Finding Critical theories using Entanglement

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In-House Symposium 2024

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February 3, 2024

Introduction

- Goal: To find possible sets of critical exponents at phase transitions.
- Recently, an approach was made to find critical ground states from an Entanglement Entropy Function $\tilde{S}$[1]

$$\tilde{S}(|\psi\rangle) := (S_{AB} + S_{BC}) - \eta(S_A + S_C) - (1-\eta)(S_B + S_{ABC}), \quad (1)$$

  where η is the cross ratio
- $(|\psi_{\alpha^*}\rangle = |GS\rangle_{critical}) \implies \left(\left.\frac{\delta(\tilde{S}(|\psi_\alpha\rangle))}{\delta\alpha}\right|_{\alpha^*} = 0 \right)$

[1] Ting Chun Lin and John McGreevy. "Conformal Field Theory Ground States as Critical Points of an Entropy Function". In: (2023). arXiv: 2303.05444 [hep-th].

Spin state ansatz

- A general state in the spin basis can be written as

$$|\psi\rangle = \sum_{s1,\ldots,s_N} \psi(s_1,\ldots,s_N)|s_1,\ldots,s_N\rangle$$

- Ansatz for ground state wavefunction[2]

$$\psi(s_1,\ldots,s_N) \propto \delta_s e^{i\pi/2\sum_{i:odd}(s_i)} \prod_{n>m}^{N} \left[sin\left(\frac{\pi(n-m)}{N}\right)\right]^{\alpha s_n s_m}$$

- Example: The *Heisenberg XXZ* spin chain Hamiltonian

$$H_{XXZ} = \sum_{i=1}^{N}(X_i X_{i+1} + Y_i Y_{i+1} + \Delta Z_i Z_{i+1})$$

$$\text{Phases of the model} = \begin{cases} \text{Ferromagnetic,} & \text{if}\Delta \leq -1 \\ \text{Gapless (c=1) CFT,} & \text{if} -1 < \Delta \leq 1 \\ \text{Gapped anti ferromagnetic} & \text{if}\Delta > 1 \end{cases}$$

[2] J. Ignacio Cirac and Germá n Sierra. "Infinite matrix product states, conformal field theory, and the Haldane-Shastry model". In: *Physical Review B* 81.10 (Mar. 2010). DOI: 10.1103/physrevb.81.104431. URL: https://doi.org/10.1103%2Fphysrevb.81.104431.

Optimized $\alpha^* = f(\Delta)$

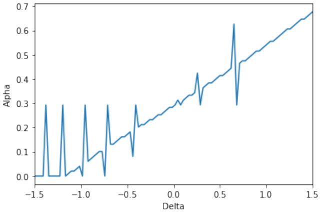


Figure: α^* vs Δ plot for N = 6

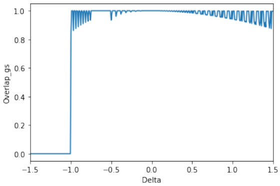


Figure: Overlap of optimized iMPS state with *exact* Ground state from ED

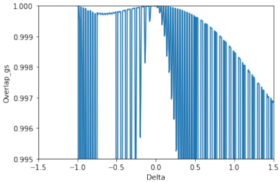


Figure: Zoomed in view of Overlap of the optimized iMPS and *exact* Ground state from ED

Entropy function S_Δ

- Energy variational optimization is specific to the Heisenberg XXZ Hamiltonian.
- Variation of Entropy function ($\tilde{S}$) to get General CFTs that admit representation in the ansatz
- Entropy Function $\tilde{S}$ vs α

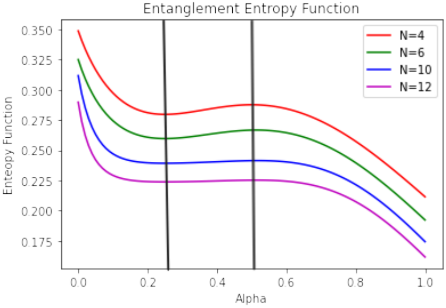


Figure: S_Δ for different system sizes

Reconstruction of Hamiltonian

- We reconstruct the CFT Hamiltonian from the critical ansatz and match its spectrum with that of the corresponding XXZ Hamiltonian
- The Entanglement Hamiltonian(EH) is defined[3]

$$K_X = -\log(\rho_X)$$

- EH written in a Local integral form[3],

$$K_{[x_1,x_2]} = 2\pi \int_{x_1}^{x_2} dx \frac{(x-x_1)(x_2-x)}{(x_2-x_1)} T_{00}(x) + \text{const.}$$

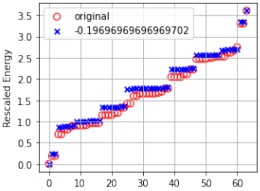
[3] John Cardy and Erik Tonni. "Entanglement Hamiltonians in two-dimensional conformal field theory". In: *Journal of Statistical Mechanics: Theory and Experiment* 2016.12 (Dec. 2016), p. 123103. ISSN: 1742-5468. DOI: 10.1088/1742-5468/2016/12/123103. URL: http://dx.doi.org/10.1088/1742-5468/2016/12/123103.

Reconstruction of Hamiltonian

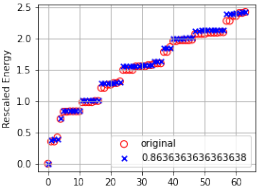
- Hamiltonian $H = \int_{-\infty}^{\infty} dx T_{00}(x)$,

$$H = \frac{1}{\pi} \sum_{j=-\infty}^{\infty} \left(K'_{[j,j+2]} - K'_{[j,j+1]} \right), \text{ where } K'_X = K_X - \langle K_X \rangle$$

- Given a state, we can find the corresponding CFT Hamiltonian and its spectrum
- Spectra of the original XXZ Hamiltonian (red circle) and the reconstructed Hamiltonian spectra (blue cross) for N = 6



$\alpha^* = 0.25$



$\alpha^* = 0.50$

c and Δ_{exc} distribution

- Using $\tilde{S}$ definition and local integral form, the $constant$ term *(in the critical phase)* gives,

$$\tilde{K}|\psi\rangle = \frac{c}{3}h(\eta)|\psi\rangle \text{ where } c \text{ is the central charge}$$

- Distribution of Central charge, $c = \dfrac{3\tilde{S}}{h(\eta)}$
- Allowed CFTs satisfy the bound $\Delta_{exc} \leq \dfrac{c}{8.503}$, where Δ_{exc} is the first excitation energy[4]

[4] Thomas Hartman et al. *Snowmass White Paper: The Analytic Conformal Bootstrap*. 2022. arXiv: 2202.11012 [hep-th].

Admissible CFTs

- Entropy function in complex α space for N=8 in Fig 1
- Allowed (Black dots) and not allowed (Red dots) CFTs in Fig 2

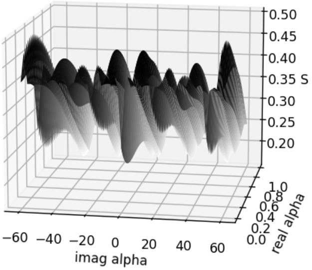


Fig 1: $\tilde{S}$ for N=8, in complex α space

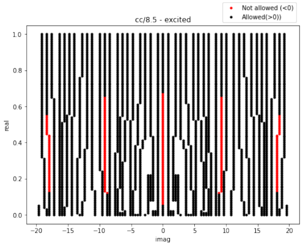
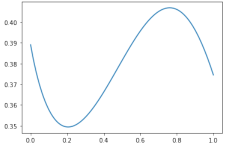


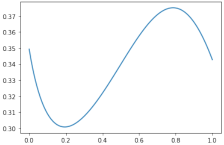
Fig 2: Admissibility of CFTs for N=8

Qutrit

- We consider spins $s_i = 0, \pm 1$ in the ansatz
- Entropy function $\tilde{S}$ for real α

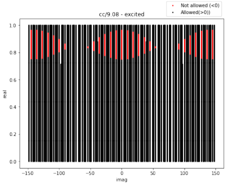


(a) N = 6 with $\alpha^* = 0.2, 0.78$

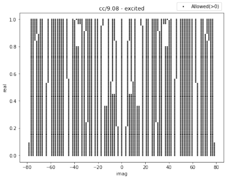


(b) N = 8 with $\alpha^* = 0.2, 0.78$

- Allowed and not allowed CFTs using qutrit



(a) N = 4



(b) N = 6

Conclusion and Future Directions

- We were able to locate and identify CFT Ground states in the XXZ spin chain which admitted the representation of the ansatz.
- However we have found solutions in the imaginary parameter sometimes violating the bootstrap bounds coming from unitarity
- Energy optimization of Spin 1 XXZ spin chain Hamiltonian using Qutrit ansatz and how it compares with entanglement optimization using qutrits
- Construct an Entropy function using Symmetry Resolved Entanglement Entropy (SREE)

Thank You