Exercise 1

Generate Gaussian distribution using uniform deviates and fit them:

1. Use the generalised transformation method in 2 dimensions.

$x_1, x_2, \cdots$ are random deviates with joint probability distributions $g(x_1, x_2, \cdots)\, dx_1\, dx_2 \cdots$

$y_1, y_2, \cdots$ each is a function of all x's (number of y's = number of x's)

$$\Rightarrow g(y_1, y_2, \cdots)dy_1 dy_2 \cdots = g(x_1, x_2, \cdots)\left|\frac{\partial(x_1, \cdots x_n)}{\partial(y_1, \cdots y_n)}\right| dx_1 dx_2 \cdots$$

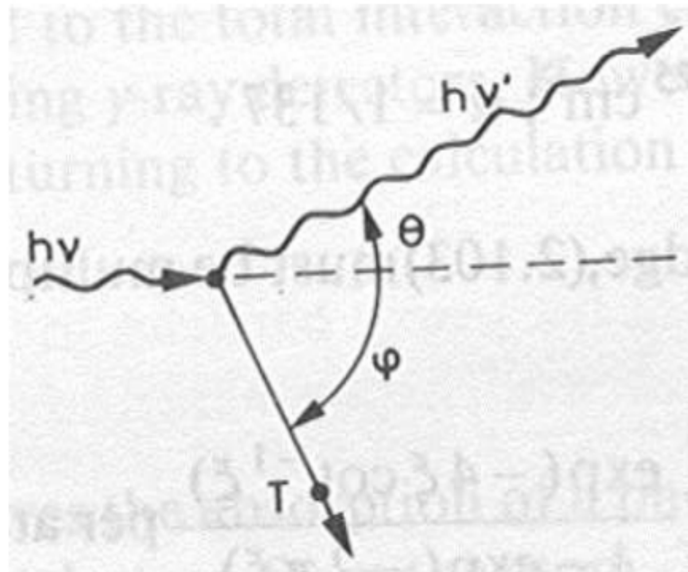
$\left|\frac{\partial(x_1, \cdots x_n)}{\partial(y_1, \cdots y_n)}\right|$ is the Jacobian determinant of x's for y's

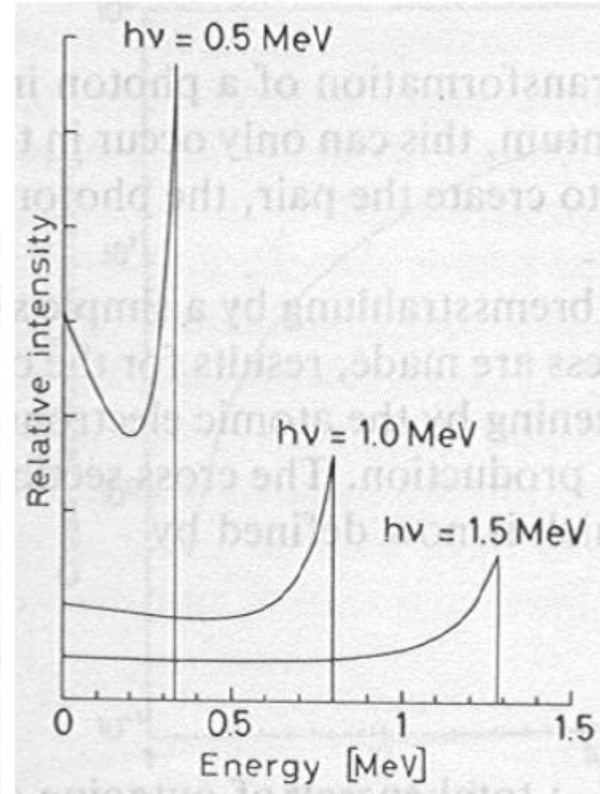
With a proper choice of y_1, y_2, generate the distribution

2. Use the Central limit theorem

3. Use acceptance and rejection method

Compton scattering





$$h\,\nu' = \frac{h\,\nu}{1+\gamma\,(1-\cos\theta)}, \qquad T = h\,\nu - h\,\nu' = h\,\nu\frac{\gamma\,(1-\cos\theta)}{1+\gamma\,(1-\cos\theta)}$$

$$\cos\theta = 1 - \frac{2}{(1+\gamma)^2\,\tan^2\,\phi + 1}, \qquad \cot\,\phi = (1+\gamma)\tan(\theta/2)$$

Klein-Nishida formula :

$$\frac{d\,\sigma}{d\,\Omega} = \frac{r_e^2}{2}\frac{1}{[1+\gamma\,(1-\cos\theta)]^2}\left[1+\cos^2\theta + \frac{\gamma^2\,(1-\cos\theta)^2}{1+\gamma\,(1-\cos\theta)}\right]$$

$$\frac{d\,\sigma}{d\,T} = \frac{\pi\,r_e^2}{m_e\,c^2\,\gamma^2}\left[2+\frac{s^2}{\gamma^2\,(1-s)^2}+\frac{s}{1-s}\left(s-\frac{2}{\gamma}\right)\right]$$

$$\frac{d\,\sigma}{d\,\epsilon} = \pi\,r_e^2\frac{m_e\,c^2}{h\,\nu}Z\left[\frac{1}{\epsilon}+\epsilon\right]\left[1-\frac{\epsilon\,\sin^2\,\theta}{1+\epsilon^2}\right]$$

$$\gamma = h\,\nu/(m_e\,c^2), \quad s = T/(h\,\nu), \quad T_{max} = h\,\nu\left(\frac{2\gamma}{1+2\gamma}\right), \quad \epsilon = \frac{h\,\nu'}{h\,\nu}$$

Generation of Compton scattering in Geant4

The value of relative energy of scattered photon, ϵ corresponding to the minimum photon energy,

$$\epsilon_0 = \frac{m_e\, c^2}{m_e\, c^2 + 2 \times h\,\nu}, \qquad hence \quad \epsilon \in [\epsilon_0, 1]$$

$$\Phi\epsilon \approx \left[\frac{1}{\epsilon} + \epsilon\right] \left[1 - \frac{\epsilon \sin^2\theta}{1+\epsilon^2}\right] = \sum_i \alpha_i \times f_i(\epsilon) \times g_i(\epsilon) = [\alpha_1\, f_1(\epsilon) + \alpha_2\, f_2(\epsilon)] \cdot g(\epsilon)$$

where,

$$\alpha_1 = ln(1/\epsilon_0), \quad f_1(\epsilon) = 1/(\alpha_1\, \epsilon), \quad \alpha_2 = (1-\epsilon_0^2)/2, \quad f_2(\epsilon) = \epsilon/\alpha_2$$

f_1 and f_2 are probability density functions defined on the interval $[\epsilon_0, 1]$, and

$$g(\epsilon) = \left[1 - \frac{\epsilon}{1+\epsilon^2}\sin^2\theta\right]$$

is the rejection function for each $\epsilon \in [\epsilon_0, 1] \Longrightarrow 0 < g(\epsilon) \leq 1$

Use three uniform random number r, r', r'',

1. if $(r < \alpha_1/(\alpha_1+\alpha_2))$ select $f_1(\epsilon)$, otherwise select $f_2(\epsilon)$
2. sample ϵ
   - for $f_1 : \epsilon = \epsilon_0^{r'} (\equiv exp(-r'\,\alpha_1)$
   - for $f_2 : \epsilon^2 = \epsilon_0^2 + (1-\epsilon_0^2)\, r'$
3. Calculate $\sin^2\theta = t\,(2-t)$, where $t \equiv (1-\cos\theta) = m_e\, c^2\,(1-\epsilon)/(h\,\nu')$
4. if $g(\epsilon) \geq r''$ accept ϵ, otherwise again generate r, r', r''

Exercise 2

- Write a program for Compton scattering and use it to generate the angular distribution for the photon and electron for incident photons of 0.5 GeV, 1.0 GeV and 1.5 GeV and fit the distributions with theoretical expectation. Also, obtain the energy distribution of the electron and compare that with the one shown in previous pages.

Exercise 3

- Consider the process of W-pair production in an electron-positron collision (the two W's going back to back and the total energy of the W's is 189 GeV) and both the W's decay to a pair of jets (use theta and phi of the first W to be theta = 60 degree and phi = 45 degree)

- Jets are measured in the calorimeter with an energy resolution of 10%, theta and phi resolutions of 0.5 degrees.

- Generate a sample of 1000 such events and then reconstruct back W-mass using energy-momentum conservation constraint

- Add a second constraint that both the W's have the same mass

- Compare the measured mass versus fitted mass from the two sets of fits

- Fit the mass distributions using Crystal Ball formula (Gaussian on one side and exponential on other side).

Exercise 4

- Generate top quark momenta and top quark decay angles (both in theta and phi) in the top quark (mass 173.34 GeV) rest frame using uniformly generated random numbers.

- Calculate the b-quark (mass 4.18 GeV) and W-boson (mass 80.37 GeV) momenta in the top rest frame (using top quark momentum and decay angles). Boost b quark and W boson momenta to the lab frame.

- Generate W boson decay angles in its rest frame again using uniformly generated random numbers.

- Then, from W momentum and decay angles, it calculates the momenta of light quarks from the W boson. Boosts light quarks to lab frame.

—> At this stage, we have all the quark momenta (q1, q2, b) in the lab frame

- Perform smearing of quark momenta (in E, theta, phi) separately for q1, q2, and b. Energy resolution of 10% and angular resolution of 1 degree.

- Perform kinematic fitting using the top quark mass constraint (up to a minimization of chi^2 or a maximum number of iterations).

- Plot quark momenta and top mass after the kinematic fit.