Contour Extrapolation of baryon density in search of the QCD critical point

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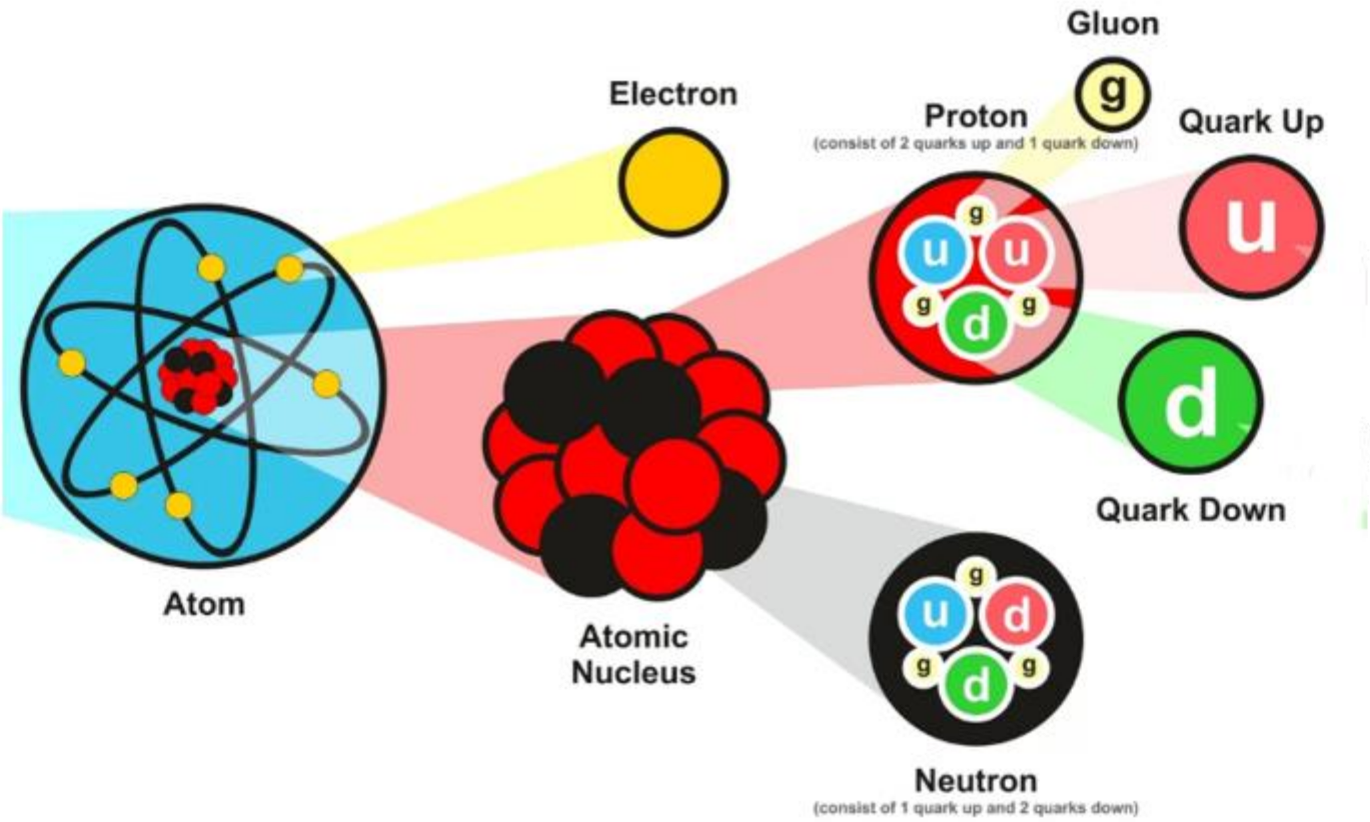
Physics Research Day - 2024

Introduction

- Quarks and Gluons
- Color charge
- Hadron :- Baryons and Mesons

➢ Baryons: consists of 3 quarks
(Protons and neutrons)

➢ Mesons: consists of quark-antiquark
(Pions and Kaons)

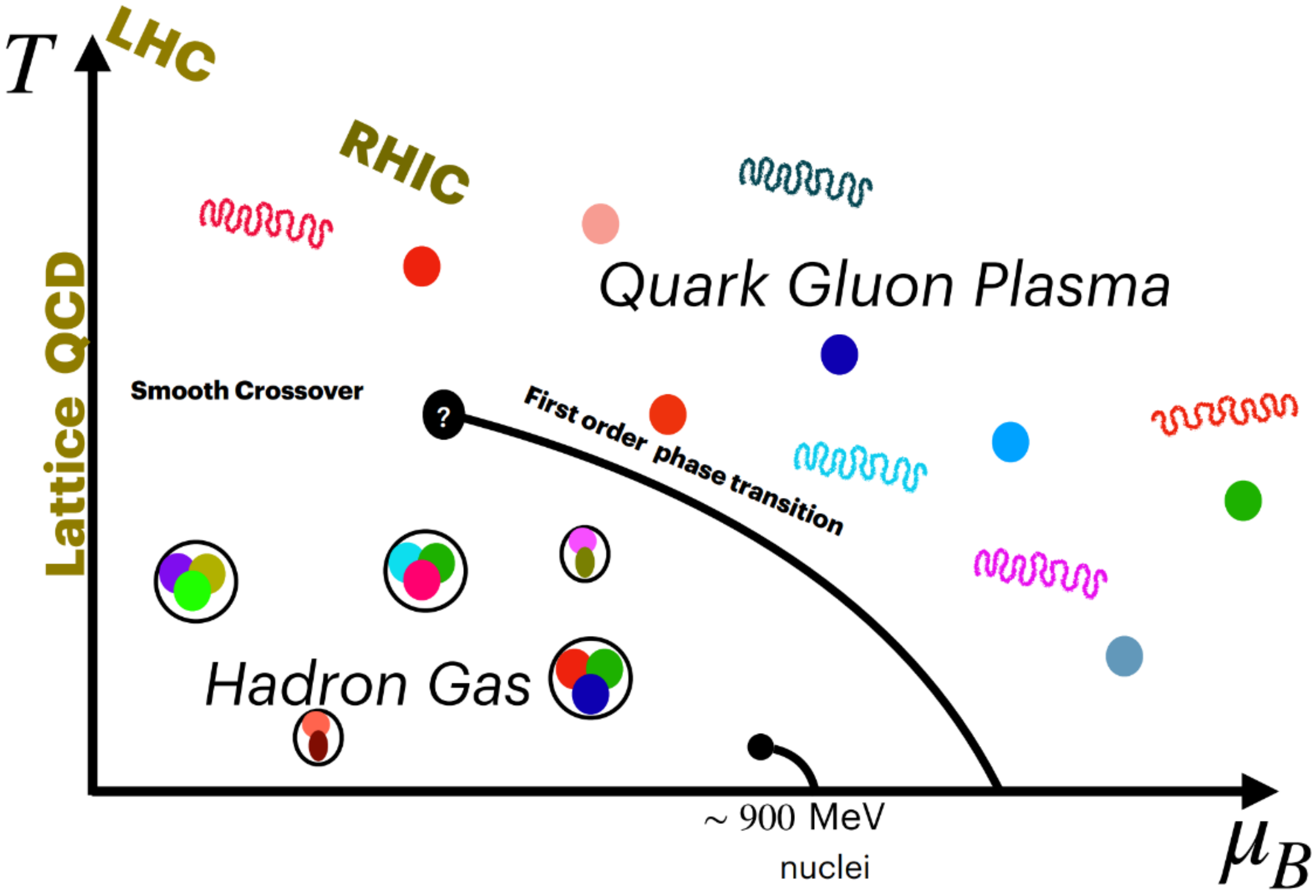


QCD Phase Diagram

- Deconfined Quark Gluon Plasma
- Analytic Crossover at

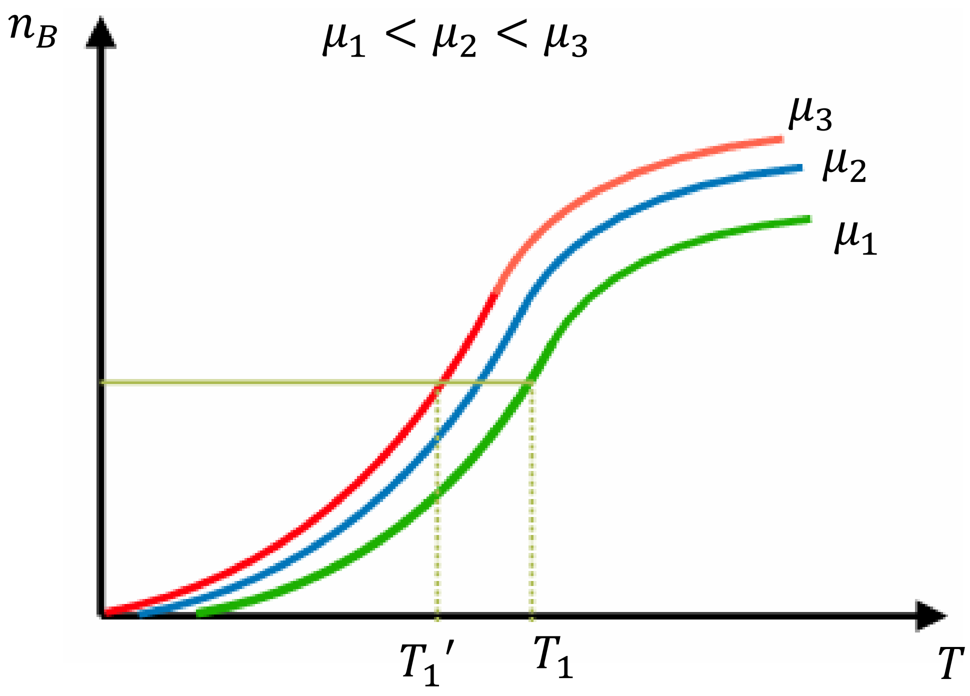
Vanishing chemical potential

- Nuclear Liquid - gas phase transition
- Is there a critical point and a first order phase transition? If yes, where?



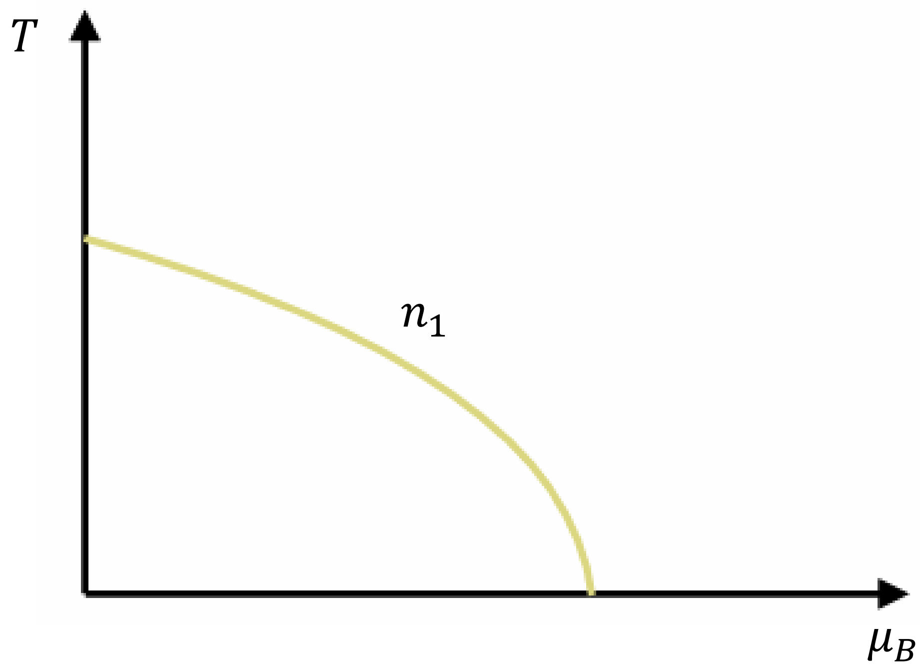
Using Contours to understand the phase transitions

No phase transition



$T_1' < T_1$

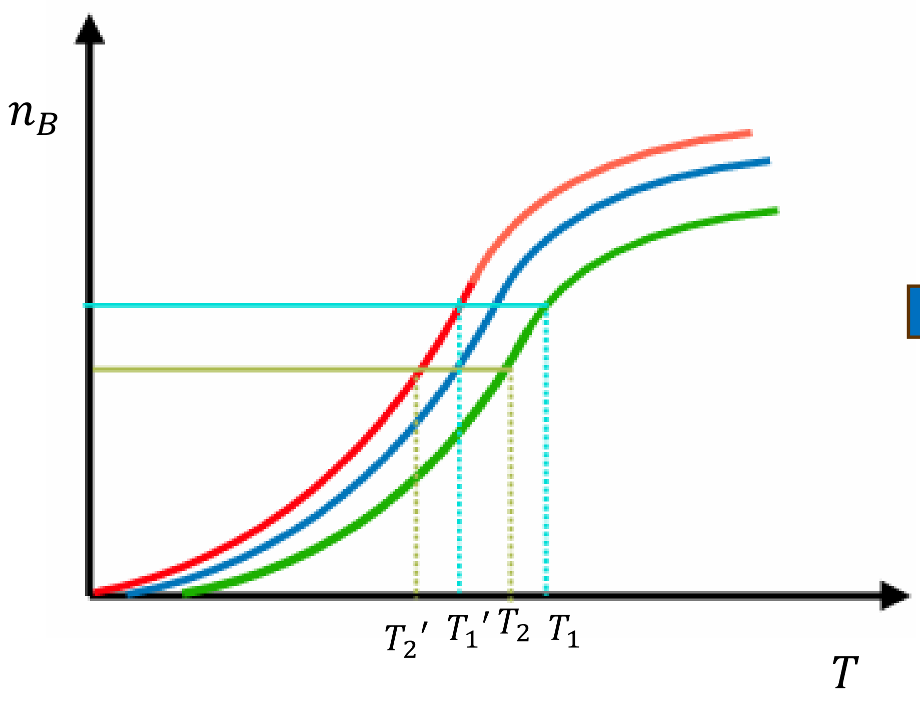
Contours of constant baryon chemical potential



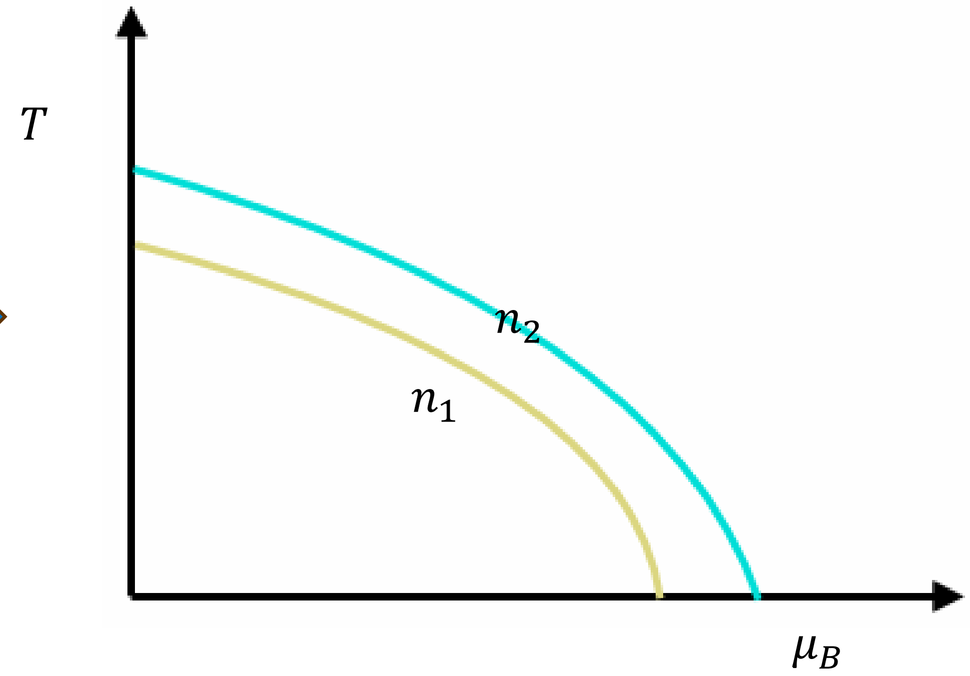
Contours of constant baryon density

Using Contours to understand the phase transitions

No phase transition



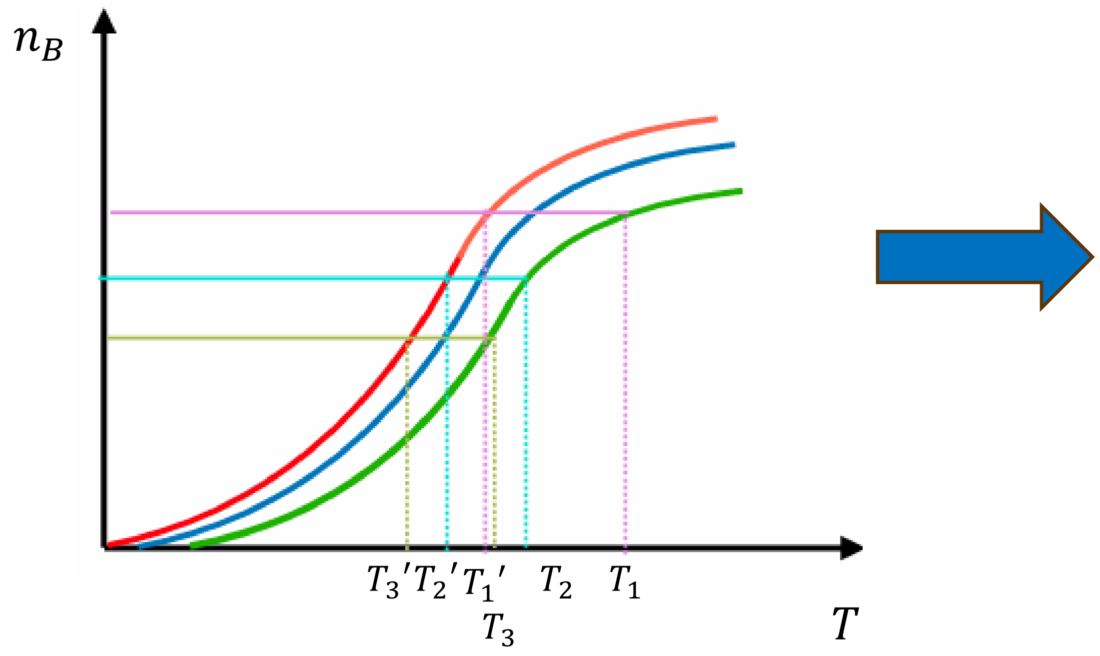
Contours of constant baryon chemical potential

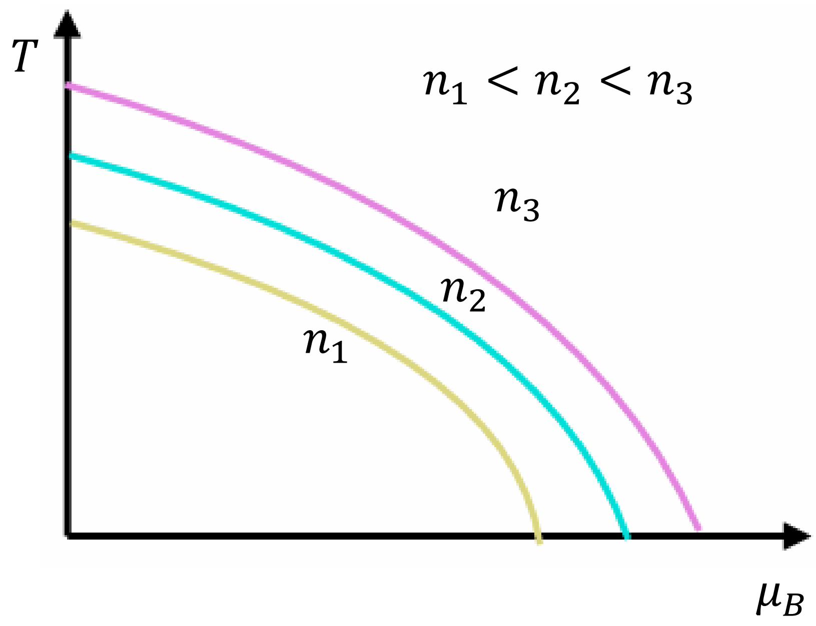


Contours of constant baryon density

Using Contours to understand the phase transitions

No phase transition

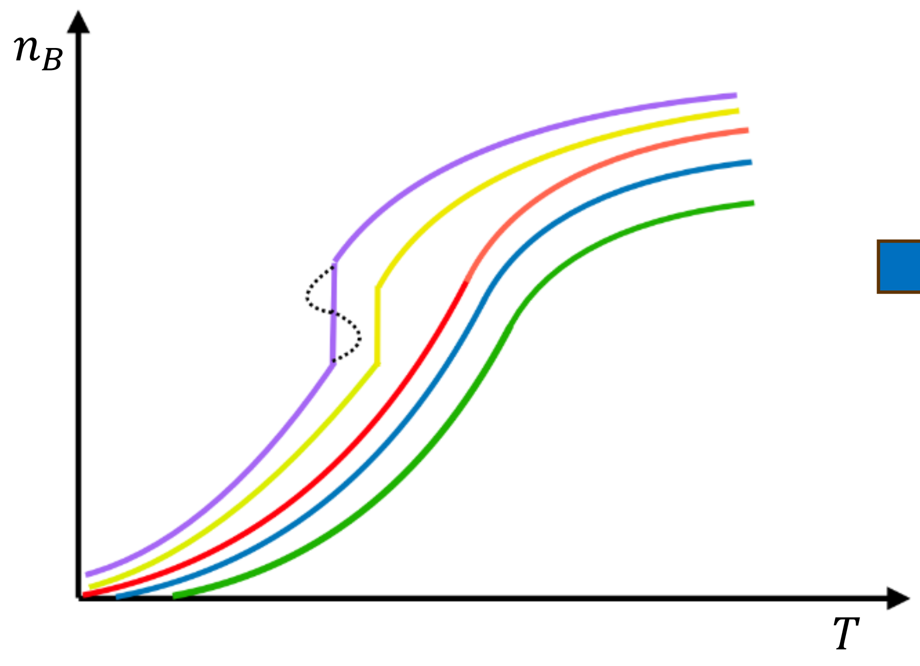




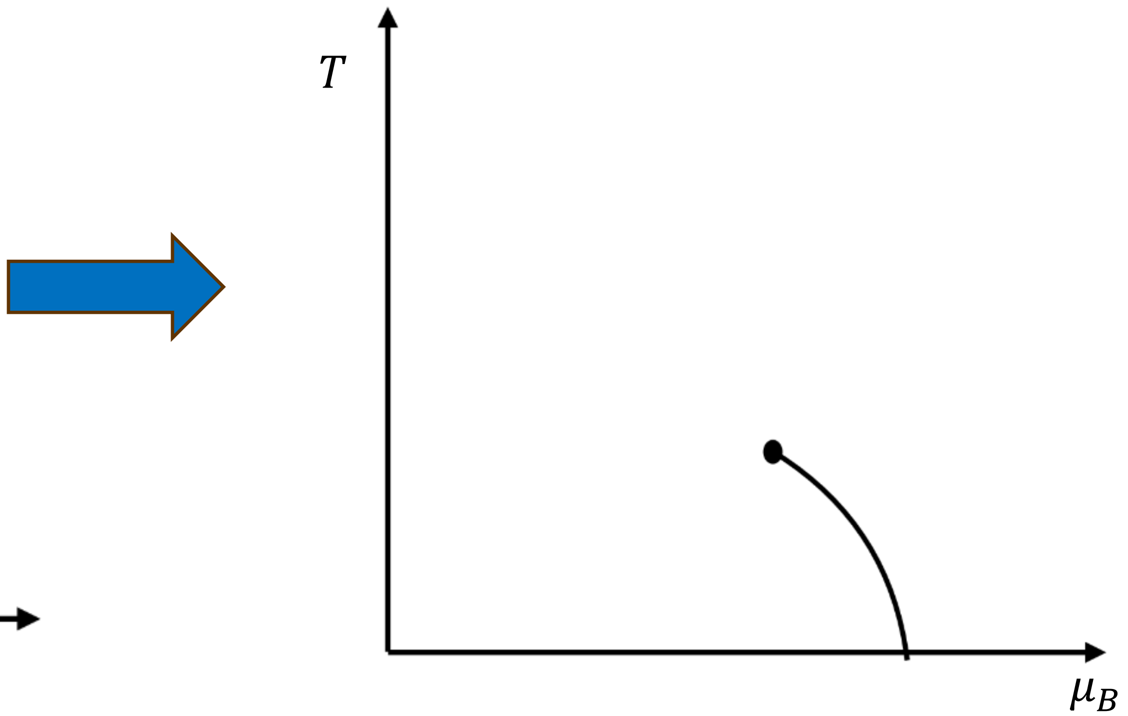
Contours of constant baryon chemical potential

Contours of constant baryon density

Using Contours to understand the phase transitions

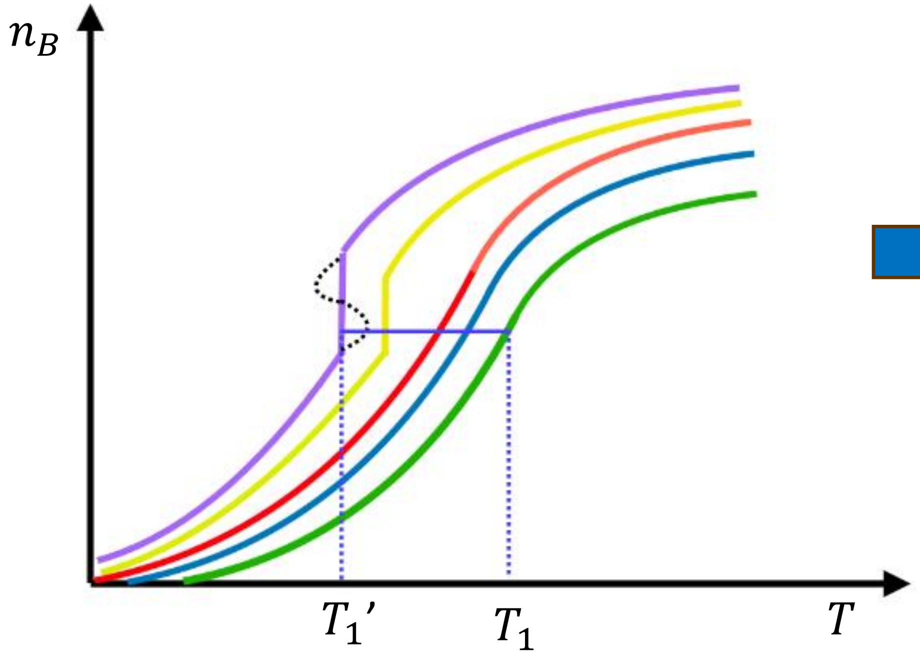


Contours of constant baryon chemical potential

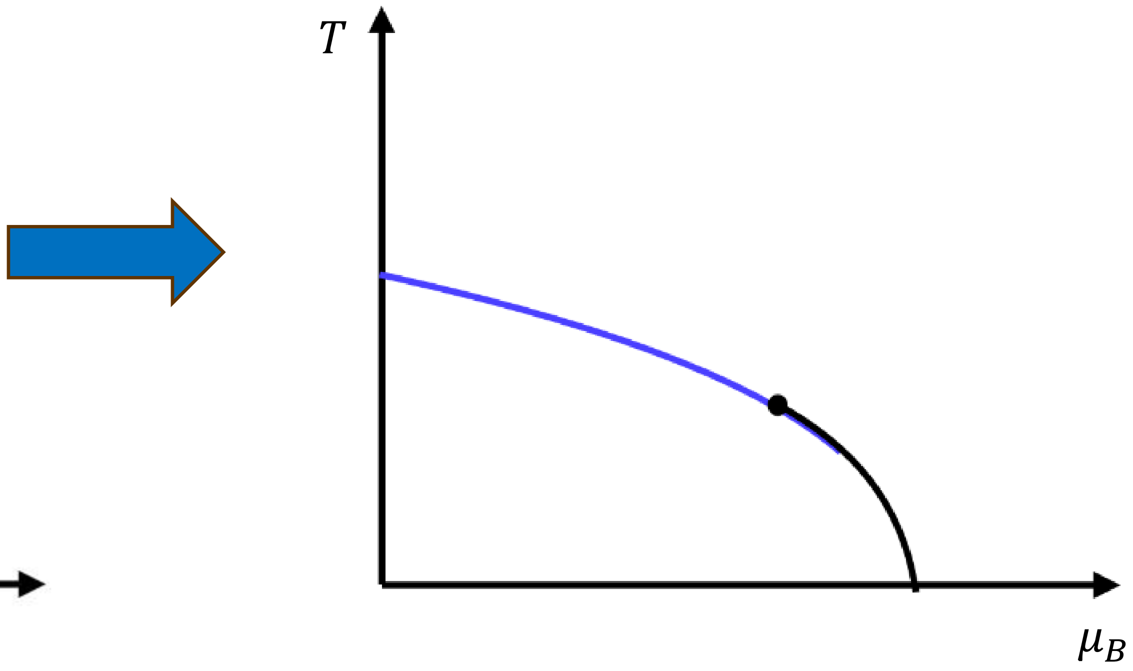


Contours of constant baryon density

Using Contours to understand the phase transitions

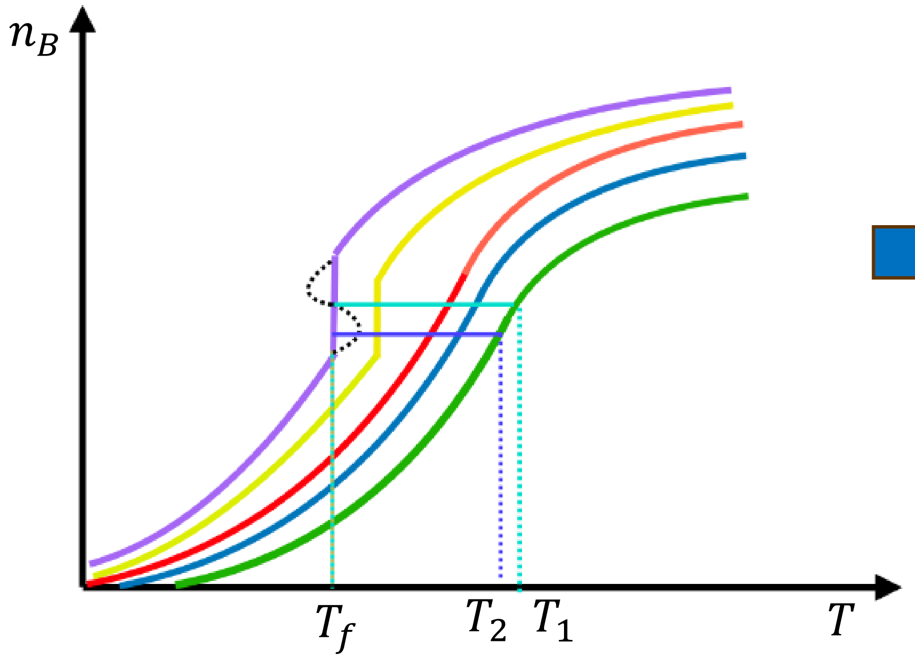


Contours of constant baryon chemical potential

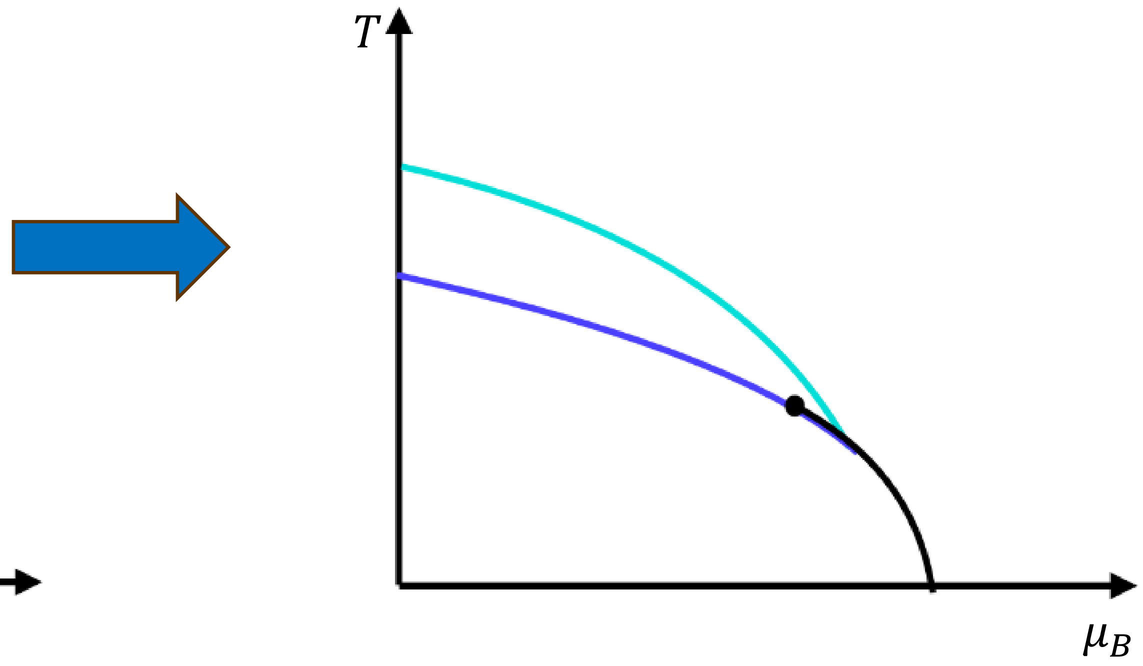


Contours of constant baryon density

Using Contours to understand the phase transitions



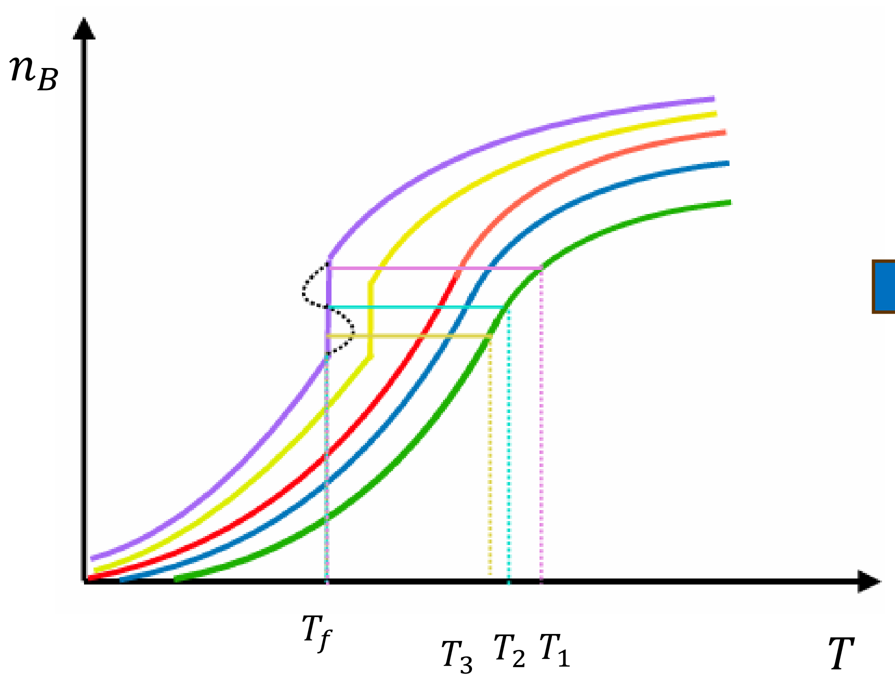
Contours of constant baryon chemical potential



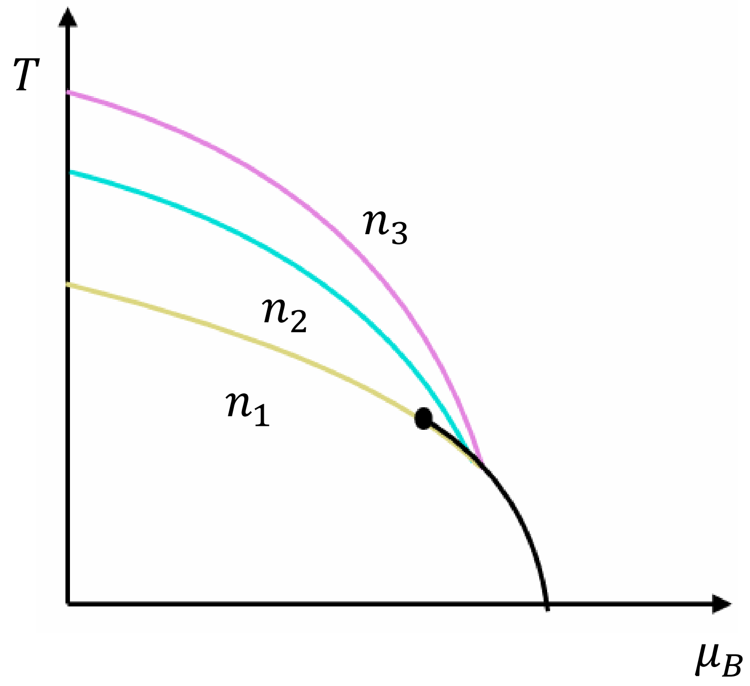
Contours of constant baryon density

Using Contours to understand the phase transitions

First Order Phase Transition



Contours of constant baryon chemical potential



Contours of constant baryon density

Extrapolation

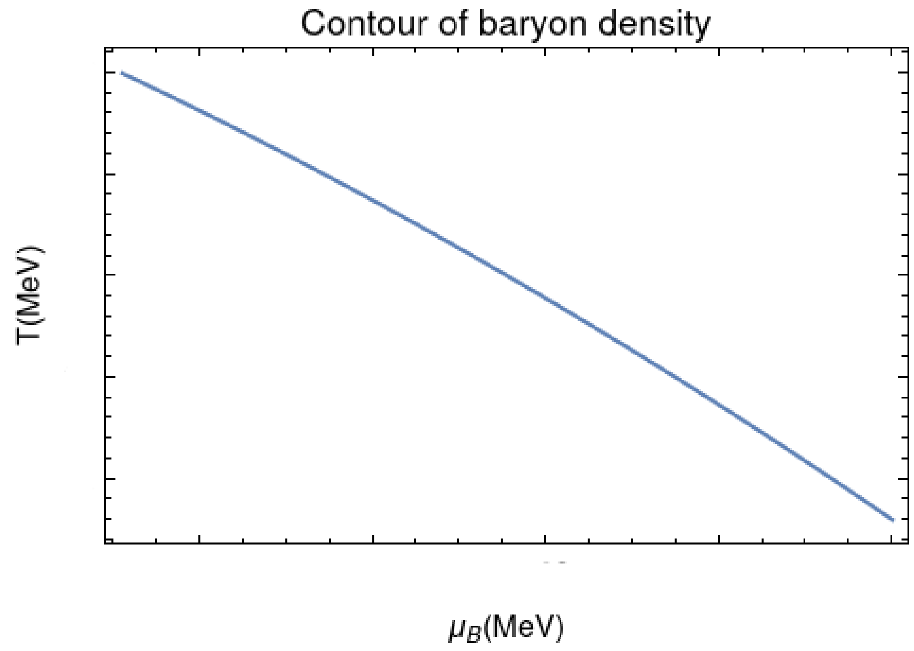
- Studying contours works if we have the entire EoS.
- We only have EoS on a vertical line of $\mu_B = 0$
- So what do we do? Extrapolate!

HOW?

Now, to extrapolate, we first analyze the contour. For this contour,

$$\frac{dn}{d\mu} = \frac{\partial n}{\partial \mu}|_T + \frac{\partial n}{\partial T}|_\mu \cdot \frac{dT}{d\mu} = 0$$

Thus, slope $F = \frac{dT}{d\mu} = -\frac{\frac{\partial n}{\partial \mu}}{\frac{\partial n}{\partial T}}$



$$T_i = T_0 + F(T_0, \mu_0) * (\mu_i - \mu_0) + \frac{dF(T_0, \mu_0)}{d\mu} * \frac{(\mu_i - \mu_0)^2}{2} + \frac{d^2F(T_0, \mu_0)}{d\mu^2} * \frac{(\mu_i - \mu_0)^3}{6}$$

Van der Waals model in CE (Single particle)

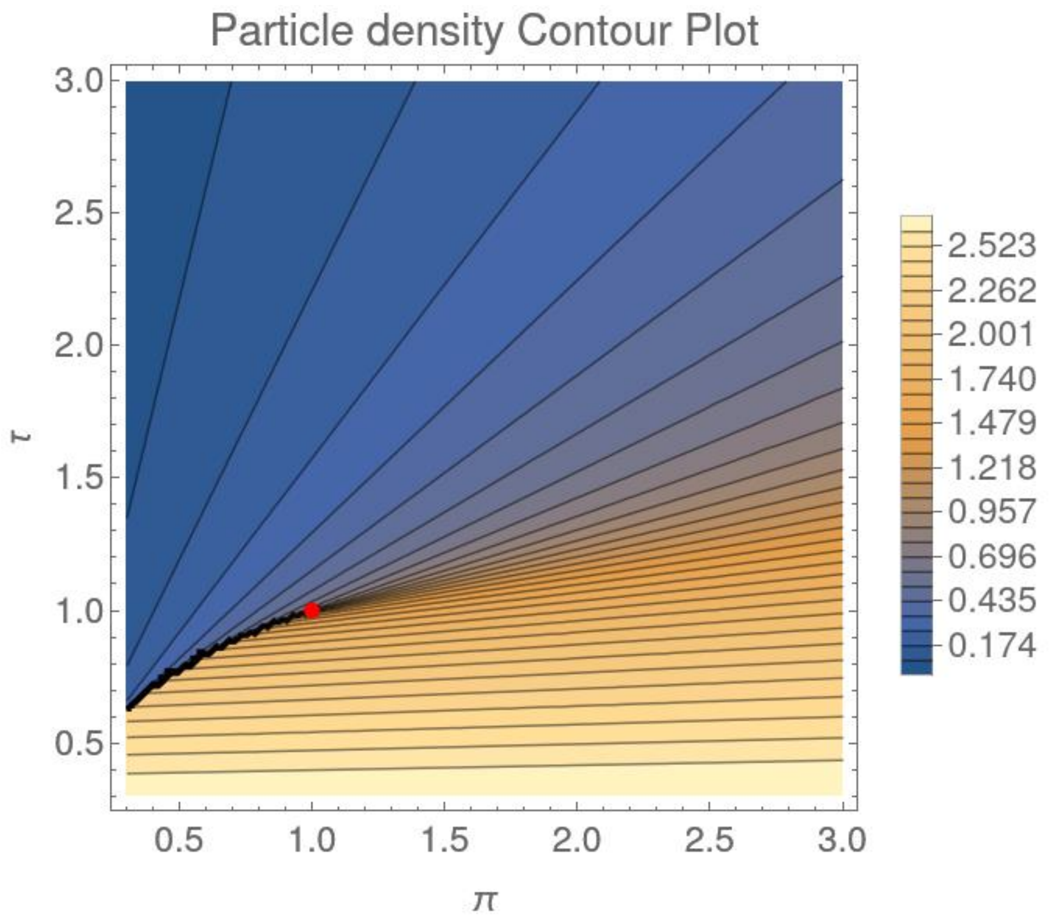
- Particles are treated as hard spheres.
- Repulsive and attractive interactions are present between the particles.
- VdW EoS has a liquid gas phase transition in the EoS.

- The equation of state is given as follows:

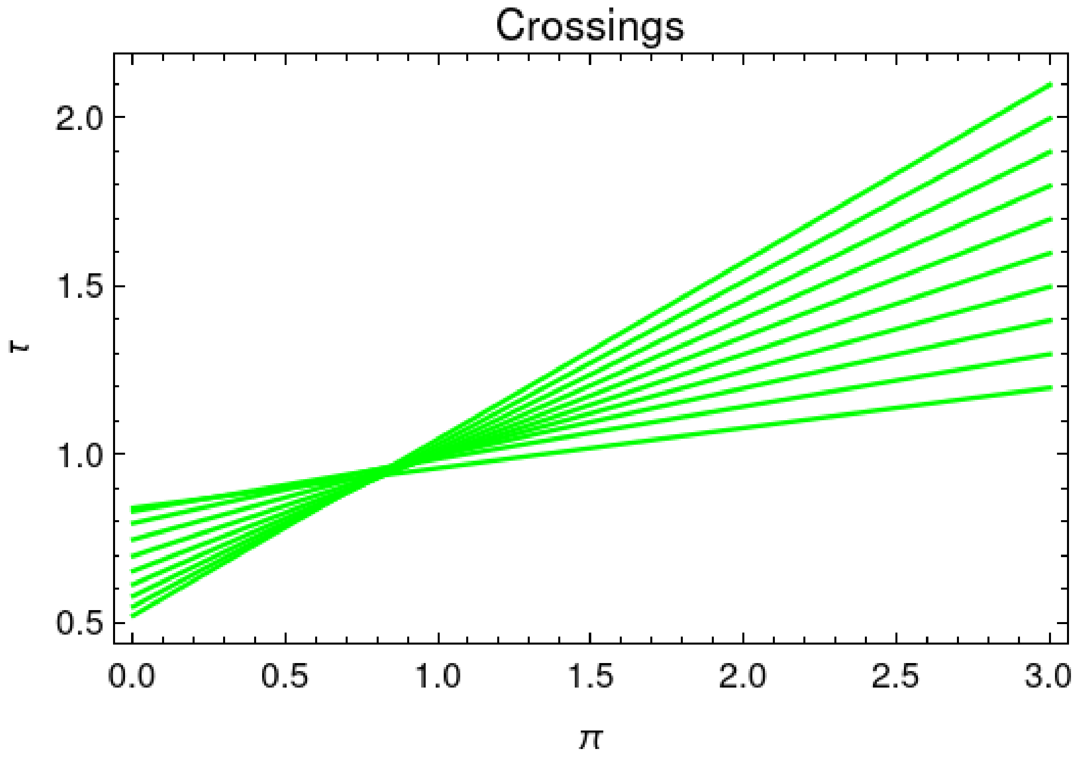
$$(P + \frac{a}{v^2})(v - b) = T \qquad \text{where } v = \frac{V}{N}$$

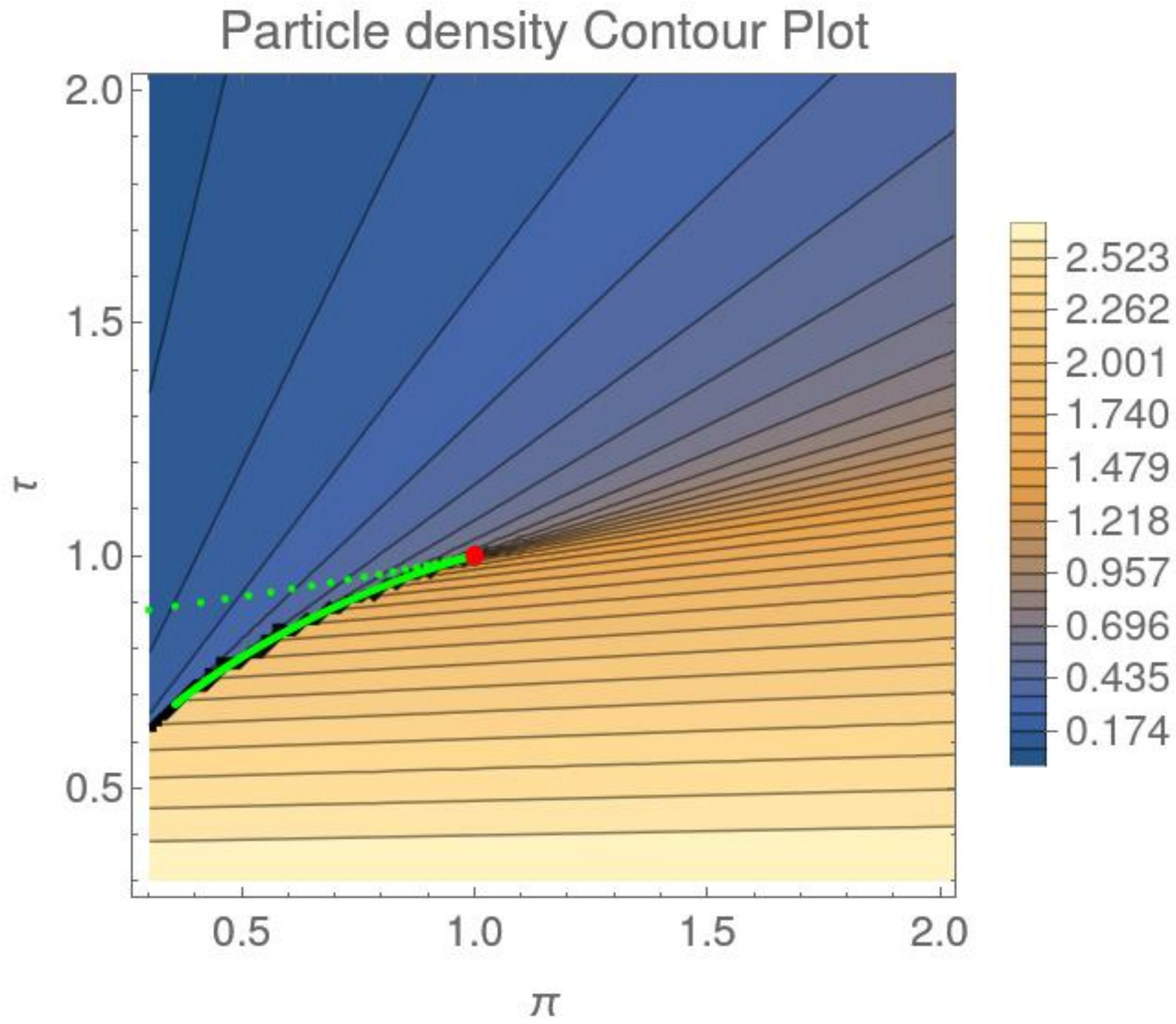
In dimensionless variables, where $\tau = \frac{T}{T_c}, \pi = \frac{P}{P_c}, \eta = \frac{n}{n_c}$

The critical point is at $(\pi, \tau) = (1,1)$.



Crossings


Crossings
2.0
1.5
1.0
0.5
τ
0.0
0.5
1.0
1.5
2.0
2.5
3.0
π

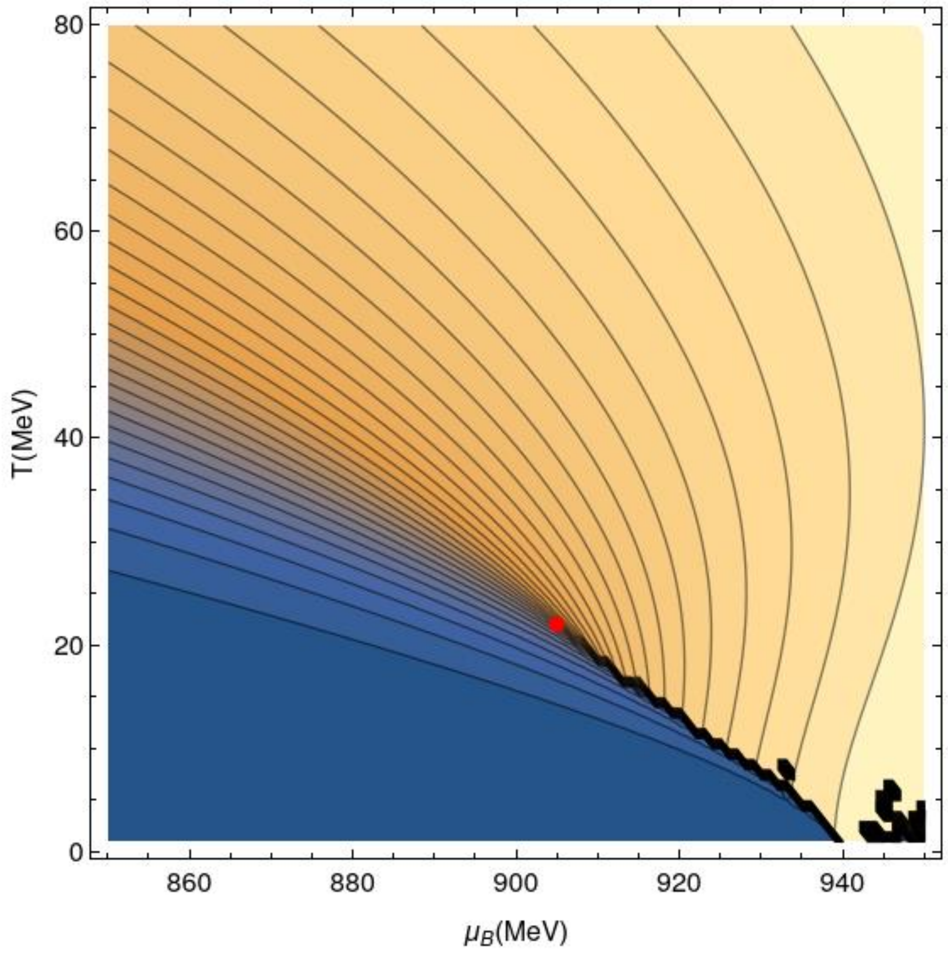

Particle density Contour Plot
2.0
1.5
1.0
0.5
τ
0.5
1.0
1.5
2.0
π
2.523
2.262
2.001
1.740
1.479
1.218
0.957
0.696
0.435
0.174

Quantum VdW in GCE

- We switch from Canonical Ensemble to Grand canonical ensemble
- Baryon chemical potential μ_B becomes a control parameter instead of pressure.
- Equation of state can be now solved using the transcendental equation below:

$$n(T,\mu) = \frac{n^{\rm id}(T,\mu^*)}{1 + b\, n^{\rm id}(T,\mu^*)}, \qquad \mu^* = \mu - T\frac{bn}{1-bn} + 2an$$

a = 329 MeV fm^3 b=3.42 fm^3



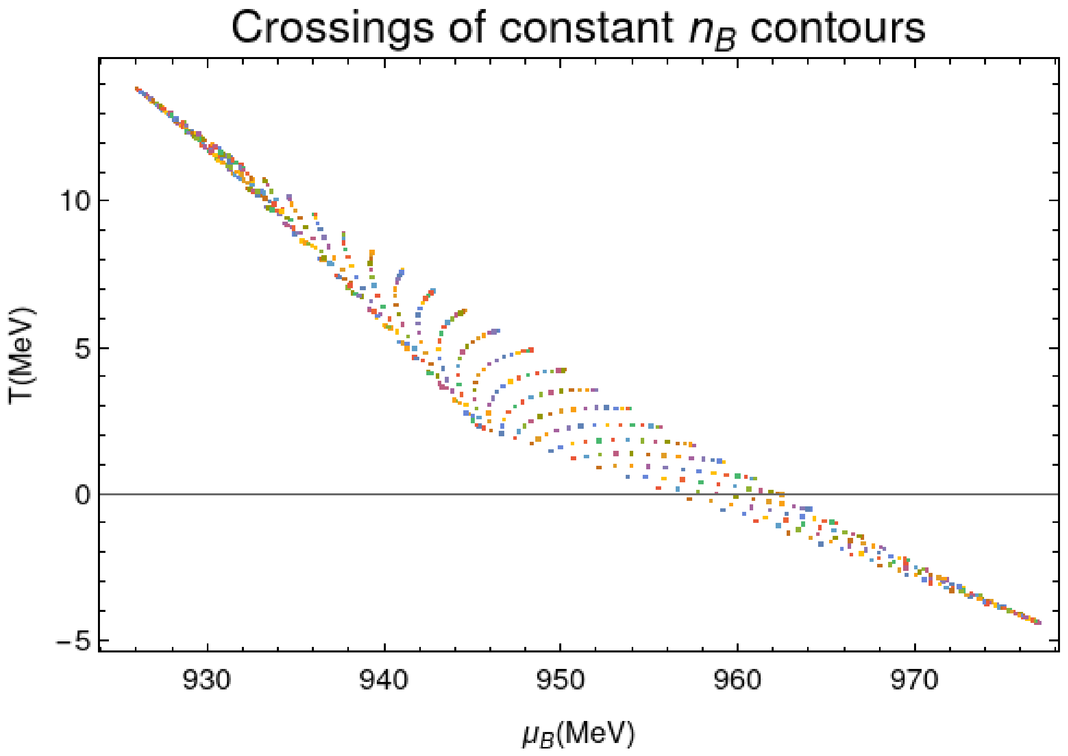
- We only have a proton and neutron in the system currently.

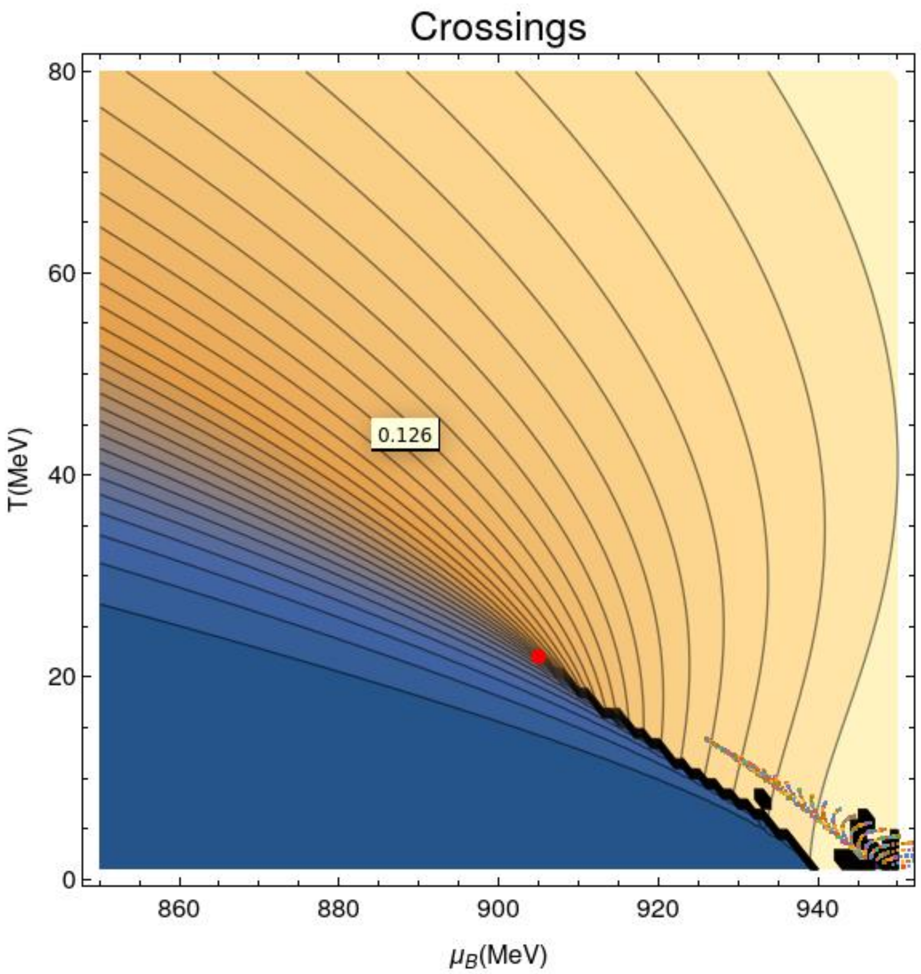
V. Vovchenko et.al., D. V. Anchishkin et. al and M. I. Gorenstein et. al. (2015)

Crossings $\mu_0 = 851MeV$

Actual Critical point :- ($\mu_{Bc} = 903\ MeV, T_c = 21\ MeV$)

Estimated Critical Point:-($\mu_{Bc} = 925MeV, T_c = 15\ MeV$)



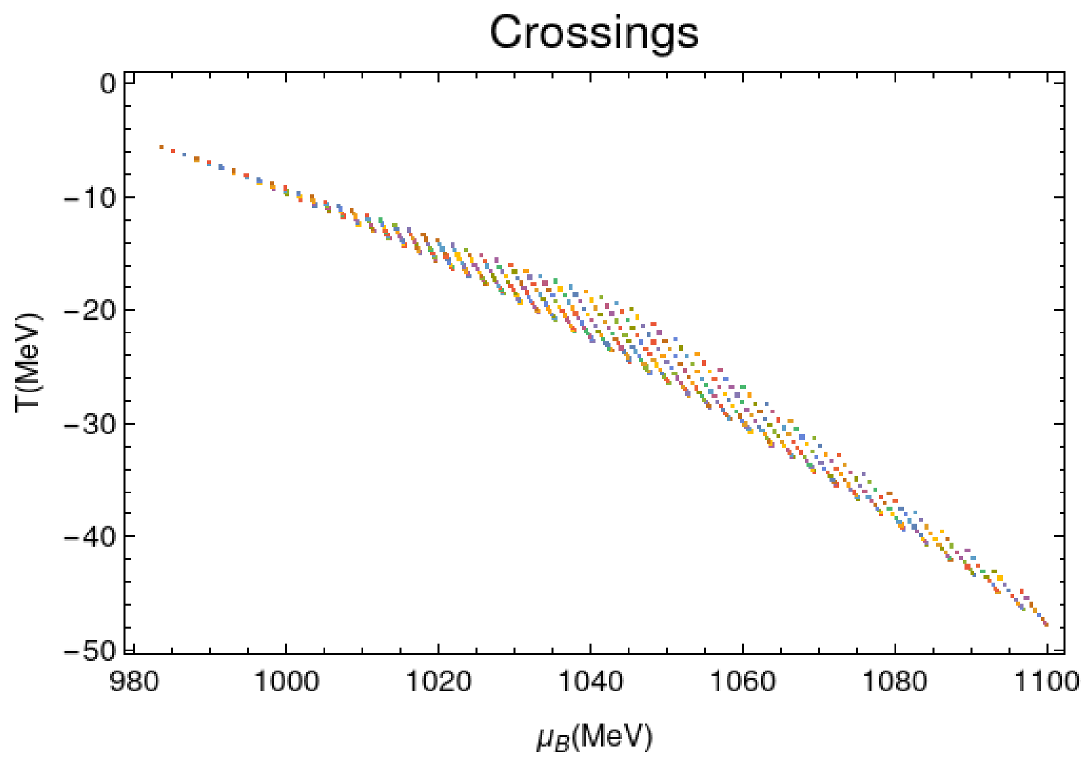


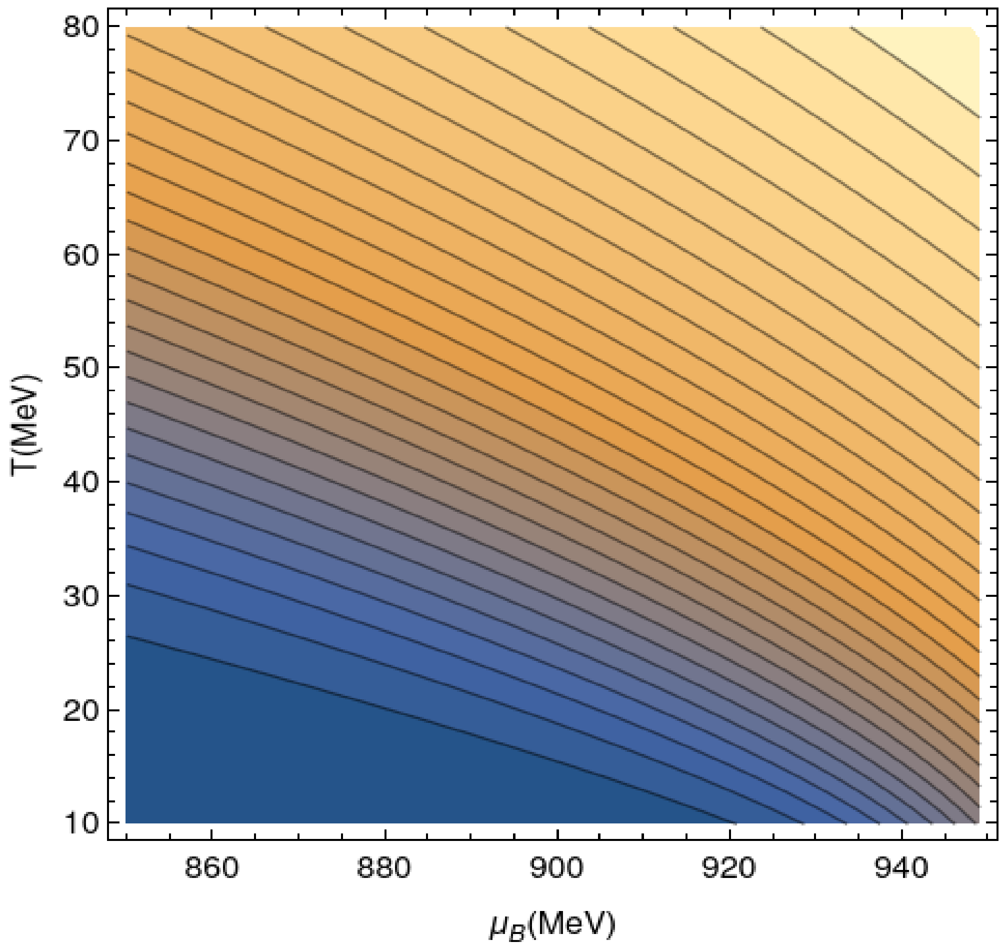
V. Vovchenko, H. Stoecker, *Thermal-FIST:*, Comput. Phys. Commun. **244**, 295 (2019)

Excluded Volume model

a = 0 MeV fm^3 b=3.42 fm^3

No attractive interactions, no phase transition





Outlook

- The extrapolation of the baryon density contours provides us whether there is a critical point or a first order phase transition in the system or not.
- Works in VdW CE model and gives the correct critical point.
- Gives no crossing for excluded volume model, while estimates a critical point for VdW GCE model.
- What next?
- Look into contours of entropy.
- Change μ_0 and see what happens to the phase transition.
- Look into Holographic Blackhole model next, which gives QCD phase transition.
- Go to Lattice QCD.

BACKUP SLIDES

Extrapolation technique

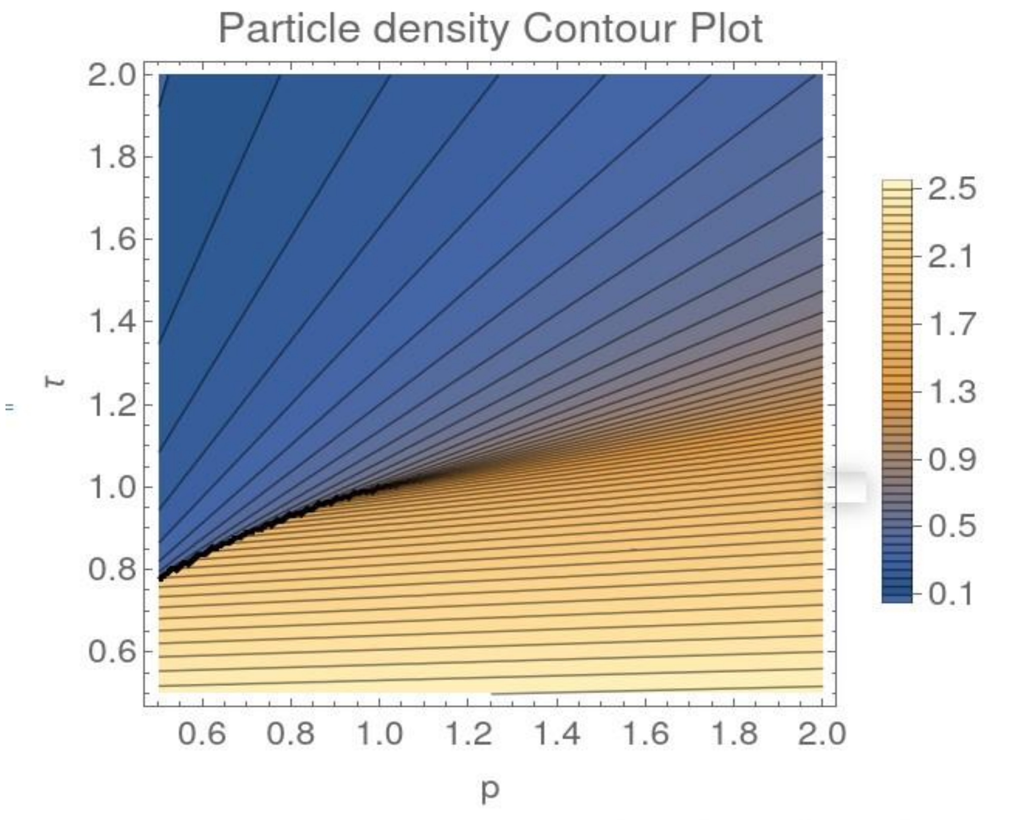
Show theory of extrapolation, crossings and lensing criterion.

$$\Delta n \equiv n_2 - n_1 < 0 \qquad \Delta \tau \equiv \tau_2 - \tau_1 > 0$$

$$\frac{dn_1}{d\pi} = \left.\frac{\partial n_1}{\partial \pi}\right|_{\tau_1} + \left.\frac{\partial n_1}{\partial \tau}\right|_{\tau_1} \frac{d\tau_1}{d\pi} = 0$$

$$\frac{d\tau}{d\pi} = -\frac{\partial \eta/\partial \pi}{\partial \eta/\partial \tau} = F(\tau, \pi)$$

$$\tau_j = \tau_0 + F(\tau_i, \pi_0) \cdot (\pi - \pi_0)$$



Ideal Hadron Resonance Gas Model

- Nucleons and Resonances are considered as point particles
- No interactions between the particles

The partition function for an ideal HRG is shown as:

$$\ln Z^{HRG}(T, V, \boldsymbol{\mu}) = \sum_{i \in PDG} \ln Z_i(T, V, \boldsymbol{\mu}) \quad = \frac{V g_i}{2\pi^2} \int_0^\infty \pm p^2 dp \ln\left[1 \pm \lambda_i(T, \boldsymbol{\mu}) \exp(-\beta \epsilon_i)\right]$$

In the GCE, the pressure is given as follows:

$$p^{\mathrm{id}}(T, \mu) = \frac{d}{3} \int \frac{d^3k}{(2\pi)^3} \frac{\boldsymbol{k}^2}{\sqrt{m^2 + \boldsymbol{k}^2}} \left[\exp\left(\frac{\sqrt{m^2 + \boldsymbol{k}^2} - \mu}{T}\right) + \eta\right]^{-1}$$

Ratti, C., & Belwied, R. (2020). The Deconfinement Transition of QCD.

Van der Waals HRG Model

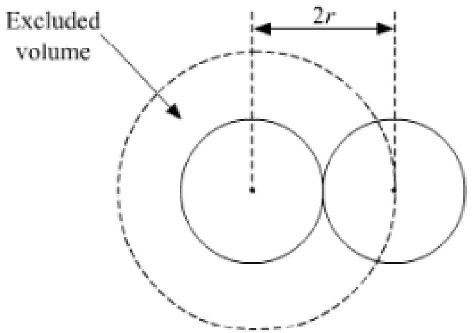
- Instead of point particles, we consider nucleons and resonances as hard spheres
- Describes the pressure function in equilibrium systems of particles with both repulsive and attractive interactions.
- Equation predicts the existence of a first-order phase transition and contains a critical point.

$$p(T,n) = \frac{NT}{V-bN} - a\frac{N^2}{V^2} \equiv \frac{nT}{1-bn} - a\,n^2$$

→ VdW equation in Canonical Ensemble

- Short-range repulsion: particles are hard spheres

$V \rightarrow V - \mathrm{bN} \quad b = 4\pi r^3/3$



- Attractive interactions in mean-field approximation

$P \rightarrow P - an^2$

V. Vovchenko et.al., D. V. Anchishkin et. al and M. I. Gorenstein et. al. (2015)

Quantum Van der Waals HRG

$$p^{\rm id}(T,\mu) \;=\; \frac{d}{3}\int \frac{d^3k}{(2\pi)^3}\,\frac{\boldsymbol{k}^2}{\sqrt{m^2+\boldsymbol{k}^2}}\left[\exp\left(\frac{\sqrt{m^2+\boldsymbol{k}^2}-\mu}{T}\right)+\eta\right]^{-1}$$

$$p(T,\mu) \;=\; p^{\rm id}(T,\mu^*) - a\,n^2$$

- η equals +1 for Fermi statistics, -1 for Bose statistics, and 0 for the Boltzmann approximation

- The VdW equation of state in the GCE is obtained in the form of a transcendental equation for particle number density n ≡ n(T, μ) as a function of T and μ

$$n(T,\mu) \;=\; \frac{n^{\rm id}(T,\mu^*)}{1\;+\;b\,n^{\rm id}(T,\mu^*)}\,, \qquad \mu^* \;=\; \mu \;-\; T\frac{bn}{1\;-\;bn} \;+\; 2an$$

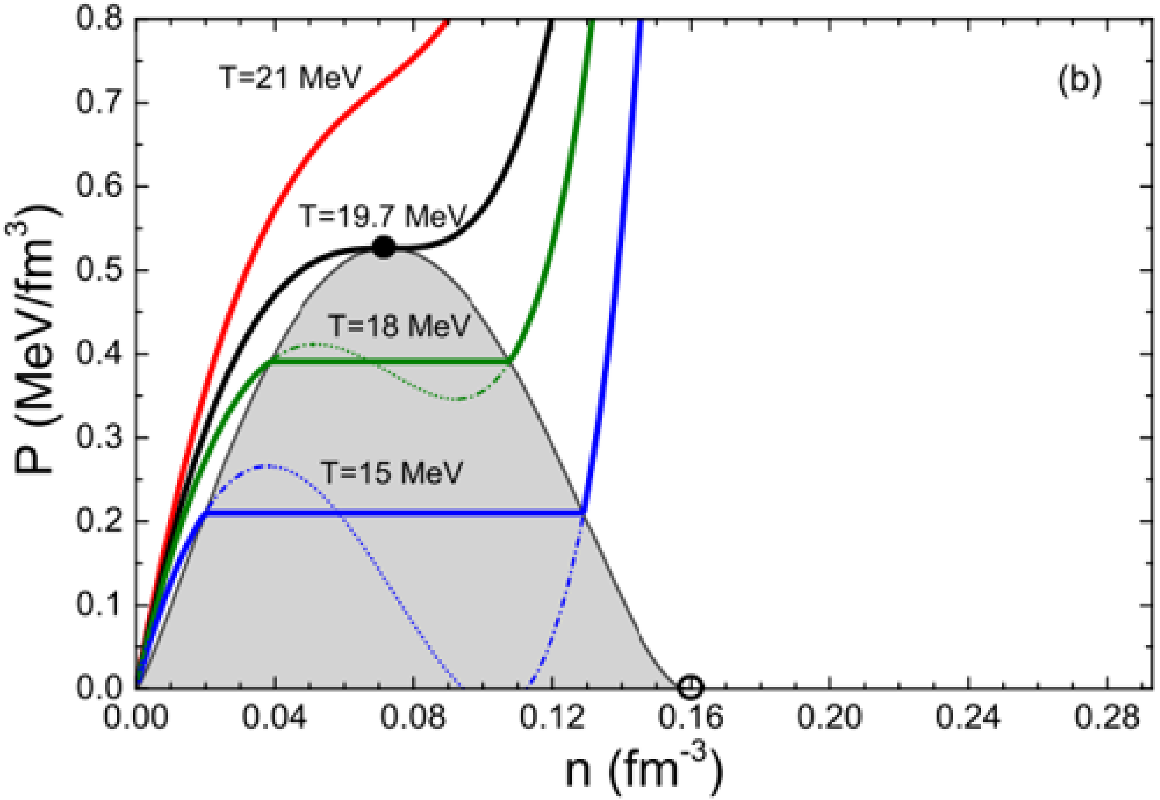
a = 329 MeV fm^3 b=3.42 fm^3

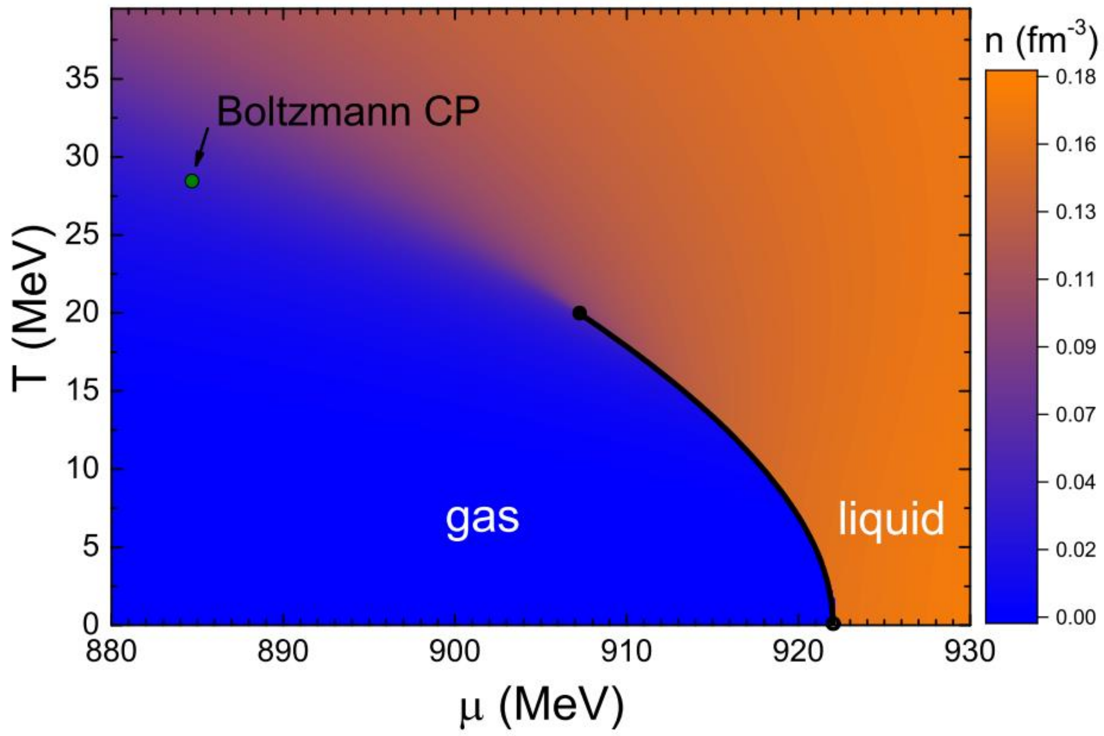
For baryons

- where n_{id} is a particle number density in the ideal Boltzmann gas
- for T< T_c gives multiple solutions , T>T_c gives unique solution

V. Vovchenko et.al., D. V. Anchishkin et. al and M. I. Gorenstein et. al. (2015)

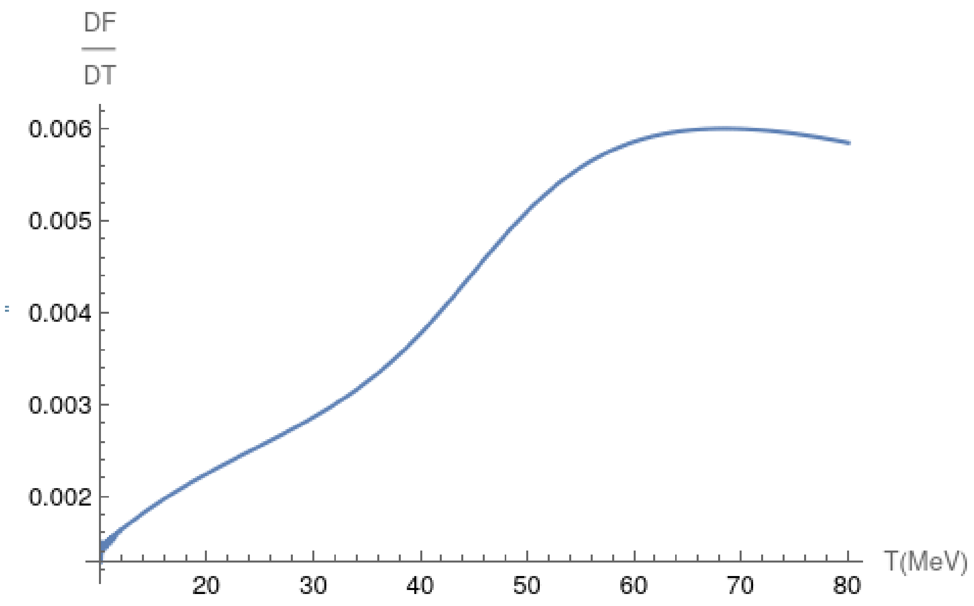
Liquid Gas Phase Transition

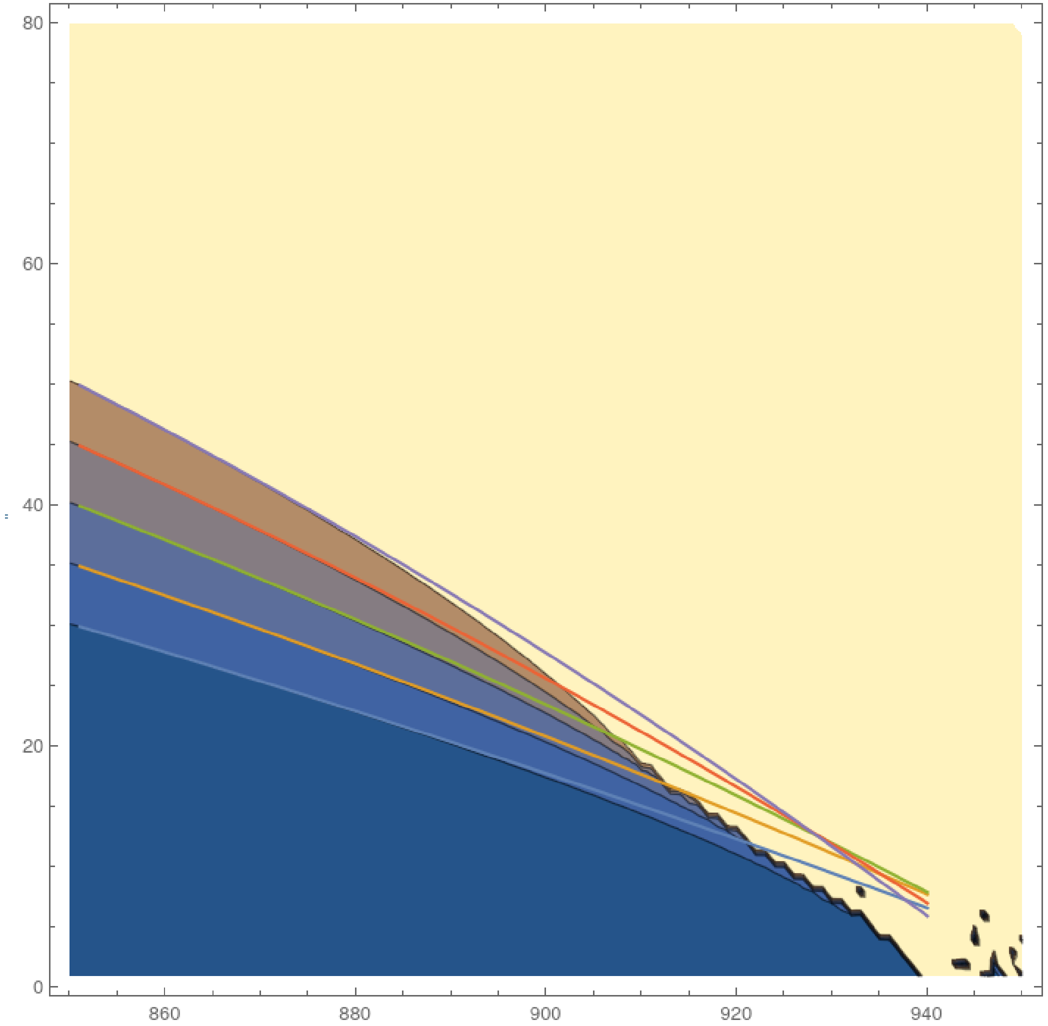


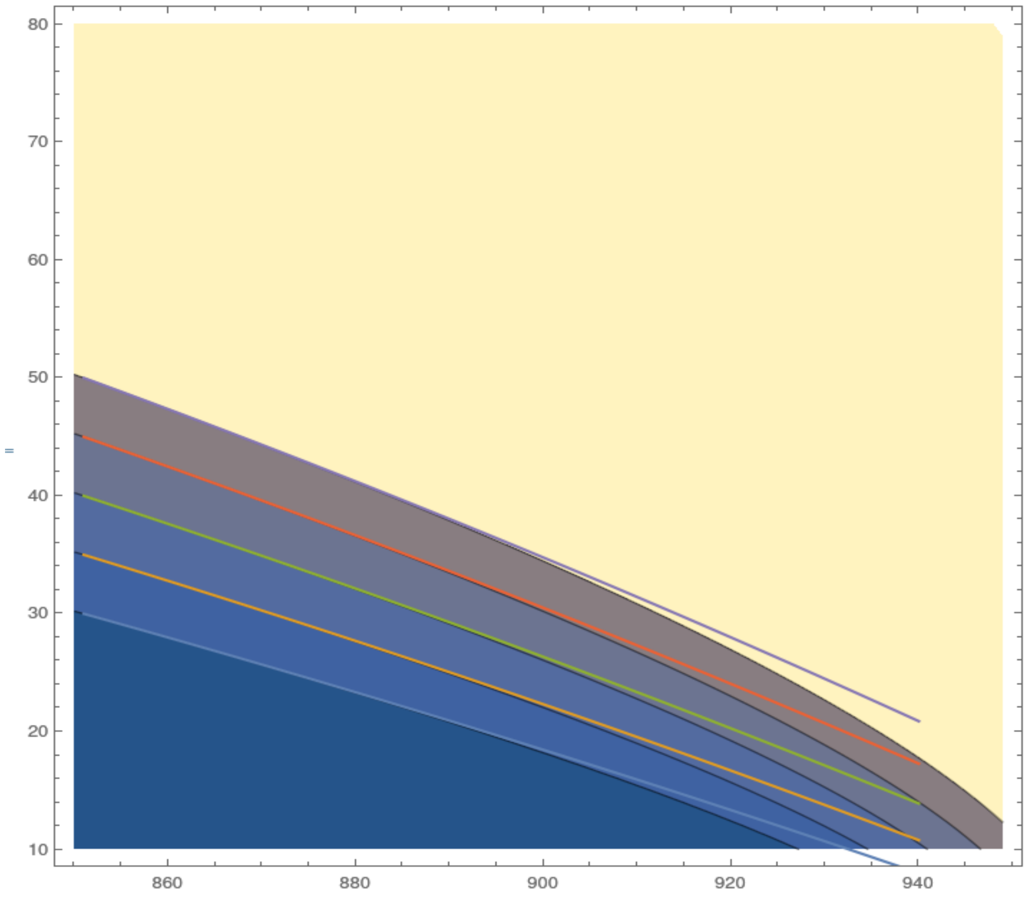


Boltzmann: $T_C = 28.5$ MeV. Classical VDW does not work!

Lensing Criterion and Contours $\mu_0 = 851MeV$






80
70
60
50
40
30
20
10
860
880
900
920
940

New scheme to do 2nd order derivatives

$F = \frac{dT}{d\mu} = -\frac{\left(\frac{\partial n}{\partial \mu}\right)}{\left(\frac{\partial n}{\partial T}\right)} = -\frac{\chi_2^B}{\frac{\partial n}{\partial T}}$ which is the slope and the first order term in our expansion.

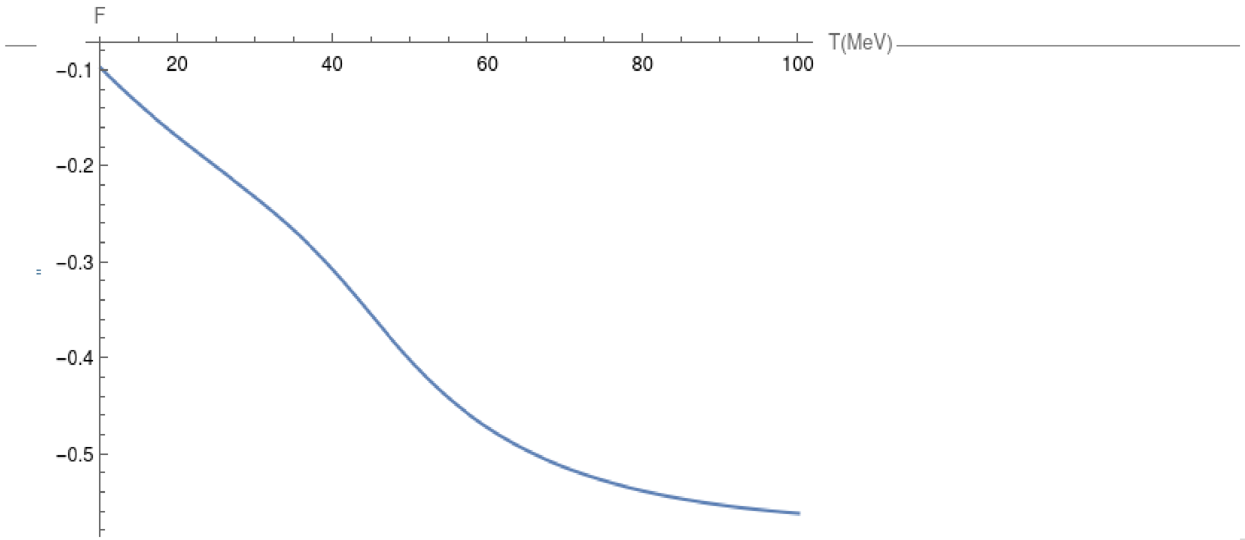
Now, the second order coefficient would be $\frac{dF}{d\mu_B} = \frac{\partial F}{\partial \mu_B} + \frac{\partial F}{\partial T} * F$

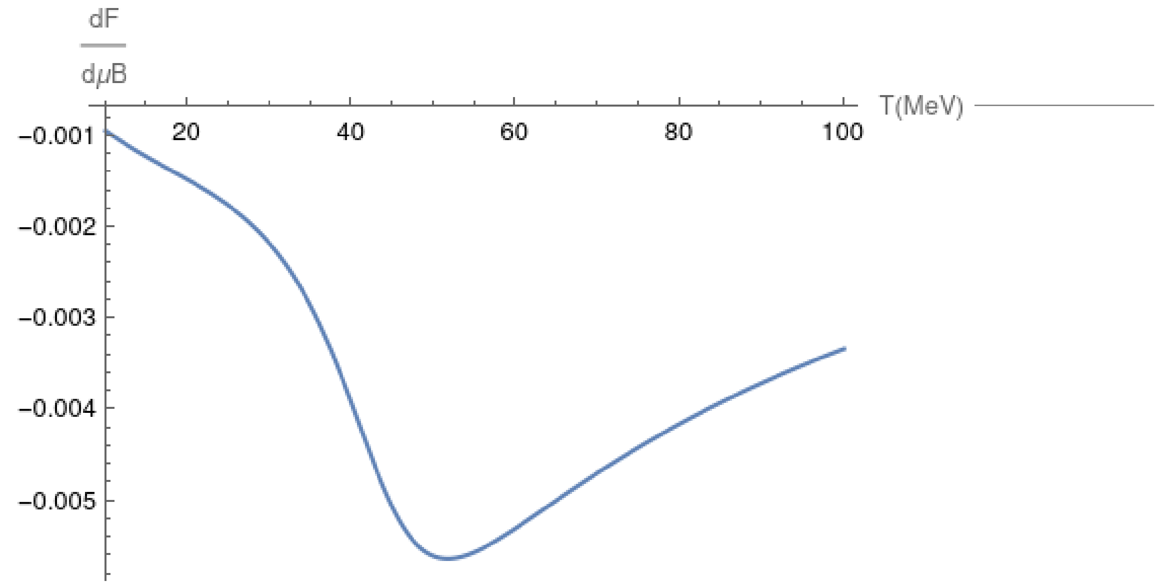
The expansion hence looks like:

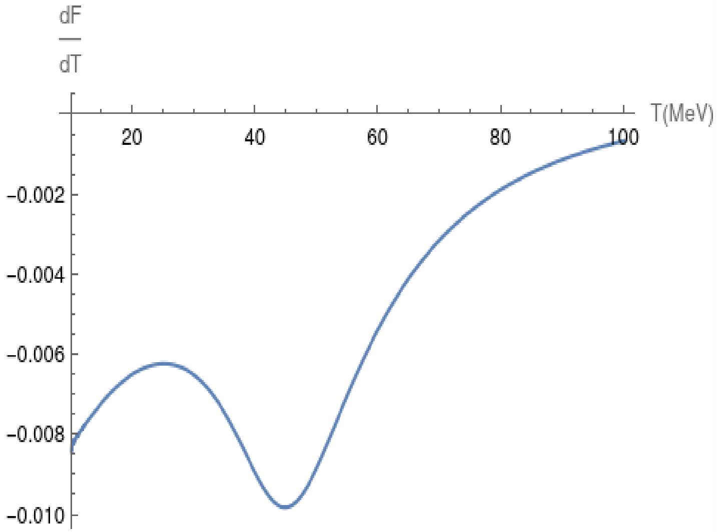
$$T_i = T_0 + F(T_0, \mu_0) * (\mu - \mu_0) + \frac{dF(T_0,\mu_0)}{d\mu} * \frac{(\mu-\mu_0)^2}{2} + \frac{d^2F(T_0,\mu_0)}{d\mu^2} * \frac{(\mu-\mu_0)^3}{6}$$

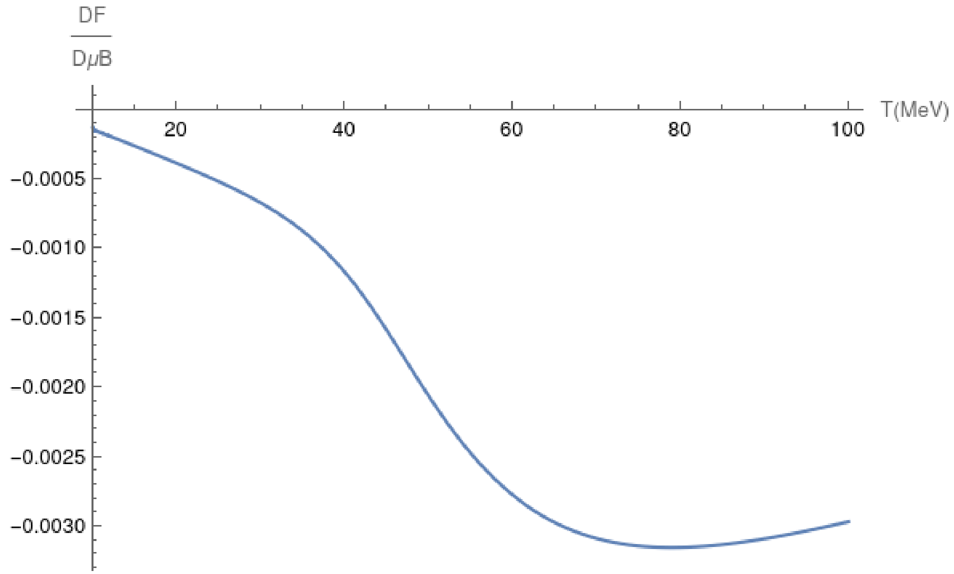
$$\frac{\partial F}{\partial \mu_B} = -\frac{\chi_3^B}{\frac{\partial n}{\partial T}} + \frac{\frac{\partial}{\partial \mu_B}\frac{\partial n}{\partial T}}{\left(\frac{\partial n}{\partial T}\right)^2} = -\frac{\chi_3^B}{\frac{\partial n}{\partial T}} + \frac{\chi_2^B\frac{\partial \chi_2^B}{\partial T}}{\left(\frac{\partial n}{\partial T}\right)^2} \text{ and } \frac{\partial F}{\partial T} = -\frac{\frac{\partial \chi_2^B}{\partial T}}{\frac{\partial n}{\partial T}} + \frac{\frac{\partial}{\partial \mu_B}\frac{\partial n}{\partial T}}{\left(\frac{\partial n}{\partial T}\right)^2} = -\frac{\frac{\partial \chi_2^B}{\partial T}}{\frac{\partial n}{\partial T}} + \frac{\chi_2^B\frac{\partial^2 n}{\partial T^2}}{\left(\frac{\partial n}{\partial T}\right)^2}$$

Increasing Precision VdW

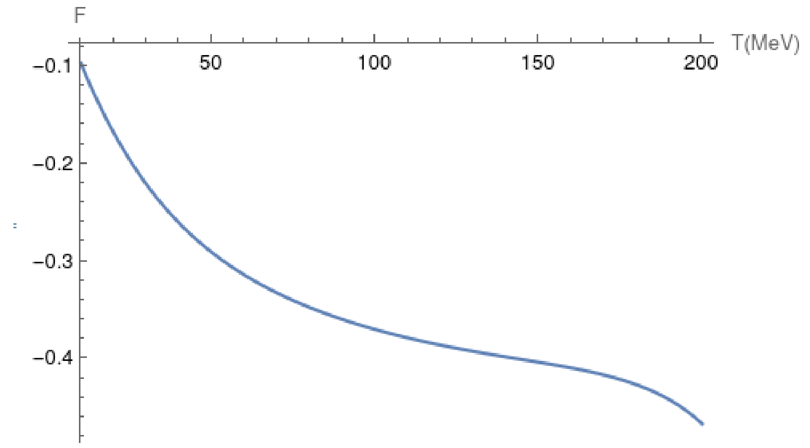


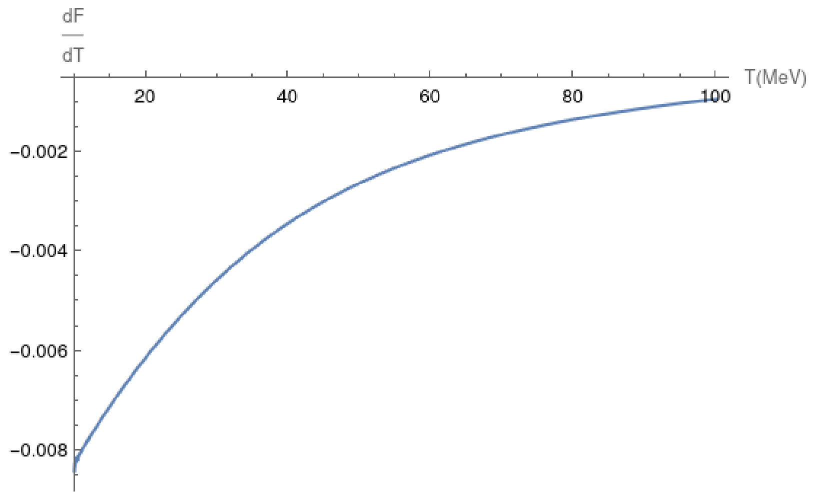


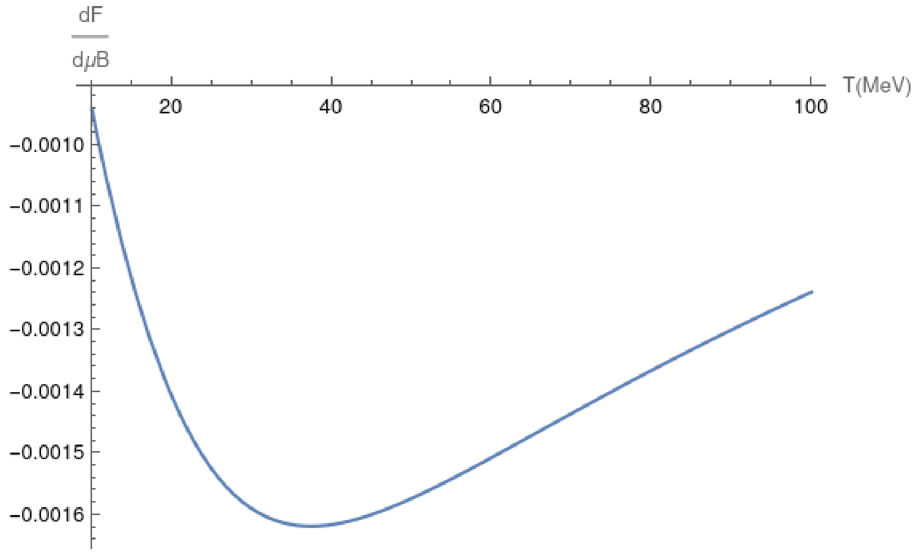


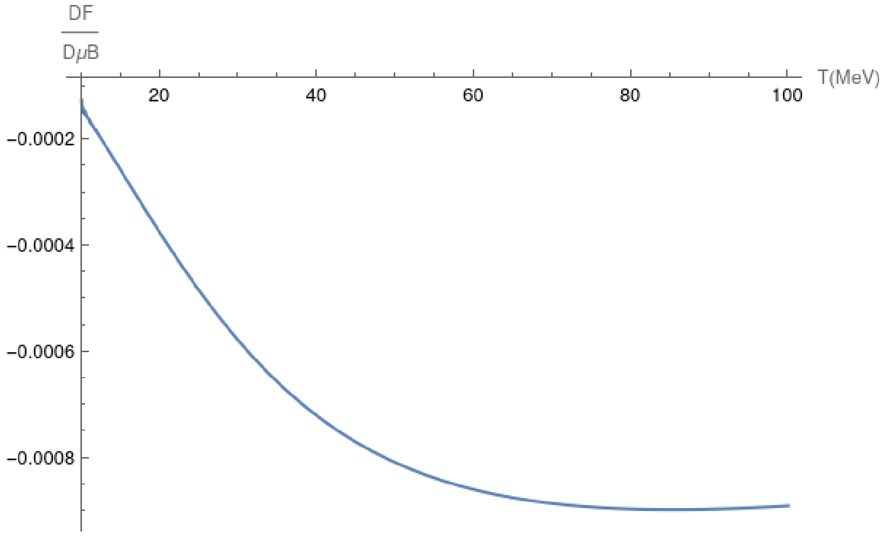


EV p&n

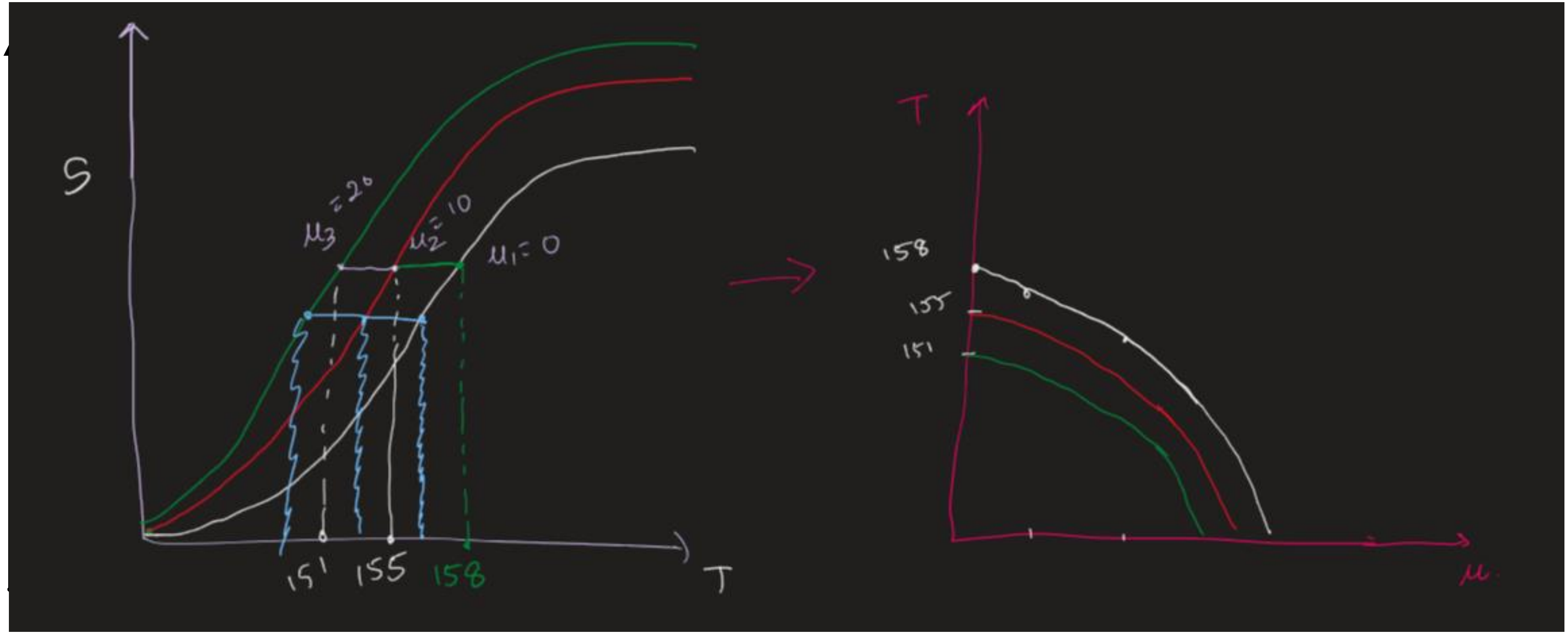

F
T(MeV)
50
100
150
200
−0.1
−0.2
−0.3
−0.4

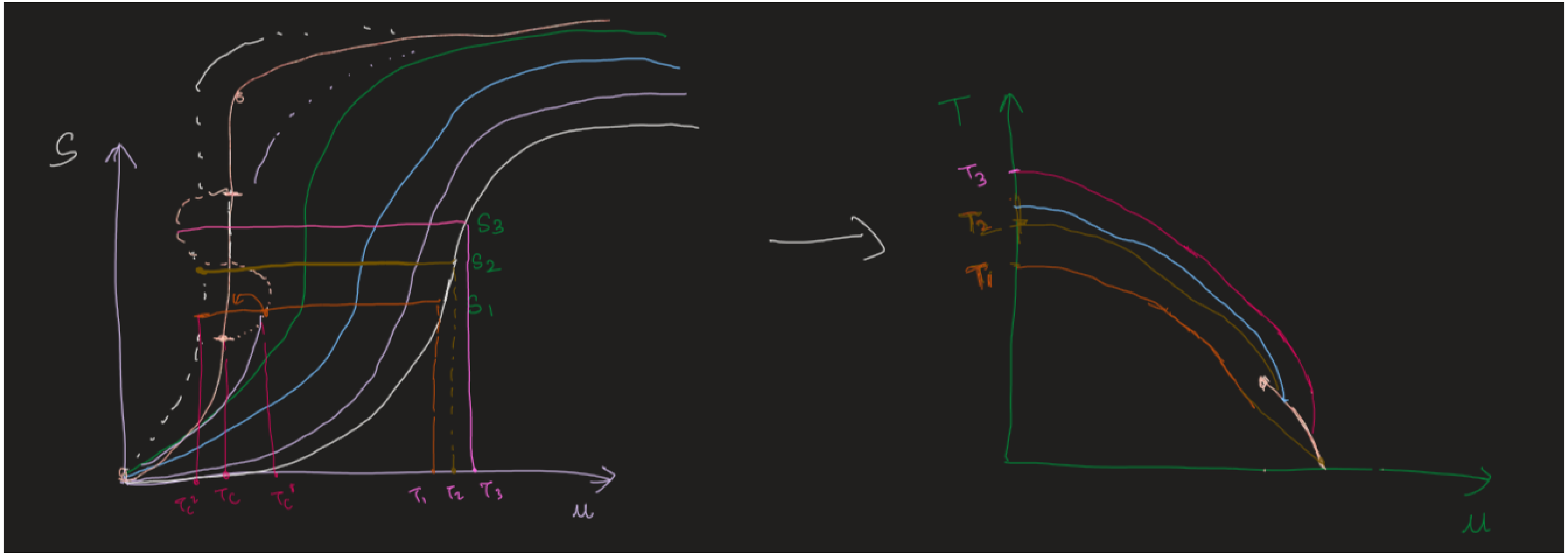

dF
dT
T(MeV)
20
40
60
80
100
−0.002
−0.004
−0.006
−0.008


dF
dμB
T(MeV)
20
40
60
80
100
−0.0010
−0.0011
−0.0012
−0.0013
−0.0014
−0.0015
−0.0016


DF
DμB
T(MeV)
20
40
60
80
100
−0.0002
−0.0004
−0.0006
−0.0008

But, is there a critical point and if yes, where?





S
S3
S2
S1
T1
T2
T3
u
T
T3
T2
T1
u

Outline

- Introduction
  - Quantum Chromodynamics
  - QCD Phase Diagram
- Concept
  - Contours
  - Extrapolation scheme
- Toy Models
  - Van der Waals Classical CE
  - VdW model GCE (p and n)
  - Excluded Volume GCE
- Summary