Ultra Slow Roll in Warm Inflation

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Outline

- Cold Inflation
- Slow Roll Approximation in CI
- Ultra Slow Roll in CI
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- Ultra Slow Roll in WI

Cold Inflation

- In 1981, Alan Guth proposed a solution to the flatness problem and the horizon problem[1]. This solution, known as **inflation**, describes a period of rapid and accelerated expansion in the early universe, i.e. the scale factor of the universe was accelerating

$$INFLATION \implies \ddot{a} > 0.$$

- The acceleration equation gives

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3p\right),$$

so, taking the convention of positive energy density, for $\ddot{a}$ to be greater than zero we need $p < -\frac{\rho}{3}$.

[1] PhysRevD.23.347

- The condition for cold inflation can be obtained by using a scalar field, also known as the inflaton ϕ.
- The action for cold inflation is

$$S = \int d^4x\sqrt{-g}\mathcal{L} = \int d^4x\sqrt{-g}\left[-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right].$$

Euler-Lagrange equation gives

$$\ddot{\phi} + 3H\dot{\phi} - \frac{\nabla^2\phi}{a^2} + V_{,\phi} = 0,$$

where $V_{,\phi} \equiv \frac{dV}{d\phi}$, and $3H\dot{\phi}$ acts as a friction term.

- From the energy-momentum tensor we get

$$\rho_\phi = T^0_0 = \frac{\dot{\phi}^2}{2} + V(\phi) + \frac{(\nabla\phi)^2}{2a^2}$$

$$p_\phi = \frac{T^i_i}{3} = \frac{\dot{\phi}^2}{2} - V(\phi) - \frac{(\nabla\phi)^2}{6a^2}.$$

- Hence the condition for inflation is $V(\phi) \gg \dot{\phi}^2 \gg \frac{(\nabla\phi)^2}{2a^2}$.

- So the equation of motion becomes

$$\boxed{\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0}.$$

- The FRLW scale factor $a(t)$ evolves according to Friedmann equations

$$3M_{\rm Pl}^2 H^2 = \frac{\dot{\phi}^2}{2} + V(\phi)$$
$$-2M_{\rm Pl}^2 \dot{H} = \dot{\phi}^2,$$

where $M_{\rm Pl}$ is the reduced Plank mass.

Slow Roll Approximation in CI

- In Slow Roll with the requirement $|V| \gg \dot{\phi}^2$, we also need $|\ddot{\phi}| \ll H|\dot{\phi}|$.
- The equations of motion are

$$\dot{\phi} \simeq -\frac{V_{,\phi}}{3H}$$

and first Friedmann equations

$$3M_{\rm Pl}^2 H^2 \simeq V.$$

- We define the parameters

$$\epsilon_1 \equiv -\frac{\dot{H}}{H^2}$$
$$\epsilon_2 \equiv \frac{\dot{\epsilon}}{H\epsilon} = \frac{\ddot{H}}{H\dot{H}} + 2\epsilon_1.$$

- For inflation we need $\epsilon_1 < 1$, but for SR we need $\epsilon_1 \ll 1$, as well as $\epsilon_2 \ll 1$.

Ultra Slow Roll in CI

- Ultra Slow Roll is described by the situation where the inflaton field has to traverse an extremely flat part of the scalar field potential[2]. In that case, even though we still have $|V| \gg \dot{\phi}^2$ the slope of the potential becomes negligible.
- The equation of motion becomes

$$\ddot{\phi} + 3H\dot{\phi} \simeq 0$$

- We still need $\epsilon \ll 1$ for inflation to continue, but now we have $|\epsilon_2| \sim -6 + \epsilon_1$.
- Since we still have $\epsilon \ll 1$, and the potential term dominates over the kinetic term, the inflation continues. However, since $|\epsilon_2| > 1$, a pronounced departure from SR becomes evident, marking the onset of a new phase in the evolution of scalar field dynamics known as Ultra Slow Roll.

[2] physletb.2017.10.066

Warm Inflation(WI)

- In 1995 Arjun Barera and Li-Zhi-Fang came up with a new model of inflation[3] where the inflaton field is no longer assumed to be isolated, during the inflation period, rather it's interacting with thermalized radiation causing dissipation of energy out of the inflation system, maintaining a non-negligible radiation energy density ρ_r. The equations governing the dynamics of the inflaton field ϕ, and the radiation bath, ρ_r, in WI can be written as

$$\boxed{\begin{aligned} \ddot{\phi} + 3H\dot{\phi} + V_{,\phi} &= -\Gamma\dot{\phi} \\ \dot{\rho}_r + 4H\rho_r &= \Gamma\dot{\phi}^2, \end{aligned}}$$

where Γ is the dissipative term, which can depend on the amplitude of the inflaton field, ϕ, as well as the temperature of the radiation bath, T.

[3] PhysRevLett.74.1912

- It is assumed that the radiation bath, resulting from the dissipation of the inflaton field, maintains near thermal equilibrium throughout the WI phase, and thus a temperature T can be defined.
- The dissipative coefficient Γ has the general form

$$\Gamma(\phi, T) = C_\Gamma T^p \phi^c M^{(1-p-c)}.$$

  where C_Γ is a dimensionless constant, and M is some appropriate mass scale, so that the dimensionality of the dissipative coefficient is preserved, $[\Gamma] = [\text{mass}]$.
- We define the dimensionless quantity Q as

$$\boxed{Q \equiv \frac{\Gamma}{3H}}.$$

- If $Q > 1$, we call it strong dissipation, and if $Q < 1$ we call it weak dissipation.

- In the SR regime of WI we again assume $|\ddot{\phi}|$ to be negligible with respect to all the other terms, and the equation of motion for the field becomes

$$3H(1+Q)\dot{\phi} + V_{,\phi} \approx 0.$$

- But for inflation to continue we still need potential energy density to dominate over the kinetic term and the radiation energy density during WI, the Friedmann equation is then given by

$$3M_{\rm Pl}^2 H^2 \approx V(\phi)$$

- As during WI a constant radiation bath is maintained by the dissipation of energy of the inflaton field into the radiation bath, we can assume $\dot{\rho}_r \approx 0$, hence we get

$$4H\rho_r \approx \Gamma\dot{\phi}^2.$$

Also

$$\rho_r = \frac{\pi^2}{30} g_* T^4,$$

where g_* is the relativistic d.o.f of the radiation bath.

Ultra Slow Roll in WI

- We ask the question what happens in the case of WI, when the potential becomes extremely flat? Can WI continue while maintaining thermal equilibrium with the radiation bath?
- When potential becomes extremely flat the equation of motion becomes

$$\ddot{\phi} + 3H\dot{\phi}(1+Q) \simeq 0.$$

- We find that during this part, the temperature evolves as

$$\frac{1}{T}\frac{dT}{dN} \simeq \frac{1}{4-p}\left[c\frac{\dot{\phi}}{H\phi} - 6(1+Q)\right].$$

and

$$\epsilon_1 = -\frac{\dot{H}}{H^2} = \frac{3}{2}\frac{(1+Q)\dot{\phi}^2}{V(\phi)} \ll 1$$

$$\epsilon_2 = -6\left(1+\frac{4Q}{4-p}\right) + \frac{4}{4-p}\left[c\frac{\dot{\phi}}{H\phi} + \epsilon_1\right]\frac{Q}{1+Q} + 2\epsilon_1.$$

Linear Potential: $V(\phi) = V_0 + M_0^3\phi$, $\Gamma = C_\Gamma \frac{T^3}{\phi^2}$

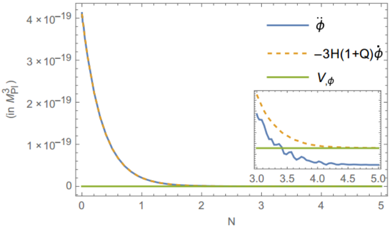

Figure: The figure depicts the numerical evolution of the acceleration term $\ddot{\phi}$, the friction term $3H(1+Q)\dot{\phi}$, and the slope term $V_{,\phi}$ present in the equation of motion of the inflaton field through an ultraslow-roll phase in the case of linear potential.We have chosen the parameters as follows: $V_0 = (10^{-4}M_{Pl})^4$, $M_0 = 2.5 \times 10^{-8}M_{Pl}$, $C_\Gamma = 10$, and $g* = 106.75$.

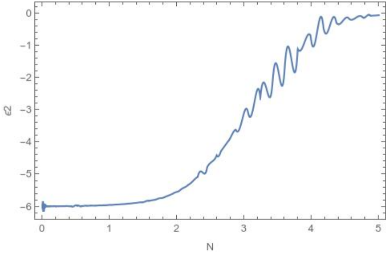

Figure: Evolution of the second Hubble slow-roll parameter, ϵ_2, during ultraslow roll in the case of linear potential.

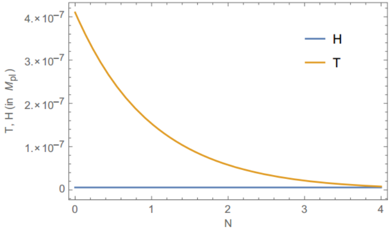

Figure: Evolution of the temperature and the Hubble parameter, during ultraslow roll in the case of linear potential.

Linear Potential: $V(\phi) = V_0 \left[1 + \left(\frac{\phi}{\phi_0}\right)^3\right]$, $\Gamma = C_\Gamma \frac{T^3}{\phi^2}$

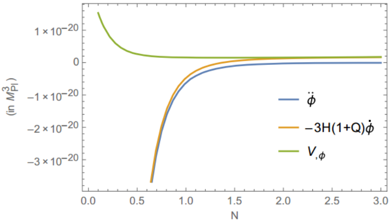

Figure: The figure depicts the numerical evolution of the acceleration term $\ddot{\phi}$, the friction term $3H(1+Q)\dot{\phi}$, and the slope term $V_{,\phi}$ present in the equation of motion of the inflaton field through an ultraslow-roll phase in the case of cubic potential.We have chosen the parameters as follows: $V_0 = (10^{-4} M_{Pl})^4$, $\phi_0 = 2.5 \times 10^{-1} M_{Pl}$, $C_\Gamma = 10^4$, and $g* = 106.75$.

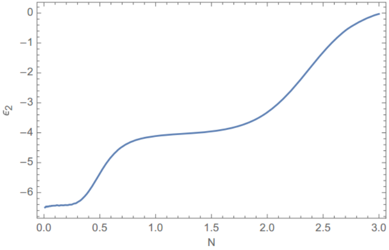

Figure: Evolution of the second Hubble slow-roll parameter, ϵ_2, during ultraslow roll in the case of cubic potential.

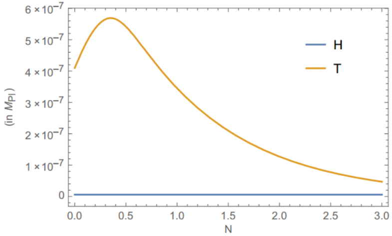

Figure: Evolution of the temperature and the Hubble parameter, during ultraslow roll in the case of linear potential.

Thank You