Amplitudes, Feynman integrals, and mathematics

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overview of activities in the Amplitudes group

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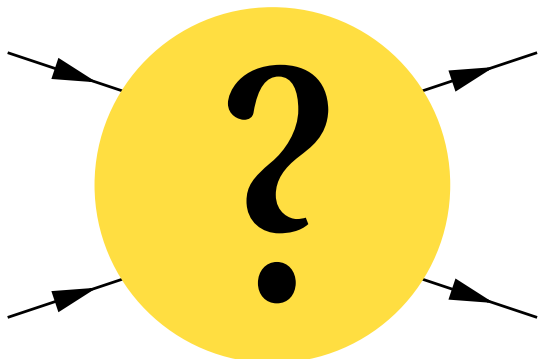
Old Library, All Souls College

Amplitudes

$$\mathcal{A}_{n+m} = \langle \text{out}|\mathcal{S}|\text{in}\rangle = \langle p_{n+m} \dots p_{n+1}|\mathcal{S}|p_n \dots p_1\rangle$$



- How to compute them?
- What are their properties?
- What can we learn about mathematics from them?

AdS/CFT duality

$$\left\{\begin{array}{c}\text{tree-level graviton scattering}\\ \mathcal{A}_4\\ \text{in IIB superstrings on } AdS_5\times S_5\end{array}\right\}\leftrightarrow\left\{\begin{array}{c}\text{correlator}\\ \langle\mathcal{O}_2(x_1)\mathcal{O}_2(x_2)\mathcal{O}_2(x_3)\mathcal{O}_2(x_4)\rangle\\ \text{at large charge in } \mathcal{N}=4\end{array}\right\}$$

$$A^{(1)}(S,T)=\int_{\mathbb{C}}\mathrm{d}^2z\;|z|^{-2S-2}|1-z|^{-2T-2}L^{\mathrm{sv}}(S,T,z).$$

- *The AdS Virasoro-Shapiro Amplitude* (arXiv:2306.12786) [Luis F. Alday, Tobias Hansen]
- *High Energy String Scattering in AdS* (arXiv:2312.02261) [...+Maria Nocchi]
- *Emergent world-sheet for the AdS Virasoro-Shapiro amplitude* (arXiv:2305.03593) [...+Joao A. Silva]
- *Six-Point AdS Gluon Amplitudes from Flat Space and Factorization* (arXiv:2307.06884) [Luis F. Alday, Vasco Gonçalves, Maria Nocchi, Xinan Zhou]

⇒ world-sheet description of scattering in $AdS_5\times S_5$

Single-valued multiple polylogarithms

$$\frac{\mathrm{d}}{\mathrm{d}z}\,\mathrm{Li}_n(z) = \frac{1}{z}\,\mathrm{Li}_{n-1}(z) \qquad \mathrm{Li}_n(1) = \zeta(n) = \sum_{k=1}^{\infty}\frac{1}{k^n}$$

$$\frac{\partial}{\partial z}\,\mathrm{Li}^{\mathrm{SV}}_n(z,\bar{z}) = \frac{1}{z}\,\mathrm{Li}^{\mathrm{SV}}_{n-1}(z,\bar{z}) \qquad \mathrm{Li}^{\mathrm{SV}}_n(1) = \zeta^{\mathrm{SV}}_n(1)$$

- *Single-valued integration and superstring amplitudes in genus zero* (`arXiv:1910.01107`)
- *Single-valued integration and double copy* (`arXiv:1810.07682`)

  [Francis Brown, Clément Dupont]
- *Motivic coaction and single-valued map of polylogarithms from zeta generators* (`arXiv:2312.00697`)

  [Hadleigh Frost, Martijn Hidding, Deepak Kamlesh, Carlos Rodriguez, Oliver Schlotterer, Bram Verbeek]

elliptic polylogarithms

$$\int_0^z f_1(t_1)\mathrm{d}t_1 \int_0^{t_1} f_2(t_2)\mathrm{d}t_2 \int_0^{t_2} \cdots$$

- *The cosmic Galois group, the sunrise Feynman integral, and the relative completion of* $\Gamma_1(6)$ (`arXiv:2303.17534`)

  [Matija Tapušković]
- *A double integral of dlog forms which is not polylogarithmic* (`arXiv:2006.09413`)

  [Francis Brown, Claude Duhr]

Applications of twistor theory:

Strong coupling

$$\mathcal{A} = \langle \mathcal{W}_\gamma \rangle = \int_{\partial\Sigma=\gamma} \mathcal{D}[\Sigma \subset AdS_5 \times S^5] e^{-S/\alpha'} \sim e^{-\text{Area}(\Sigma)/\alpha'}$$

null momenta p_k in $\mathcal{A} \mapsto$ polygonal Wilson loop $\mathcal{W}_\gamma \mapsto$ minimal surface Σ

- *Amplitudes at strong coupling as hyperkähler scalars* (arXiv:2306.17044)
  [Hadleigh Frost, Ömer Gürdogan, Lionel Mason]

flat space holography

- *Carrollian Amplitudes and Celestial Symmetries* (arXiv:2312.10138)
  [Lionel Mason, Romain Ruzziconi, Akshay Yelleshpur Srikant]

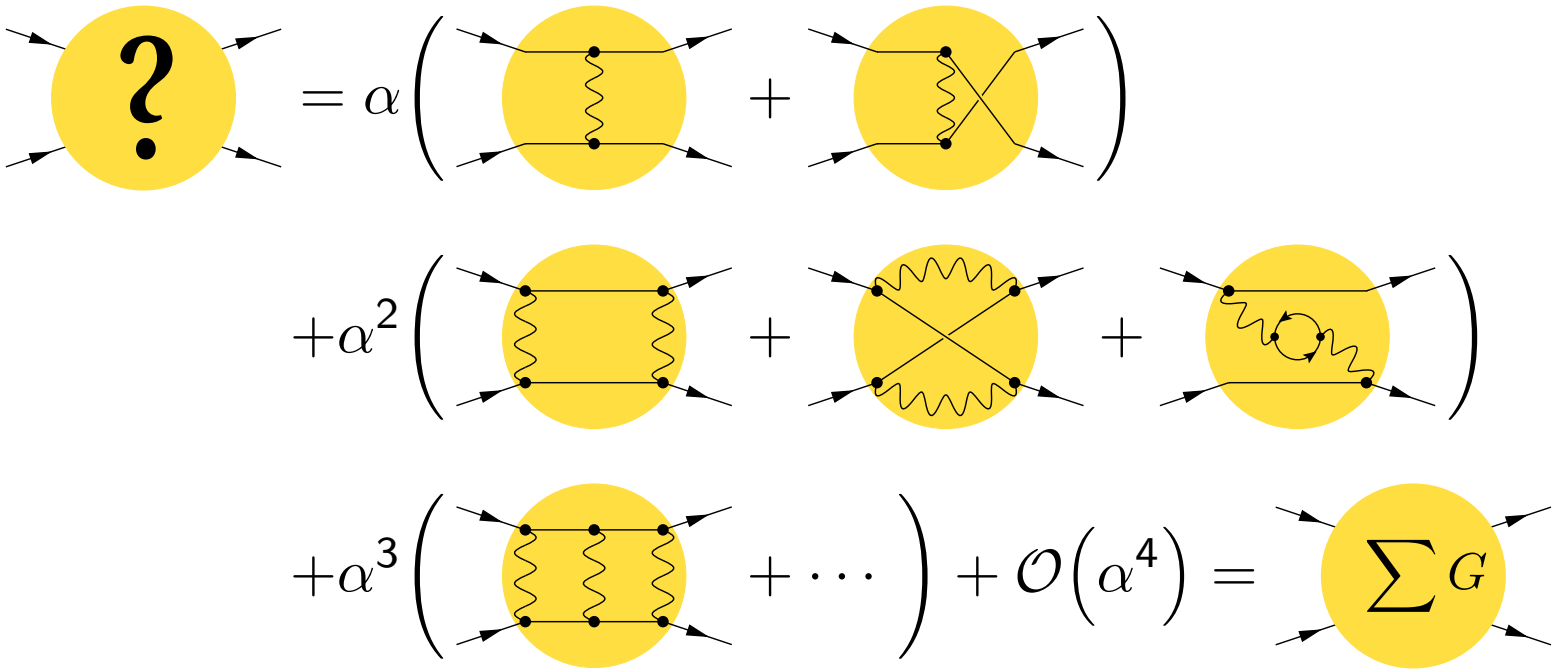
curved backgrounds

- *Yang-Mills form factors on self-dual backgrounds* (arXiv:2305.07542)
  [Giuseppe Bogna, Lionel Mason]
- *Scattering on self-dual Taub-NUT* (arXiv:2309.03834) [. . . +Tim Adamo, Atul Sharma]

- *All Loop Scattering For All Multiplicity* (arXiv:2311.09284)
  [Nima Arkani-Hamed, Hadleigh Frost, Giulio Salvatori, Pierre-Guy Plamondon, Hugh Thomas]

Feynman integrals

$$I_G(m^2, p^2) = \left(\prod_k \int_{\mathbb{R}^D} \mathrm{d}\ell_k \right) \prod_e \frac{1}{m_e^2 - k_e^2}$$

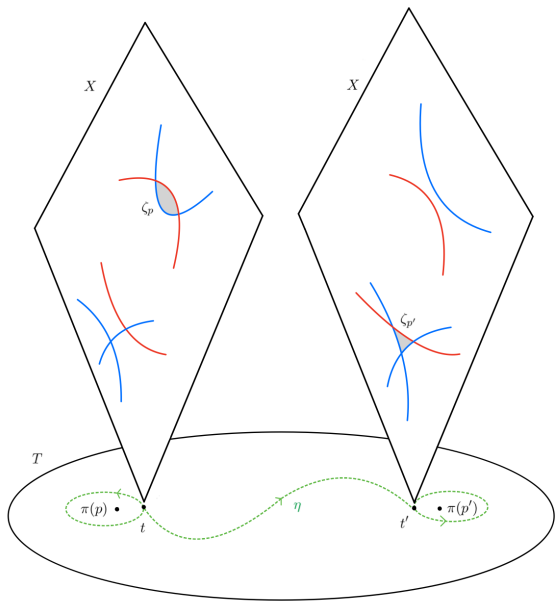


perturbation theory

Symbolic integration:

- *Two-loop mixed QCD-EW corrections to $q\bar{q} \to Hg$, $qg \to Hq$, and $\bar{q}g \to H\bar{q}$* (arXiv:2203.17202)
  [Marco Bonetti, Erik Panzer, Lorenzo Tancredi]
- *Four-loop collinear anomalous dimensions in QCD and $\mathcal{N} = 4$ super Yang-Mills* (arXiv:2102.09725)
  [Bakul Agarwal, Andreas von Manteuffel, Erik Panzer, Robert M. Schabinger]

(Landau) singularities and monodromies (Steinmann relations):



- *Hierarchies in relative Picard-Lefschetz theory* (arXiv:2212.06661)
  [Marko Berghoff, Erik Panzer]

Motivic coactions ("symbols"):

- *Motivic Galois coaction and one-loop Feynman graphs* (1911.01540)
  [Matija Tapušković]

Graph complexes:

- *The wheel classes in the locally finite homology of* $\mathrm{GL}_n(\mathbb{Z})$*, canonical integrals and zeta values* (arXiv:2402.06757)
  [Francis Brown, Oliver Schnetz]
- *Bordifications of the moduli spaces of tropical curves and abelian varieties, and unstable cohomology of* $\mathrm{GL}_g(\mathbb{Z})$ *and* $\mathrm{SL}_g(\mathbb{Z})$ (arXiv:2309.12753)
  [Francis Brown]
- *Generalised graph Laplacians and canonical Feynman integrals with kinematics* (arXiv:2205.10094)
  [Francis Brown]
- *Invariant Differential Forms on Complexes of Graphs and Feynman Integrals* (arXiv:2101.04419)
  [Francis Brown]

Regularization:

- *Regularized integrals and manifolds with log corners* (arXiv:2312.17720)
  [Clément Dupont, Erik Panzer, Brent Pym]

Combinatorial Feynman integrals:

- *Feynman symmetries of the Martin and* c_2 *invariants of regular graphs* (arXiv:2304.05299)
  [Erik Panzer, Karen Yeats]
- *Hepp's bound for Feynman graphs and matroids* (arXiv:1908.09820) [Erik Panzer]