Finite density QCD equation of state: critical point and lattice-based T′-expansion (Ising-TExS)

Micheal Kahangirwe[1], Stefan A. Bass[2], Elena Bratkovskaya[3], Johannes Jahan[1], Pierre Moreau[2], Paolo Parotto[4,5], Damien Price[1], Claudia Ratti[1], Olga Soloveva[3] and Mikhail Stephanov[6].

[1]University of Houston, [2]Duke University, [3]Goethe-Universit¨at, [4]Pennsylvania State University, [5]Universit`a di Torino and INFN Torino, [6]University of Illinois at Chicago

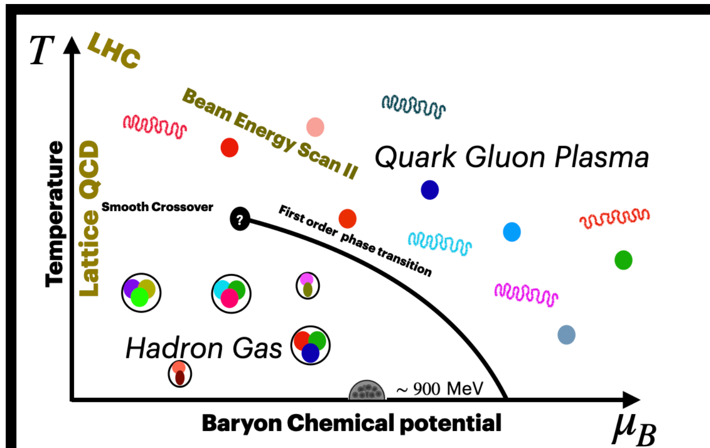
Email: mkahangi@central.uh.edu

Motivation

- Hydrodynamics simulations of heavy ion collisions require an **equation of state (EoS)** to comprehend experimental results.

Goal: To provide an equation of state (EoS) for large μ_B with a critical point, that matches lattice results.

Phase diagram



- At $\mu_B = 0$ QCD phase diagram is well established.
- At finite μ_B a critical point is expected but has not yet been observed.

Taylor Expansion

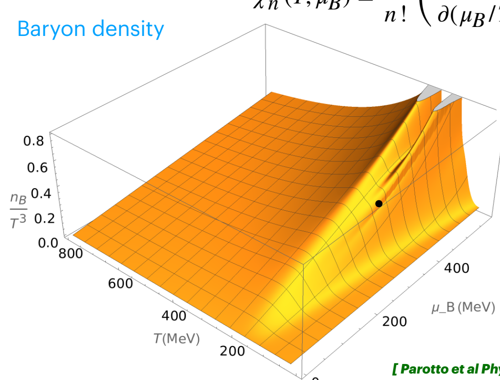
Previous Approach :

- The EoS with a critical point from Taylor expansion is available and works well for low μ_B.

$$n_B(T,\mu_B) = T^3 \sum_{n=1}^{2} \frac{1}{(2n-1)!} \chi_{2n}^{\text{non-Ising}}(T) \left(\frac{\mu_B}{T}\right)^{2n-1} + \frac{T_C^4}{T} n_B^{\text{Ising}}(T,\mu_B)$$

$$\chi_n^B(T,\mu_B) = \frac{1}{n!}\left(\frac{\partial}{\partial(\mu_B/T)}\right)^n (P/T^4)$$

Baryon density



BEST COLLABORATION

[Parotto et al PhysRevC.101.034901(2020)]

- For $\mu_B > 450$ MeV, thermodynamics observables have unphysical wiggles due to limitations of truncated Taylor expansion, hindering critical phenomena studies

T′- Expansion Scheme

- As a solution, the Wuppertal-Budapest collaboration developed a T'- expansion scheme that exhibits smooth behavior at high μ_B and copes well with QCD transition temperature.

$$T\frac{\chi_1^B(T,\mu_B)}{\mu_B} = \chi_2^B(T',0)$$

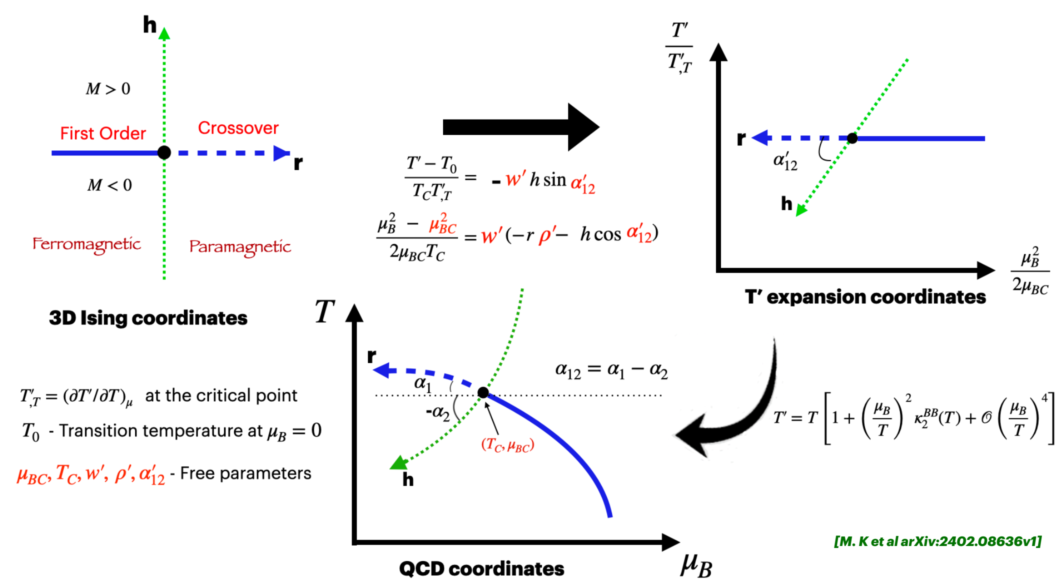
$$T'(T,\mu_B) = T\left[1 + \kappa_2^{BB}(T)\left(\frac{\mu_B}{T}\right)^2 + \kappa_4^{BB}(T)\left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}\left(\frac{\mu_B}{T}\right)^6\right]$$

[Borsányi, S et al PRL. 108(1), 101.034901(2021)]

- From the T′-expansion scheme, as long as $T\chi_1^B/\mu_B$ is smooth, then finite density physics, such as the critical point can be encoded in T'.

Mapping 3D-Ising to QCD

- If the critical point in QCD exists, then it must be in 3D-Ising model Universality class



- This mapping allows us to transport any physical observable from 3D-Ising to QCD with free parameters determined by the current physics knowledge.

Merging 3D-Ising with T′-Expansion

Methodology:

$$\frac{n_B(T,\mu_B)}{T^3} = \chi_1^B(T,\mu_B) = \left(\frac{\mu_B}{T}\right)\chi_{2,lat}^B(T',0)$$

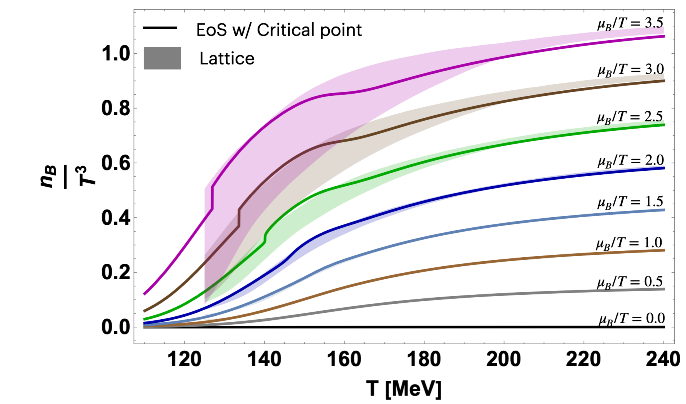
- We introduce the critical point in T' by separating into critical T'_{crit} and non-critical parts

$$T' = \underbrace{T'_{lat}(T,\mu_B)}_{\text{lower order in } \left(\frac{\mu_B}{T}\right)} + \underbrace{T'_{crit}(T,\mu_B) - Taylor[T'_{crit}(T,\mu_B)]}_{\text{higher orders in } \left(\frac{\mu_B}{T}\right)}$$

$$T'_{crit}(T,\mu_B) \approx \left(\frac{\partial\chi_{2,lat}^B(T,0)}{\partial T}\bigg|_{T=T_0}\right)^{-1} \frac{n_B^{crit}(T,\mu_B)/T^3}{(\mu_B/T)}$$

[M. K et al arXiv:2402.08636v1]

Results



$\mu_{BC} = 350\ MeV,\ \alpha_{12} = 90,\ w = 2,\ \rho = 2$ [M. K et al arXiv:2402.08636v1]

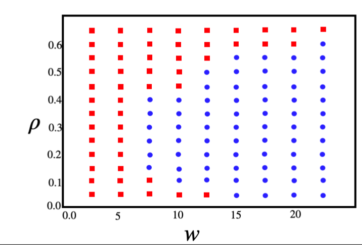
- Our EoS with the critical point for some choice of parameters is within the error band of Extrapolated lattice QCD results.

Constraints

- Transition line: Choosing μ_{BC} fixes T_C and α_1
- Physical quark masses: $\alpha_{12} = \alpha_1$

[Pradeep, M. S., & Stephanov, M PhysRevD . 100(5), 056003.(2019)]

- Stability and causality: fix w and ρ



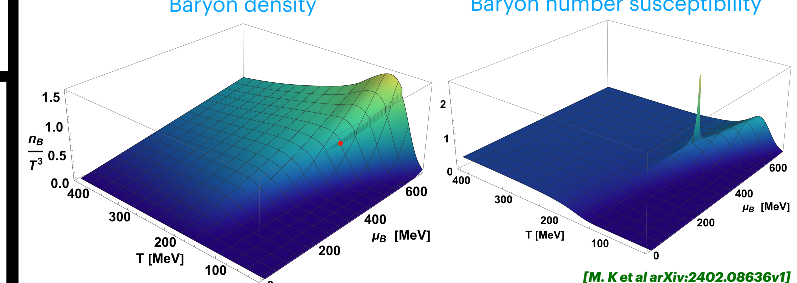
$\mu_{BC} = 500\ MeV,\ \alpha_{12} = \alpha_1$

- Blue point -> acceptable EoS
- Red point -> unacceptable EoS

[M. K et al arXiv:2402.08636v1]

Thermodynamics

Baryon density — Baryon number susceptibility



[M. K et al arXiv:2402.08636v1]

$\mu_{BC} = 500\ MeV,\ \alpha_{12} = \alpha_1,\ w = 15,\ \rho = 0.3$

Summary

Disclaimer! : We don't predict the location of the critical point

- We offer a family of EoS with an enhance coverage ranging from $\mu_B = 0$ to 700 MeV with adjustable parameters
- Our EoS can be tuned to reproduce physical quark masses