Stress testing the Standard Model at colliders: from Amplitudes to Events

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Oxford Theoretical Studies of Particles and Strings retreat, March 2024

The QCD&collider group

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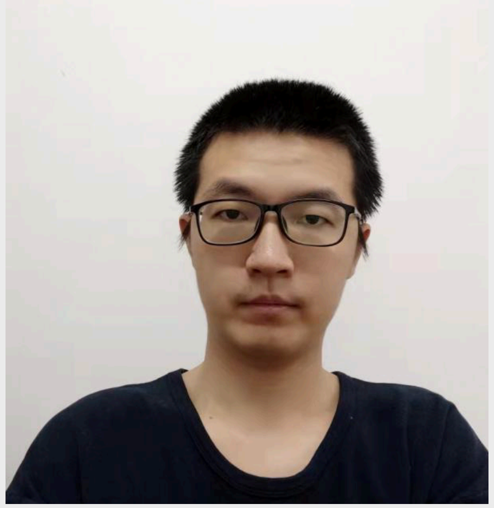
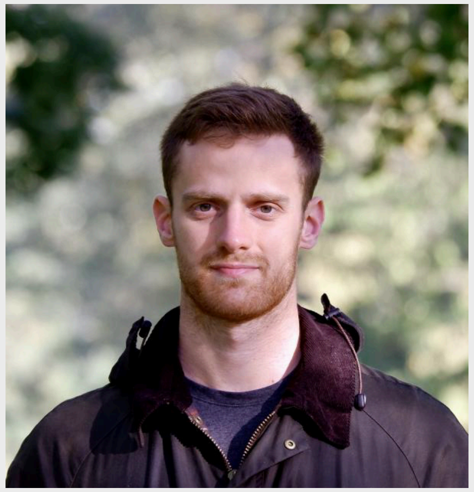


Xiao



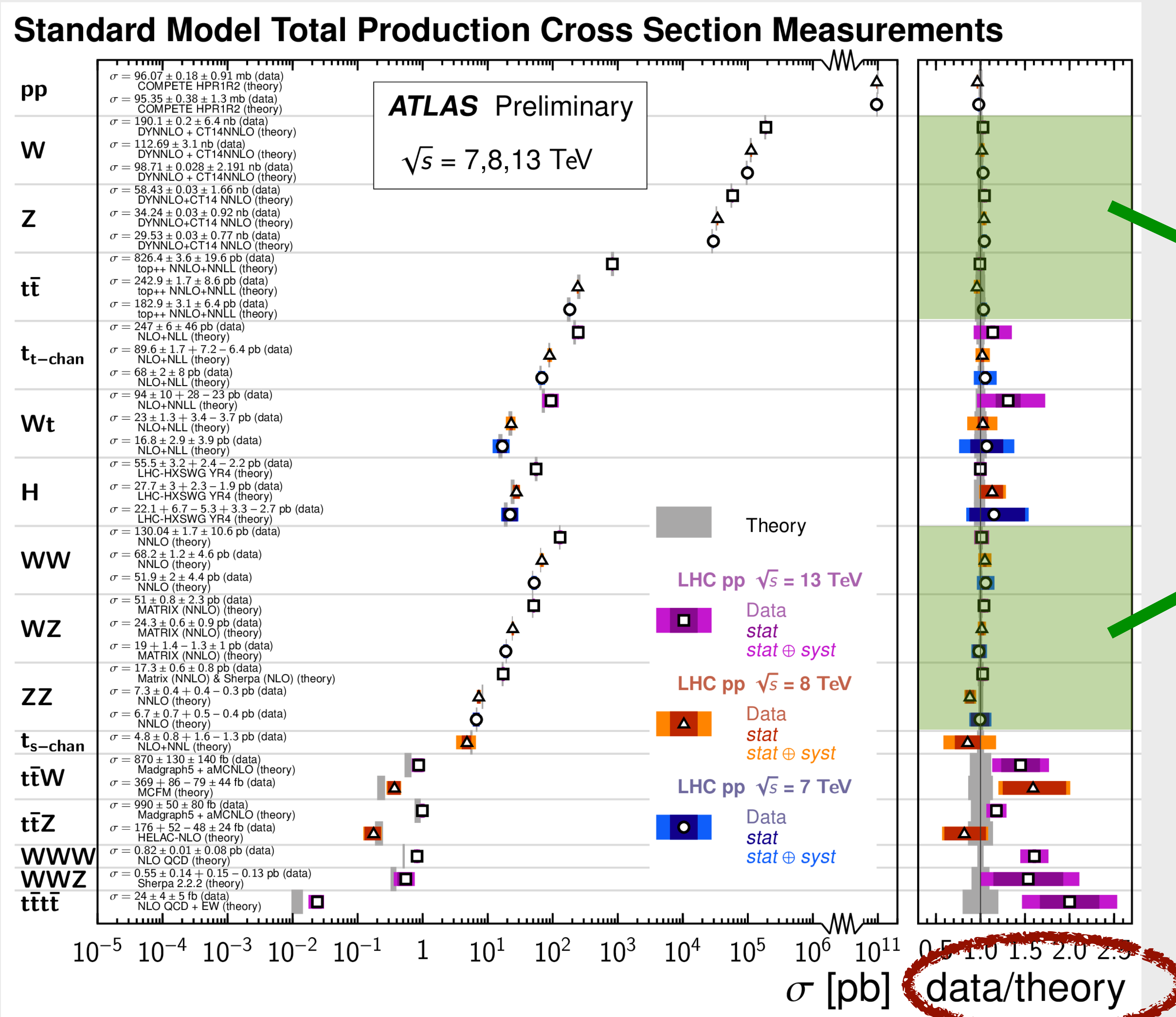
Giulio

Peter

The LHC: stress-testing the Standard Model

What have we learned so far

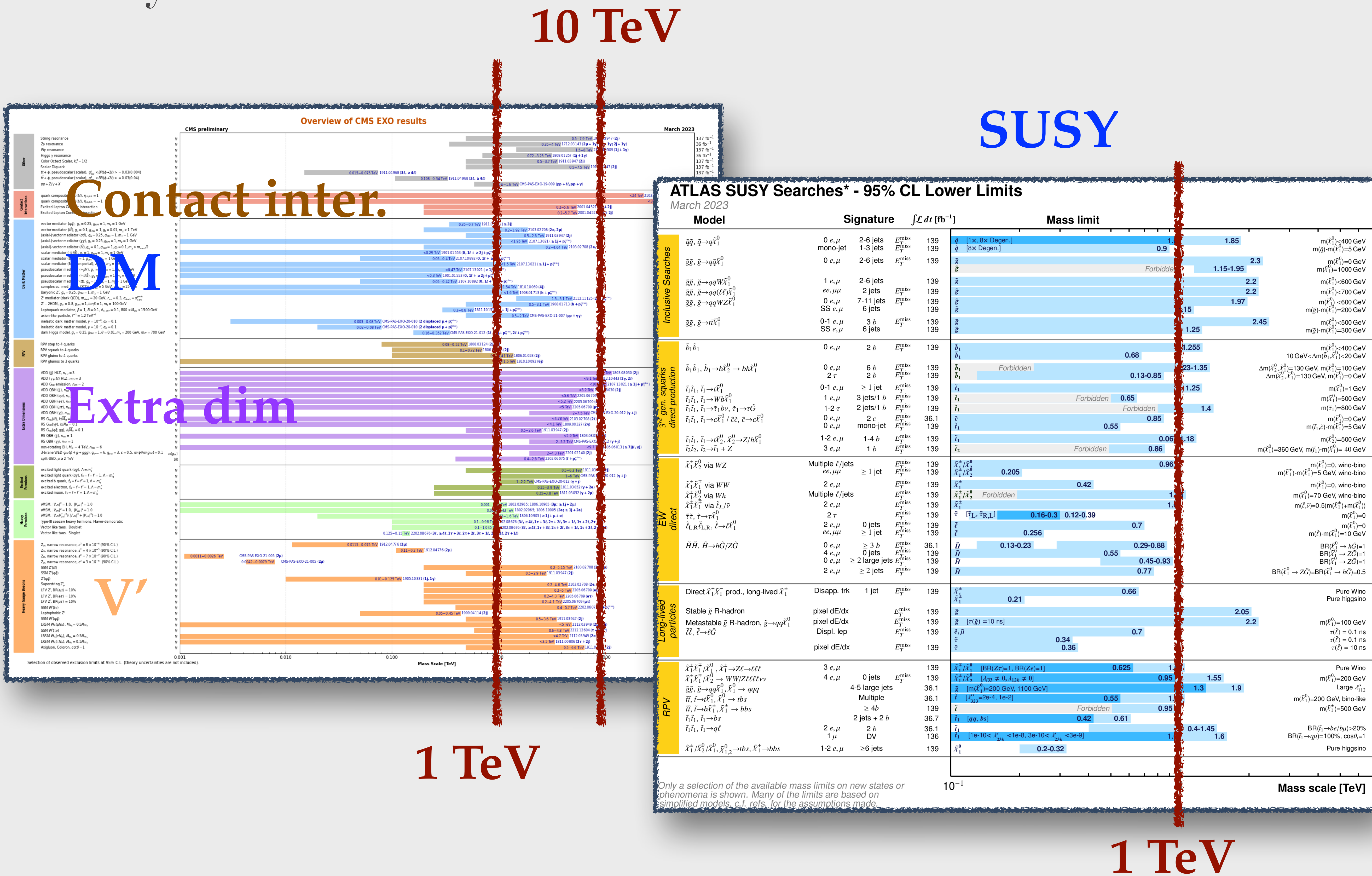
Overall, the Standard Model works very well



Vector boson(s), top: high precision, now

What have we learned so far

Many “vanilla” BSM scenarios excluded at the EW scale

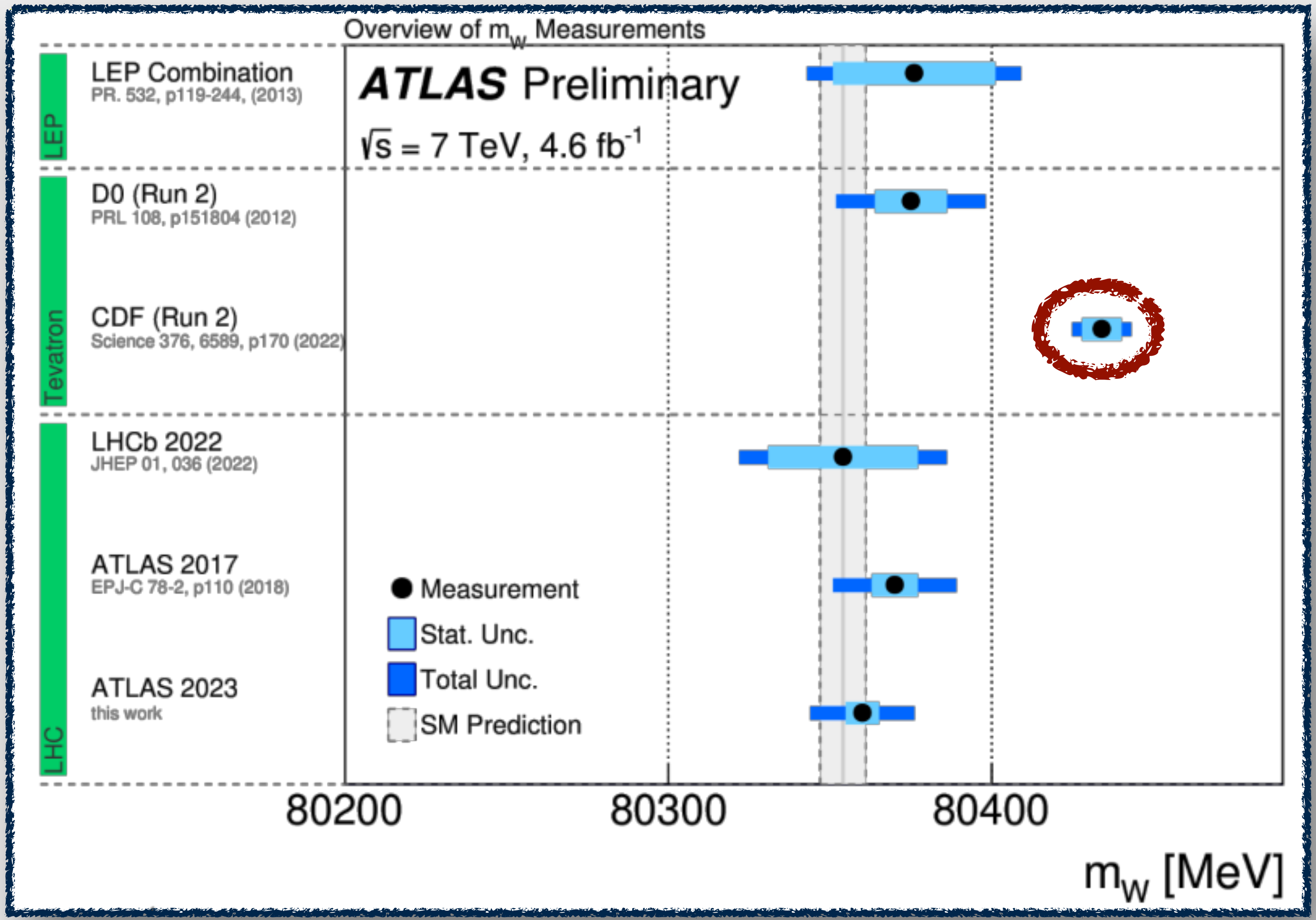


Is the SM established? Can we go home?

Ultra-precise data / theory comparison: some tensions

Example: W mass

Input in the SM. But enters radiative corrections → access through global fit to precision observables

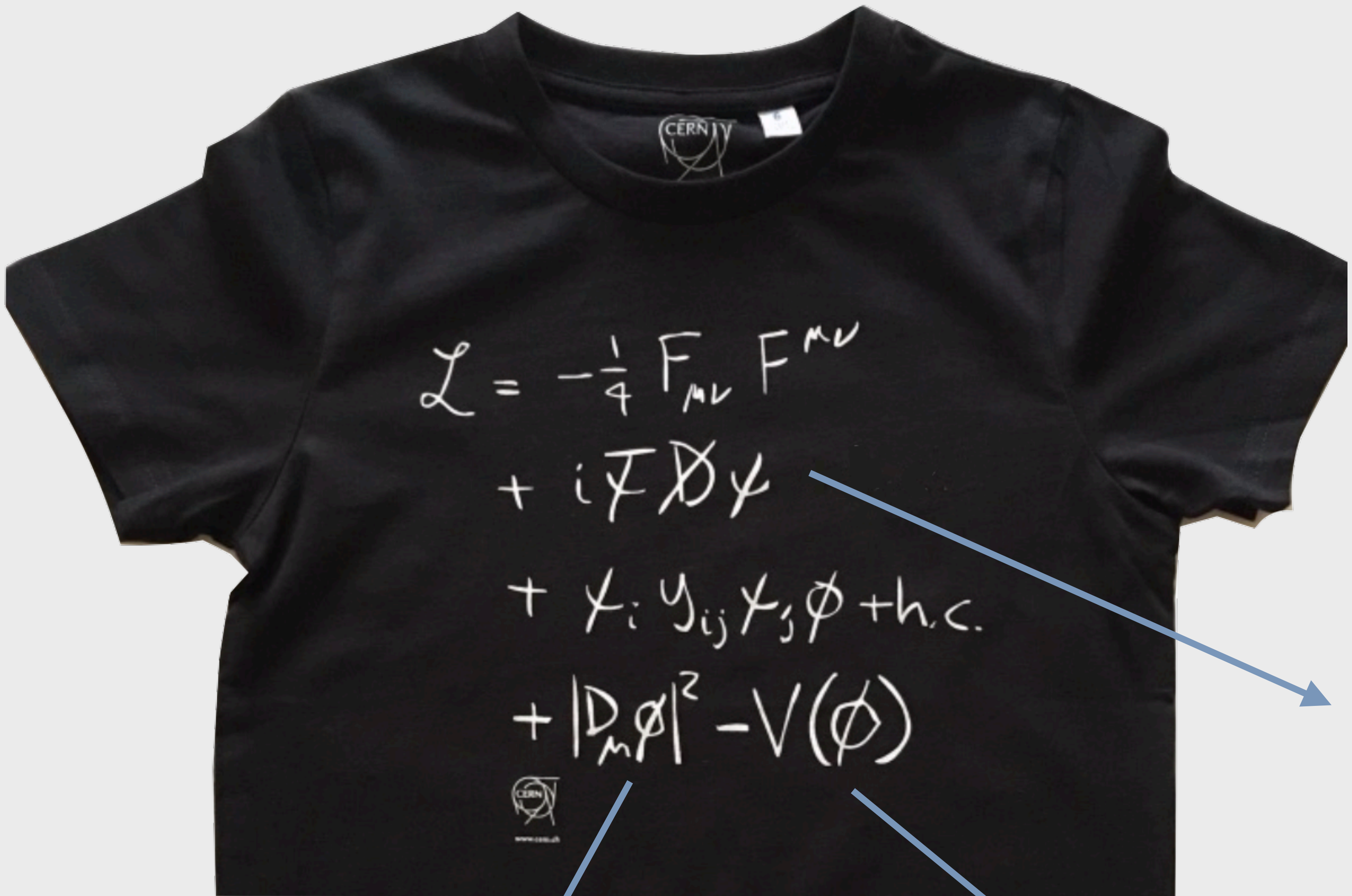


Require exquisite theoretical control over Z/W bosons predictions

Testing ground for "precision" programme

similar situation: $(g\text{-}2)_\mu$

Is the SM established? Can we go home?



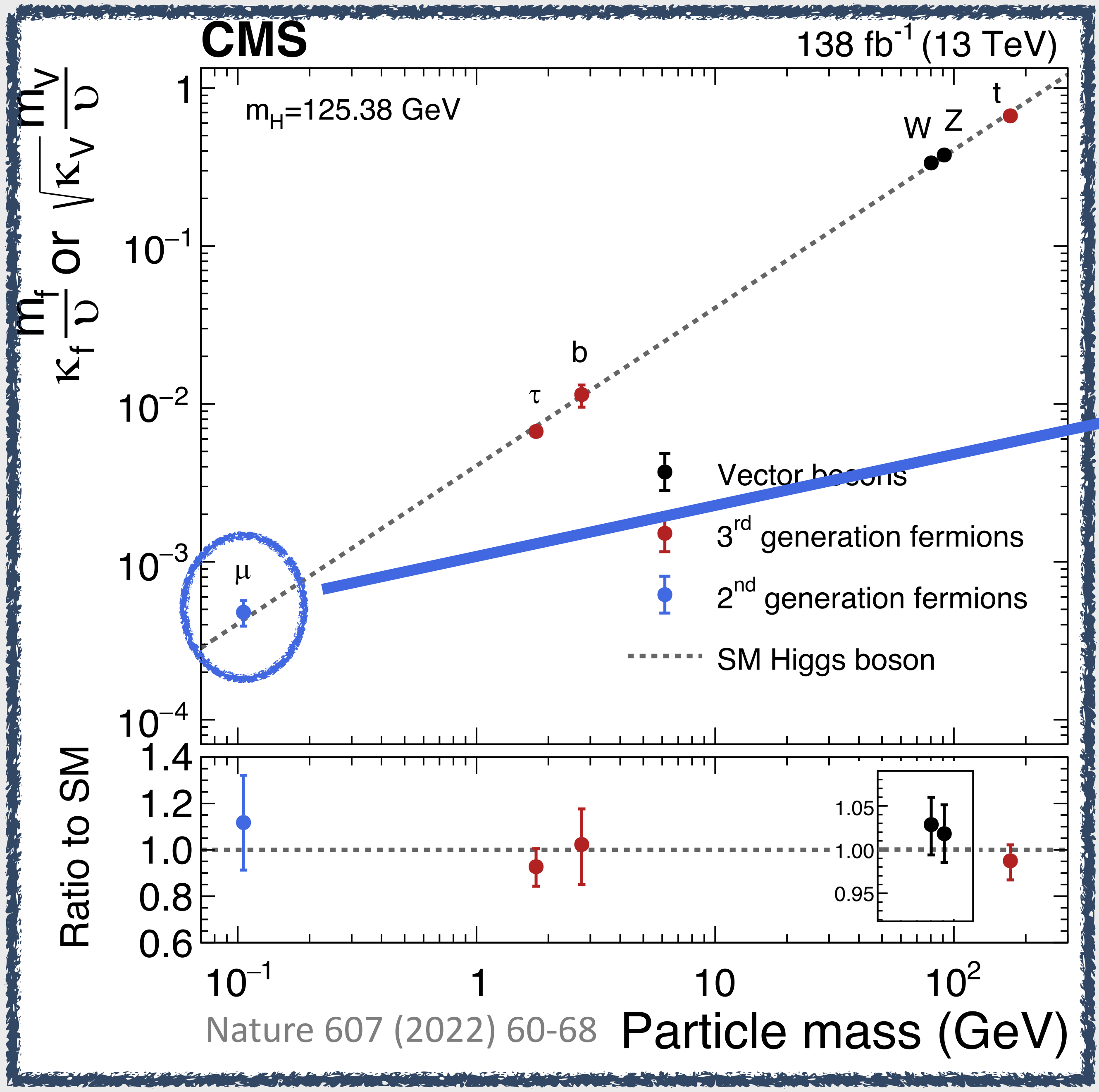
Yukawa sector: exploration only just beginning

Gauge sector: well-studied, but now with a scalar field

Higgs potential: basically unexplored

What have we learned so far

Higgs sector: in many cases, SM to within ~10% or better

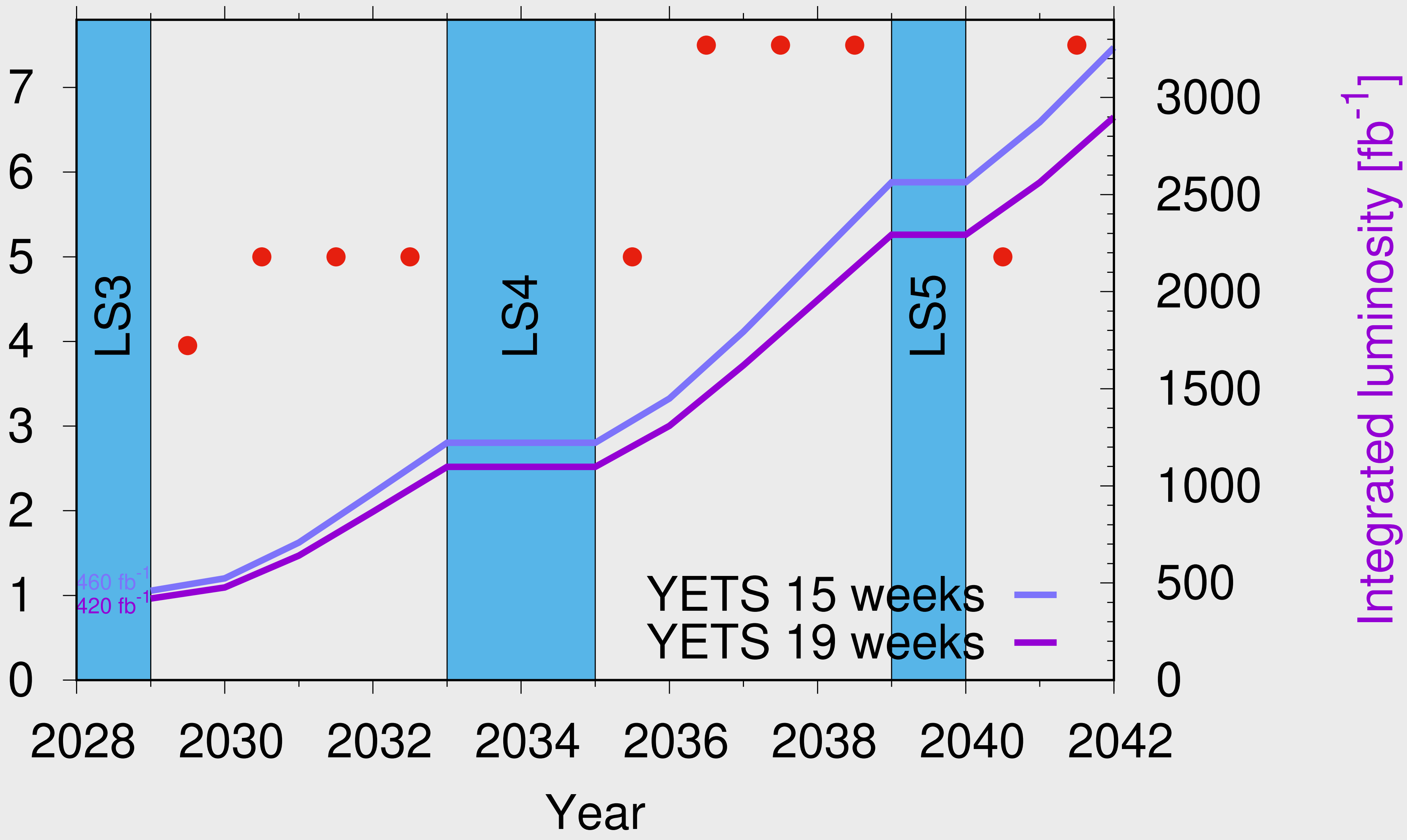


μ: starting to probe Higgs/matter interactions for 2nd generation family!

Still a lot to be done: Higgs/matter interactions, Higgs potential…

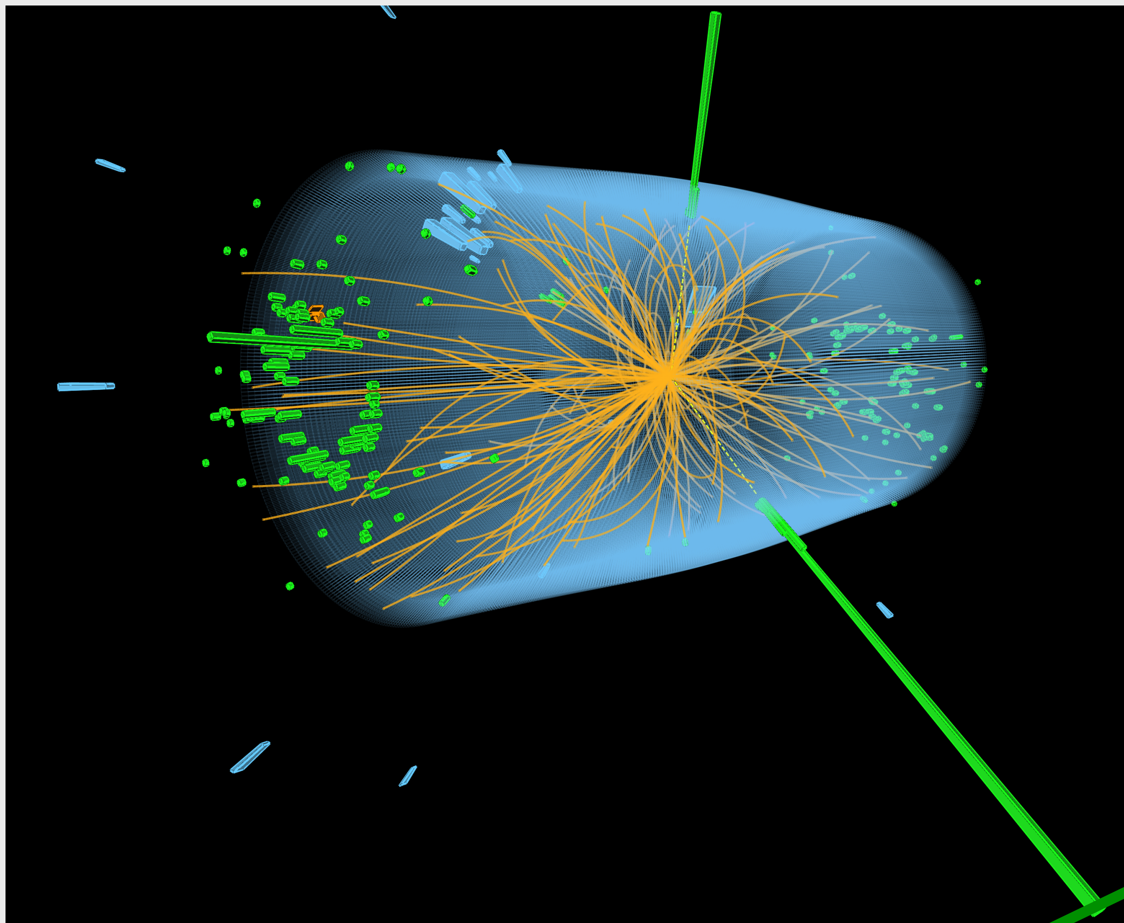
Moving forward

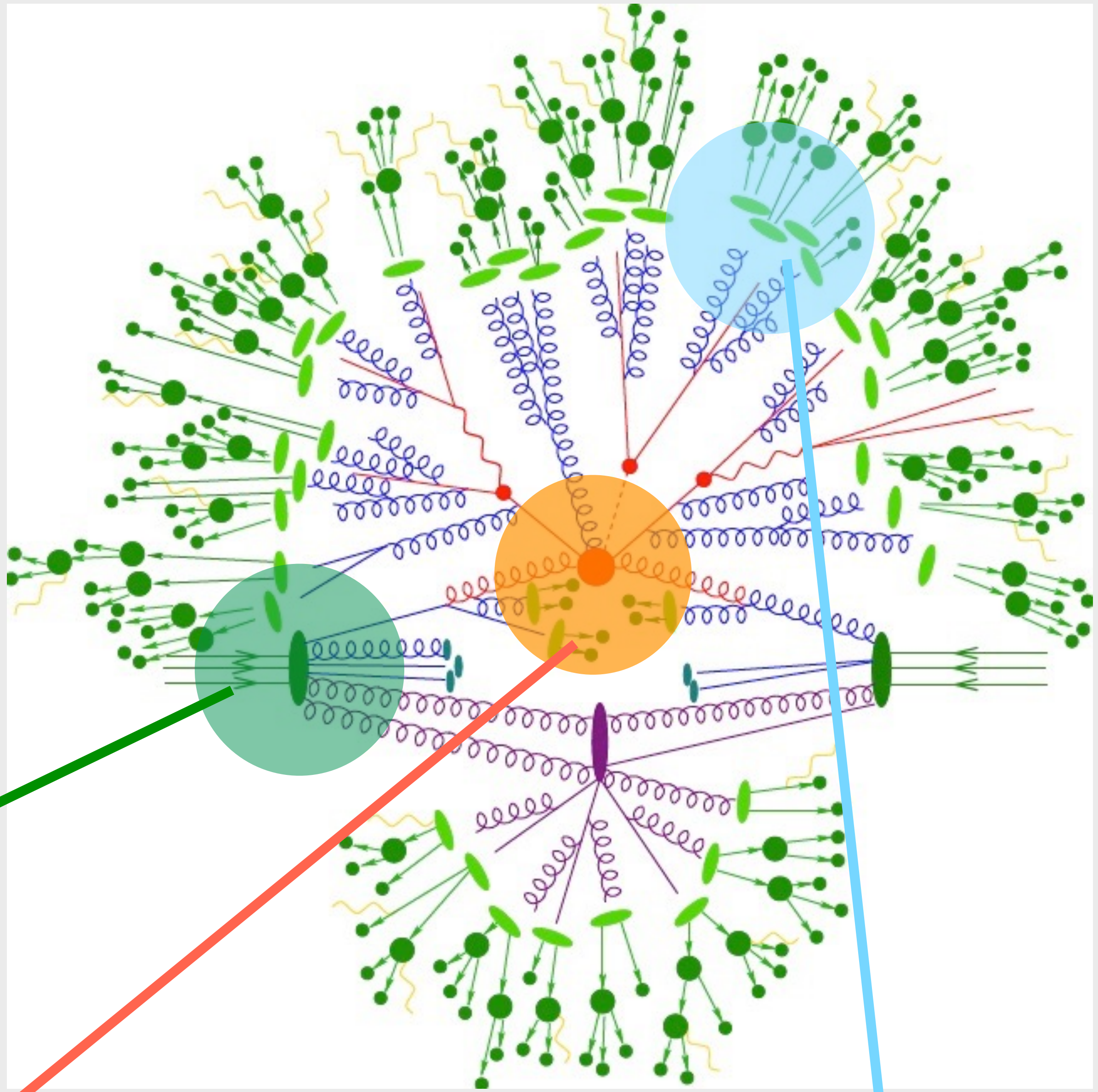
We are only at the beginning of the LHC programme



- So far: only about 5% of data
- Big jump in statistics, no big jump in energy → precision data/theory comparison more and more important

Collider predictions: how to get there



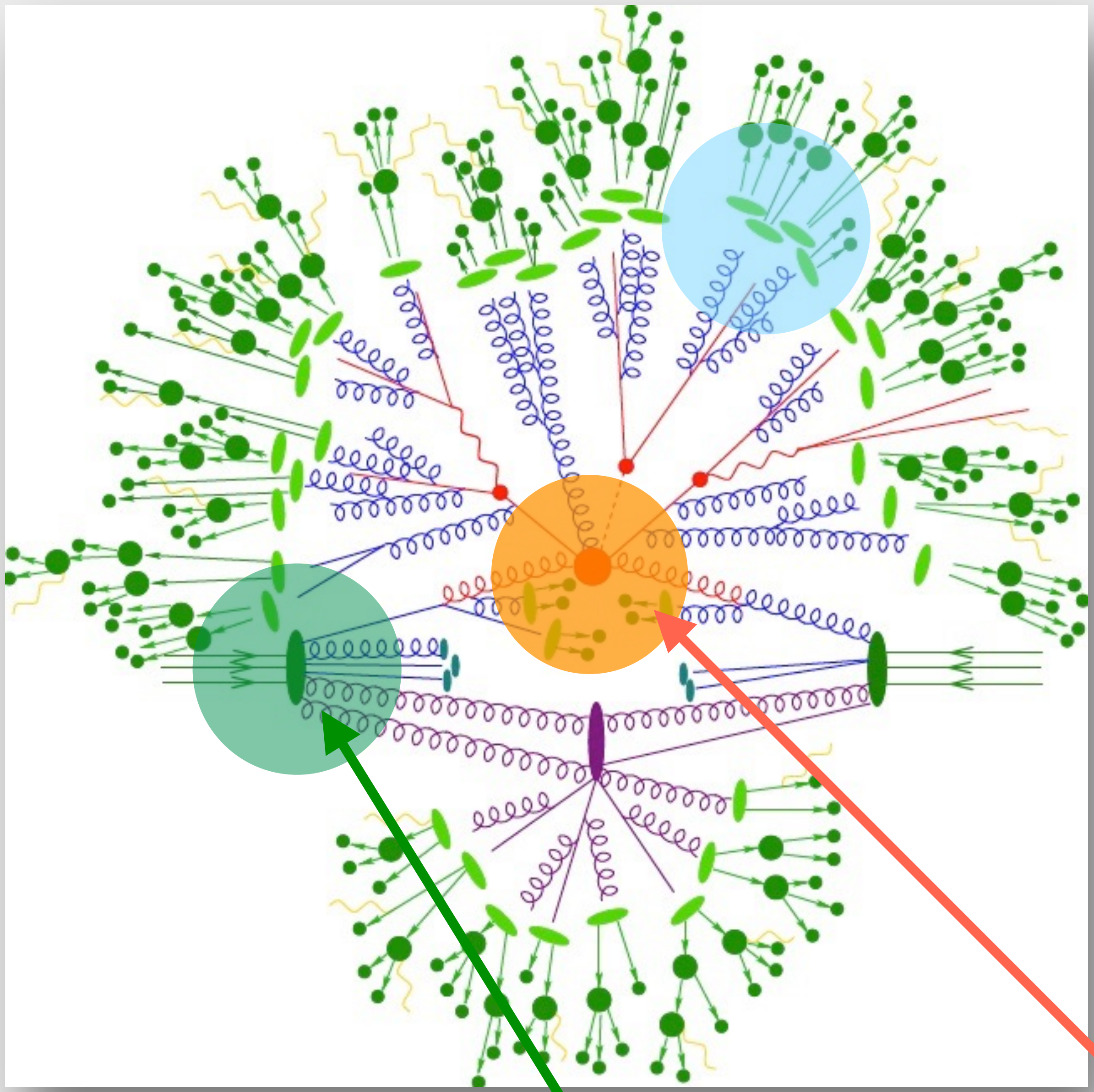


Structure of the proton, Q~GeV, non perturbative

High-energy scattering, perturbative

From high-energy to hadrons: PS → Silvia

Collider predictions: how to get there



Key: physics at very different scales, can be separated

Non-perturbative

$$d\sigma = \int dx_1 dx_2 f(x_1) f(x_2) d\sigma_{\text{part}}(x_1, x_2) F_J (1 + \mathcal{O}(\Lambda^n_{\text{QCD}}/Q^n))$$

The hard process: amplitudes & co

[Graphics: Giulio Gambuti]

Collider predictions: how to get there

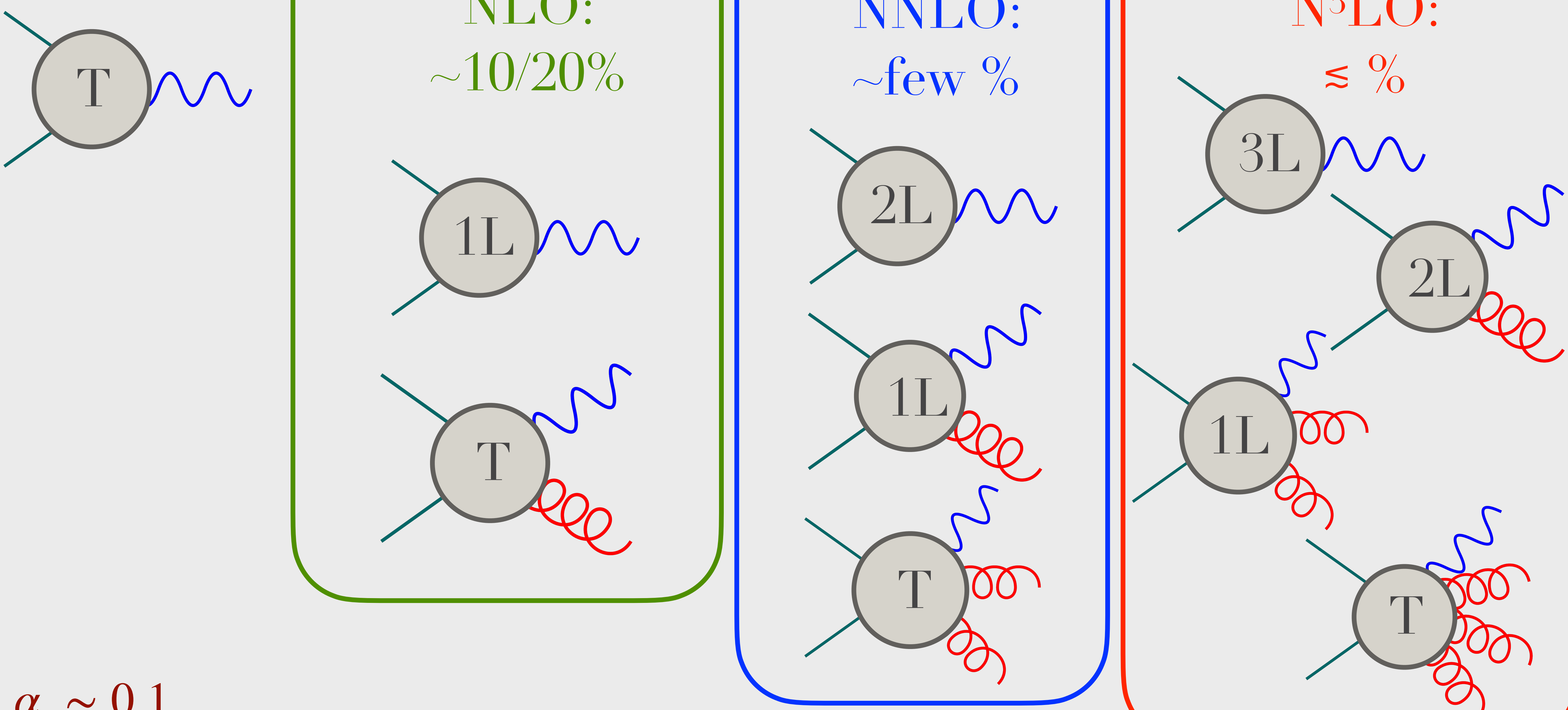
$$\mathrm{d}\sigma = \int \mathrm{d}x_1 \mathrm{d}x_2 f(x_1) f(x_2) \mathrm{d}\sigma_{\mathrm{part}}(x_1, x_2) F_J (1 + \mathcal{O}(\Lambda^n_{\mathrm{QCD}}/Q^n))$$

$$\sigma_{\mathrm{part}} = \sigma_{\mathrm{LO}} \left(1 + \alpha_s \Delta\sigma_{\mathrm{NLO}} + \alpha_s^2 \Delta\sigma_{\mathrm{NNLO}} + \alpha_s^3 \Delta\sigma_{\mathrm{N^3LO}} + \ldots\right.$$

NLO: ~10/20%

NNLO: ~few %

N^3LO: ≲ %

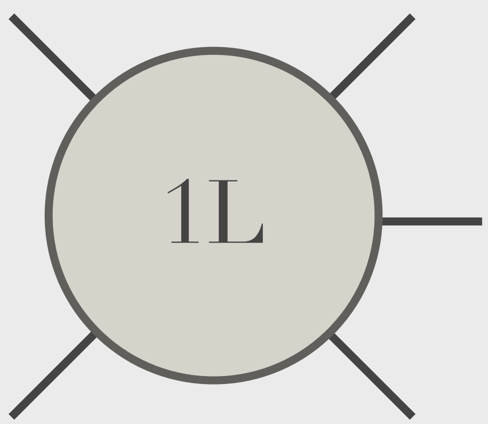


$\alpha_s \sim 0.1$

From "nice" theories to real-life QCD

"Nice" ≡ N=4 sYM, in the planar limit

Issue: QCD is non (dual) (super) conformal → most cool N=4 tricks don't work



$$\text{A0}\left(\frac{2}{9}+4\left(-2-\frac{5}{2\,\text{ep}}+\frac{1}{2}\left(-\text{Log}\left[-\frac{\mu^2}{s_{1\times 5}}\right]-\text{Log}\left[-\frac{\mu^2}{s_{3\times 4}}\right]\right)\right)+\frac{1}{3}\left(2+\frac{5}{2\,\text{ep}}+\frac{1}{2}\left(\text{Log}\left[-\frac{\mu^2}{s_{1\times 5}}\right]+\text{Log}\left[-\frac{\mu^2}{s_{3\times 4}}\right]\right)\right)\right)+$$

$$\text{A0}\left(-\frac{1}{3}+\frac{5\pi^2}{6}+\text{Log}\left[\frac{s_{1\times 5}}{s_{1\times 2}}\right]\text{Log}\left[\frac{s_{2\times 3}}{s_{3\times 4}}\right]+\text{Log}\left[\frac{s_{1\times 5}}{s_{1\times 2}}\right]\text{Log}\left[\frac{s_{3\times 4}}{s_{4\times 5}}\right]+\text{Log}\left[\frac{s_{1\times 2}}{s_{2\times 3}}\right]\text{Log}\left[\frac{s_{3\times 4}}{s_{4\times 5}}\right]+\right.$$

$$\left.\text{Log}\left[\frac{s_{1\times 2}}{s_{2\times 3}}\right]\text{Log}\left[\frac{s_{4\times 5}}{s_{1\times 5}}\right]+\text{Log}\left[\frac{s_{2\times 3}}{s_{3\times 4}}\right]\text{Log}\left[\frac{s_{4\times 5}}{s_{1\times 5}}\right]-\frac{\left(-\frac{\mu^2}{s_{1\times 2}}\right)^{\text{ep}}+\left(-\frac{\mu^2}{s_{1\times 5}}\right)^{\text{ep}}+\left(-\frac{\mu^2}{s_{2\times 3}}\right)^{\text{ep}}+\left(-\frac{\mu^2}{s_{3\times 4}}\right)^{\text{ep}}+\left(-\frac{\mu^2}{s_{4\times 5}}\right)^{\text{ep}}}{\text{ep}^2}\right)+$$

N=4 bit

$$i\left(\frac{\left(\text{L1}\left[\frac{s_{2\times 3}}{s_{1\times 5}}\right]+\text{L1}\left[\frac{s_{3\times 4}}{s_{1\times 5}}\right]+2\,\text{Ls1}\left[\frac{s_{2\times 3}}{s_{1\times 5}},\frac{s_{3\times 4}}{s_{1\times 5}}\right]\right)\langle 1|2\rangle\langle 1|4\rangle^2\langle 2|3\rangle\langle 3|4\rangle[4|2]^2}{\langle 1|5\rangle\langle 2|4\rangle^2\langle 4|5\rangle}-\frac{2\,\text{L2}\left[\frac{s_{2\times 3}}{s_{1\times 5}}\right]\langle 1|4\rangle^3\langle 2|3\rangle^2[4|2]^3}{3\,s_{1\times 5}^3\langle 1|5\rangle\langle 2|4\rangle\langle 4|5\rangle}+\right.$$

$$\frac{\langle 1|3\rangle^2[4|2]\,[5|2]}{6\,s_{1\times 5}\,s_{3\times 4}\langle 4|5\rangle}-\frac{\left(\text{L1}\left[\frac{s_{1\times 2}}{s_{3\times 4}}\right]+\text{L1}\left[\frac{s_{1\times 5}}{s_{3\times 4}}\right]+2\,\text{Ls1}\left[\frac{s_{1\times 2}}{s_{3\times 4}}\right]\right)\langle 1|2\rangle\langle 1|5\rangle\langle 2|3\rangle\langle 3|5\rangle^2[5|2]^2}{\langle 2|4\rangle^2\langle 3|4\rangle\langle 4|5\rangle}-$$

$$\frac{[4|2]^2[5|2]^2}{3\langle 4|5\rangle[2|1]\,[3|2]\,[4|3]\,[5|1]}-\frac{2\,\text{L2}\left[\frac{s_{1\times 2}}{s_{3\times 4}}\right]\langle 1|2\rangle^2\langle 3|5\rangle^3[5|2]^3}{3\,s_{3\times 4}^3\langle 2|5\rangle\langle 3|4\rangle\langle 4|5\rangle}-$$

$$\frac{\langle 1|2\rangle\langle 1|4\rangle^2\langle 1|5\rangle\langle 2|4\rangle[5|2]^3}{3\,s_{1\times 5}\langle 4|5\rangle[3|2]\,[4|3]}-\frac{\langle 2|3\rangle\langle 3|5\rangle^2[5|2]^3}{3\,s_{3\times 4}\langle 2|5\rangle\langle 3|4\rangle\langle 4|5\rangle[2|1]\,[5|1]}-$$

$$\frac{\text{L0}\left[\frac{s_{3\times 4}}{s_{1\times 5}}\right]\langle 1|3\rangle^3(\langle 1|2\rangle\langle 3|4\rangle[4|2]+\langle 1|5\rangle\langle 2|3\rangle[5|2])}{6\,s_{1\times 5}\langle 1|2\rangle\langle 1|5\rangle\langle 2|3\rangle\langle 3|4\rangle\langle 4|5\rangle}+4\left(-\frac{\text{Ls1}\left[\frac{s_{2\times 3}}{s_{1\times 5}},\frac{s_{3\times 4}}{s_{1\times 5}}\right]\langle 1|3\rangle^2\langle 1|4\rangle[4|2]^2}{s_{1\times 5}^2\langle 1|5\rangle\langle 4|5\rangle}-\right.$$

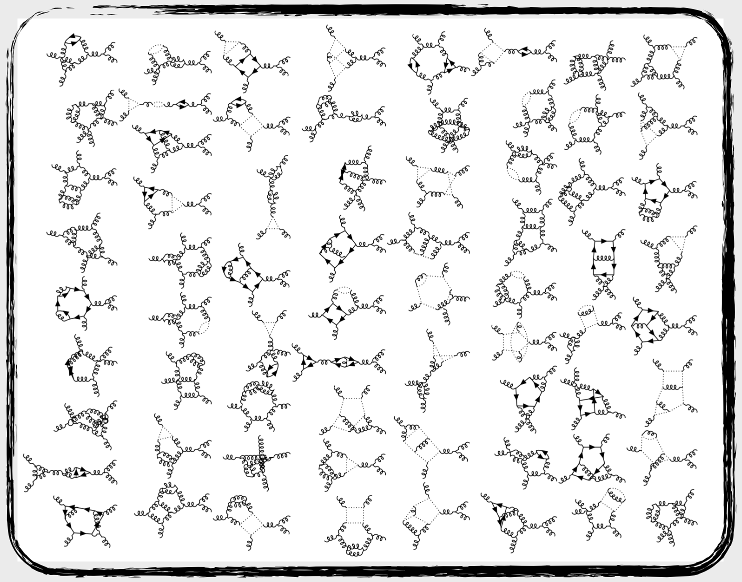
$$\left.\frac{\text{Ls1}\left[\frac{s_{1\times 2}}{s_{3\times 4}},\frac{s_{1\times 5}}{s_{3\times 4}}\right]\langle 1|3\rangle^2\langle 3|5\rangle[5|2]^2}{s_{3\times 4}^2\langle 3|4\rangle\langle 4|5\rangle}+\frac{\text{L0}\left[\frac{s_{3\times 4}}{s_{1\times 5}}\right]\langle 1|3\rangle^3(\langle 1|2\rangle\langle 3|4\rangle[4|2]+\langle 1|5\rangle\langle 2|3\rangle[5|2])}{2\langle 1|2\rangle\langle 1|5\rangle\langle 2|3\rangle\langle 3|4\rangle\langle 4|5\rangle}\right)+$$

$$\left.\frac{\text{L2}\left[\frac{s_{3\times 4}}{s_{1\times 5}}\right]\left(-\frac{2\langle 1|2\rangle^2\langle 1|4\rangle\langle 3|4\rangle^2[4|2]^3}{3\langle 1|5\rangle\langle 2|4\rangle\langle 4|5\rangle}-\frac{2\langle 1|5\rangle^2\langle 2|3\rangle^2\langle 3|5\rangle[5|2]^3}{3\langle 2|5\rangle\langle 3|4\rangle\langle 4|5\rangle}+\frac{\langle 1|3\rangle[4|2]\,[5|2]\,(\langle 1|2\rangle\langle 3|4\rangle[4|2]+\langle 1|5\rangle\langle 2|3\rangle[5|2])}{3\langle 4|5\rangle}\right)}{s_{1\times 5}^3}\right)$$

QCD amplitudes: how to compute them

No "N=4"-like tricks (nice integrands…) → "textbook" way ☹️

$$\mathscr{A} = \sum$$



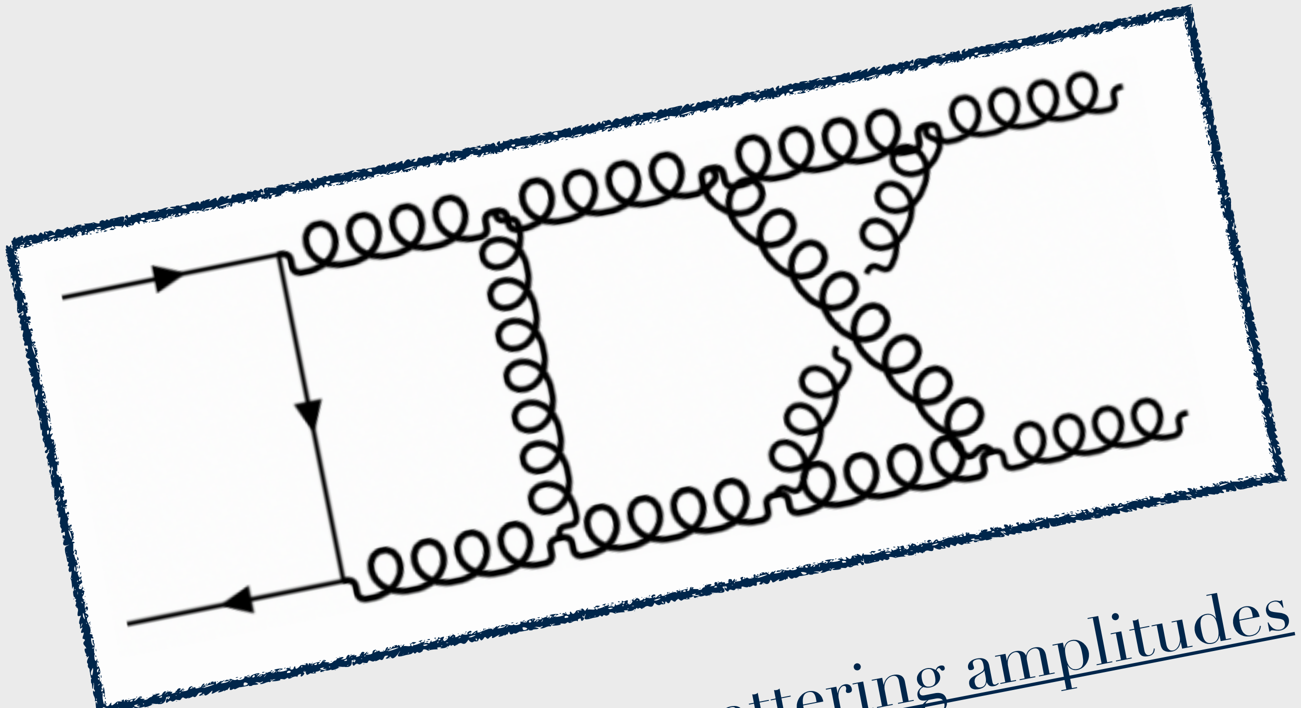
$$\int \mathrm{d}^d k \;\; \partial_\mu [v^\mu \mathscr{I}(k)] \; = \; 0$$

Integr<u>al</u> reduction, IBPs

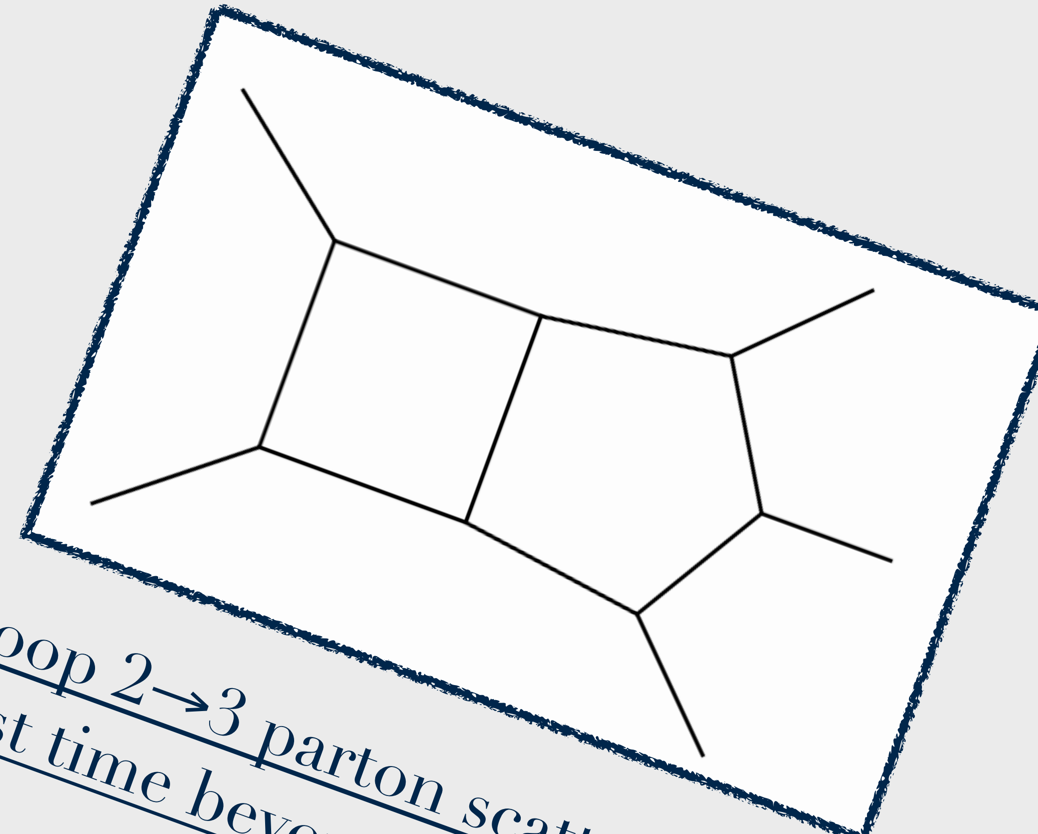
Major bottleneck

$$\mathscr{A} = \sum c_i \times \mathrm{MI}_i$$

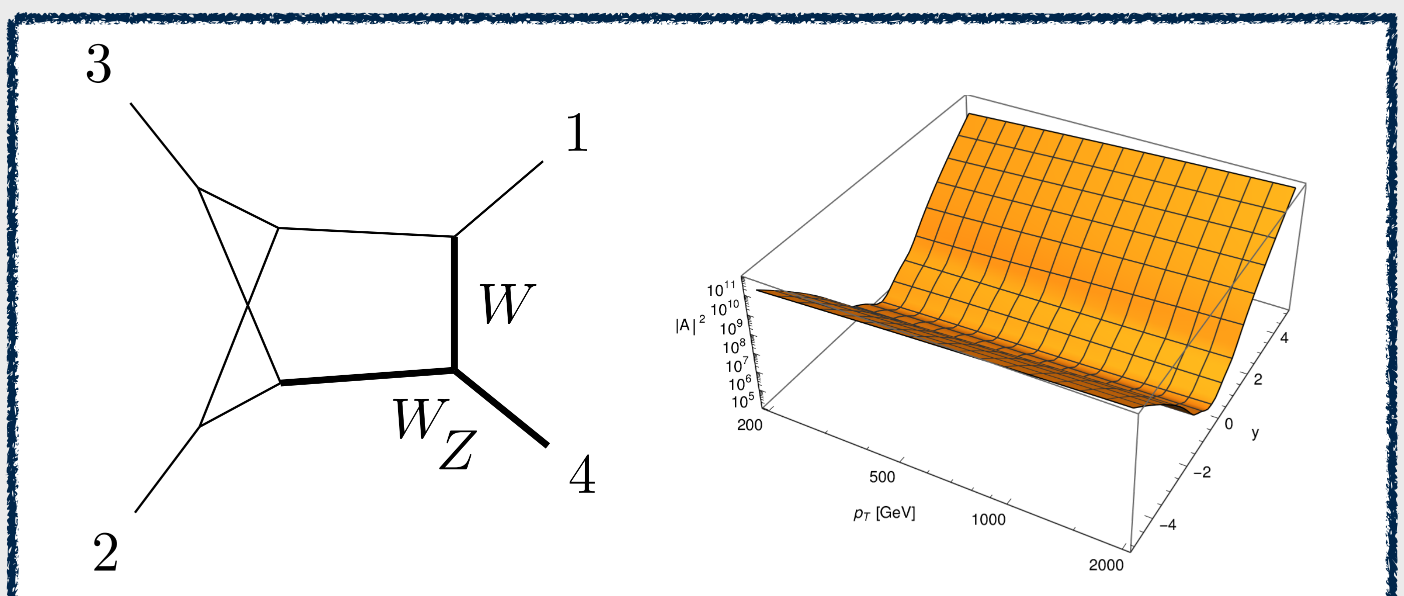
Some recent results from us & friends



First QCD 3-loop scattering amplitudes



2-loop 2→3 parton scattering, for the first time beyond the large-N_c limit

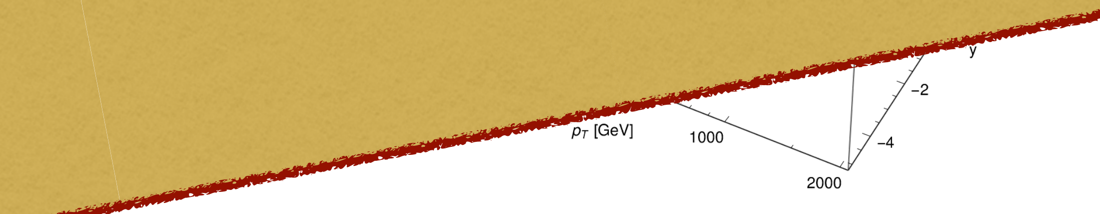


Numerical methods for complex Electroweak amplitudes

Some recent results from us & frien

Sub-leading N_c results open the way to many interesting investigations:

- High-energy limit: sensitive to “Regge cuts” in the complex angular momentum plane (observable@3L)
- Soft physics beyond the dipole picture and potential breaking of collinear factorisation (2L&3L)



[Numerical methods for complex ElectroWeak amplitudes]

The problem with IBPs

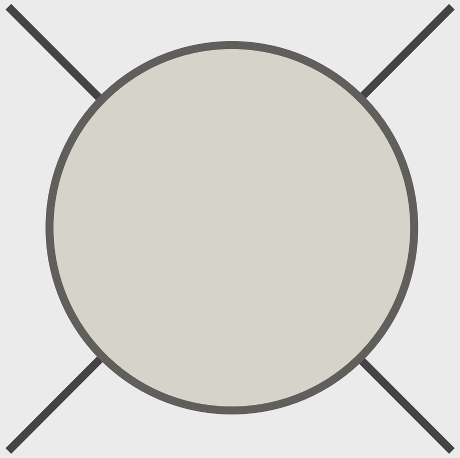
$$\int d^d k \;\; \partial_\mu [v^\mu \mathcal{I}(k)] \; = \; 0 \qquad\qquad \mathcal{I}(k) = \frac{\mathcal{N}(k, \{p_i\})}{D_1 \ldots D_n}$$

Issue:

- IBPs generate a lot of irrelevant junk
- IR/UV mixing

→

- Huge intermediate results
- Underlying physics obscured



	1L	2L	3L
Number of diagrams	6	138	3299
Number of integral topologies	1	2	3
Number of integrals before IBPs and symmetries	209	20935	4370070
Number of master integrals	6	39	486
Size of the Feynman diagrams list [kB]	4	90	2820
Size of the result before integral reduction [kB]	276	54364	19734644
Size of the result in terms of MIs [kB]	12	562	304409
Size of the result in terms of HPLs [kB]	136	380	1195

Interesting way forward: intersection theory

$$\mathscr{A} = \sum c_i \times \mathrm{MI}_i$$

Can we "project" over MIs? $c_i = \langle \widetilde{\mathrm{MI}}_i \,|\, \mathscr{A} \rangle$

$$I = \int \prod d^d k_i \frac{\mathcal{N}}{D_1 \ldots D_n} \longrightarrow \int \frac{dz_1 \ldots dz_n\, f(\mathbf{z})}{z_1^{n_1} \ldots z_n^{n_n}} \times u(\mathbf{z}) = \int_{\mathbf{C_R}} \mathbf{u}(\mathbf{z}) \varphi_{\mathbf{L}}(\mathbf{z})$$

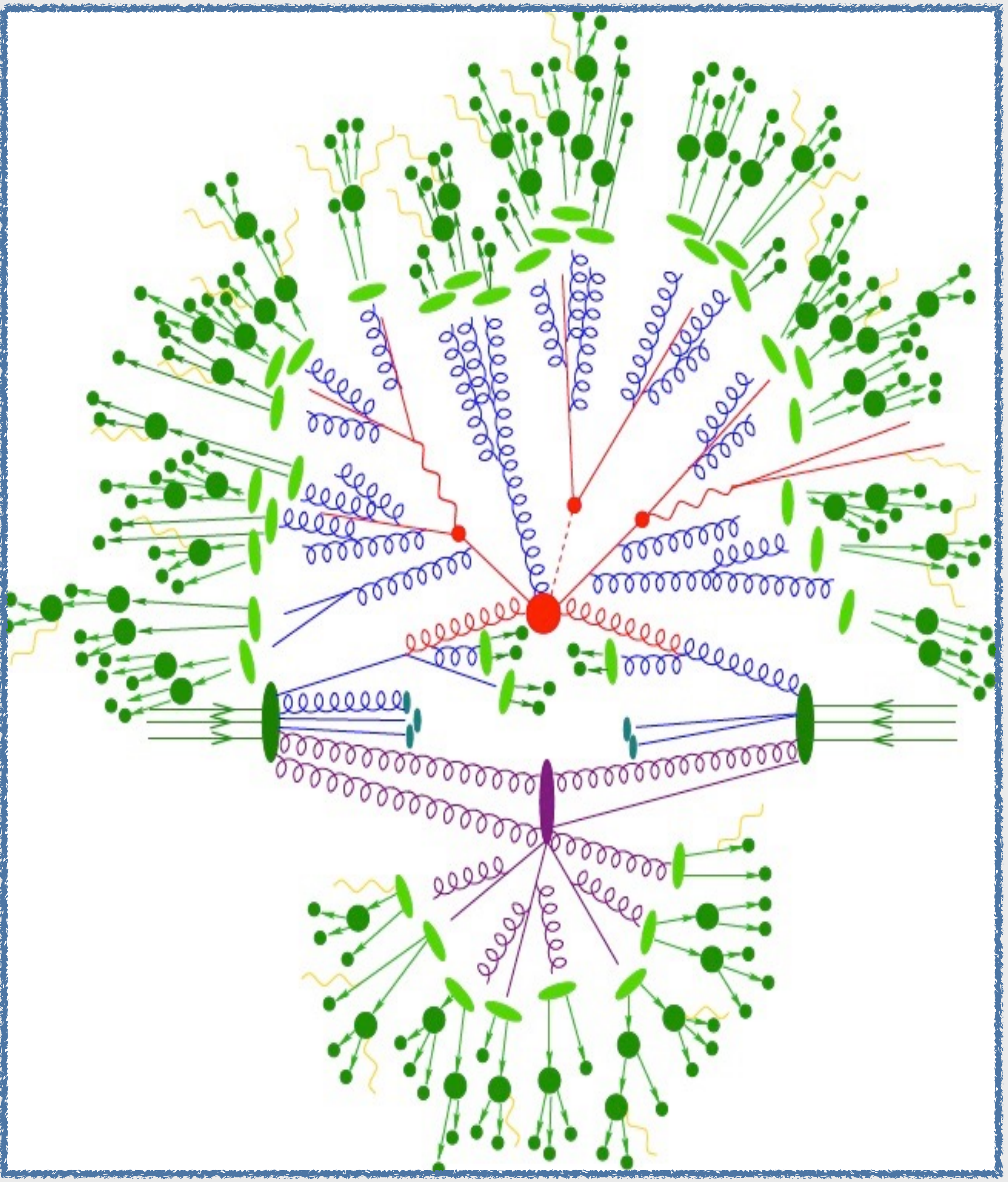
$$\varphi \sim \varphi_L + (d + \omega\wedge)\xi, \quad \omega = d\ln(u) \longrightarrow \varphi_L \in H^n_\omega$$

$$I = \langle \varphi_L C_R] = \sum c_i \langle e_i C_R] \longrightarrow c_i = \langle \varphi_L e_j \rangle [\langle e_j e_i \rangle]^{-1}, \quad | e_i \rangle \in H^n_{-\omega}$$

From high-energy scattering to observable final states: parton showers

[Graphics: Gavin Salam]

Parton showers in a nutshell



- Hard process: high-scale, perturbative, few legs…
- … triggering a cascade of QCD radiation, down to hadronic scale
- Up to the very last step, computable from pQCD
- ~ Markov chain semi (but not entirely) classical process

$$d\mathscr{P}_{\text{split}} \sim \alpha_s(k_t)\frac{dk_t^2}{k_t^2}\frac{dz}{z}$$

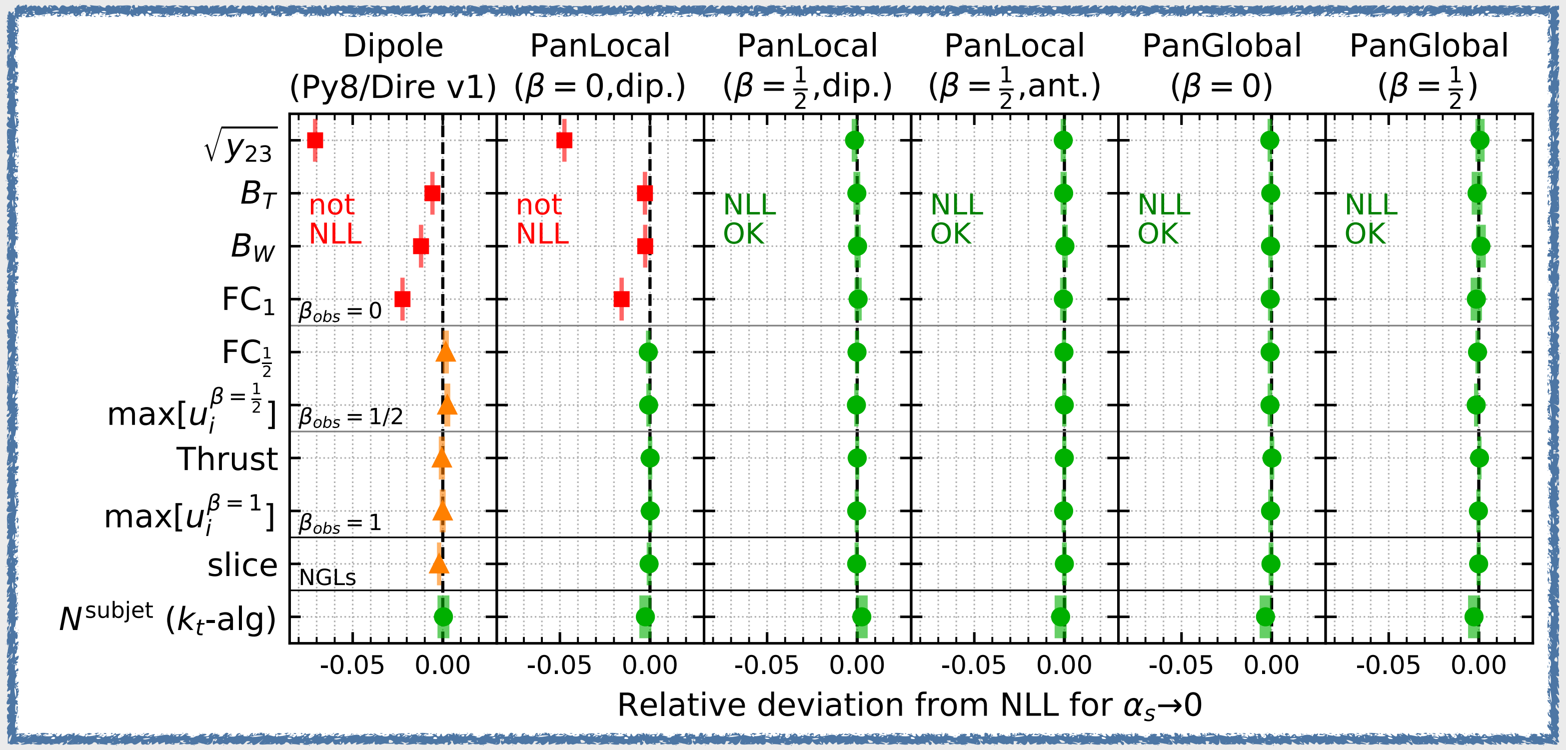
The two faces of parton showers

- PS 1: a highly tuneable device, able to accurately reproduces datas over a multitude of configurations (*"with 4 parameters I can fit an elephant"*)
  - huge amount of intuition, ingenuity; a*b initio* control not required

- PS 2: a highly predictive tool, till the hadronic scale. Exquisite control *ab initio* mandatory
  - more and more crucial with the rise of AI/ML: PS predictions are "the truth" that we feed the machines…
  - highly non-trivial, both technically and conceptually, to go beyond the standard "(N)LL" paradigm

Recent results from the group & friends

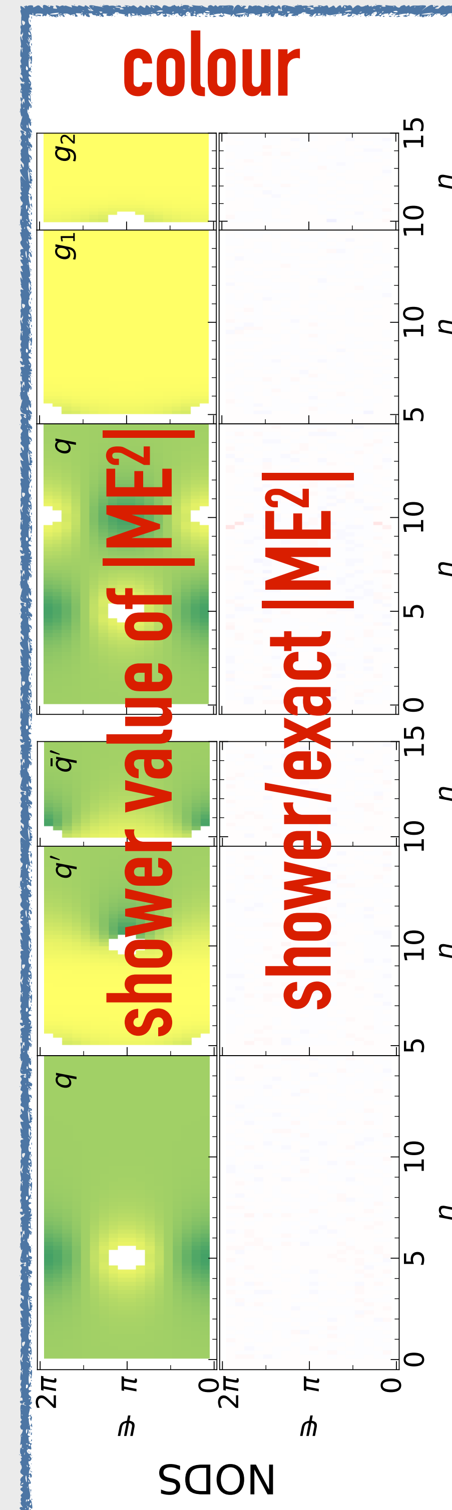
A robust theoretical definition for "PS accuracy" + the first NLL shower



Steps towards higher-order PS

$$
\begin{aligned}
\mathbb{K}_q^{\rm R}[G_q, G_g] = & \sum_{(A)} \frac{1}{S_2} \int d\Phi_3^{(A)} P_{1\to3}^{(A)} \Big\{ G_{f_1}(x\, z_p\, (1-z), t_{1,2})\, G_{f_2}(x\,(1-z_p)\,(1-z), t_{1,2}) \\
& \times G_q(x\, z, t_{12,3}) - G_{f_{12}}(x\,(1-z), t_{12,3})\, G_q(x\,z, t_{12,3}) \Big\} \frac{\Delta_q(t)}{\Delta_q(t_{1,2})} \\
& + \int d\Phi_3^{(B)} P_{1\to3}^{(B)} \Big\{ G_g(x\,(1-z), t_{1,23})\, G_g(x\, z\,(1-z_p), t_{2,3}) \\
& \times G_q(x\, z\, z_p, t_{2,3}) - G_g(x\,(1-z), t_{1,23})\, G_q(x\,z, t_{1,23}) \Big\} \frac{\Delta_q(t)}{\Delta_q(t_{2,3})}\, \Theta(t_{2,3} - t_{1,3})\,, \qquad \text{(C.2)}
\end{aligned}
$$

Accurate PS: beyond large-N_c



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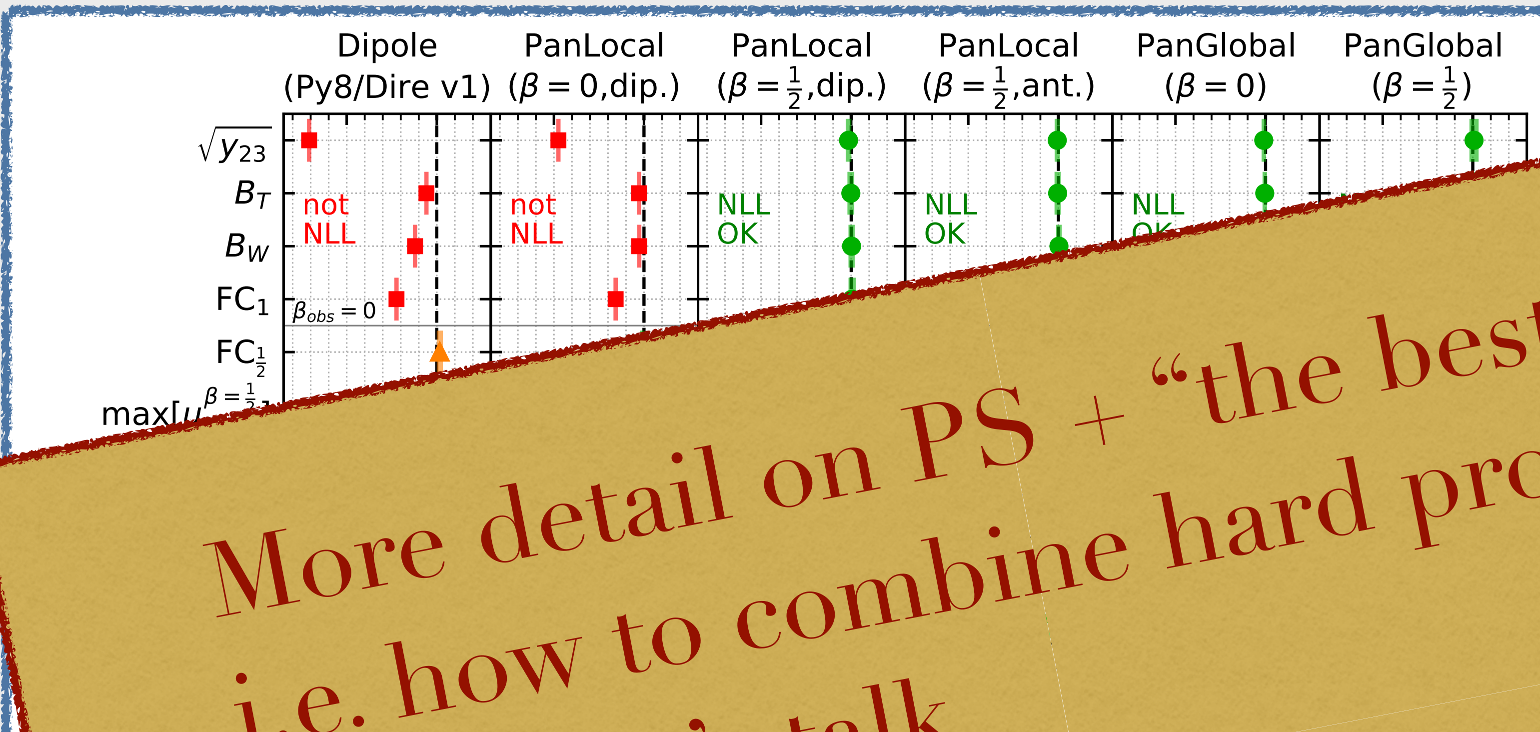


More detail on PS + "the best of both worlds",
i.e. how to combine hard process + PS →
see Silvia's talk

…gher-order PS

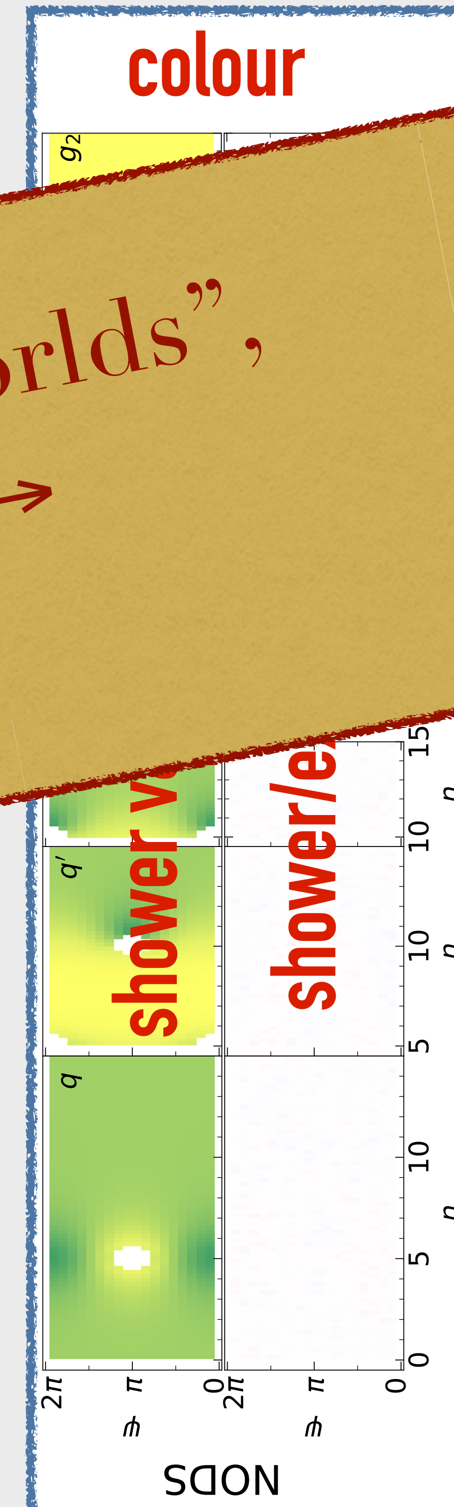
$$\Sigma_{(A)} \frac{1}{S_2} \int d\Phi_3^{(A)} P_{1\to 3}^{(A)} \Big\{ G_{f_1}(x\, z_p\, (1-z), t_{1,2})\, G_{f_2}(x\,(1-z_p)\,(1-z), t_{1,2})$$

$$\times G_q(x\, z, t_{12,3}) - G_{f_{12}}(x\,(1-z), t_{12,3})\, G_q(x\, z, t_{12,3}) \Big\} \frac{\Delta_q(t)}{\Delta_q(t_{1,2})}$$

$$+ \int d\Phi_3^{(B)} P_{1\to 3}^{(B)} \Big\{ G_g(x\,(1-z), t_{1,23})\, G_g(x\, z\,(1-z_p), t_{2,3})$$

$$\times G_q(x\, z\, z_p, t_{2,3}) - G_g(x\,(1-z), t_{1,23})\, G_q(x\, z, t_{1,23}) \Big\} \frac{\Delta_q(t)}{\Delta_q(t_{2,3})}\, \Theta(t_{2,3} - t_{1,3})\,, \qquad \text{(C.2)}$$

Accurate PS:



Making use of all of this: pheno studies

An example: jet flavour

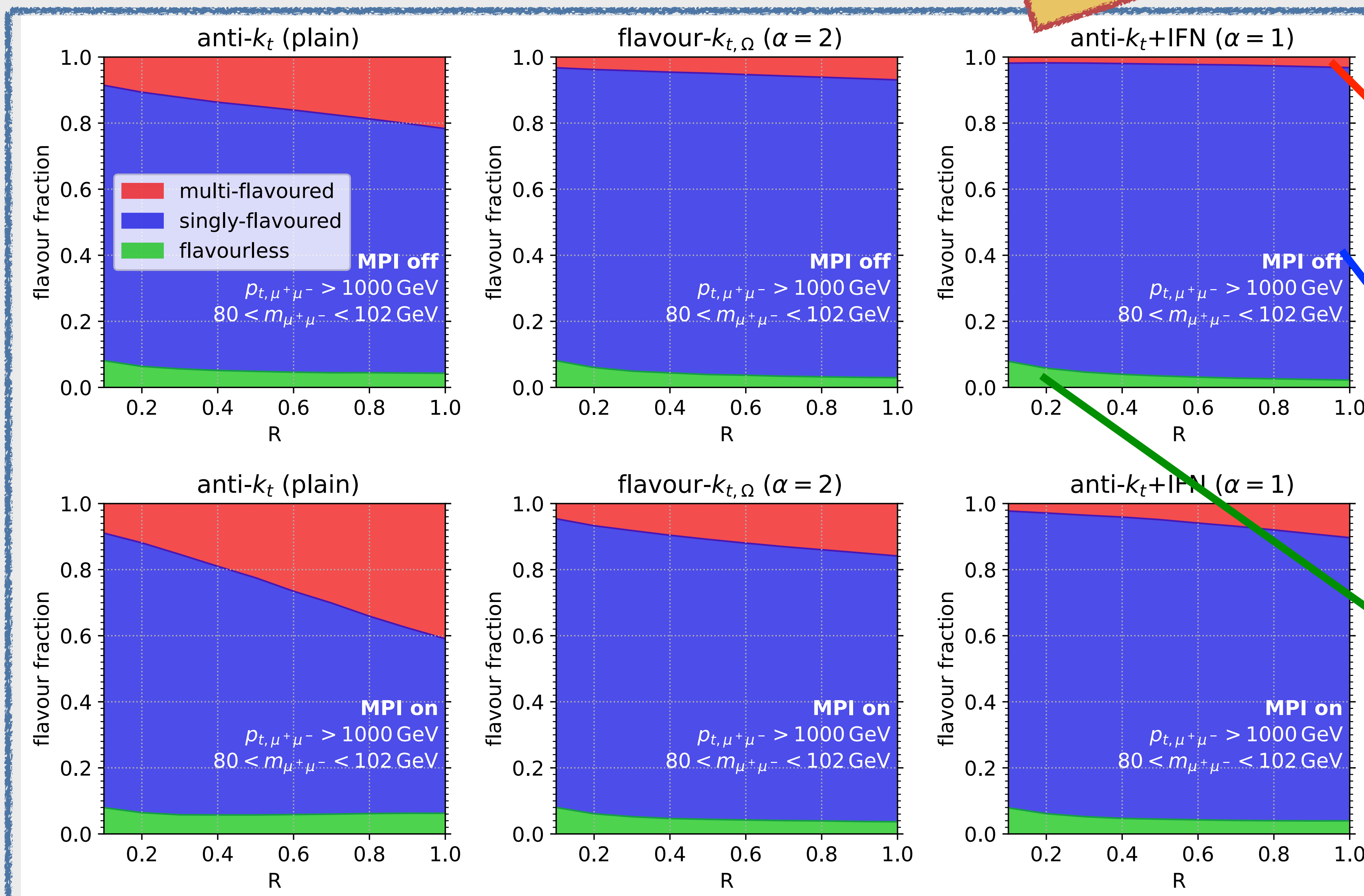
- What we want
  - back to the start: we want to measure Higgs Yukawa couplings → H → bb(cc…) decay → need to measure "b-quarks"
  - many (B)SM resonances decay to quarks/gluons → want to measure them
- The obvious problem
  - quark and gluons are not asymptotic states… *"what is a b-quark?"*
- The "in principle" solution…
  - find observables which are strongly correlated to "I had a b-quark in the hard process", yet insensitive to IR physics. *"Jet flavour"*
- The "good" solution
  - something you can compute, to high precision
  - something that experimentalists can measure, to high precision
  - something that is flexible enough for a wide array of studies

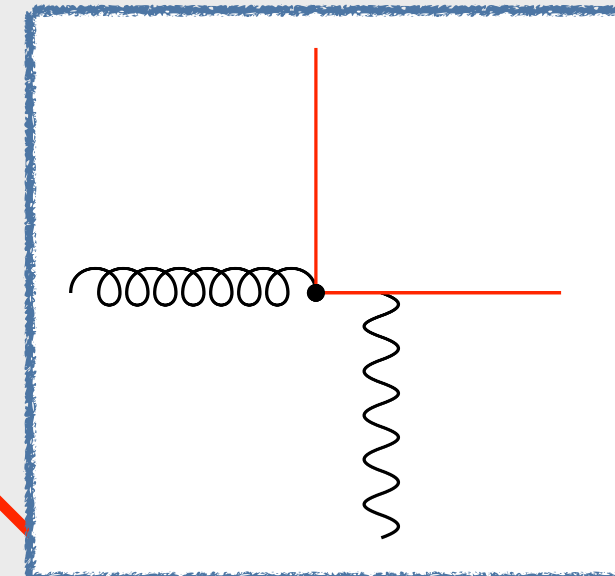
A “jet flavour” for the modern era

EXP 😀 EXP ☹️

TH ☹️ TH 😀

!!!NEW!!!





contamination

b-quark

flavourless

A final disclaimer: only a subset of what the group is doing, more stuff going on

(heavy-ions, IR subtractions, non-perturbative effects, Higgs physics…)

If you want to hear more: ask us/ come to our Friday 1pm-2pm Journal Club (also on zoom)

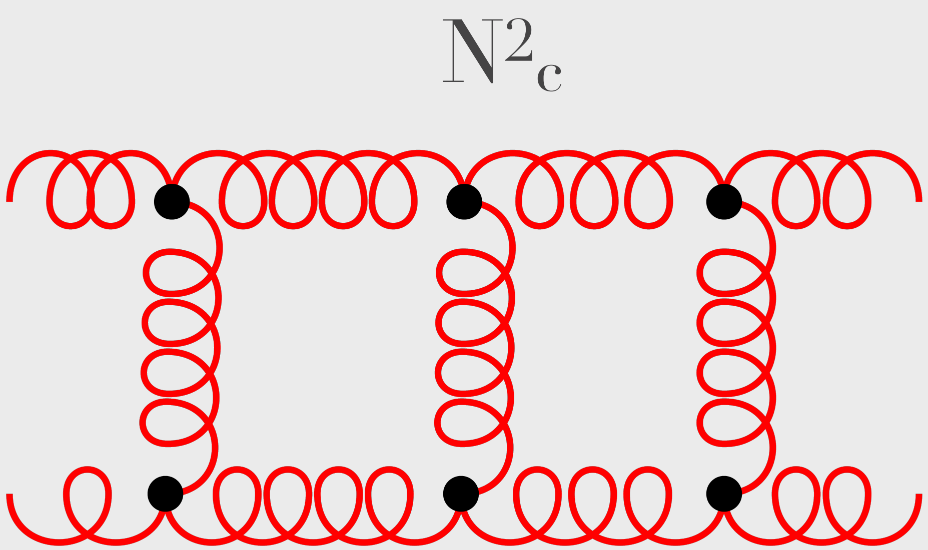
Thank you very much for your attention

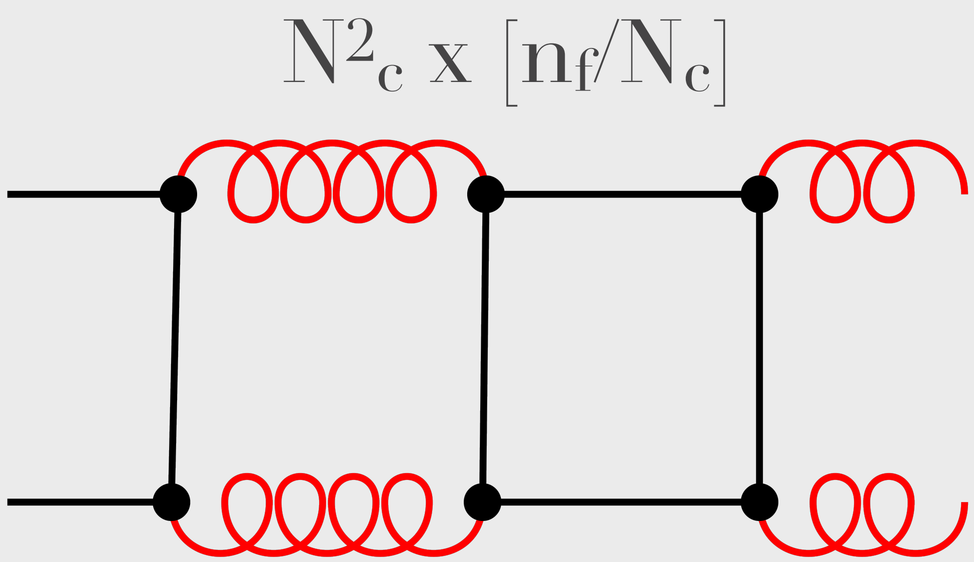
From “nice” theories to real-life QCD: issues

“Nice” ≡ N=4 sYM, in the planar limit

Issue: QCD is non (dual) (super) conformal → most cool N=4 tricks don’t work

Issue: EW particles → leading-N_c ≠ planar





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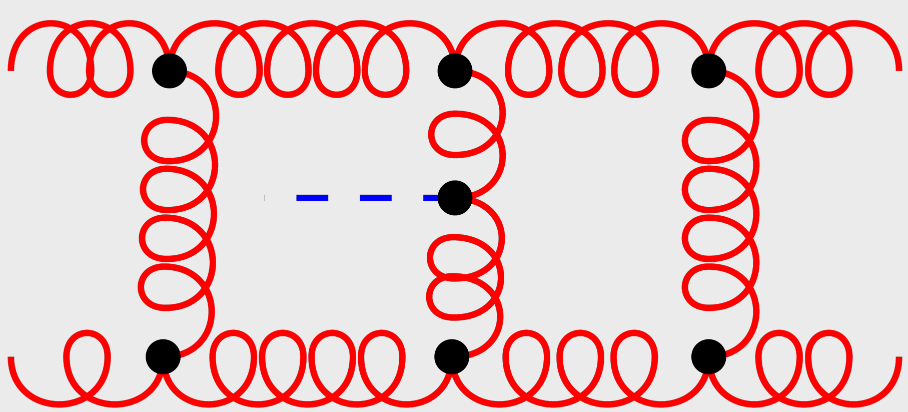
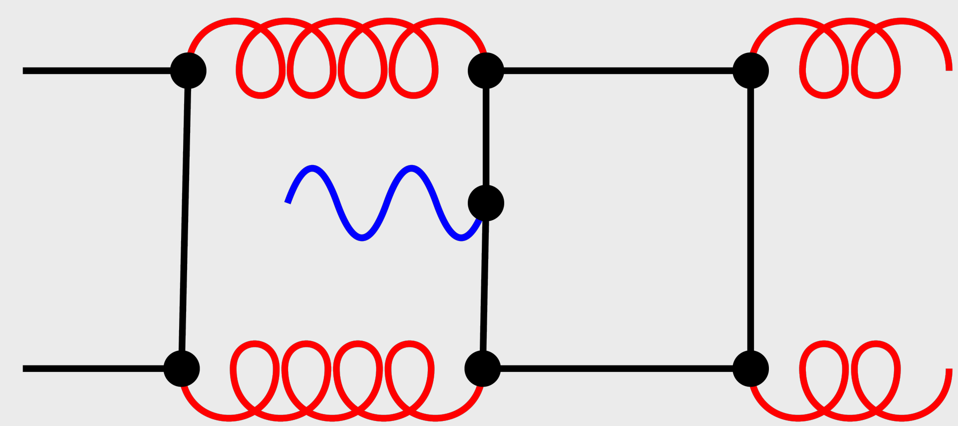
Issue: EW particles → leading-N_c ≠ planar

N^2_c



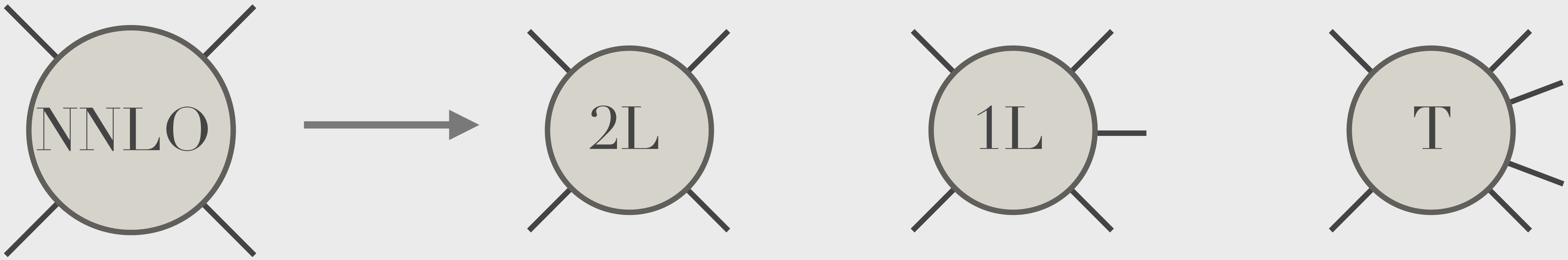
N^2_c x $[n_f/N_c]$

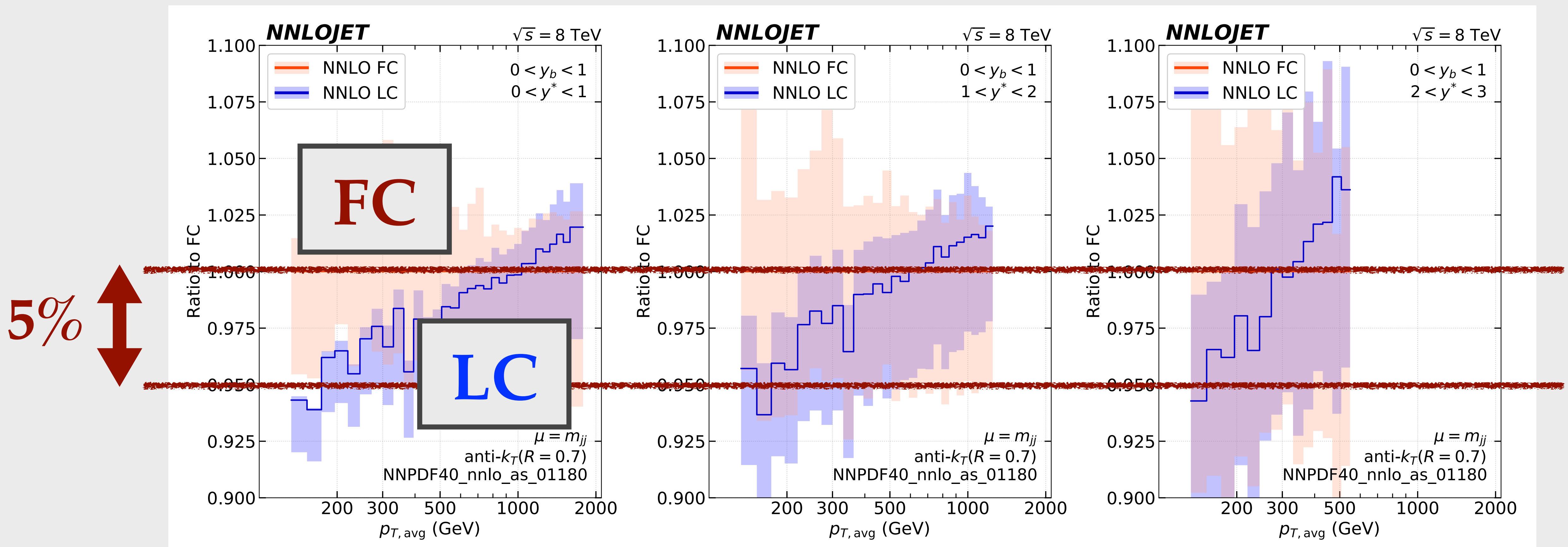


No parametric enhancement of the "simple" contributions

Beyond leading colour: 2

Even in pure QCD: sub-leading N_c can be enhanced in regions of the phase-space

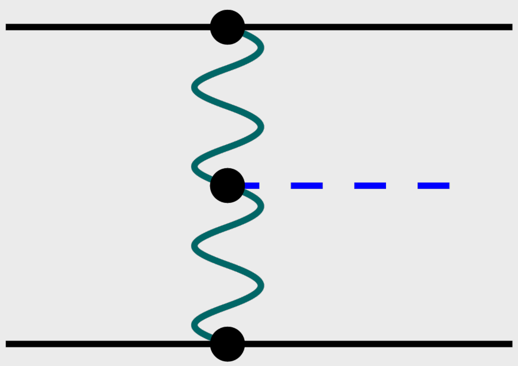




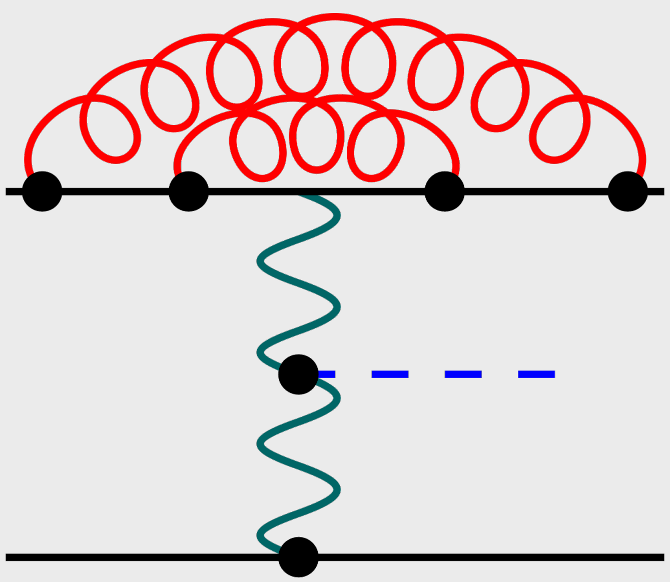
[Chen, Gehrmann, Glover, Huss, Mo (2022)]

Beyond leading colour: 3

In some cases: $1/N_c^2$ x $[i\pi]^2 \rightarrow$ no longer suppressed

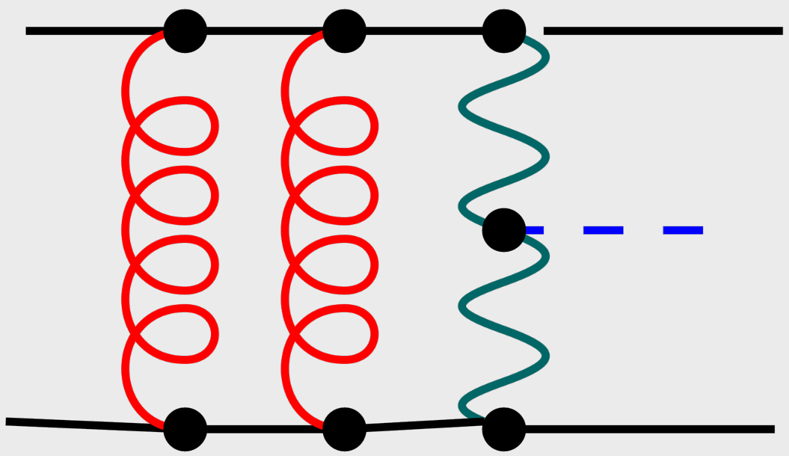


x



VS

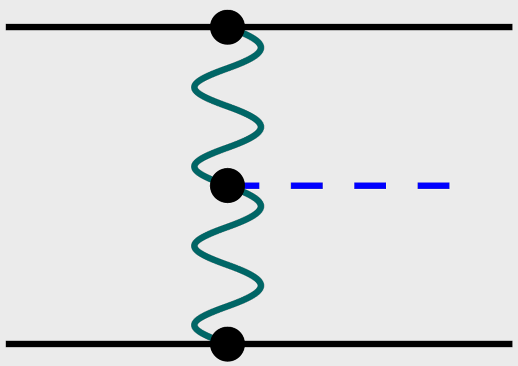
$1/N_c^2$



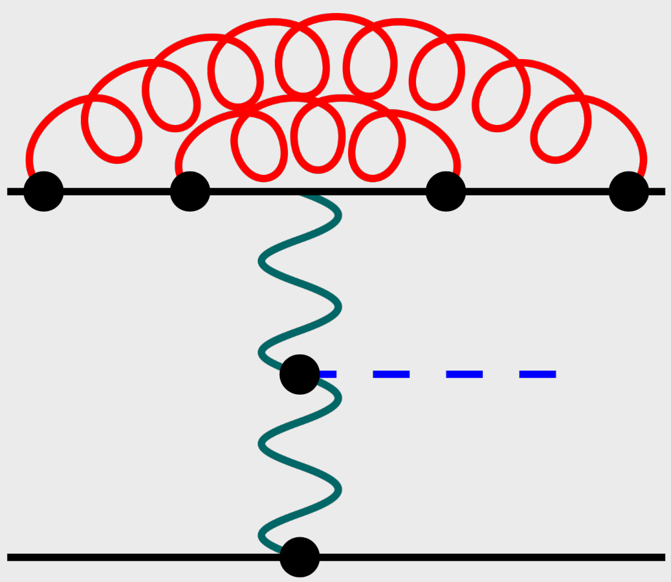
δNNLO/NLO: 3%

Beyond leading colour: 3

In some cases: $1/N_c^2$ x $[i\pi]^2 \rightarrow$ no longer suppressed

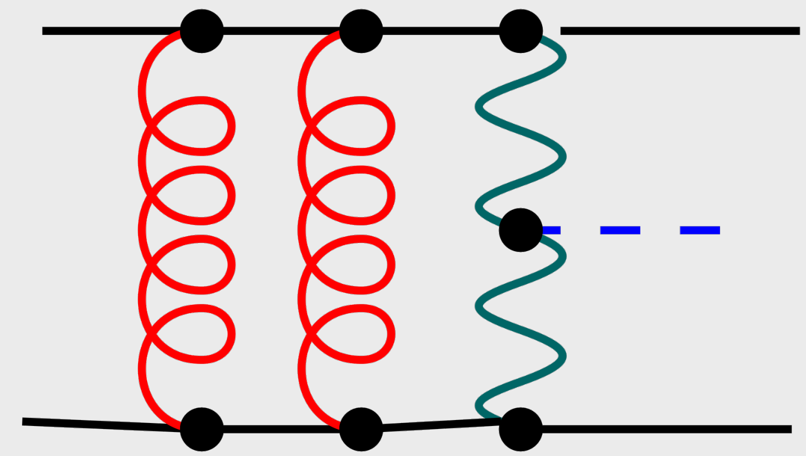


x



VS

$1/N_c^2$ x π^2

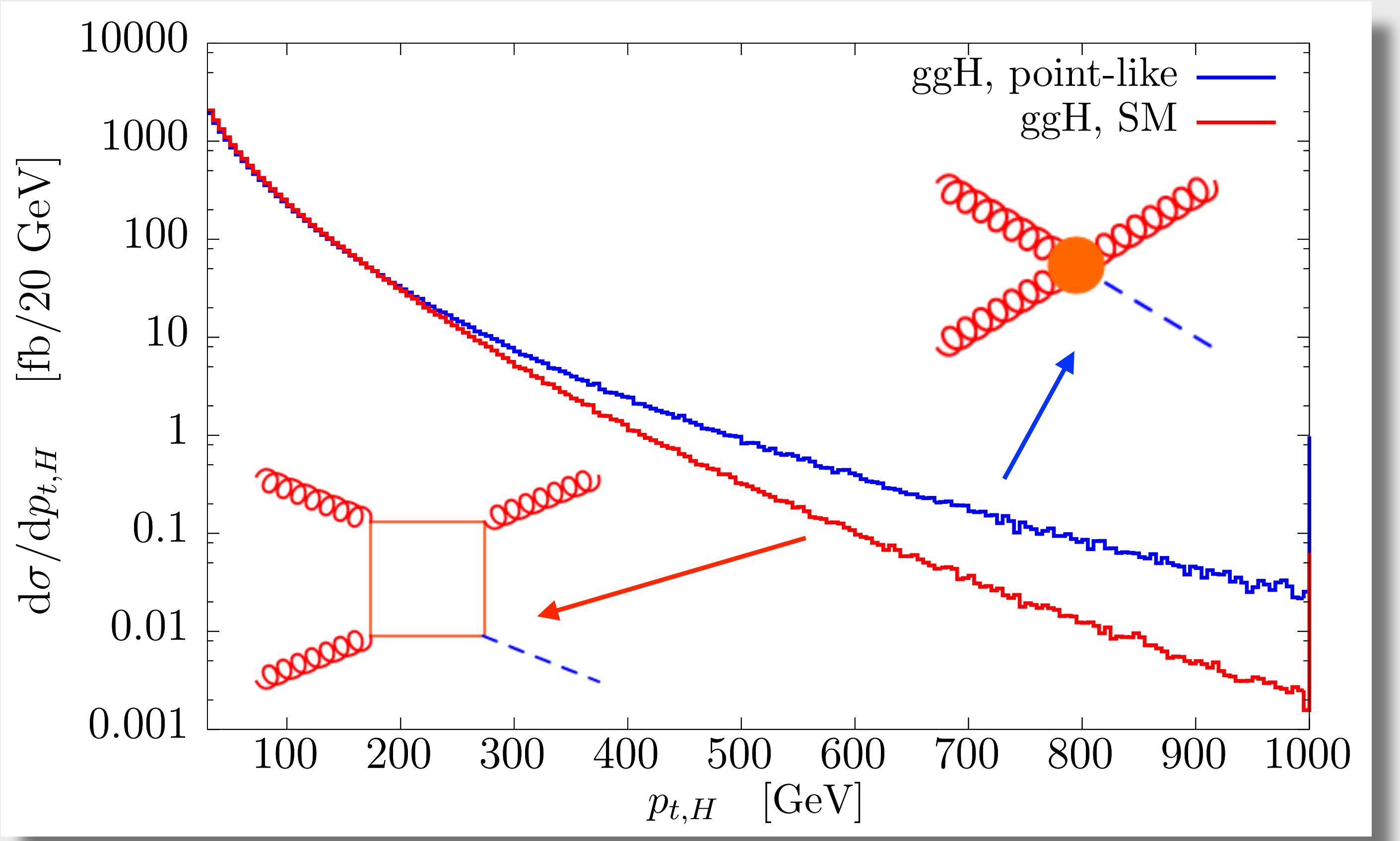


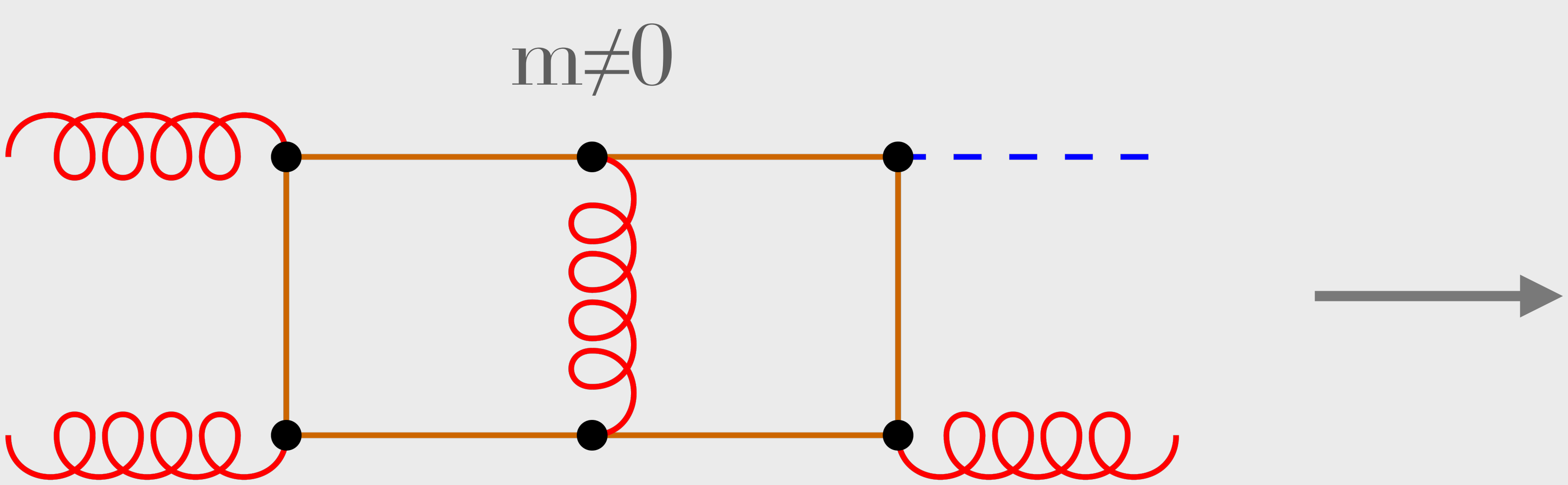
δNNLO/NLO: 3%

δNNLO/NLO: up to 1.5%

[Liu, Melnikov, Penin (2019)]

High scale: top quark effects





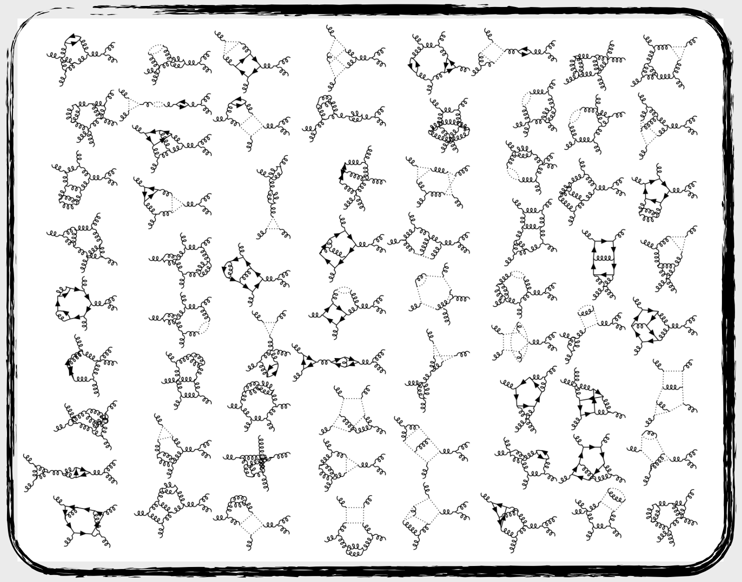
Elliptic sectors

similar problems for EW corrections, massive W/Z

QCD amplitudes: how to compute them

No "N=4"-like tricks (nice integrands…) → "textbook" way ☹️

$$\mathscr{A} = \sum$$



$$\int \mathrm{d}^d k \;\; \partial_\mu [v^\mu \mathscr{I}(k)] \;=\; 0$$

Integral reduction, IBPs

see T. Peraro's talk

$$\mathscr{A} = \sum c_i \times \mathrm{MI}_i$$

QCD amplitudes: how to compute them

$$\mathscr{A} = \sum c_i \times \mathrm{MI}_i$$

Computing MIs: binary situation

1. They are GPLs → 😀
2. They are not (elliptic, CY…) → hic sunt leones 😞

QCD amplitudes: how to compute them

$$\mathscr{A} = \sum c_i \times \mathrm{MI}_i$$

Computing MIs: binary situation

1. They are GPLs → 😀
2. They are not (elliptic, CY…) → hic sunt mansuefacti leones

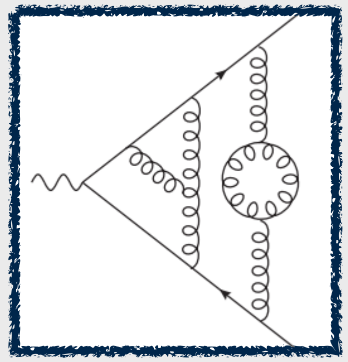
Lot of progress in understanding space of required functions, and in providing fast and reliable numerical evaluations

$$\mathrm{E}_4\left(\begin{smallmatrix} n_1 & \dots & n_m \\ c_1 & \dots & c_m \end{smallmatrix}; z, \vec{q}\right) = \int_0^z dt\, \psi_{n_1}(c_1, t, \vec{q})\, \mathrm{E}_4\left(\begin{smallmatrix} n_2 & \dots & n_m \\ c_2 & \dots & c_m \end{smallmatrix}; t, \vec{q}\right)$$

…ask the experts in the audience!

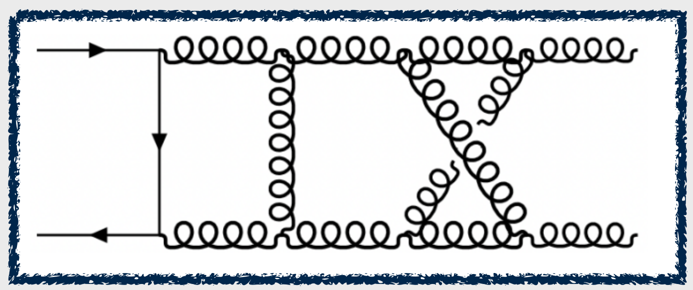
QCD amplitudes: status

Form factor: 4 loops



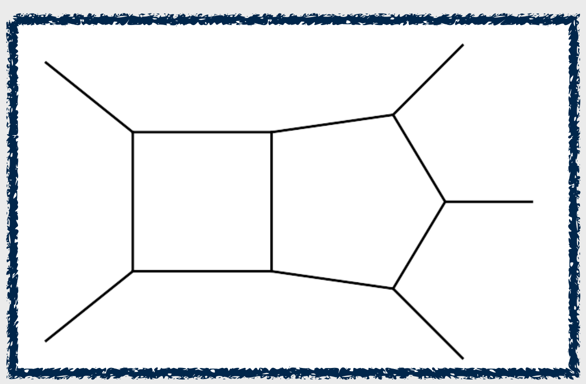
Lee, von Manteuffel, Schabinger, Smirnov, Smirnov, Steinhauser (2022)

2→2 scattering: 3 loops



[Bargiela, Chakraborty, FC, von Manteuffel, Tancredi]

2→3: 2 loops



[Abreu, Badger, Brønnum-Hansen, Bargiela, Borowka, Buccioni, Chawdhry, Chen, Chicherin, Czakon, de Laurentis, Dormans, Duhr, Dunbar, Febres-Cordero, Frellesvig, Gambuti, Gehrmann, Gluza, Hartanto, Heinrich, Henn, Ita, Jones, Jehu, Kajda, Kosower, Liu, Lo Presti, Manteuffel, Ma, Maître, Mitev, Mitov, Page, Peraro, Perkins, Poncelet, Schabinger, Sotnikov, Tancredi, Usovitch, Wasser, Weinzierl, Zhang…]

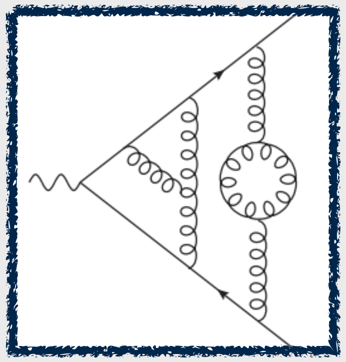
2→n≥4: 1-loop, numerical

OpenLoops, Collier, MadLoops, Recola, GoSam, Ninja, Blackhat, Rocket… up to 20 (!) gluons

QCD amplitudes: status

Form factor:

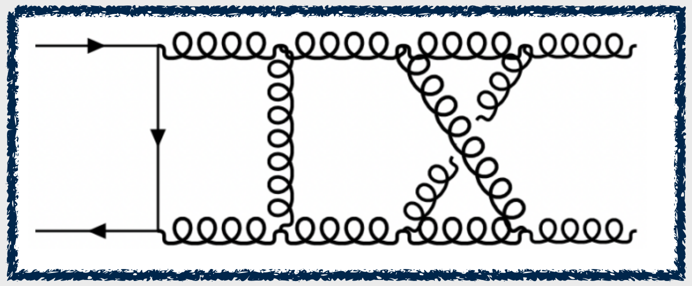
4 loops



Lee, von Manteuffel, Schabinger, Smirnov, Smirnov, Steinhauser (2022)

2→2 scattering:

3 loops

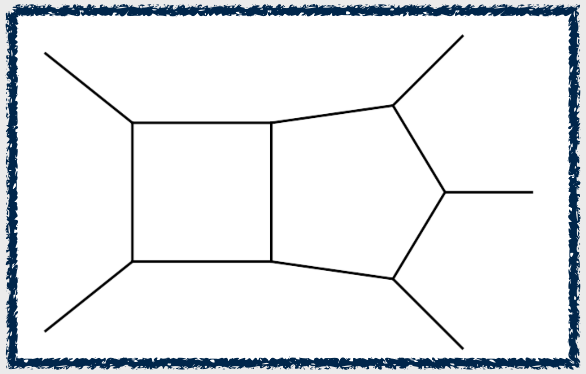


NEW: planar result with one off-shell leg

Gehrmann, Jacubčík, Mella, Syrrakos, Tancredi (2023)

2→3:

2 loops

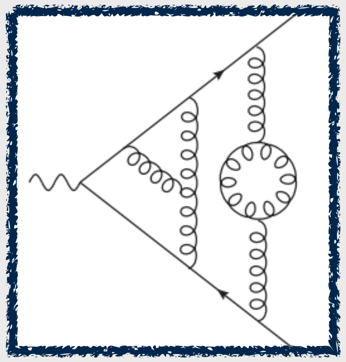


2→n≥4: 1-loop, numerical

OpenLoops, Collier, MadLoops, Recola, GoSam, Ninja, Blackhat, Rocket… up to 20 (!) gluons

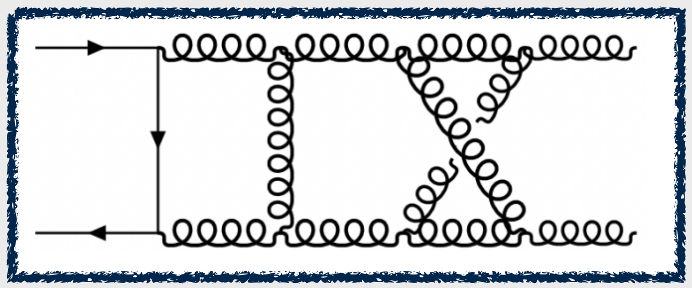
QCD amplitudes: status

Form factor: 4 loops

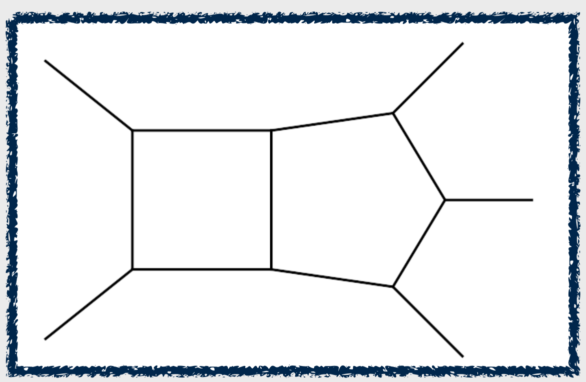


Lee, von Manteuffel, Schabinger, Smirnov, Smirnov, Steinhauser (2022)

2→2 scattering: 3 loops



2→3: 2 loops



NEW: fast and efficient basis functions with one of-shell leg, planar and non-planar

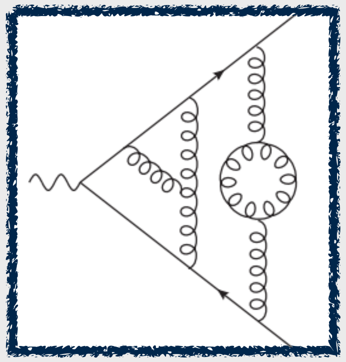
Abreu, Chicherin, Ita, Page, Sotnikov, Tschernow, Zoia (2023)

2→n≥4: 1-loop, numerical

OpenLoops, Collier, MadLoops, Recola, GoSam, Ninja, Blackhat, Rocket… up to 20 (!) gluons

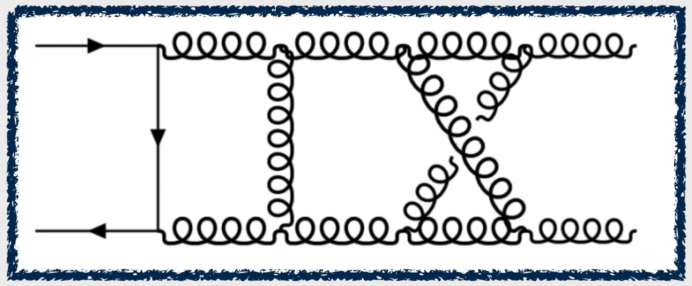
QCD amplitudes: status

Form factor: 4 loops



Lee, von Manteuffel, Schabinger, Smirnov, Smirnov, Steinhauser (2022)

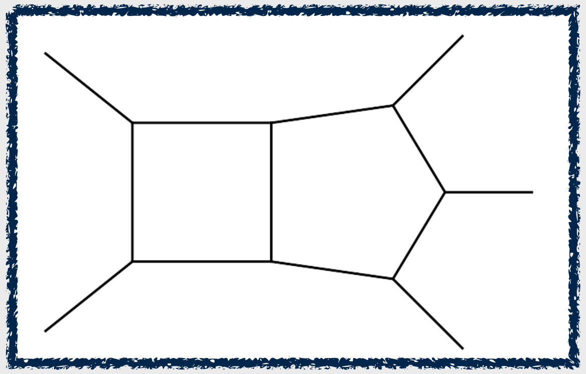
2→2 scattering: 3 loops



NEW: non-planar MIs with one off-shell leg (new letters, ~~adjacency~~)

Henn, Lim, Bobadilla (2023)

2→3: 2 loops

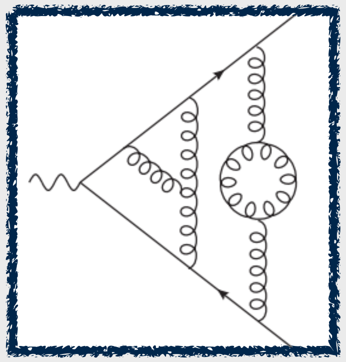


2→n≥4: 1-loop, numerical

OpenLoops, Collier, MadLoops, Recola, GoSam, Ninja, Blackhat, Rocket… up to 20 (!) gluons

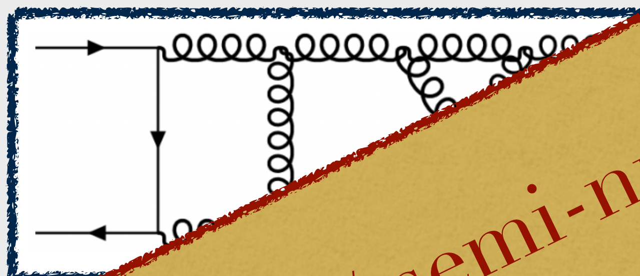
QCD amplitudes: status

Form factor: 4 loops



Lee, von Manteuffel, Schabinger, Smirnov, Smirnov, Steinhauser (2022)

2→2 scattering: 3 loops



Bobadilla (2023)

+numerical / semi-numerical results,
especially for 2→2 (sector decomposition,
numerical diff. eq., Fröbenius expansions…)

2→3:

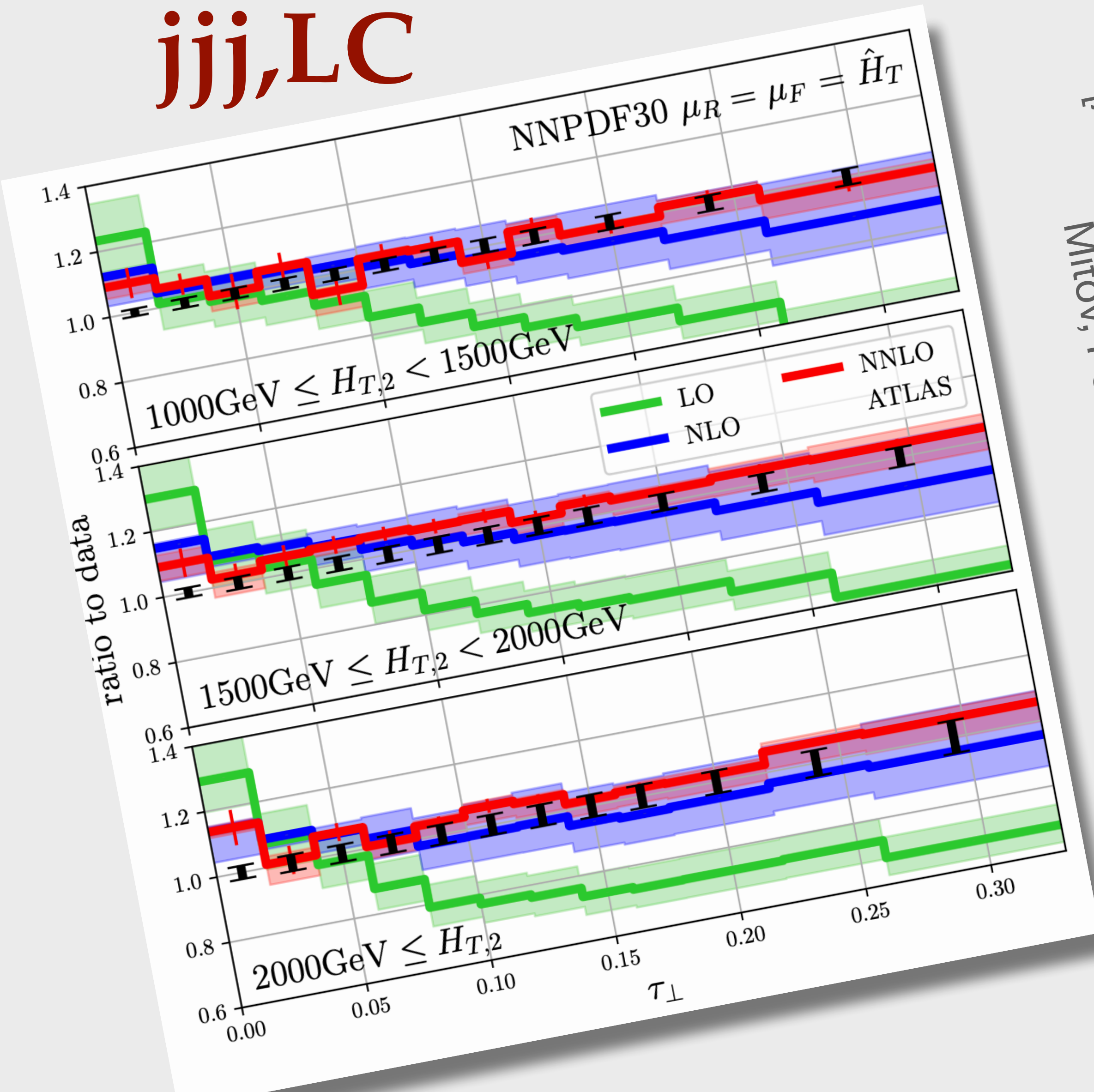
2→n≥4: 1-loop, numerical

OpenLoops, Collier, MadLoops, Recola, GoSam, Ninja, Blackhat, Rocket… up to 20 (!) gluons

From amplitudes to phenomenology

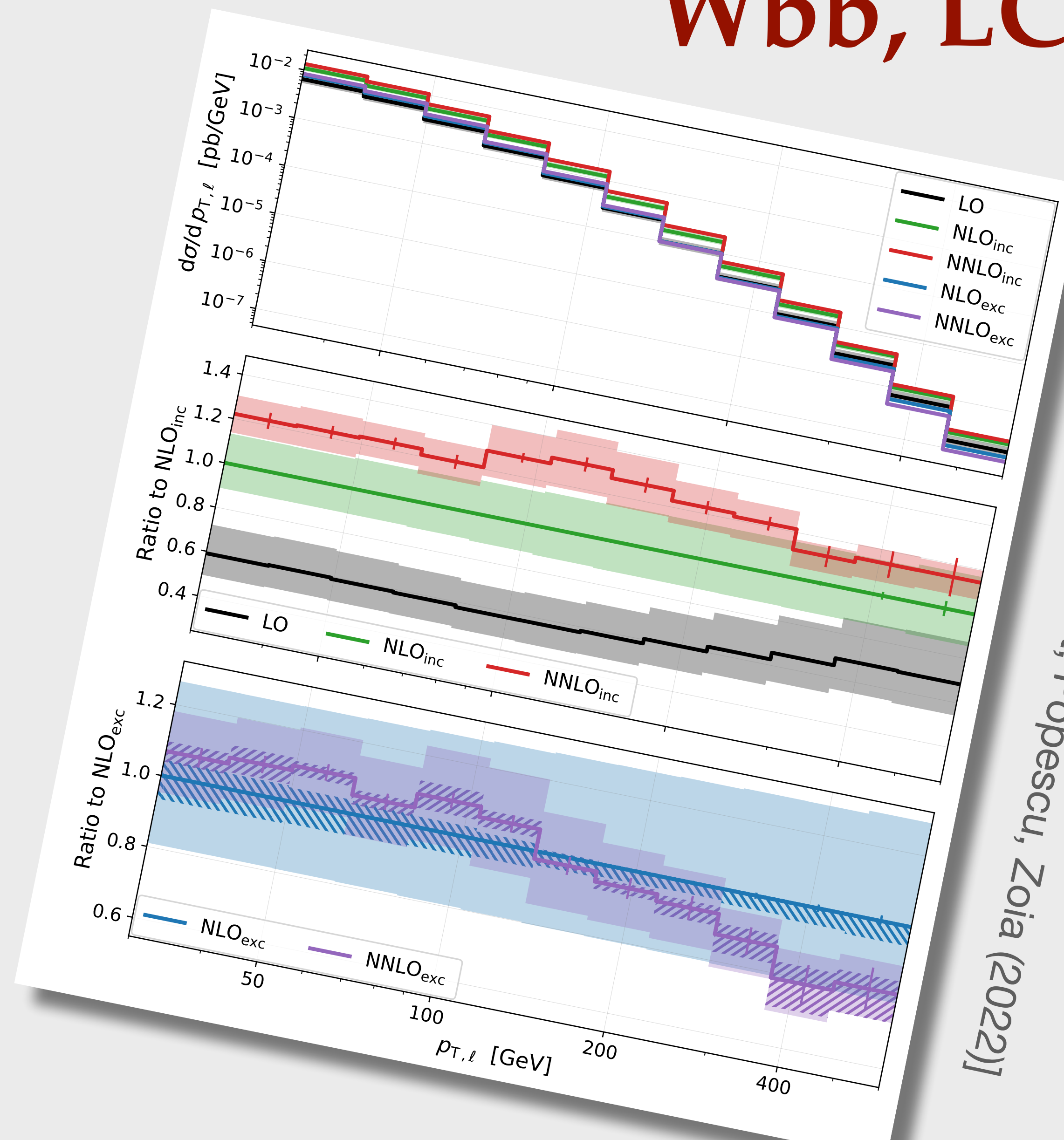
A glimpse of recent results: exploring the 2→3 frontier

jjj,LC



[Alvarez, Cantero, Czakon, Llorente, Mitov, Poncelet (2023)]

Wbb, LC



[Hartanto, Poncelet, Popescu, Zoia (2022)]

A rich phenomenology, to be explored

QCD amplitudes: some open questions

Some obvious questions:

- better ways of approaching QCD amplitudes
- how to get more complicated ones (e.g. 5pt, 2 legs off-shell $\rightarrow$ di-boson)

QCD amplitudes: some open questions

Perhaps less obvious questions:

- are we really minimising non-physical information?

$$\mathscr{A} = \mathscr{Z}_{\mathrm{IR}} \mathscr{A}_{\mathrm{IR-finite}}$$

[Structure: see G. Gambuti's talk]

$\mathscr{A}$ → what we compute

$\mathscr{Z}_{\mathrm{IR}}$ → Universal IR divergences. Cancel against real emission (KLN)

$\mathscr{A}_{\mathrm{IR-finite}}$ → Genuine "non-trivial" information

- Z_{IR}: ambiguous. "Optimal" choice?
- In simple cases, $A_{\mathrm{IR-finite}}$ much simpler than A → accident or hint?
- Would likely require a better understanding of (quasi) soft/collinear regions

QCD amplitudes: some open questions

Perhaps less obvious questions:

- are we choosing the right MIs?

One-loop amplitudes for e^+e^- to four partons

Zvi Bern[a,1], Lance Dixon[b,2], David A. Kosower[c,3]

[a] *Department of Physics, University of California, Los Angeles, Los Angeles, CA 90024, USA*
[b] *Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309, USA*
[c] *Service de Physique Théorique, Centre d'Etudes de Saclay, F-91191 Gif-sur-Yvette cedex, France*[4]

Received 13 August 1997; accepted 20 October 1997

1L: "optimal" QCD MIs are not UT…

$$\mathrm{L}_0(r) = \frac{\ln(r)}{1-r}\,, \qquad \mathrm{L}_1(r) = \frac{\mathrm{L}_0(r)+1}{1-r}\,,$$

$$\mathrm{Ls}_{-1}(r_1,r_2) = \mathrm{Li}_2(1-r_1) + \mathrm{Li}_2(1-r_2) + \ln r_1 \, \ln r_2 - \frac{\pi^2}{6}\,,$$

$$\mathrm{Ls}_0(r_1,r_2) = \frac{1}{(1-r_1-r_2)}\,\mathrm{Ls}_{-1}(r_1,r_2)\,,$$

$$\mathrm{Ls}_1(r_1,r_2) = \frac{1}{(1-r_1-r_2)}\left[\mathrm{Ls}_0(r_1,r_2) + \mathrm{L}_0(r_1) + \mathrm{L}_0(r_2)\right]\,,$$

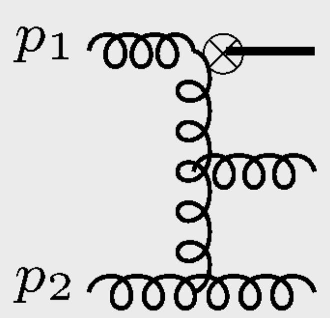
Guiding principle: singularity structure of the QCD amplitude (which is not uniform transcendental…)

Beyond amplitudes

Amplitude techniques: more general

A “revolution” in precision phenomenology: reverse unitarity

> **Higgs boson production at hadron colliders in NNLO QCD**
>
> Charalampos Anastasiou, Kirill Melnikov
>
> *Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309, USA*
>
> Received 9 July 2002; received in revised form 4 September 2002; accepted 19 September 2002



$$\left|\ \cdots\ \right|^2 \sim \int \frac{\mathrm{d}^d q_1\, \mathrm{d}^d q_2\, \delta(q_1^2)\delta(q_2^2)\delta(q_H^2 - m_H^2)[\cdots]}{[(q_H - p_1)^2]^2[(q_2 - p_2)^2]^2}.$$

$$2i\pi\delta(p^2 - m^2) \to \frac{1}{p^2 - m^2 + i0} - \frac{1}{p^2 - m^2 - i0}.$$

Maps phase-space integration into loop integrals → amplitude technology

Amplitude techniques: more general

Dilepton Rapidity Distribution in the Drell-Yan Process at Next-to-Next-to-Leading Order in QCD

Charalampos Anastasiou,[1] Lance Dixon,[1] Kirill Melnikov,[2] and Frank Petriello[1]
[1]*Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309, USA*
[2]*Department of Physics & Astronomy, University of Hawaii, Honolulu, Hawaii 96822, USA*
(Received 25 June 2003; published 31 October 2003)

$$\frac{d\sigma_{ij}}{2e^{2Y}dY} = \int d\Pi_f |\mathcal{M}_{ij}|^2 \delta\left(e^{2Y} - \frac{E + p_z}{E - p_z}\right).$$

"Observable": constraint in the integration domain

$$\delta(x) \to \frac{1}{2\pi i}\left[\frac{1}{x - i0} - \frac{1}{x + i0}\right].$$

Reverse unitarity trick still works, but generalised propagators (also: θ functions…)

Can we understand the properties of these "generalised" Feynman integrals (e.g. regularity conditions to fix b.c. of diff. eq. etc)

Amplitude techniques: more general

Understanding the structure of QCD amplitudes: crucial beyond amplitudes themselves

For example

- Soft/collinear regions: insight into large $\alpha_s \ln^k(v \ll 1)$ contributions to the cross section. Starting from 2L: non-trivial limits in the non-planar sector [see G. Gambuti's talk]

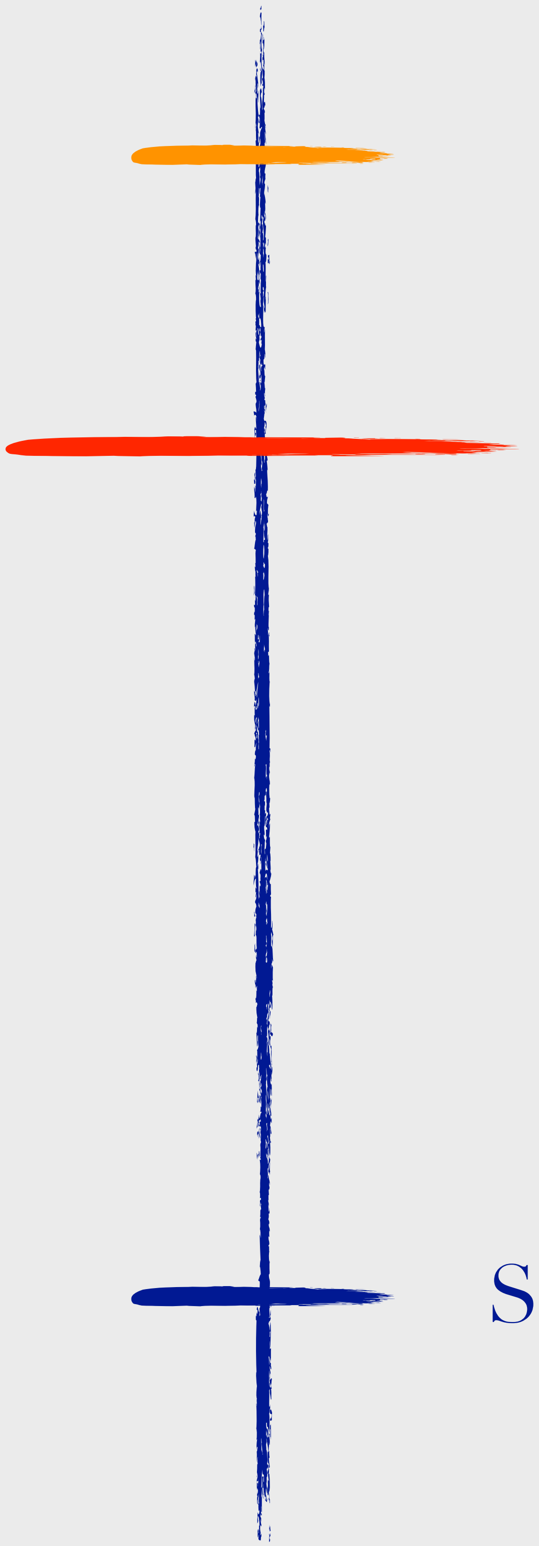
- Soft/collinear limits beyond leading power: insight into NP regime of QCD

- (Multi)-Regge limit: insight into physics of high-density QCD / NNLL important information for parton distribution functions at N^3LO [see E.Gardi's talk]

- Non-planar soft/collinear interplay at 2 loops and beyond: expose "Glauber" modes, responsible for potential breaking of collinear factorisation (foundation of perturbative QCD at hadron colliders!) [see G. Gambuti's talk]

and many more!

"Precision": back-of-the envelope



New physics at a heavy scale Λ_{NP}

Typical modification to observable w.r.t. standard model prediction:

$$\delta O \sim Q^2/\Lambda_{NP}{}^2$$

$$Q \sim 100 \text{ GeV}, \Lambda_{NP} \gtrsim 1 \text{ TeV} \rightarrow$$

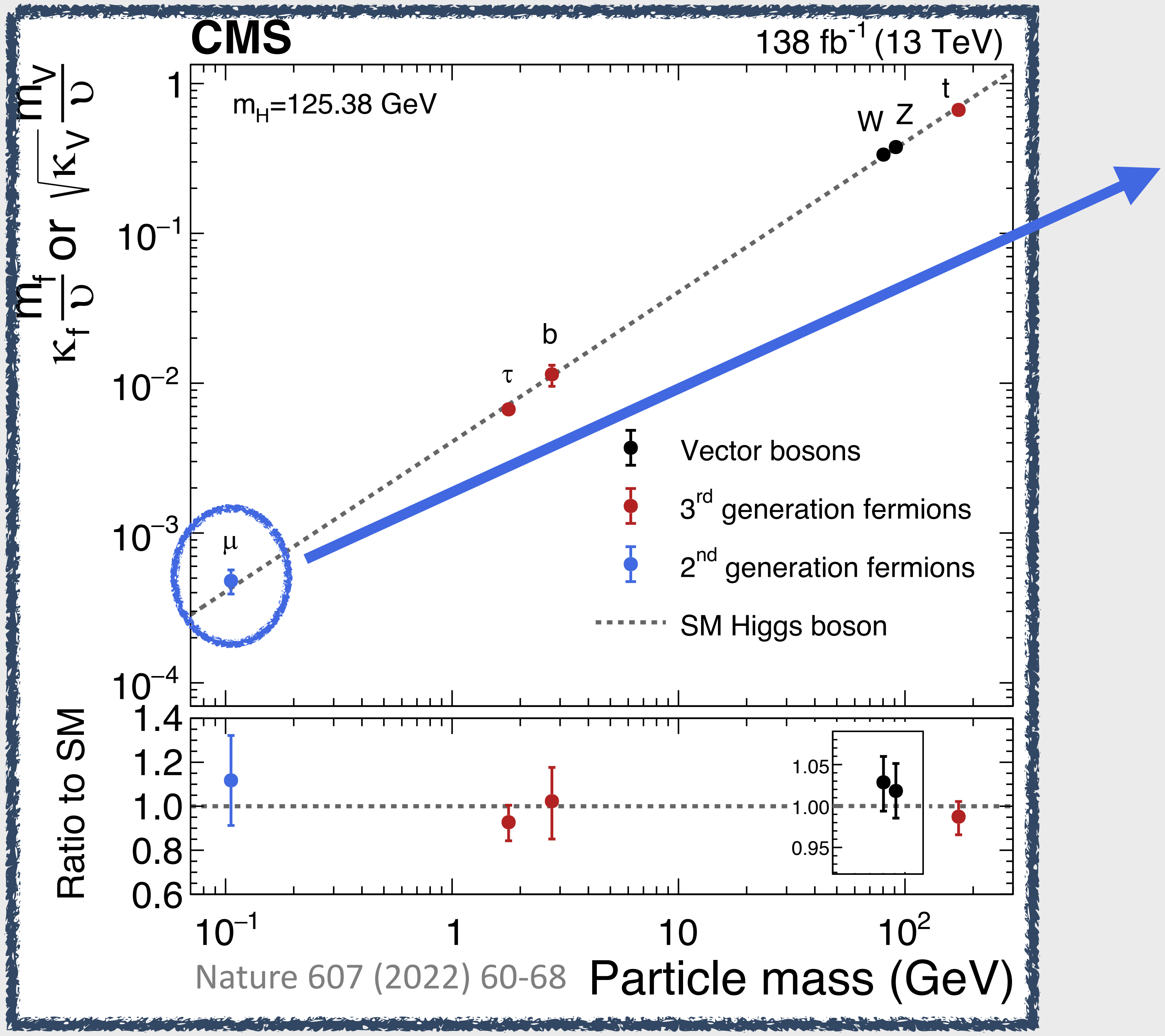
"Few percent" deviation from SM

Indirect probe for NP

… also few percent: ~ α → SM at the quantum level (e.g. W-mass)

Higgs couplings at a glance

Higgs sector: in many cases, SM to within ~10% or better



Nature 607 (2022) 60-68

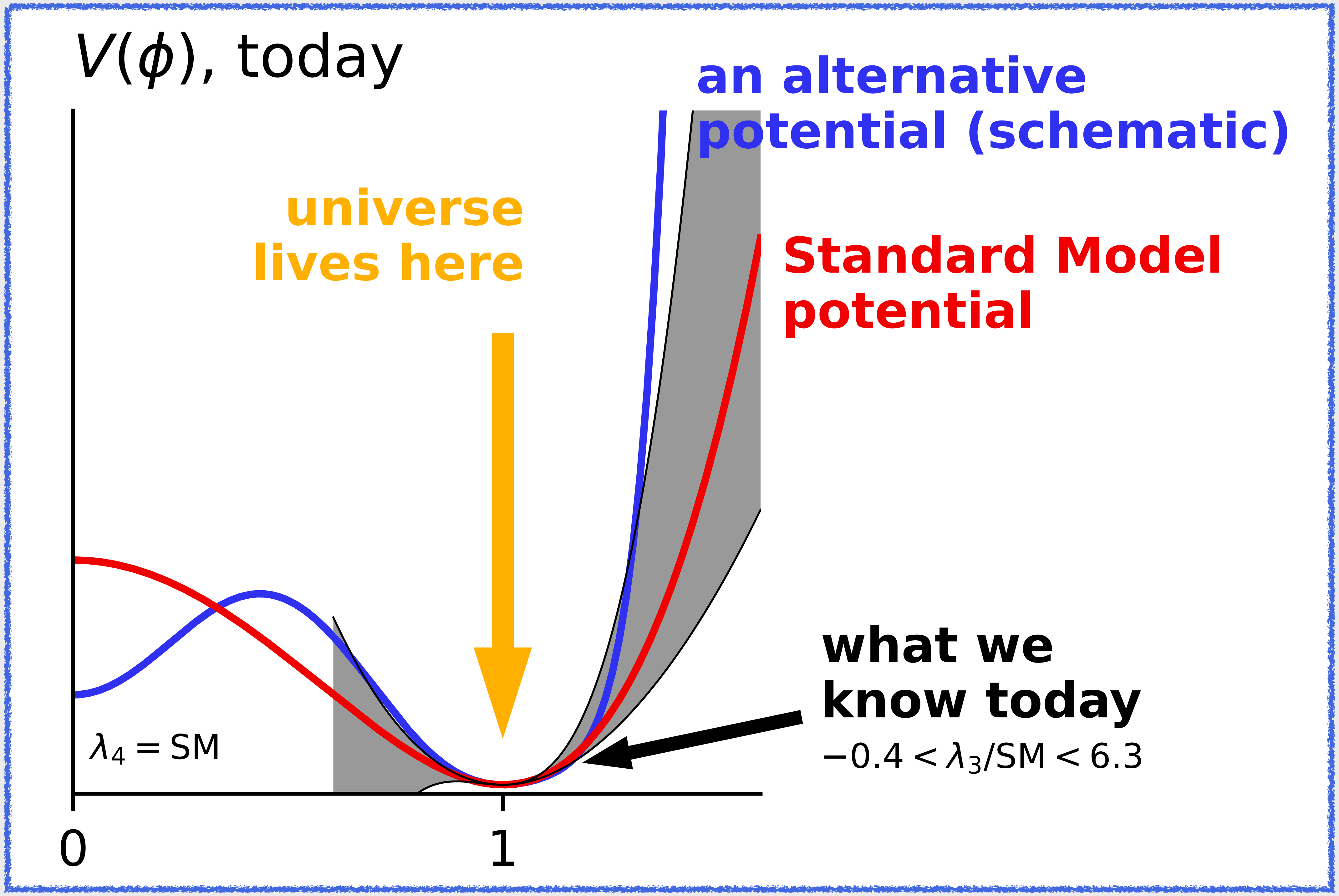
μ: starting to probe Higgs/matter interactions for 2nd generation family!

Charm: much more difficult

Lighter particles → new colliders

The Higgs potential: a long journey

$$V = \lambda(H^\dagger H - v^2)^2 \rightarrow 2\lambda v^2 h^2 + \sqrt{2}\lambda v h^3 + \lambda h^4/4$$



The Higgs potential: a long journey

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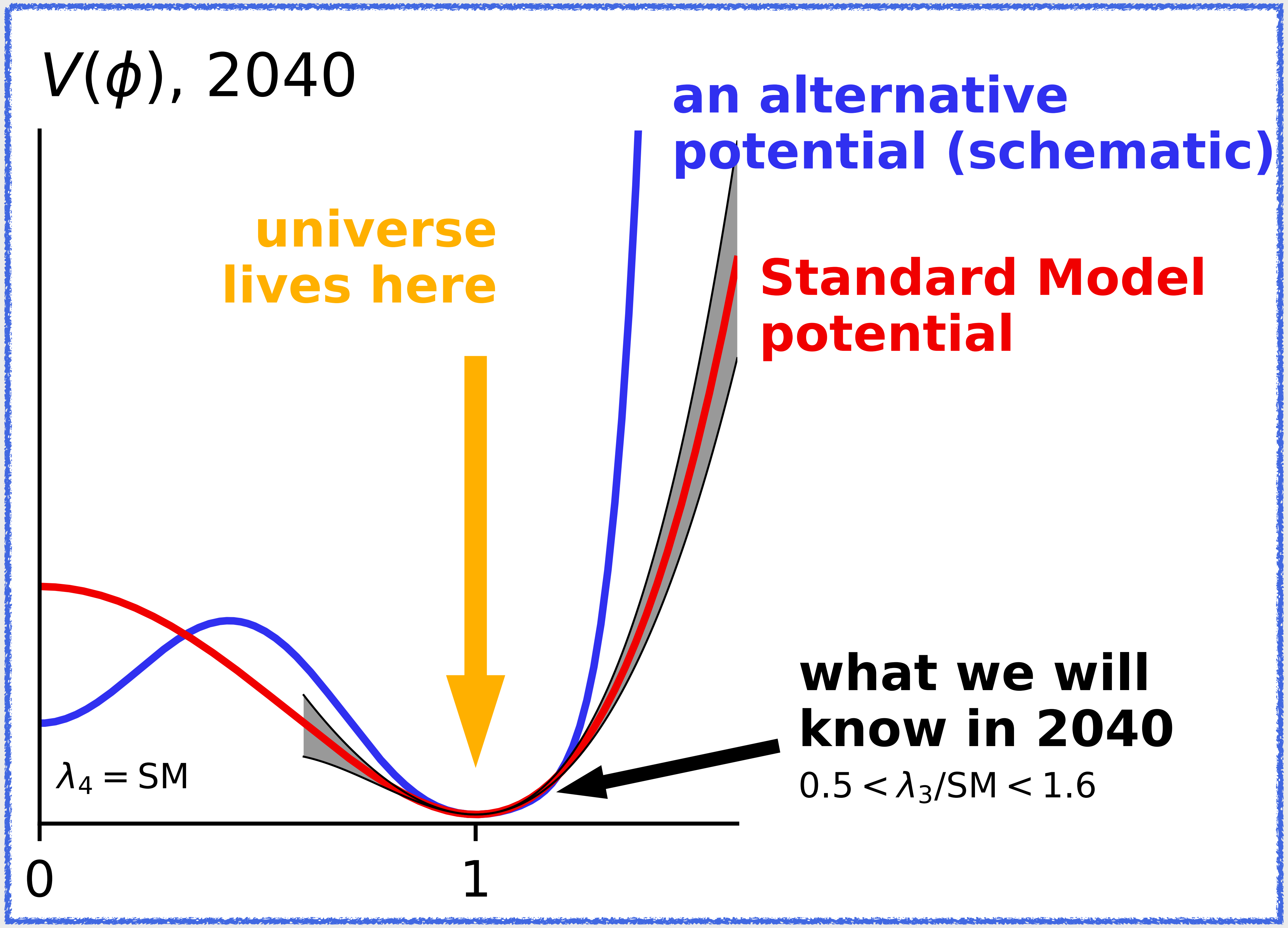

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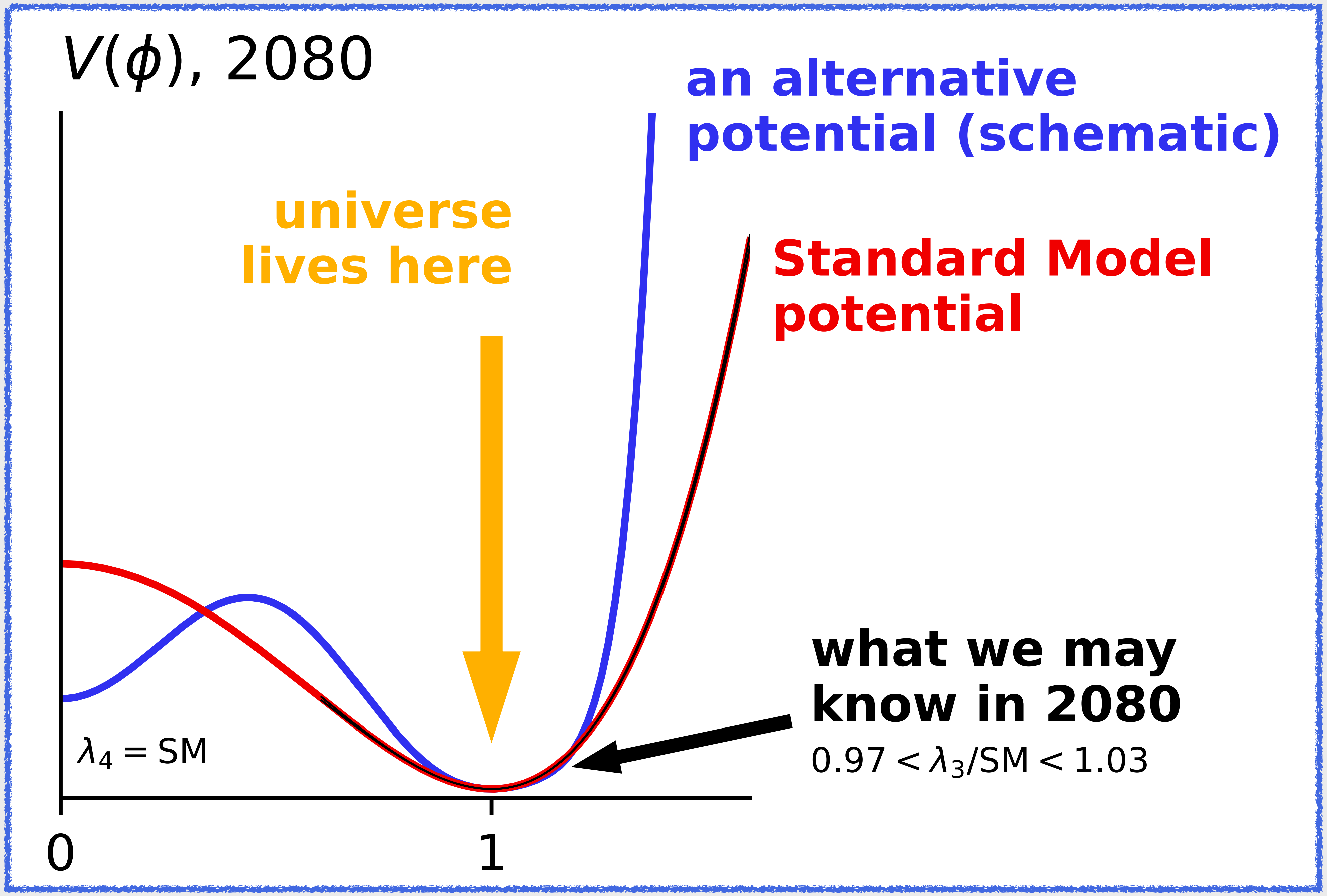
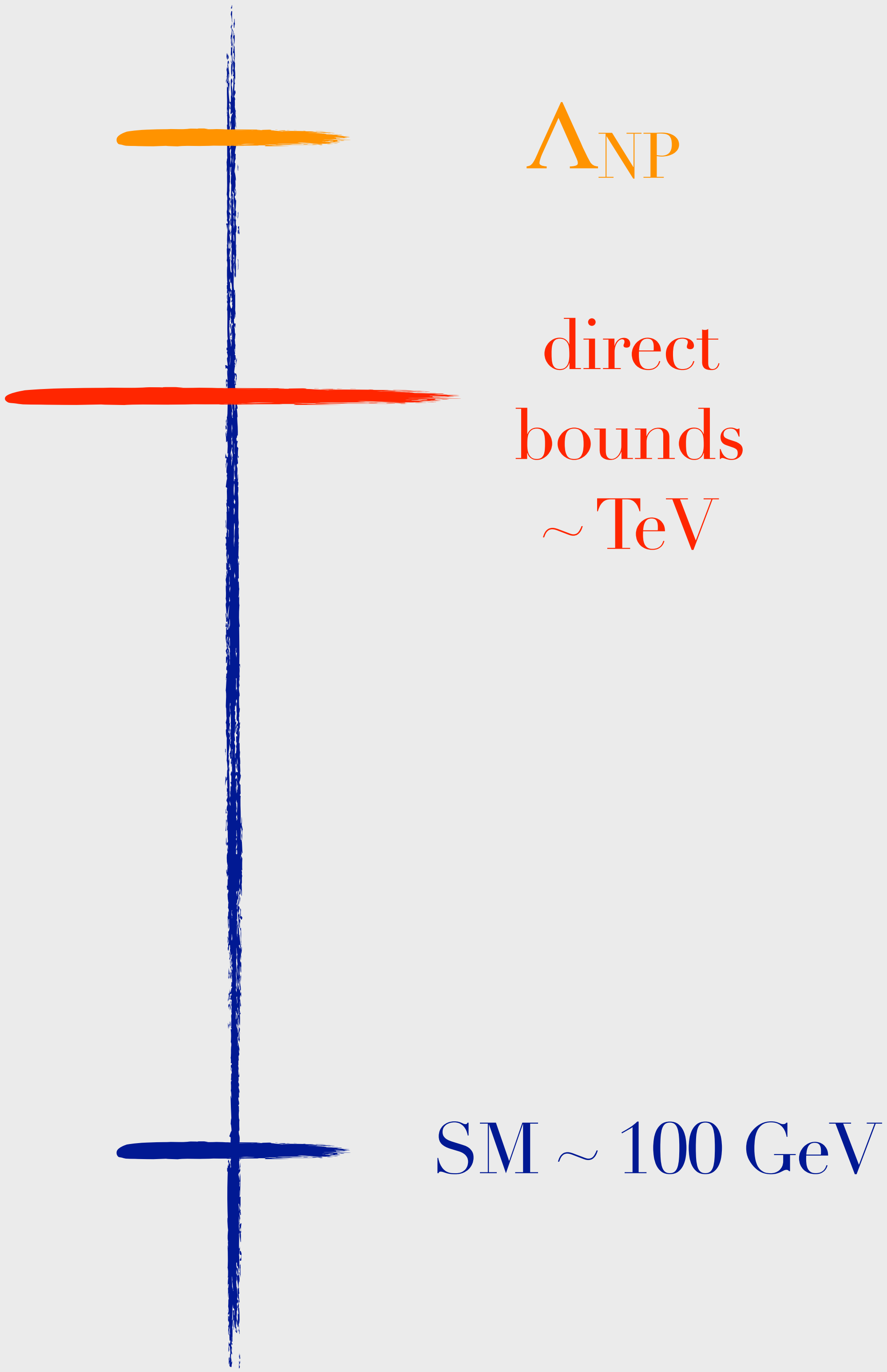


Predictions: back-of-the-envelope



Imagine BSM at a heavy scale Λ_{NP}

Typical modification to observable w.r.t. standard model prediction:

$$\delta O \sim Q^2/\Lambda_{NP}{}^2$$

$$Q \sim 100 \text{ GeV}, \Lambda_{NP} \gtrsim 1 \text{ TeV} \rightarrow$$

"Few percent" deviation from SM

Indirect probe for NP

… also few percent: ~ α → SM at the quantum level (e.g. W-mass)

$$\varphi_L(\mathbf{z}) = \frac{f(\mathbf{z})}{z_1^{n_1}\dots z_N^{n_N}} dz_1 \dots dz_N$$

$$0 = \int_{C_R} d(u\xi_L) = \int_{C_R} du \wedge \xi_L + ud\xi_L \qquad 0 = \int_{C_R} u\left(\frac{du}{u} \wedge \xi_L + d\right)\xi_L = \int_{C_R} u\boldsymbol{\nabla}_\omega \xi_L$$

$$\int_{C_R} u(\mathbf{z})\varphi_L = \int_{C_R} u(\mathbf{z})(\varphi_L + \boldsymbol{\nabla}_\omega \xi_L) \qquad I = \int_{C_R} u(\mathbf{z})\varphi_L(\mathbf{z}) = \langle \varphi_L | \, C_R]$$

$$\langle \varphi_L | \varphi_R \rangle = \frac{1}{(2\pi i)^n} \int_X \varphi_L \wedge \varphi_R \qquad \left\langle \varphi_L^{(\mathbf{1})} \middle| \varphi_R^{(\mathbf{1})} \right\rangle = \sum_{p \in \mathcal{P}_1} \mathrm{Res}_{z_1 = p} \left(\psi^{(p)} \varphi_R^{(\mathbf{1})} \right) \tag{2.21}$$

where $\mathcal{P}_1$ is the set of poles of $\omega_1 = \frac{\partial \log u(z_1)}{\partial z_1}$ (including infinity), and $\psi_p^{(1)}$ is the solution to the differential equation

$$\boldsymbol{\nabla}_\omega \psi^{(p)} = d\psi^{(p)} + \omega_1 \wedge \psi^{(p)} = \varphi_L^{(\mathbf{1})} \tag{2.22}$$