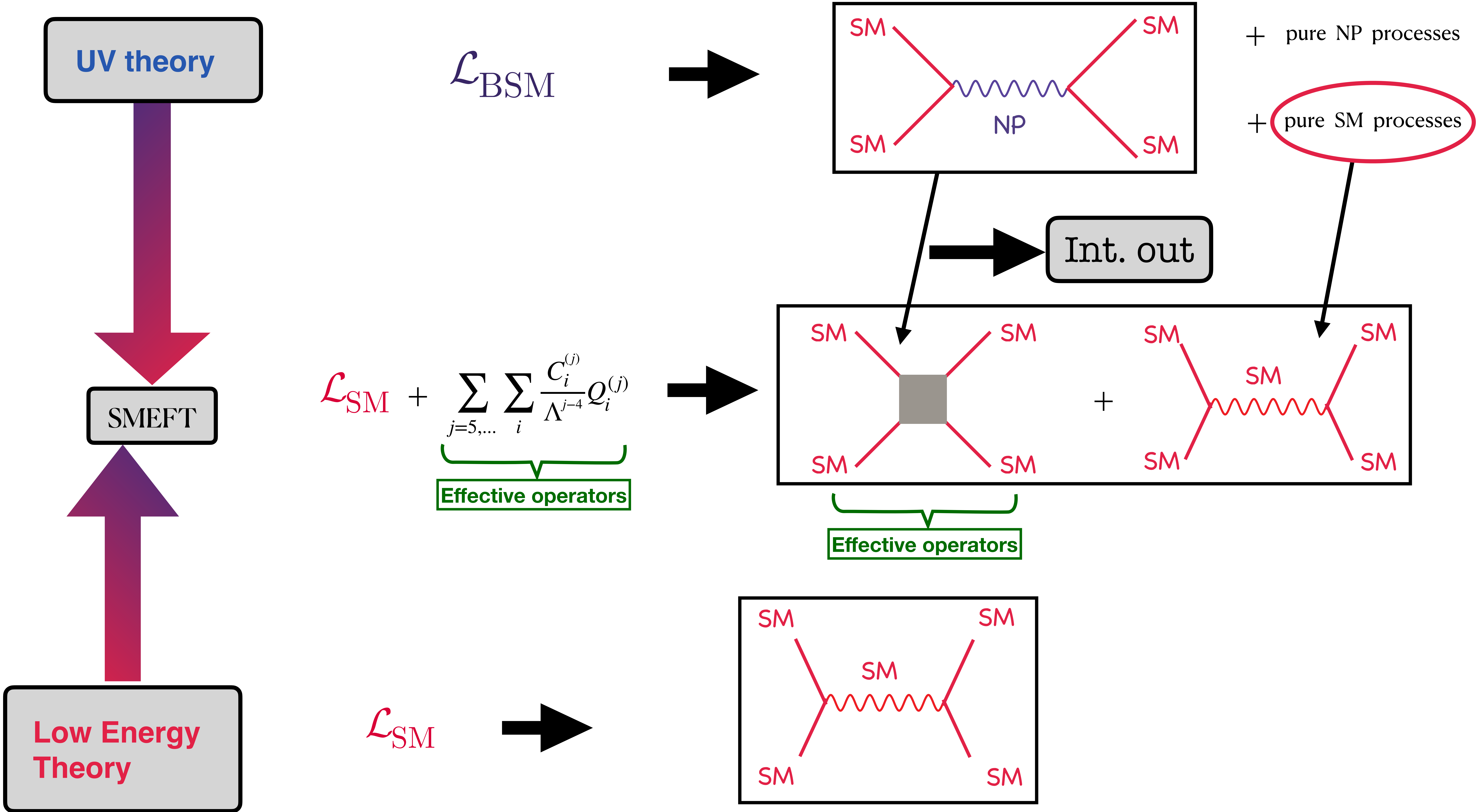
Our plan: EFT driven new physics search



Three Important Areas : to be developed



Top-Down vs Bottom-Up

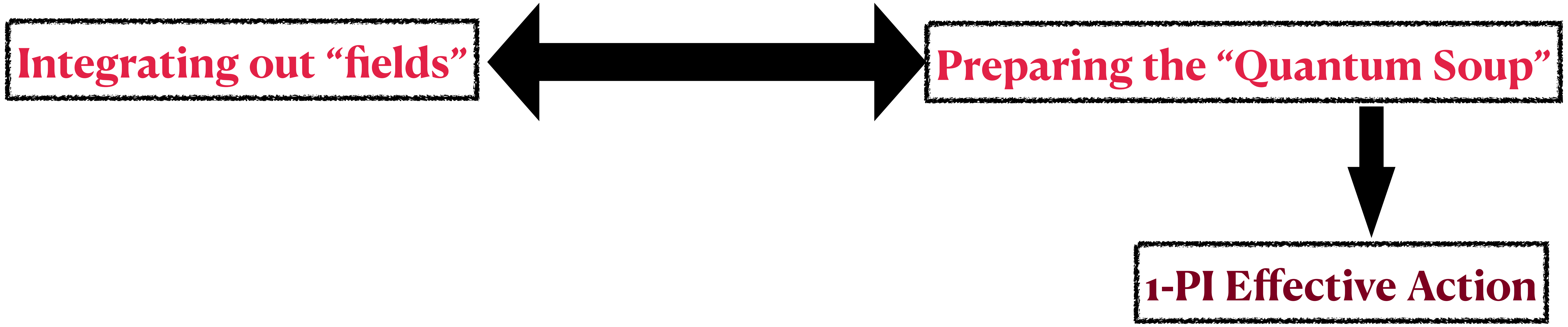


Top-down and a few queries:

Decoupling!

EFT vs Full Theory!

EFT Truncation!



Effective Action up to ***** ??

Up to Dim-6, 8, 10, ... !!

Up to one-Loop, two-Loop, ... !!

Heat-Kernel and Effective Action

$$\mathcal{L}^{\Phi} = \Phi^{\dagger}(D^2 + U + M^2)\Phi = \Phi^{\dagger}(\Delta)\Phi,$$

Effective Action

$$\mathcal{L}_{eff} = c_s \; tr\left[\int_0^{\infty} \frac{dt}{t} K(t, x, x, \Delta)\right]$$

Heat Equation

$$(\partial_t + \Delta_x)\, K(t, x, y, \Delta) = 0,$$

Initial condition

$$K(0, x, y, \Delta) = \delta(x - y)$$

Heat-Kernel

$$K(t, x, y, \Delta) = K_0(t, x, y)\; H(t, x, y, \Delta).$$

Degenerate masses

Free HK

Interaction part

Heat-Kernel and Effective Action

Heat-Kernel

$$K(t,x,y,\Delta) = K_0(t,x,y)\; H(t,x,y,\Delta).$$

Free HK

Interaction part

$$H(t,x,y,\Delta) = \sum_k \frac{(-t)^k}{k\,!} b_k(x,y), \quad K_0(t,x,y) = (4\pi t)^{-d/2}\; \mathrm{Exp}\left[\frac{z^2}{4t} - t\,M^2\right],$$

Effective Action

$$\mathcal{L}_{eff} = \frac{c_s}{(4\pi)^{d/2}} \sum_{k=0}^{\infty} M^{d-2k} \frac{(-1)^k}{k!}\; \Gamma[k-d/2]\; tr[b_k].$$

Universal One-Loop Effective Action up to D8

$$
\begin{aligned}
\mathcal{L}_{\text{eff}}^{d\leq 8} &= \frac{c_s}{(4\pi)^2} M^4 \left[-\frac{1}{2}\left(\ln\left[\frac{M^2}{\mu^2}\right]-\frac{3}{2}\right)\right] + \frac{c_s}{(4\pi)^2}\,\mathrm{tr}\left\{ M^2\left[-\left(\ln\left[\frac{M^2}{\mu^2}\right]-1\right) U\right]\right. \\
&+ M^0\,\frac{1}{2}\left[-\ln\left[\frac{M^2}{\mu^2}\right] U^2 - \frac{1}{6}\ln\left[\frac{M^2}{\mu^2}\right](G_{\mu\nu})^2\right] \\
&+ \frac{1}{M^2}\frac{1}{6}\left[-U^3-\frac{1}{2}(P_\mu U)^2-\frac{1}{2}U\,(G_{\mu\nu})^2-\frac{1}{10}(J_\nu)^2+\frac{1}{15}G_{\mu\nu}\,G_{\nu\rho}\,G_{\rho\mu}\right] \\
&+ \frac{1}{M^4}\frac{1}{24}\left[U^4-U^2(P^2U)+\frac{4}{5}U^2(G_{\mu\nu})^2+\frac{1}{5}(U\,G_{\mu\nu})^2+\frac{1}{5}(P^2U)^2\right. \\
&\qquad -\frac{2}{5}U\,(P_\mu U)\,J_\mu+\frac{2}{5}U(J_\mu)^2-\frac{2}{15}(P^2U)(G_{\rho\sigma})^2+\frac{1}{35}(P_\nu J_\mu)^2 \\
&\qquad -\frac{4}{15}U\,G_{\mu\nu}G_{\nu\rho}G_{\rho\mu}-\frac{8}{15}(P_\mu P_\nu U)\,G_{\rho\mu}G_{\rho\nu}+\frac{16}{105}G_{\mu\nu}J_\mu J_\nu \\
&\qquad +\frac{1}{420}(G_{\mu\nu}G_{\rho\sigma})^2+\frac{17}{210}(G_{\mu\nu})^2(G_{\rho\sigma})^2+\frac{2}{35}(G_{\mu\nu}G_{\nu\rho})^2 \\
&\qquad \left.+\frac{1}{105}G_{\mu\nu}G_{\nu\rho}G_{\rho\sigma}G_{\sigma\mu}+\frac{16}{105}(P_\mu J_\nu)G_{\nu\sigma}G_{\sigma\mu}\right] \\
&+\frac{1}{M^6}\frac{1}{60}\left[-U^5+2\,U^3(P^2U)+U^2(P_\mu U)^2-\frac{2}{3}U^2G_{\mu\nu}U\,G_{\mu\nu}-U^3(G_{\mu\nu})^2\right. \\
&\qquad +\frac{1}{3}U^2(P_\mu U)J_\mu-\frac{1}{3}U\,(P_\mu U)(P_\nu U)\,G_{\mu\nu}-\frac{1}{3}U^2J_\mu(P_\mu U) \\
&\qquad -\frac{1}{3}U\,G_{\mu\nu}(P_\mu U)(P_\nu U)-U\,(P^2U)^2-\frac{2}{3}(P^2U)(P_\nu U)^2-\frac{1}{7}((P_\mu U)G_{\mu\alpha})^2 \\
&\qquad +\frac{2}{7}U^2G_{\mu\nu}G_{\nu\alpha}G_{\alpha\mu}+\frac{8}{21}U\,G_{\mu\nu}U\,G_{\nu\alpha}G_{\alpha\mu}-\frac{4}{7}U^2(J_\mu)^2-\frac{3}{7}(U\,J_\mu)^2 \\
&\qquad +\frac{4}{7}U\,(P^2U)(G_{\mu\nu})^2+\frac{4}{7}(P^2U)U(G_{\mu\nu})^2-\frac{2}{7}U\,(P_\mu U)J_\nu G_{\mu\nu} \\
&\qquad -\frac{2}{7}(P_\mu U)U\,G_{\mu\nu}J_\nu-\frac{4}{7}U\,(P_\mu U)G_{\mu\nu}J_\nu-\frac{4}{7}(P_\mu U)U\,J_\nu G_{\mu\nu} \\
&\qquad +\frac{4}{21}U\,G_{\mu\nu}(P^2U)G_{\mu\nu}+\frac{11}{21}(P_\alpha U)^2(G_{\mu\nu})^2-\frac{10}{21}(P_\mu U)J_\nu U\,G_{\mu\nu} \\
&\qquad -\frac{10}{21}(P_\mu U)G_{\mu\nu}U\,J_\nu-\frac{2}{21}(P_\mu U)(P_\nu U)G_{\mu\alpha}G_{\alpha\nu}+\frac{10}{21}(P_\nu U)(P_\mu U)G_{\mu\alpha}G_{\alpha\nu} \\
&\qquad -\frac{1}{7}(G_{\alpha\mu}(P_\mu U))^2-\frac{1}{42}((P_\alpha U)G_{\mu\nu})^2-\frac{1}{14}(P_\mu P^2U)^2-\frac{4}{21}(P^2U)(P_\mu U)J_\mu \\
&\qquad \left.+\frac{4}{21}(P_\mu U)(P^2U)J_\mu+\frac{2}{21}(P_\mu U)(P_\nu U)(P_\mu J_\nu)-\frac{2}{21}(P_\nu U)(P_\mu U)(P_\mu J_\nu)\right] \\
&+\frac{1}{M^8}\frac{1}{120}\left[U^6-3\,U^4(P^2U)-2\,U^3(P_\nu U)^2+\frac{12}{7}U^2(P_\mu P_\nu U)(P_\nu P_\mu U)\right. \\
&\qquad +\frac{26}{7}(P_\mu P_\nu U)U\,(P_\mu U)(P_\nu U)+\frac{26}{7}(P_\mu P_\nu U)(P_\mu U)(P_\nu U)U+\frac{9}{7}(P_\mu U)^2(P_\nu U)^2 \\
&\qquad +\frac{9}{7}U\,(P_\mu P_\nu U)U\,(P_\nu P_\mu U)+\frac{17}{14}((P_\mu U)(P_\nu U))^2+\frac{8}{7}U^3G_{\mu\nu}U\,G_{\mu\nu} \\
&\qquad +\frac{5}{7}U^4(G_{\mu\nu})^2+\frac{18}{7}G_{\mu\nu}(P_\mu U)U^2(P_\nu U)+\frac{9}{14}(U^2G_{\mu\nu})^2 \\
&\qquad +\frac{18}{7}G_{\mu\nu}U\,(P_\mu U)(P_\nu U)U+\frac{18}{7}(P_\mu P_\nu U)(P_\mu U)U\,(P_\nu U) \\
&\qquad +\left(\frac{8}{7}G_{\mu\nu}U\,(P_\mu U)U\,(P_\nu U)+\frac{26}{7}G_{\mu\nu}(P_\mu U)U\,(P_\nu U)U\right) \\
&\qquad \left.+\left(\frac{24}{7}G_{\mu\nu}(P_\mu U)(P_\nu U)U^2-\frac{2}{7}G_{\mu\nu}U^2(P_\mu U)(P_\nu U)\right)\right] \\
&+\frac{1}{M^{10}}\frac{1}{210}\left[-U^7-5\,U^4(P_\nu U)^2-8\,U^3(P_\mu U)U(P_\mu U)-\frac{9}{2}(U^2(P_\mu U))^2\right] \\
&\left.+\frac{1}{M^{12}}\frac{1}{336}\left[U^8\right]\right\}.
\end{aligned}
$$

$$U_{ij} = \frac{\delta^2 \mathcal{L}_{UV}}{\delta\Phi_i\delta\Phi_j}$$

$$J_\mu = P_\nu G_{\nu\mu} = [P_\nu,[P_\nu,P_\mu]].$$

$$G_{\mu\nu} = [P_\mu, P_\nu],$$

How to Read it

	Scalar	Fermion
c_s	1 or 1/2	-1/2
$\mathcal{P}_\mu$	P_μ	$P_\mu - i\gamma^5\gamma_\mu R$
U	U_s	$U_f = Y + 2M\Sigma,$ $Y = -\frac{1}{2}\sigma_{\mu\nu}G_{\mu\nu} + S^2 + 3R^2 - (\not{P}S) + i\gamma^5(RS + SR),\ \Sigma = S + i\gamma^5 R$
$G_{\mu\nu}$	$F_{\mu\nu}$	$F_{\mu\nu} + \Gamma_{\mu\nu},$ $\Gamma_{\mu\nu} = i\gamma^5\gamma_\mu(P_\nu R) - i\gamma^5\gamma_\nu(P_\mu R) + 2\sigma_{\mu\nu}R^2$

Generalisation of one-loop effective Lagrangian.

1. **One-loop Effective Action up to Dimension Eight: Integrating out Heavy Scalar(s)**
   Upalaparna Banerjee, Joydeep Chakrabortty, Shakeel Ur Rahaman, Kaanapuli Ramkumar
   e-Print: 2306.09103 [hep-ph].

2. **One-loop Effective Action up to Dimension Eight: Integrating out Heavy Fermion(s)**
   Upalaparna Banerjee, Joydeep Chakrabortty, Shakeel Ur Rahaman, Kaanapuli Ramkumar
   e-Print: 2308.03849 [hep-ph].

Up to any mass-dimension

with any number of non-degenerate fields!

Heat-Kernel and Effective Action: Non-Degenerate Spectrum

Heat-Kernel

$$K(t,x,y,\Delta) = K_0(t,x,y)\, H(t,x,y,\Delta).$$

$$S_{\text{eff,1-loop}} = i\, c_s\, \mathrm{Tr} \log\big(D^2 + M^2 + U\big),$$

$$\mathcal{L}_{\text{eff}} = c_s\, \mathrm{tr} \int_0^\infty \frac{dt}{t} \int \frac{d^4p}{(2\pi)^4\, t^2}\, e^{p^2}\, e^{-M^2 t} \left[1 + \sum_{n=1}^{\infty} (-1)^n f_n(t,\mathcal{A})\right]$$

$$f_n(t,\mathcal{A}) = \int_0^t ds_1 \int_0^{s_1} ds_2 \cdots \int_0^{s_{n-1}} ds_n\, \mathcal{A}(s_1)\mathcal{A}(s_2)\cdots\mathcal{A}(s_n).$$

$$\mathcal{A}(t) = e^{M^2 t}\big(D^2 + 2i\, p\cdot D/\sqrt{t} + U\big)e^{-M^2 t},$$

Effective Action: Non-Deg. Mass-> Deg. Mass -> Light-Heavy Mixing

$$\mathcal{L}_{\text{eff}} = c_s \,\text{tr} \int_0^\infty \frac{dt}{t} \int \frac{d^4 p}{(2\pi)^4\, t^2}\; e^{p^2}\, e^{-M^2 t} \left[1 + \sum_{n=1}^{\infty} (-1)^n f_n(t, \mathcal{A})\right]$$

Features: One-Loop 1PI Effective Action up to any mass dimension

A Mathematica package is attached.

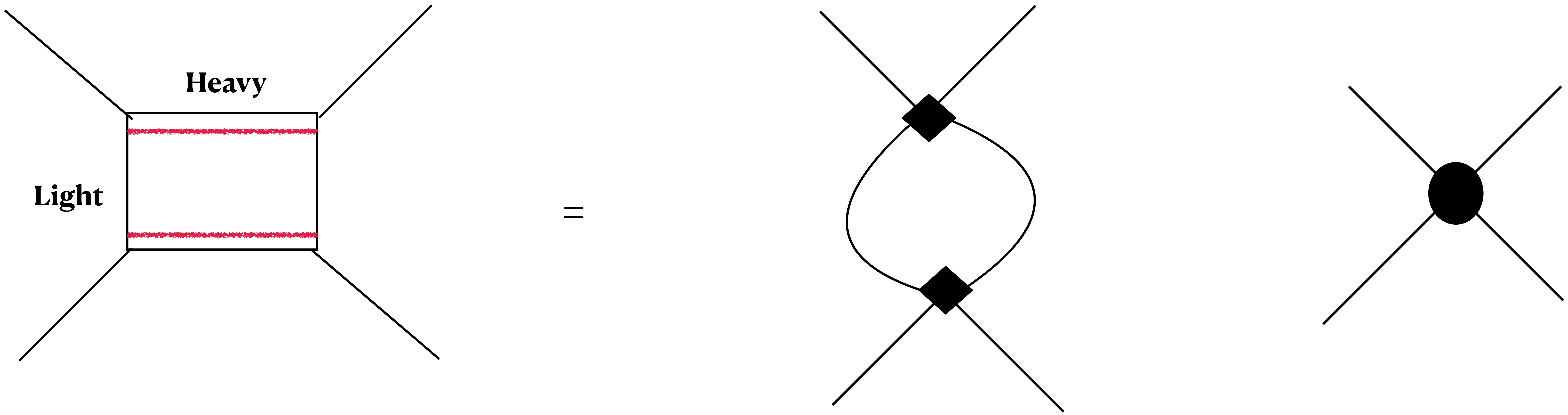
Applicable for Scalars and Fermions both

One-loop Effective Action up to any Mass-dimension for Non-degenerate Scalars and Fermions including Light-Heavy Mixing

Upalaparna Banerjee, Joydeep Chakrabortty, Shakeel Ur Rahaman, Kaanapuli Ramkumar

ArXiv: 2311.12757

Effective Action: Non-Deg. Mass-> Deg. Mass -> Light-Heavy Mixing



Non-Degenerate Masses —> Sub-Degeneracy: Just take the "Limits" (Mi —> Mk)

Non-Degenerate Masses —> Light-Heavy Mixing: Just take the "Limits" (mi —> 0)

And ensure the theory is "IR"-safe

One-loop Effective Action up to any Mass-dimension for Non-degenerate Scalars and Fermions including Light-Heavy Mixing
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ArXiv: 2311.12757

Beyond one-loop

with any number of non-degenerate fields!

Paving the path to L-Loop Renormalization: Scalar field theory

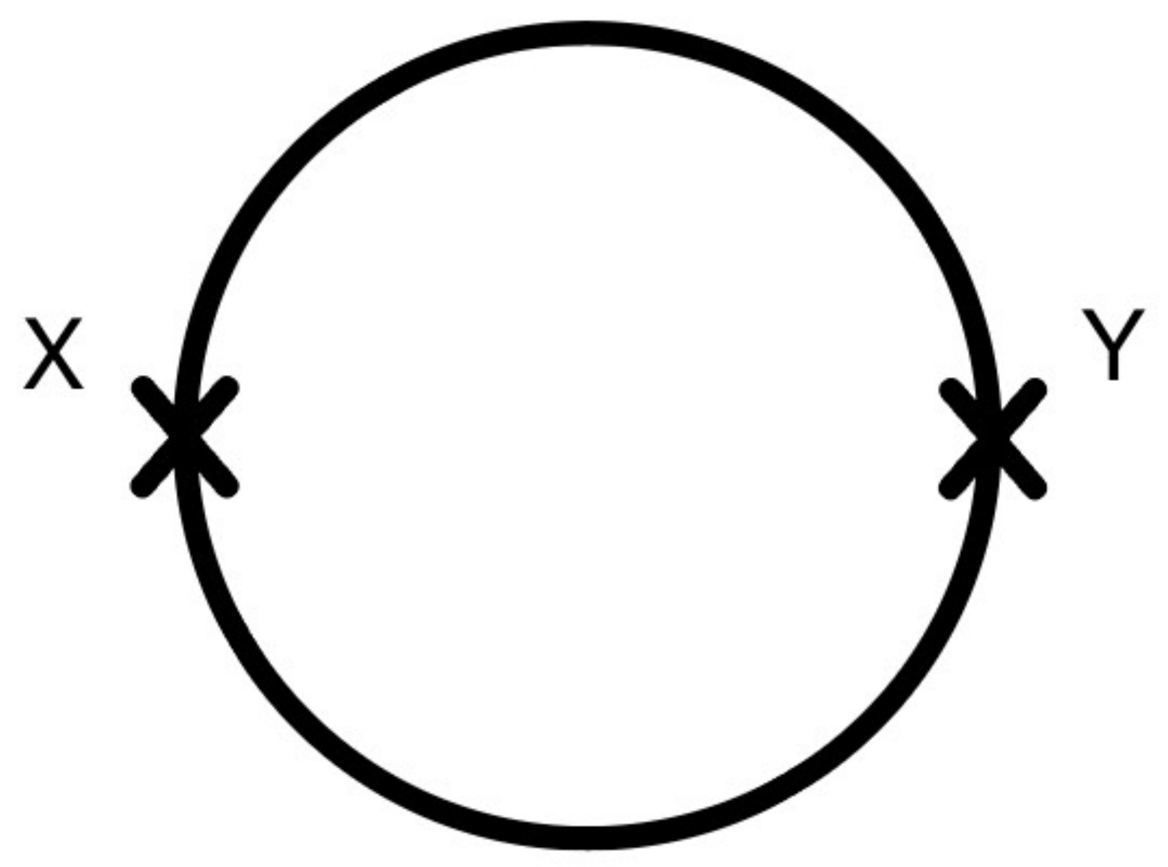
$$G(x,y) = \int_0^\infty dt\ K(t,x,y,\Delta).$$

$$G(x,y) = \sum_{n=0}^{\infty} g_n(x,y)\tilde{b}_n(x,y),$$

$$\begin{aligned} g_n(x,y) &= \int_0^\infty dt\ \frac{1}{(4\pi t)^{d/2}} e^{\frac{z^2}{4t}} e^{-M^2 t} \frac{(-t)^n}{n!} \\ &= \frac{2^{\frac{d}{2}-2}}{(4\pi)^{\frac{d}{2}}} \left(\frac{M}{z}\right)^{\frac{d}{2}-n-1} \mathcal{K}_{\frac{d}{2}-n-1}(Mz) \end{aligned}$$

$$\begin{aligned} G(x,y) &= g_0(x,y)\,\tilde{b}_0 + g_1(x,y)\,\tilde{b}_1 + g_2(x,y)\,\tilde{b}_2 + \alpha R(x,y) \\ g_0(x,y) &= \alpha\Big[2^{4-d}M^{d-2}\Gamma[1-d/2] - 2^{2-d}z^2M^d\Gamma[-d/2] + \frac{1}{8}M^4z^{6-d}\Gamma[d/2-3] \\ &\qquad - M^2z^{4-d}\Gamma[d/2-2] + 4z^{2-d}\Gamma[d/2-1]\Big] + \mathcal{O}(z^4) \\ g_1(x,y) &= \alpha\Big[-2^{2-d}z^2M^{d-2}\Gamma[1-d/2] + 2^{4-d}M^{d-4}\Gamma[2-d/2] - \frac{1}{4}M^2z^{6-d}\Gamma[d/2-3] \\ &\qquad + z^{4-d}\Gamma[d/2-2]\Big] + \mathcal{O}(z^4) \\ g_2(x,y) &= \alpha\Big[-2^{2-d}z^2M^{d-4}\Gamma[2-d/2] + 2^{4-d}M^{d-6}\Gamma[3-d/2] + \frac{1}{4}z^{6-d}\Gamma[d/2-3]\Big] + \mathcal{O}(z^4). \end{aligned}$$

One-Loop

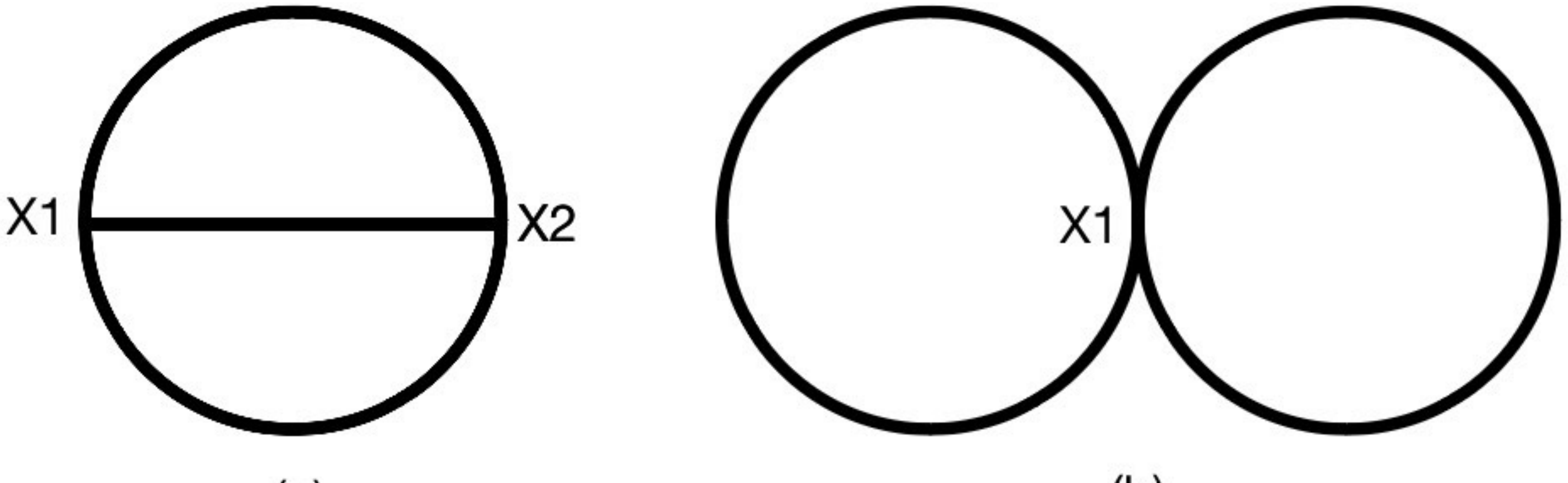


Sample diagram !

$$\mathcal{L}_{1-loop} = \frac{1}{2}\int \frac{dt}{t} K(t, x, x, \Delta)$$

$$\mathcal{L}_{(1)} = \frac{\alpha}{2}\left(\Gamma[\epsilon/2-2]M^4 - \Gamma[\epsilon/2-1]M^2\,\tilde{b}_1 + \frac{1}{2}\Gamma[\epsilon/2]\tilde{b}_2\right)$$

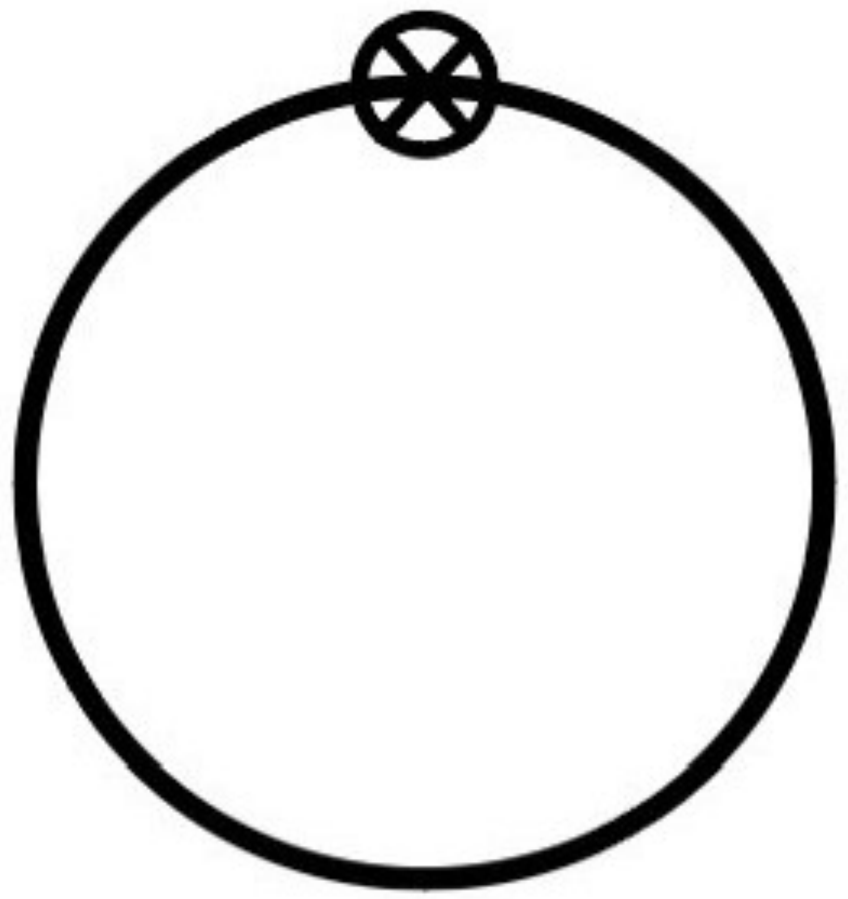
Two-Loop



(a) (b)

$$\mathcal{L}^{a}_{(2)} = S_f \int d^dx\, d^dy\, V_{(3)}(x)\, G(x,y)^3\, V_{(3)}(y),$$

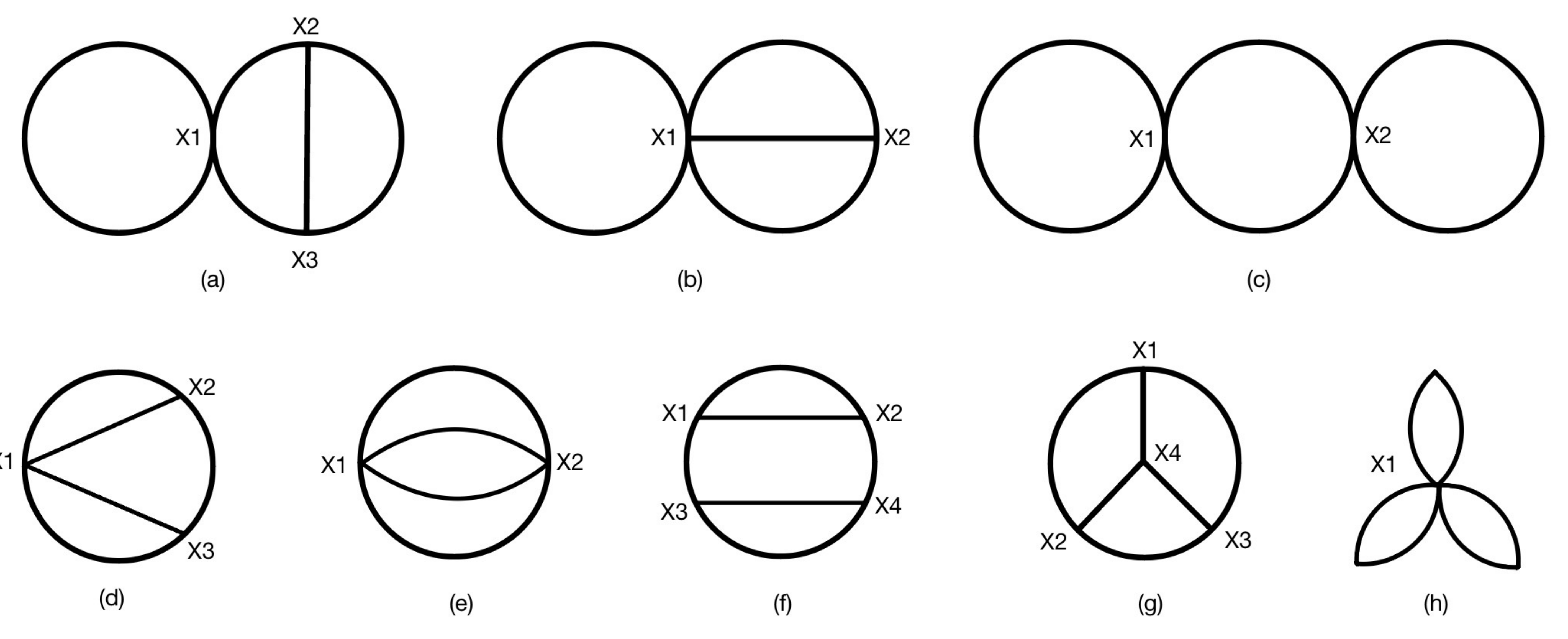
$$\mathcal{L}^{b}_{(2)} = S_f \int d^dx\, V_{(4)}(x)\, G(x,x)^2.$$

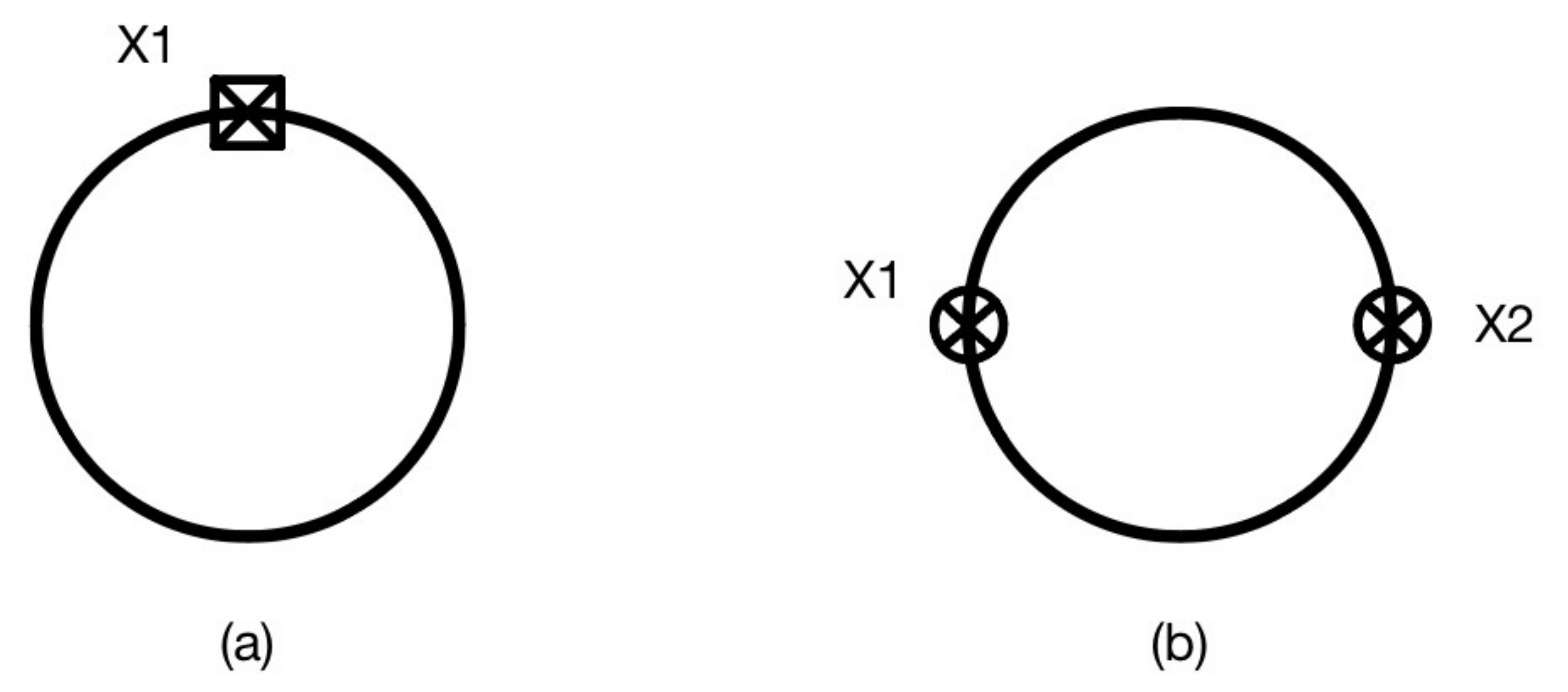


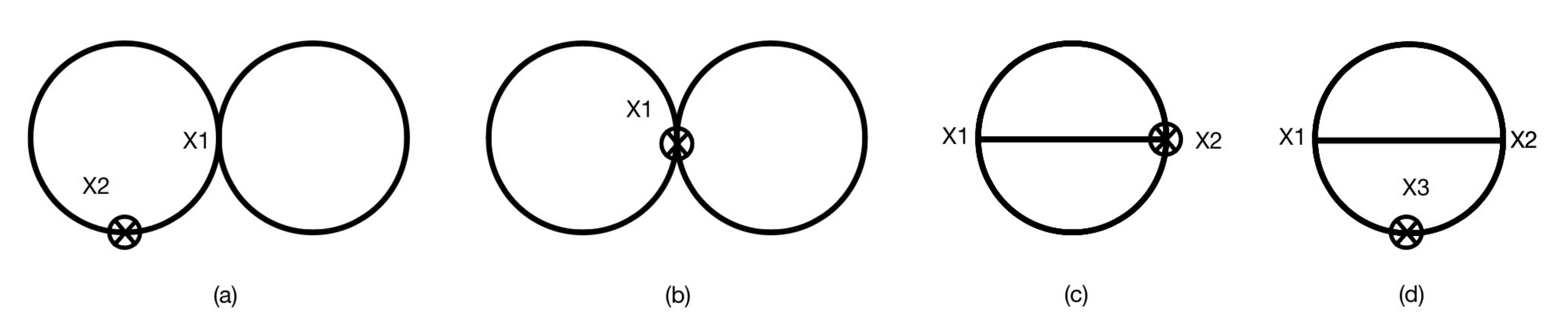
$$\mathcal{L}^{ct}_{(2)} = \int d^dx\, V^{(ct-1)}_{(2)}(x)\, G(x,x).$$

$$V^{(ct-m)}_{(n)}(x) = \frac{1}{n!}\frac{\partial^n \mathcal{L}_{(m)}}{\partial \phi^n}$$

Three-Loop







$$\mathcal{L}_{(L)} = S_f \int d^d x_1 \ldots \int d^d x_n \, V_{(m_1)}(x_1) \ldots V_{(m_n)}(x_n) G(x_1, x_1)^{p_1} \ldots G(x_n, x_n)^{p_n} G(x_1, x_n)^{q_{1n}}.$$

$$V^{(n)}(x) = \frac{1}{n!} \frac{\partial^n \mathcal{L}}{\partial \phi^n}$$

Task: Algebric Singularity

$$g_0(x,y)^2 = \alpha\frac{\Gamma[d/2-2]}{1-d/2}\delta(z)$$

$$g_0(x,y)^2 g_1(x,y) = \alpha^2\frac{\Gamma[d/2-2]}{1-d/2}\left(\frac{3}{2}\Gamma[d/2-2]+3\,\Gamma[2-d/2]\right)\delta(z)$$

$$g_0(x,y)^3 = \alpha^2\frac{\Gamma[d/2-2]}{1-d/2}\left[-\frac{2}{d}\Gamma[d/2-1]D^2+3M^2\left(\Gamma[1-d/2]-\frac{1}{2}\Gamma[d/2-2]\right)\right]\delta(z)$$

$$\begin{aligned} g_0(x,y)^4 = \alpha^3\Bigg[&-\frac{2}{3}\frac{\Gamma[1-\epsilon/2]^3\Gamma[-\epsilon/2]}{\Gamma[4-\epsilon/2]}D^4+\frac{4M^2}{3}\frac{\Gamma[1-\epsilon/2]^2\Gamma[-\epsilon/2]}{\Gamma[3-\epsilon/2]}\Big(2\Gamma[-\epsilon/2]-3\Gamma[\epsilon/2-1]\Big)D^2 \\ &-2M^4\frac{\Gamma[1-\epsilon/2]\Gamma[-\epsilon/2]}{\Gamma[2-\epsilon/2]}\Big(\Gamma[-\epsilon/2]^2-3\Gamma[\epsilon/2-1]\Gamma[-\epsilon/2]+3\Gamma[\epsilon/2-1]^2\Big) \\ &-\frac{2M^4}{3}\frac{\Gamma[1-\epsilon/2]^2\Gamma[-\epsilon/2]}{\Gamma[2-\epsilon/2]}\Big(\Gamma[-1-\epsilon/2]-3\Gamma[\epsilon/2-2]\Big)\Bigg]\delta(z) \end{aligned}$$

$$\begin{aligned} g_0(x,y)^3 g_1(x,y) = \alpha^3\Bigg[&-\frac{\Gamma[1-\epsilon/2]^2\Gamma[-\epsilon/2]}{\Gamma[3-\epsilon/2]}\left(\Gamma[\epsilon/2]+\frac{2}{3}\Gamma[-\epsilon/2]\right)D^2 \\ &-M^2\frac{\Gamma[1-\epsilon/2]\Gamma[-\epsilon/2]}{\Gamma[2-\epsilon/2]}\Bigg(\Gamma[-\epsilon/2]^2+\frac{3}{2}\Gamma[\epsilon/2]\Gamma[-\epsilon/2]-\frac{3}{2}\Gamma[\epsilon/2-1]\Gamma[-\epsilon/2] \\ &-3\Gamma[\epsilon/2-1]\Gamma[\epsilon/2]+\frac{1}{2}\Gamma[\epsilon/2-1]\Gamma[1-\epsilon/2]+\frac{1}{3}\Gamma[-\epsilon/2-1]\Gamma[1-\epsilon/2]\Bigg)\Bigg]\delta(z) \end{aligned}$$

$$g_0(x,y)^2 g_1(x,y)^2 = -\alpha^3\frac{\Gamma[1-\epsilon/2]\Gamma[-\epsilon/2]}{\Gamma[2-\epsilon/2]}\left(\Gamma[\epsilon/2]^2+\Gamma[-\epsilon/2]\Gamma[\epsilon/2]+\frac{1}{3}\Gamma[-\epsilon/2]^2\right)\delta(z)$$

$$\begin{aligned} g_0(x,y)^3 g_2(x,y) = -\alpha^3\Bigg[&\frac{\Gamma[1-\epsilon/2]^2\Gamma[\epsilon/2+1]\Gamma[-\epsilon/2]}{M^2\Gamma[3-\epsilon/2]}D^2+\frac{\Gamma[1-\epsilon/2]^2\Gamma[-\epsilon/2]}{\Gamma[2-\epsilon/2]}\left(\frac{1}{3}\Gamma[-\epsilon/2-1]-\frac{1}{2}\Gamma[\epsilon/2]\right) \\ &+\frac{\Gamma[1-\epsilon/2]\Gamma[\epsilon/2+1]\Gamma[-\epsilon/2]}{\Gamma[2-\epsilon/2]}\left(3\Gamma[\epsilon/2-1]-\frac{3}{2}\Gamma[-\epsilon/2]\right)\Bigg]\delta(z) \end{aligned}$$

Example

Let us consider the case of real scalar Lagrangian with ϕ^6 operator given by,

$$\mathcal{L} = \frac{1}{2}\phi\, D^2\,\phi + \frac{1}{2}M^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \frac{c_6}{6!}\phi^6.$$

The Δ operator in the HK is defined from the action as,

$$\Delta = \frac{\partial^2\mathcal{L}}{\partial\phi^2} = D^2 + M^2 + \frac{\lambda}{2}\phi^2 + \frac{c_6}{4!}\phi^4.$$

$$U = \frac{\lambda}{2}\phi^2 + \frac{c_6}{4!}\phi^4,$$

$$\tilde{b}_0(x,x) = I, \quad \tilde{b}_1(x,x) = U, \quad \tilde{b}_2(x,x) = U^2.$$

$$\begin{aligned}\mathcal{L}_{(1)} &= \frac{\alpha}{2}\left(\Gamma[\epsilon/2-2]M^4 - \Gamma[\epsilon/2-1]M^2\,U + \frac{1}{2}\Gamma[\epsilon/2]\,U^2\right),\\ &= \frac{\alpha}{2}\left(\Gamma[\epsilon/2-2]M^4 - \Gamma[\epsilon/2-1]M^2\left[\frac{\lambda}{2}\phi^2 + \frac{c_6}{4!}\phi^4\right] + \frac{1}{2}\Gamma[\epsilon/2]\left[\frac{\lambda^2}{4}\phi^4 + \frac{\lambda c_6}{4!}\phi^6\right]\right).\end{aligned}$$

$$V_{(3)}(x) = \frac{1}{3!}\left[\lambda\phi + \frac{c_2}{3!}\phi^3\right].$$

$$V_{(4)}(x) = \frac{1}{4!}\left[\lambda + \frac{c_2}{2}\phi^2\right].$$

$$\begin{aligned}V_{(2)}^{(ct-1)}(x) &= \frac{1}{2!}\frac{\partial^2\mathcal{L}_{(1)}}{\partial\phi^2}\\ &= \frac{\alpha}{4}\left(-\Gamma[\epsilon/2-1]M^2\left[\lambda + \frac{c_6}{2}\phi^2\right] + \Gamma[\epsilon/2]\left[\frac{3\lambda^2}{2}\phi^2 + \frac{5\lambda c_6}{8}\phi^4\right]\right)\end{aligned}$$

Example contd...

$$\phi D^2\phi\Big|_\epsilon = \alpha^2 \frac{\lambda^2}{24\epsilon}$$

$$\phi^2\Big|_\epsilon = \alpha^2\left[\frac{M^2\lambda^2}{4\epsilon} - \frac{1}{\epsilon^2}\left(M^2\lambda^2 + \frac{M^4 c_6}{4}\right)\right]$$

$$\phi^4\Big|_\epsilon = \alpha^2\left[\frac{1}{\epsilon}\left(\frac{\lambda^3}{8} + \frac{M^2\lambda c_6}{12}\right) - \frac{1}{24\epsilon^2}\left(9\lambda^3 + 11\,M^2\lambda c_6\right)\right]$$

$$\phi^6\Big|_\epsilon = \alpha^2\left[\frac{5\lambda^2 c_6}{96\epsilon} - \frac{3\lambda^2 c_6}{16\epsilon^2}\right]$$

$$\phi^3 D^2\phi\Big|_\epsilon = \alpha^2 \frac{\lambda c_6}{72\epsilon}.$$

$$D^2\phi = -M^2\phi - \frac{\lambda}{3!}\phi^3 - \frac{c_6}{5!}\phi^5.$$

$$\phi^2\Big|_\epsilon = \alpha^2\left[\frac{M^2\lambda^2}{4\epsilon} - \frac{1}{\epsilon^2}\left(M^2\lambda^2 + \frac{M^4 c_6}{4}\right)\right]$$

$$\phi^4\Big|_\epsilon = \alpha^2\left[\frac{1}{\epsilon}\left(\frac{\lambda^3}{8} + \frac{5\,M^2\lambda c_6}{72}\right) - \frac{1}{24\epsilon^2}\left(9\lambda^3 + 11\,M^2\lambda c_6\right)\right]$$

$$\phi^6\Big|_\epsilon = \alpha^2\left[\frac{43\lambda^2 c_6}{864\epsilon} - \frac{3\lambda^2 c_6}{16\epsilon^2}\right].$$

❖ Take home messages

One-Loop Universal Effective Action up to any mass dimension

Compact formula upto Dim-8 is ready to be fed into CoDEx!

Equally Applicable for Scalars and Fermions as well.

Can Deal with any number of non-degenerate fields.

Effective Action is IR-safe — Light-Heavy mixing is easily computed from general non-degenerate result as a limiting case

Formalism for L-loop Renormalization: SQFT

Formalism for L-loop Renormalization: FQFT, Mixed-stat QFT ??