

# Coalescence studies for light nuclei

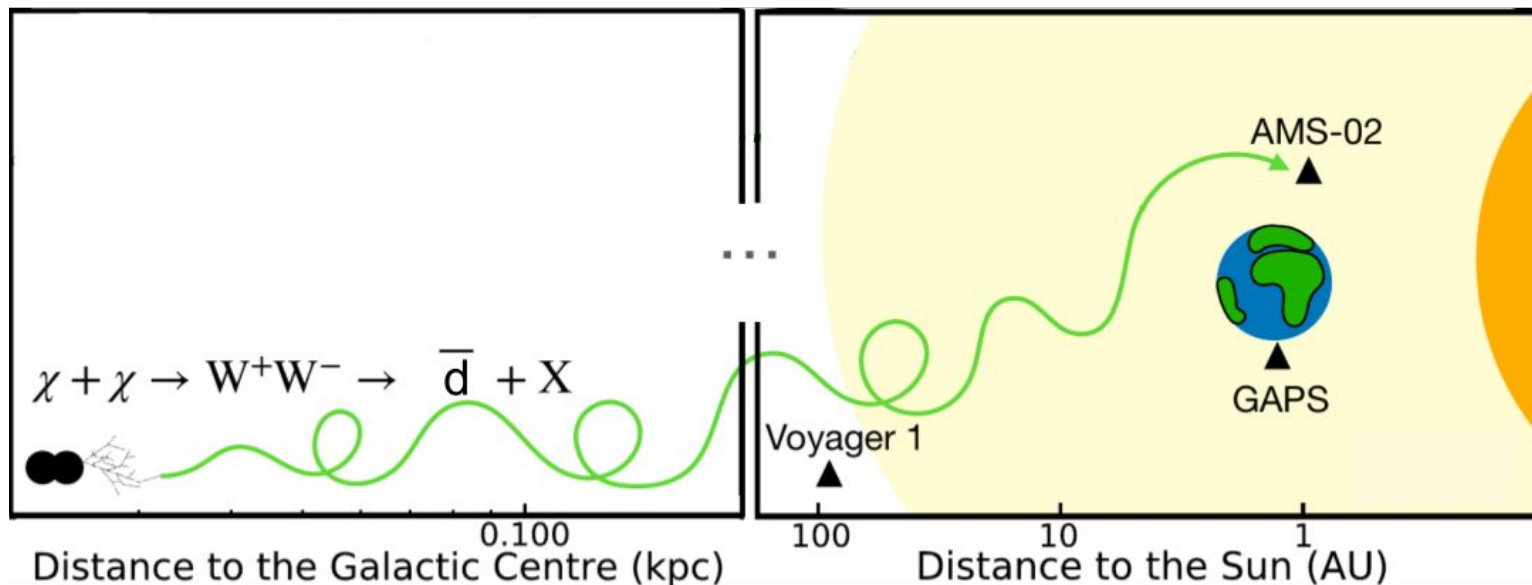
Maximilian Mahlein, Laura Fabbietti, Bhawani  
Singh, Chiara Pinto, Michele Viviani

Based on: arXiv:2404.03352 (accepted by EPJC)

*Technical University Munich*

# Cosmic Rays

## Antinuclei in Cosmic Rays

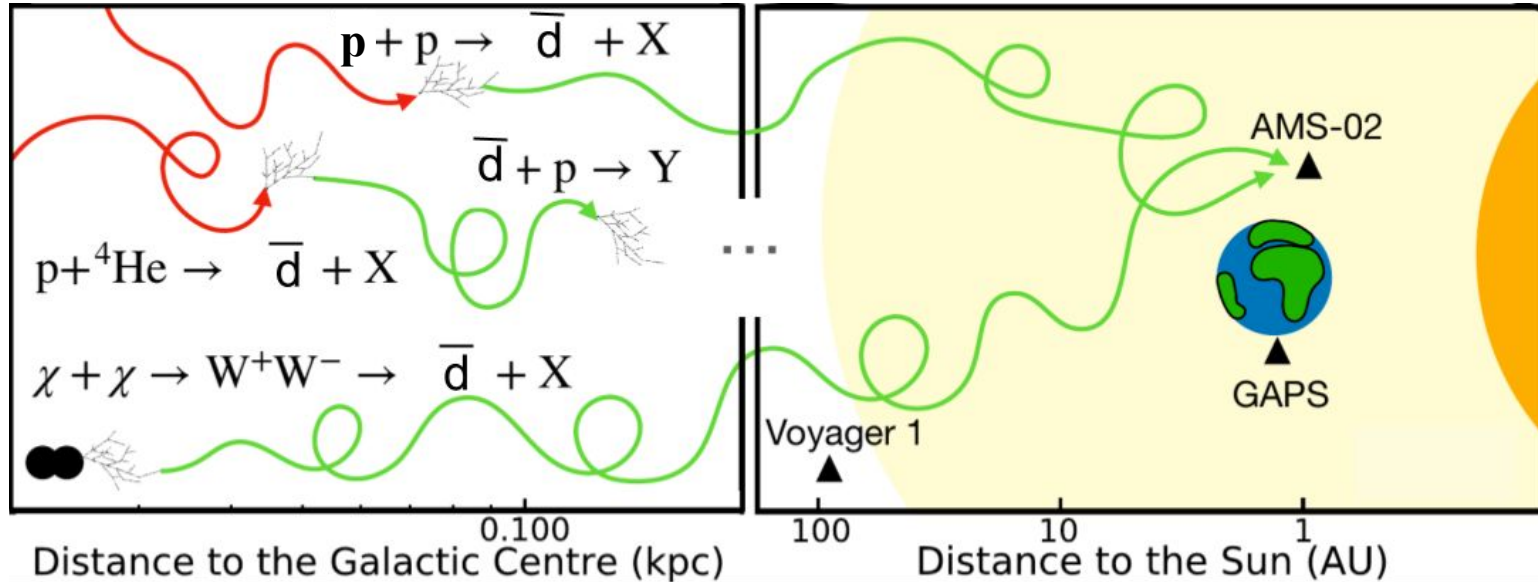


ALICE Collaboration, Nat. Phys. 19, 61–71 (2023)

- Antinuclei could be a probe for indirect Dark Matter searches

# Cosmic Rays

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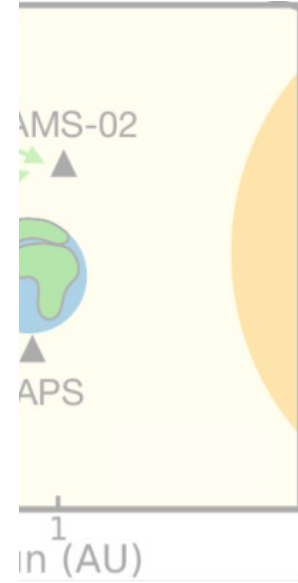
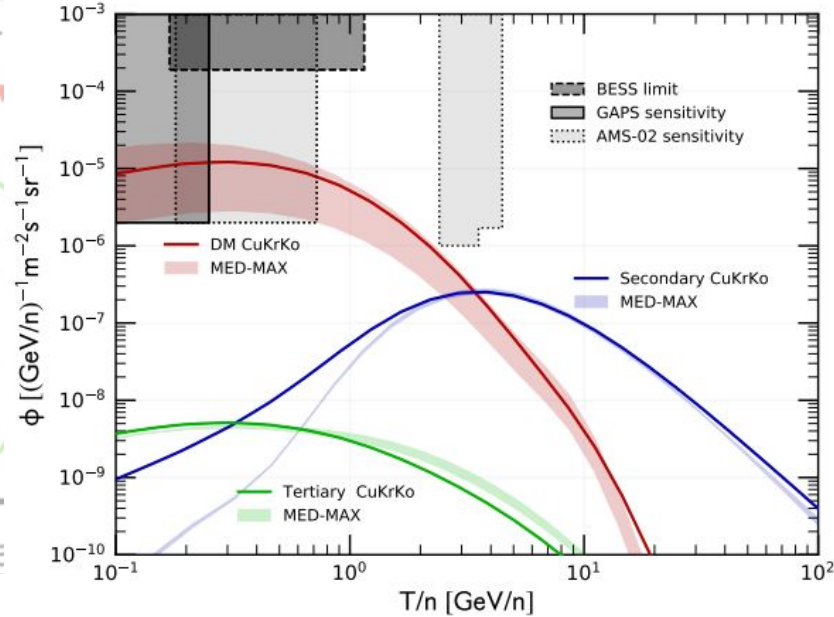
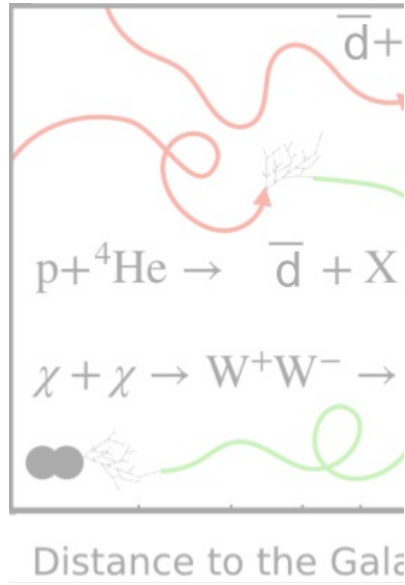


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- Antinuclei could be a probe for indirect Dark Matter searches
- However: Astrophysical background from cosmic rays expected

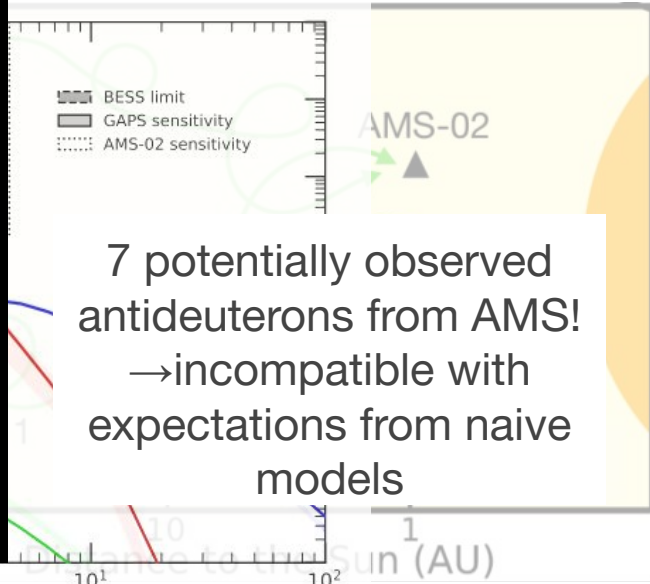
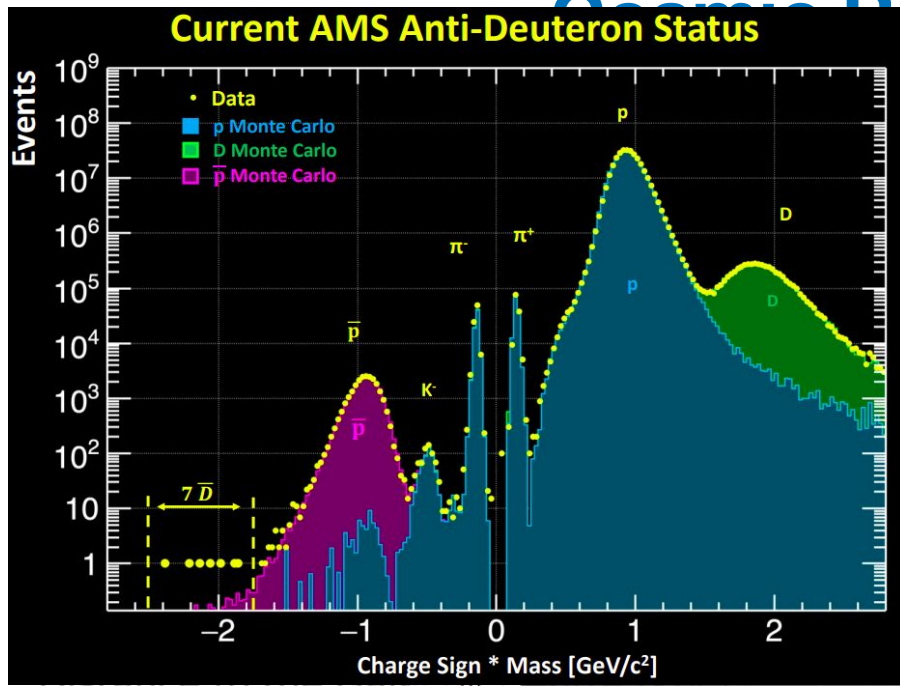
# Cosmic Rays

## Antinuclei in Cosmic Rays



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- Antinuclei could be a probe for indirect Dark Matter searches
- However: Astrophysical background from cosmic rays expected
- High Signal/Noise ratio ( $\sim 10^2$ - $10^4$ ) at low  $E_{\text{kin}}$  expected by many models!



T/n [GeV/n]

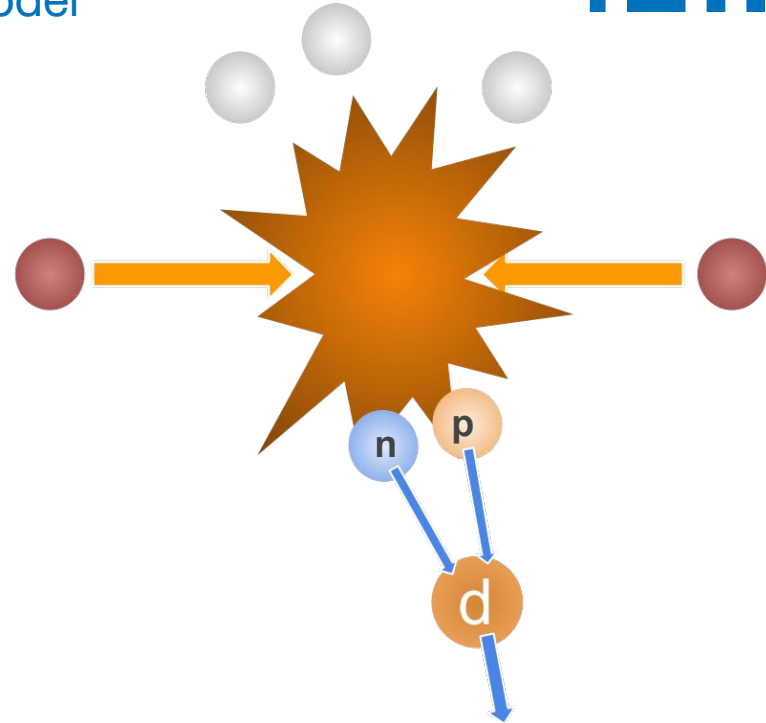
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# Modelling (Anti)nuclei Production

## The Coalescence Model

- Nucleons bind after freeze-out if they are close in phase-space

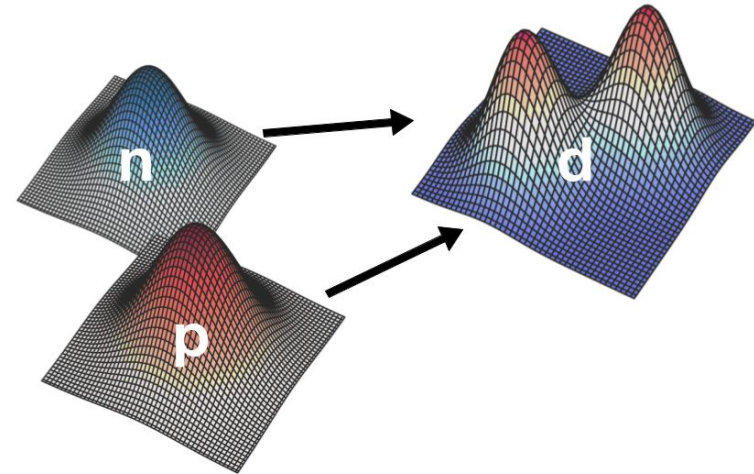


# Modelling (Anti)nuclei Production

## The Coalescence Model

- Nucleons bind after freeze-out if they are close in phase-space
- Wigner function formalism:

$$\frac{dN_d}{d^3P} = S_d \int d^3x_1 \int d^3x_2 \int d^3x'_1 \int d^3x'_2 \Psi_d^*(\vec{x}'_1, \vec{x}'_2) \times \Psi_d(\vec{x}_1, \vec{x}_2) \langle \Psi_2^\dagger(\vec{x}'_2) \Psi_1^\dagger(\vec{x}'_1) \Psi_1(\vec{x}_1) \Psi_2(\vec{x}_2) \rangle$$



$$\mathcal{P}(q, \sigma) = \frac{S_2}{(2\pi)^3 \sigma^6} \int d^3r_p d^3r_n D(q, r) e^{-\frac{r_p^2 + r_n^2}{2\sigma^2}}$$

$$= \int d^3\zeta \Psi(\vec{r} + \vec{\zeta}/2) \Psi^*(\vec{r} - \vec{\zeta}/2) \exp(i\vec{q} \cdot \vec{\zeta})$$

$$= \frac{1}{(2\pi\sigma^2)^3} \exp\left(-\frac{\vec{r}_n^2 + \vec{r}_p^2}{2\sigma^2}\right)$$

Nucleus wave function

Relative momenta of nucleons

Source size

Kachelriess et al EPJA (2020)56: 4, MM et al .Eur.Phys.J.C 83 (2023) 9, 804

# Modelling (Anti)nuclei Production

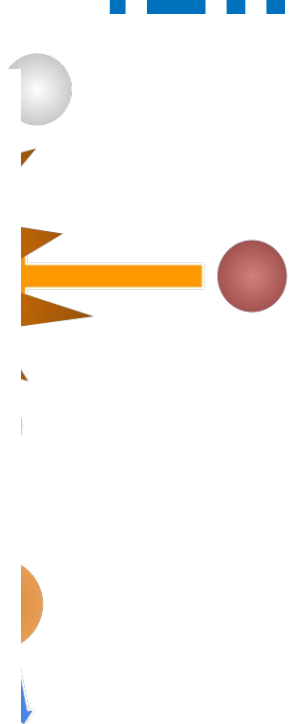
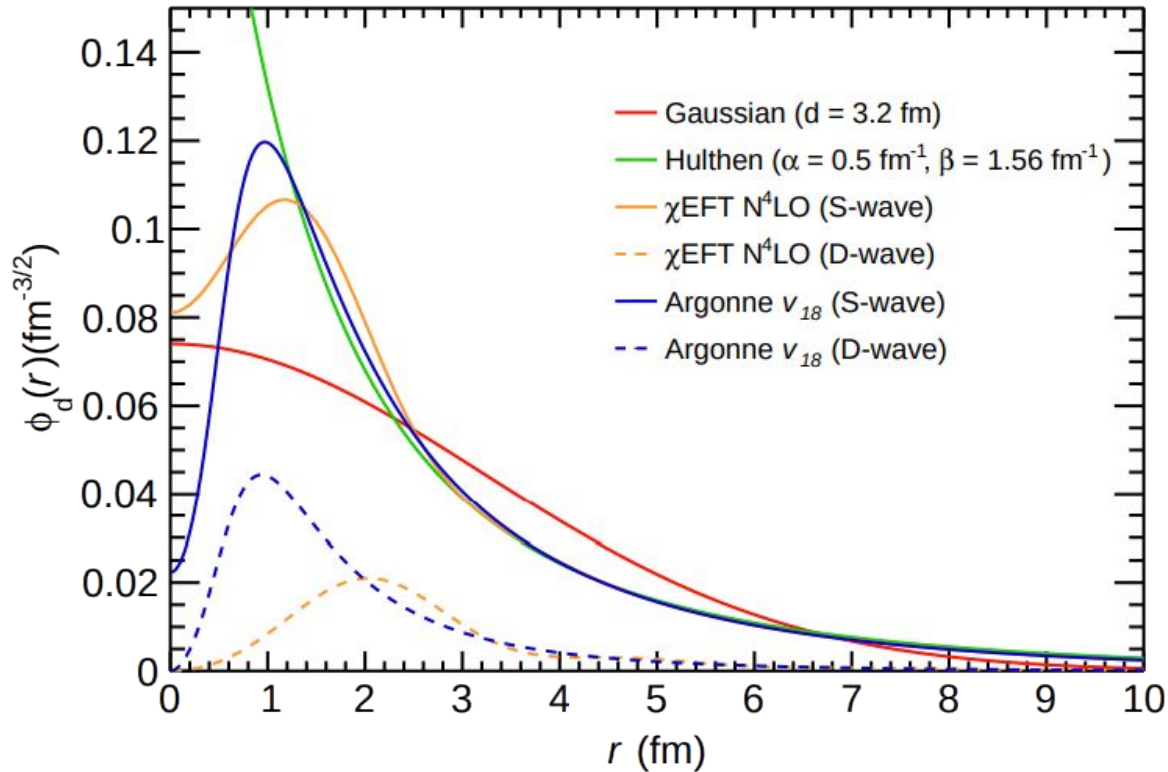
## The Coalescence Model

- Nucleons bind are close in phase space
- Wigner function

$$\frac{dN_d}{d^3P} = S_d \int d^3x_1 \int d^3x_2 \dots \times \Psi_d(\vec{x}_1, \dots, \vec{x}_d)$$

$$\mathcal{P}(q, \sigma) = \frac{S_2}{(2\pi)^3}$$

$$\int d^3\zeta \Psi(\vec{r} + \vec{\zeta}/2) \Psi^*(\vec{r} - \vec{\zeta}/2)$$



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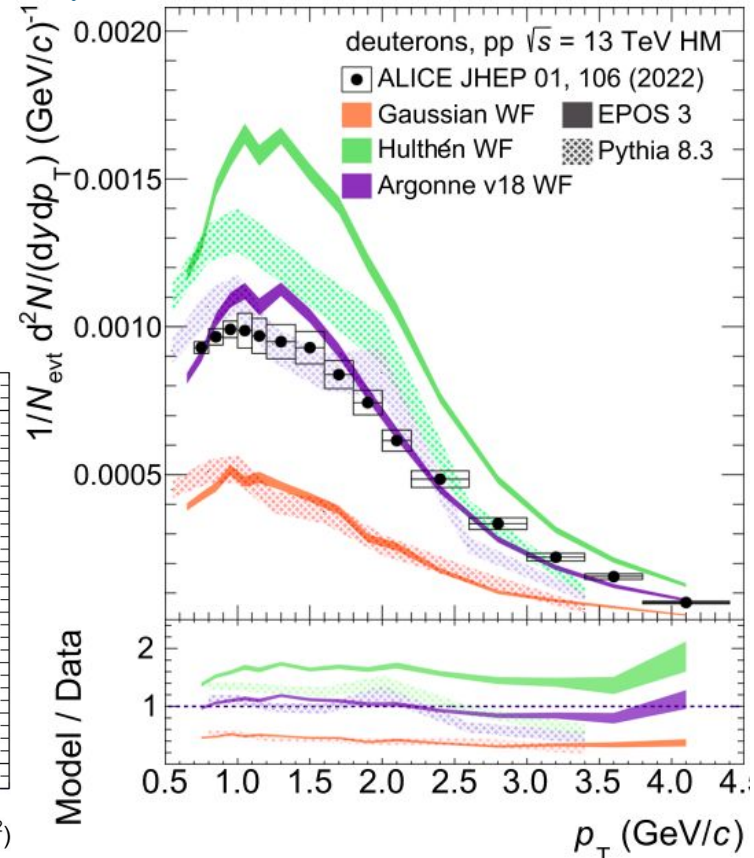
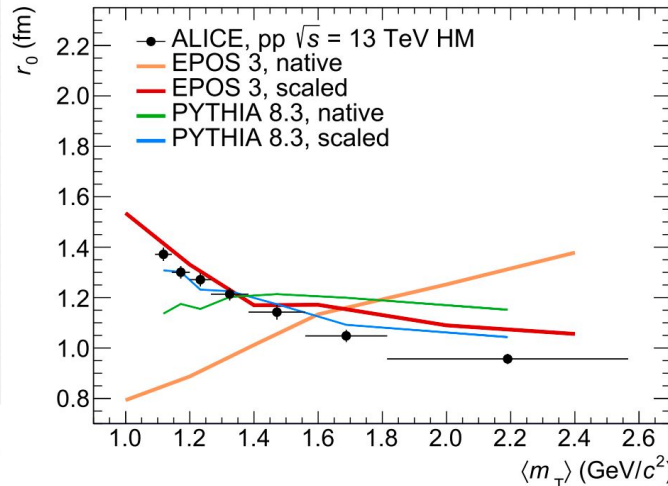
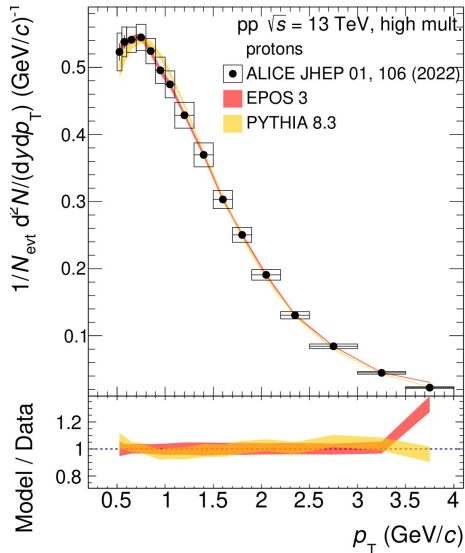


# Coalescence Results EPOS & Pythia

## Deuteron spectra

MM et al. Eur.Phys.J.C 83 (2023) 9, 804

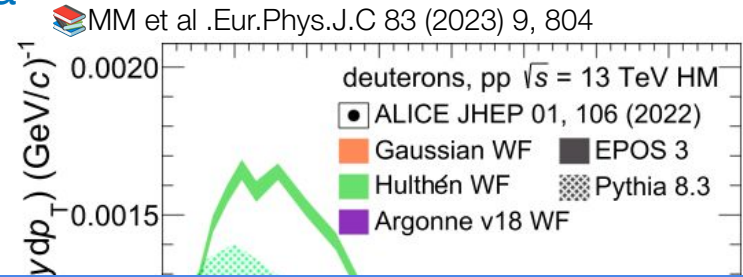
- Corrections to Protons, Source, Multiplicity
- Wavefunctions: Gaussian, Hulthén and Argonne  $v_{18}$
- $AV_{18}$  reproduces data to  $\sim 10\%$



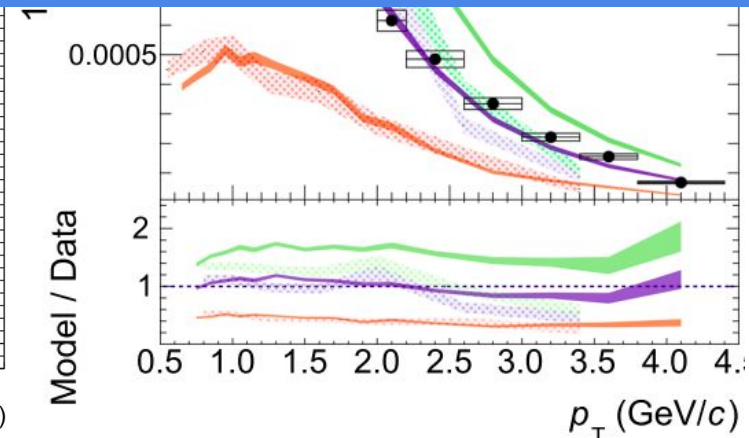
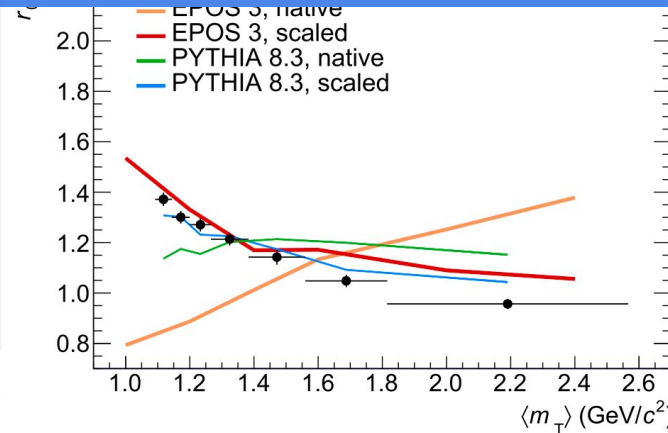
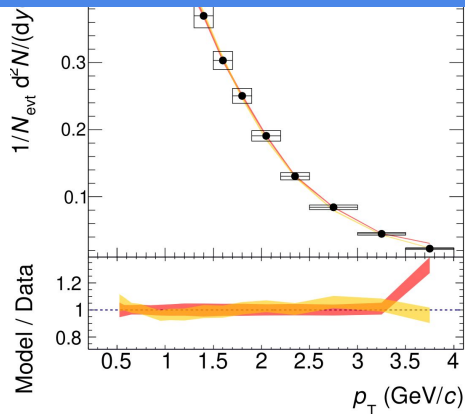
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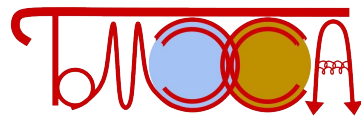


Develop a purpose built Event Generator to apply this model



# The ToMCCA Model

A Toy Monte Carlo Coalescence Afterburner



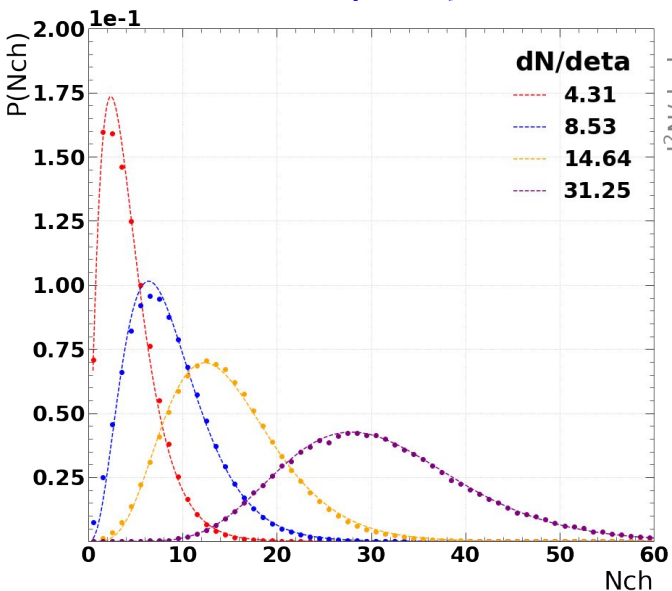
Main Inputs: Multiplicity, momentum distributions, source size

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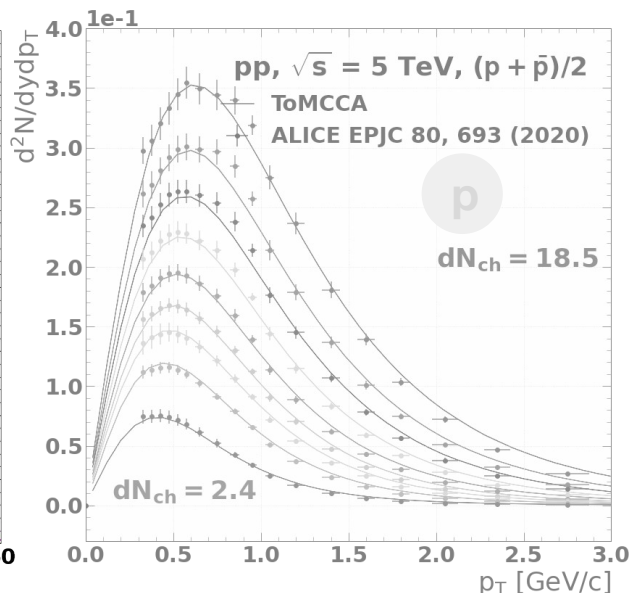
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## Multiplicity



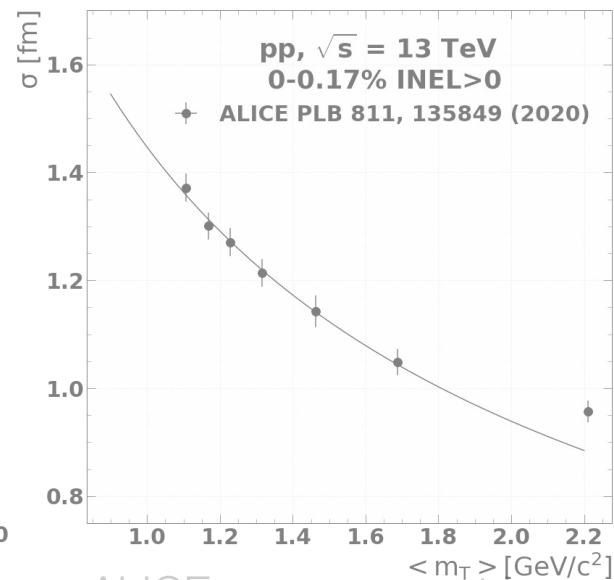
Erlang distribution fitted to EPOS3 simulations

## Momentum distribution



ALICE measurements over a large multiplicity range

## Source size



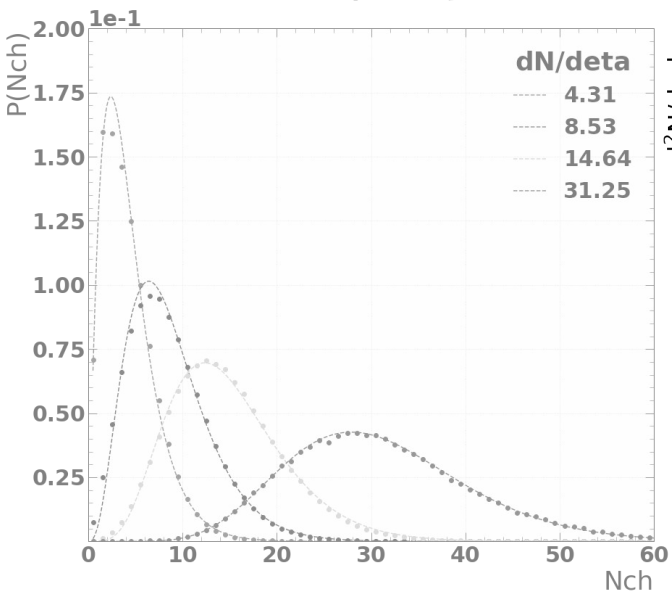
ALICE measurement in high-multiplicity collisions

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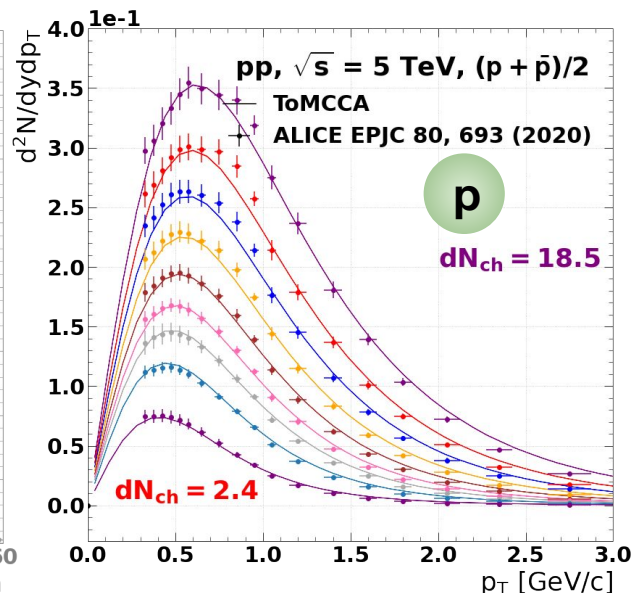
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Multiplicity



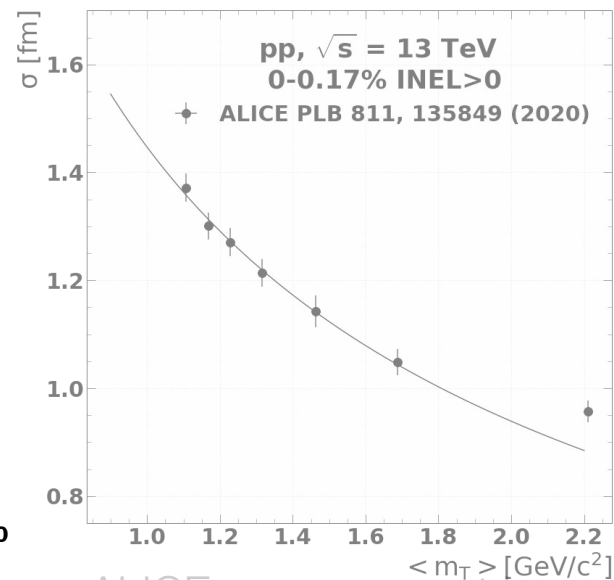
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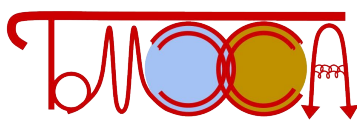


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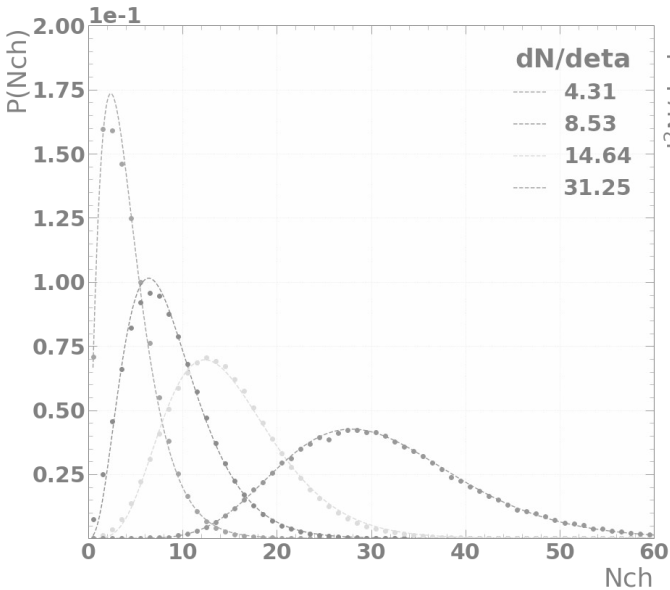


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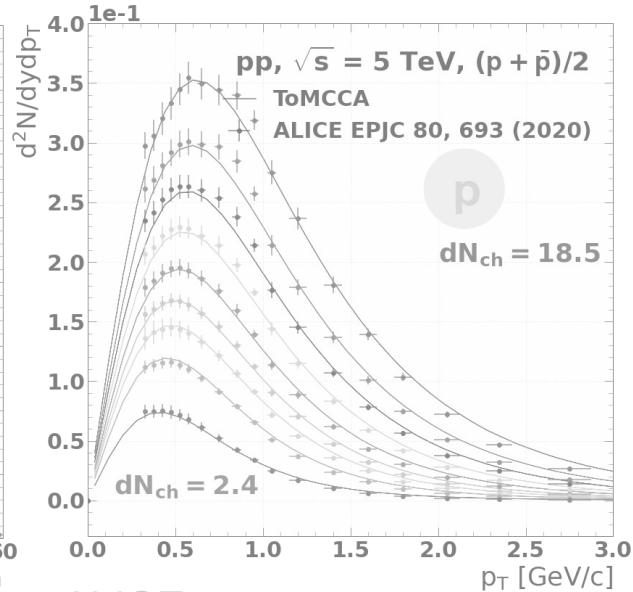
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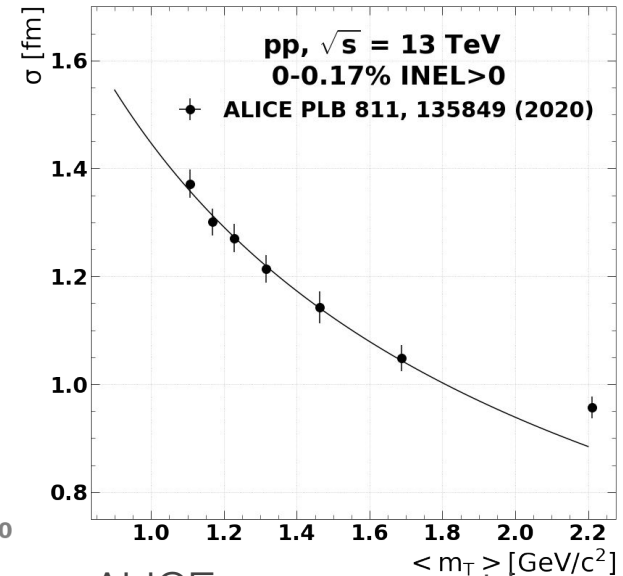
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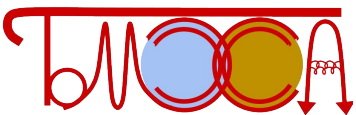
Source size



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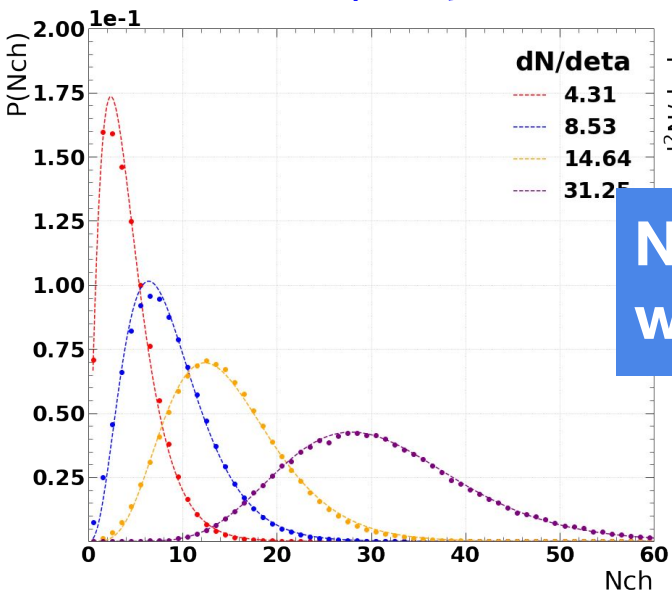
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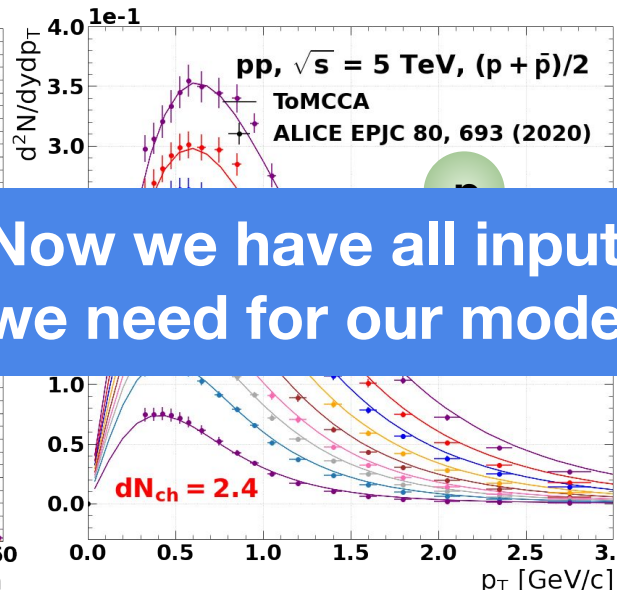
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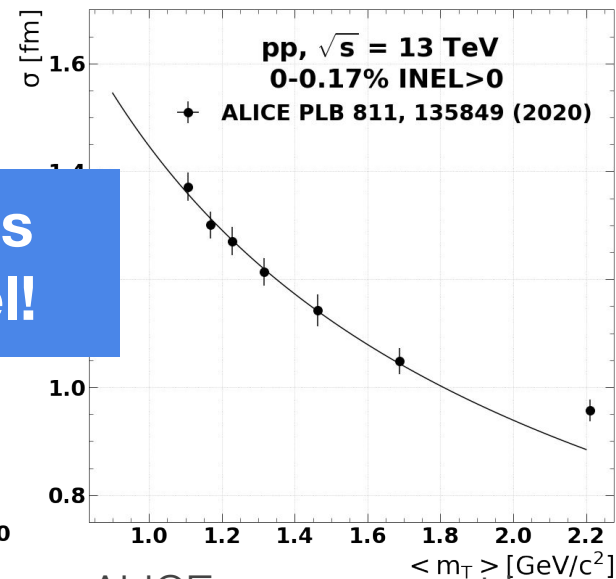
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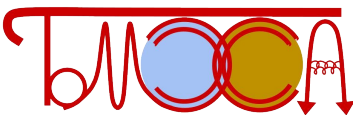


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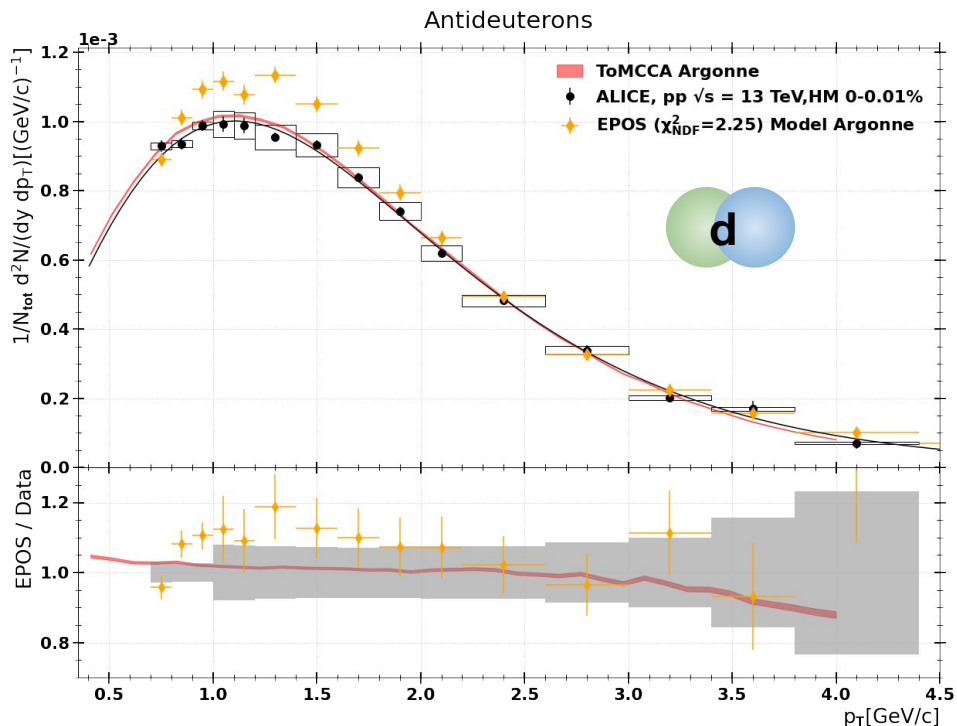
Now we have all inputs we need for our model!

# Deuteron Spectra

## ToMCCA Model in HM pp Collisions



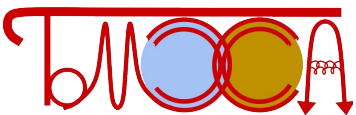
- Using ToMCCA for 13 TeV HM collisions ( $(dN_{ch}/dn)_{|\eta|<0.8} \sim 31$ ) we can reproduce measured spectra
- No free parameters!





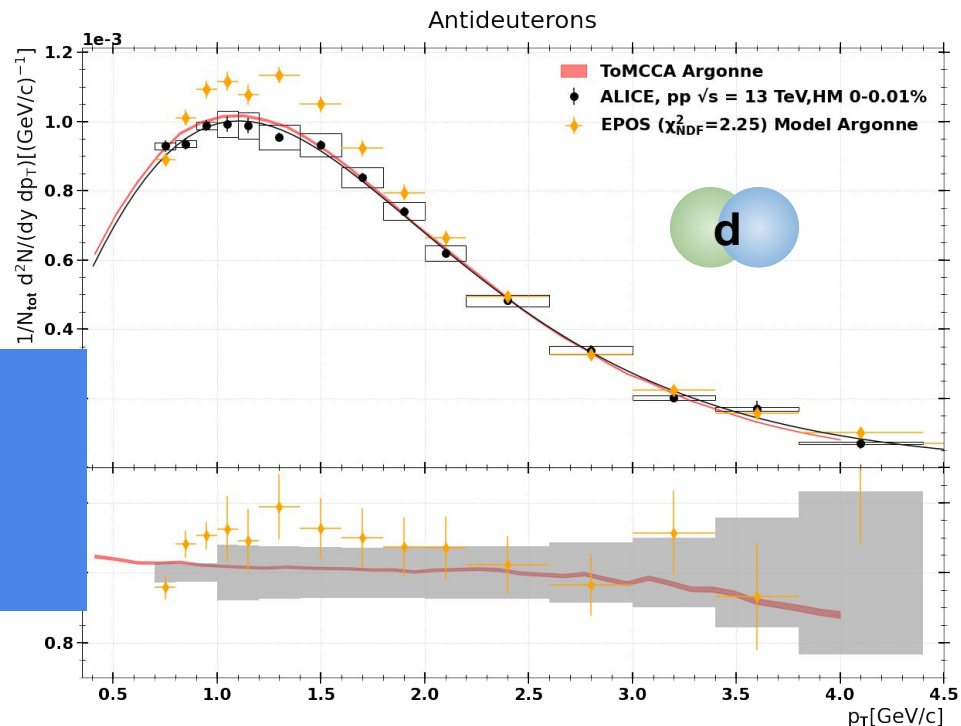
# Deuteron Spectra

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Does this help us in predicting Cosmic Ray fluxes?

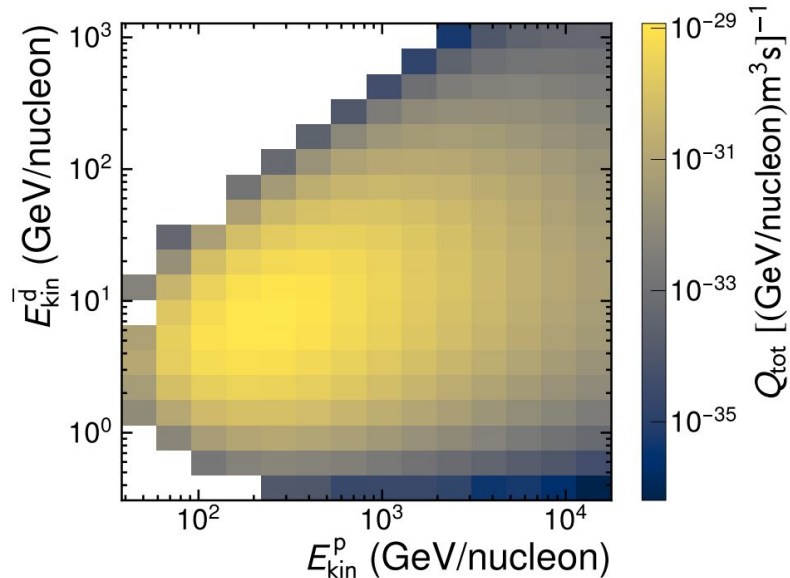


# Cosmic Rays

## Production energy of antinuclei

- Antideuteron production predominantly for protons of  $E_{\text{kin}} \sim 200\text{-}500$  GeV ( $\sqrt{s} \sim \mathbf{19\text{-}30}$  GeV for p-H)

- Extrapolation to lower energies via event multiplicity



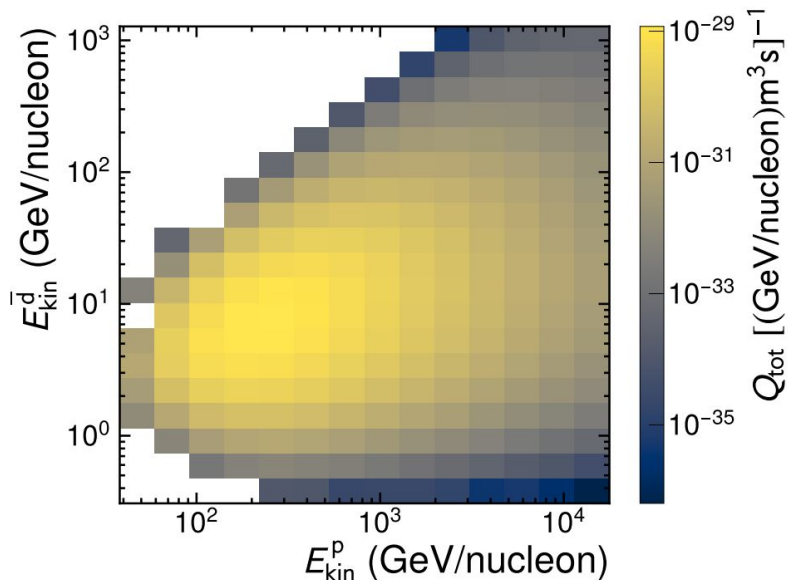
 Šerkšnytė, et al. PRD 105, 083021 (2022)

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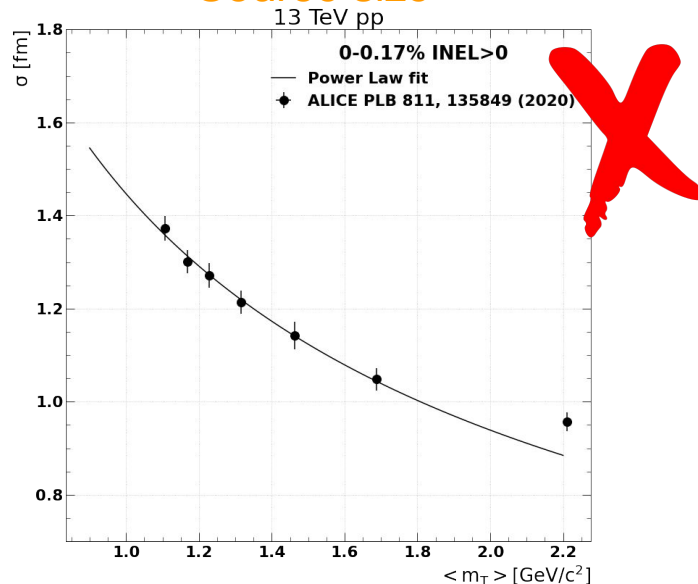
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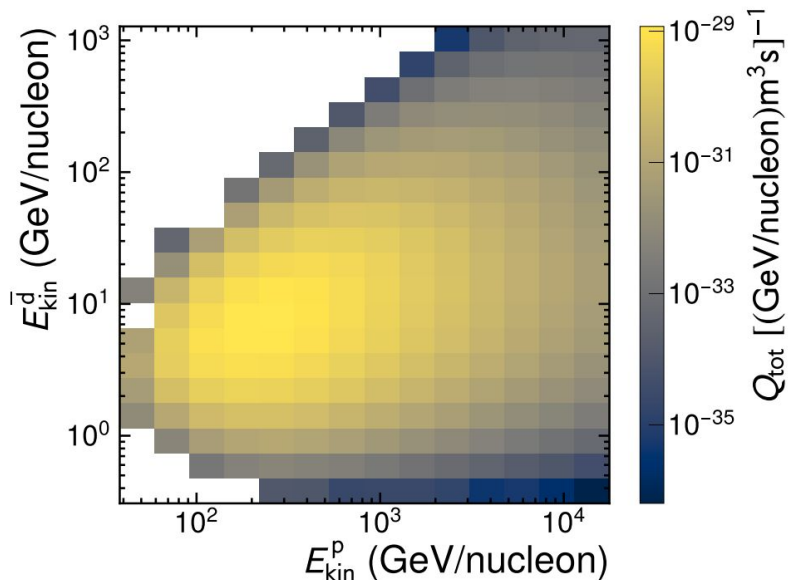


ALICE measurement in high-multiplicity collisions

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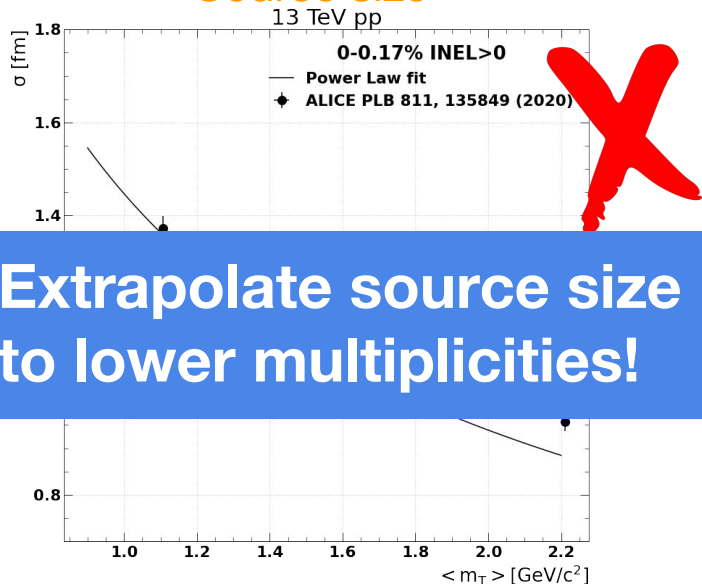
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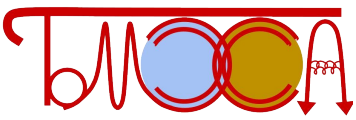


**Extrapolate source size to lower multiplicities!**

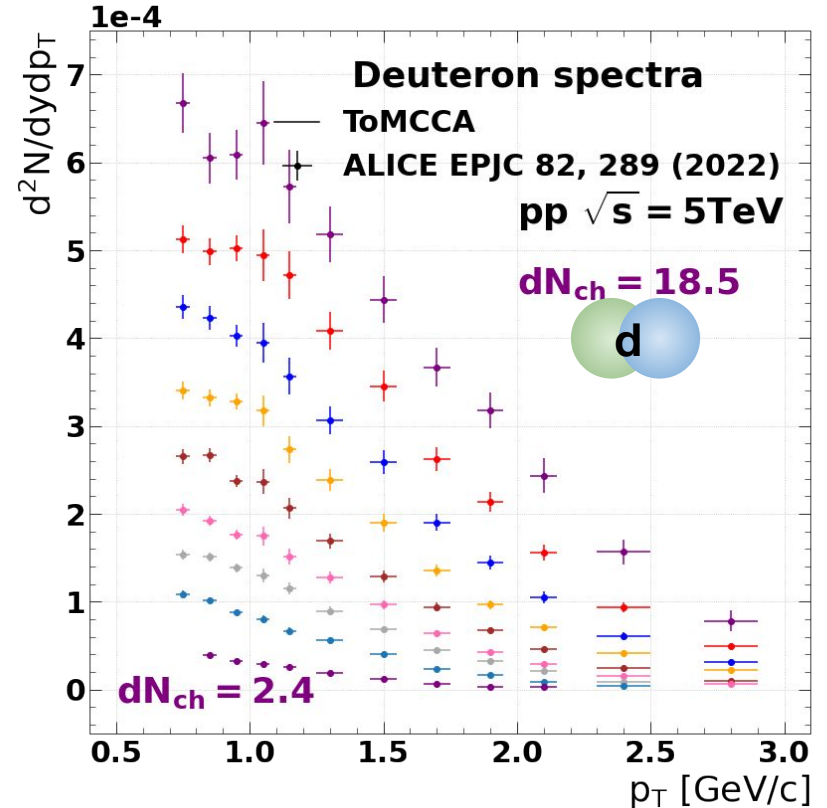
ALICE measurement in high-multiplicity collisions

# Extrapolating the Source

Using ToMCCA as a fitting tool



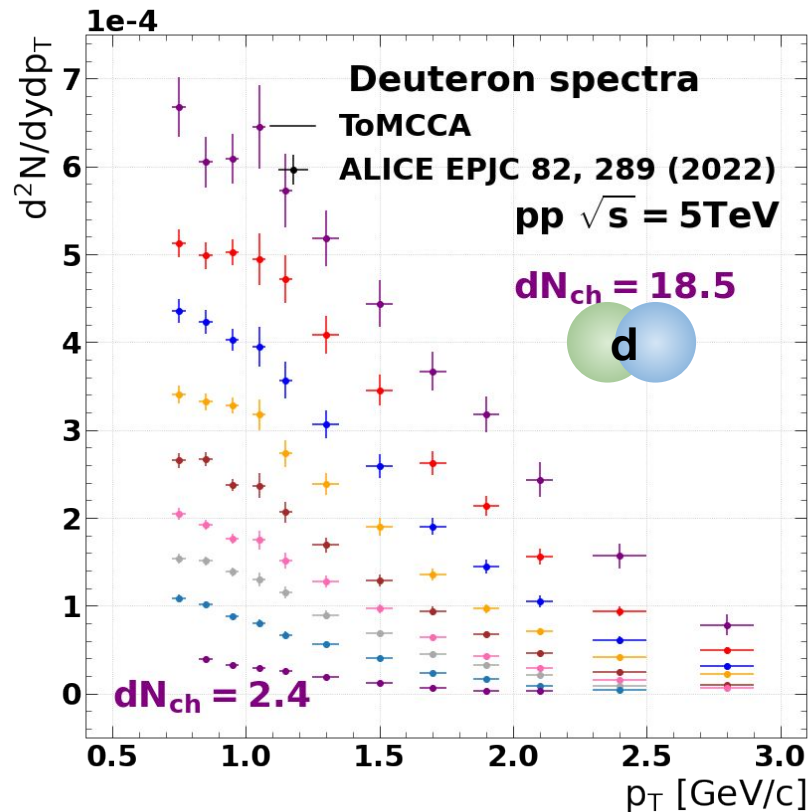
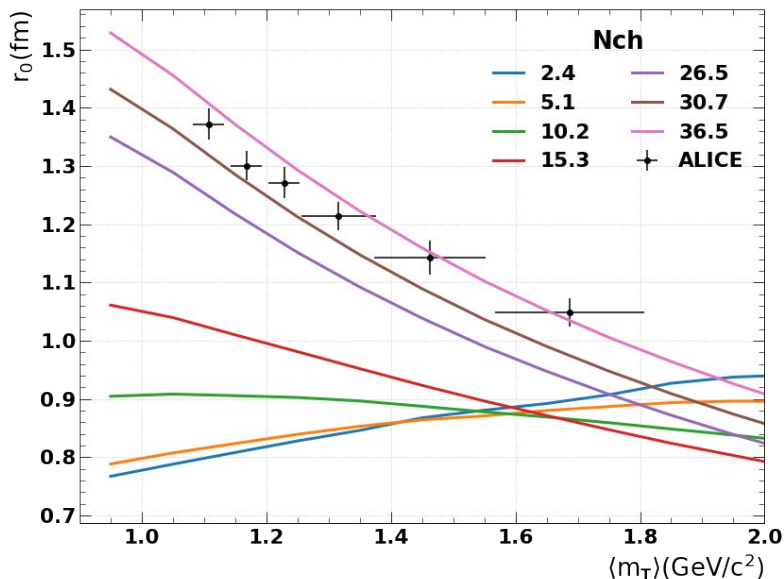
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- Fit source size and scaling with  $m_T$  to measured data
- Cross check at different energies



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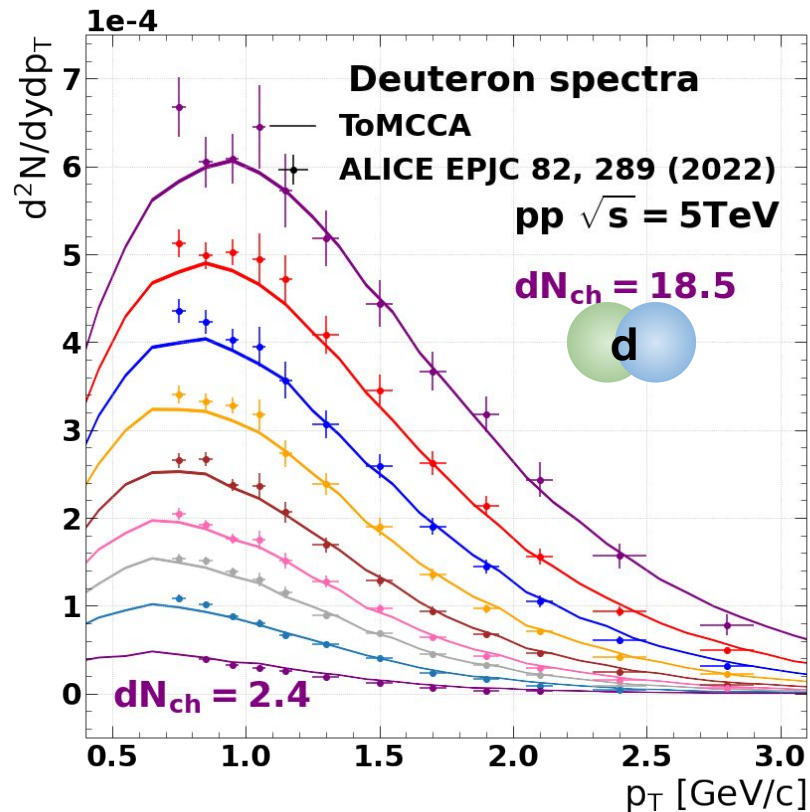
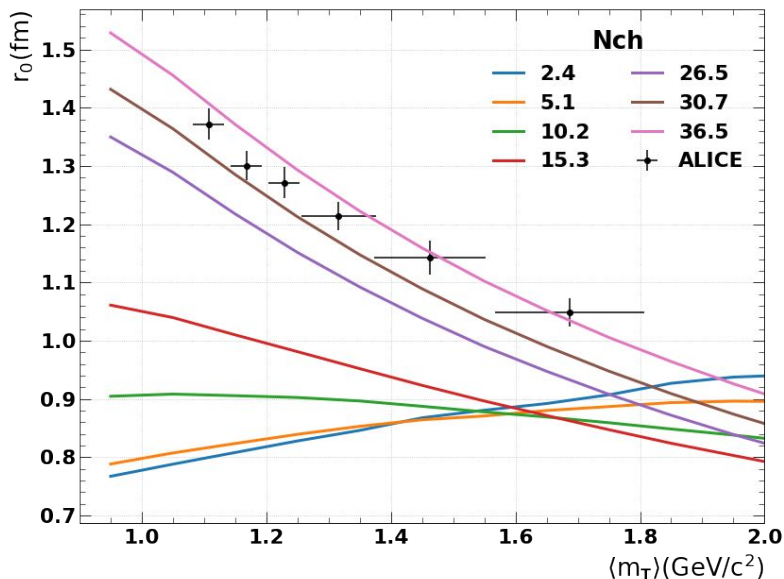
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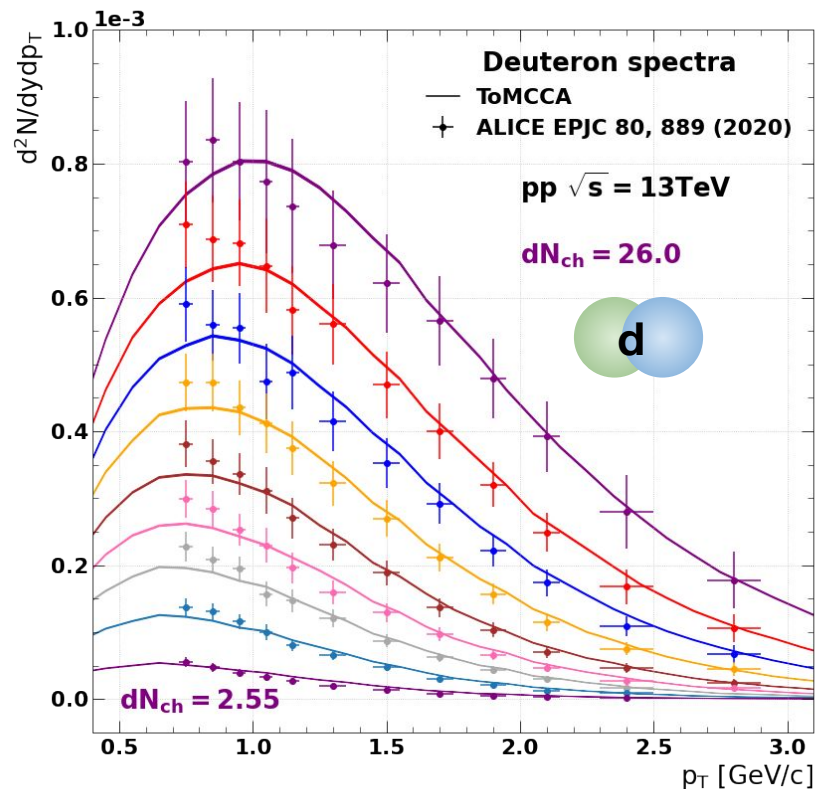
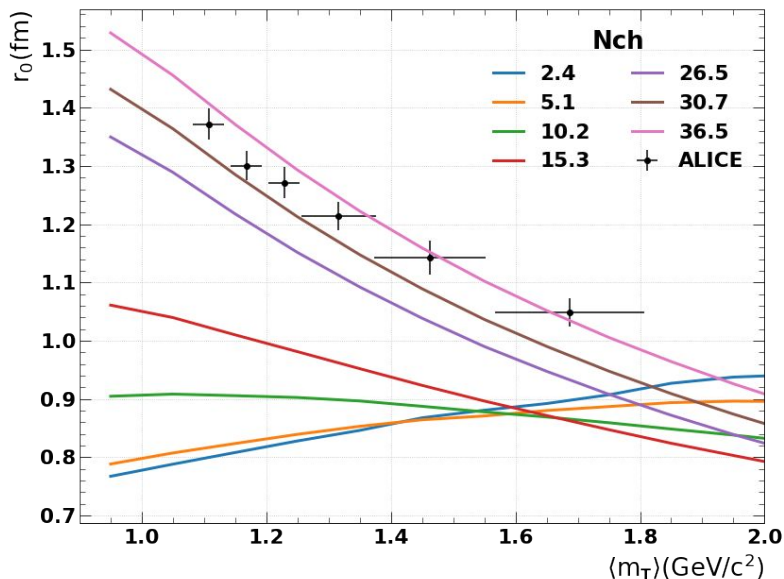
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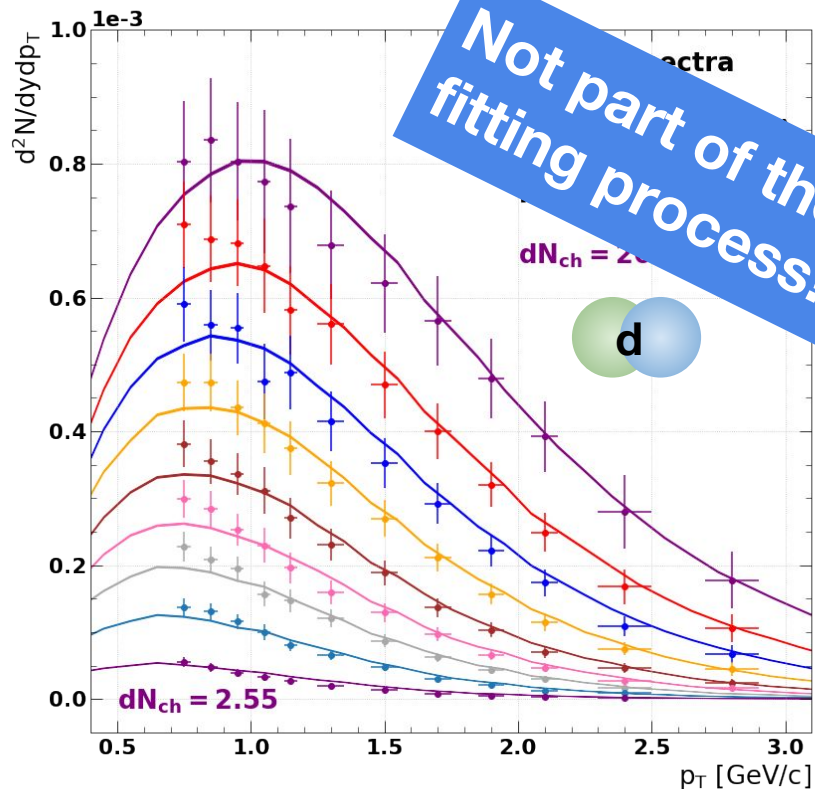
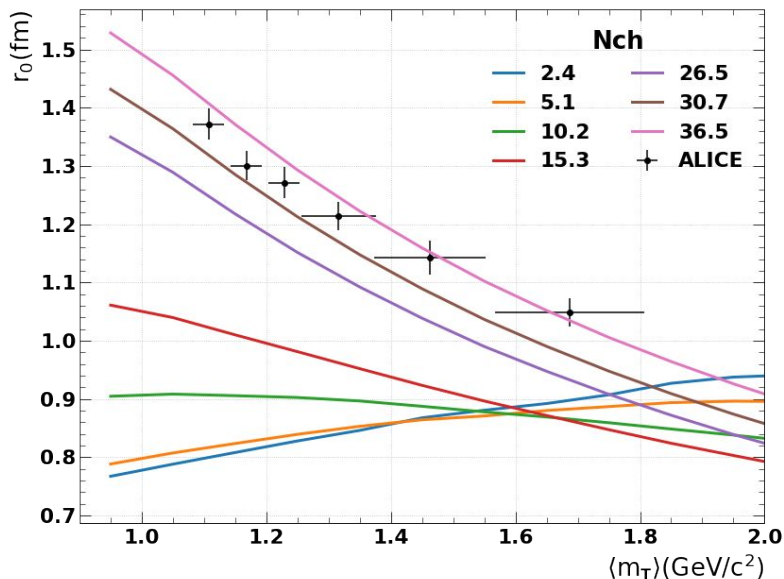




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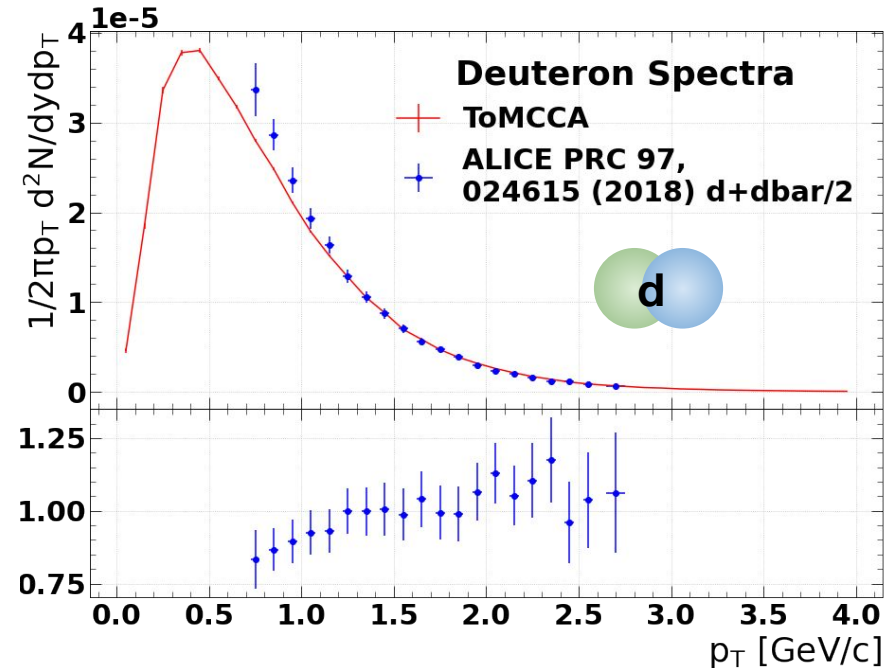
Not part of the fitting process!



# Deuteron results

Minimum bias 7 TeV

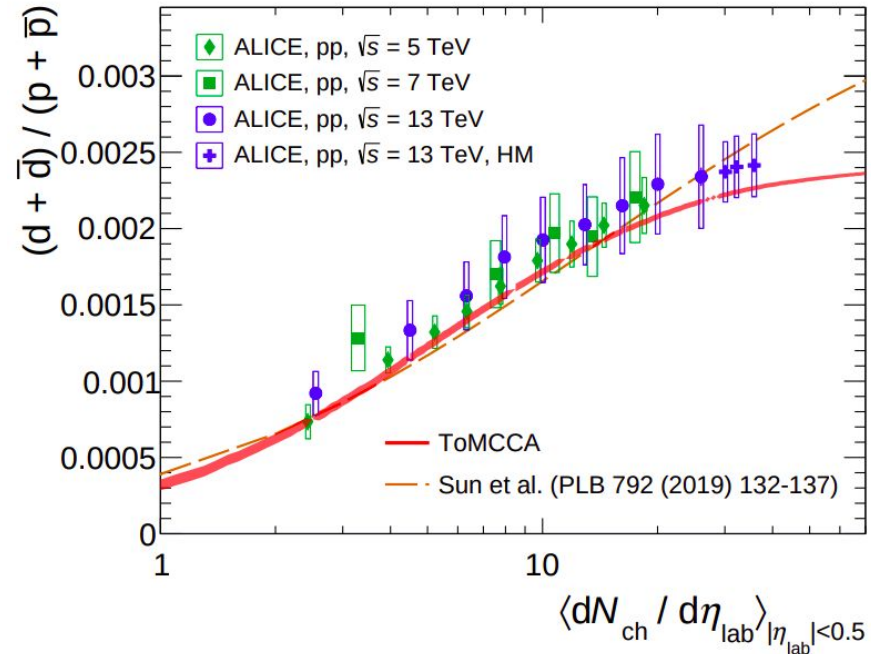
- Deuterons were also measured by ALICE Collab. for different multiplicities
- Fit source size and scaling with  $m_T$  to measured data
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- Minimum Bias works well



# Deuteron results

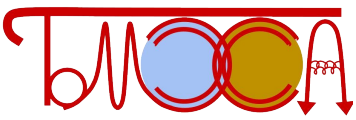
d/p ratio

- Deuterons were also measured by ALICE Collab. for different multiplicities
- Fit source size and scaling with  $m_T$  to measured data
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- Minimum Bias works well
- d/p ratio reproduces data well, tension to previous predictions at high multiplicity



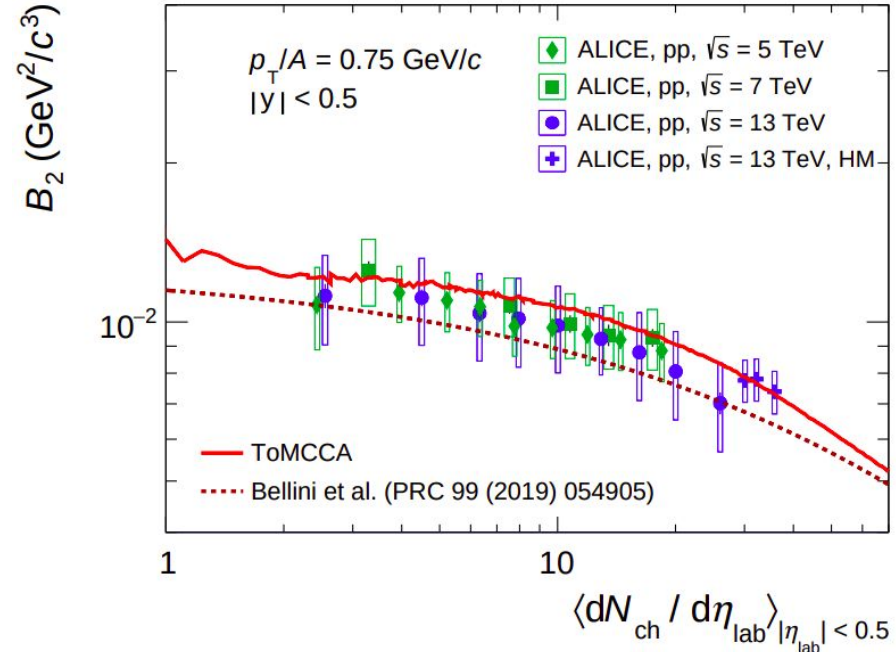
# Deuteron results

$B_2$  parameter



- Deuterons were also measured by ALICE Collab. for different multiplicities
- Fit source size and scaling with  $m_T$  to measured data
- Cross check at different energies
- Minimum Bias works well
- $d/p$  ratio reproduces data well, tension to previous predictions at high multiplicity
- $B_2$  also reproduced well

$$B_A(p_T^p) = E_A \frac{d^3 N_A}{dp_A^3} \Big/ \left( E_p \frac{d^3 N_p}{dp_p^3} \right)^A$$



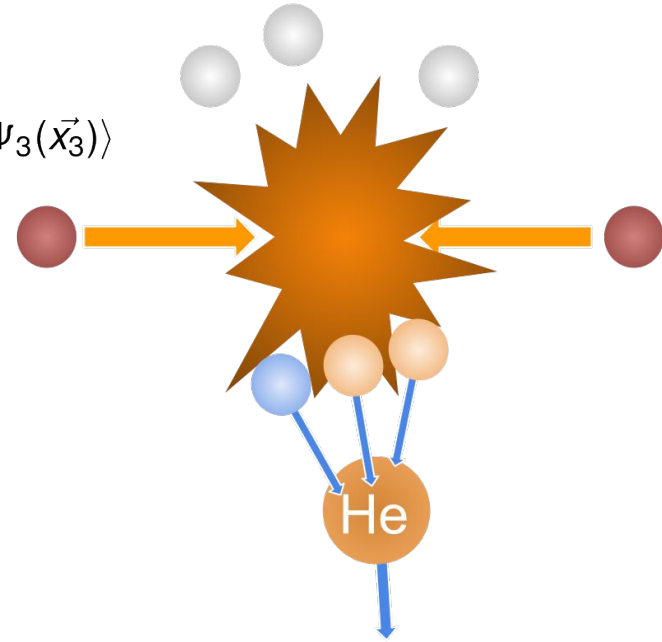
Add 3rd particle to basic formalism

$$\frac{dN_{He}}{d^3P} = S_{He} \int d^3x_1 \int d^3x_2 \int d^3x_3 \int d^3x'_1 \int d^3x'_2 \int d^3x'_3$$

$$\times \Psi_{He}^* (\vec{x}'_1, \vec{x}'_2, \vec{x}'_3) \Psi_{He} (\vec{x}_1, \vec{x}_2, \vec{x}_3) \langle \Psi_3^\dagger(\vec{x}'_3) \Psi_2^\dagger(\vec{x}'_2) \Psi_1^\dagger(\vec{x}'_1) \Psi_1(\vec{x}_1) \Psi_2(\vec{x}_2) \Psi_3(\vec{x}_3) \rangle$$

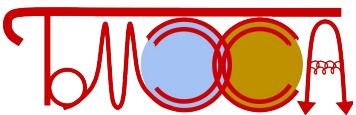
Similarly the probability can be expressed as

$$\mathcal{P}(q_1, q_2, \sigma) = \frac{S_d}{(2\pi)^3 2^3 \sigma^6} \int d^3r_1 d^3r_2 \mathcal{D}(q_1, q_2, r_1, r_2) e^{-\frac{r_1^2 + r_2^2}{4\sigma^2}}$$



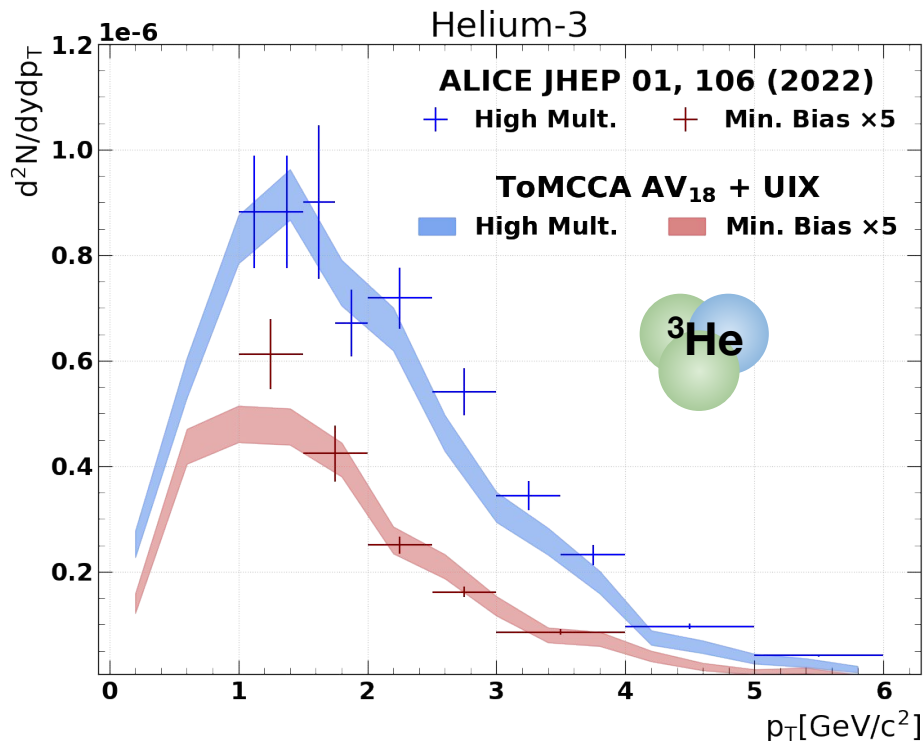
# Extension to A=3

Helium-3



Extension to A=3 coalescence

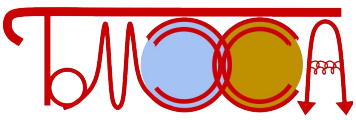
- Use 2-body source size
  - Assign every pair a distance
  - Geometric mean of distance for coalescence probability
- 3-body angular correlations built from 2-body
- Wavefunction based on Argonne  $v_{18}$  (2-body) + Urbana IX (3-body)<sup>1</sup>
- Fully numeric calculation of Probability



<sup>1</sup> Provided by Michele Viviani, INFN Pisa

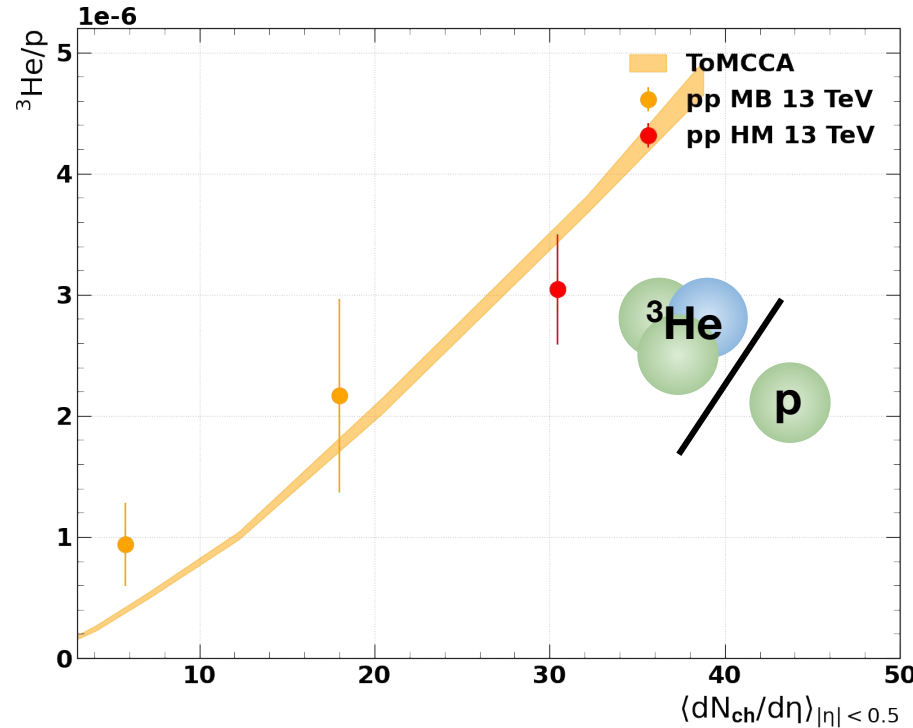
# Extension to A=3

## Helium-3



Extension to A=3 coalescence

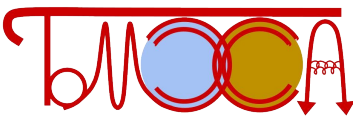
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# Extension to $A=3$

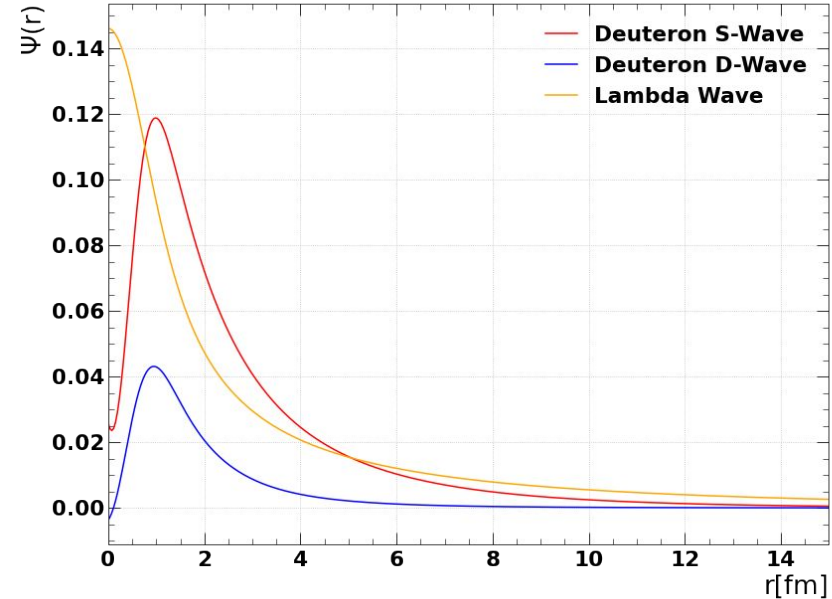
## Hypertriton



- Congleton<sup>1</sup> wavefunction

$$\psi_{\Lambda}(q) = N \frac{\exp[-(q/\Lambda)^2]}{q^2 + \alpha^2}$$

- Assumes factorization of Hypertriton wavefunction into deuteron+ $\Lambda$
- Scattering parameters retuned to latest Hypertriton formfactor calculations<sup>2</sup> by Hildenbrand & Hammer<sup>3</sup>



<sup>1</sup> J G Congleton 1992 J. Phys. G: Nucl. Part. Phys. 18 339

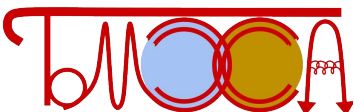
<sup>2</sup> F. Bellini et al.: Phys.Rev.C 103, 1 (2021)

<sup>3</sup> F. Hildenbrand and H.-W. Hammer: Phys. Rev. C 100, 034002

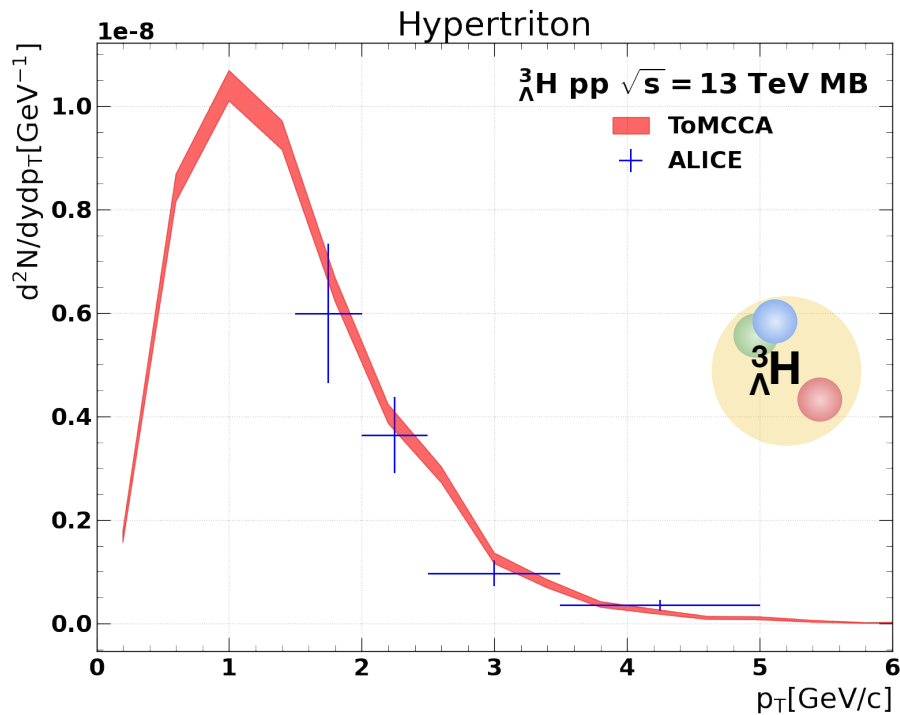


# Extension to $A=3$

## Hypertriton

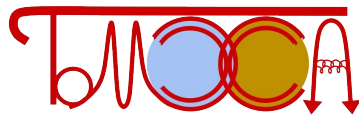


- Latest ALICE measurements of  ${}^3\Lambda\text{H}$  in 13 TeV MB

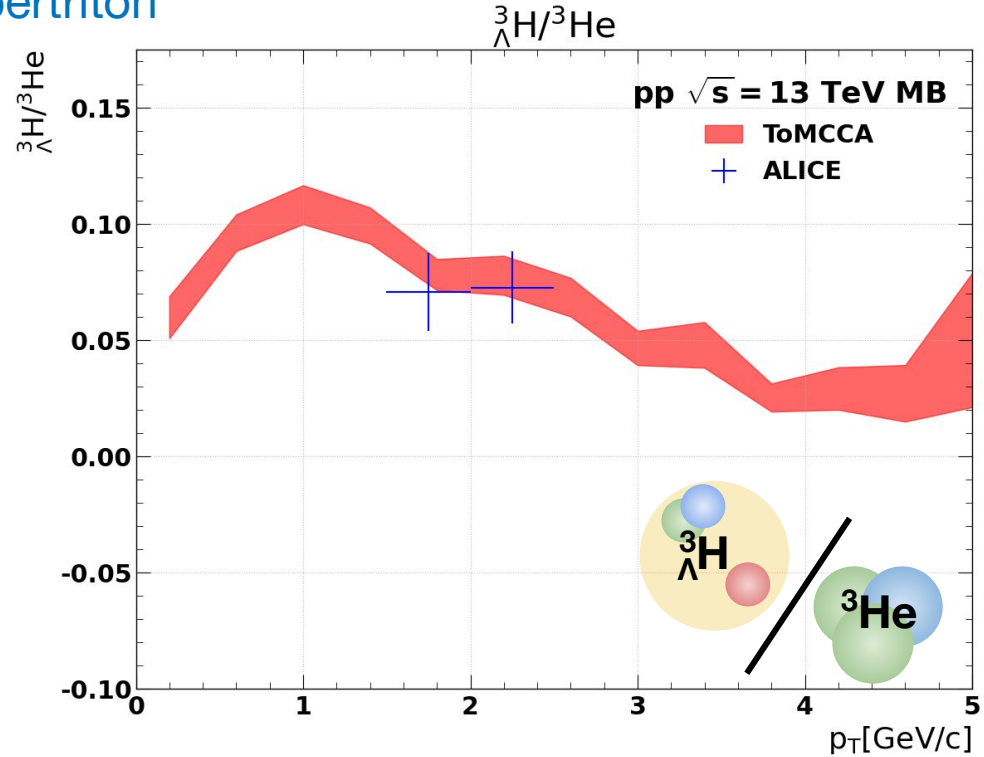


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## Hypertriton

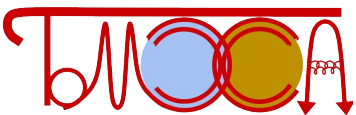


- Latest ALICE measurements of  ${}^3_{\Lambda}\text{H}$  in 13 TeV MB
- ${}^3_{\Lambda}\text{H}/{}^3\text{He}$  Ratio falling off for large  $p_T$



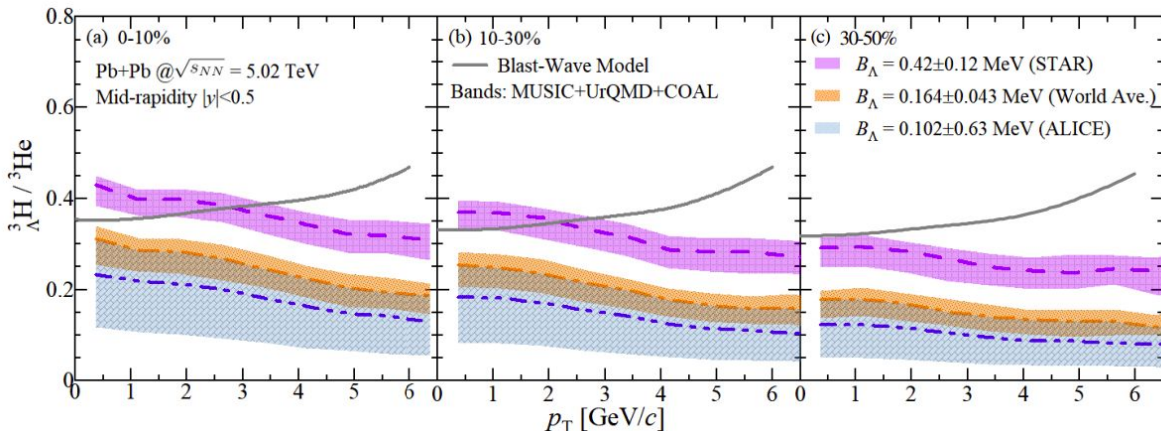
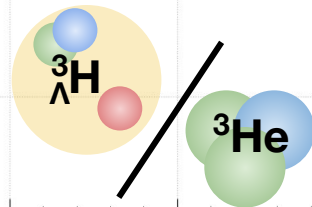
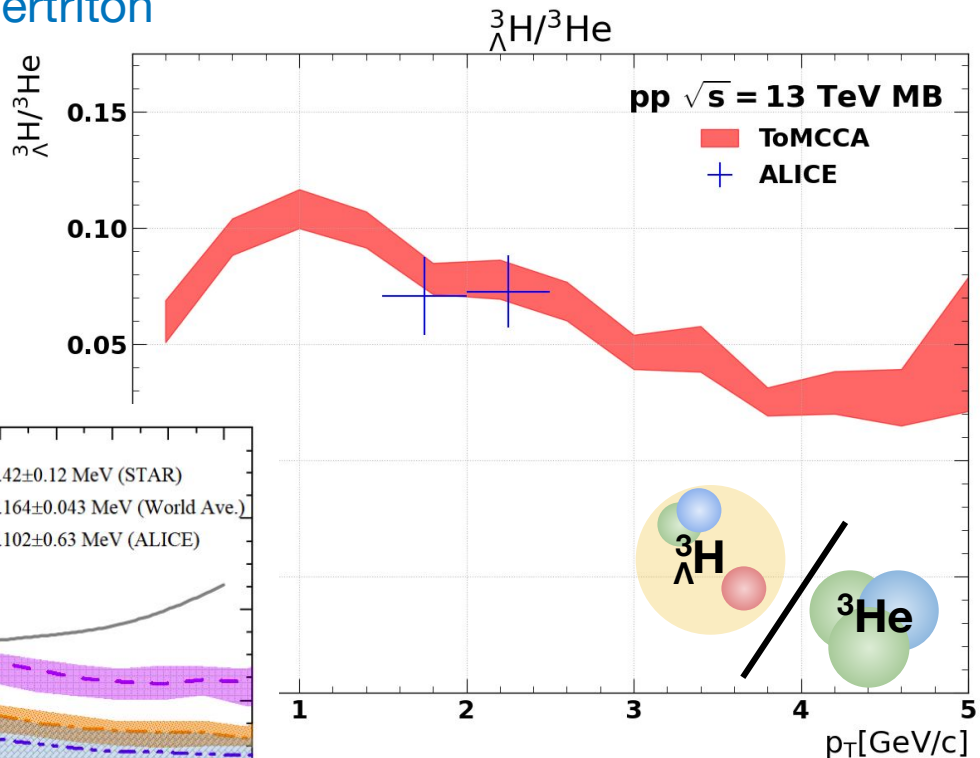
# Extension to $A=3$

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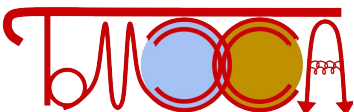
Only available predictions in Heavy Ions<sup>1</sup>:



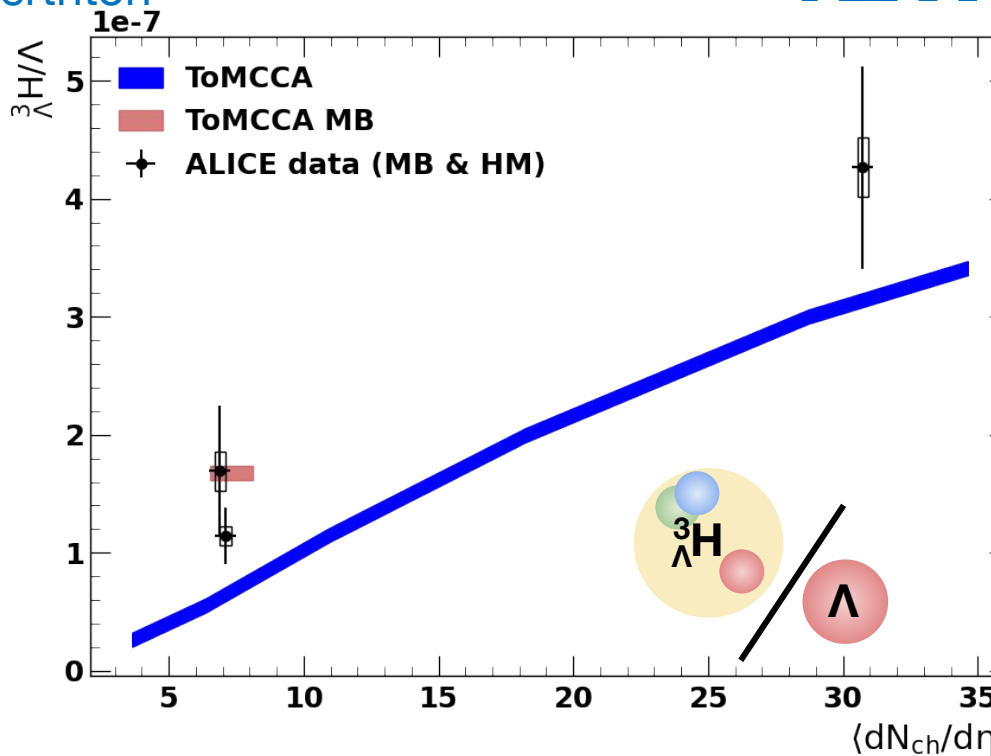
<sup>1</sup>K.-J. Sun et.al. arXiv:2404.02701

# Extension to $A=3$

## Hypertriton



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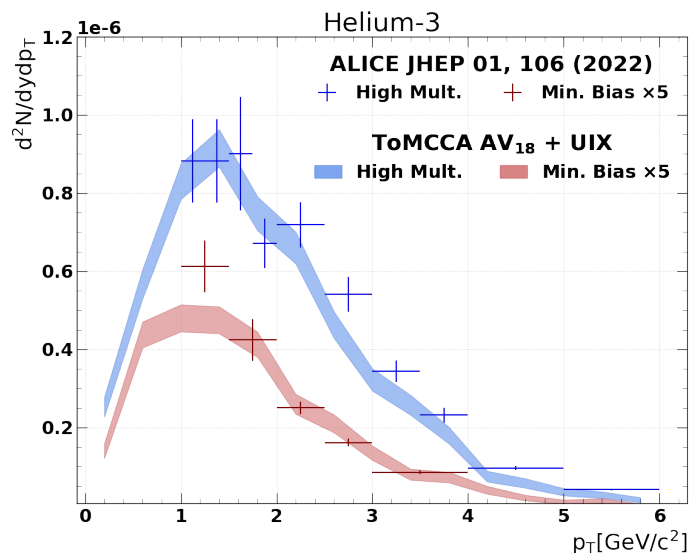
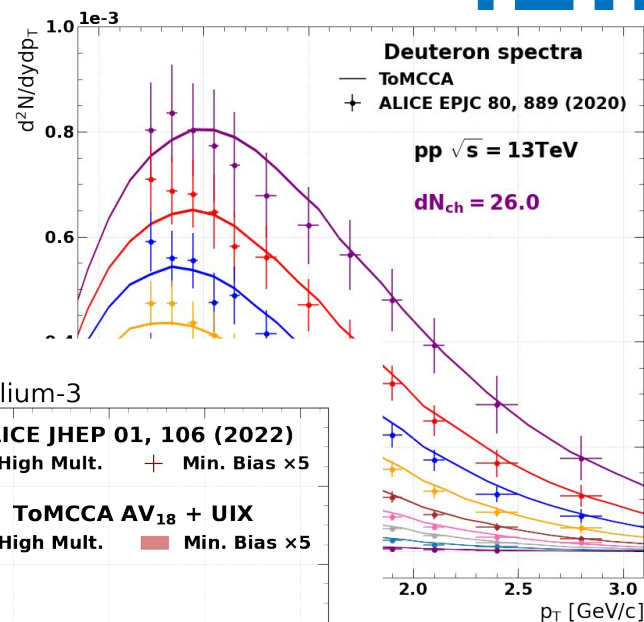
## Deuterons:

- Coalescence model reproduces data with no free parameters
- Realistic wavefunction required
- ToMCCA allows for an extension to arbitrary multiplicities

## A=3 Coalescence

- Successful extension of the model to A=3
- Nuclei and *Hyper*nuclei
- Realistic wavefunctions required

ToMCCA is available under:  
<https://github.com/horstma/tomcca-publi>



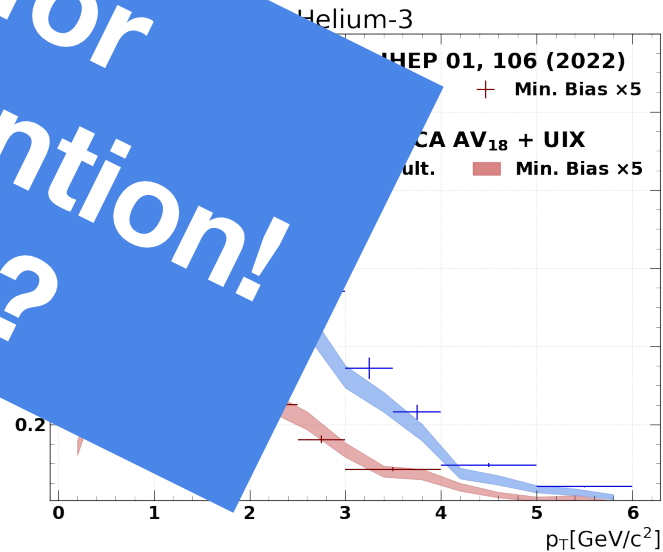
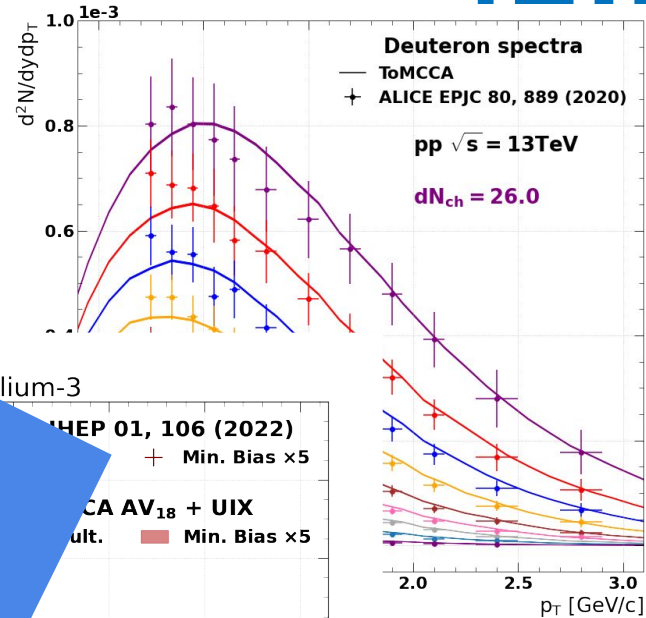
Deuterons:

- Coalescence model fits ALICE data with no free parameters
- Realistic wavefunctions
- ToMCCA allows for arbitrary multiplicity

A=3 Coalescence:

- Successful fit to ALICE data A=3
- Nuclei and Hypernuclei
- Realistic wavefunctions

Thanks for your attention!  
Questions?



ToMCCA is available under:  
<https://github.com/horstma/tomcca-public>

# BACKUP

## Advantages:

- Model extremely complex phenomena and particle correlations
- Easy to use ('Plug and play')
- Trivial extrapolation to different energies, multiplicities (and Collision systems)

## Disadvantages:

- Convoluted Code, hard to adjust
- Hard to distill influence of single mechanism on the final result
- Long simulation times
- No nuclei production



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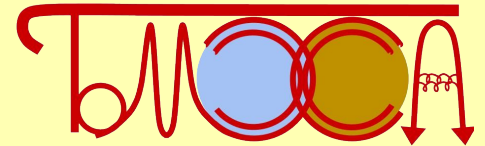
## Disadvantages:

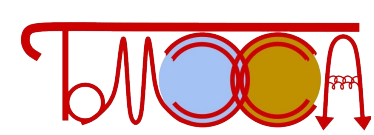
- Convoluted Code, hard to adjust
- Hard to distill influence of single mechanism on the final result
- Long simulation times
- No nuclei production

➔ Build Toy Monte Carlo that uses only the necessary mechanisms for nuclei production

Requirements: Fast simulation, easy to adjust to end-users needs

**T**oy **M**onte **C**arlo **C**oalescence **A**fterburner: ToMCCA





## Speed:

Slowest parts of Event generators: *Hadronization, Hadronic Cascade*

- ➔ Fully omit Hadronization, start from a statistical distribution of nucleons (no mesons)
- ➔ No Rescattering, Flow, Jets, ...

## Correlations:

No ab-initio correlations, built in fully by hand

- ➔ can be easily deactivated or adjusted

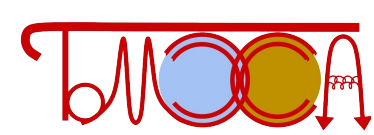
## User-Friendly:

All of ToMCCA is ~800 lines of Code

- ➔ Easy to find code responsible for specific effect

Run-in-place configuration

- ➔ Download (<https://github.com/HorstMa/ToMCCA-Public>) and run immediately



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Slowest parts of Event generators: *Hadronization, Hadronic Cascade*

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**But a Toy Model needs measured inputs...**

## User-Friendly:

All of ToMCCA is ~800 lines of Code

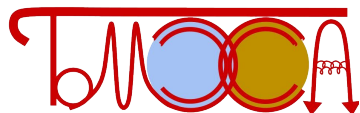
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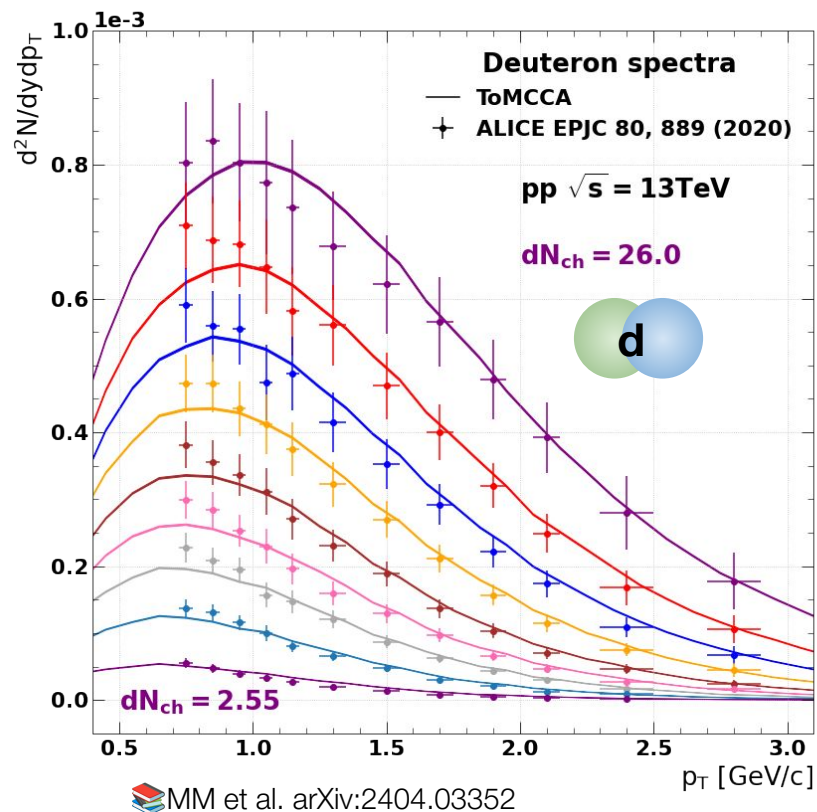
- ➔ Download (<https://github.com/HorstMa/ToMCCA-Public>) and run immediately

# Conclusion

## Deuteron production



- Understanding nuclei formation on earth can open a window to **indirect dark matter** searches
- **Wigner function formalism** can predict nuclei yields with no free parameters
- ToMCCA allows us to extrapolate to arbitrary multiplicities

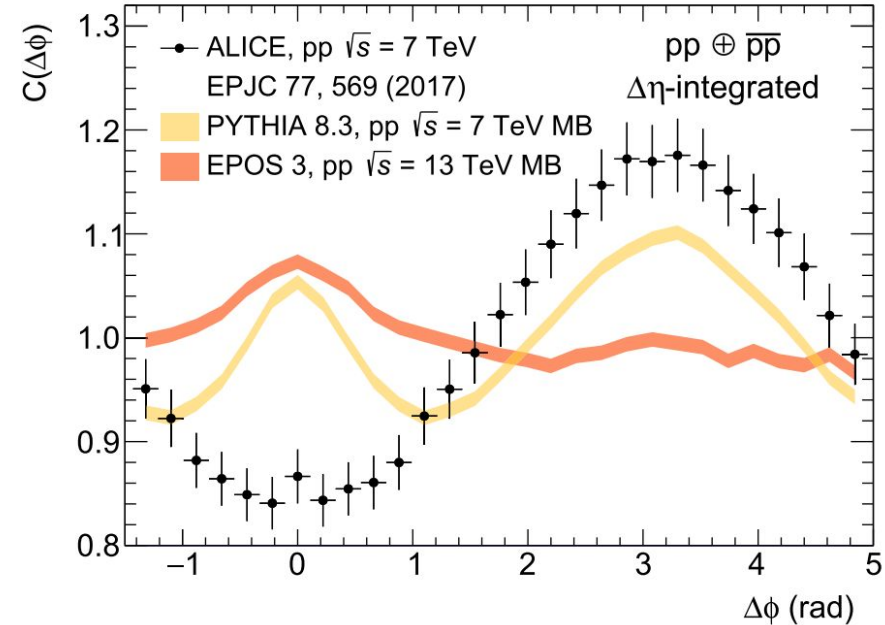


# Coalescence Results EPOS

## Angular correlations

- $\Delta\phi$  of pp (pn) pairs
- Not reproduced by EPOS or Pythia
- No real control over these behaviours in general purpose event generators

MM et al. Eur.Phys.J.C 83 (2023) 9, 804



# Comparison to previous predictions

- Important observable in accelerator measurements:  $\mathbf{B}_A$

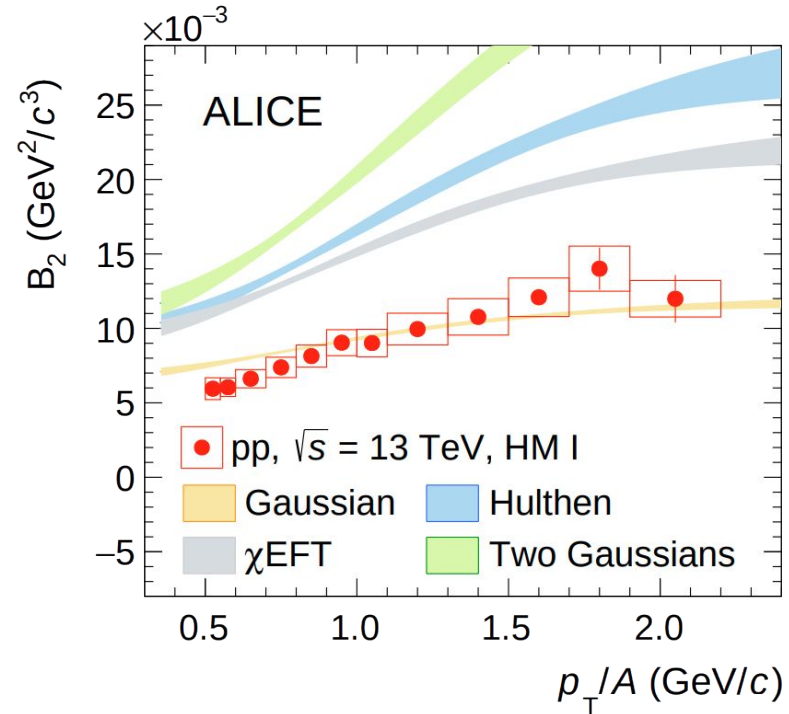
$$B_A(p_T^p) = E_A \frac{d^3 N_A}{dp_A^3} \bigg/ \left( E_p \frac{d^3 N_p}{dp_p^3} \right)^A$$

- Theoretical prediction [1]

$$B_2(\vec{p}) \approx \frac{3}{2m} \int d^3 q D(\vec{q}) e^{-R^2(p_T) q^2}$$

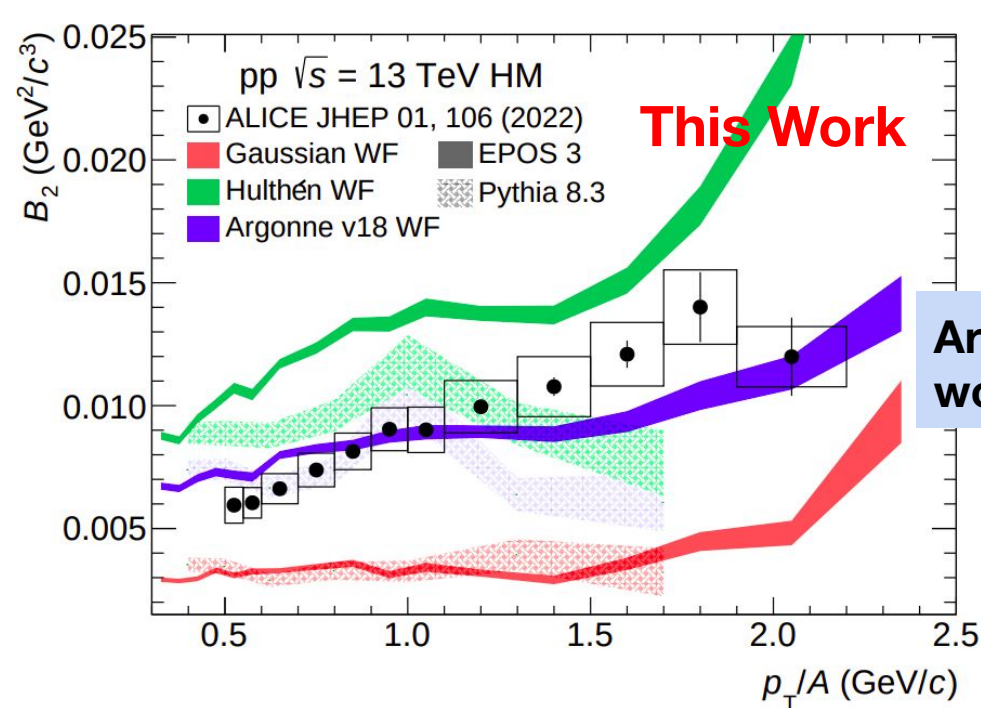
$$D(\vec{q}) = \int d^3 r |\phi_d(\vec{r})|^2 e^{-i\vec{q}\cdot\vec{r}}$$

- This neglects momentum difference between Nucleons
- approximate to 10% in Pb–Pb, factor 2 in pp

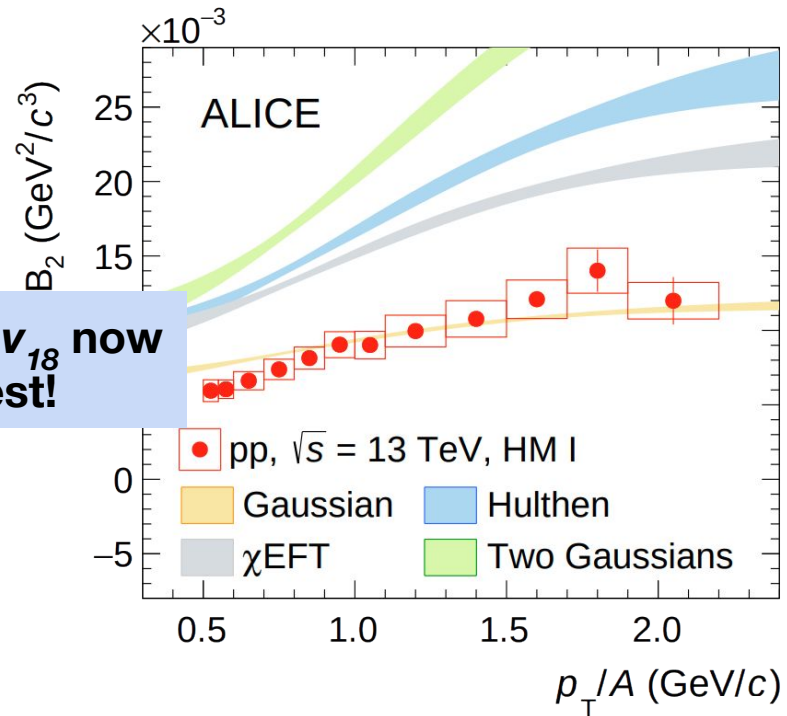


[1] Blum, Takimoto, PRC 99 (2019) 044913

# Comparison to previous predictions



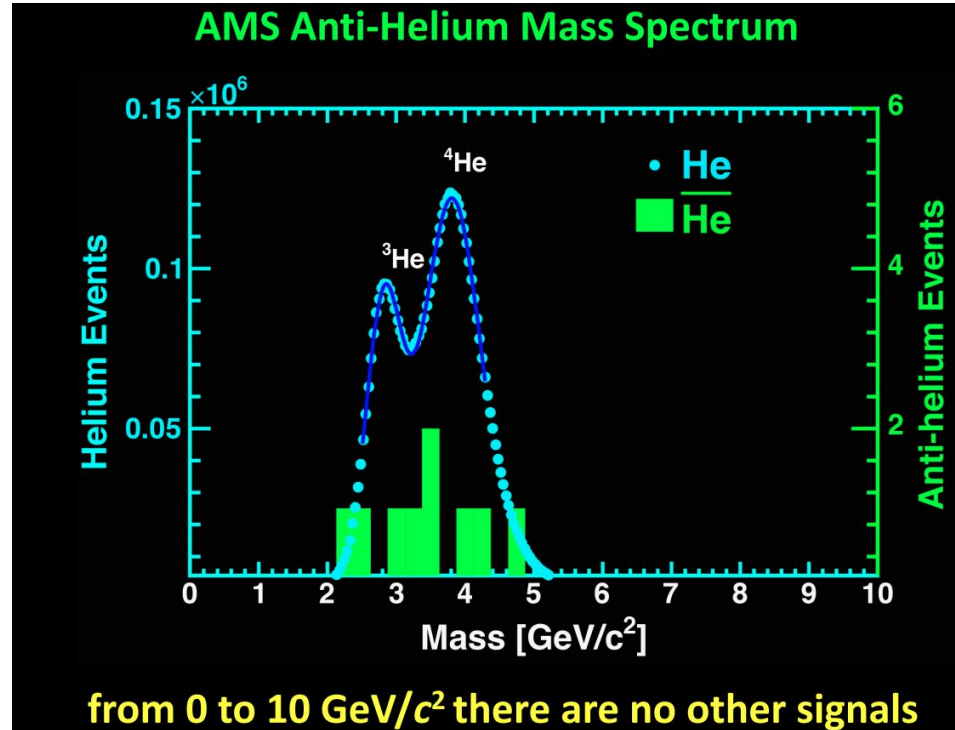
Argonne  $v_{18}$  now works best!



# Cosmic Rays

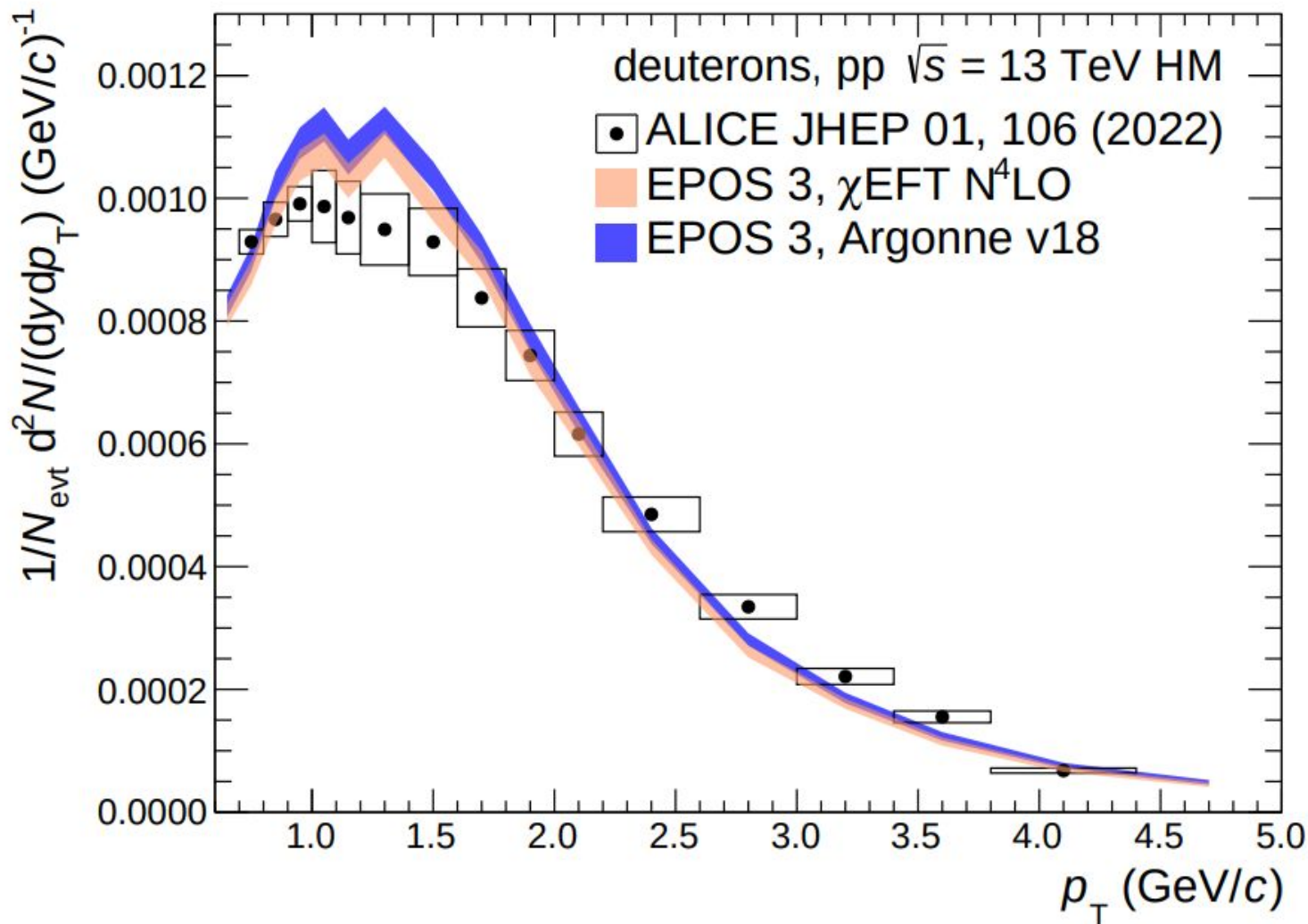
## Antinuclei in Cosmic Rays?

- AMS-02 @ ISS has measured 9 antihelium candidates
- Not yet published
- What could be the origin of these antinuclei?



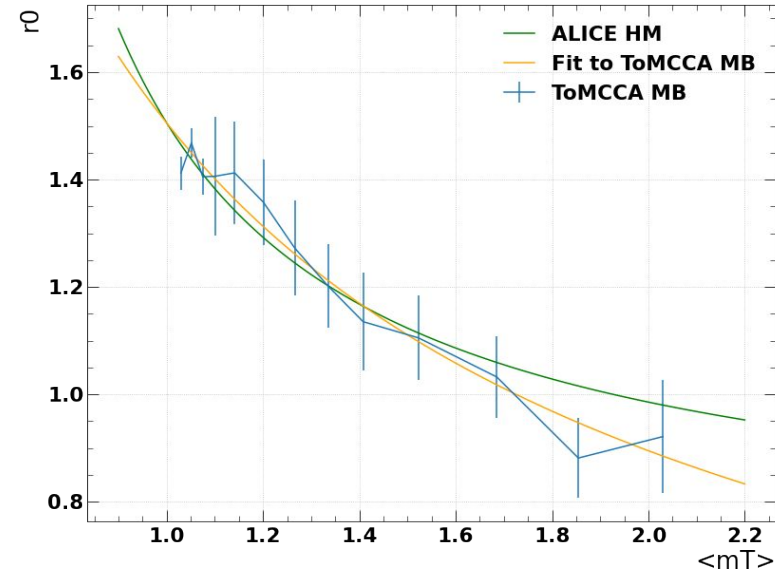
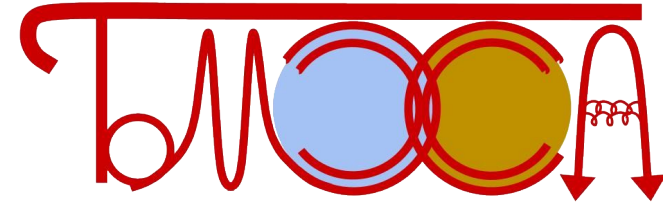
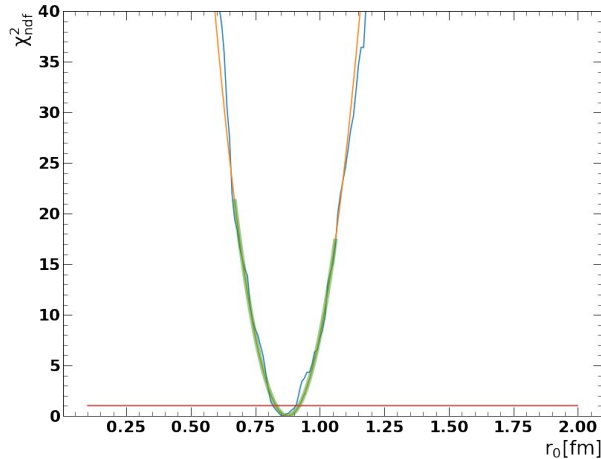
Pauolo Zuccon for AMS-02 Collaboration at MIAPP workshop 2022





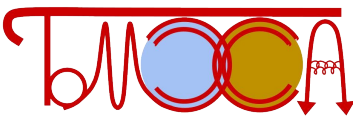
Fitting Procedure:

- Run ToMCCA with a fixed source size (e.g. 1.8 fm, flat in  $m_T$ )
- For the resulting deuteron spectra calculate the  $\chi^2$  for each bin and save it
- Reduce source size
- Repeat until source size is 0

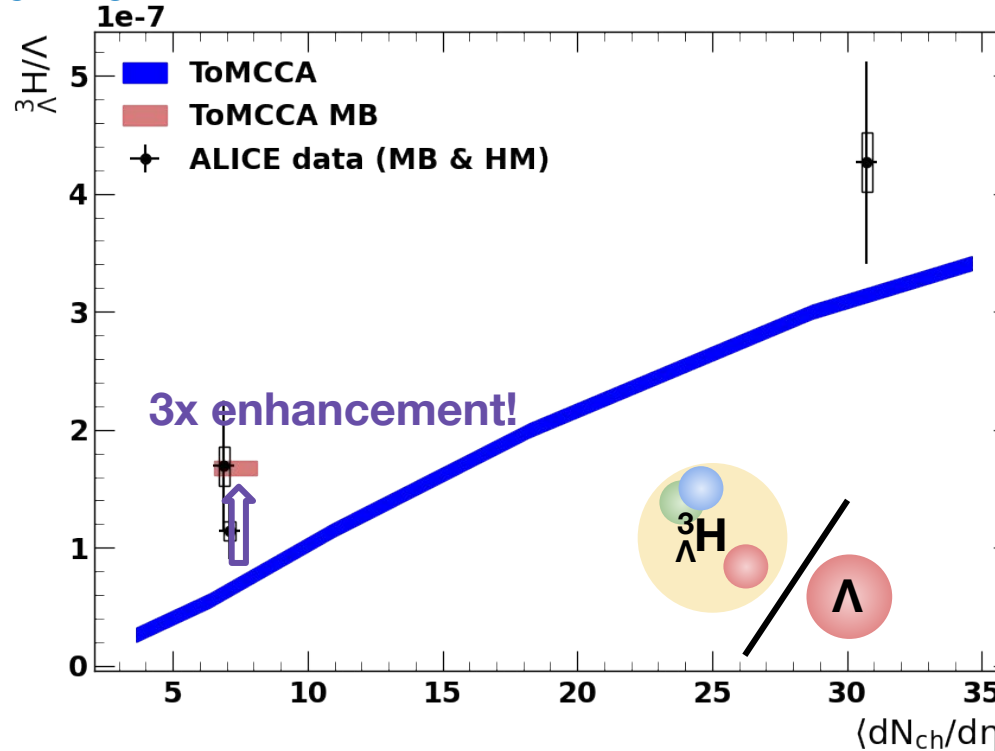


# Extension to A=3

## Hypertriton



- Latest ALICE measurements of  ${}^3\Lambda\text{H}$  in 13 TeV MB
- ${}^3\Lambda\text{H}/{}^3\text{He}$  Ratio falling off for large  $p_T$
- ${}^3\Lambda\text{H}/\Lambda$  Ratio as a function of Multiplicity
- Important Note:  
Minimum Bias Data is not comparable this way! *3x enhancement from wide multiplicity distribution*

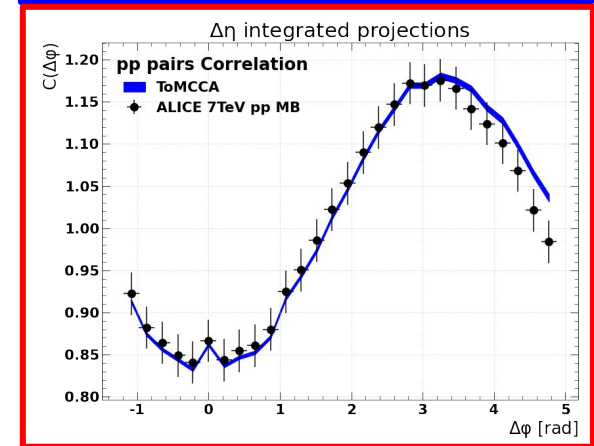
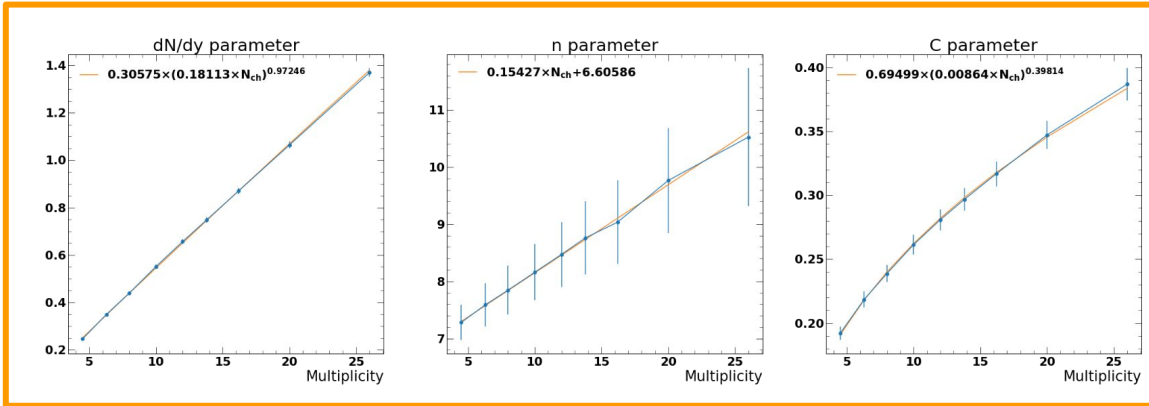
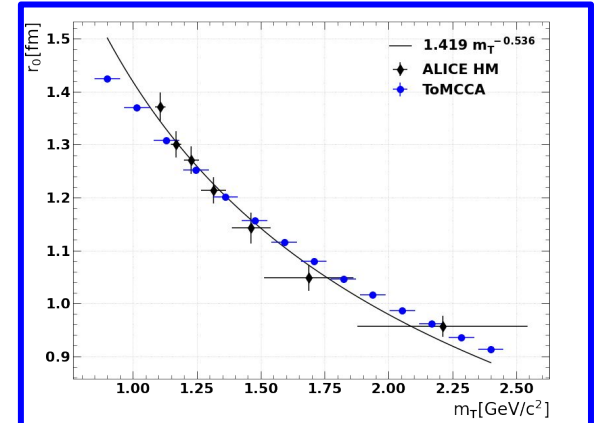


<sup>1</sup>K.-J. Sun et.al. arXiv:2404.02701

# Recap: ToMCCA

## Inputs

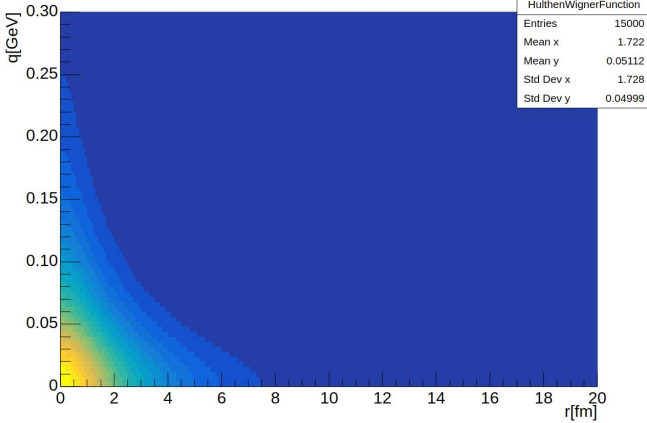
- ToMCCA is a Toy Monte Carlo → it requires everything as an **input**:
  - **Momentum distribution** → Fully parameterized
  - **Multiplicity** → Poissonian/Event Generator
  - **Angular distribution** → From Measurement
  - **Source Size** → ALICE Measurement



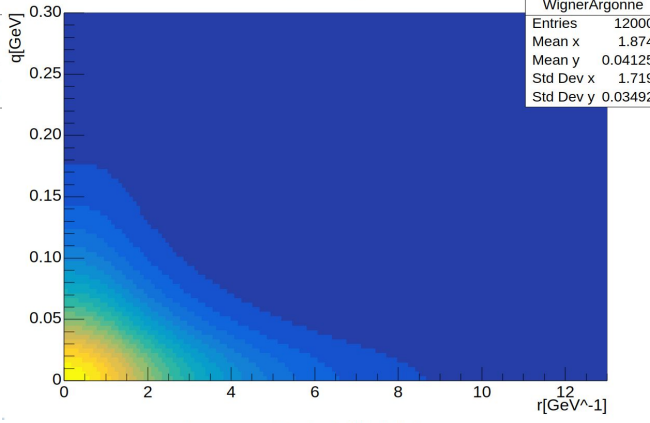
$$\frac{d^2N}{dy dp_T} = \frac{dN}{dy} \frac{p_T(n-1)(n-2)}{nC[nC+m_p(n-2)]} \left(1 + \frac{m_T - m_p}{nC}\right)^{-n}$$

# New Wigner functions/Probabilities

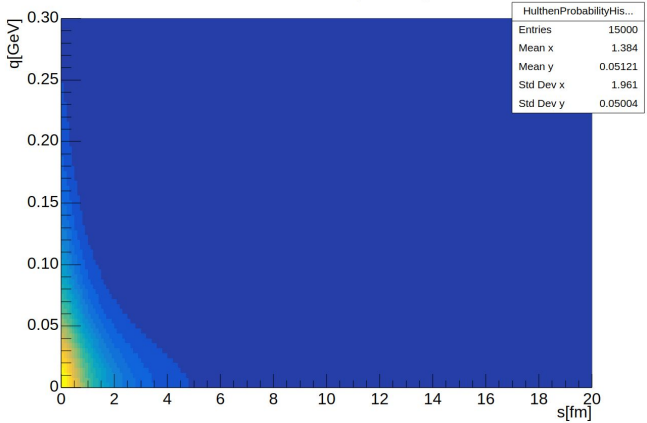
HulthenWignerFunction



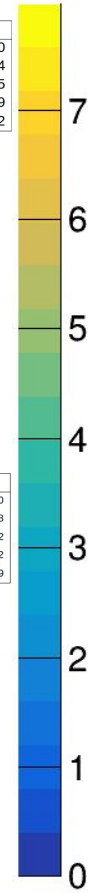
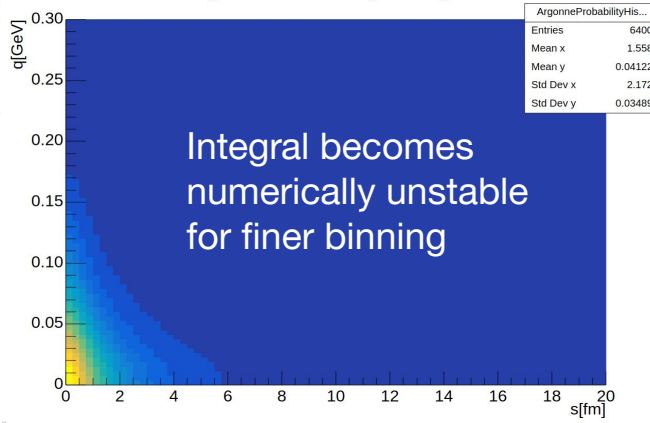
WignerArgonne



HulthenProbabilityHistogram

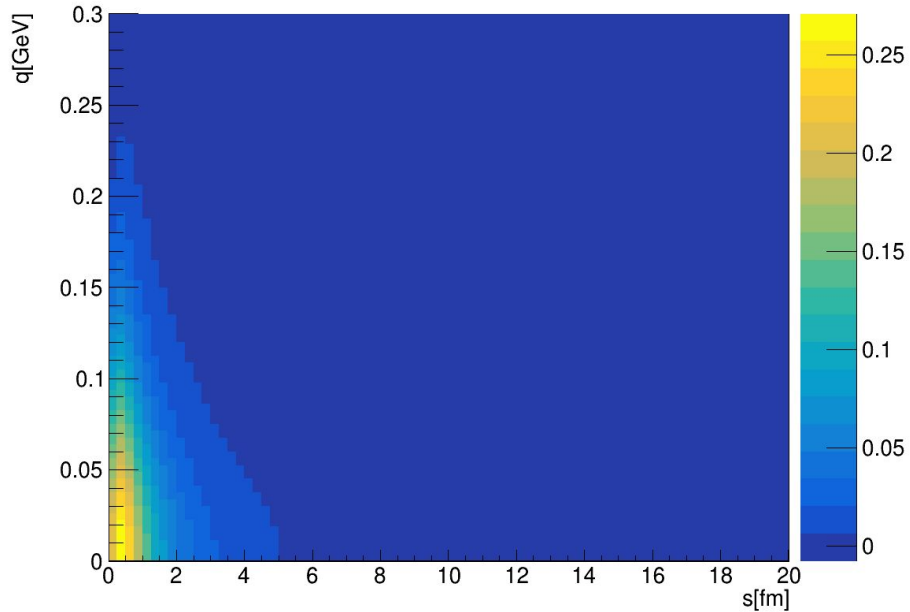


ArgonneProbabilityHistogram

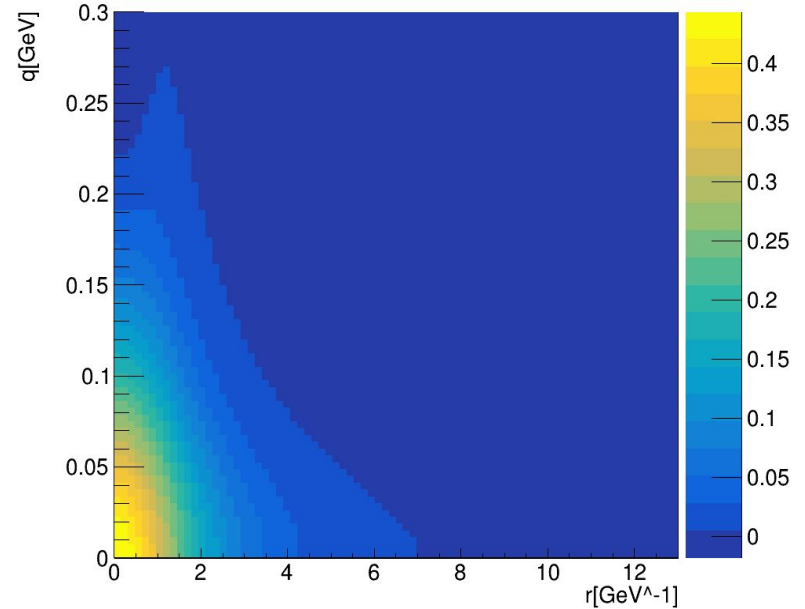


# Argonne D-State probability

ArgonneProbabilityHistogramDWave



WignerArgonne\_D\_Wave



D-State probability is 6% → Maximum ~11% effect