Pathirana and Gurevich, Front. Electr. Mat. 3, 1246016 (2023)

# Superheating field in superconductors with nanostructured surfaces

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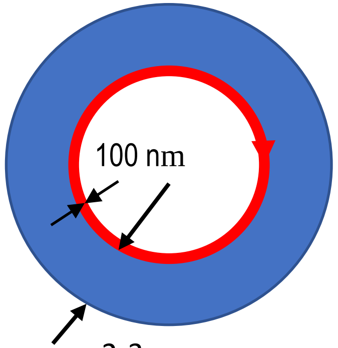
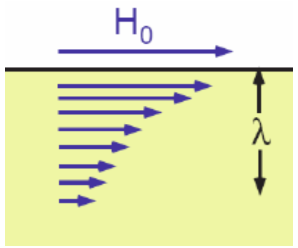
# What are the SRF field limits and how could they be raised?

- The widely used GL theory of dc superheating field $H_s(T)$ is applicable near $T_c$ but not at $T << T_c$.
- Only $H_s(0) = 0.84H_c$ at $\kappa = \lambda/\xi \gg 1$ in the clean limit (Galaiko 1966, Catelani and Sethna, 2008) and for arbitrary impurity concentration (Lin and Gurevich, 2012) have been calculated.
- How different can the dynamic superheating field $H_d(T,f)$ be from the static $H_s(T)$ at GHz frequencies?
  $H_d(T,f) \to \sqrt{2}H_s(T), \quad T \approx T_c$ (Sheikhzada and Gurevich, 2020)
- Increase of $H_s$ by surface nanostructuring: dirty layers, SIS or SNS multilayers for which $H_s$ has only been calculated in the limit of $\kappa \to \infty$
- We report GL numerical calculations of $H_s$ at realistic finite $\kappa$ for:
  - Dirty surface layers
  - SIS structures: dirty Nb-I-clean Nb, $Nb_3Sn$-I-Nb and others
- A significant boost of $H_s$ can be achieved by optimizing SIS parameters.

# Superfluid pairbreaking velocity




Everything has been said before, but since nobody listens we have to keep going back and beginning all over again.

**Andre Gide**

RF field induces supercurrent density carried by Cooper pairs

$$H(z) = H_p e^{-z/\lambda},$$
$$J(z) = (H_p/\lambda)e^{-z/\lambda}$$

$$J = e n_s v_s$$

Depairing current density at the surface:

**$J_d = e n v_c$ = 500 MA/cm$^2$**

Superfluid velocity $v_s$ cannot exceed the critical value $v_c$ above which the superconducting state breaks down. In the clean limit:

$$J_d \simeq H_c/\lambda$$

$$v_c = \frac{\Delta}{p_F}$$

Thermodynamic critical field

**$H_c$ = 200 mT, $\lambda$ = 40 nm.** In Nb cavities the dc depairing limit at 2 GHz and 2K has been achieved on "industrial scale"

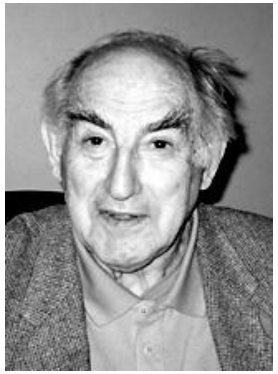
At $v = v_c \simeq 1\mathrm{km/s}$ for Nb the moving condensate of Cooper pairs becomes unstable but the density of Cooper pairs does not vanish.

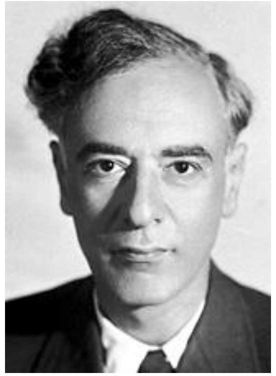
**Bardeen, Rev. Mod. Phys. 34, 667 (1962), deGennes, 1968**

# Ginzburg-Landau equations

$$\xi^2\left(\nabla + \frac{2\pi i}{\phi_0}\vec{A}\right)^2\psi + \psi - \psi|\psi|^2 = 0,$$

$$\nabla\times\nabla\times\vec{A} = \vec{J}_s = -\frac{|\psi|^2}{\lambda^2}\left(\frac{\phi_0}{2\pi}\nabla\theta + \vec{A}\right)$$





- Phenomenological Ginzburg-Landau theory **(1950, Nobel prize 2003)**

- Coupled nonlinear PDEs for the pair wave function $\psi(\mathbf{r}) = e^{i\theta}\Delta/\Delta_0$ and magnetic vector-potential A(r).
- Coherence length $\xi$ and magnetic penetration depth $\lambda$
- Thermodynamic critical field:

$$B_c = \frac{\phi_0}{2^{3/2}\pi\lambda\xi}$$

- Boundary condition between a superconductor and vacuum $J_s = 0$: $$\left(\nabla + \frac{2\pi i}{\phi_0}\vec{A}\right)\psi\vec{n} = 0$$

# GL depairing current density

- Uniform current-carrying state with $\psi = \psi_0 \exp(-iqx)$, where q is proportional to the velocity of the Cooper pairs. The GL equations give:

$$\psi_0^2 = 1 - \xi^2 q^2, \qquad J = \frac{\psi_0^2 \phi_0 q}{2\pi\lambda^2 \mu_0}$$

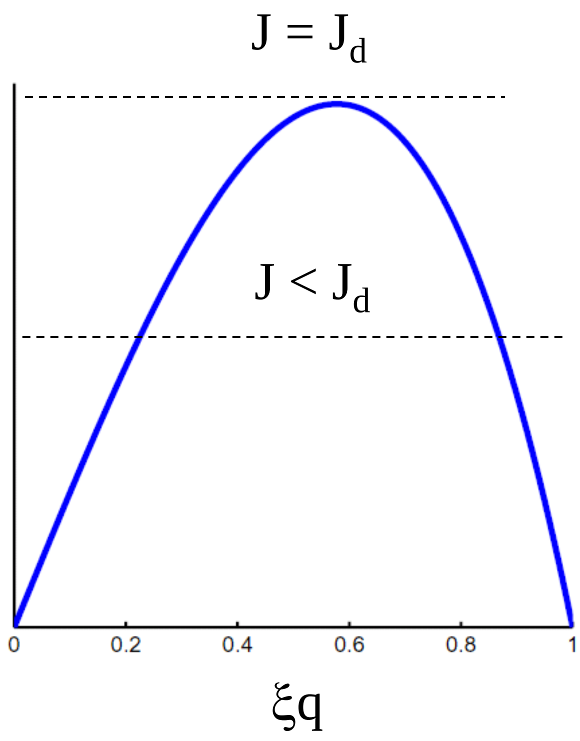
- Current density as a function of q:

$$J = \frac{\phi_0 q}{2\pi\lambda^2 \mu_0}(1 - \xi^2 q^2)$$

← Suppression of $n_s$ by current

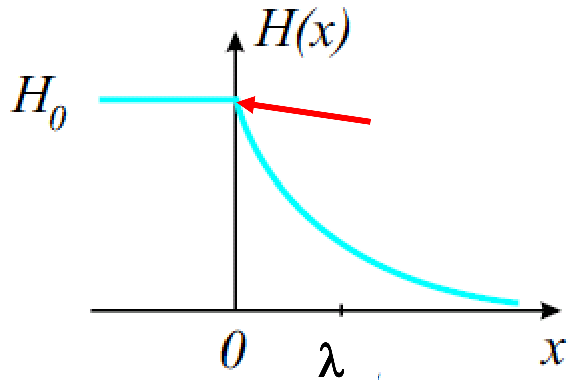
- Maximum J at $q = 1/\xi\sqrt{3}$ defines the GL depairing current density:

$$J_d = \frac{\phi_0}{3\sqrt{3}\pi\mu_0\lambda^2\xi} \cong 0.54\frac{H_c}{\lambda} \propto \left(1 - \frac{T}{T_c}\right)^{3/2}$$




At $J = J_d$ the superfluid density $n_c(J\_d) = (2/3)n_s$ does not vanish

# DC superheating field




- Meissner state becomes unstable above the superheating field $H > H_s$ as the velocity of Cooper pairs exceed the critical velocity

- GL region, $T \approx T_c$: **(Matricon and Saint-James, 1967, Chapman 1995)**

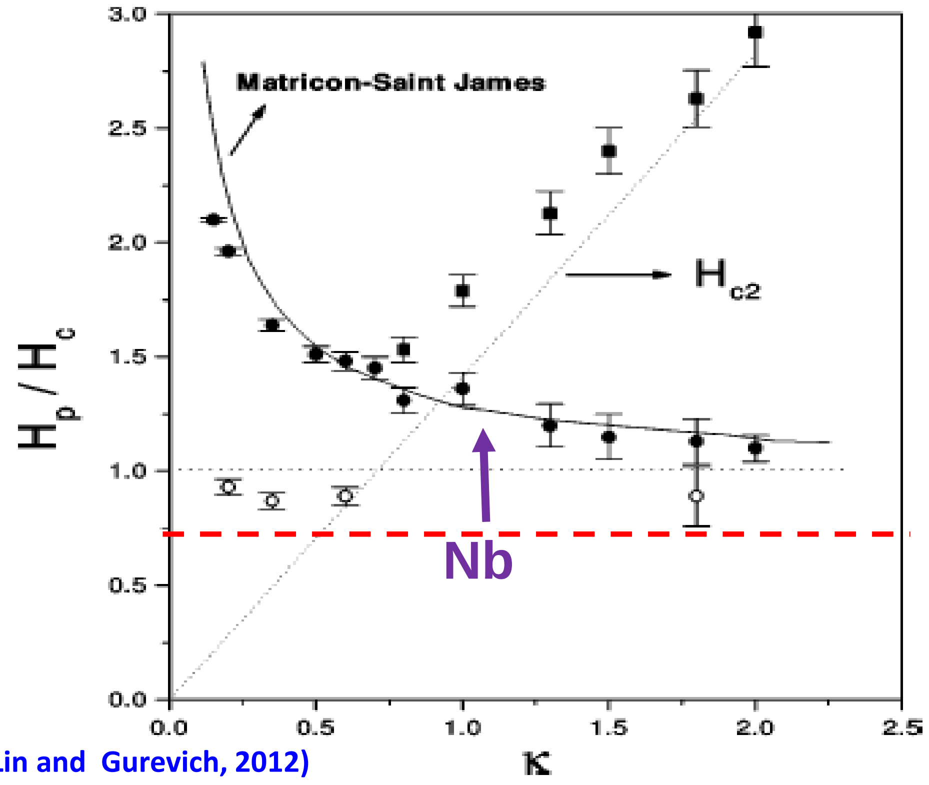
$$B_s \approx 1.2 B_c, \qquad \kappa \approx 1,$$
$$B_s \approx 0.745 B_c, \qquad \kappa \gg 1$$

- $B_s$ decreases as the GL parameter $\kappa = \lambda/\xi$ increases and the material gets dirtier.

**Why?**

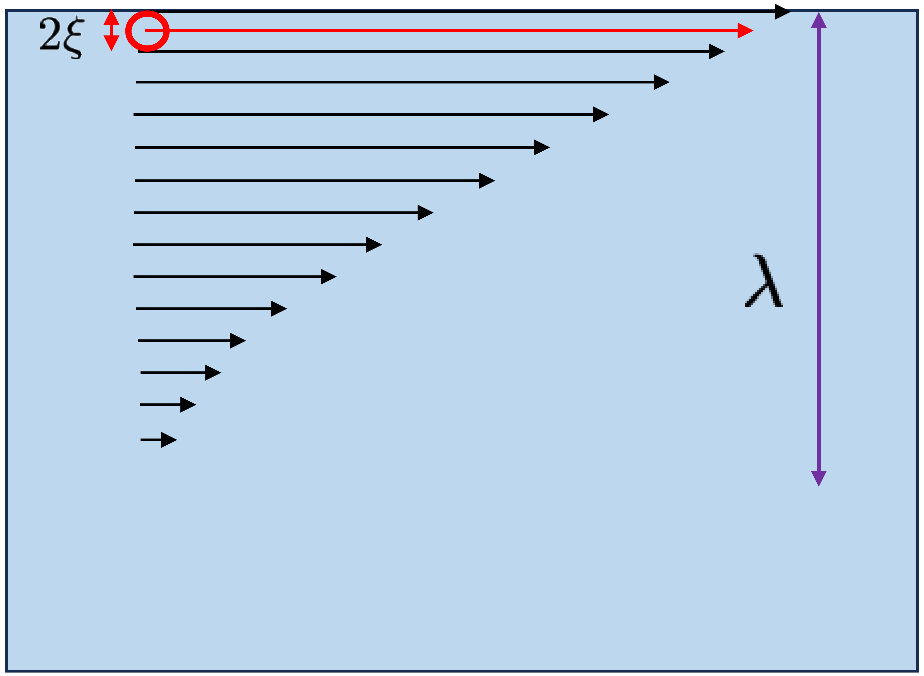
- No theory of $H_s(T)$ at low T and $\kappa = \lambda/\xi \sim 1$

Extrapolation of GL results to $T \ll T_c$ underestimates **$H_s(0) = 0.84H_c$** at $\kappa \gg 1$ in the clean limit **(Galaiko 1966, Catelani and Sethna, 2008)** and for arbitrary concentration of nonmagnetic and magnetic impurities **(Lin and Gurevich, 2012)**

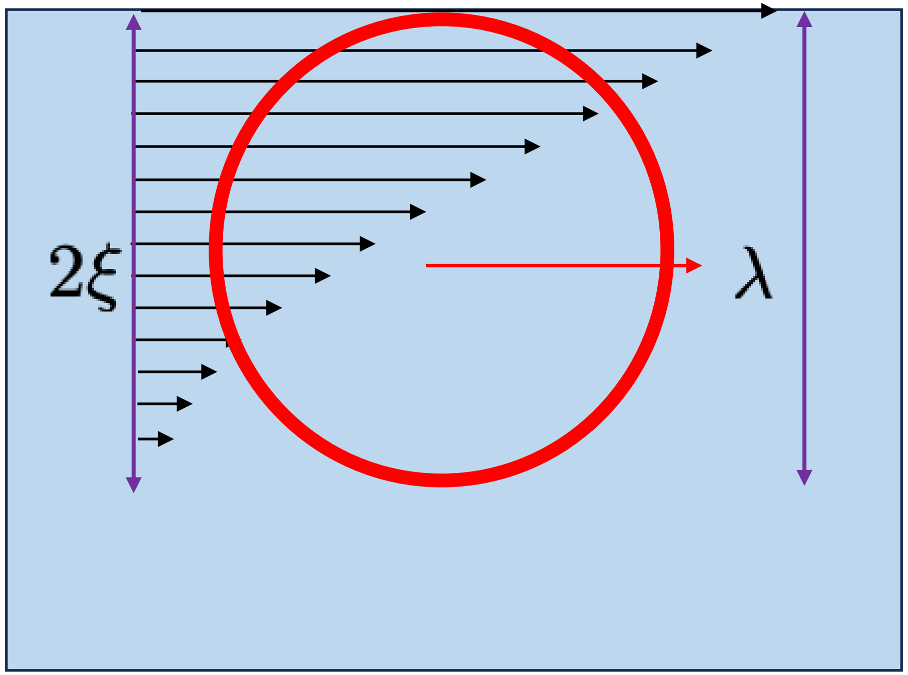


# Effect of $\kappa = \lambda/\xi$ on $H_s$

**Local EM response**




**Nonlocal EM response**

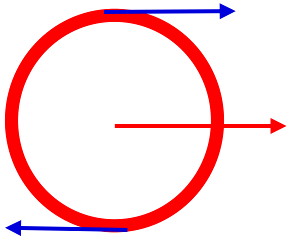



Large $\kappa = \lambda/\xi$ : Cooper pair reaches the critical velocity at the surface, where $J(0) \simeq H_s/\lambda \to J_d$. This yields the minimum superheating field at $\kappa \gg 1$:

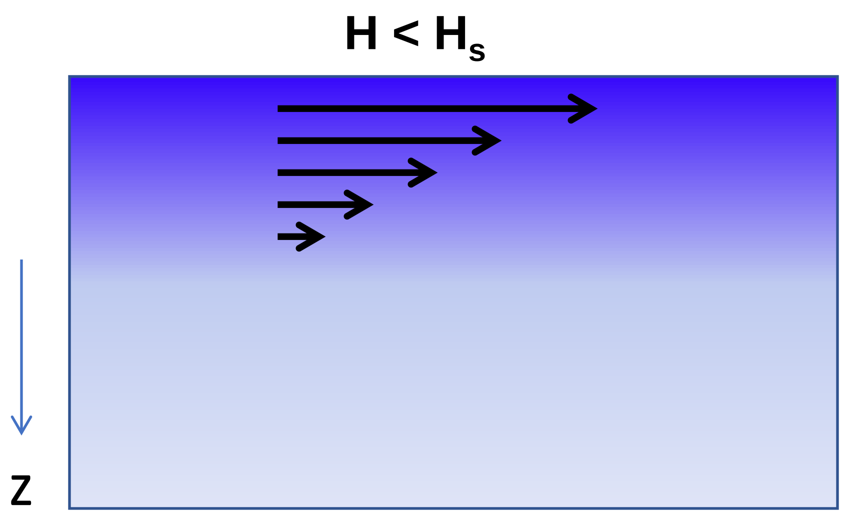
$$H_s = (\sqrt{5}/3)H_c = 0.745H_c$$

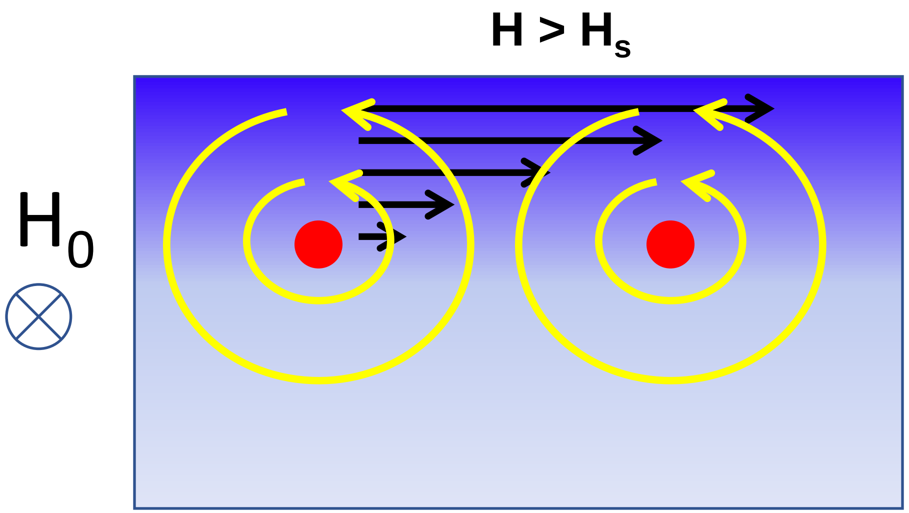
$\kappa \sim 1$ : Center of mass of Cooper pair reaches the critical velocity at J(0) > $j_d$ at higher H at the surface. As a result, $H_s(\kappa)$ Increases as $\kappa$ decreases.

Spinning Cooper pair → vortices.



# Vortices protect superconductivity at H > $H_s$




At H < $H_s$ screening currents are not strong enough to break the Meissner state at J(0) < $J_d$

At H > $H_s$ vortices reduce the current density at the surface allowing SC at higher fields

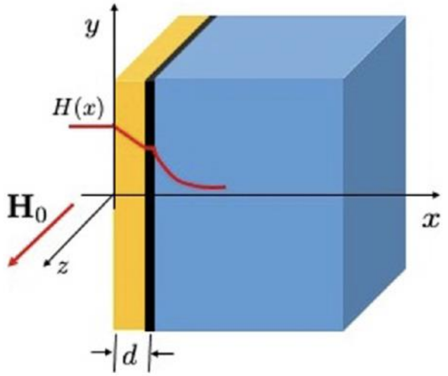
Counterflow of vortex currents against the Meissner current reduces J(0) at the surface below $J_d$ allowing SC to survive up to high fields $H_{c2}$ at which the vortex cores overlap

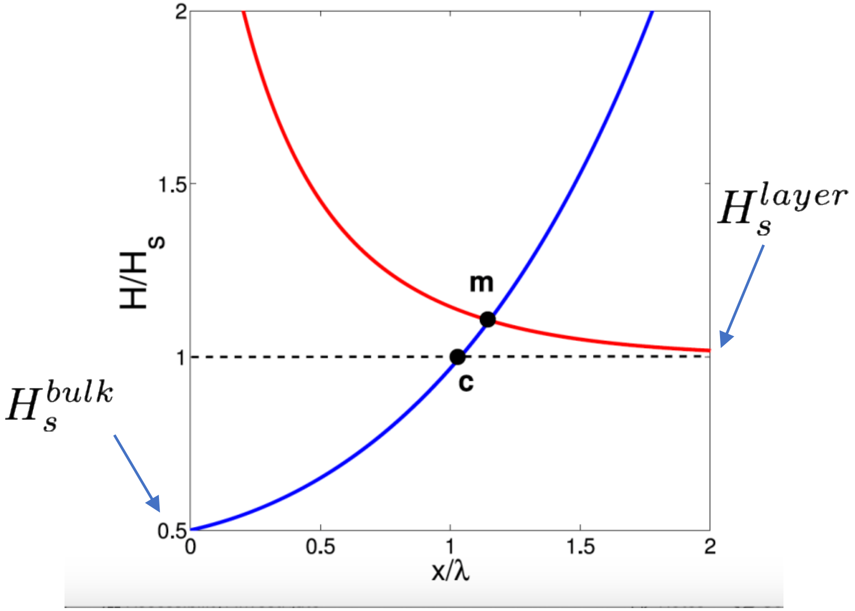
- The SRF breakdown is limited by $H_s$ but not $H_{c2}$
- Vortices penetrate with supersonic velocities $v_d \sim 10 - 20\,\mathrm{km/s}$ in $t_d \sim \xi/v_d \sim 4 \cdot 10^{-12}\,s \ll 10^{-9}\,s$
  **Embon et al Nat. Commun. 8, 85 (2017); Dobrovolskiy et al, Nat. Commun. 11, 3291 (2020)**
- Delay with penetration of vortices at H > $H_s$ does not increase the SRF field limit but destroys SC

# $H_s$ for nanostructured surfaces in the limit of $\kappa \to \infty$

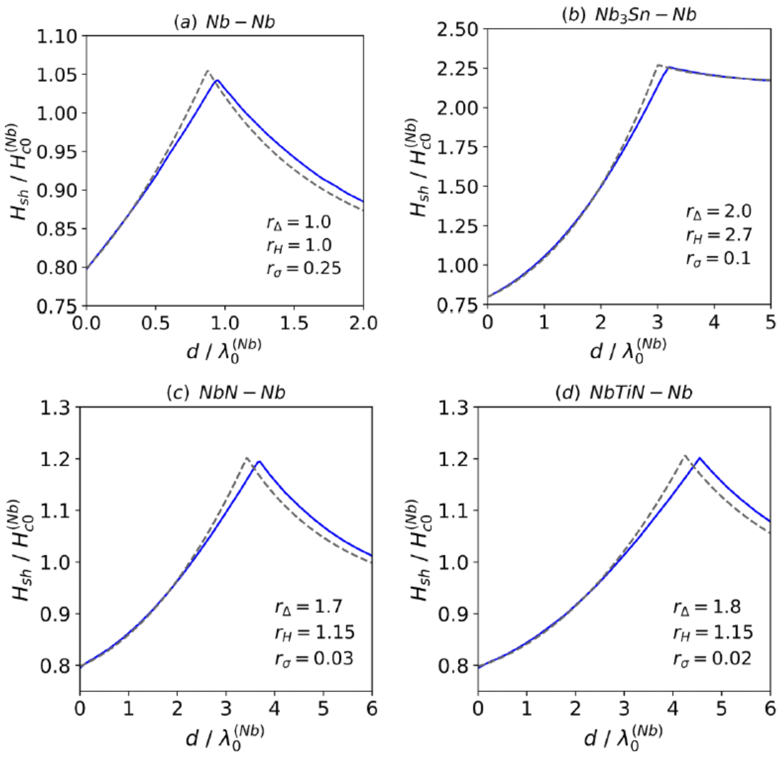
SIS London model

**Enhancement of $H_s$ by current counterflow induced in the overlaayer**



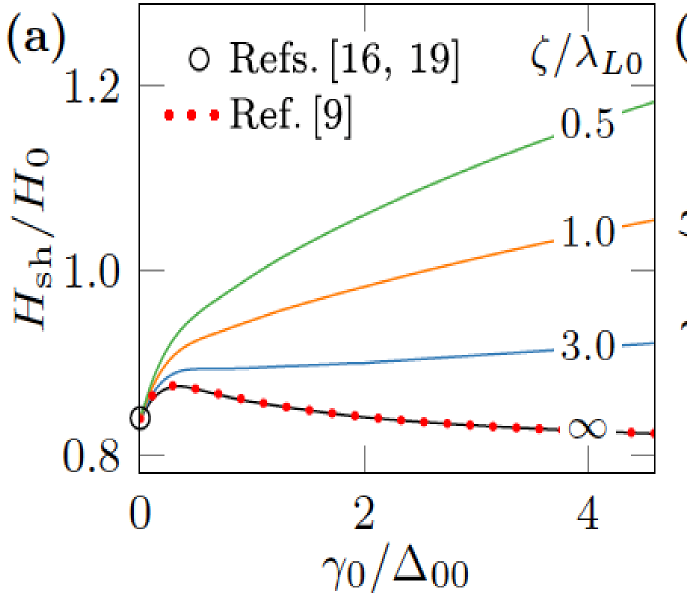




SIS, dirty limit, Usadel eqs




**Kubo, SUST 34, 045006 (2021)**

Dirty surface layer




Inhomogeneous impurity scattering rate at the surface

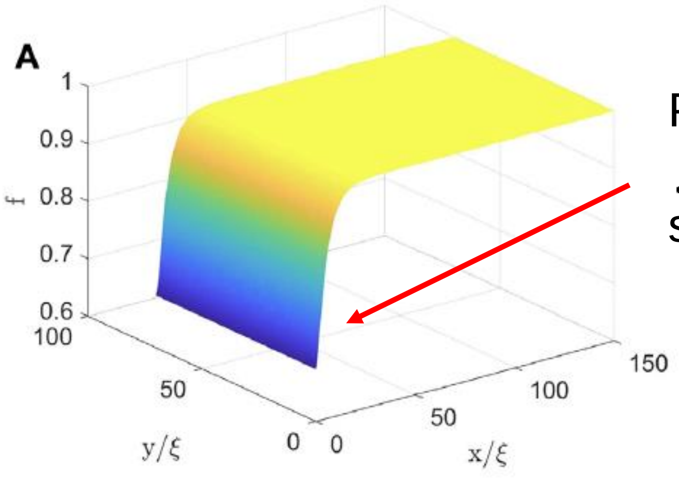
$$\gamma(x) = \gamma_0 e^{-x/\zeta}$$

**Ngampruetikorn and Sauls, Phys. Rev. Res. 1, 012015 (2019)**

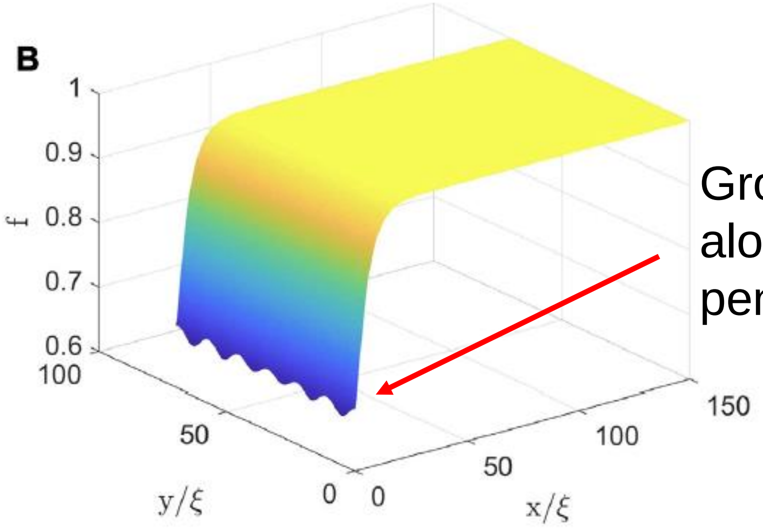
**Gurevich, APL 88, 012511 (2006); AIP Adv. 5. 117112 (2015)**
**Kubo, Iwashita, Saeki, APL 104, 045006 (2014)**

# Instability of Meissner screening at H = $H_s$ at finite $\kappa$

GL numerical simulations for Nb:
Pathirana and Gurevich, Front. Electr. Mat. 3, 1246016 (2023)




Partial suppression of $f(x) = \Delta(x)/\Delta_0$ by screening current at H < Hs




Growth of periodic perturbations along the surface preceding penetration of vortices at H > Hs

Christiansen, Solid Stat. Comm, 7, 727 (1969); Chapman SIAM J. Appl. Math , 55, 1233 (1995); Transtrum et al, PRB 83, 094505 (2011), Liarte et al, SUST 30, 033002 (2017)

GL results for $\kappa \gg 1$

$$H_s = H_c\left(\frac{\sqrt{5}}{3} + \frac{0.545}{\sqrt{\kappa}}\right)$$

Period along the surface

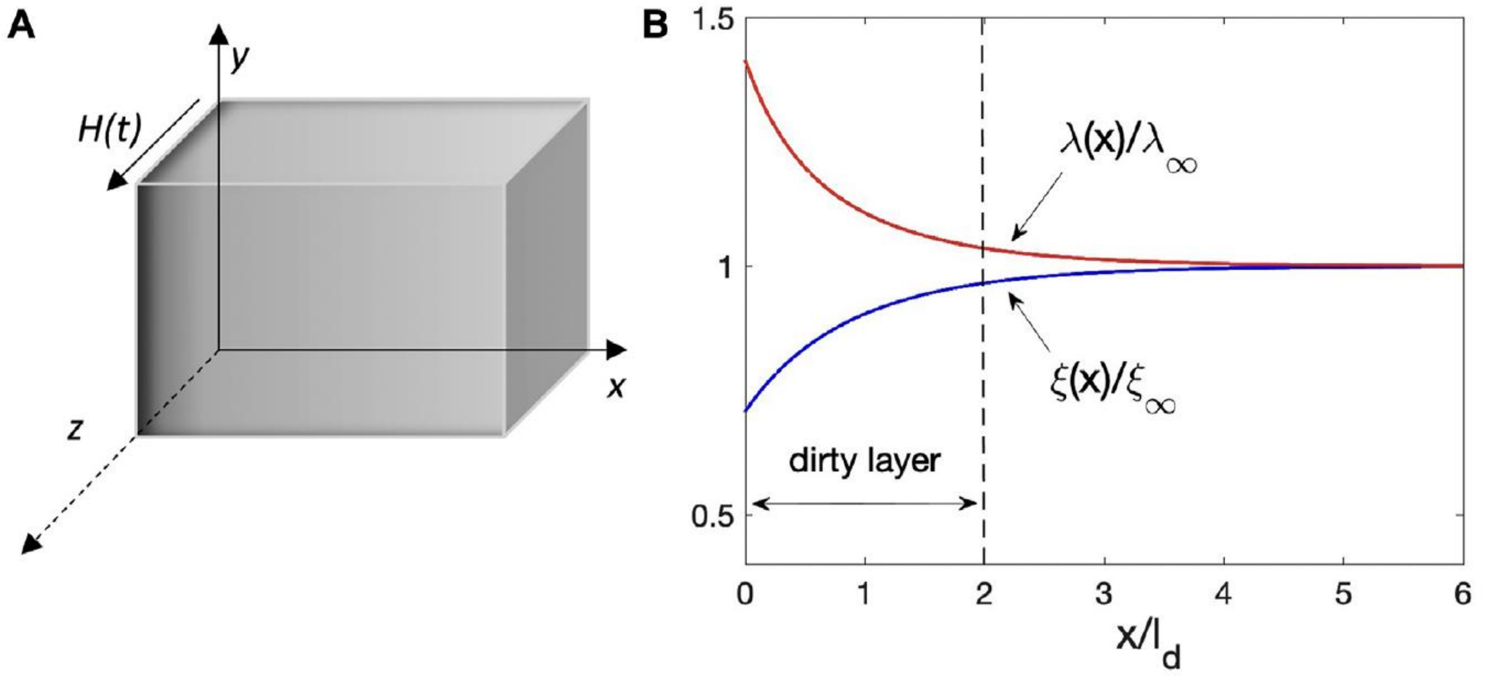
$$l_{\|} \simeq \frac{6.6\lambda}{\kappa^{3/4}}$$

Decay length perpendicular to the surface

$$l_{\perp} \sim \frac{\lambda}{\kappa^{1/2}}$$

**How can the instability lengths affect Hs for nanostructured surfaces (SIS etc)?**

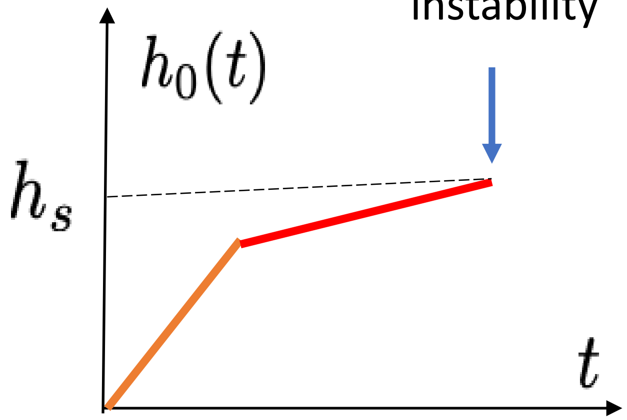
# Dirty layer at the surface




$$S(x) = \frac{\lambda^2(x)}{\lambda_\infty^2} = \frac{\xi_\infty^2}{\xi^2(x)}$$
$$= 1 - \alpha \exp(-x/l_d)$$

Slowly ramping magnetic field:

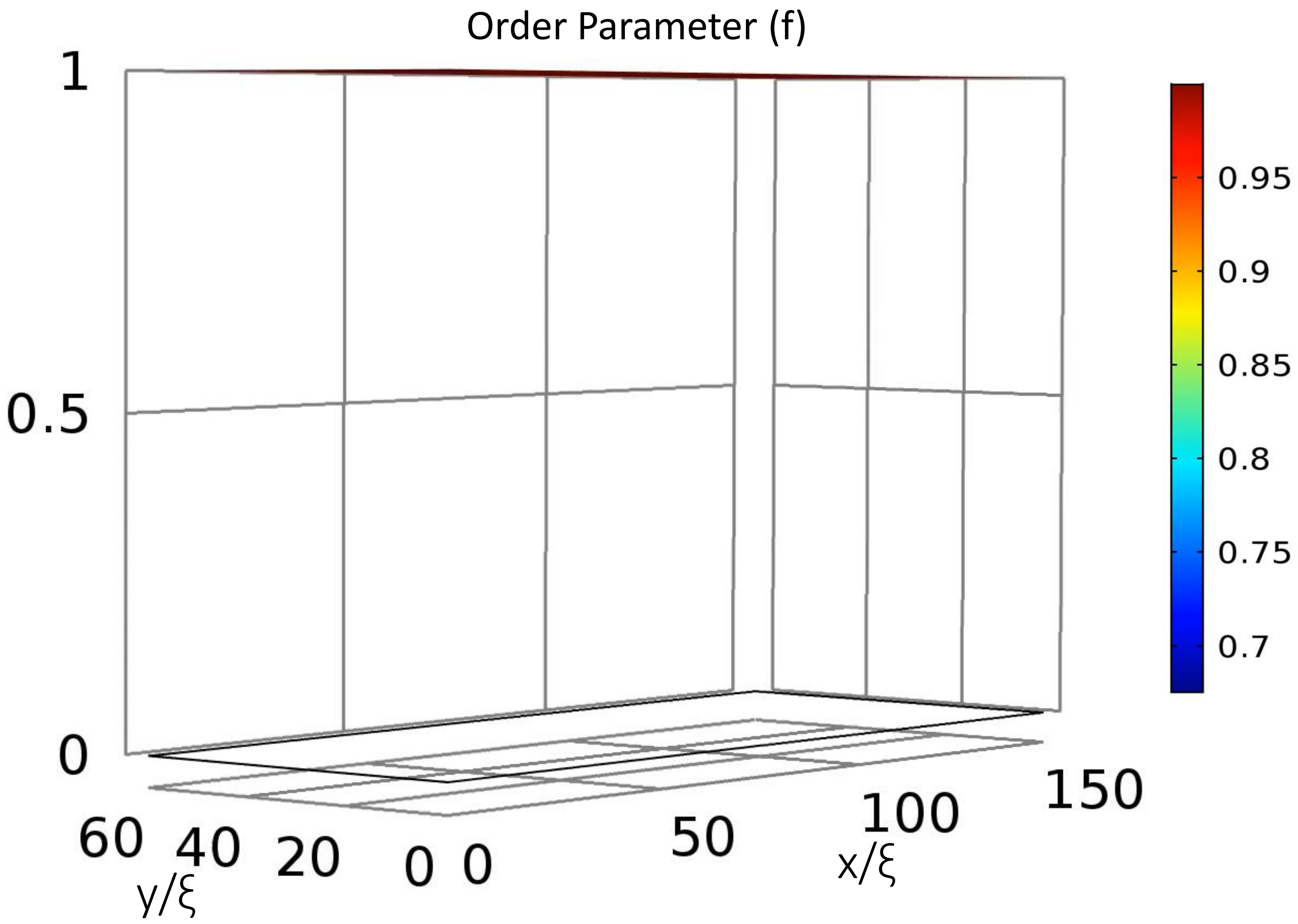
$$\dot{h}_0 = \dot{H}_0/\sqrt{2}H_c \ll 1$$



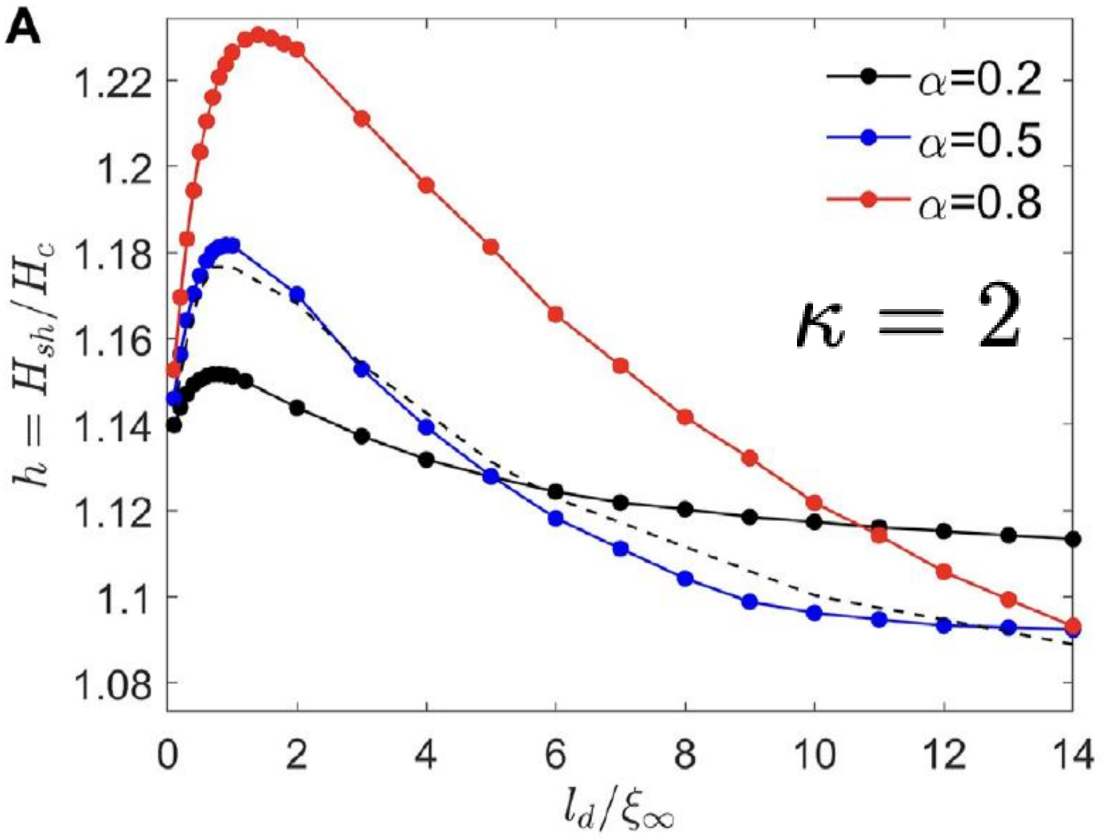

$$\dot{f} = f - f^3 + \nabla\cdot(S\nabla f) - \frac{\kappa^2}{Sf^3}\left[(\partial_x h)^2 + (\partial_y h)^2\right]$$

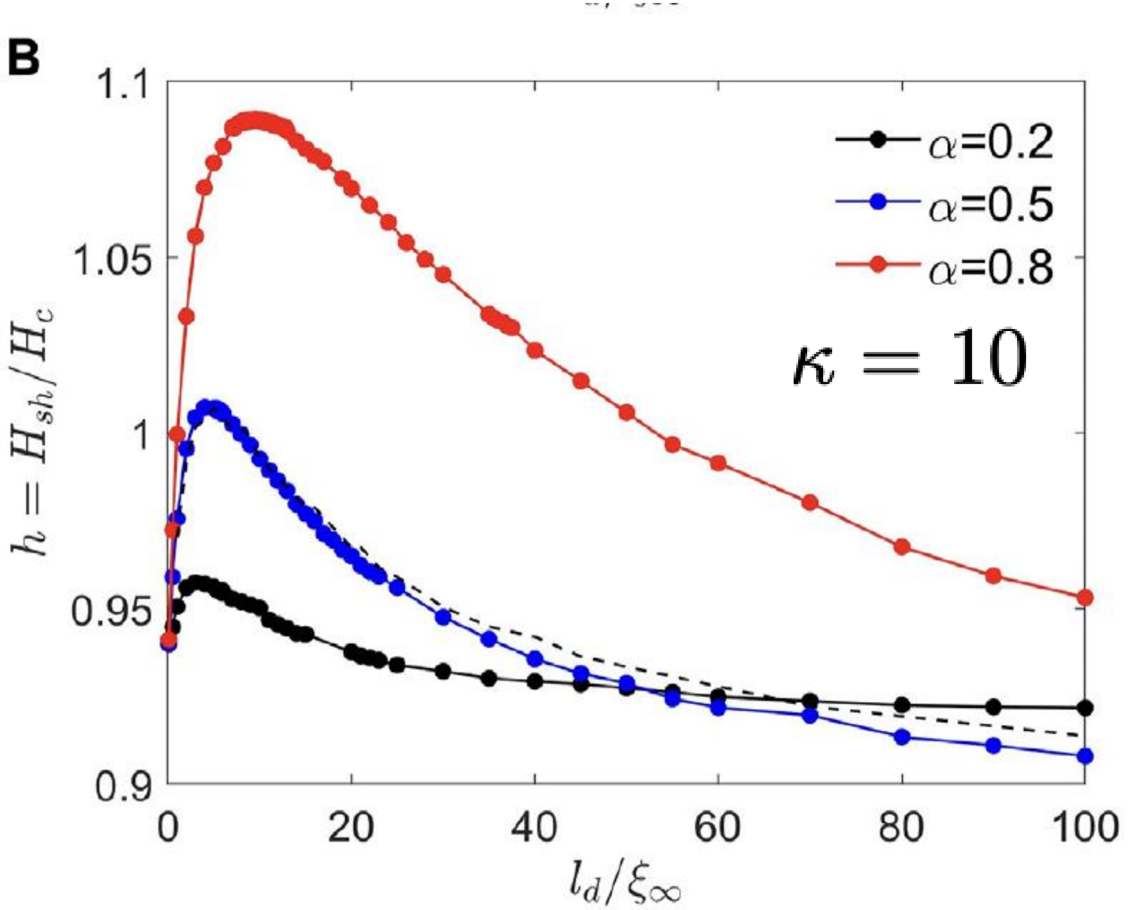
$$\nabla\cdot\left(\frac{\nabla h}{Sf^2}\right) - \frac{h}{\kappa^2} = 0$$

# Detecting superheating field



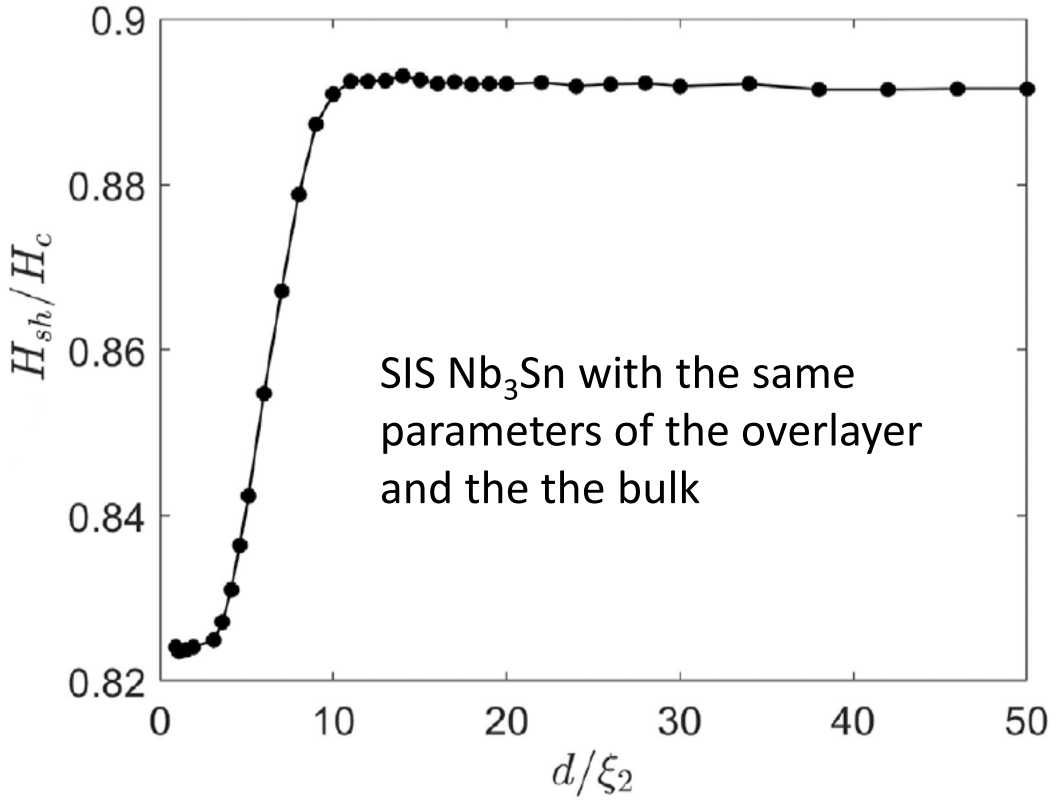
# Dirty surface layer, results

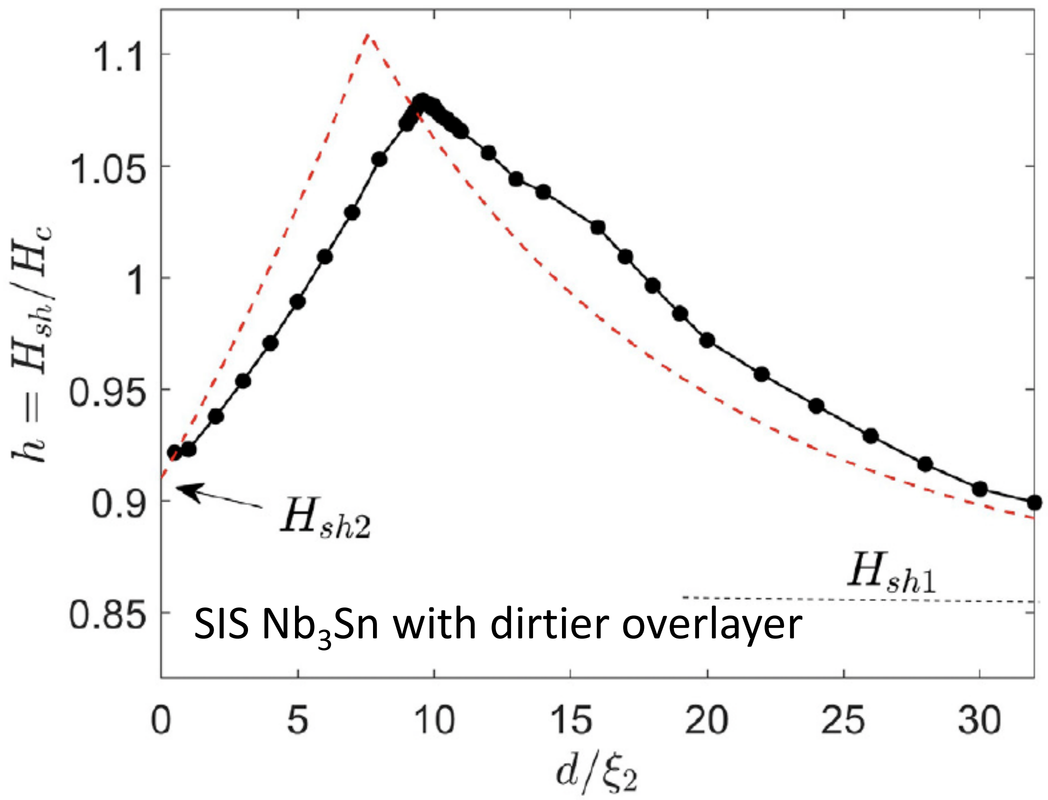



Maximum in $H_s(l_d)$ results from current counterflow induced in the dirty surface layer with larger $\lambda_0 = \lambda_\infty/\sqrt{1-\alpha}$ by the cleaner bulk with smaller $\lambda_\infty$ similar to that in SIS multilayers. Reduced Hs at larger ld is due to larger GL parameter of the dirty layer, $\kappa_0 = \kappa_\infty/(1-\alpha)$

# SIS with the same materials




Dip in $H_s(d)$ at small d results from suppression of vortex perturbations by I layer over $l_\perp \sim \sqrt{\xi\lambda}$ perpendicular to the surface

Red dashed line shows $H_s(d)$ calculated in the London model

# $H_s$ of SIS in the London model

Gurevich, AIP Adv. 5. 117112 (2015)

$$H_s(d) = \left[\cosh(d/\lambda_1) + (\lambda_2/\lambda_1)\sinh(d/\lambda_1)\right] H_{s2}, \qquad d < d_m$$

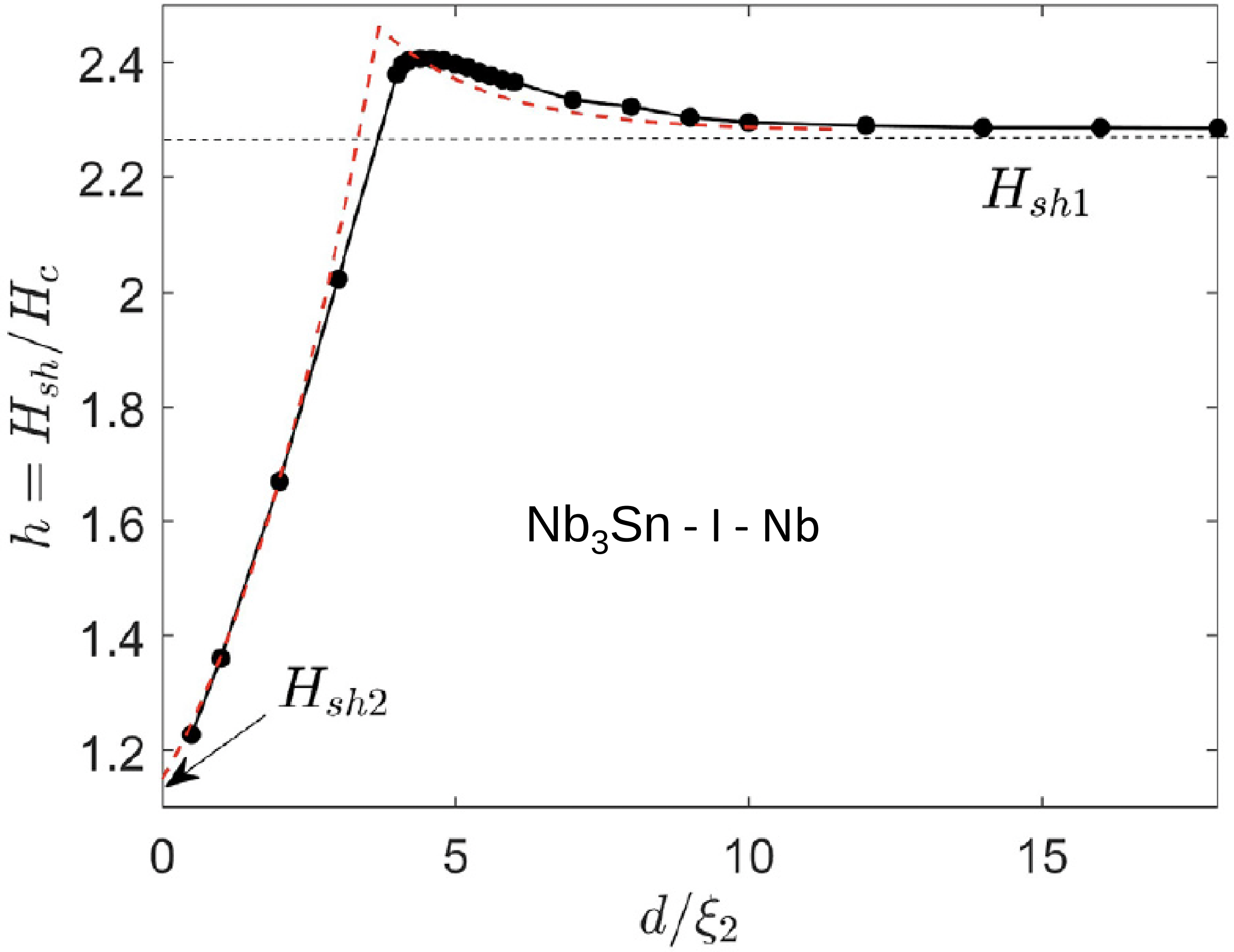
$$H_s(d) = \left[\frac{\lambda_1 + \lambda_2 \tanh(d/\lambda_1)}{\lambda_1 \tanh(d/\lambda_1) + \lambda_2}\right] H_{s1}, \qquad d > d_m$$

Cusp-like maximum Hs caused by the current counterflow effect is at the optimum overlayer thickness:

$$d_m = \lambda_1 \left[\frac{\lambda_1}{\lambda_1 + \lambda_2}\left(\frac{H_{s1}}{H_{s2}} + \sqrt{\frac{H_{s1}^2}{H_{s2}^2} + 1 - \frac{\lambda_2^2}{\lambda_1^2}}\right)\right]$$

Here $H_{s1}$ and $H_{s2}$ are the superheating fields for the bulk and overlayer materials

# High-Tc overlayer in SIS




- Thin Nb3Sn overlayer more than doubles the dc superheating field
- London model works surprisingly well
- I interlayer is necessary to prevent avalanche penetration of vortices

# Nonequilibrium SRF

Sheikhzada and Gurevich, PRB 102, 104507 (2020)

Numerical simulations of equations of nonequilibrium superconductivity

Kramer and Watts-Tobin, PRL, 40. 1041 (1978)
Watts-Tobin, Krahenbuhl and Kramer, JLTP 42, 459 (1980)

1. Kinetic equations for quasiparticles and dynamic eqs for SC condensate
2. Time-dependent GL equations for a gapped dirty superconductor

$$\gamma = 2\Delta_0 \tau_E/\hbar$$

$$\frac{\tau_\Delta}{\sqrt{1+\gamma^2|\psi|^2}}\left(\frac{\partial}{\partial t}+\frac{\gamma^2}{2}\frac{\partial|\psi|^2}{\partial t}\right)\psi = \xi^2\left(\nabla+\frac{2\pi i\mathbf{A}}{\phi_0}\right)^2\psi+(1-|\psi|^2)\psi$$

$$\nabla\times\nabla\times\mathbf{A} = -\frac{1}{\lambda^2}\mathrm{Im}\{\psi^*\left(\frac{\phi_0}{2\pi}\nabla+i\mathbf{A}\right)\psi\}-\mu_0\sigma_n\frac{\partial\mathbf{A}}{\partial t}$$

Relaxation time of SC condensate:

$$\tau_\Delta = \frac{\pi\hbar}{8k_B(T_c-T)}$$

Relaxation time constant of quasiparticles can be much larger than the RF period

For Nb at 7K, we have:

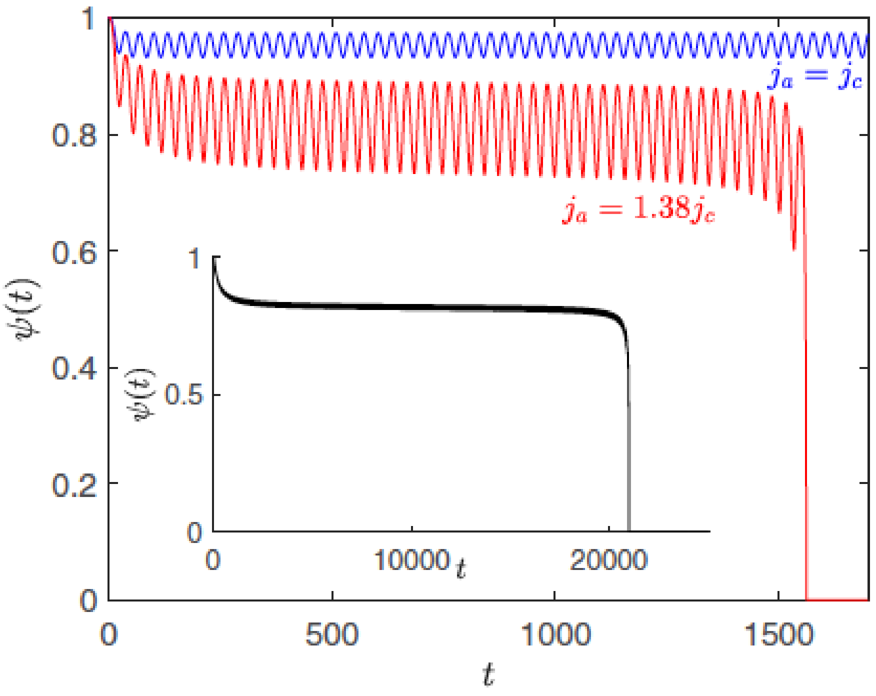
$$\tau_\Delta \approx 1.4\times 10^{-12}s \ll 1/f \sim 10^{-9}s \qquad \tau_E \approx 5\times 10^{-10}s \simeq 1/f$$

- **Nearly instantaneous reaction of SC condensate to RF field.**
- **Much slower relaxation of quasiparticles, which slows down greatly as T decreases**

Relaxation time of quasiparticles due to inelastic electron-phonon collisions:

$$\tau_E \approx \frac{\hbar}{\zeta k_B T}\left(\frac{c_s}{v_F}\right)^2\left(\frac{T_F}{T}\right)^2$$

# Dynamic SRF breakdown

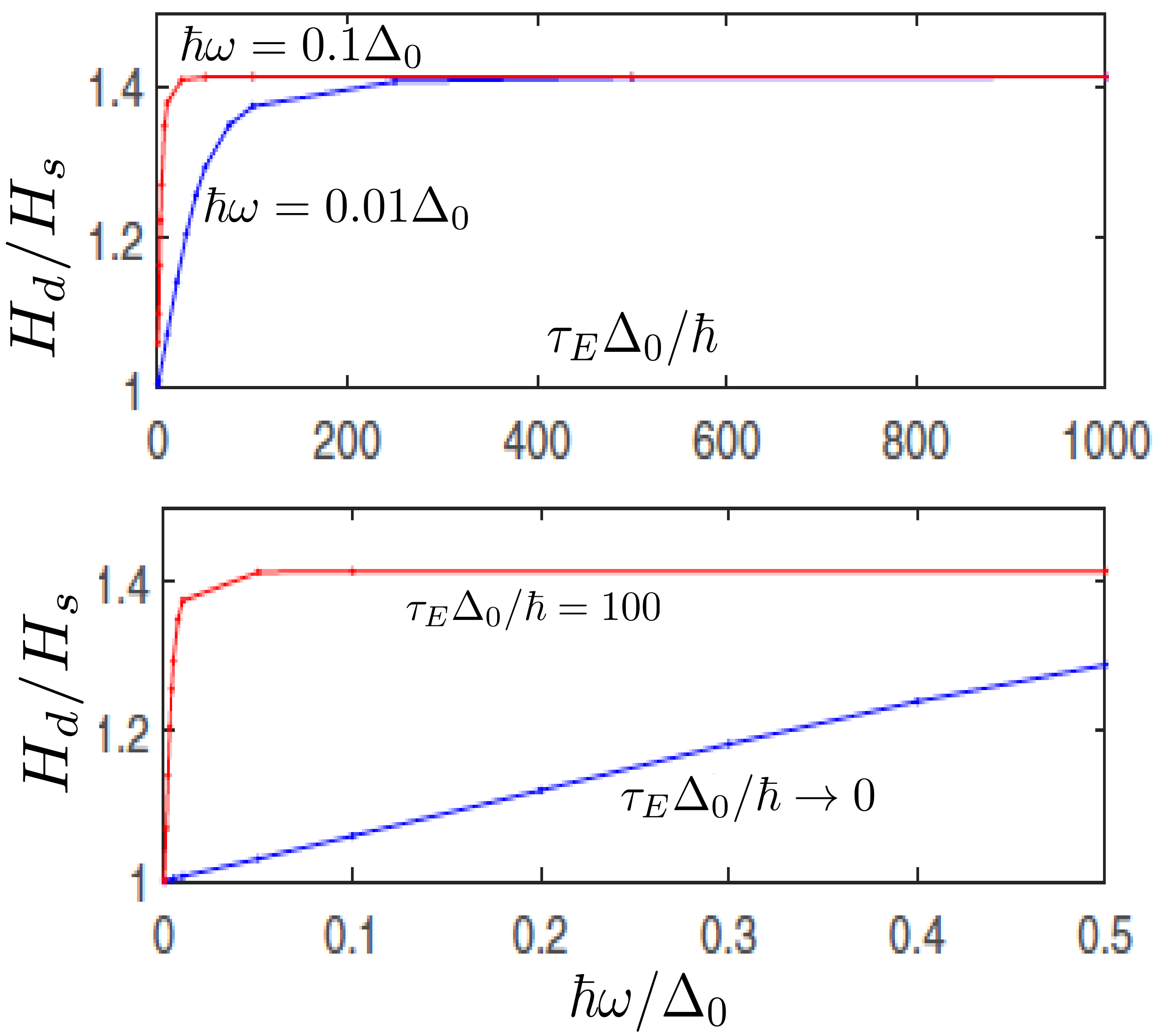



- SC breakdown field $H_d$ (T,f) increases with $\omega$ up to the limit:

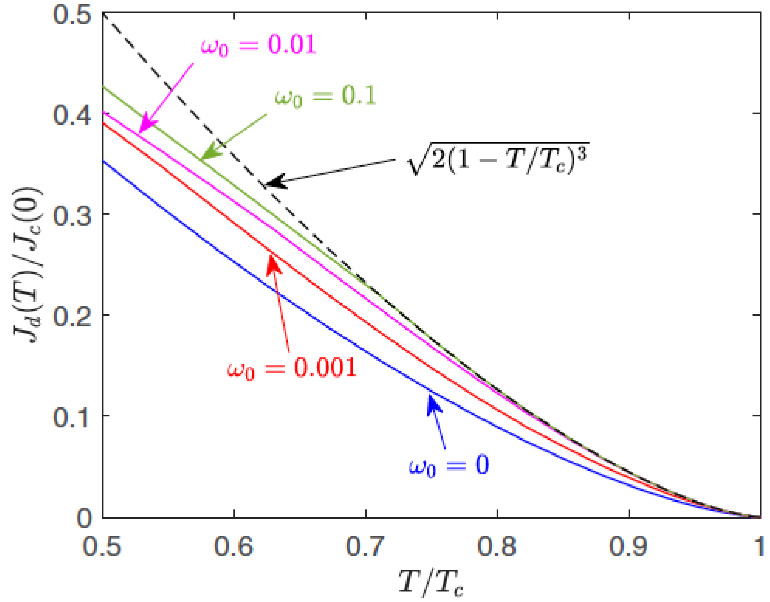
$$H_d(T,\omega) \to \sqrt{2} H_s(T), \qquad \omega\tau_E \gg 1$$

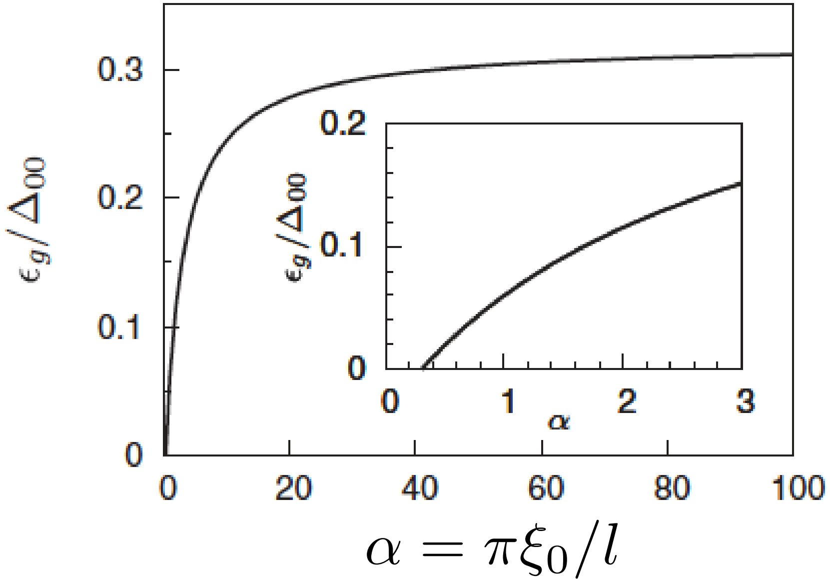
- SC breakdown caused by the time averaged squared RF field

$$\langle H_a^2 \sin^2 \omega t \rangle = H_a^2/2$$



# Dynamic superheating field




High temperatures: enhancement of $H_d$ due to slow qp relaxation:

$$H_d(T,\omega) \to H_s(T). \qquad \omega\tau_E \ll 1,$$

$$H_d(T,\omega) \to \sqrt{2}H_s(T). \qquad \omega\tau_E \gg 1,$$

$$H_s(T) = \left(\frac{\sqrt{5}}{3} + \frac{0.545}{\kappa}\right) H_c(T).$$

Low temperatures: T < 0.5Tc: the relaxation time $\tau_E(T)$ increases greatly but the density of quasiparticles decreases exponentially in the dirty limit.
No effect of quasiparticles on SC breakdown if $l \lesssim 6\xi_0$

$$n_c(T) \propto \exp(-\epsilon_g/k_B T), \qquad H = H_s$$

$$H_d(\omega,T) \to H_s(T), \qquad \hbar\omega \ll \Delta_0$$

In the clean limit $l \gtrsim 6\xi_0$ the quasiparticle gap $\epsilon_g(H)$ vanishes at H < $H_s$, so the dynamic superheating field may be enhanced by slow relaxation of quasiparticles

Sheikhzada and Gurevich, PRB 102, 104507 (2020)

Lin and Gurevich, PRB 85, 054513 (2012)

# Conclusions

- GL calculations of $H_s$ for surface dirty layer and SIS structures were performed for arbitrary GL parameter.

- In all cases we observed a maximum in $H_s$ at optimal thickness for which $H_s$ exceeds the superheating fields of both the overlayer and the bulk materials

- Our GL results for $H_s(d)$ at finite $\kappa$ are qualitatively consistent with previous calculations in the London model

- At GHz frequencies the dynamic superheating field in the dirty limit is close to the static $H_s$ but in the clean limit the relation between $H_d$ and $H_s$ has not been investigated