Entanglement Entropy in p-p Collisions at LHC Energies

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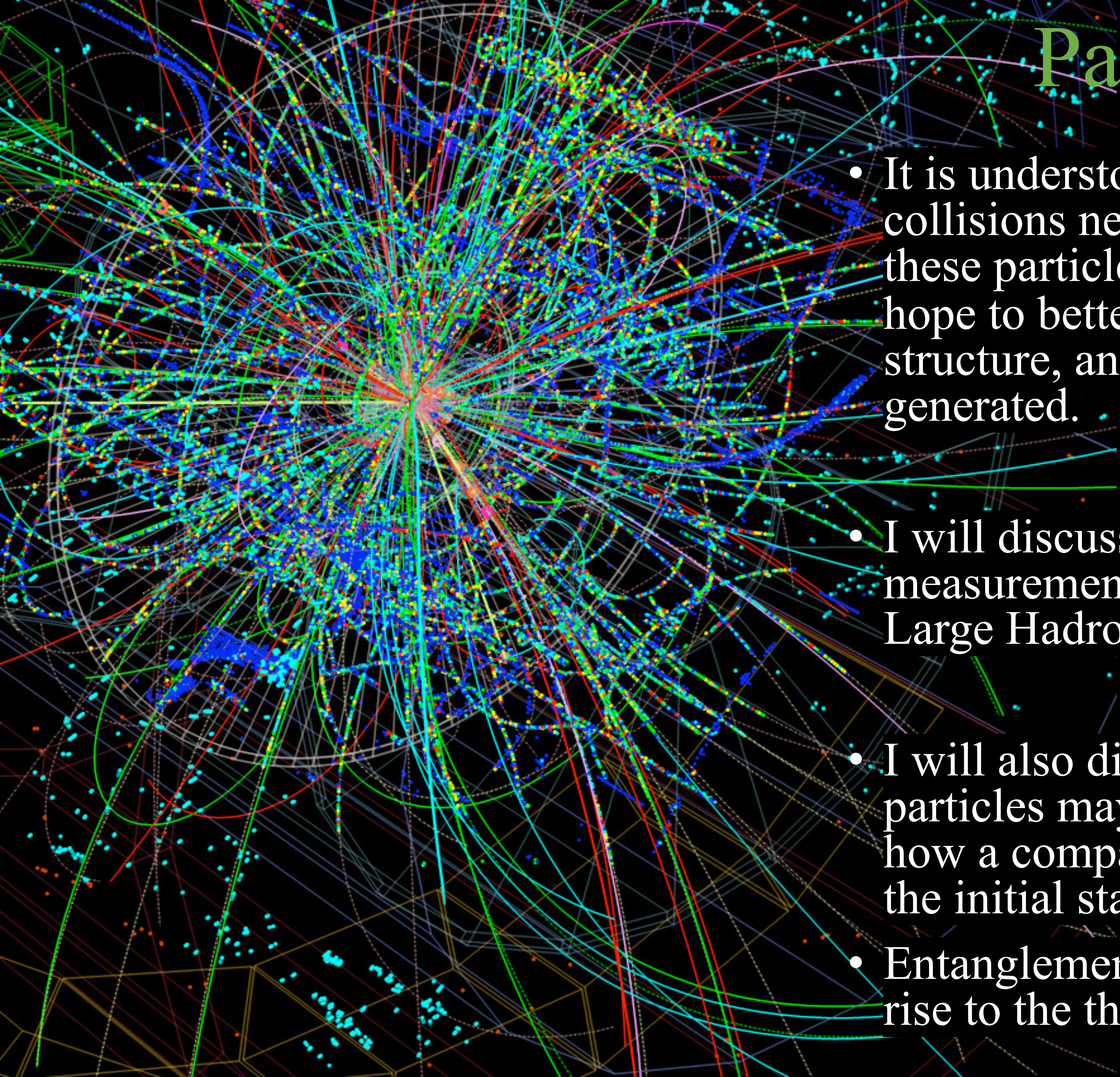
Introduction

Particle Production

- It is understood during high energy particle collisions new hadrons are formed. By studying how these particles populate difference phase spaces we hope to better understand the nature of the proton’s structure, and the process of how matter is generated.
- I will discuss charged particle multiplicity measurements taken using the ALICE detector at the Large Hadron Collider.
- I will also discuss a mechanism for how these particles may be generated using entanglement and how a comparison can be made between entropy in the initial state and entropy in the final state.

Particle Production



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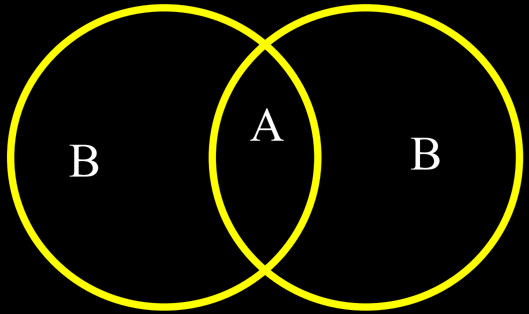
- I will discuss charged particle multiplicity measurements taken using the ALICE detector at the Large Hadron Collider.
- I will also discuss a mechanism for how these particles may be generated using entanglement and how a comparison can be made between entropy in the initial state and entropy in the final state.
- Entanglement in the initial state may be what gives rise to the thermal like behavior of larger systems.



Entanglement Entropy (initial-state)

- Entanglement entropy is a metric for understanding the level of entanglement between two or more quantum systems.
- It quantifies the amount of information that is shared between particles (partons) and can be used to characterize their degree of correlation.
- A proton represents a pure state; meaning the partons are fully correlated and maximally entangled.
- The interacting proton can be segmented into an interaction region (A) and a region that remains unprobed (B). Entanglement entropy arises between these two regions.

$$S_{\mathrm{A}} = -tr[\hat{\rho}_{\mathrm{A}} \ln \hat{\rho}_{\mathrm{A}}] = S_{\mathrm{B}} = -tr[\hat{\rho}_{\mathrm{B}} \ln \hat{\rho}_{\mathrm{B}}]$$

Gibbs Entropy (final-state)

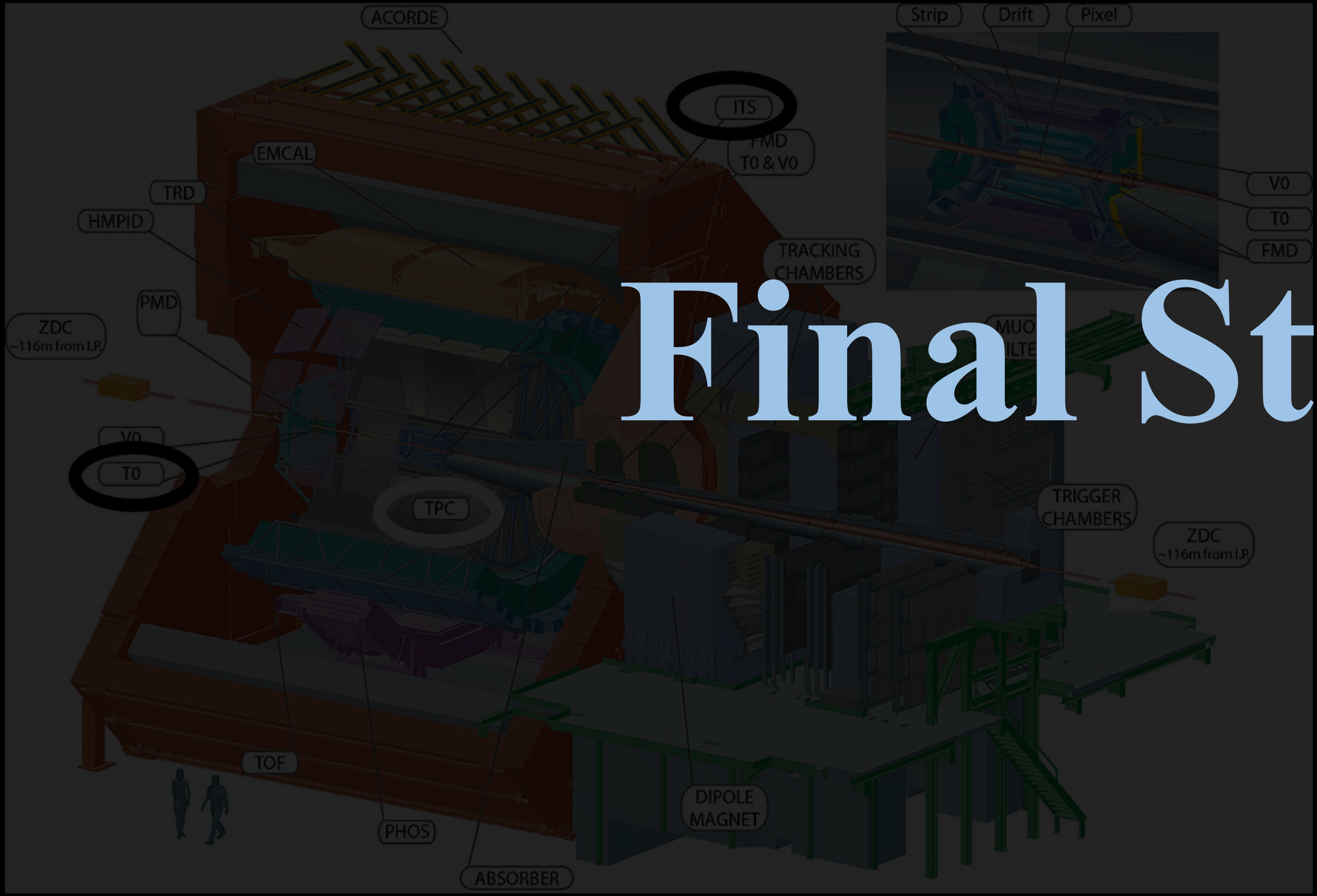
- Gibbs entropy is a thermodynamic quantity that measures the degree of disorder, or randomness, in a system.
- This is calculated using a probability distribution of the number of microstates. In this case the number of hadrons
- The more hadrons produced the higher the entropy.
- While the Gibbs entropy is defined under different context than entanglement entropy, each of these definitions attempts to quantify the disorder of the system. And each is proportional to the number of particles.

$$S_{\text{hadron}} = -\sum \mathrm{P(N)} \ln \mathrm{P(N)}$$

The goal of this analysis is to show that entanglement survives the system evolution and these two entropies can be related, even to the extent that their values should be equal. This would demonstrate that we are not dealing with a thermal system but rather a quantum many body system that appears thermal.

Final State

A Large Ion Collider Experiment (ALICE)



- Measurements on charged particles produced in collisions are taken using the ALICE detector, one of four detectors at the LHC
- The primary subsystems used for charged particle tracking are the Inner Tracking System (ITS), and Time Projection Chamber (TPC).
- For physics selection the Fast Interaction Trigger (FT0) timing information is also used.

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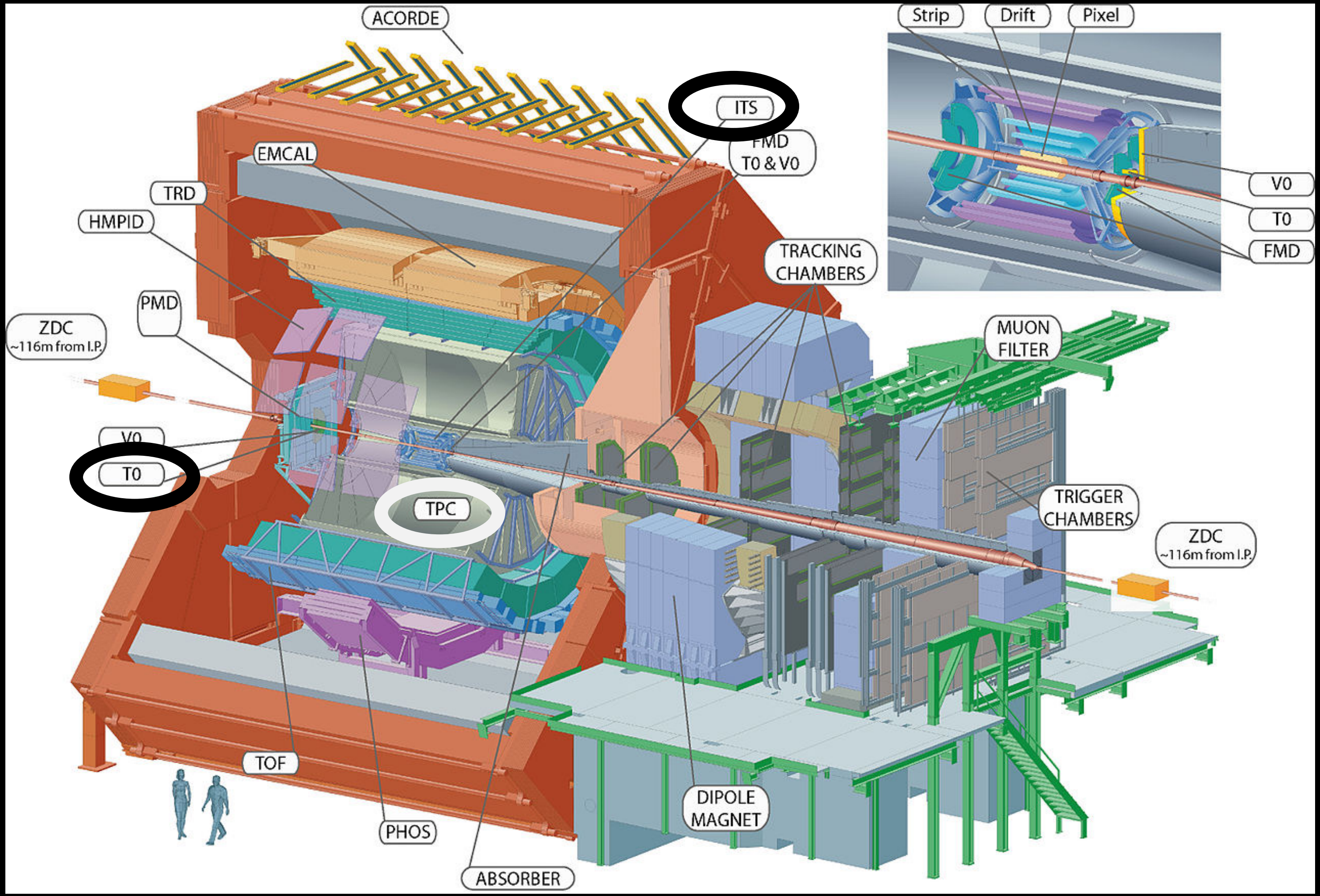

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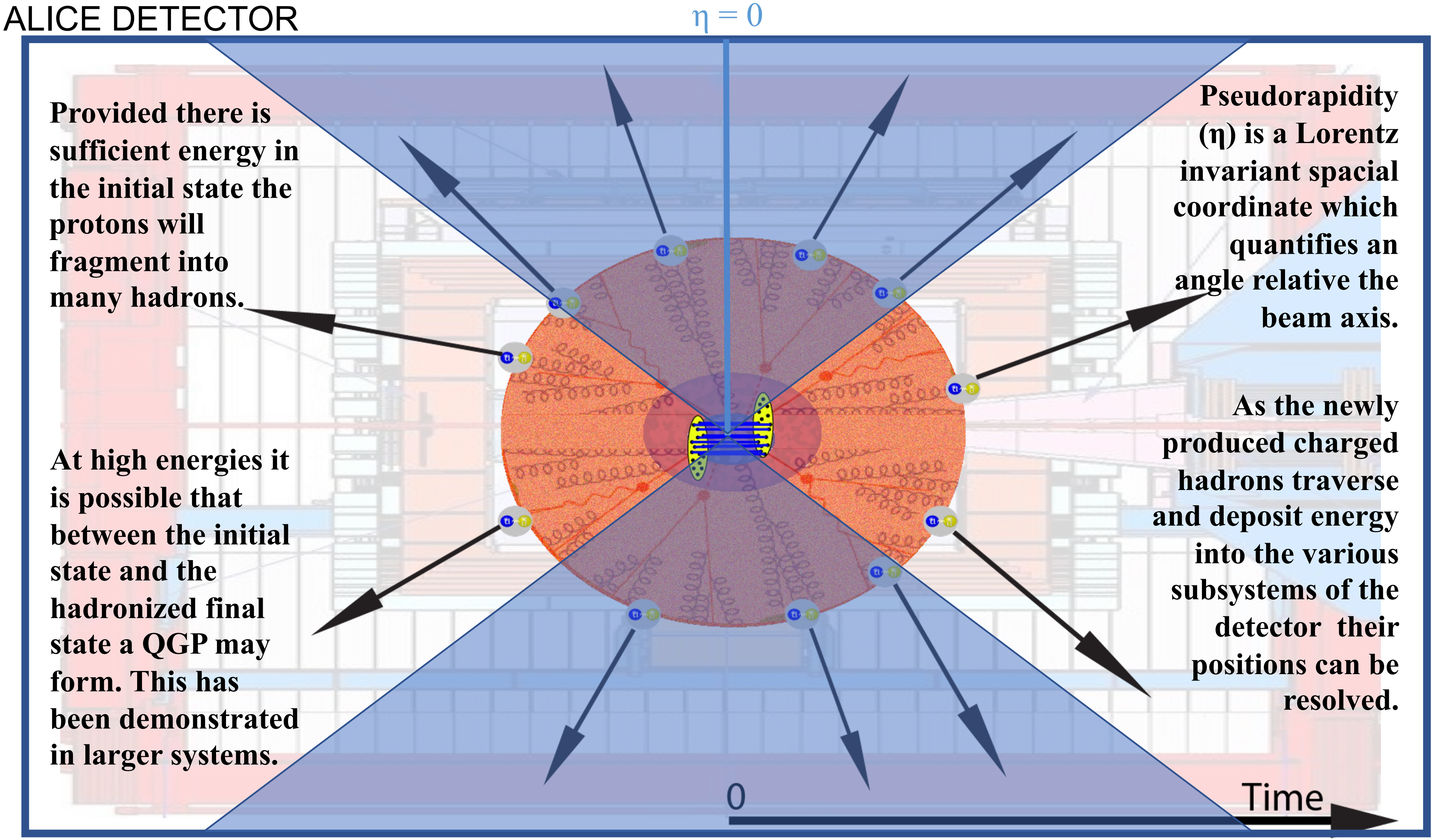
ALICE DETECTOR



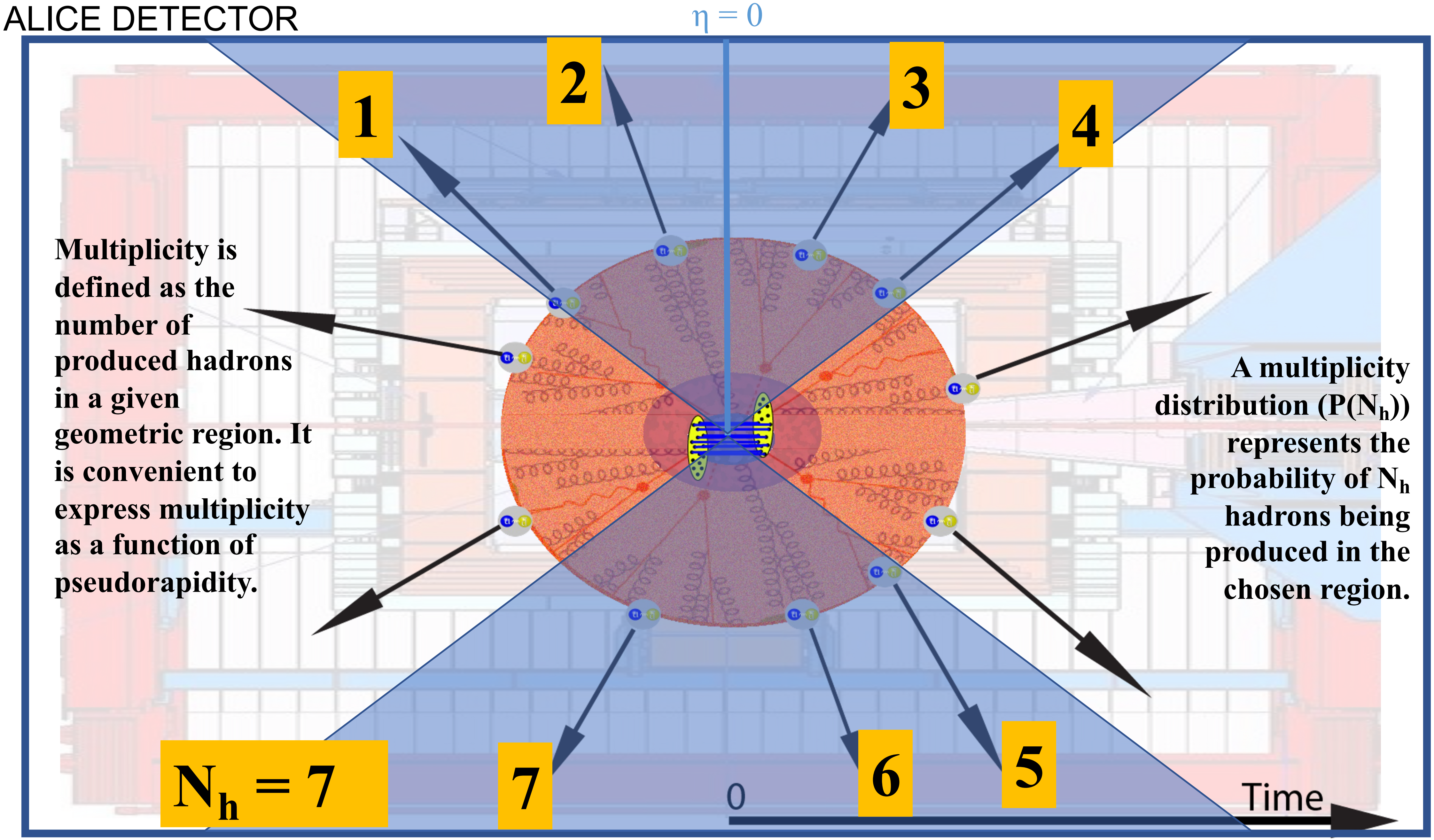
Provided there is sufficient energy in the initial state the protons will fragment into many hadrons.

At high energies it is possible that between the initial state and the hadronized final state a QGP may form. This has been demonstrated in larger systems.

Pseudorapidity (η) is a Lorentz invariant spacial coordinate which quantifies an angle relative the beam axis.

As the newly produced charged hadrons traverse and deposit energy into the various subsystems of the detector their positions can be resolved.

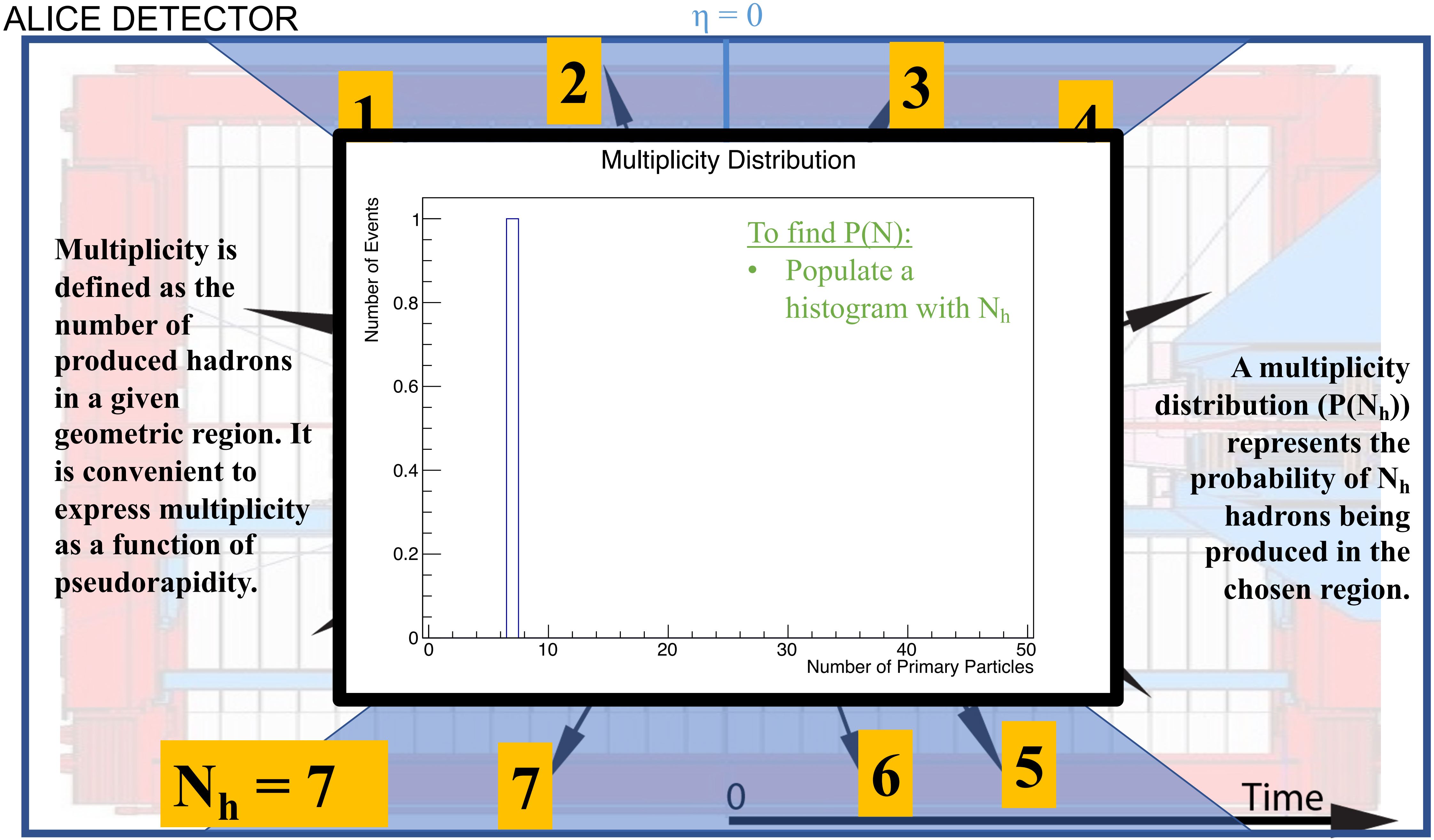
ALICE DETECTOR



Multiplicity is defined as the number of produced hadrons in a given geometric region. It is convenient to express multiplicity as a function of pseudorapidity.

A multiplicity distribution ($P(N_h)$) represents the probability of N_h hadrons being produced in the chosen region.

ALICE DETECTOR



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To find P(N):

- Populate a histogram with N_h

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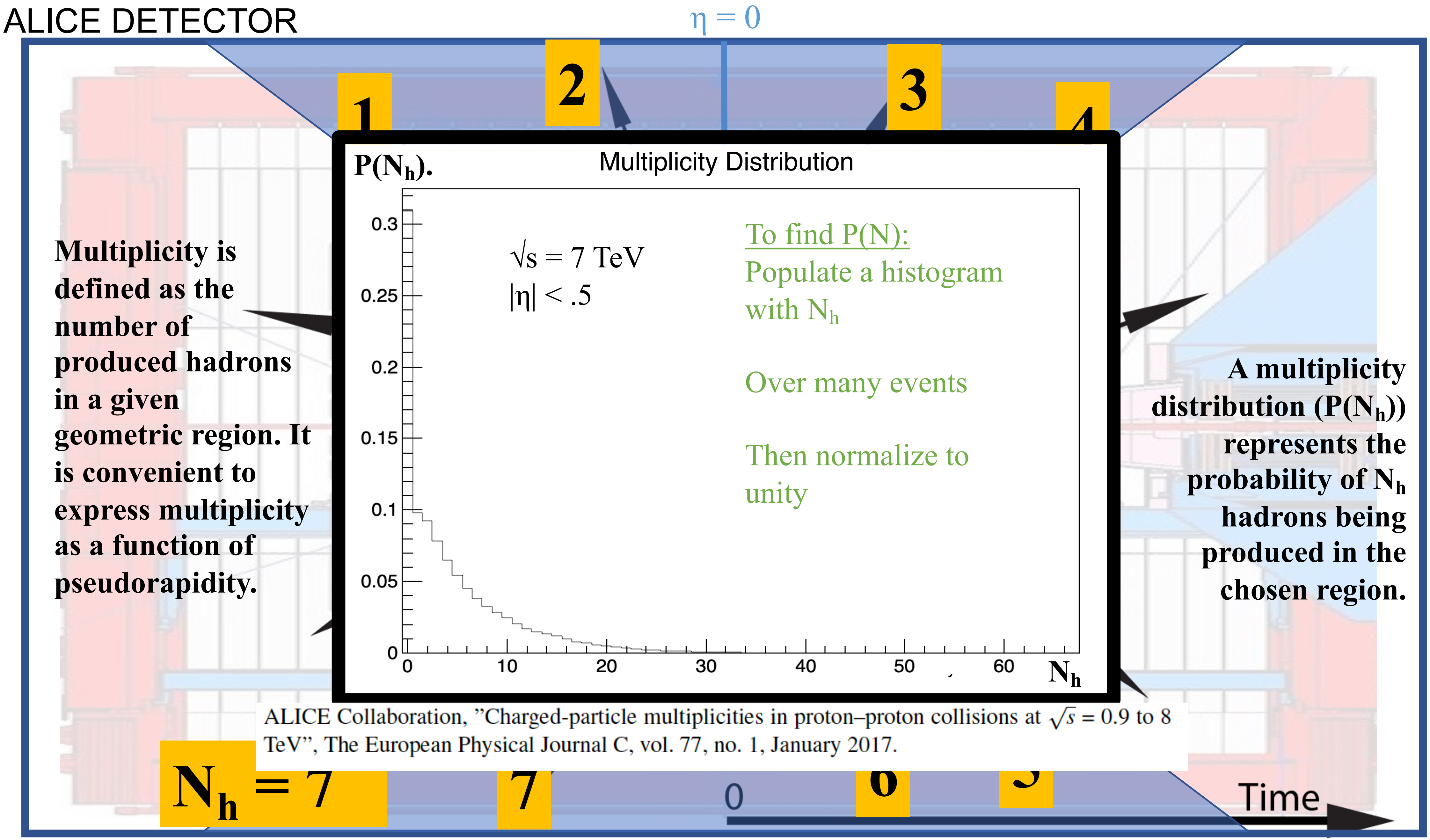


Multiplicity is defined as the number of produced hadrons in a given geometric region. It is convenient to express multiplicity as a function of pseudorapidity.

To find P(N):

Populate a histogram with N_h

Over many events

Then normalize to unity

A multiplicity distribution ($P(N_h)$) represents the probability of N_h hadrons being produced in the chosen region.

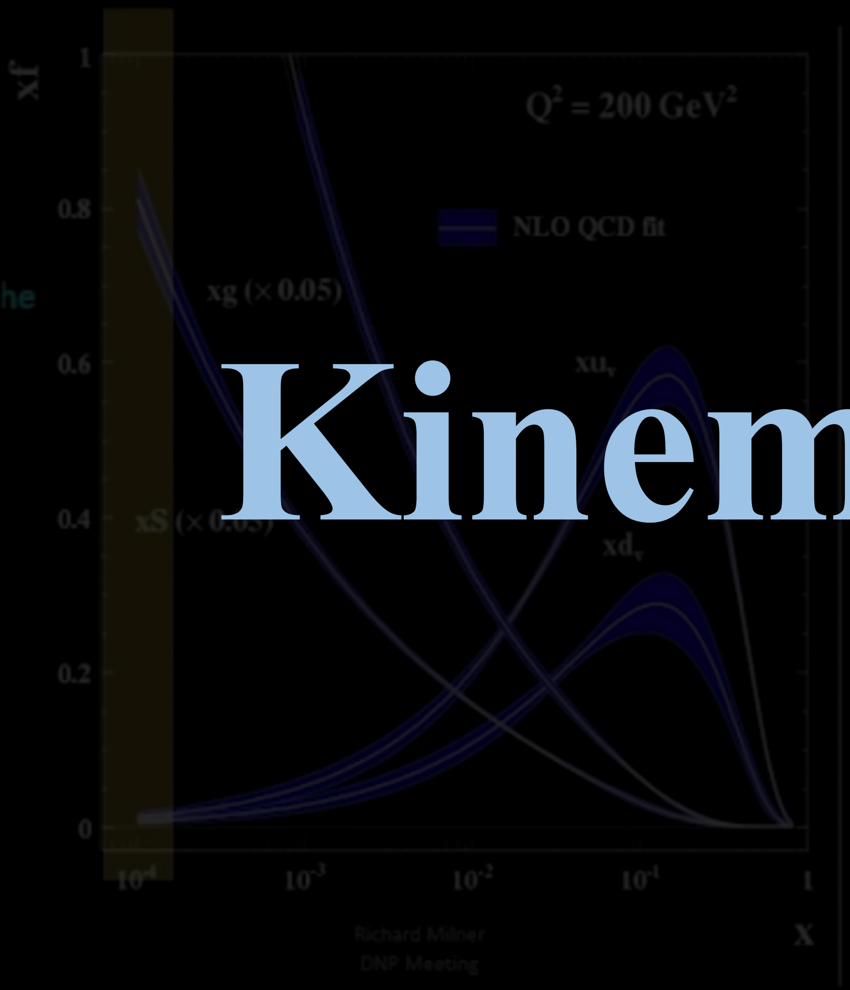
ALICE Collaboration, "Charged-particle multiplicities in proton–proton collisions at $\sqrt{s}$ = 0.9 to 8 TeV", The European Physical Journal C, vol. 77, no. 1, January 2017.

Kinematics

- The **Bjorken-x** is the key kinematic variable used in this analysis to map particles in the initial state to a spatial region in the final state.
- x represents the fraction of the proton’s momentum carried by a struck parton.

- At very low-x the protons wavefunction is dominated by gluons. Meaning that all the interactions in the low-x limit are gluon-gluon interactions.

- For an initial understanding of entanglement, it is helpful to restrict our study to this region where the wavefunction is much simpler.



”Visualizing the Proton” YouTube, uploaded by Arts at MIT, https://www.youtube.com/watch?v=G-9I0buDi4s.

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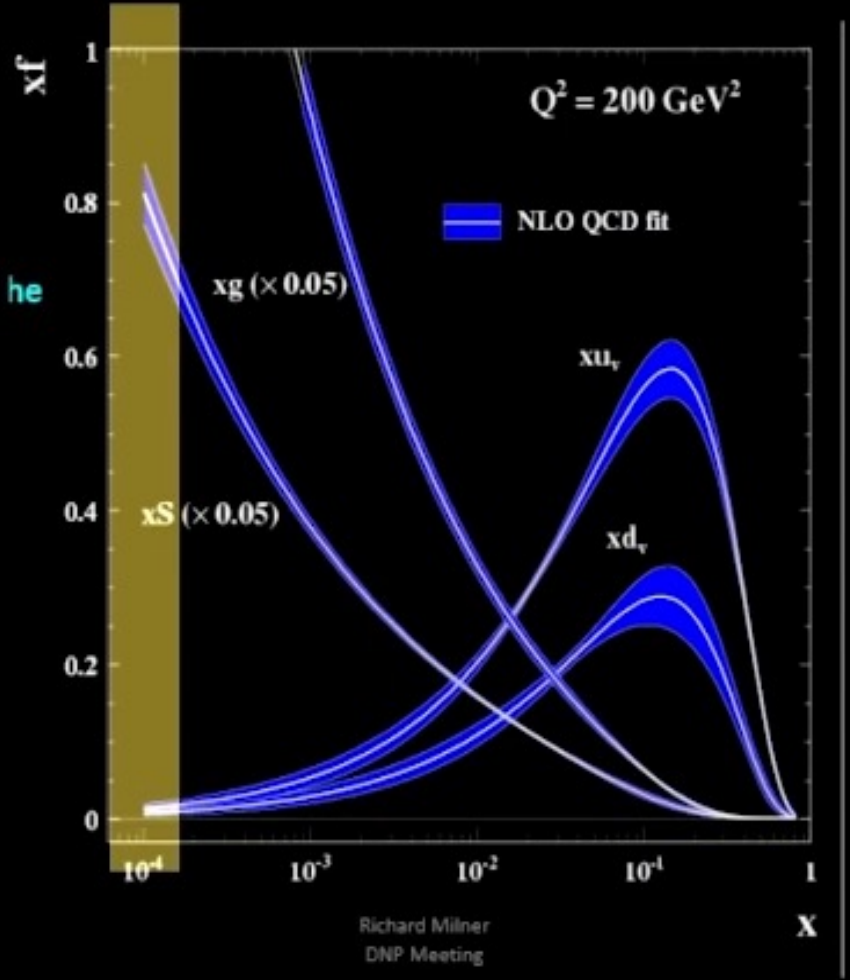

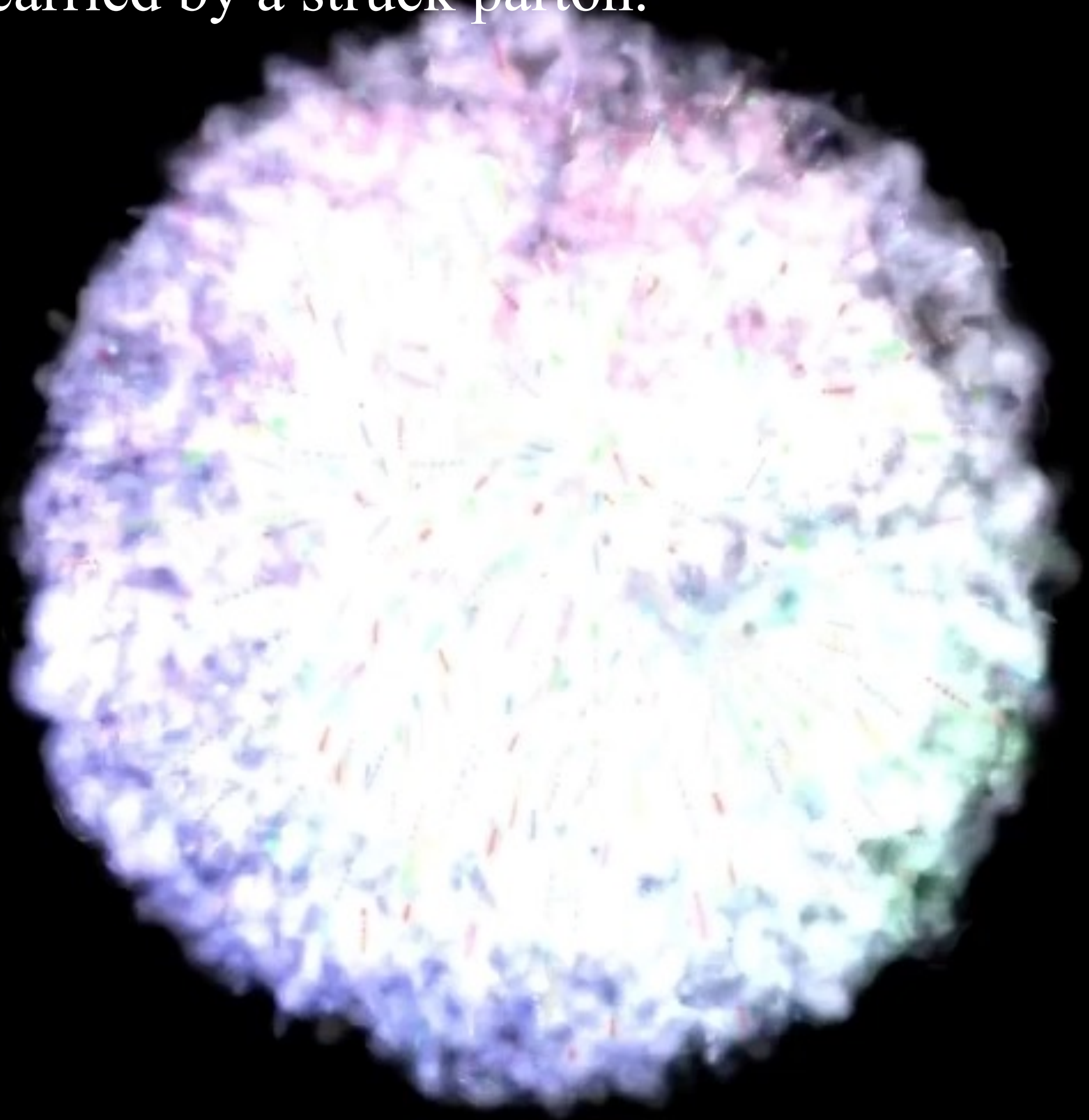

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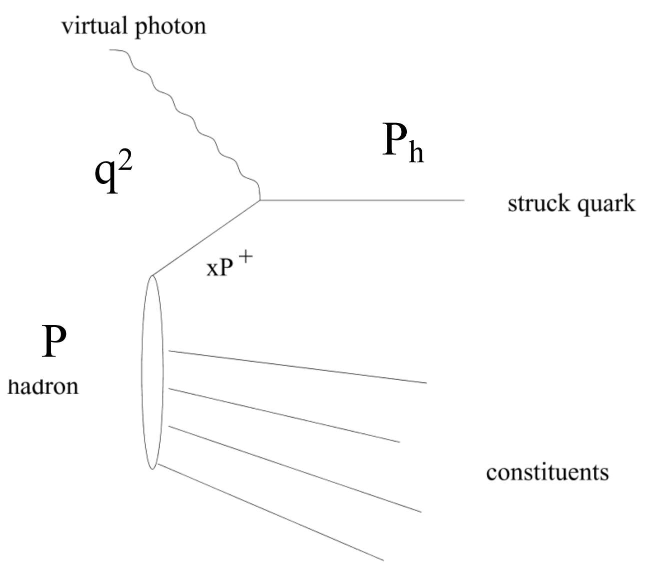
The formalism that allows us to relate x (not measurable in our system) to pseudorapidity (a measurable quantity) we will borrow the formalism of DIS.



DIS uses an e-p system where the quantity x and the energy exchange q^2 can be measured.

$$y_h \simeq \eta_h$$

$$x = e^{-y_p} e^{\eta_h}$$

$$y_p = \frac{1}{2}\ln\left(\frac{E_p + p_{zp}}{E_p - p_{zp}}\right) \tag{1}$$

$$y_h = \frac{1}{2}\ln\left(\frac{E_h + p_{zh}}{E_h - p_{zh}}\right) \tag{2}$$

$$q = P - P_h \tag{3}$$

$$x = -\frac{q^2}{2P \cdot q} \tag{4}$$

$$P \cdot q = P \cdot (P - P_h) = (P \cdot P) - (P \cdot P_h) \tag{5}$$

$$x = -\frac{(P_h \cdot P)}{(P \cdot P)} \tag{6}$$

$$\frac{p_{zh}}{p_{zp}} = \frac{(P_h \cdot P)}{(P \cdot P)} \tag{7}$$

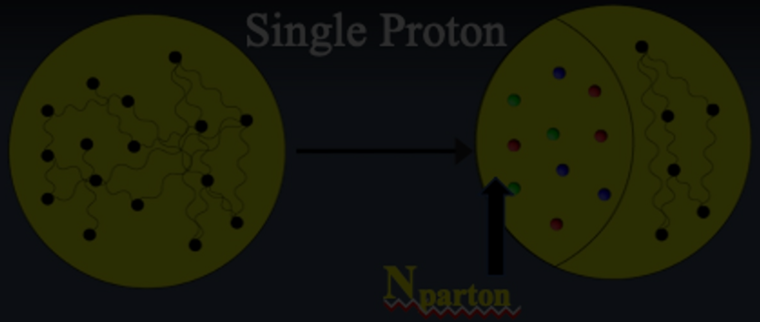
$$x = \frac{1}{\frac{p_{zh}}{p_{zp}}} \tag{8}$$

$$\ln(1/x) = \ln\left(\frac{p_{zh}}{p_{zp}}\right) = \ln\left(\frac{E_h + p_{zh}}{E_h - p_{zh}}\right) - \ln\left(\frac{E_p + p_{zp}}{E_p - p_{zp}}\right) \tag{9}$$

$$\ln(1/x) = y_p - y_h \tag{10}$$

Initial State

Proton before collision
- Pure quantum state
- Maximally entangled
- 0 von Neumann entropy



Coherence between the region probed by the other proton and the part left alone is lost. Giving rise to increase in entanglement entropy

In the region of low x where the proton can be described as a mass of indistinguishable gluons we estimate the entanglement entropy as:

$$S_{parton} = \ln(N_{parton})$$

Parton model places the proton in a Lorenz contracted high momentum frame where partons are seen as quasi-free by some external probe

Non-uniform color fields generated by the interaction can lead to the formation of flux tubes, which are regions of high color field strength that extend between partons. These flux tubes can be thought of as one-dimensional objects, or strings.

String formation adds a third dimension thereby making entropy an extensive quantity.

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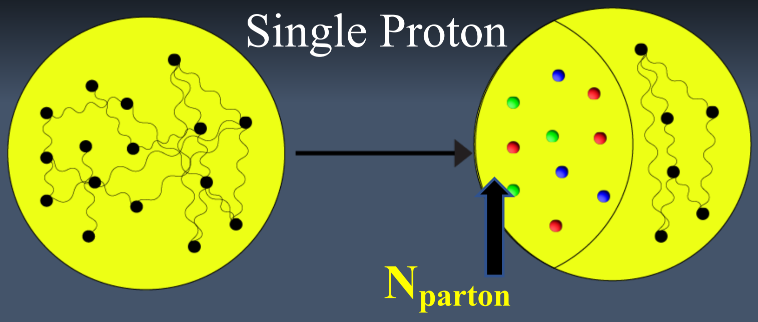

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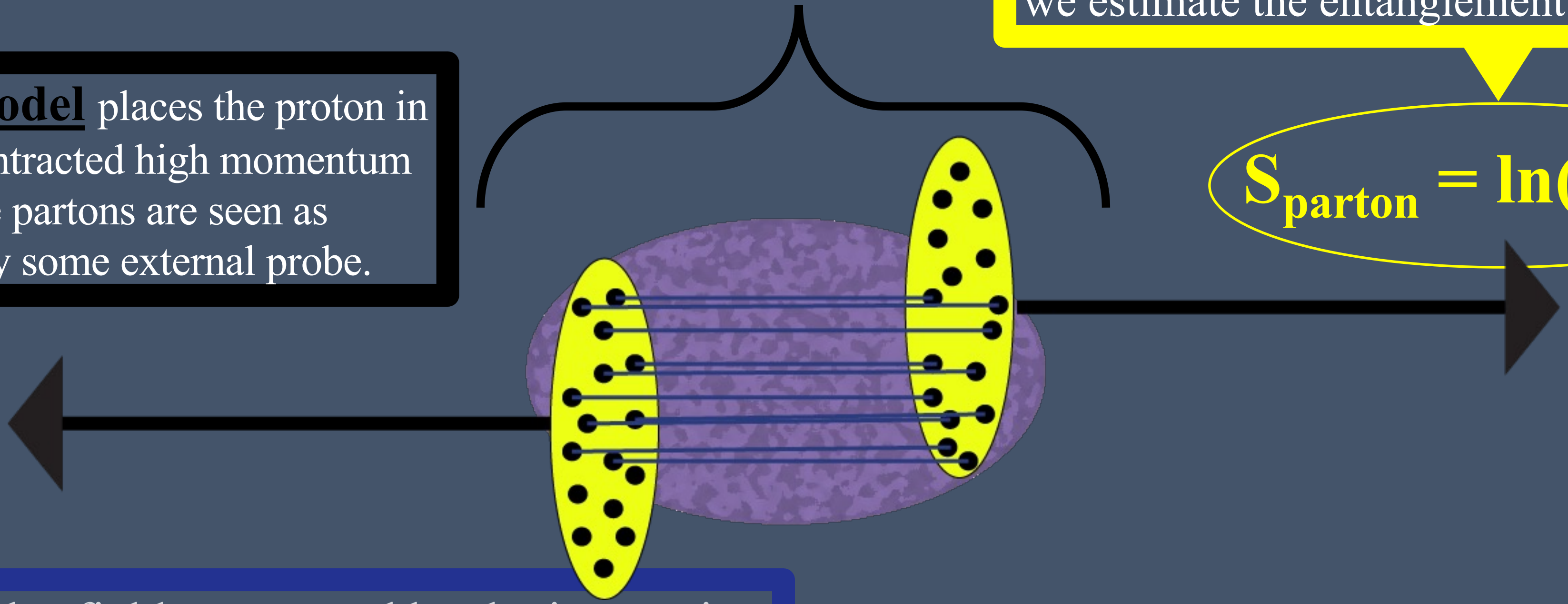

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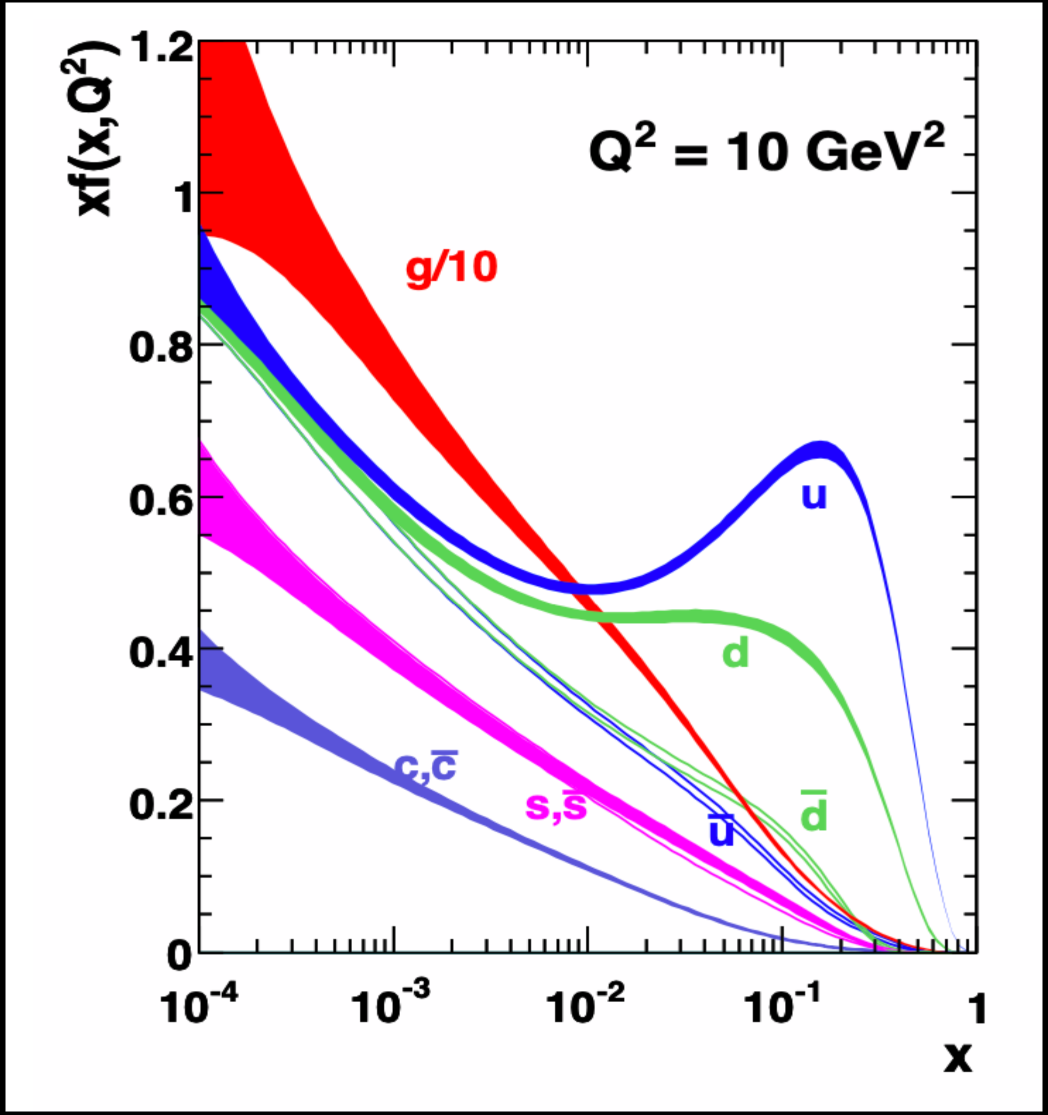
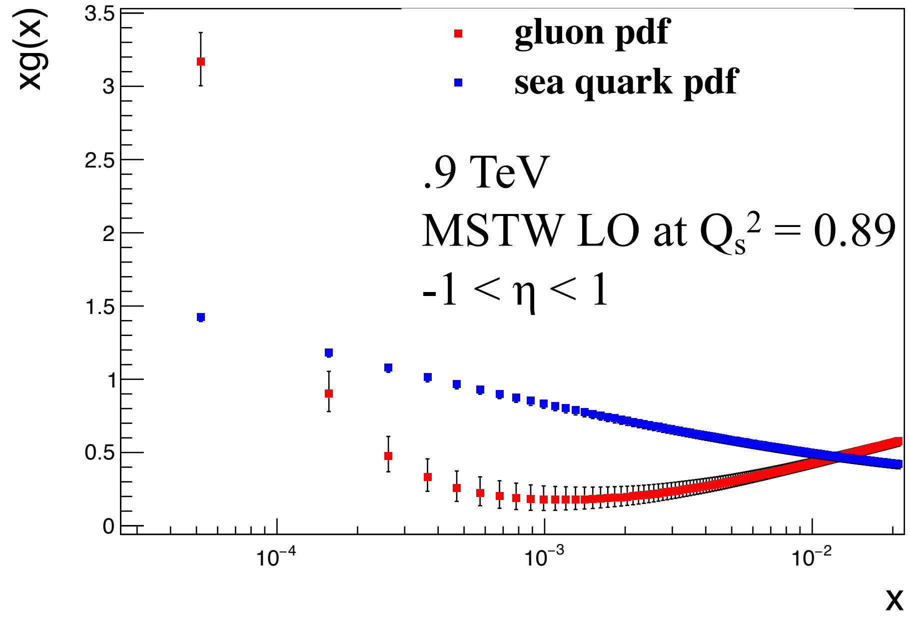
Parton Distribution Functions



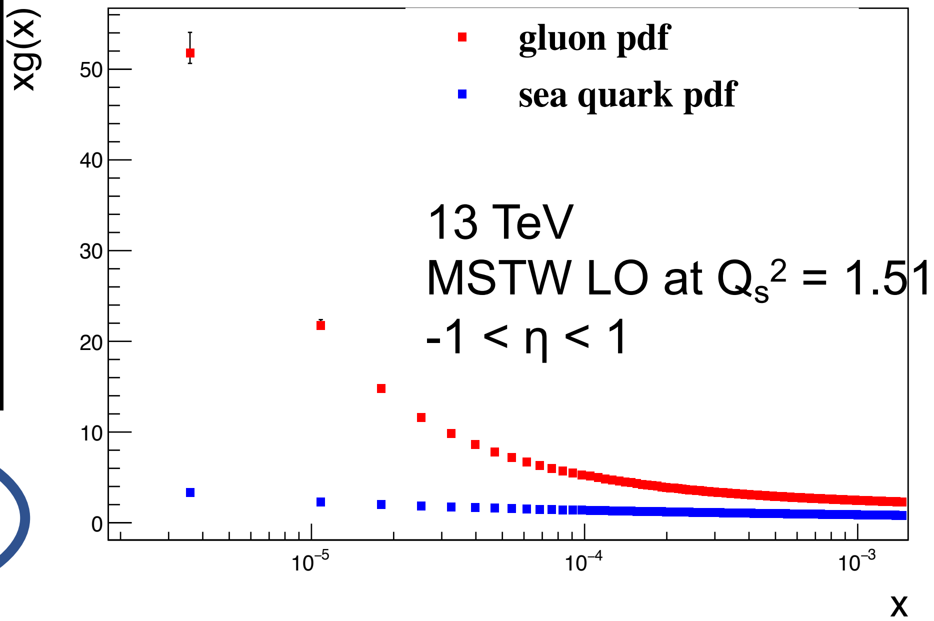
A. D. Martin, W. J. Stirling, R. S. Thorne, and G. Watt, Eur. Phys. J. C63, 189 (2009), arXiv:0901.0002 [hep-ph].

- Parton distribution functions f(x) describe the probability to find a particular parton with some fraction of the protons momentum characterized by x.
- PDF’s are dependent on the scale at which the proton is probed Q^2 called the factorization scale.
- Found using pQCD by fitting DIS cross sections from data.
- Pdf’s are fit using a third order polynomial and integrated over to find the number of particles.



To calculate the number of partons in the initial state we integrate over the appropriate range in x.

$$N_g = \int_{x_1}^{x_2} \frac{xG(x)dx}{x}$$



- **LHAPDF** is an open-source library from which one can extract and extrapolate published PDF sets.
- These PDF sets are fitted to data by various collaborations (e.g. HERAPDF).
- Errors are calculated using a Hessian approach

Factorization Scale

For average p-p collisions this scale is characterized by the transverse momentum of partons, which is dependent on the density of partons. Parton density is said to saturate at low x so the corresponding scale is called the **saturation scale** Q_s^2.

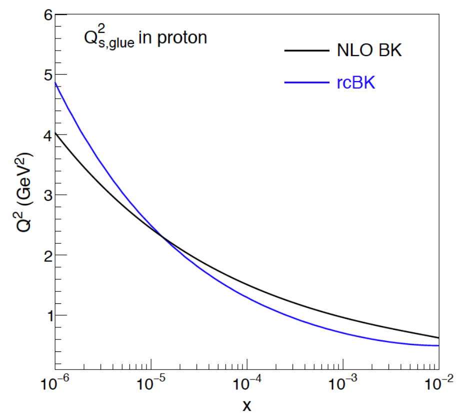
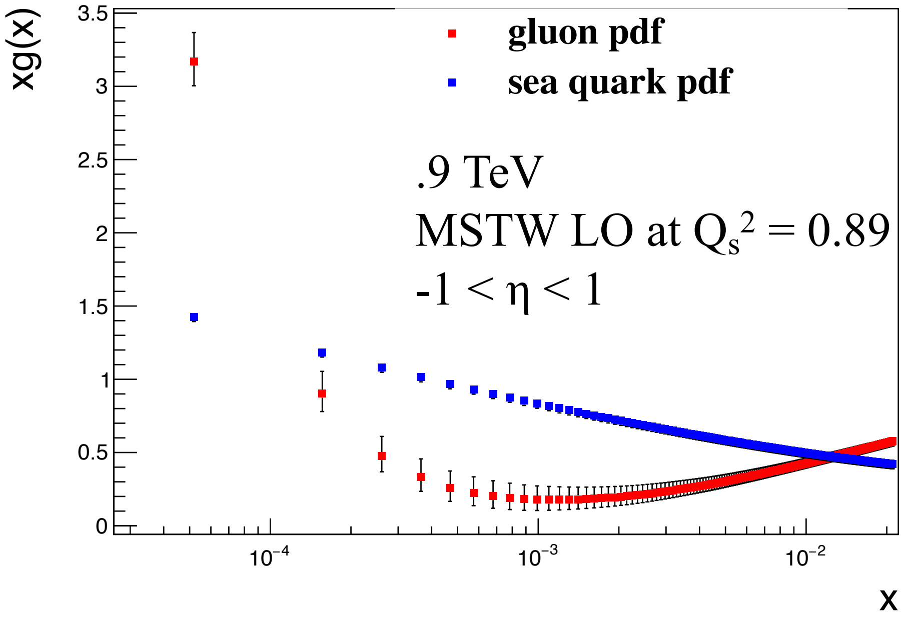


FIG. 5. Saturation scale Q_s^2 as a function of x. The saturation scale Q_s^2 as predicted by rcBK [28] (blue curve) and NLO BK [27] (black curve) calculations are shown as a function of x. For our main result we used NLO BK.



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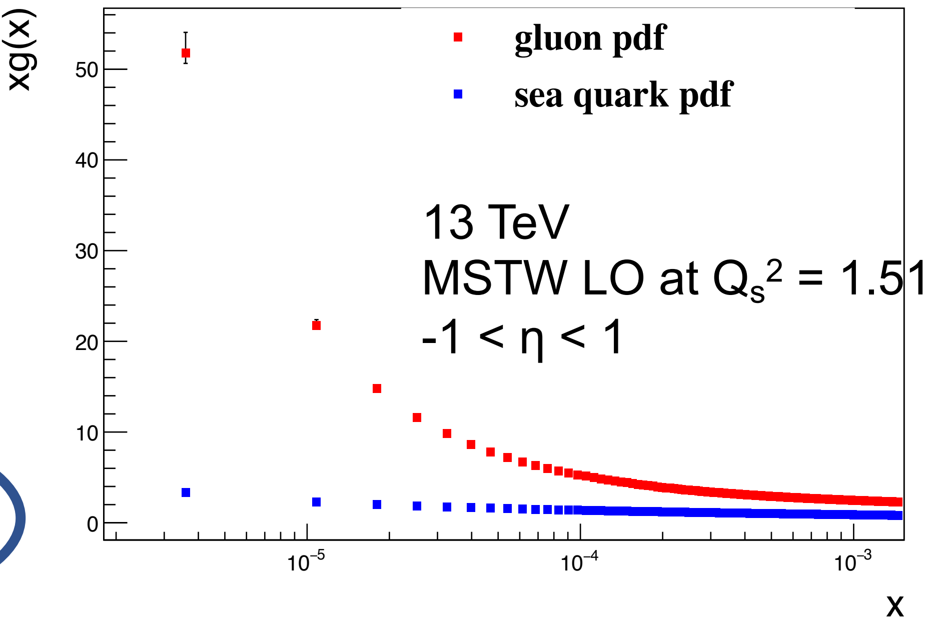


$$S_{\text{parton}} = \ln(N_{\text{parton}})$$

Final vs. Initial Entropy

<u>One Complication:</u> The initial state entropy is derived from PDFs calculated using DIS data where only one distribution of partons is present. In order to make a meaningful comparison to the initial sate we must extrapolate our distribution to reflect a single proton.

Tu, Z., Kharzeev, D. E., & Ullrich, T. (2020).

Einstein-Podolsky-Rosen Paradox and Quantum Entanglement at Subnucleonic Scales.

Physical Review Letters, 124(6), 062001. doi: 10.1103/physrevlett.124.062001.

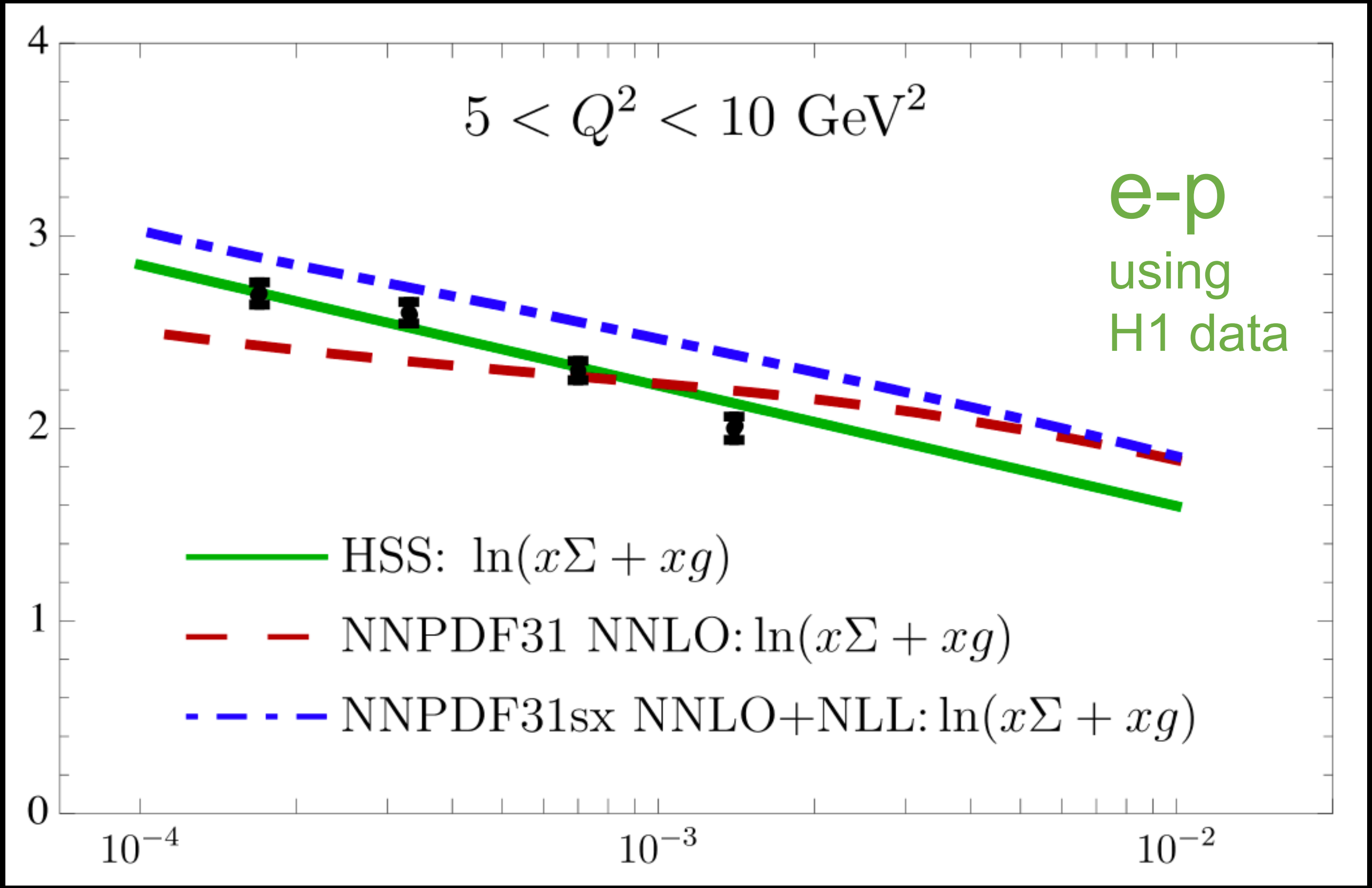
A method for doing this extrapolation is proposed in the paper that established most of the theoretical framework behind this study.

A transformation on the distribution must be made to correct for this. To make this transformation the authors made one assumption:

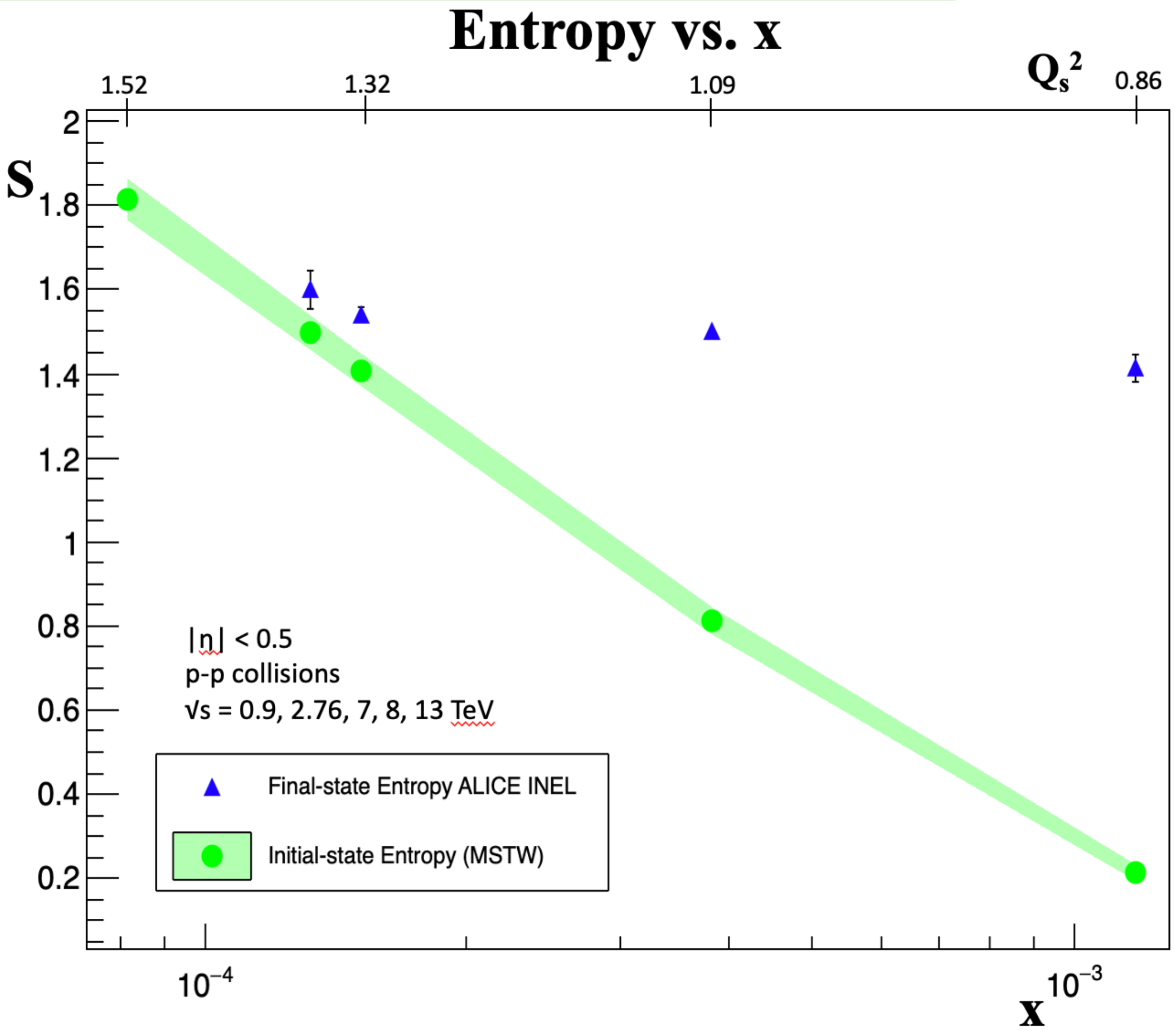
<u>twice the partons in the initial state = twice the hadrons produced in the final state</u>

3 STEPS

Equivalence shown in e-p systems



Hentschinski, M., & Kutak, K. (2022). Evidence for the maximally entangled low x proton in Deep Inelastic Scattering from H1 data. The European Physical Journal C, 82(2), 56. doi: 10.1140/epjc/s10052-022-10056-y.



As we approach x=0 (gluon dominate) the initial and final state entropies are increasing and closer in value.

The significant difference between the initial and final state at high-x may be attributed to an increase in quarks contributing to the wavefunction.

A better understanding of how to extrapolate our data to that of a single proton, may help further studies. We will continue to explore other considerations in this extrapolation.

There may also be other sources of entropy generation in the collision and a more robust theory may be necessary to demonstrate this equivalence. Particularly an understanding of the longitudinal components of entanglement may be necessary to explain fluctuations and other phenomena associated with particle production. In the future we would like to measure this in different kinematic regimes and systems. It would be particularly interesting to see these measurements done in systems which have been shown to generate QGP.

Conclusion and Future Steps

- While it seems evident that there is a relation between entanglement entropy in the initial state and thermodynamic entropy in the final state in e-p systems.
- It is not yet clear whether this holds true in hadron collisions.
- A better understanding of how to extrapolate our data to that of a single proton, may help further studies. We will continue to explore other considerations in this extrapolation.
- There may also be other sources of entropy generation in the collision and a more robust theory may be necessary to demonstrate this equivalence.
- Furthermore, we would like to measure this in different kinematic regimes and systems.

References

[1] ALICE Collaboration, ”Charged-particle multiplicities in proton–proton collisions at $\sqrt{s} = 0.9$ to 8 TeV”, The European Physical Journal C, vol. 77, no. 1, January 2017.

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[3] Zhoudunming Tu, Dmitri E. Kharzeev, Thomas Ullrich, ”Einstein-Podolsky-Rosen Paradox and Quantum Entanglement at Subnucleonic Scales”, Physical Review Letters, vol. 124, no. 6, February 2020.

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[8] Hannu Paukkunen and Pia Zurita. PDF reweighting in the Hessian matrix approach. *Journal of High Energy Physics*, 2014(12), Dec. 2014. DOI: 10.1007/jhep12(2014)100