Critical fluctuations from molecular dynamics with expansion

Volodymyr Kuznietsov

Research supervisor: Prof. Volodymyr Vovchenko

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UNIVERSITY OF HOUSTON

QCD matter

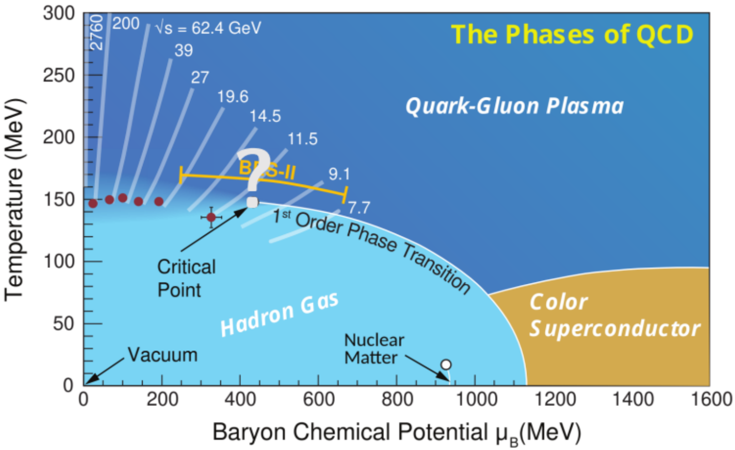


Figure from Bzdak et al., Phys. Rept. 2020 & 2015 Long Range plan

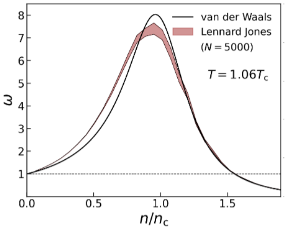
Fluctuations as CP signature

In GCE density cumulants shows singularity behaviour in the critical point.

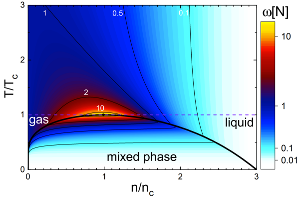
$$\ln Z^{\mathrm{gce}}(T,V,\mu) = \ln\left[\sum_N e^{\mu N} Z^{\mathrm{ce}}(T,V,N)\right], \quad (1)$$

$$\kappa_n \cong \frac{\partial^n(\ln Z^{\mathrm{gce}})}{\partial(\mu_N)^n}. \quad (2)$$

The real expression for Z^{gce} is unknown in QCD matter.



$$\kappa_2 \sim \xi^2$$



Connection to the experiment

Theory

- Coordinate and/or momentum space
- In contact with the heat bath
- Conserved charges
- Uniform
- Fixed volume

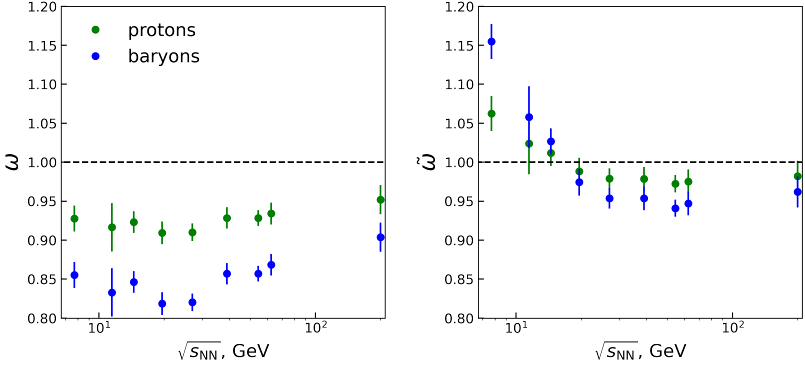
Experiment

- Momentum space
- Expanding in vacuum
- Non-conserved particle numbers
- Inhomogenous
- Fluctuating volume

Need microscopic description of fluctuations

STAR data

Data: **M. S. Abdallah et al., Phys. Rev. C 104, 024902 (2021)**



Left panel – raw data, right panel – corrected for B cons., $\tilde{\omega} = \omega/(1-\alpha)$

What does deviation from the unity means? Could it be a critical point?

Lennard-Jones potential

The Lennard-Jones potential reads

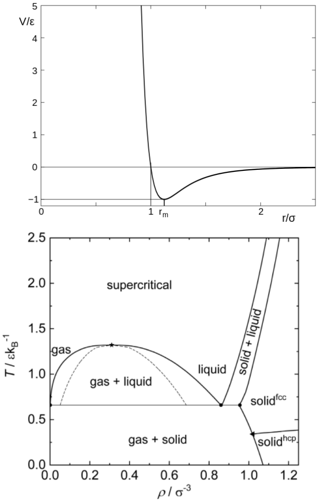
$$V_{\mathrm{LJ}} = 4\varepsilon \left[\left(\frac{\sigma}{r} \right)^{6} - \left(\frac{\sigma}{r} \right)^{12} \right], \quad (3)$$

In reduced dimensionless variables it can be rewritten as

$$\tilde{V}_{\mathrm{LJ}} = 4[\tilde{r}^{-12} - \tilde{r}^{-6}], \quad (4)$$

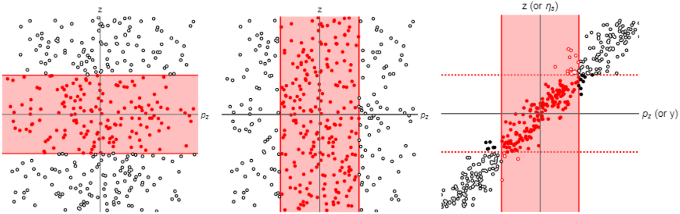
where the reduced variables are used: $\tilde{r} = r/\sigma$ and $\tilde{V}_{\mathrm{LJ}} = V_{\mathrm{LJ}}/\varepsilon$.

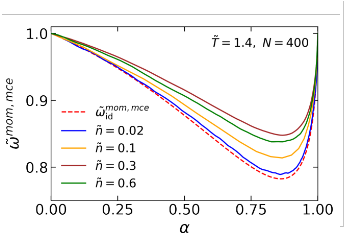
The LJ fluid contains a critical point in the 3D Ising universality class.

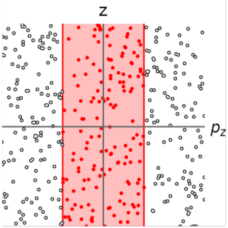


Momentum space subsystem ($\tilde{T} = 1.4 = 1.06\,T_c$)

Signal disappears in momentum space

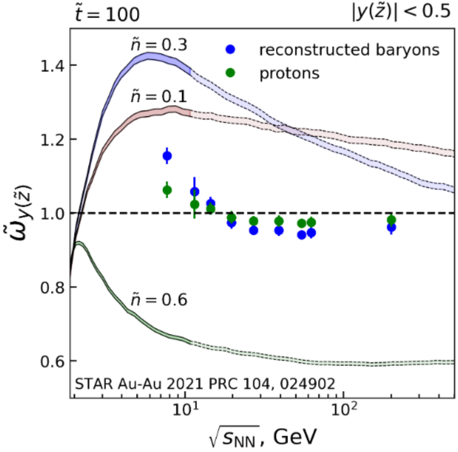






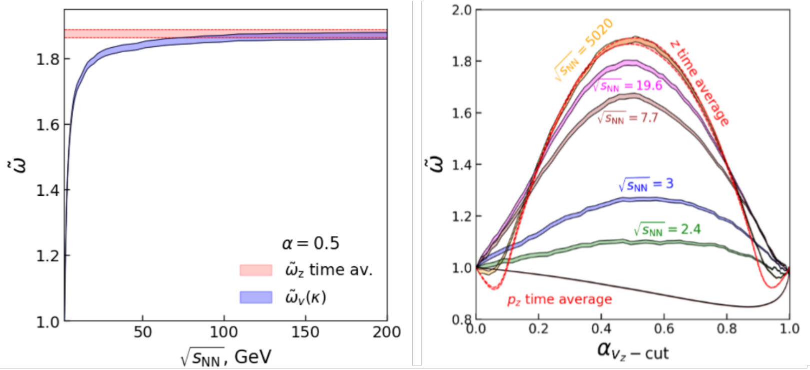
Fluctuations for constant rapidity cut

$$\alpha_y = \alpha_y(\sqrt{s_{\rm NN}}) = \frac{\langle N \rangle_y}{N_0}, \quad y_{z-{\rm cut}} = 0.5 \tag{5}$$



Fluctuations for constant α

$$\alpha_y = \text{const} = \frac{\langle N \rangle_y}{N_0} = 0.5, \quad y_{z-\text{cut}} = y_{z-\text{cut}}(\sqrt{s_{\text{NN}}}) \tag{6}$$



Summary

1. Ergodic hypothesis is shown to work for 2nd-order fluctuations along the $\tilde{T} = 1.4 \simeq 1.06\,T_c$ isotherm, including the vicinity of the critical point $\rightarrow$ *good for HICs*
2. The collective flow effect is implemented and is shown to allow us to see the enhancement of fluctuations in the momentum space $\rightarrow$ *good for HICs measurements*
3. Fluctuations in realistic rapidity acceptance $|y| < 0.5$ is studied as a function of collision energy and the maximum of fluctuations observed for $\sqrt{s_{\rm NN}} \simeq 5$ GeV $\rightarrow$ *good for upcoming HIC data*

Outlook

- Comparing our results to hydrodynamics simulations.
- Assessment of the antiparticle contribution at higher energies.
- Higher-order cumulants (with bigger statistics).
- Study of the mixed phase with the expansion. Test of the ergodicity in the mixed phase.

THANK YOU FOR ATTENTION!

Questions?

Ergodicity and ensemble averaging

Time average **[V. A. Kuznietsov et al., PRC 105, 044903 (2022)]**

$$\langle A\rangle_\tau = \frac{1}{\tau}\int\limits_{\tilde{t}_{\mathrm{eq}}}^{\tilde{t}_{\mathrm{eq}}+\tau} A(t)dt, \tag{7}$$

versus ensemble average:

$$\langle A\rangle_M = \frac{1}{M}\sum_{i=0}^{M} A_i \tag{8}$$

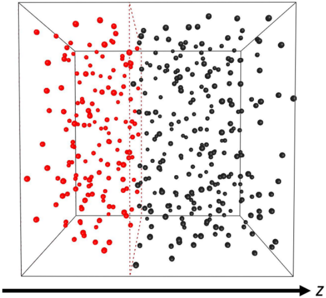
Ergodic hypothesis:

$$\lim_{\tau\to\infty}\langle A\rangle_\tau = \lim_{M\to\infty}\langle A\rangle_M \tag{9}$$

Simulation setup

$$m\frac{d^2\tilde{\mathbf{r}}_{\rm ij}}{d\tilde{t}^2} = -\vec{\nabla}\tilde{V}_{\rm LJ}(\tilde{\mathbf{r}}_{\rm ij}) \quad (10)$$

- Three points on the phase diagram, $\tilde{n} = 0.1 \approx 0.3 n_c$, $\tilde{n} = 0.3 \approx 0.95 n_c$, $\tilde{n} = 0.6 \approx 2 n_c$, all for $T = 1.06 T_c$
- $N_{\rm ev} \simeq 32000$ events at each density
- Initialize each event with random initial coordinates and momenta
- Run each event for long time ($\tilde{t} = 100$), write snapshots to file at regular time intervals
- Calculate observables as event-by-event (ensemble) average



The simulations are performed on PhysGPU cluster. Code is available at: **https://github.com/vlvovch/lennard-jones-cuda**

Time vs ensemble average