Spread Complexity and 2 Point Measurements

Ankit Gill[1]

[1]IIT Kanpur

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In collaboration with **Tapobrata Sarkar, Kunal Pal, Kuntal Pal**

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Lanczos Algorithm and Krylov Basis

- Consider the auto-correlation $\langle O|e^{iH_0 u}|O\rangle$ with a hermitian operator H_0 and a quantum state $\big|O\big\rangle$.
- Krylov basis is generated with $\big|O\big\rangle = \big|\tilde{K}_0\big\rangle$ as the first vector,

$$\big|\tilde{K}_{n+1}\big\rangle = \frac{1}{\tilde{b}_{n+1}}\big[(H_0 - \tilde{a}_n)\big|\tilde{K}_n\big\rangle - \tilde{b}_n\big|\tilde{K}_{n-1}\big\rangle\big] \ . \qquad (1)$$

- Where $\tilde{a}_n = \langle\tilde{K}_n|H_0|\tilde{K}_n\rangle$,
  $\tilde{b}_n = \big|\big|\big[(H_0 - \tilde{a}_n)\big|\tilde{K}_n\big\rangle - \tilde{b}_n\big|\tilde{K}_{n-1}\big\rangle\big]\big|\big|$.
- Algorithm is stopped when $\tilde{b}_n = 0$ for any particular step.

Spread Complexity

- Goal of the Algorithm is to Orthonormalise the basis $\{|\tilde{K}_0\rangle, H_0|\tilde{K}_0\rangle, H_0^2|\tilde{K}_0\rangle, ...\}$.
- In Krylov basis Hamiltonian takes Tri-diagonal form

$$H_0|\tilde{K}_n\rangle = \tilde{a}_n|\tilde{K}_n\rangle + \tilde{b}_n|\tilde{K}_{n-1}\rangle + \tilde{b}_{n+1}|\tilde{K}_{n+1}\rangle. \tag{2}$$

- Spread Complexity is defined as

$$\mathcal{C}(u) = \sum_n n|\langle O|e^{iH_0 u}|\tilde{K}_n\rangle|^2 = \sum_n n|\tilde{\phi}_n(u)|^2 \ . \tag{3}$$

- n is the weight associated with the state $|\tilde{K}_n\rangle$.

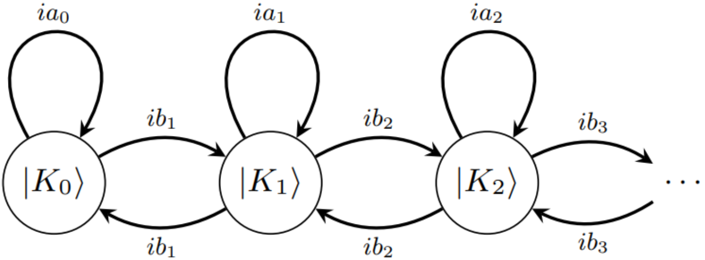

Figure: Schematic for evolution in Krylov basis - Vijay B. et al. PRD 106, 046007

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OTOC and Two Point Measurement

- Consider the following correlator for two point measurement

$$G(u,\tau) = \left\langle O_0 \middle| W_\tau^\dagger V^\dagger W_\tau V \middle| O_0 \right\rangle, \tag{4}$$

- where $V = e^{-iuO}$ and $W_\tau = e^{-iH_1\tau} W e^{iH_1\tau}$.
- eq. 4 can be obtained from a two point measurement protocol.

$$\begin{aligned} G(u,\tau) &= \sum_m \left\langle O_0 \middle| W_\tau^\dagger V^\dagger \middle| O_m \right\rangle \left\langle O_m \middle| W_\tau V \middle| O_0 \right\rangle, \\ & \sum_m e^{i(O_m - O_n)} \left\langle O_0 \middle| W_\tau^\dagger \middle| O_m \right\rangle \left\langle O_m \middle| W_\tau \middle| O_0 \right\rangle \ . \end{aligned} \tag{5}$$

- Applying the Lanczos algorithm with $|\tilde{K_0}\rangle = |E_{00}, \tau\rangle$ and H_0 where $|E_{00}, \tau\rangle = W_\tau |E_{00}\rangle$.
- One can compute the $\tilde{b_1}$

$$\begin{aligned} \tilde{b}_1^2 &= \left\langle (\Delta E_0)^2 \right\rangle - (\left\langle \Delta E_0 \right\rangle)^2 \\ &= \left\langle E_{00}, \tau \middle| H_0^2 \middle| E_{00}, \tau \right\rangle - \left\langle E_{00}, \tau \middle| H_0 \middle| E_{00}, \tau \right\rangle^2 . \end{aligned} \tag{6}$$

- The first piece is Squared Commutator between W_τ and H_0

$$\begin{aligned} \left\langle (\Delta E_0)^2 \right\rangle &= \left\langle E_{00} \middle| [W_\tau, H_0]^\dagger [W_\tau, H_0] \middle| E_{00} \right\rangle \\ &= \left\langle E_{00}, \tau \middle| H_0^2 \middle| E_{00}, \tau \right\rangle . \end{aligned} \tag{7}$$

- Squared commutator measures how much two operators fails to commute with each other.

Spread Complexity for Integrable and Chaotic Systems

- Spread Complexity for two point scheme for $\left\langle E_{00}, \tau \middle| e^{-iH_0 u} \middle| E_{00}, \tau \right\rangle$ amplitude

$$\mathcal{C}(\tau, u) = \sum_n n|\langle E_{00}, \tau, u|\tilde{K}_n\rangle|^2 = \sum_n n|\tilde{\phi}_n(\tau, u)|^2 \,. \tag{8}$$

- Hamiltonian for Integrable and Chaotic spin chain

$$H_{\text{int}}(J, h) = -\sum_{i=1}^{N} \left[J\sigma^z_{i+1}\sigma^z_i + h\sigma^x_i \right] , \tag{9}$$

and

$$H_{\text{cha}}(J, h, g) = -\sum_{i=1}^{N} \left[J\sigma^z_{i+1}\sigma^z_i + h\sigma^x_i + g\sigma^z_i \right] . \tag{10}$$

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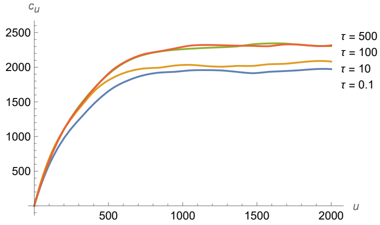
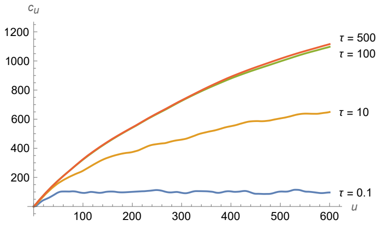
Spread Complexity for Integrable and Chaotic Systems



(a) H_0 and H_1 are both chaotic.



(b) H_0 is integrable and H_1 is chaotic.

Figure: Spread Complexity for u evolution.

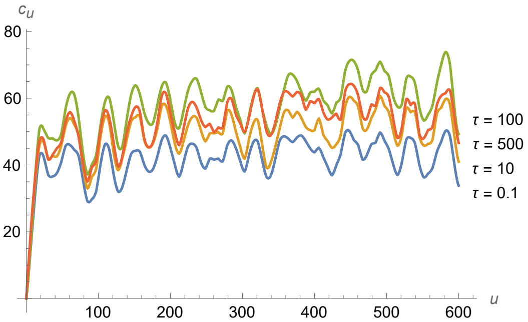


Figure: H_0 and H_1 are both integrable.

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Summary

- Identified $\tilde{b_1}$ as the Squared Commutator between the Perturbing Operator and the Hamiltonian.
- $\tilde{b_1}$ can be used measure the scrambling of the Quantum information.
- Perturbation Operator evolved under chaotic H_1 saturate the maximum value of spread complexity.
- The initial growth of Spread Complexity is dependent on coherence of initial state with respect to the eigen basis of H_0.

Thank You