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Generalized Free Cumulants for Quantum Chaotic Systems

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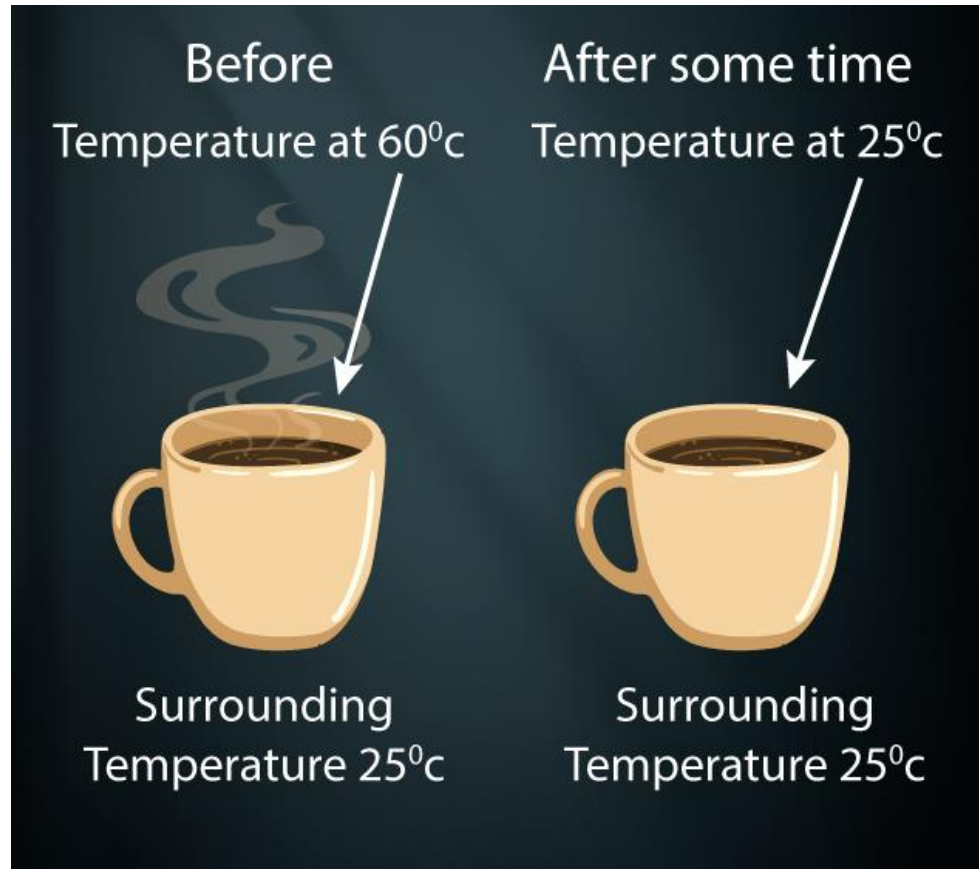
Generalized Free Cumulants for Quantum Chaotic Systems

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arXiv:2401.13829

Thermodynamics



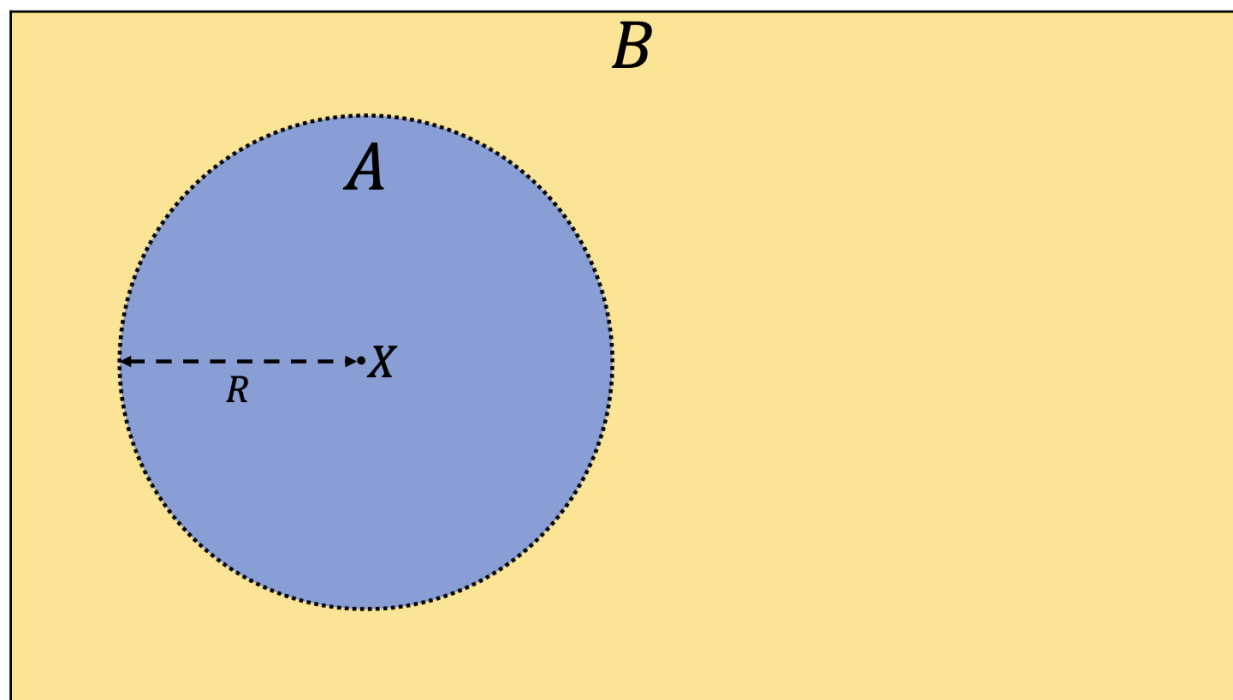
Irreversible

Quantum Mechanics

$$|\psi(t)\rangle = e^{-iHt} |\psi(0)\rangle$$

Reversible

Does the coffee cup thermalize?



$$\rho_A \rightarrow \rho_A^{\text{Gibbs}}$$

Postulate: Many-body Berry's conjecture

Eigenstates are (pseudo-)random vectors up to the symmetry constraints of the system

$$H = H_0 + V$$

$$\mathcal{O}(1) \gg \langle V \rangle \gg \epsilon \sim \mathcal{O}(e^{-S})$$

Physical properties of the system should be robust to microscopic perturbations (Deutsch 1991)

Eigenstate thermalization (ETH)

Assuming Berry's conjecture:

$$X_{ij} = f_1(E_{ij})\delta_{ij} + \tilde{R}_{ij}$$

(Feingold & Peres 1986, Deutsch 1991, Srednicki 1994)

$f_1(E_{ij})$ microcanonical average

\tilde{R}_{ij} uncorrelated gaussian random numbers

Describes the emergence of equilibrium ensembles but not the thermalization process

Ergodic bipartition (EB)

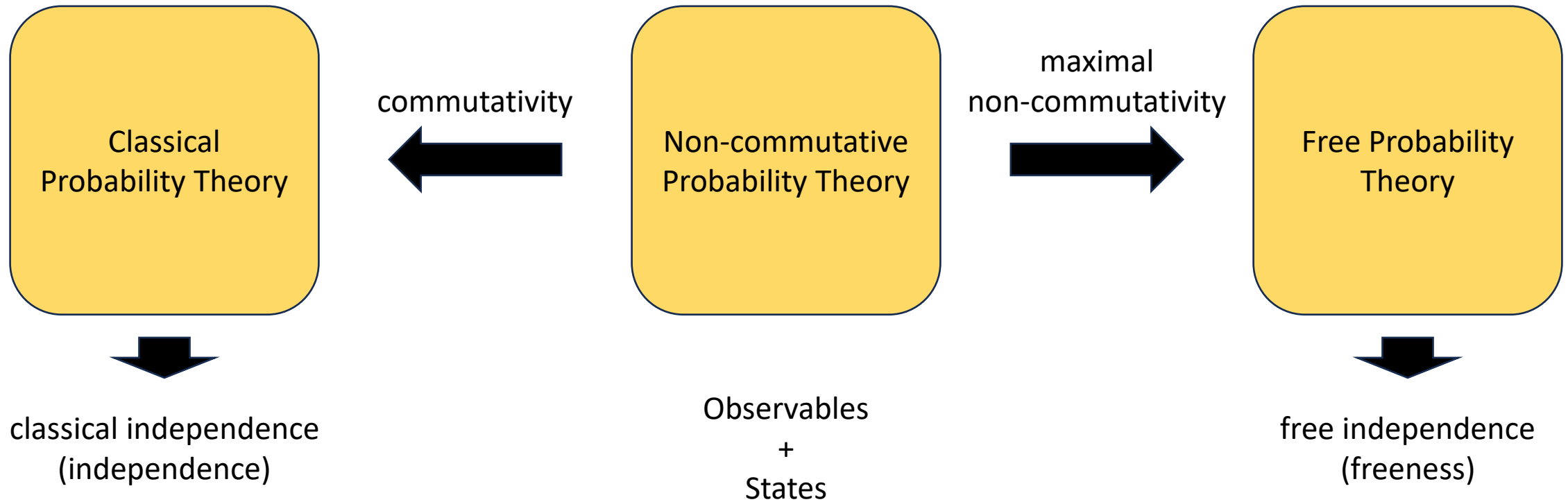
$$|i\rangle = \sum_{ab} c_{ab}^i |a\rangle \otimes |b\rangle \quad \overline{|c_{ab}^i|^2} = e^{-S(E_i)} F(E_i - E_a - E_b)$$

Recovers the Page curve for entanglement entropy (Deutsch 2010, Lu & Grover 2018, Murthy & Srednicki 2019)

Analogous to MBBC/ETH but computes distinct quantities

Previously only used to compute static quantities

Non-commutative probability



Eigenstate thermalization

$$\overline{X_{i_1 i_2} X_{i_2 i_3} \cdots X_{i_n i_1}} = e^{-(n-1)S(\bar{E})} f_n(\bar{E}; \vec{\omega})$$

(i_m distinct) (Foini & Kurchan 2018)

f_n *planar* connected n-point correlators a.k.a *free cumulants*

(Pappalardi, Foini, & Kurchan 2022)

Can be applied to distinct operators

Distinguishability of eigenstates

Using a free probability-inspired analysis,

(Kudler-Flam, Narovlansky, Ryu 2021)

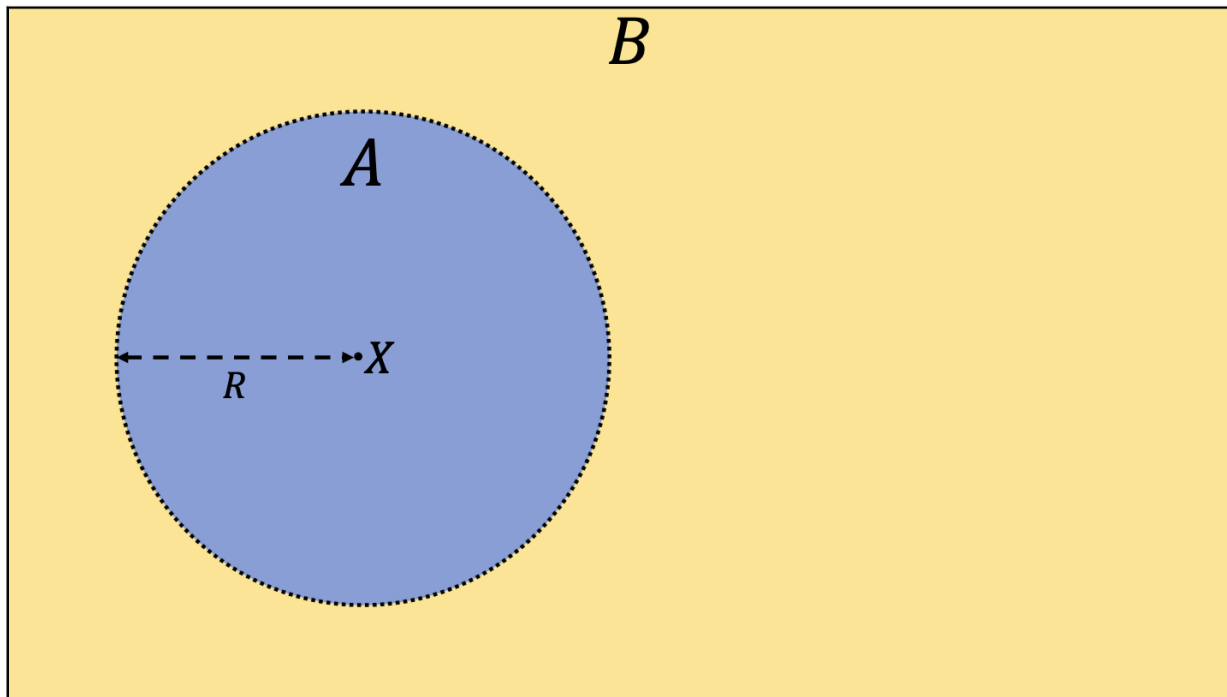
EB \Rightarrow subsystems are indistinguishable when $< \frac{1}{2}$ size \Rightarrow diagonal ETH
interpreted as matrix on lower indices, upper index is a sample

Diagrammatic Approach

$$\overline{\text{Tr} [\rho_A^2]} = a_2 \left[\text{Diagram 1} \right] a_1 = \left[\text{Diagram 2} \right] + \left[\text{Diagram 3} \right] + \dots$$

The diagrammatic approach shows the expansion of the trace of the squared density matrix, $\overline{\text{Tr} [\rho_A^2]}$, into a series of terms. The first term is a diagram with a central circle labeled i . It is surrounded by four black triangles (two pointing up, two pointing down) and four blue vertical lines. A yellow line labeled b_2 connects the top and bottom triangles on the left and right. A dashed green line connects the top and bottom triangles on the left and right, passing through the central circle i . The second term is a diagram with the same structure as the first, but the dashed green line is now a solid yellow line. The third term is a diagram with the same structure as the first, but the dashed green line is now a solid blue line. The series continues with an ellipsis.

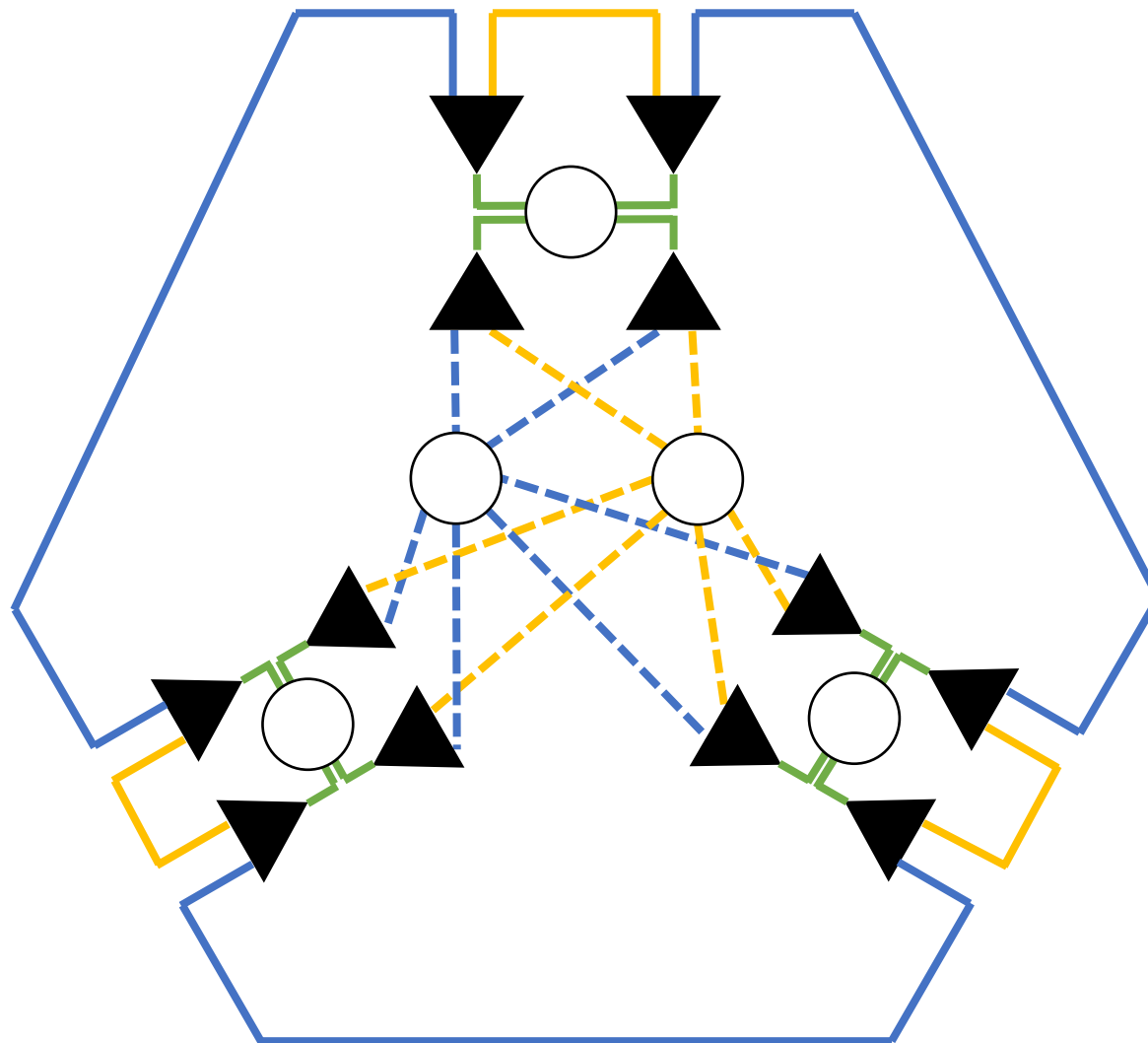
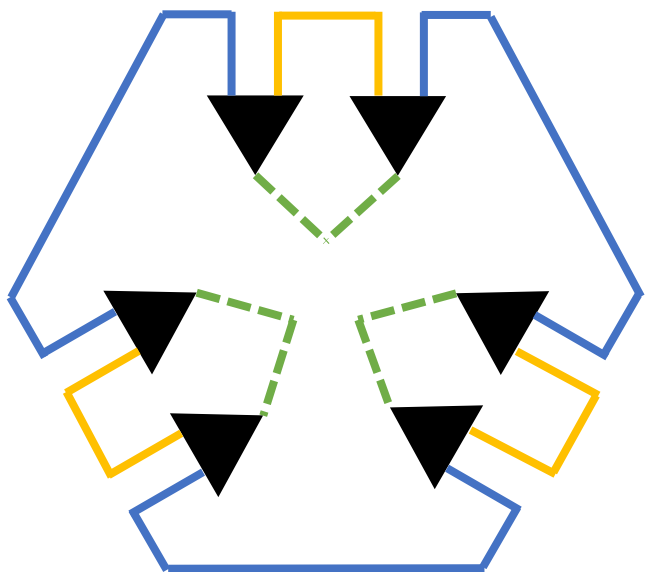
Q: Does the coffee cup thermalize?



$$\rho_A \rightarrow \rho_A^{\text{Gibbs}}$$

Does the coffee cup thermalize? Yes!

$$\rho_A \rightarrow \rho_A^{\text{Gibbs}}$$



Some other technical results

- Criteria for validity of ensemble averaging
- Calculation of Page curve for entanglement entropy
- Arguments for gaussianity of random matrix terms
- Eigenstate correlations encode entanglement velocities
- Operator-eigenstate correlations encode butterfly velocities

Summary and future work

- The fundamental postulate of the ETH is that eigenstates are random vectors **How much of quantum chaos does the MBBC encompass?**
- Ideas from free probability and random matrix theory are central to quantum chaos **Can further techniques be drawn?**
- Ergodic bipartition enables strong statements about chaotic systems **Can this be extended to higher-order correlators?**
- The ETH and EB are unified into a powerful calculational tool



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