Texas Center for Superconductivity
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Generalized Free Cumulants for Quantum Chaotic Systems

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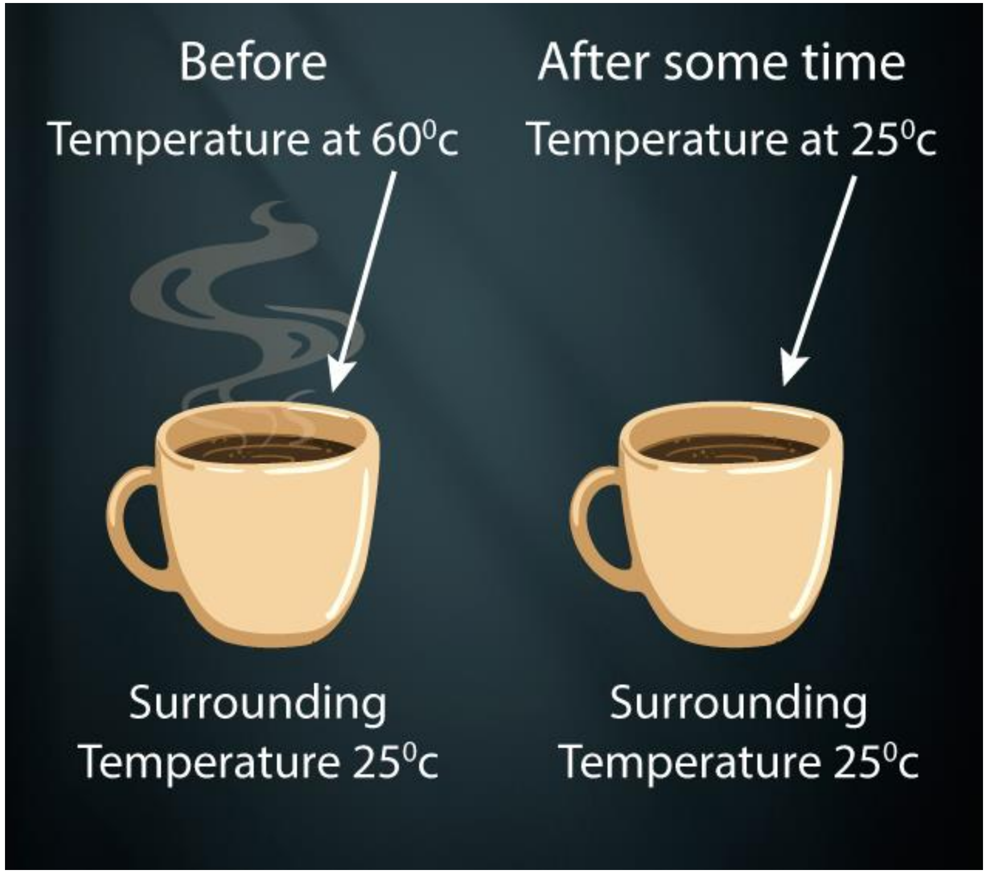
Thermodynamics

Quantum Mechanics

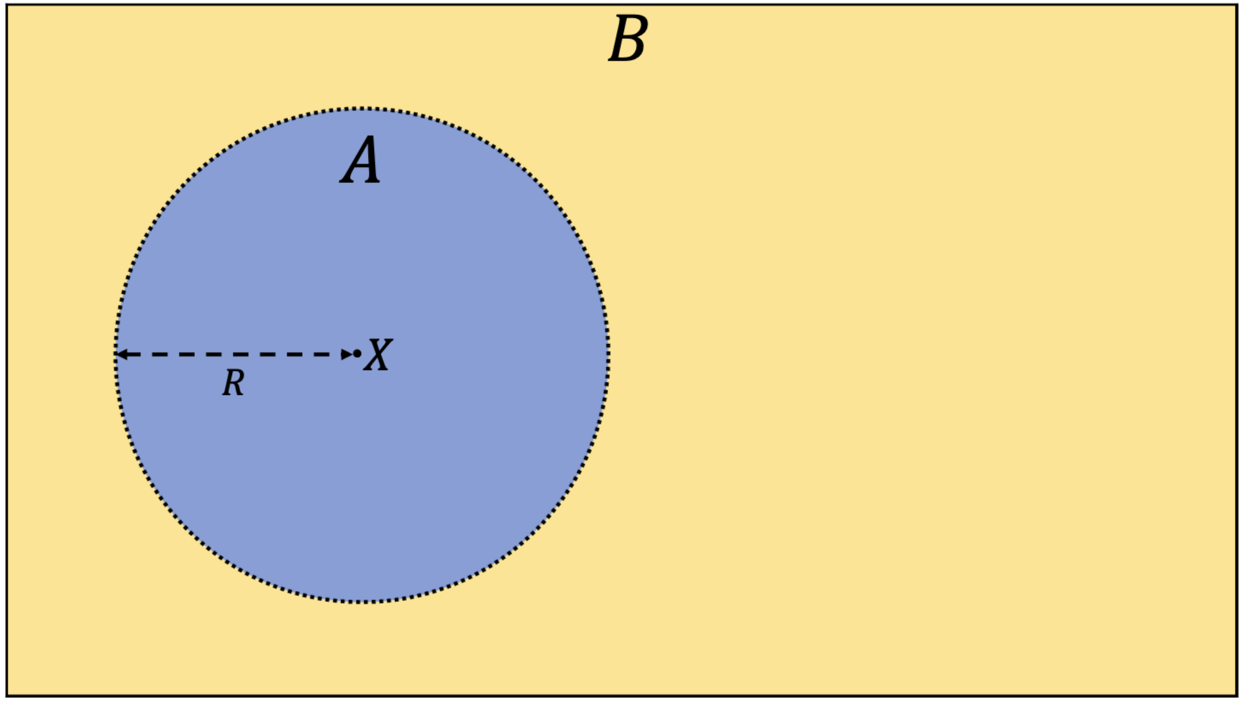


$$|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$$

Irreversible

Reversible

Does the coffee cup thermalize?



$$\rho_A \to \rho_A^{\mathrm{Gibbs}}$$

Postulate: Many-body Berry's conjecture

Eigenstates are (pseudo-)random vectors up to the symmetry constraints of the system

$$H = H_0 + V$$

$$\mathcal{O}(1) \gg \langle V \rangle \gg \epsilon \sim \mathcal{O}(e^{-S})$$

Physical properties of the system should be robust to microscopic perturbations (Deutsch 1991)

Eigenstate thermalization (ETH)

Assuming Berry's conjecture:

$$X_{ij} = f_1(E_{ij})\delta_{ij} + \tilde{R}_{ij}$$

(Feingold & Peres 1986, Deutsch 1991, Srednicki 1994)

$f_1(E_{ij})$ microcanonical average

$\tilde{R}_{ij}$ uncorrelated gaussian random numbers

Describes the emergence of equilibrium ensembles but not the thermalization process

Ergodic bipartition (EB)

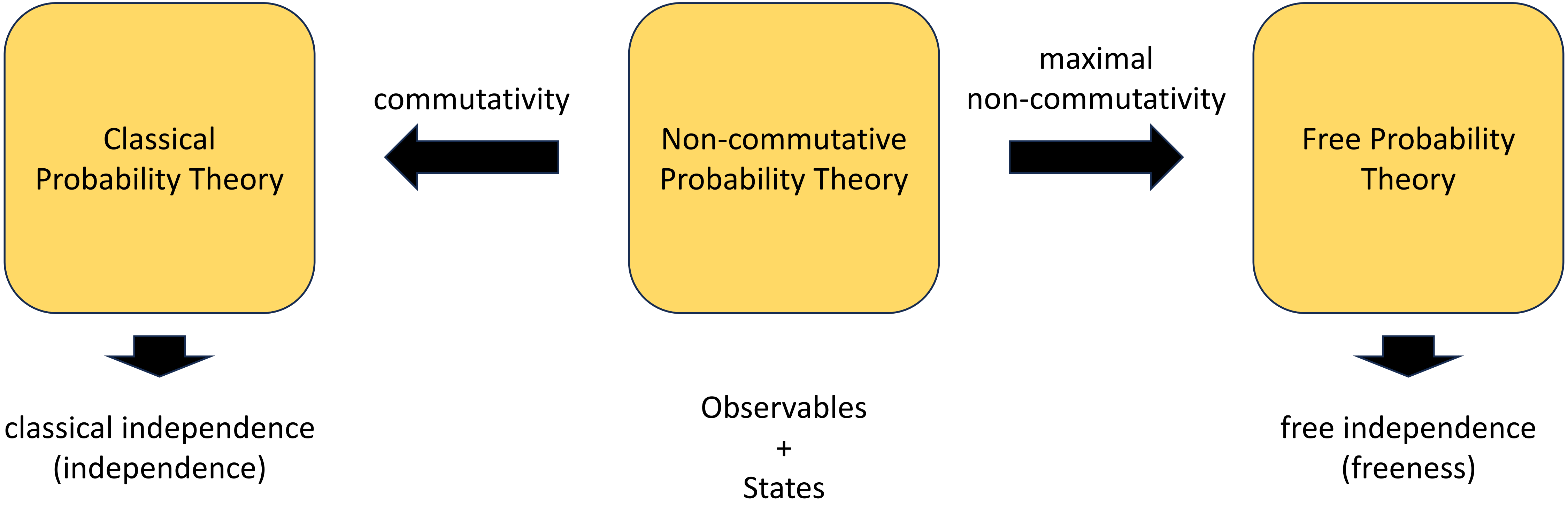
$$|i\rangle = \sum_{ab} c^i_{ab} |a\rangle \otimes |b\rangle \qquad \overline{|c^i_{ab}|^2} = e^{-S(E_i)} F(E_i - E_a - E_b)$$

Recovers the Page curve for entanglement entropy (Deutsch 2010, Lu & Grover 2018, Murthy & Srednicki 2019)

Analogous to MBBC/ETH but computes distinct quantities

Previously only used to compute static quantities

Non-commutative probability



Eigenstate thermalization

$$\overline{X_{i_1 i_2} X_{i_2 i_3} \cdots X_{i_n i_1}} = e^{-(n-1)S(\bar{E})} f_n(\bar{E}; \vec{\omega})$$

(i_m distinct) (Foini & Kurchan 2018)

f_n *planar* connected n-point correlators a.k.a *free cumulants*

(Pappalardi, Foini, & Kurchan 2022)

Can be applied to distinct operators

Distinguishability of eigenstates

Using a free probability-inspired analysis,

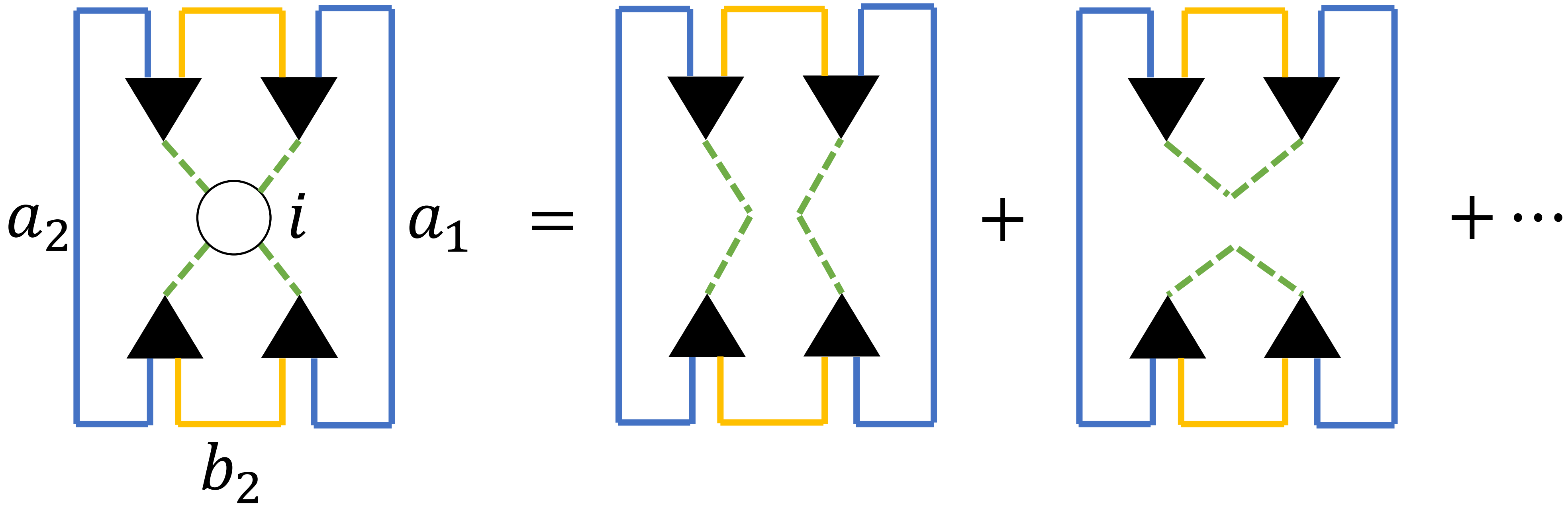
(Kudler-Flam, Narovlansky, Ryu 2021)

EB => subsystems are indistinguishable when < ½ size => diagonal ETH

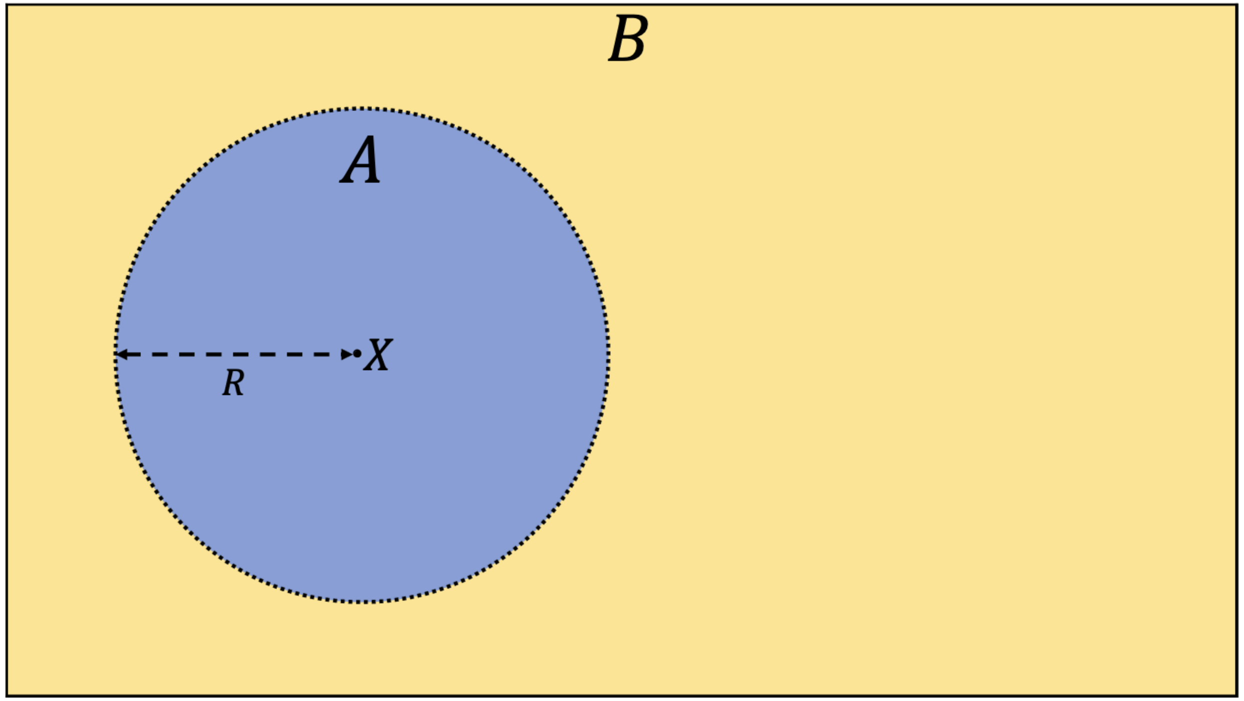
interpreted as matrix on lower indices, upper index is a sample

Diagrammatic Approach

$$\overline{\mathrm{Tr}\left[\rho_A^2\right]} =$$



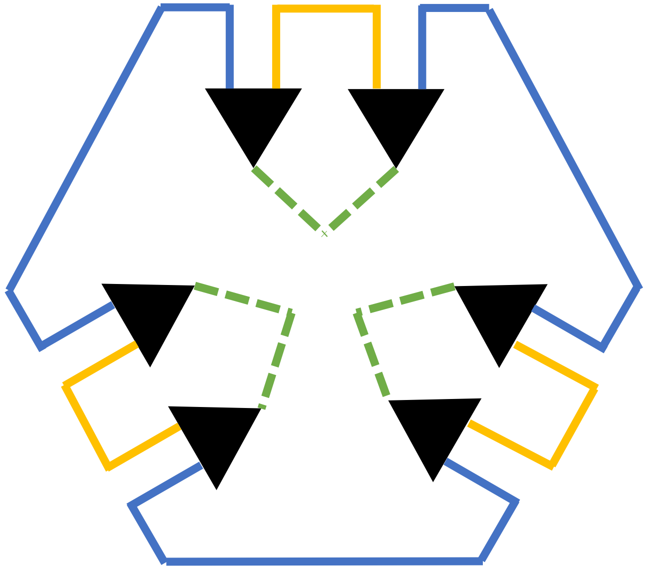
Q: Does the coffee cup thermalize?

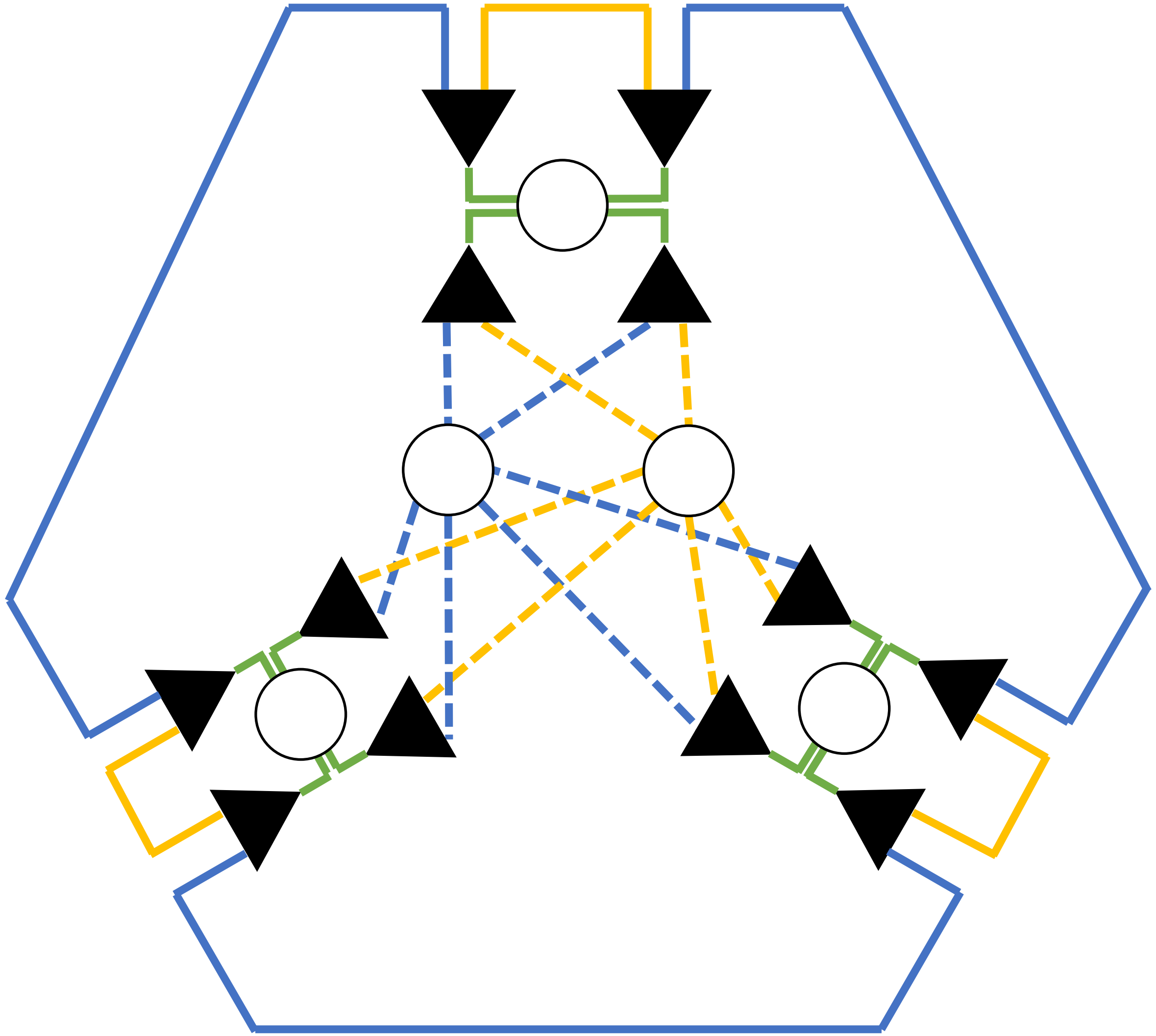


$$\rho_A \to \rho_A^{\mathrm{Gibbs}}$$

Does the coffee cup thermalize? Yes!

$$\rho_A \rightarrow \rho_A^{\mathrm{Gibbs}}$$





Some other technical results

- Criteria for validity of ensemble averaging
- Calculation of Page curve for entanglement entropy
- Arguments for gaussianity of random matrix terms
- Eigenstate correlations encode entanglement velocities
- Operator-eigenstate correlations encode butterfly velocities

Summary and future work

- The fundamental postulate of the ETH is that eigenstates are random vectors How much of quantum chaos does the MBBC encompass?
- Ideas from free probability and random matrix theory are central to quantum chaos Can further techniques be drawn?
- Ergodic bipartition enables strong statements about chaotic systems Can this be extended to higher-order correlators?
- The ETH and EB are unified into a powerful calculational tool

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