FACULTY OF MATHEMATICS, PHYSICS AND INFORMATICS
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Bratislava

Local Limit Disorder Characteristics of Superconducting Radio Frequency Cavities II.

arXiv:2409.04203

Resonant Frequency Shift in the Vicinity of T_c and Quality within the Dynes Superconductor Theory Approach

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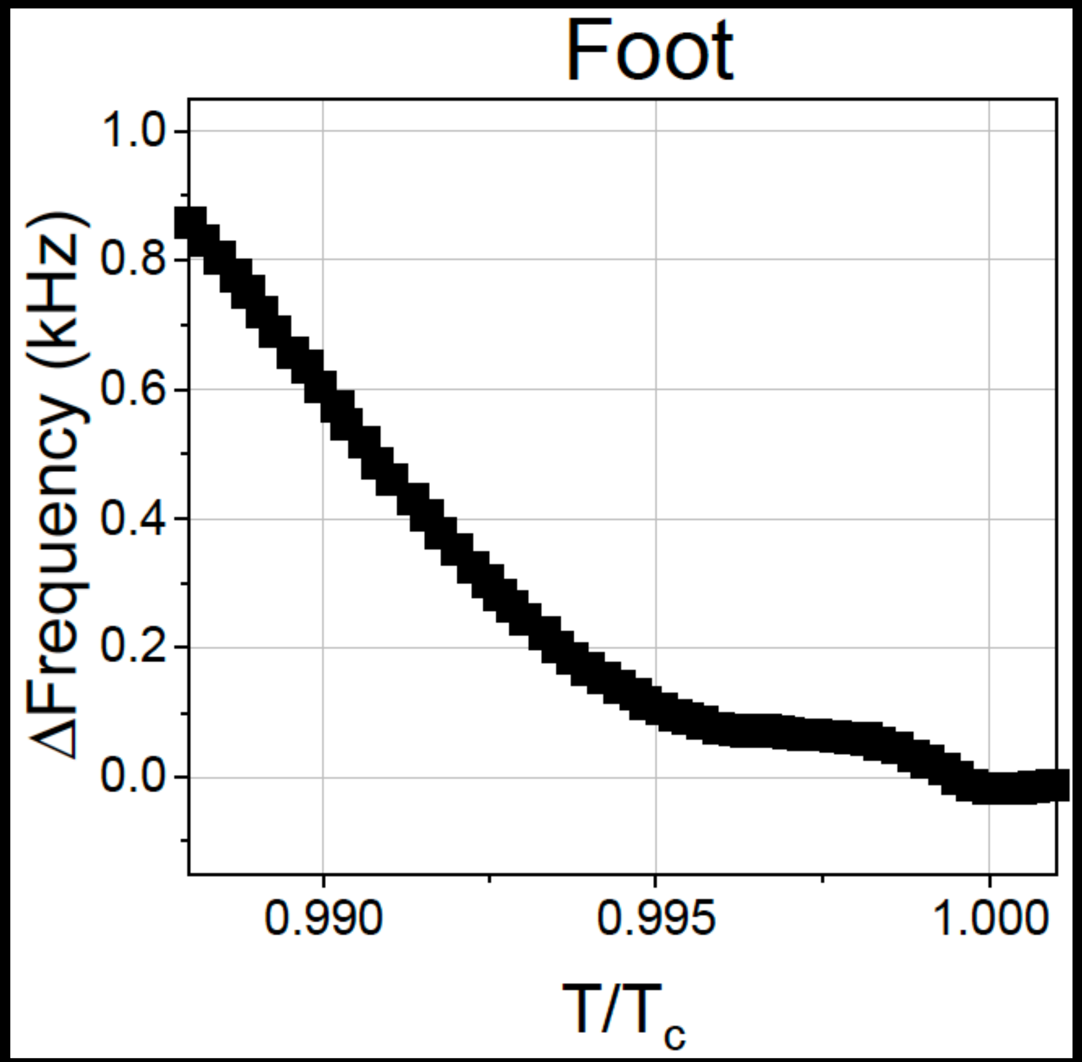
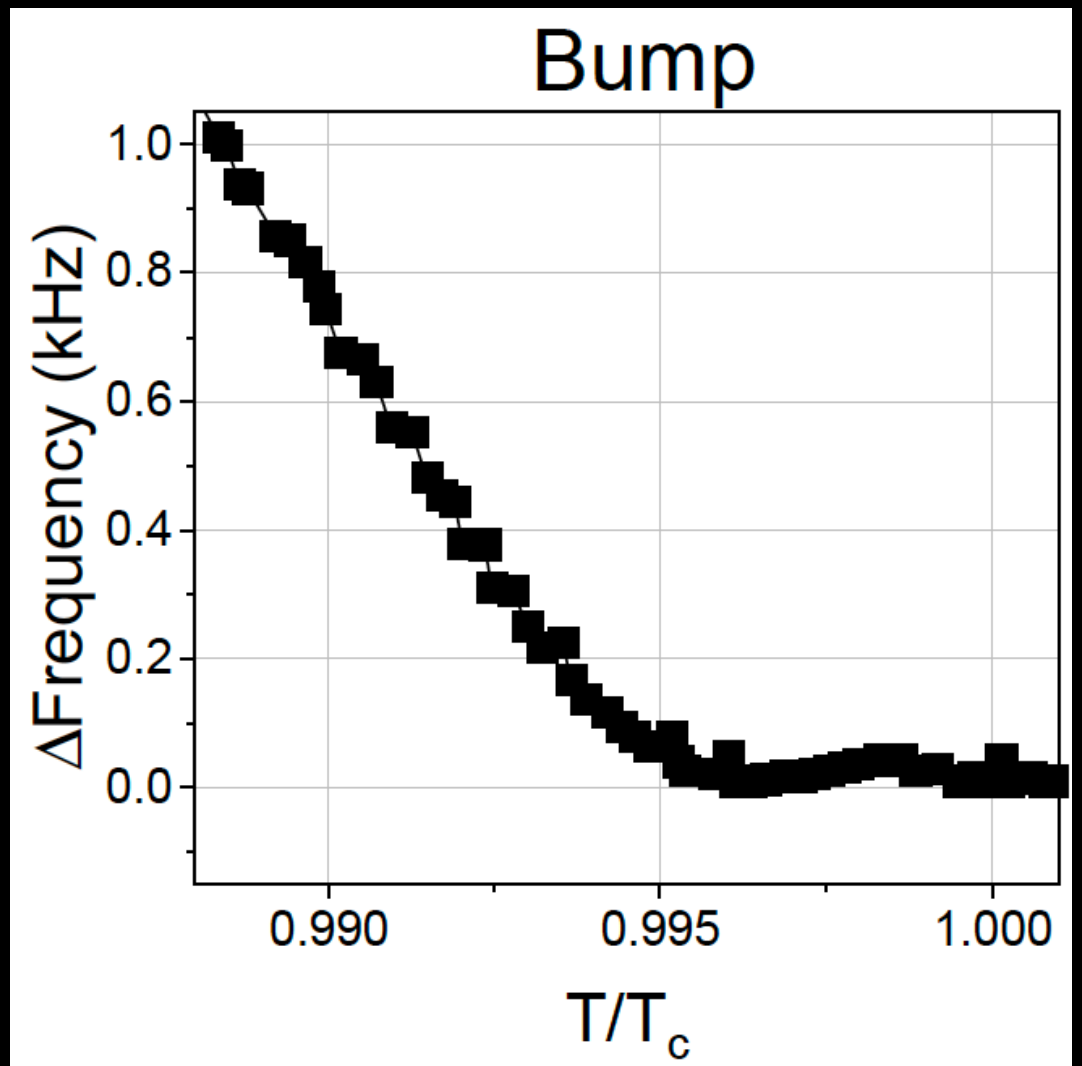
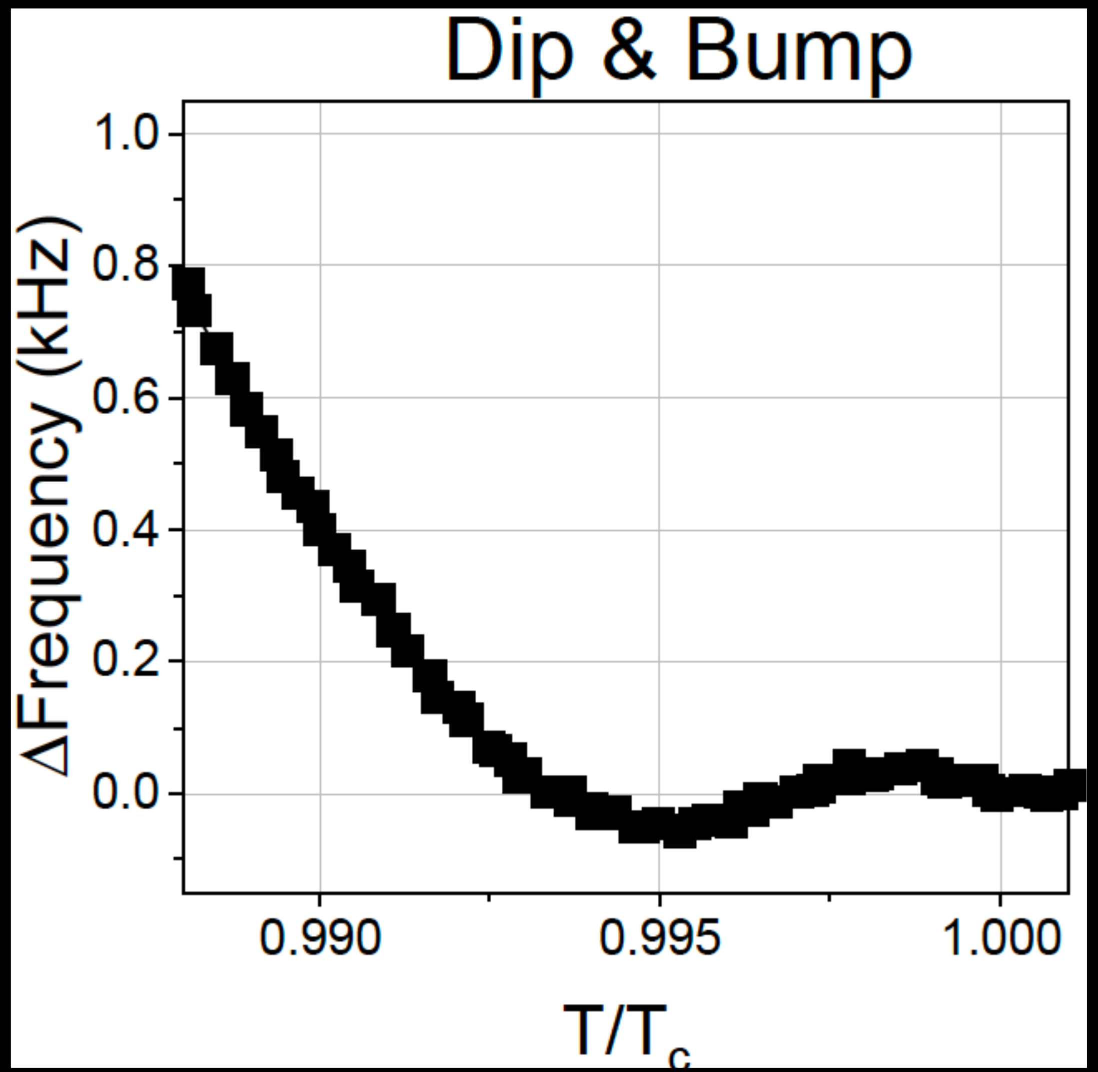
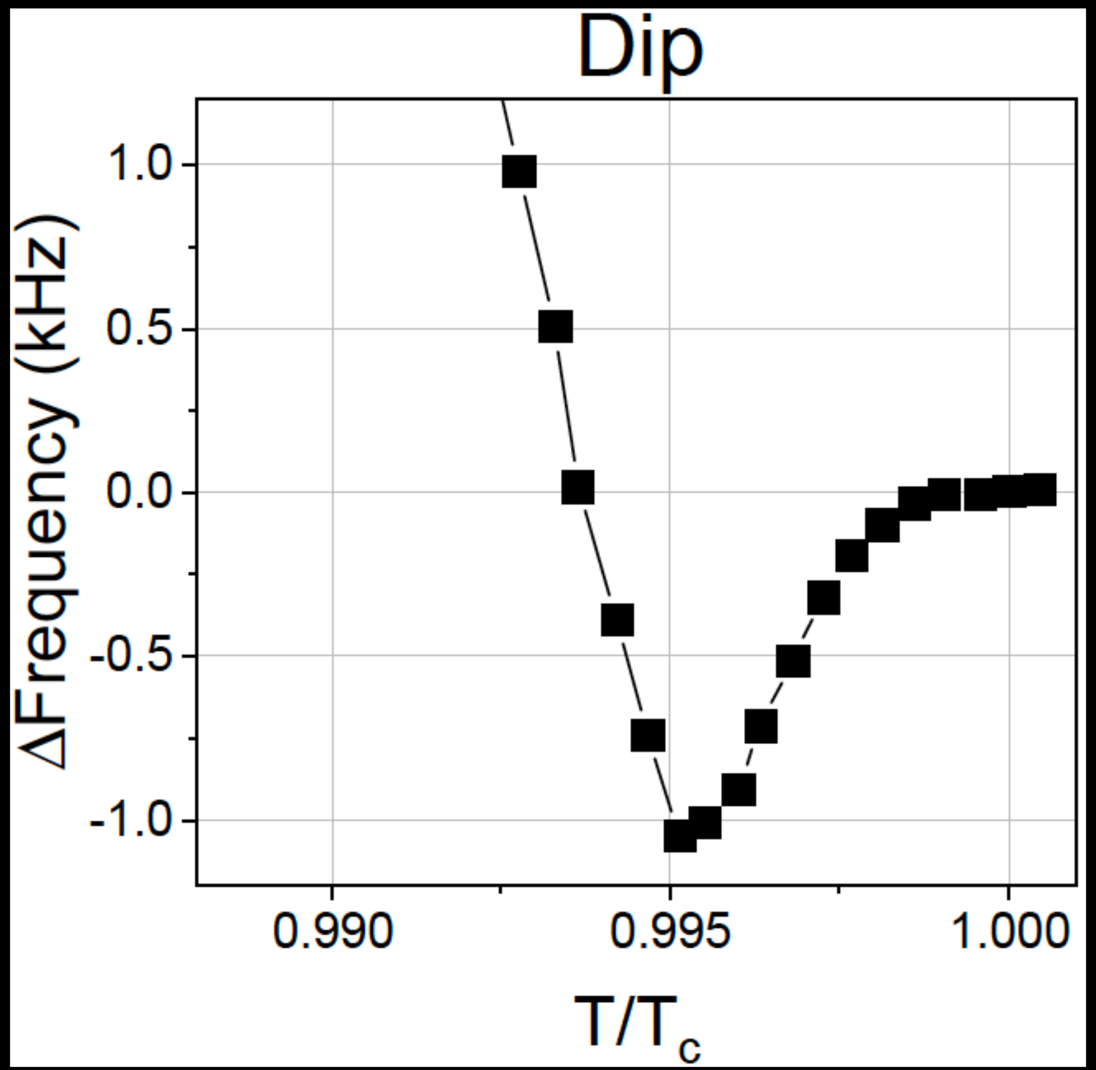
Motivation: Conclusions from TTCM-Fermilab

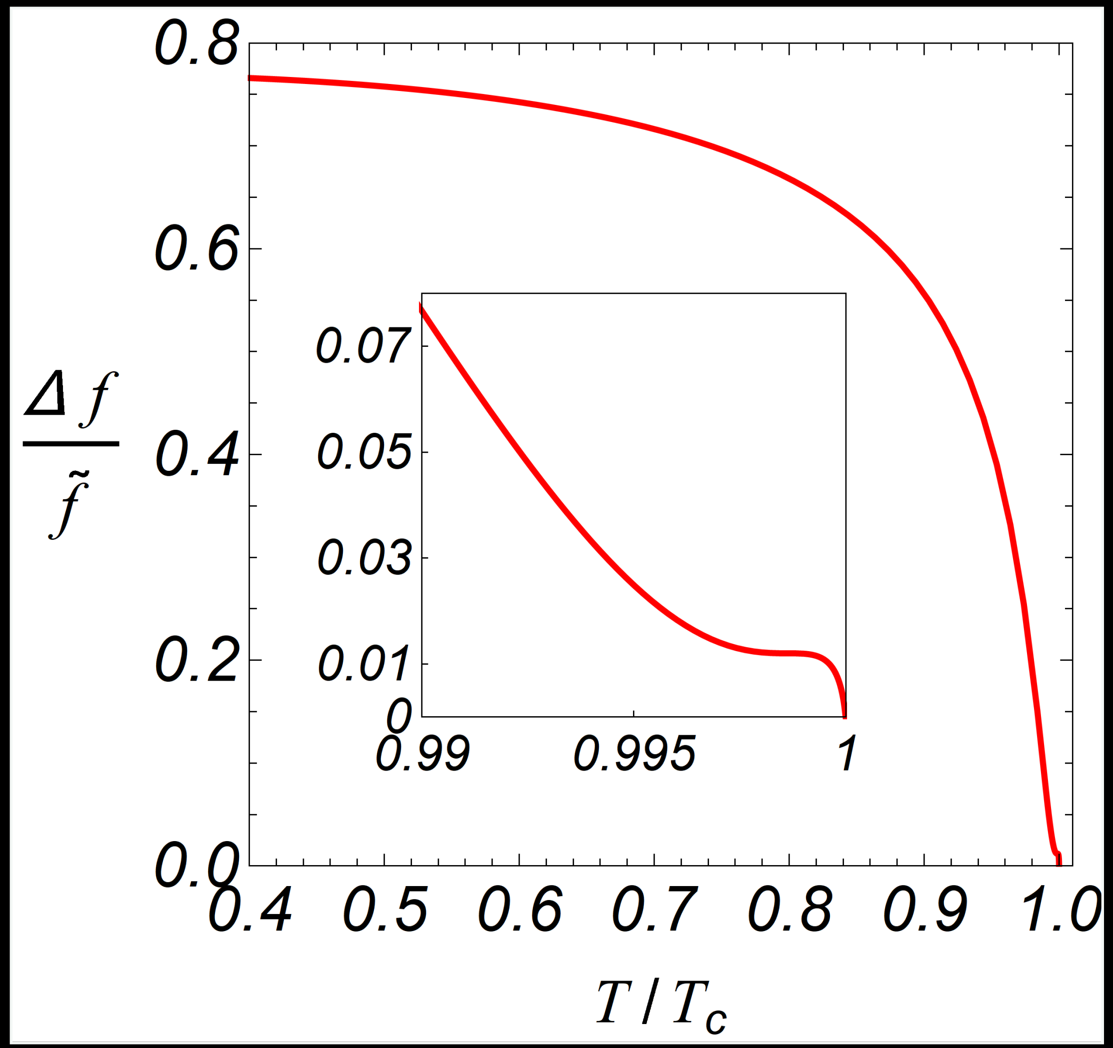
D. Bafia et al., arXiv:2103.10601v2

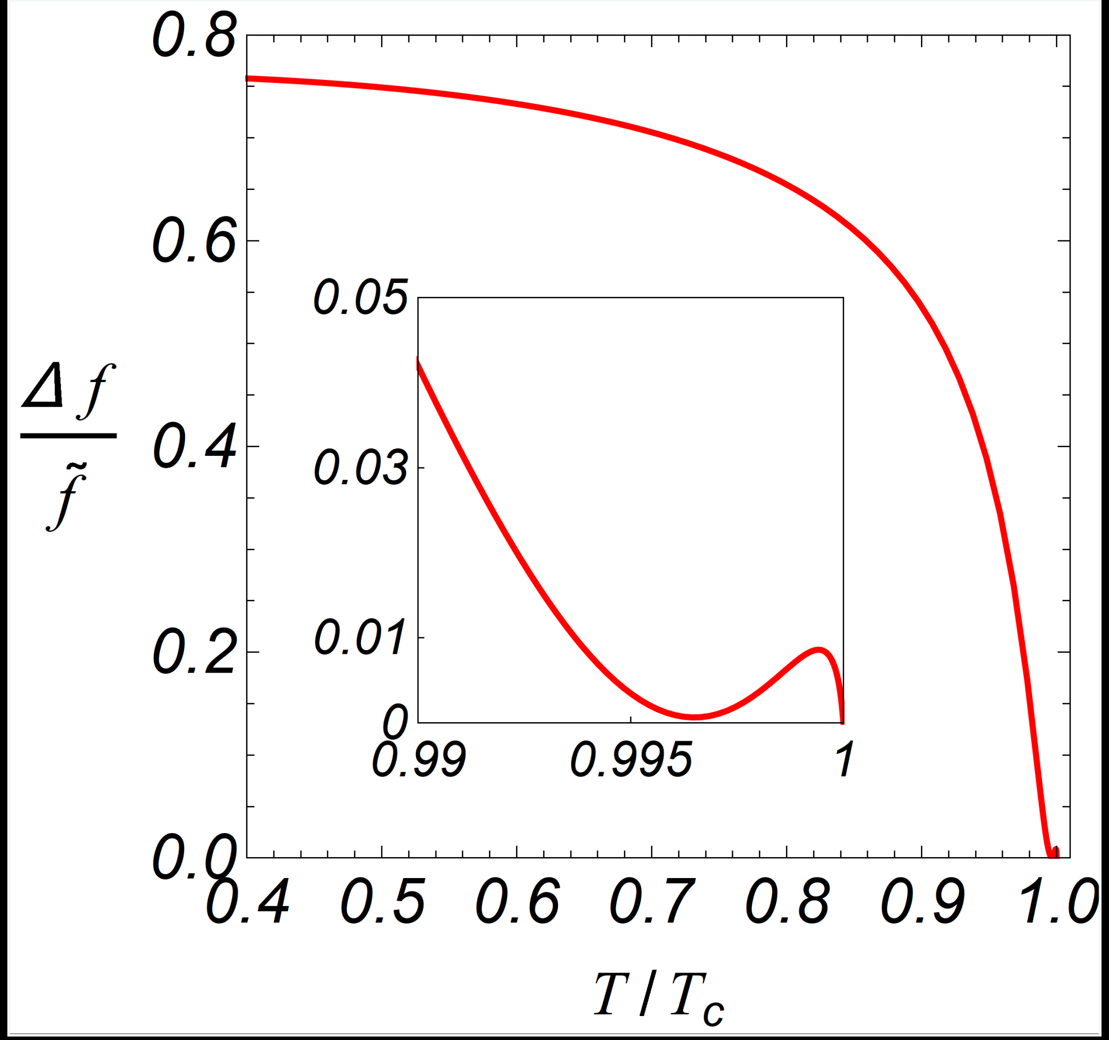


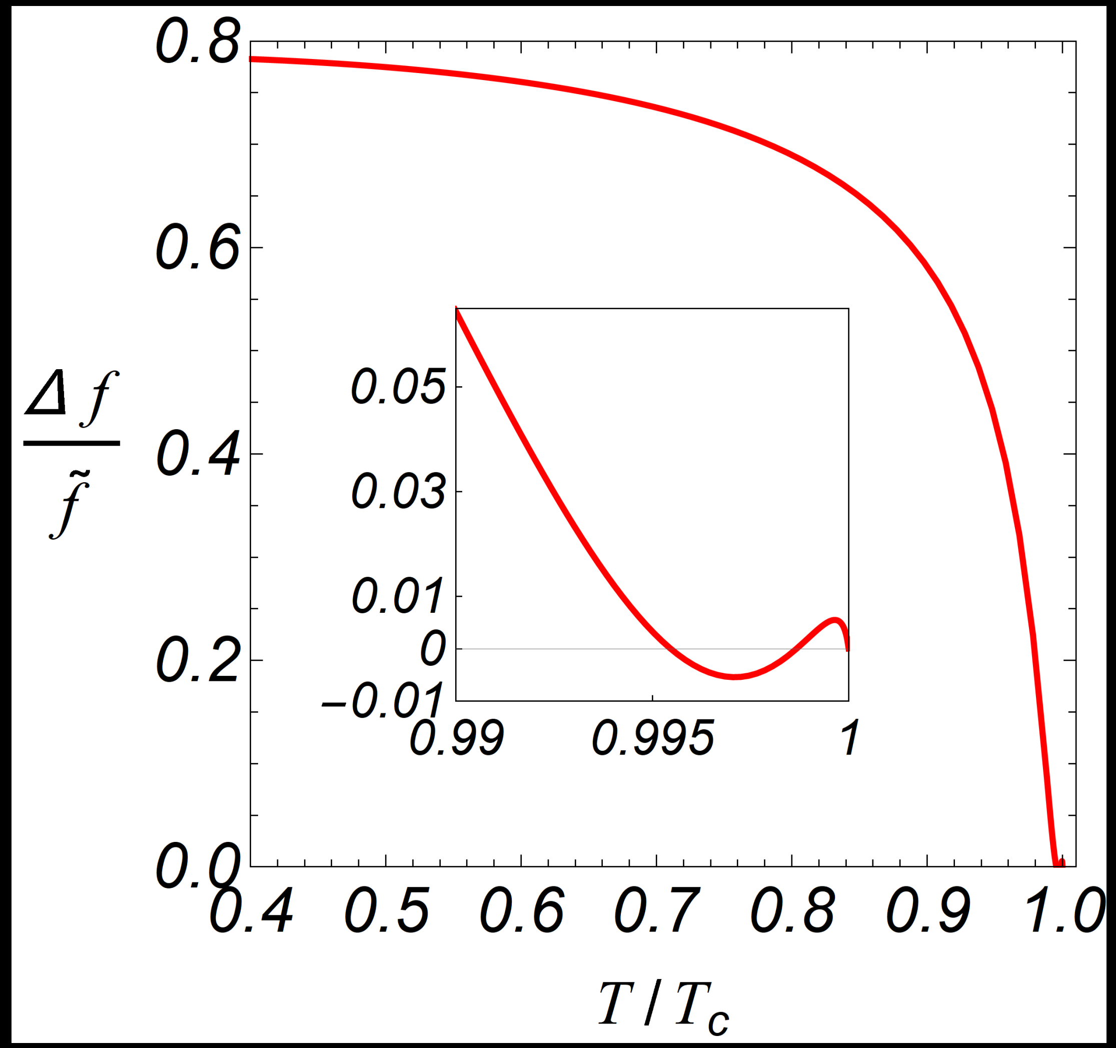


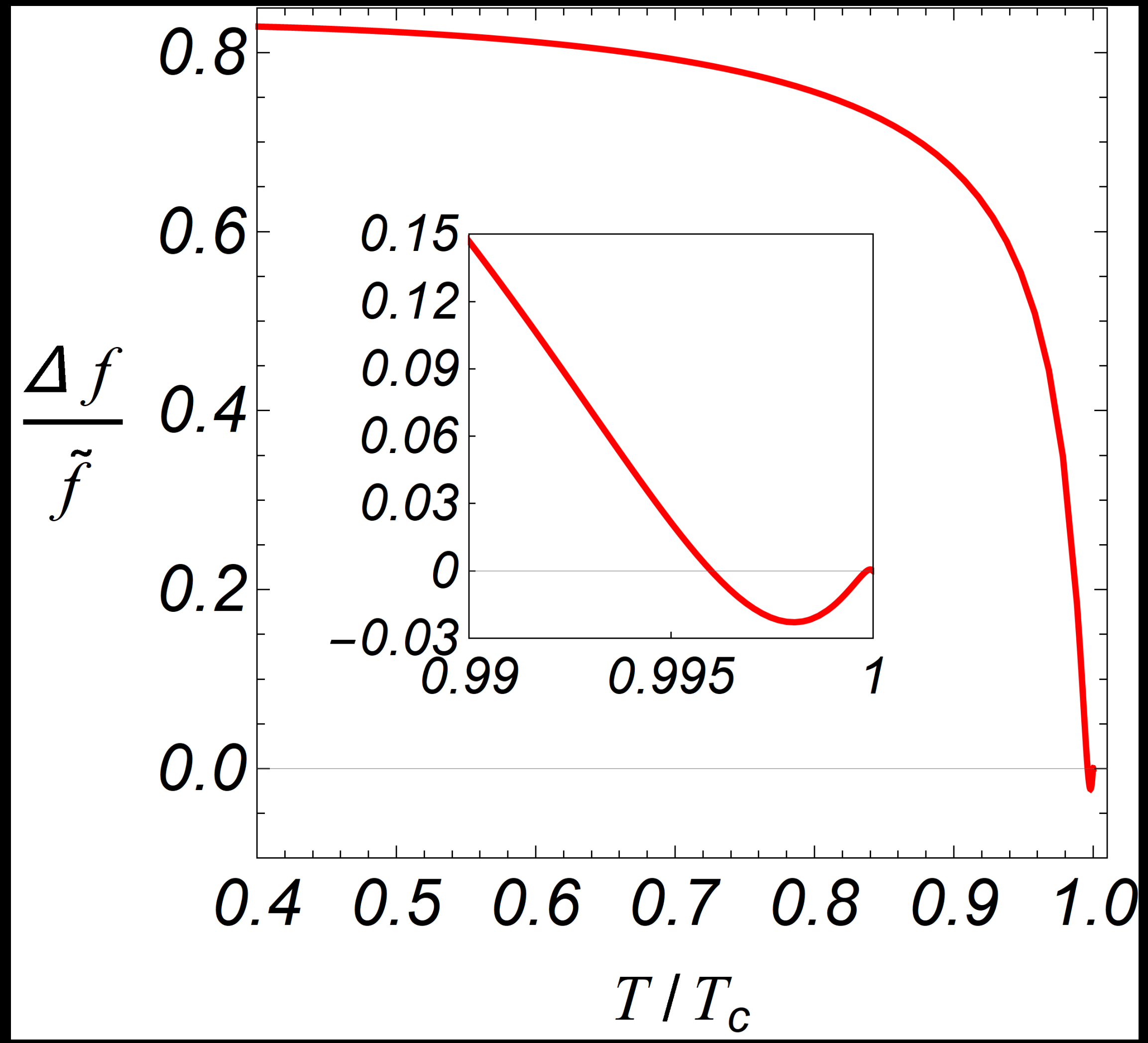












arXiv:2409.04203

Dynes Superconductor

- Internally consistent extension of the BCS superconductor including (simultaneously present) pair-breaking and pair-conserving scattering processes in general case.
- Capable to describe electromagnetic and thermodynamic response.
- Applicable at low (density of states) and also higher (coherence peak) temperatures.
- Applicable at high disorder (MoC - superconductor-insulator transition) and low disorder (Nb-SRF motivated)
- Mathematical formulation using Green function method:

$$\hat{G}(\mathbf{k},\omega) = \frac{1}{2}\delta \ln\left[\varepsilon_{\mathbf{k}}^2 - \epsilon(\omega)^2\right],$$

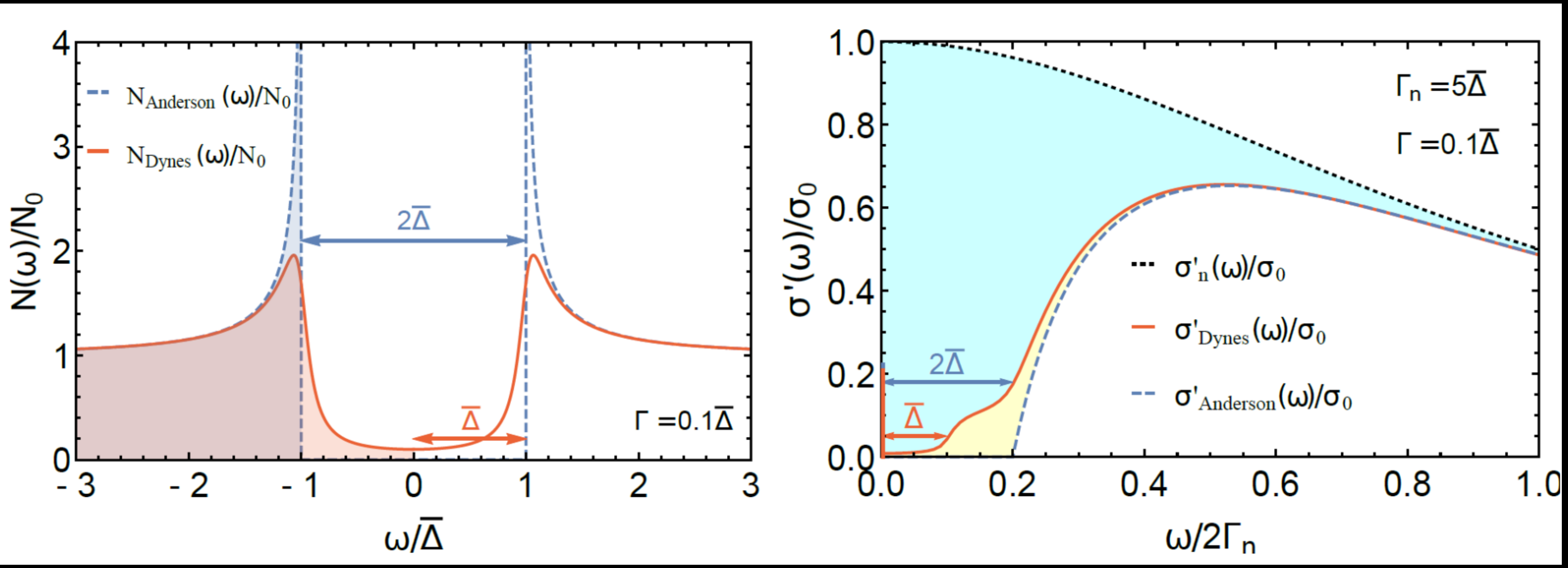
$$\delta = \tau_0\partial_\omega - \tau_1\partial_\Delta - \tau_3\partial_{\varepsilon_{\mathbf{k}}},$$

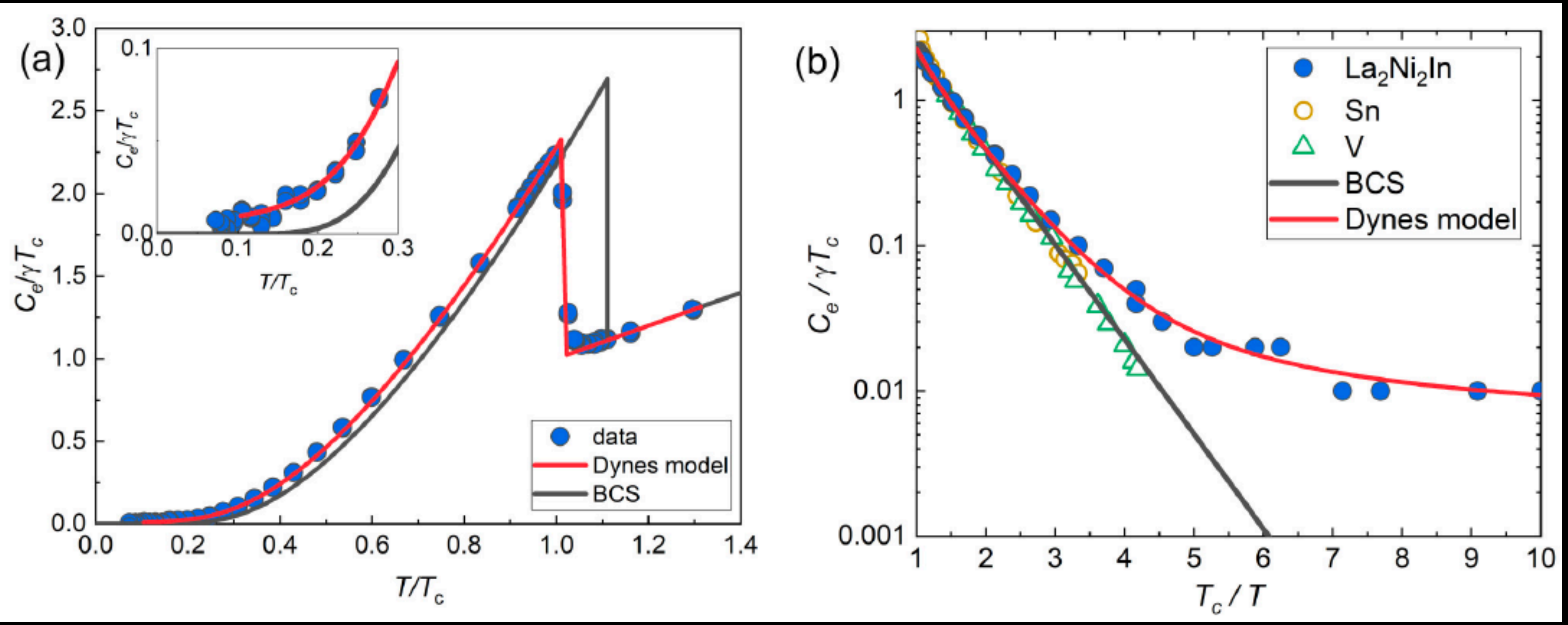
$$\epsilon(\omega) = \sqrt{(\omega + i\Gamma)^2 - \Delta^2 + i\Gamma_s}.$$

and temperature = 5 energy scales.

Electromagnetic properties and optical conductivity

Phys. Rev. B **96**, 014509





Maiwald et al., PRB **102**, 165125 (2020)

Dynes Superconductor

Electromagnetic properties and optical conductivity, formulation

- Green function parametrisation:

$$\Delta(\omega) = \bar{\Delta} \Big/ \left(1 + \frac{i\Gamma}{\omega}\right), \; \mathrm{Z}(\omega) = \left(1 + \frac{i\Gamma}{\omega}\right)\left(1 + \frac{i\Gamma_s}{\Omega(\omega)}\right)$$

$$\Omega(\omega) = \sqrt{(\omega + i\Gamma)^2 - \bar{\Delta}^2} \quad \text{and} \quad \bar{\Delta} = \bar{\Delta}(T)$$

- Green function parametrisation:

$$n(\omega) = n_1(\omega) + in_2(\omega) = \frac{\omega}{\sqrt{\omega^2 - \Delta^2(\omega)}} = \frac{\omega + i\Gamma}{\Omega(\omega)}$$

$$p(\omega) = p_1(\omega) + ip_2(\omega) = \frac{\Delta(\omega)}{\sqrt{\omega^2 - \Delta^2(\omega)}} = \frac{\bar{\Delta}}{\Omega(\omega)},$$

$$\epsilon(\omega) = \epsilon_1(\omega) + i\epsilon_2(\omega) = \mathrm{Z}(\omega)\sqrt{\omega^2 - \Delta^2(\omega)} = \Omega(\omega) + i\Gamma_s.$$

- Resulting formulation

$$\sigma_s(\omega) = \frac{iD_0}{\omega}\int_{-\infty}^{\infty} d\nu \tanh\left(\frac{\nu}{2T}\right) H(\nu + \omega, \nu).$$

$D_0 = ne^2/m$ is the normal-state Drude weight and

$$H_1(x,y) = \frac{1 + n(x)n^*(y) + p(x)p^*(y)}{2\left[\epsilon^*(y) - \epsilon(x)\right]},$$

$$H_2(x,y) = \frac{1 - n(x)n(y) - p(x)p(y)}{2\left[\epsilon(y) + \epsilon(x)\right]},$$

$$H(x,y) = H_1(x,y) + H_2(x,y)$$

Dynes Superconductor

Implications towards the superconductive cavities: ***Coherence peak and Imaginary part of DC Conductivity***

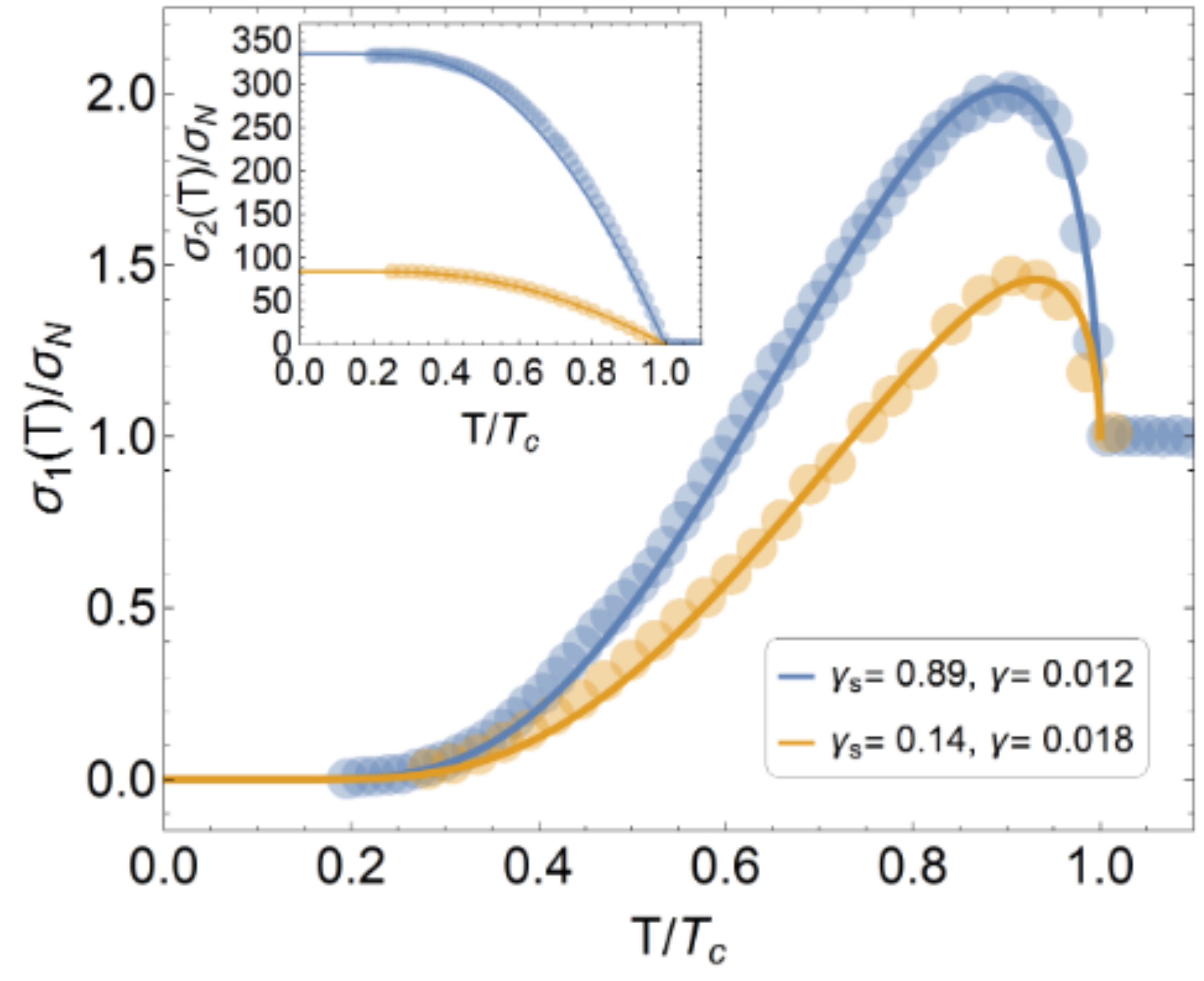


FIG. 9. Real and imaginary parts of the conductivities of the two samples from Fig. 4 in [13] (symbols), together with their fits by the theory of Dynes superconductors with the strong-coupling corrections described for both samples by $x = 1.145$ (lines).

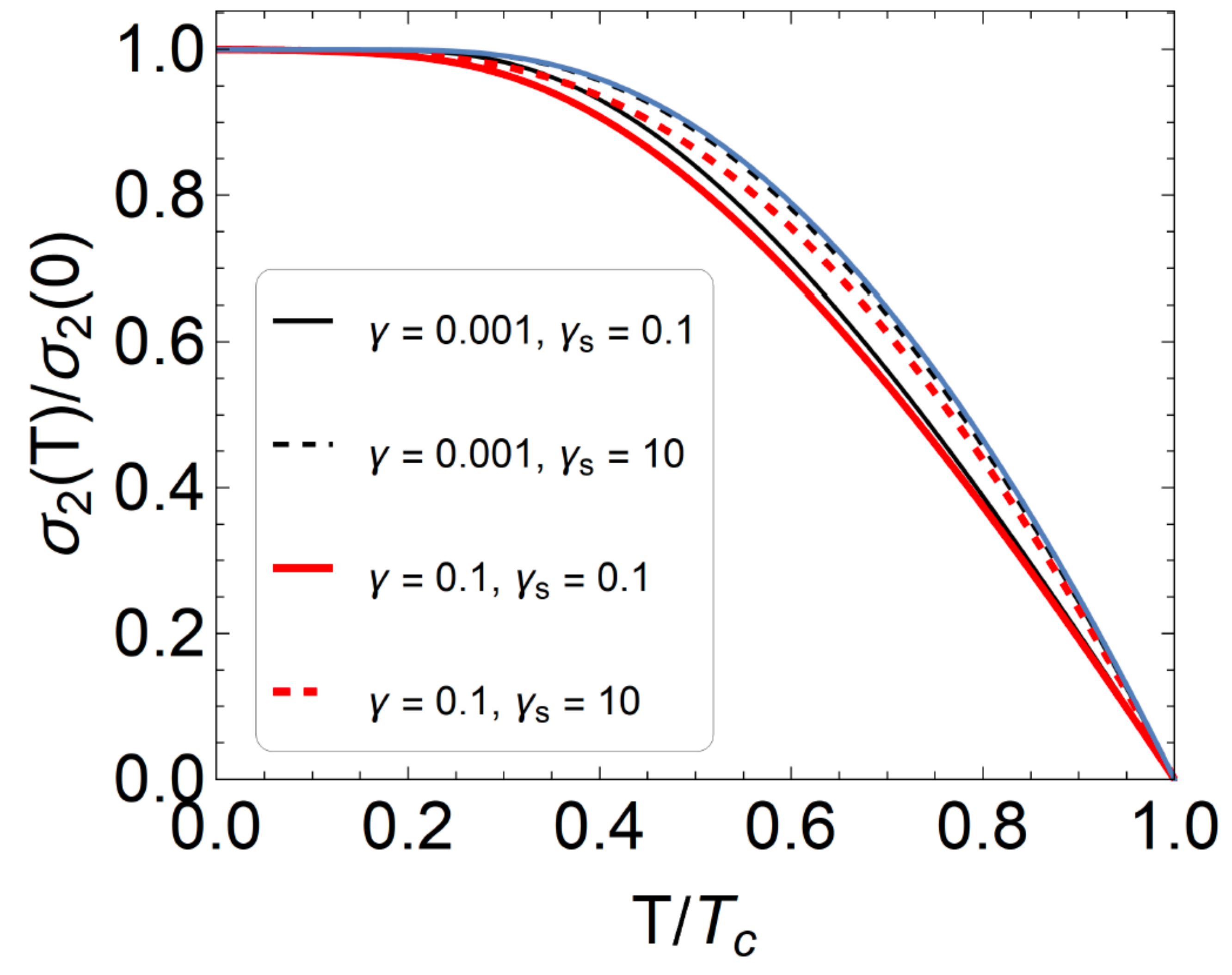


FIG. 6. Temperature dependence of the imaginary part of the microwave conductivity $\sigma_2(T)/\sigma_2(0)$ for several choices of γ and γ_s. Note that when γ_s decreases and γ increases, the curves are pushed slightly downwards with respect to Eq. (14), which is shown by the blue line.

$\Delta(T) \;=\; x\Delta_{\mathrm{BCS}}(T)$

x - strong coupling correction

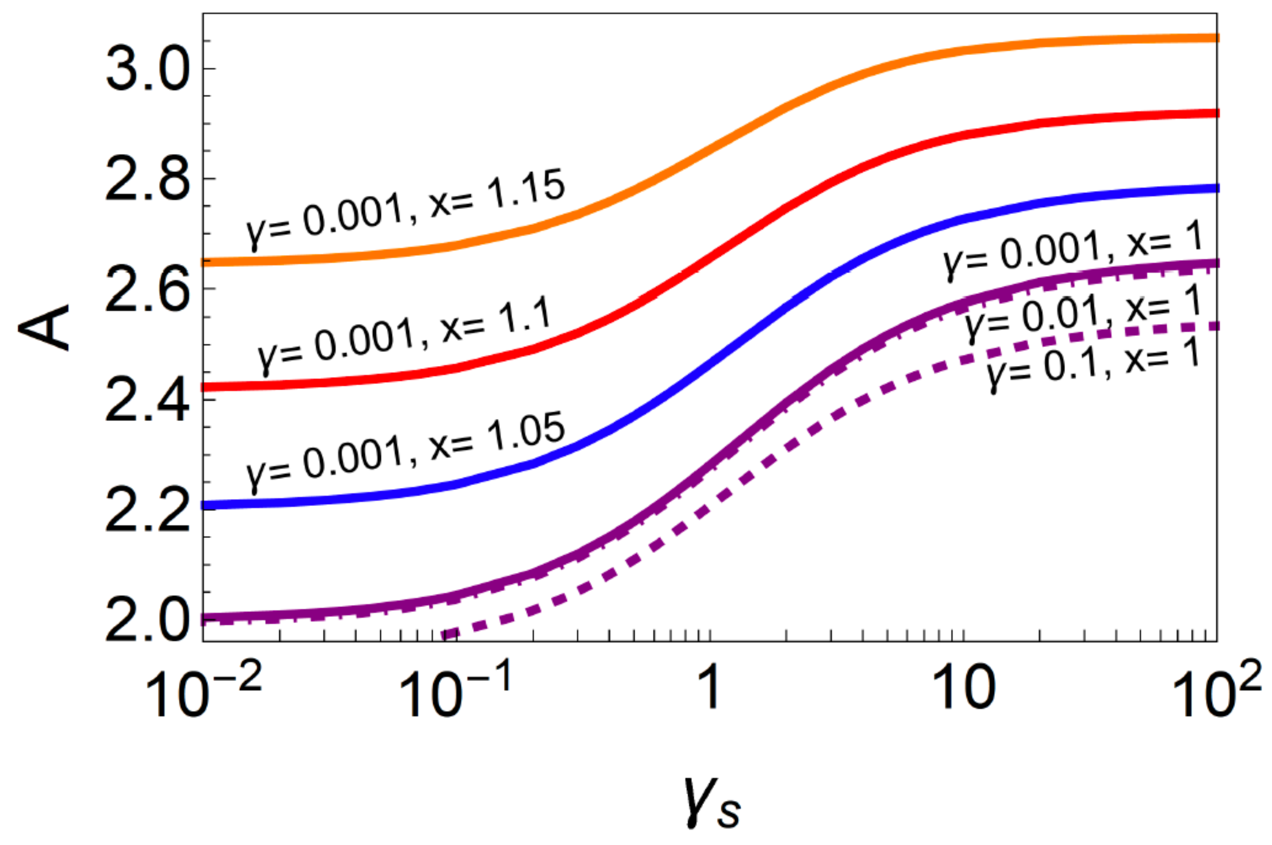


FIG. 7. Coefficient A describing the T-dependence of the imaginary part of the microwave conductivity close to T_c, $\sigma_2(T)/\sigma_2(0) = A(1 - T/T_c)$, plotted as a function of γ_s for several values of γ and x. Note that A depends only very weakly on γ in the limit of small pair breaking.

D. Bafia et al., ArXiv:2106.10601 (2021)

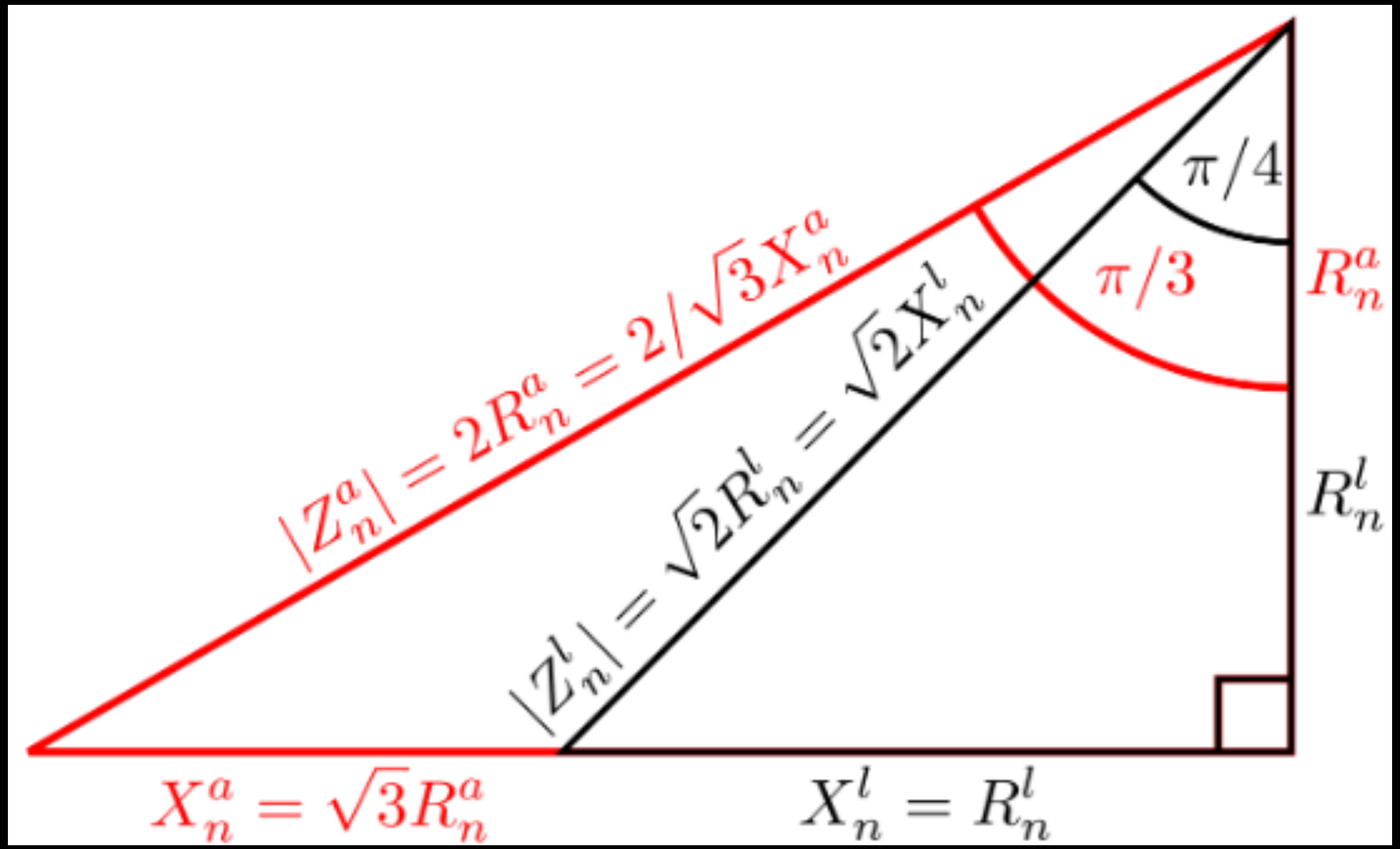
Phys. Rev. B **104**, 094519

Surface Impedance (Z) & microwave conductivity (σ)

$$\frac{Z_s}{Z_n} = \frac{R_s + iX_s}{R_n + iX_n} = \left(\frac{\sigma_1 - i\sigma_2}{\sigma_n}\right)^k$$

- R - surface resistance, X - surface reactance

- Normal state parametrization for $\omega\tau \to 0$.



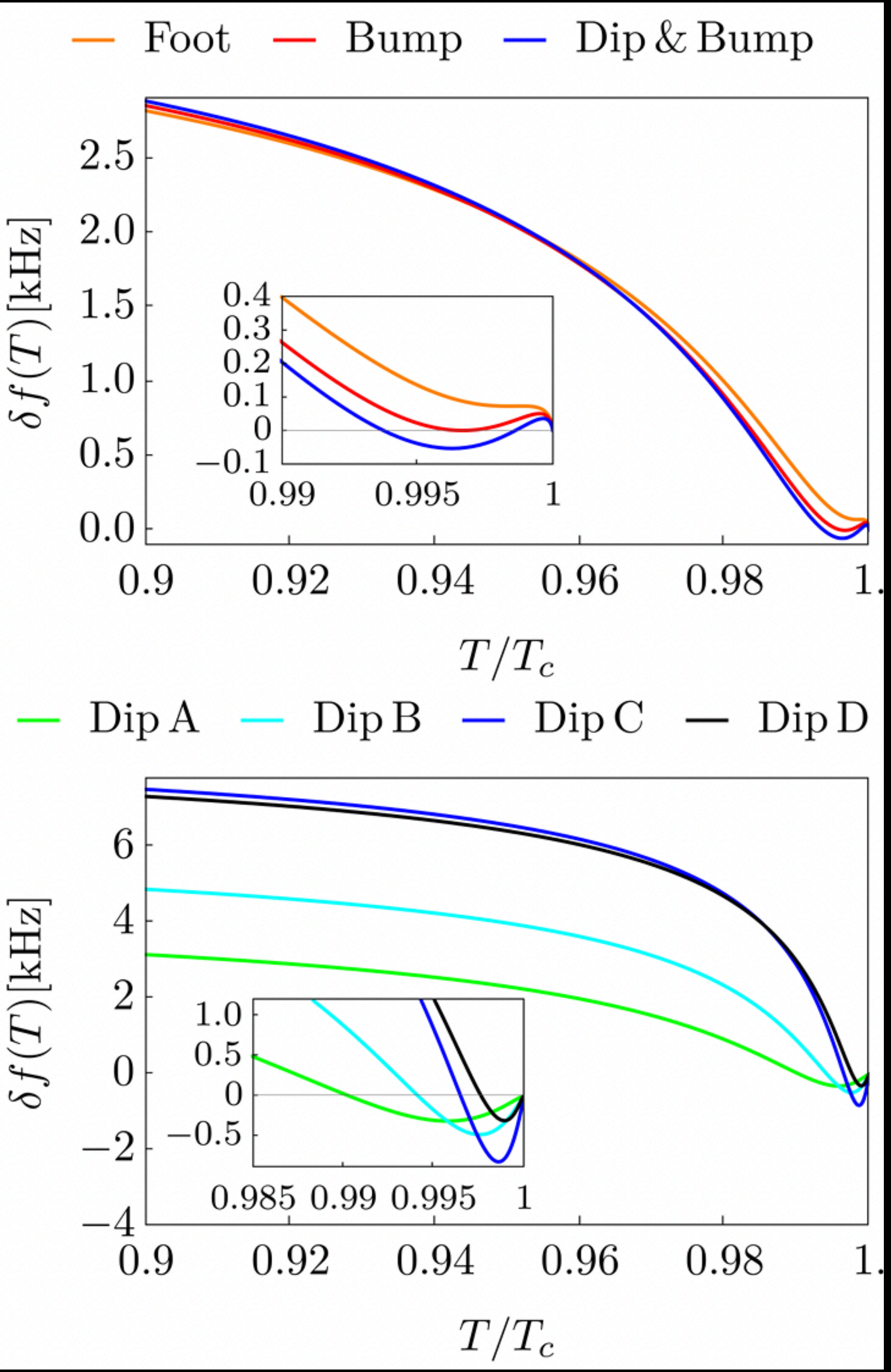
- More convenient form:

$$Z_s = |Z_n| \left(\frac{|\sigma_s|}{|\sigma_n|}\right)^k e^{i(\alpha_n - k\delta\varphi)}$$

- Simple formulation of superconducting state properties:

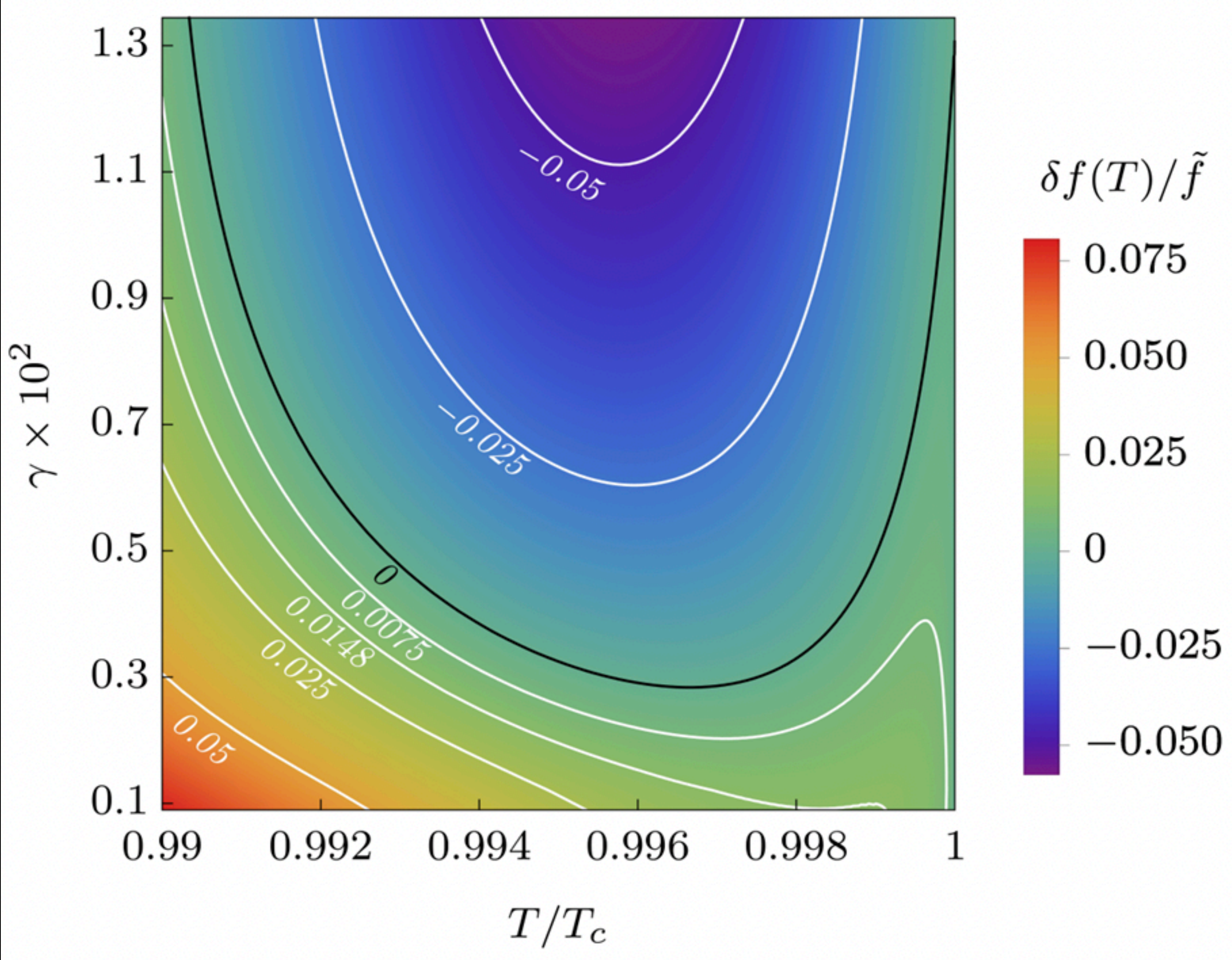
$$R_s = |Z_n| \left(\frac{|\sigma_s|}{|\sigma_n|}\right)^k \cos(\alpha_n - k\delta\varphi) \qquad X_s = |Z_n| \left(\frac{|\sigma_s|}{|\sigma_n|}\right)^k \sin(\alpha_n - k\delta\varphi)$$

Frequency Shift - different regimes

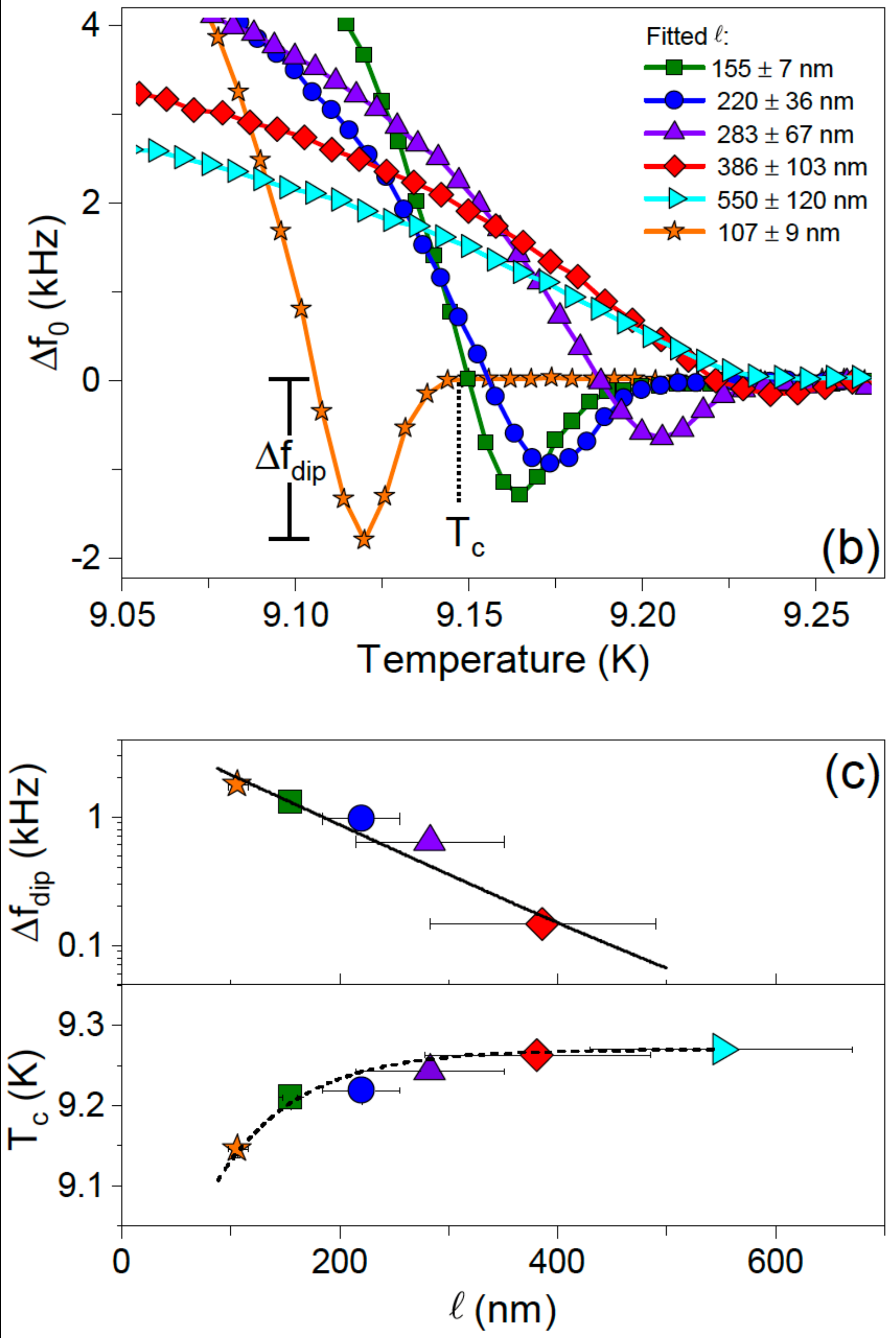


	$\Gamma[\omega]$	$\Gamma_s[\omega]$	Regime	ℓ[nm]	X_n[mΩ]	$\tilde{f}$[kHz]
Foot	0.33	26	$\Gamma \lessapprox \omega \ll \Gamma_s \ll \Delta_0$	697	2.1	4.9
Bump	1.05	26	$\Gamma \approx \omega \ll \Gamma_s \ll \Delta_0$	679	2.1	5
D&B	1.5	26	$\omega \lessapprox \Gamma \ll \Gamma_s \ll \Delta_0$	668	2.1	5.1
Dip A	5	26	$\omega \lesssim \Gamma < \Gamma_s \ll \Delta_0$	592	2.2	5.4
Dip B	5	50	$\omega \lesssim \Gamma \ll \Gamma_s \lesssim \Delta_0$	334	3	7.1
Dip C	5	100	$\omega \lesssim \Gamma \ll \Gamma_s \lesssim \Delta_0$	175	4.1	9.9
Dip D	0.33	100	$\Gamma \lessapprox \omega \ll \Gamma_s \lesssim \Delta_0$	183	4	9.7

- Various regimes, under assumption of the local limit.



- Higher values of scattering rates reduce mean free path, reduce the T_c and deepen the dip.

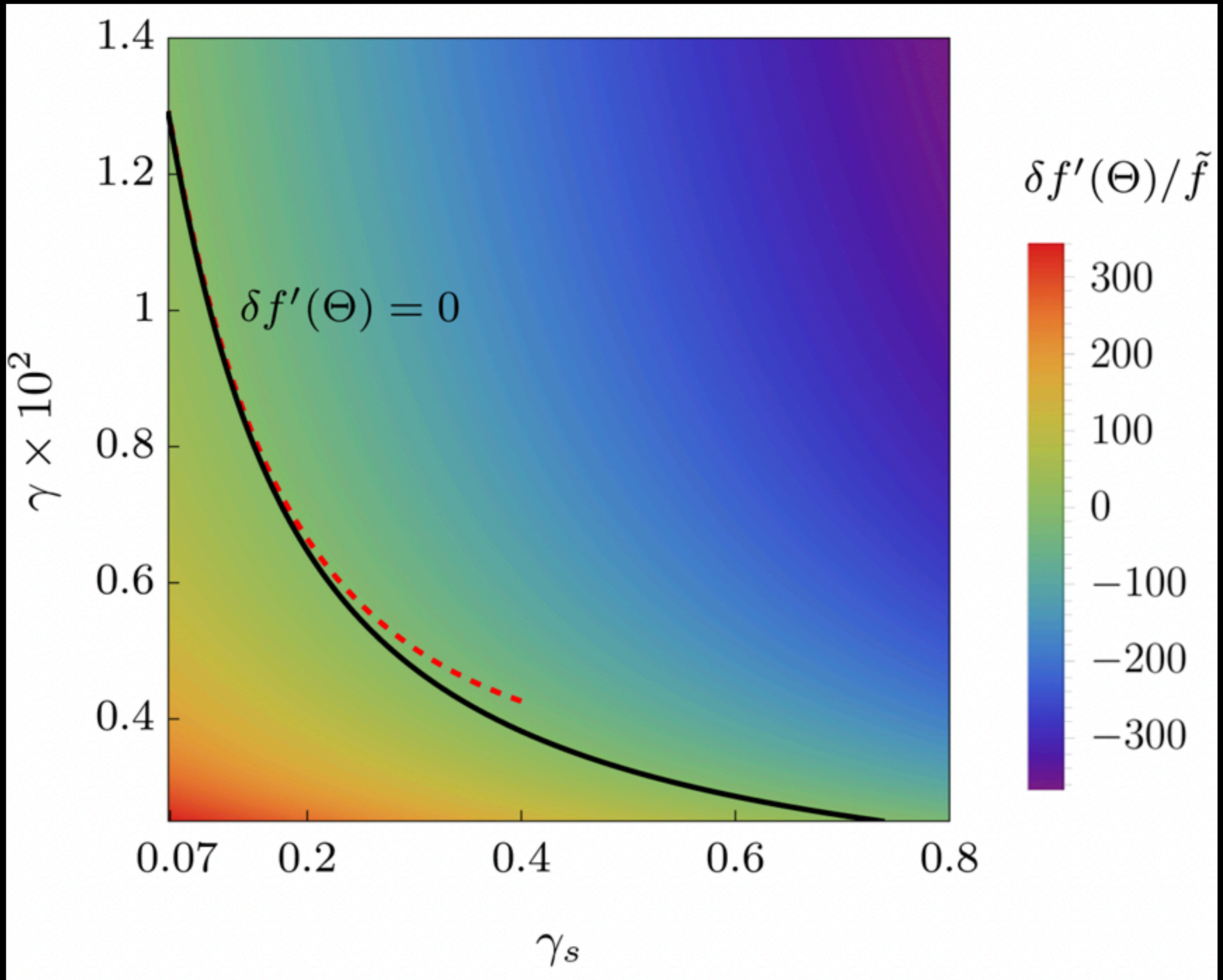


Moderately clean regime: $\hbar\omega < \Gamma \ll \Gamma_s \lesssim \Delta_0$

$$\delta f'(\Theta) \equiv \left.\frac{\partial \delta f(\Theta)}{\partial \Theta}\right|_{\Theta=0} \quad \text{where:} \quad \Theta = 1 - T/T_c$$

$$\sigma_s'(T)/\sigma_0 = 1 + f_1(\Gamma, \Gamma_s)\Theta, \quad \sigma_s''(T)/\sigma_0 = f_2(\Gamma, \Gamma_s, \omega)\Theta\,.$$

$$\delta f(T \to T_c)/\tilde{f} = \left(f_1 - f_2\right)\Theta/2$$



1) $\Gamma \ll T_c \sim \Delta_{00}$

$$f_1(\Gamma, \Gamma_s) = \frac{\gamma_e \pi^2}{7\zeta(3)} \frac{\Gamma_s}{\Gamma_n} \frac{\Delta_{00}}{\Gamma}$$

$$f_2(\Gamma, \Gamma_s, \omega) = \frac{2\Gamma_n}{\omega} \frac{n_s(0)}{n} A$$

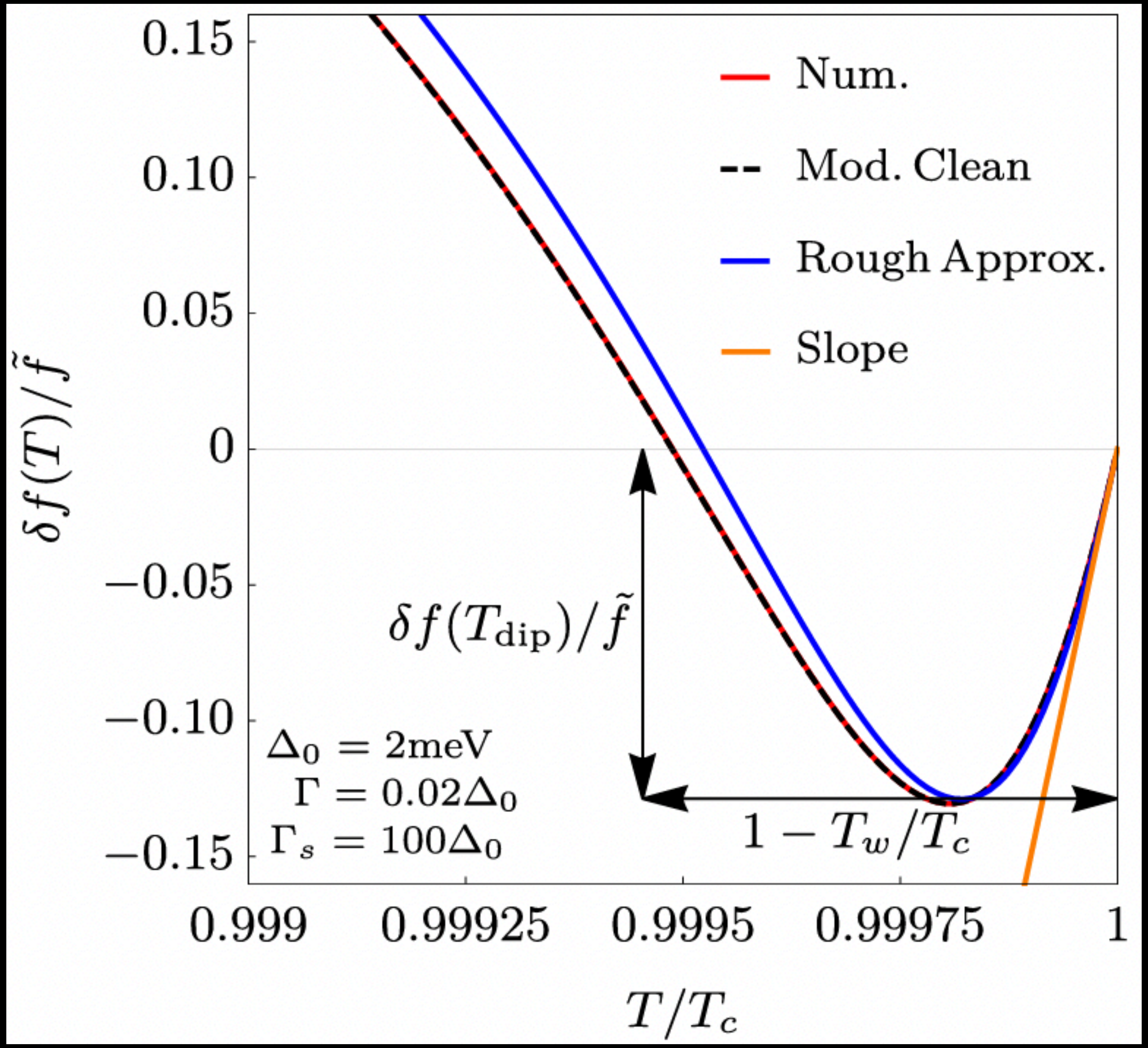
- Both signs are possible.

2) $\Gamma \gg T_c$ with arbitrary Γ_s

$$f_1 = -\frac{2\gamma_s + 3\gamma}{\gamma_s + \gamma} \frac{1}{\gamma^2} < 0$$

- Pure dip.

Dirty ideal limit: $\omega \ll \Gamma \ll \Delta(0) \approx \Delta_{00} \ll \Gamma_s$



Slope at T_{c0}:

$$\frac{\delta f(T)}{\tilde{f}} \approx \boxed{\frac{2\gamma_e \pi^2}{7\zeta(3)}} \frac{\Delta_{00}}{\omega}\left(\frac{\omega}{4\Gamma} - 1\right)\left(1 - \frac{T}{T_{c0}}\right)$$

≈ 4.18

Width of the dip: $\delta f(T_w) = 0$ for $T_w \neq T_{c0}$

$$\frac{T_w}{T_{c0}} \approx 1 - \frac{7\zeta(3)}{4\gamma_e \pi^2}\frac{\omega}{\Delta_{00}}\left(\sqrt{3} - \frac{\omega}{4\Gamma}\right)$$

Depth of the dip: $T_{dip} \approx T_w/3$

$$\frac{\delta f\left(T_{\rm dip}\right)}{\tilde{f}} \approx -0.1398 + 0.082\frac{\omega}{\Gamma}$$

Comparison & fitting the experimental dip

- No free parameters in comparison scenario.
- Pair-breaking fixed by the reduced T_c.

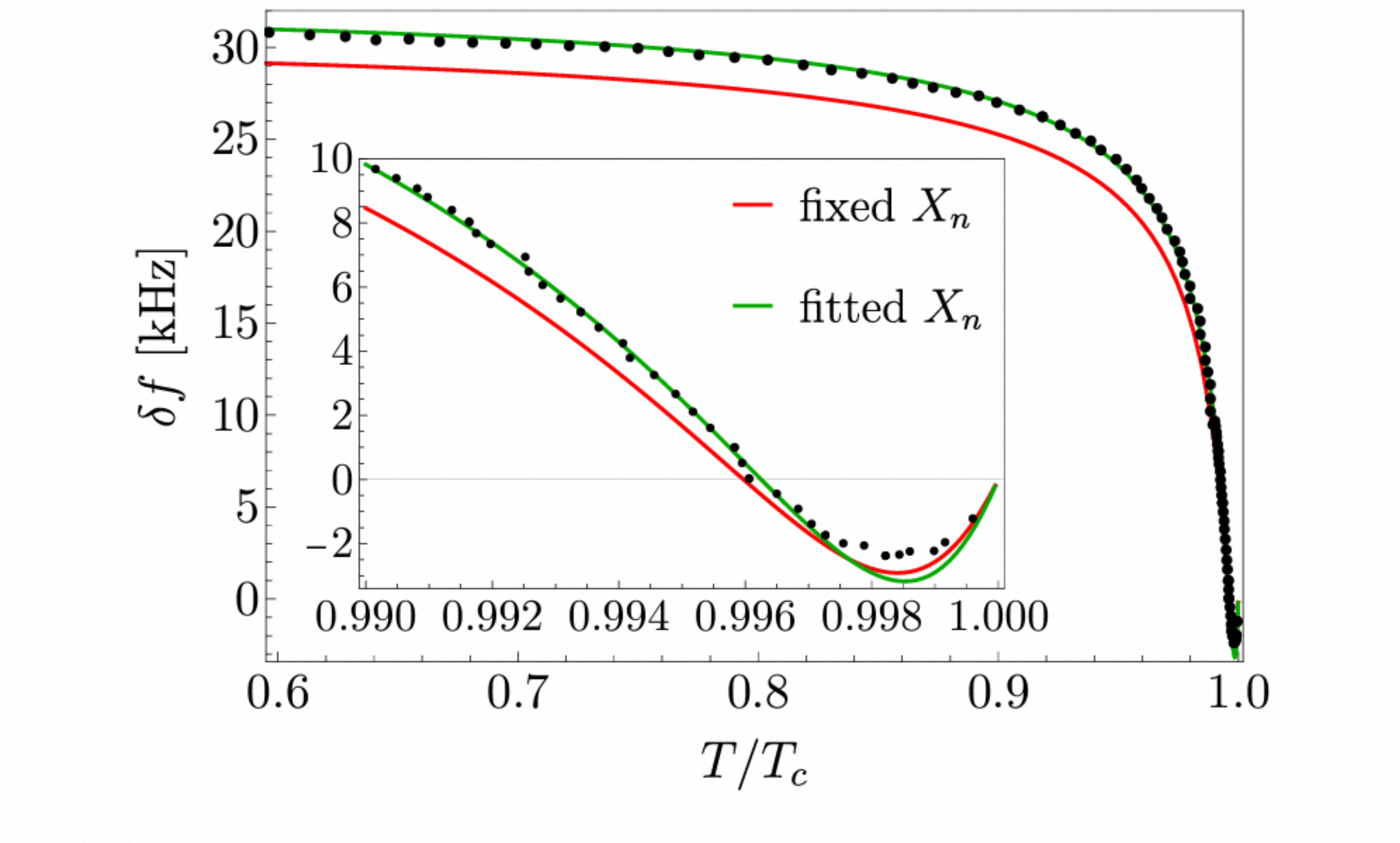


FIG. 7. Considered parameters: resonant frequency $f = 2.6\,\text{GHz}$, $\Delta_0 = 1.55\,\text{meV}$ [1], pair-conserving scattering rate corresponding to fixed $X_n = 7.1\,\text{m}\Omega$ (fitted $X_n = 7.5\,\text{m}\Omega$) is $\gamma_s = 0.52$ ($\gamma_s = 0.58$), and resulting $\ell = 121\,\text{nm}$ ($\ell = 108\,\text{nm}$).

Data: M. Zarea *et al.*, Frontiers in Superconducting Materials, 3: 1–7 (2023), arXiv:2307.07905v1

- Frequency scaling

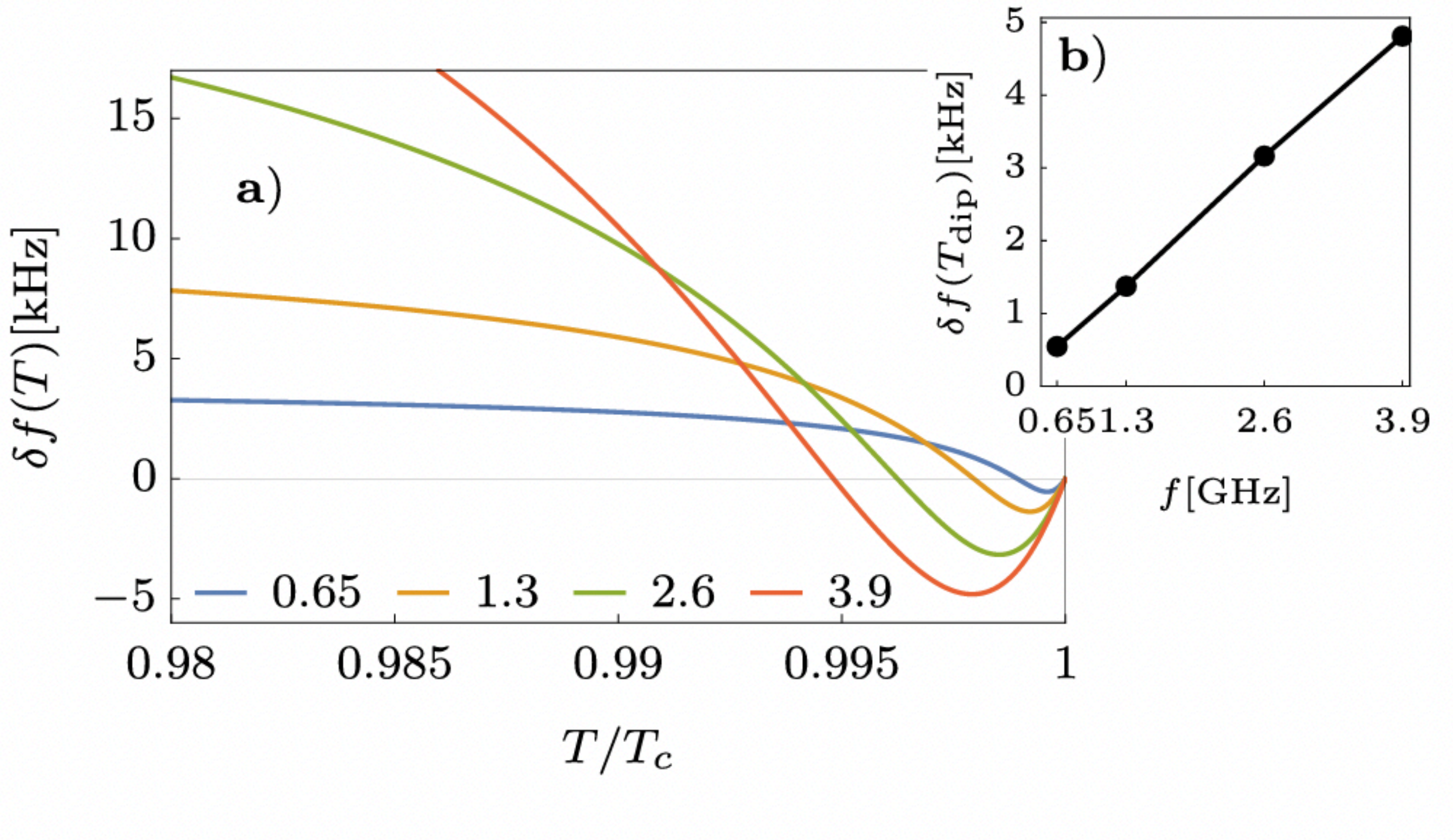
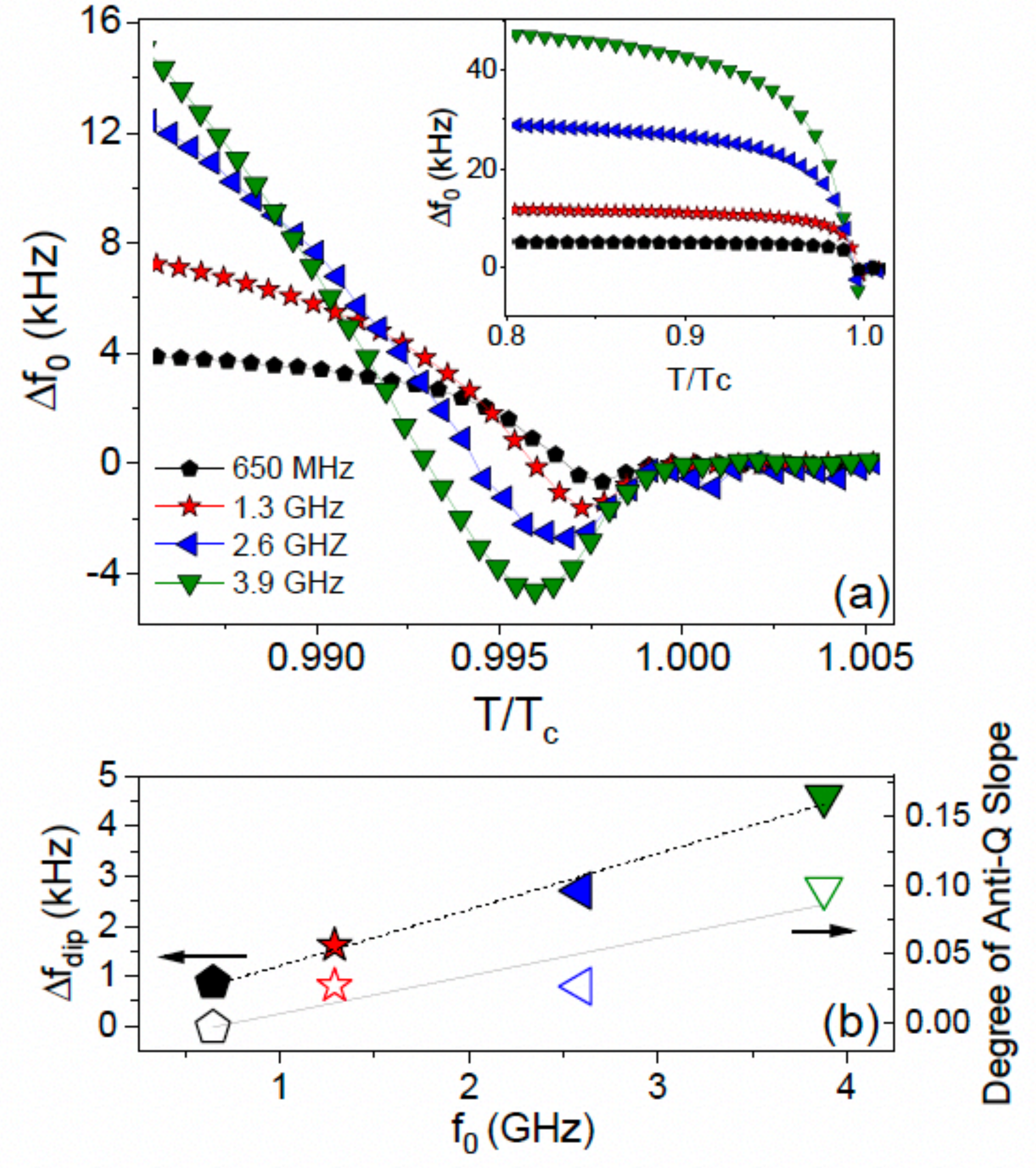


FIG. 8. Resonant frequency dependence of $\delta f(T)$ considering values of $f = \{0.65, 1.3, 2.6, 3.9\}$GHz. Considered scattering rates are $\gamma = 0.026$ and $\gamma_s = 0.58$.



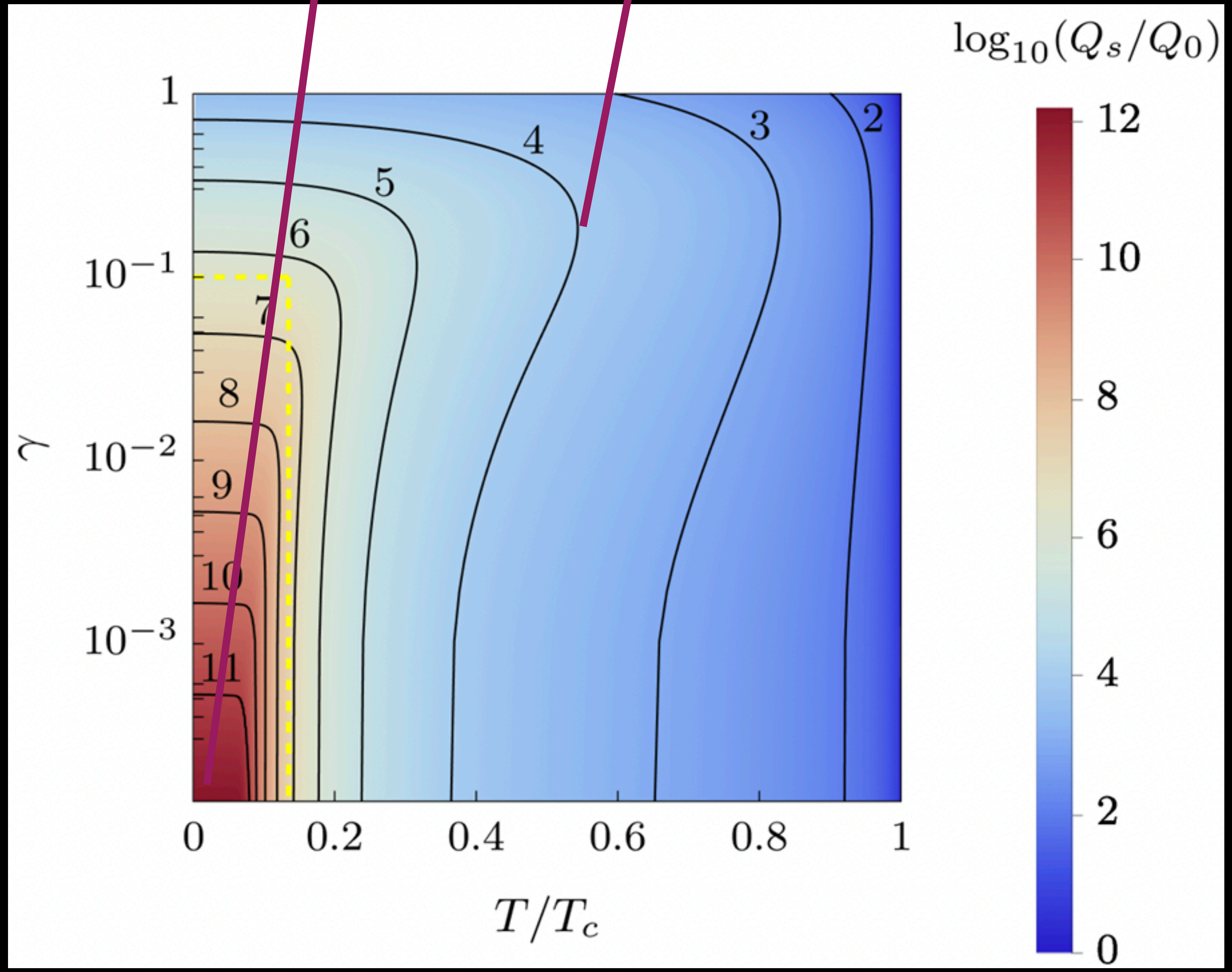
D. Bafia et al., arXiv:2103.10601v2

Quality $Q_s = G/R_s$ in local limit

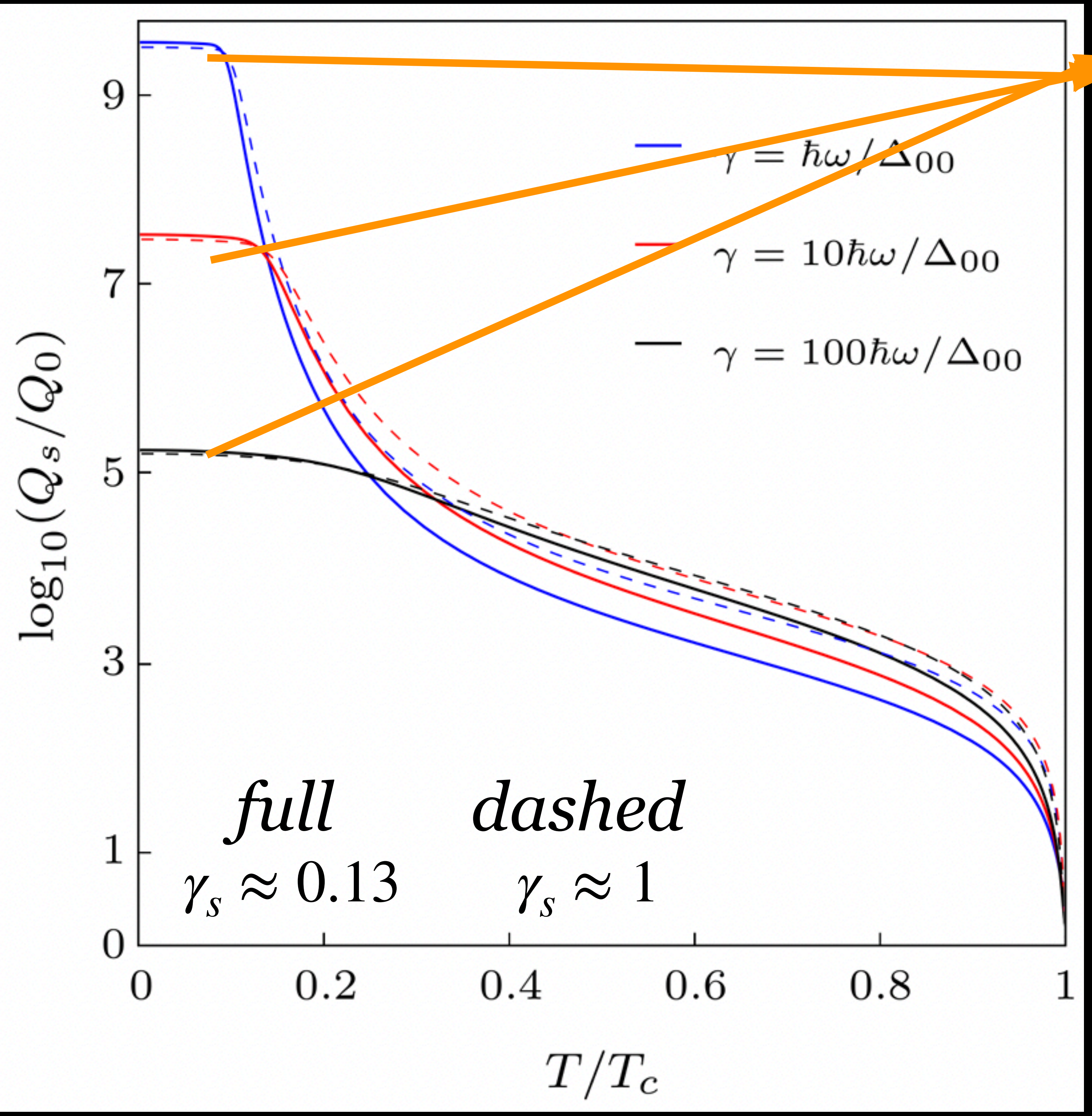
$$\frac{Q_s}{Q_n} = \left(\frac{|\sigma_s|}{|\sigma_n|}\right)^{1/2} \frac{\cos(\pi/4)}{\cos(\pi/4 + \delta\varphi/2)}$$

- Huge increase at low temperature due to low σ_s' and relatively high σ_s''.

- Absolute maxima at $T = 0K$, and $\gamma = 0$.
- Maximum for fixed $T \neq 0K$ at nonzero pair breaking.



$\gamma_s \approx 0.13, \quad f = 1.3\, GHz, \quad \Delta_{00} = 2\, meV$



- High Quality "plateaus" for $T \lesssim T_p \approx 0.1 T_{c,0}$.
- Analysis showing the role of the in-gap states and Fermi-Dirac distribution can be found in arXiv:2409.04203
- Weakly dependent on $\gamma_s \lesssim 1$.

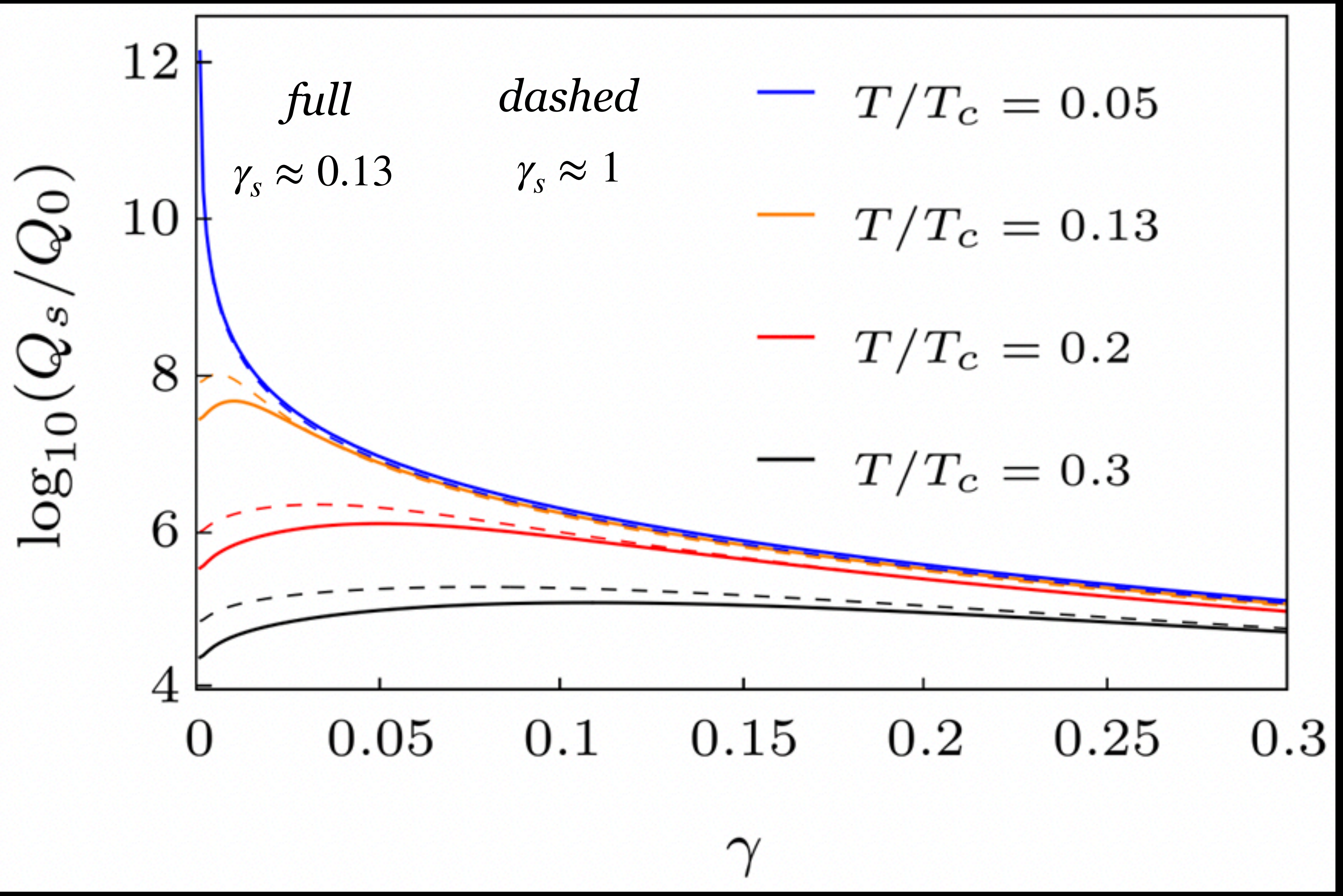
Quality at $T = 0K$

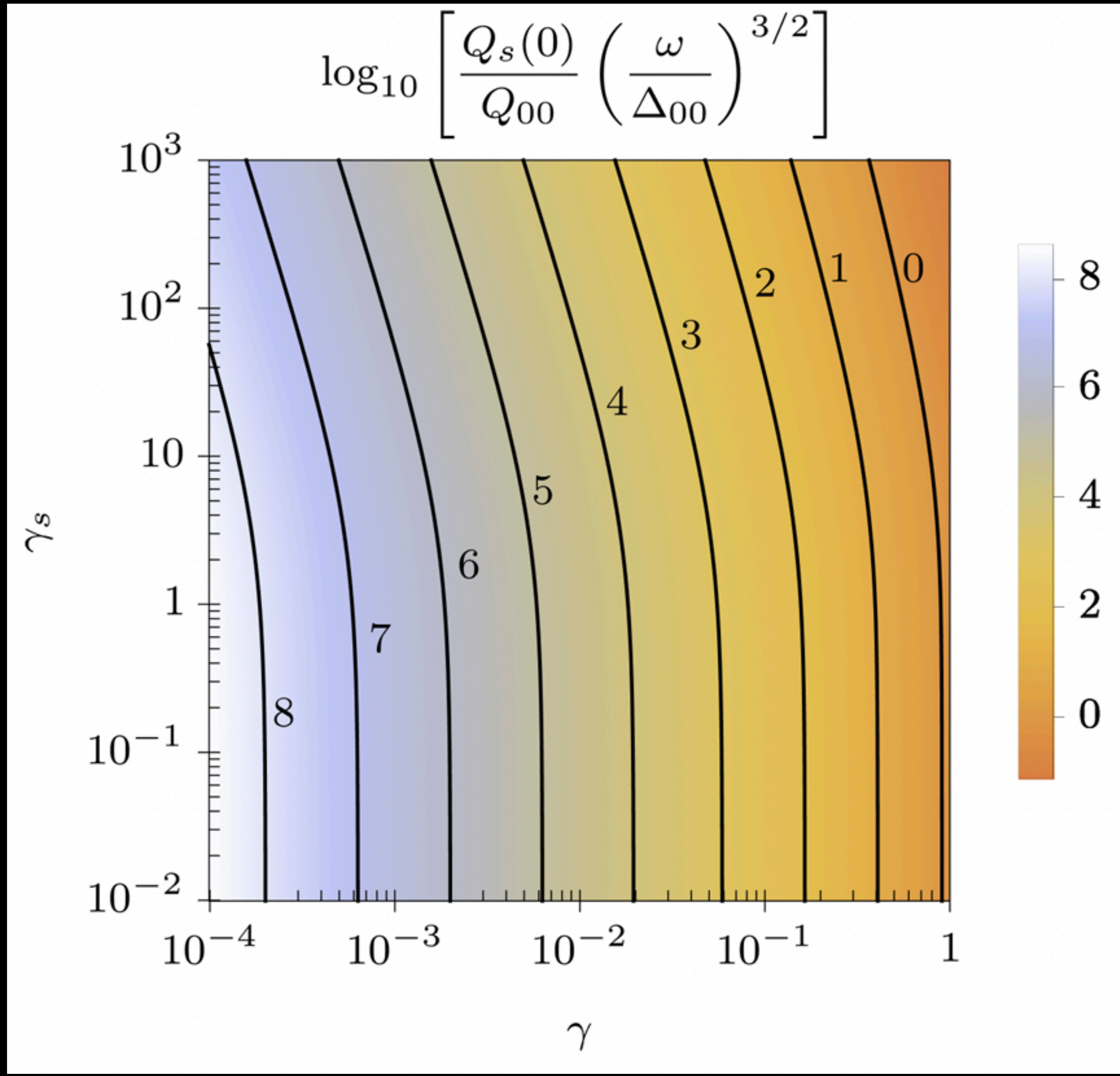
$$\frac{Q_s(0)}{Q_n} = \sqrt{\frac{2}{\sigma_0}\frac{\sigma_s''(0)^{3/2}}{\sigma_s'(0)}} = \frac{4\sigma_0}{\sigma_s'(0)}\left(\boxed{\frac{n_s(0)}{n}}\,\frac{\Gamma + \Gamma_s}{\omega}\right)^{3/2}$$

- Role of superfluid fraction.

$$\frac{\sigma'(0)}{\sigma_0} = \boxed{\frac{\gamma^2}{1+\gamma^2}} \times \frac{\gamma_s + \gamma}{\gamma_s + \sqrt{1+\gamma^2}}$$

- Pair-breaking subgap induced residual resistance.





- The different roles of pair-breaking and pair-conserving scattering.
- Role of pair-conserving disorder changes.

Superfluid fraction

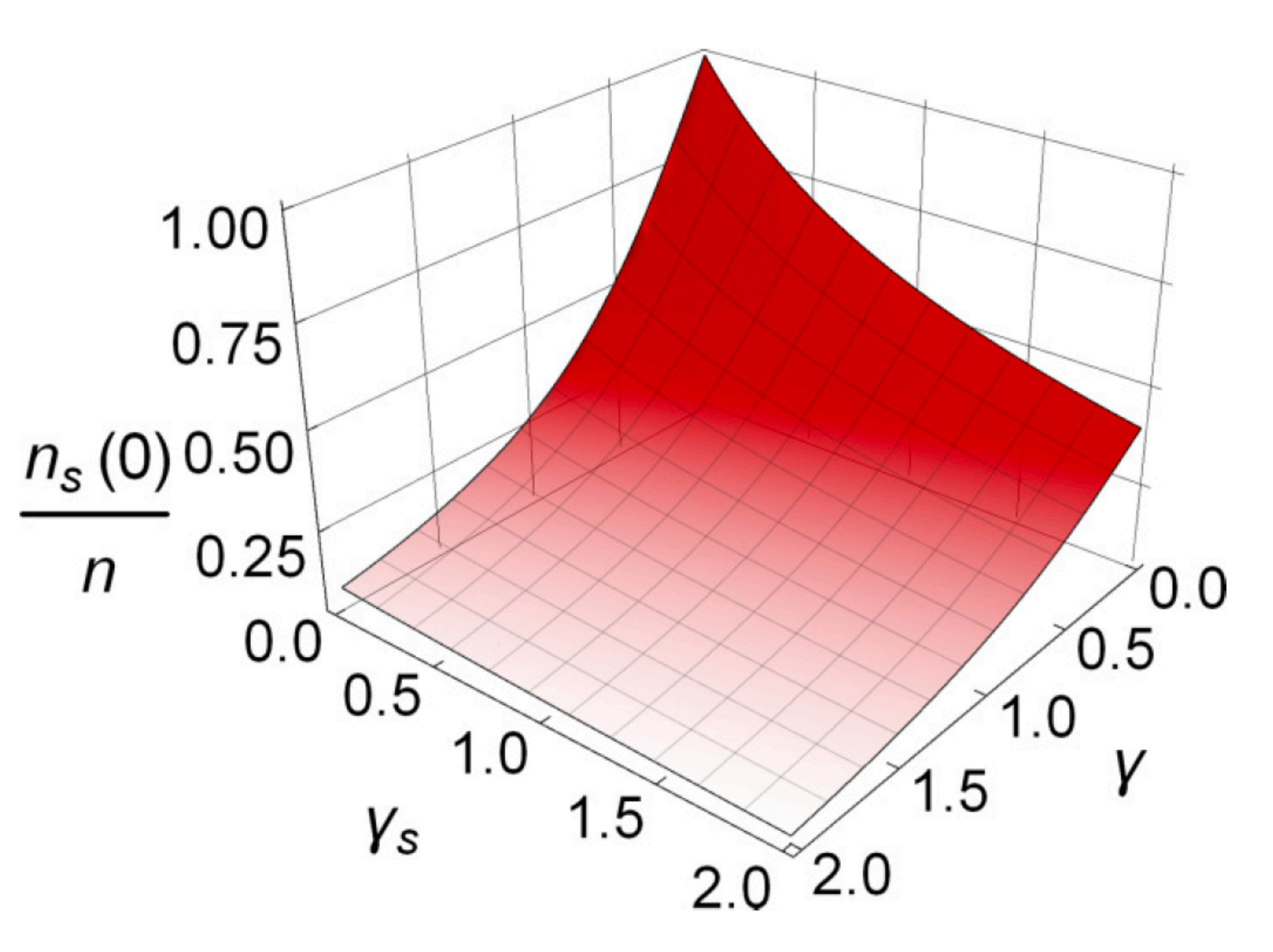
$$n_s(0)/n = \begin{cases} \frac{1}{\gamma_s}\left[\arctan(1/\gamma) - \frac{1}{\sqrt{1-\gamma_s^2}}\left(\arccos\gamma_s + \arctan\frac{\sqrt{1-\gamma_s^2}}{\gamma} - \arctan\frac{\sqrt{1-\gamma_s^2}\sqrt{1+\gamma^2}}{\gamma\gamma_s}\right)\right], \text{ if } \gamma_s < 1 \\ \frac{1}{\gamma_s}\left[\arctan(1/\gamma) - \frac{1}{\sqrt{\gamma_s^2-1}}\ln\frac{\left(\gamma_s+\sqrt{\gamma_s^2-1}\right)\left(\gamma+\sqrt{\gamma_s^2-1}\right)}{\gamma\gamma_s+\sqrt{\gamma_s^2-1}\sqrt{\gamma^2+1}}\right], \text{ if } \gamma_s \geq 1 \end{cases}$$

- Assuming $\gamma_s < 1$ and linear order of γ:

$$\frac{n_s(0)}{n} = \frac{1}{\gamma_s}\left[\frac{\pi}{2} - \frac{\arccos\gamma_s}{\sqrt{1-\gamma_s^2}}\right] - \frac{\gamma}{\gamma_s+1}$$

- Assuming $\gamma_s \ll 1$ and $\gamma \ll 1$:

$$n_s(0)/n \approx 1 - \gamma - \gamma_s\pi/4$$

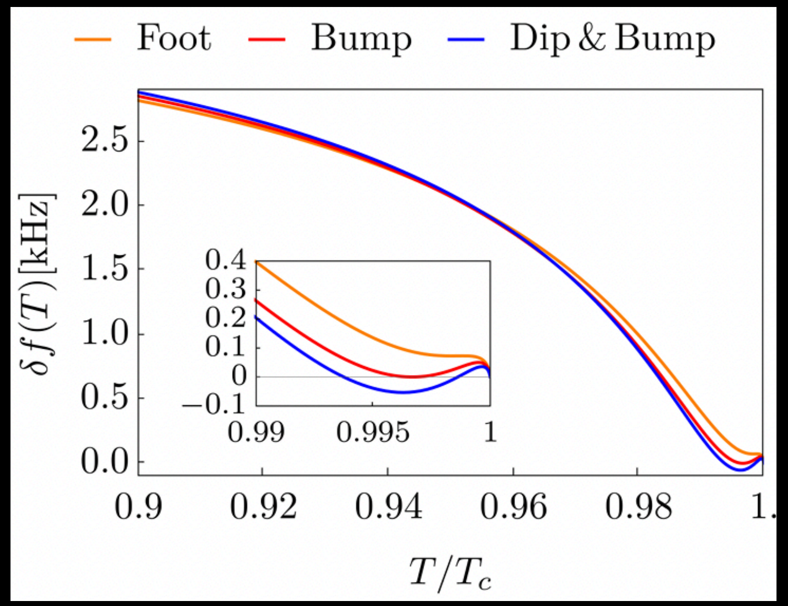


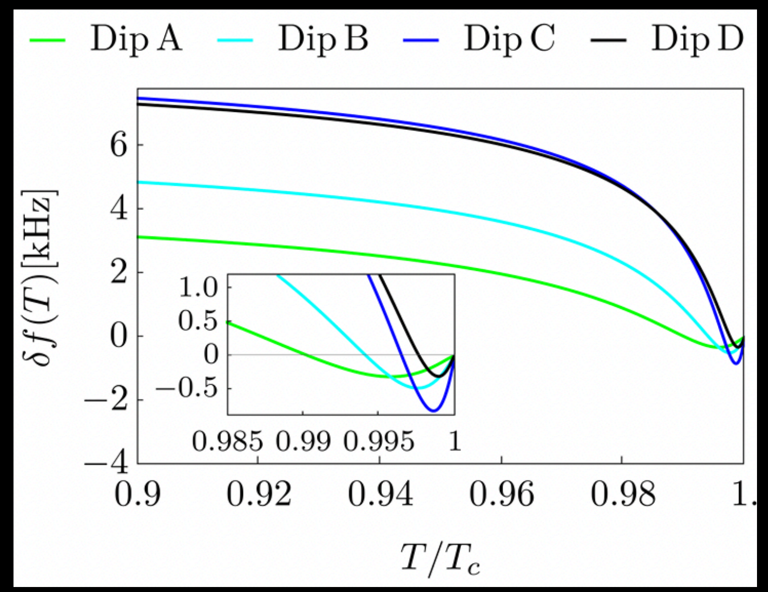
Summary & Conclusions I.

Frequency Shift

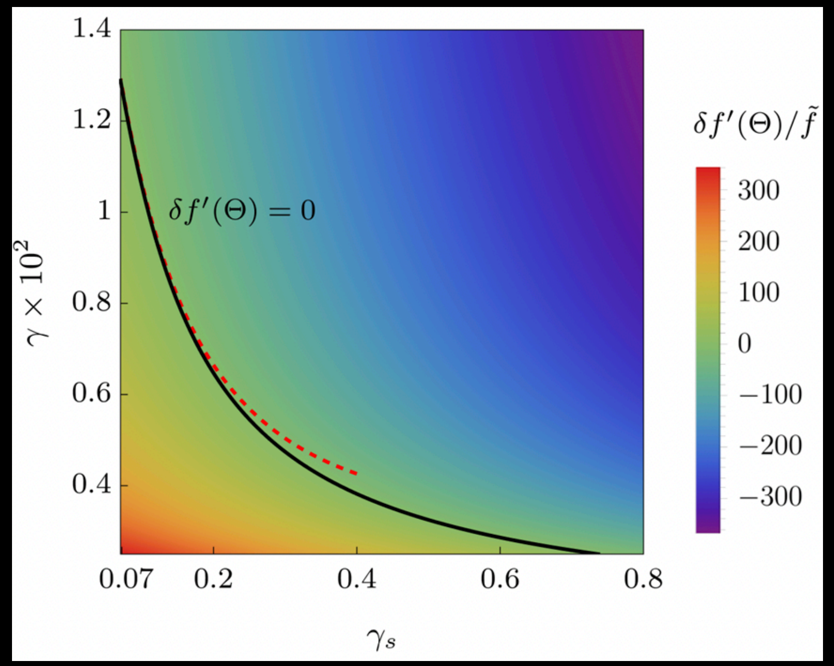
- Despite omitting the inhomogeneity effects, DS framework provides qualitatively (and quantitatively) reasonable results with the same numerical complexity as the Mattis-Bardeen theory.

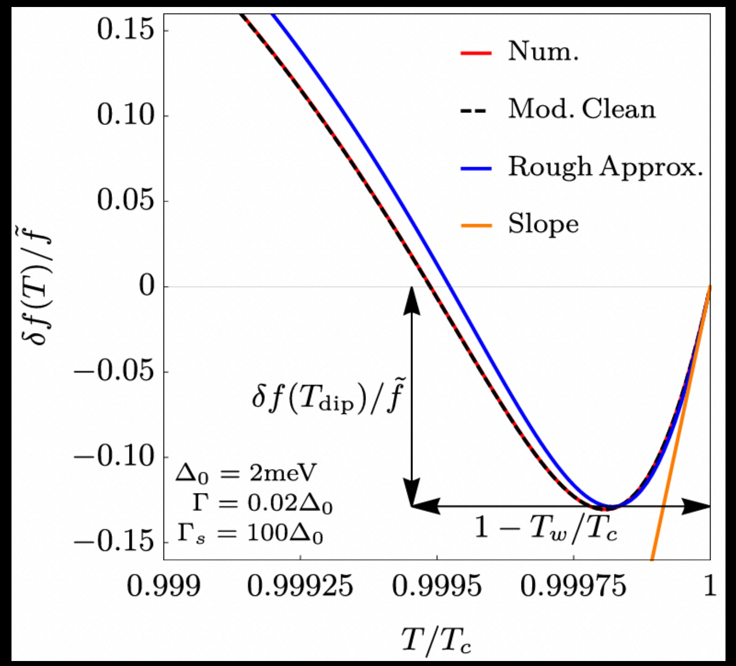
- Various frequency shift regimes (Foot, Bump, Dip & Bump and Dip) assuming low disorder can be systemically identified.



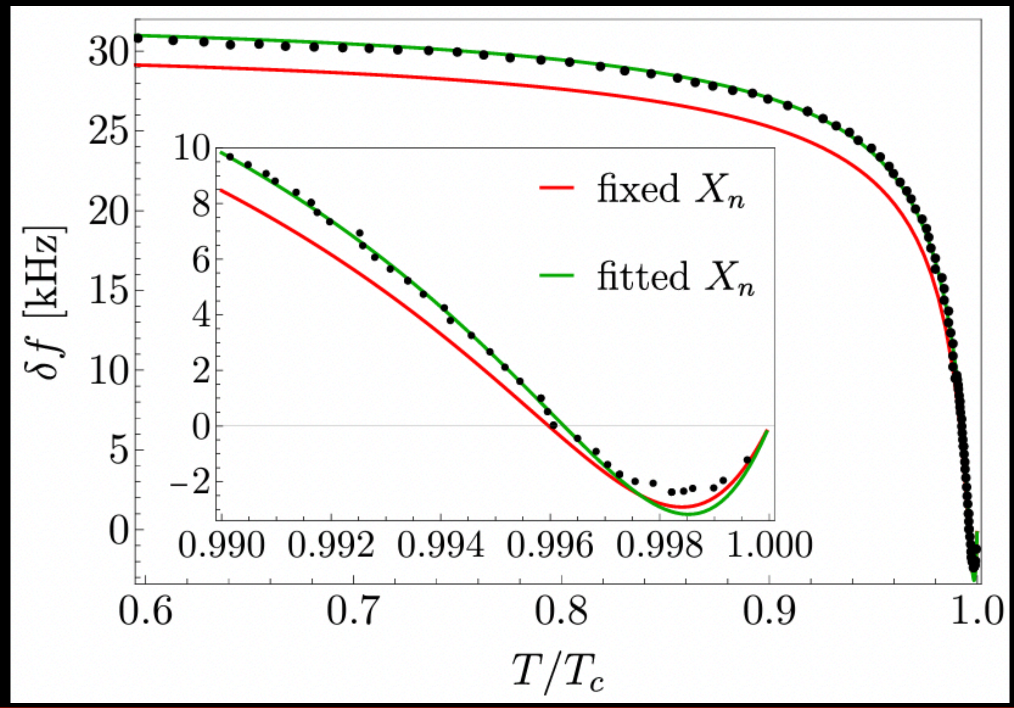


- Addressing the moderately clean regime, we describe the frequency shift slope $\delta f'(\Theta) = \partial\delta f(\Theta)/\partial\Theta|_{\Theta=0}$ with the simple function of pair-conserving and pair-breaking scattering rates.





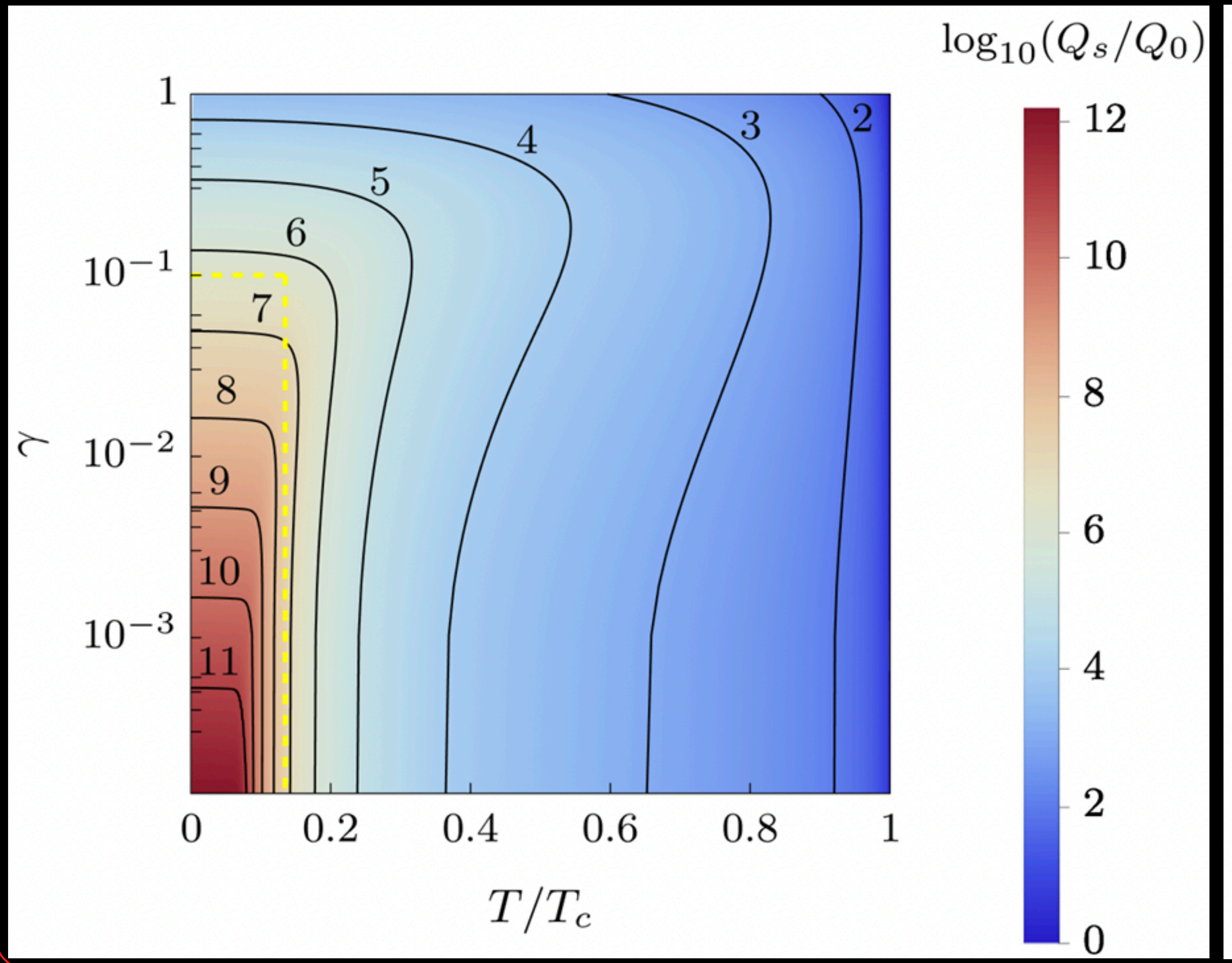
- Considering the complexity of the phenomena together with the relevant temperature scale and simplicity of our approach, we achieve compelling agreement with the experiment.

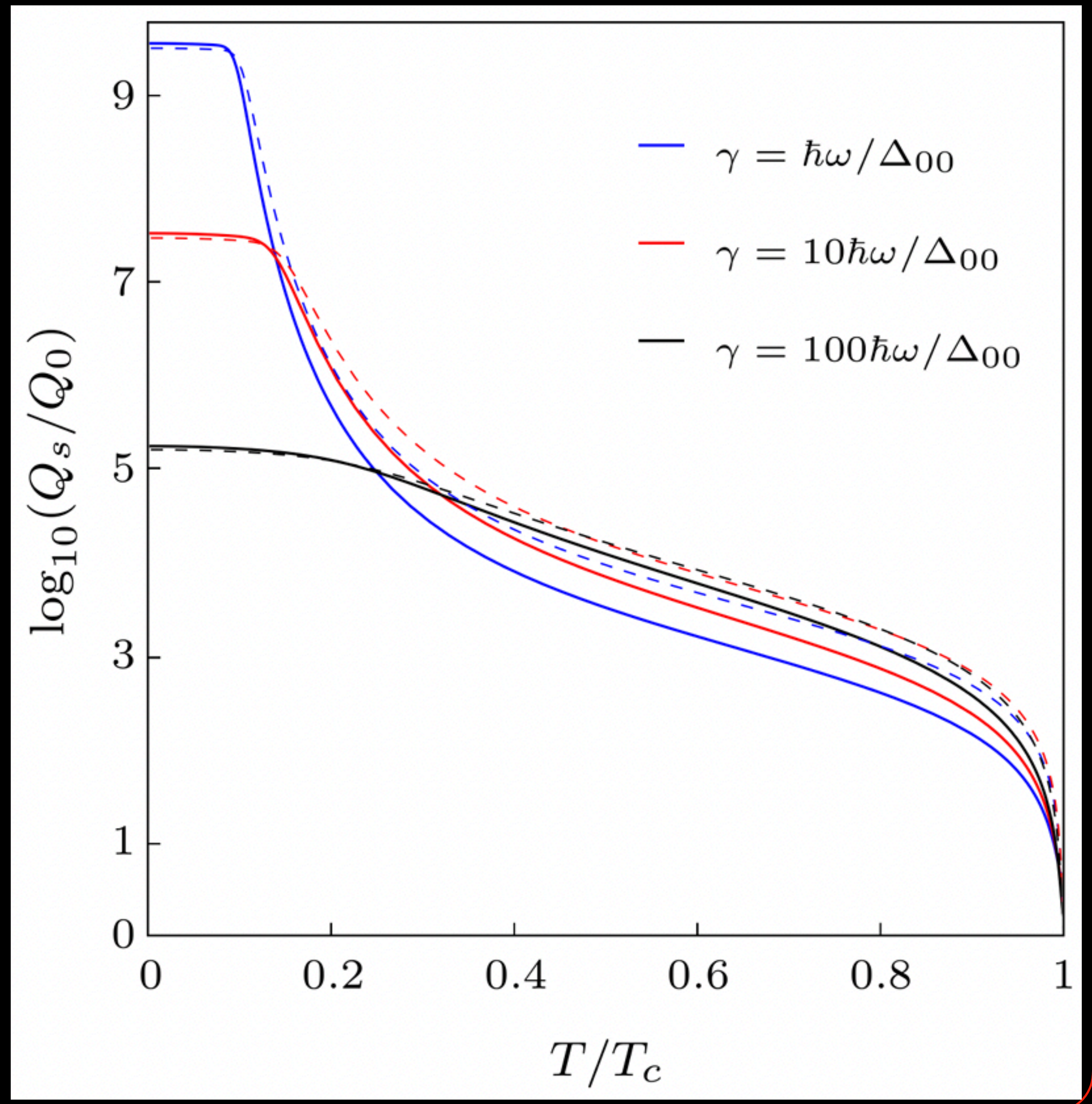


Summary & Conclusions II.

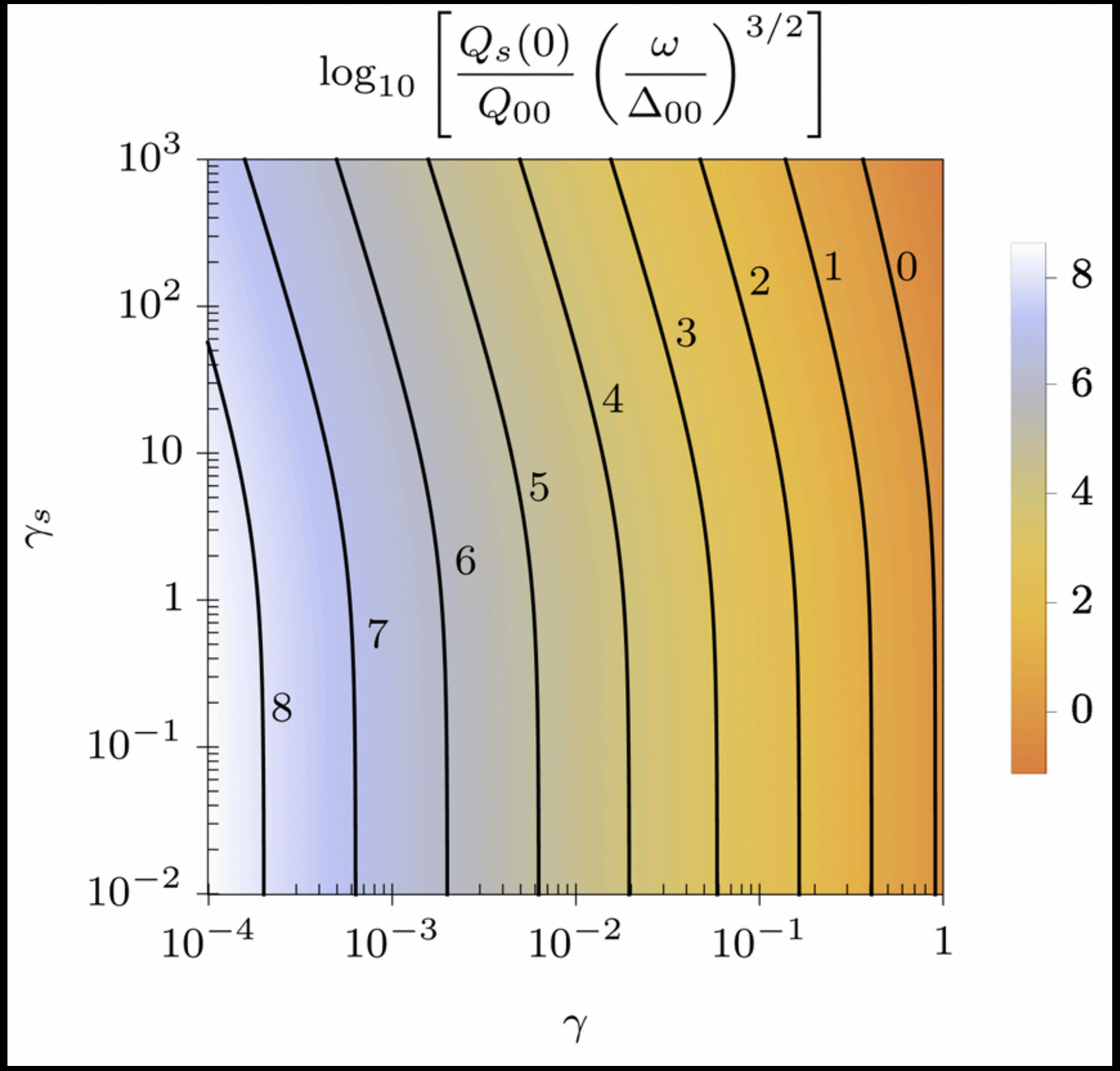
Quality

- Analysis of the high quality plateaus at low temperature and low disorder. For details see: arXiv:2409.04203.





- Analysis of the quality factor at zero Kelvin Temperature shows different influence of pair-breaking and pair-conserving disorder.



Publications Related to the Dynes superconductivity

- F. Herman and R. Hlubina,Phys. Rev. B **108**, 134518 (2023),
  Slope of Hc2 close to Tc versus the size of the Cooper pairs: The role of disorder in Dynes superconductors,
- F. Herman, Phys. Rev. B **106**, 224521 (2022),
  Advanced approach of superconducting gap function extraction from tunneling experiments,
- D. Kavický, F. Herman and R. Hlubina, Phys. Rev. B **105**, 214504 (2022),
  Model-independent determination of the gap function of nearly localized superconductors,
- F. Herman and R. Hlubina, Phys. Rev. B **104**, 094519 (2021),
  Microwave response of superconductors that obey local electrodynamics,
- F. Herman and R. Hlubina, Phys. Rev. B **97**, 014517 (2018),
  Thermodynamic properties of the DS,
- F. Herman and R. Hlubina, Phys. Rev. B **96**, 014509 (2017),
  Electromagnetic properties of impure superconductors with pair-breaking processes,
- F. Herman and R. Hlubina, Phys. Rev. B **95**, 094514 (2017),
  Consistent two-lifetime model for spectral functions of superconductors,
- F. Herman and R. Hlubina, Phys. Rev. B **94**, 144508 (2016),
  Microscopic interpretation of the Dynes formula for the tunnelling density of states

Open question

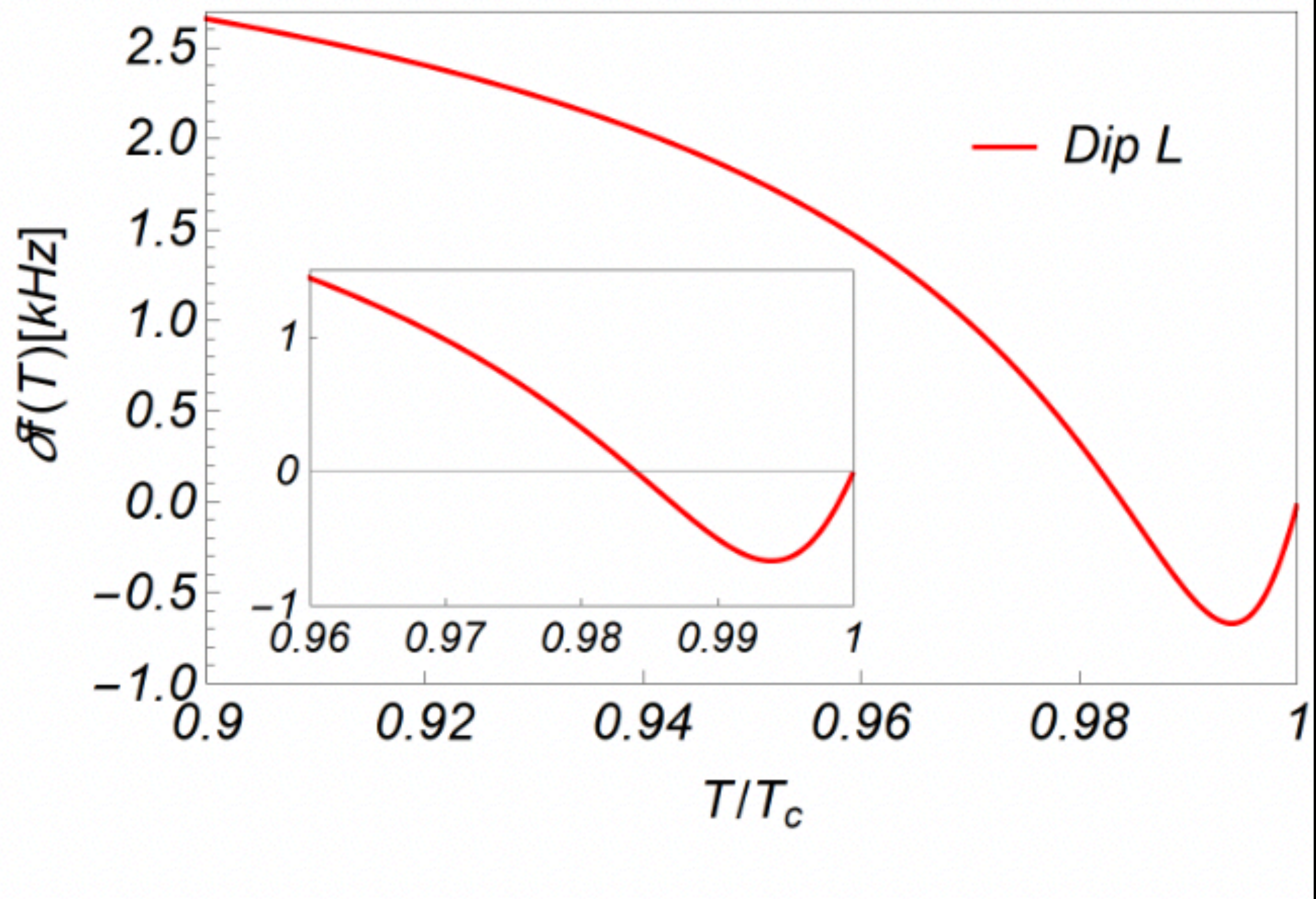
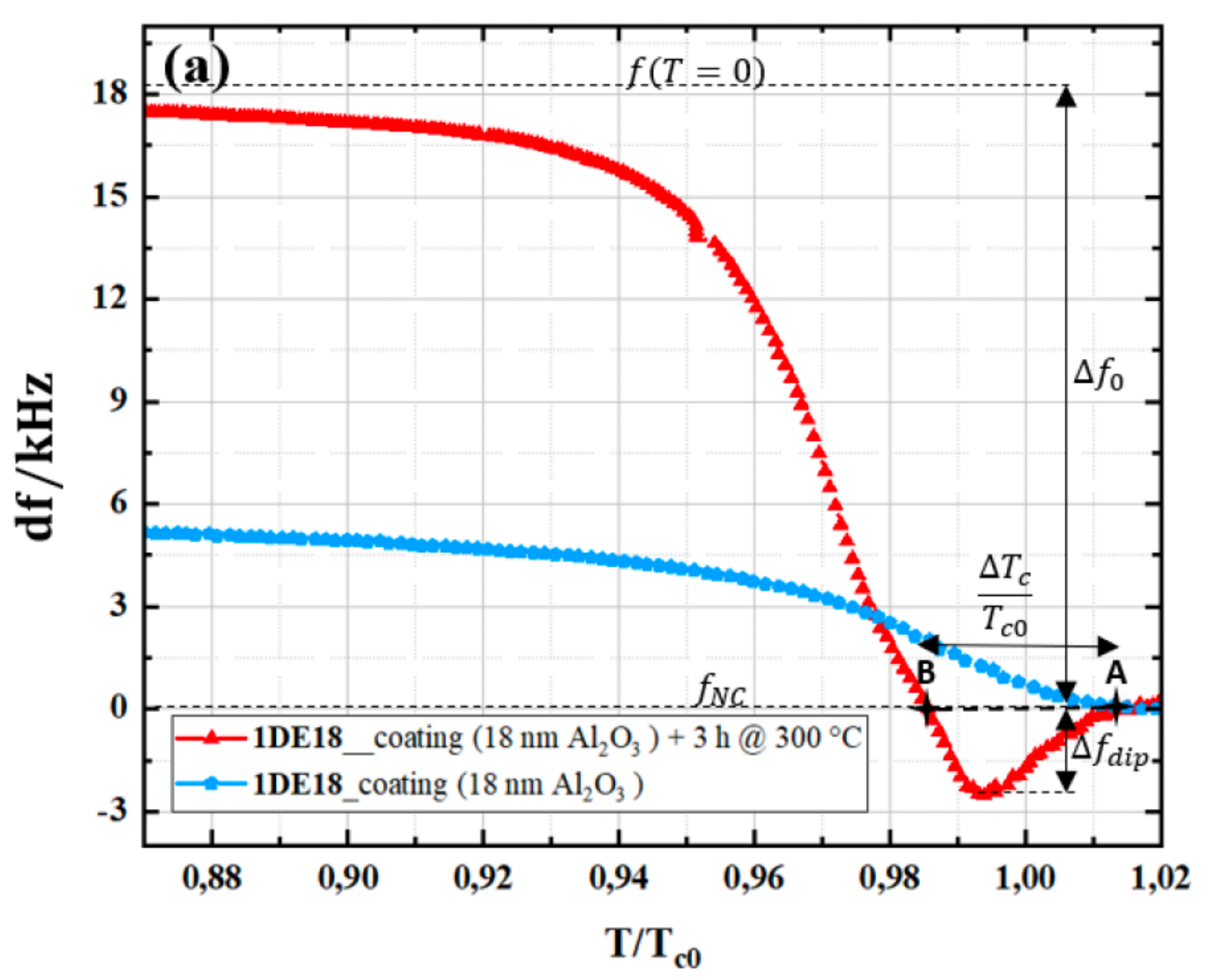


FIG. 1. Dip on the scale of $\sim 0.01 T_c$.



	$\Gamma[\omega]$	$\Gamma_s[\omega]$	Regime	ℓ[nm]	X_n[mΩ]	$\tilde{f}$[kHz]
Dip L	26	0.01	$\Gamma_s \ll \omega \ll \Gamma \lesssim \Delta_0$	706	2	4.9

TABLE I. Table of individual regimes together with its considered scattering constants Γ and Γ_s in units of the angular resonant frequency $\hbar\omega$ (for convenience $\hbar = 1$, and $f = 1.3\,\mathrm{GHz}$). For each regime, we also add the parameter inequalities as well as calculated values of mean free path ℓ, normal state reactance X_n, and the scale $\tilde{f}$.

Role of the 18 nm layer of Al2O3, or oxides in general?

Mean free path ℓ?

Comparison of Approaches