Large N gauge theories, quantum chaos, and black holes

STFC retreat

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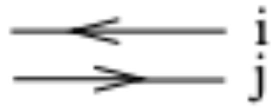
1/N expansion

Theories with NxN matrix degrees of freedom have 1/N expansion <-— useful at strong coupling

' t Hooft applied this idea to QCD:

$A_\mu^{ij}(x) \qquad i,j = 1,\cdots N$

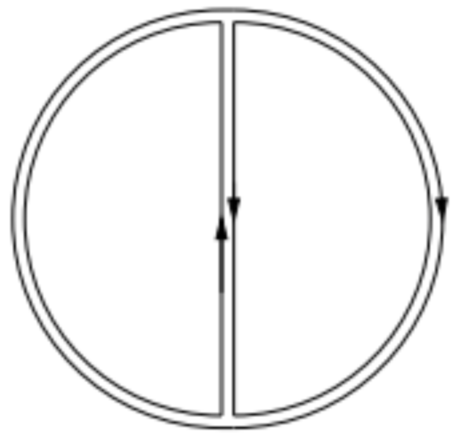


$\lambda = g_{YM}^2 N$

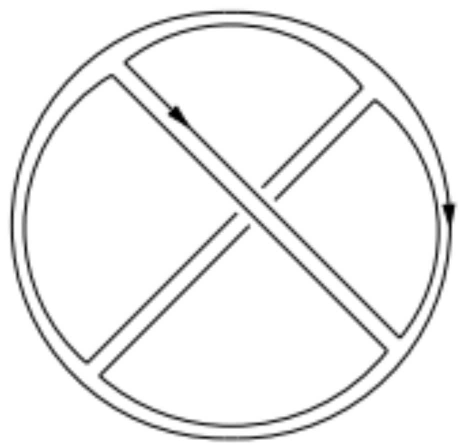
$N \to \infty$

Powers of N characterizes the topology: $N^\chi \lambda^{\text{power}}$ $\quad \chi =$ faces-edges+vertices

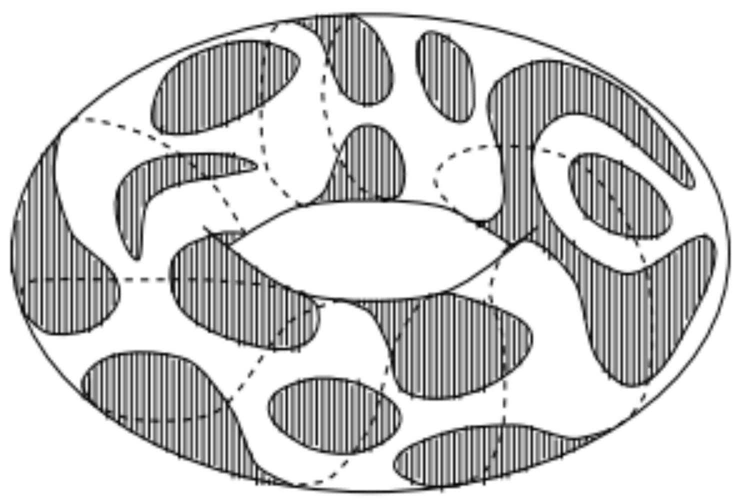
N^2

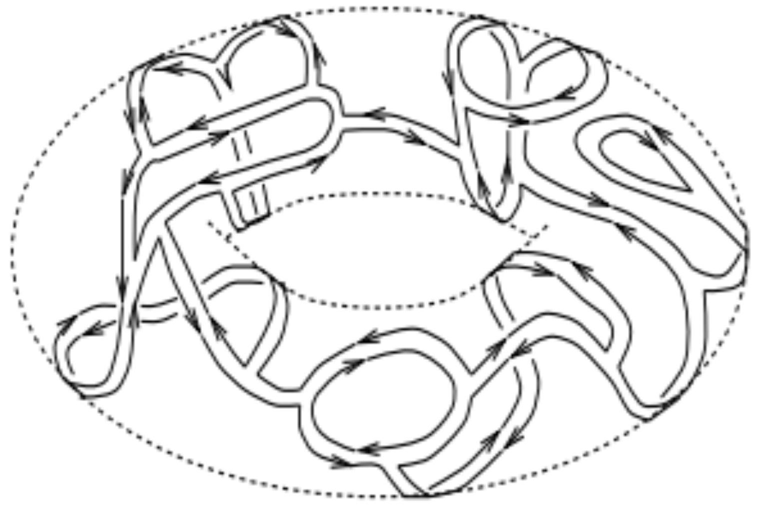


N^0



The strongly coupled theory is reformulated in terms of new perturbative degrees of freedom: these are strings:





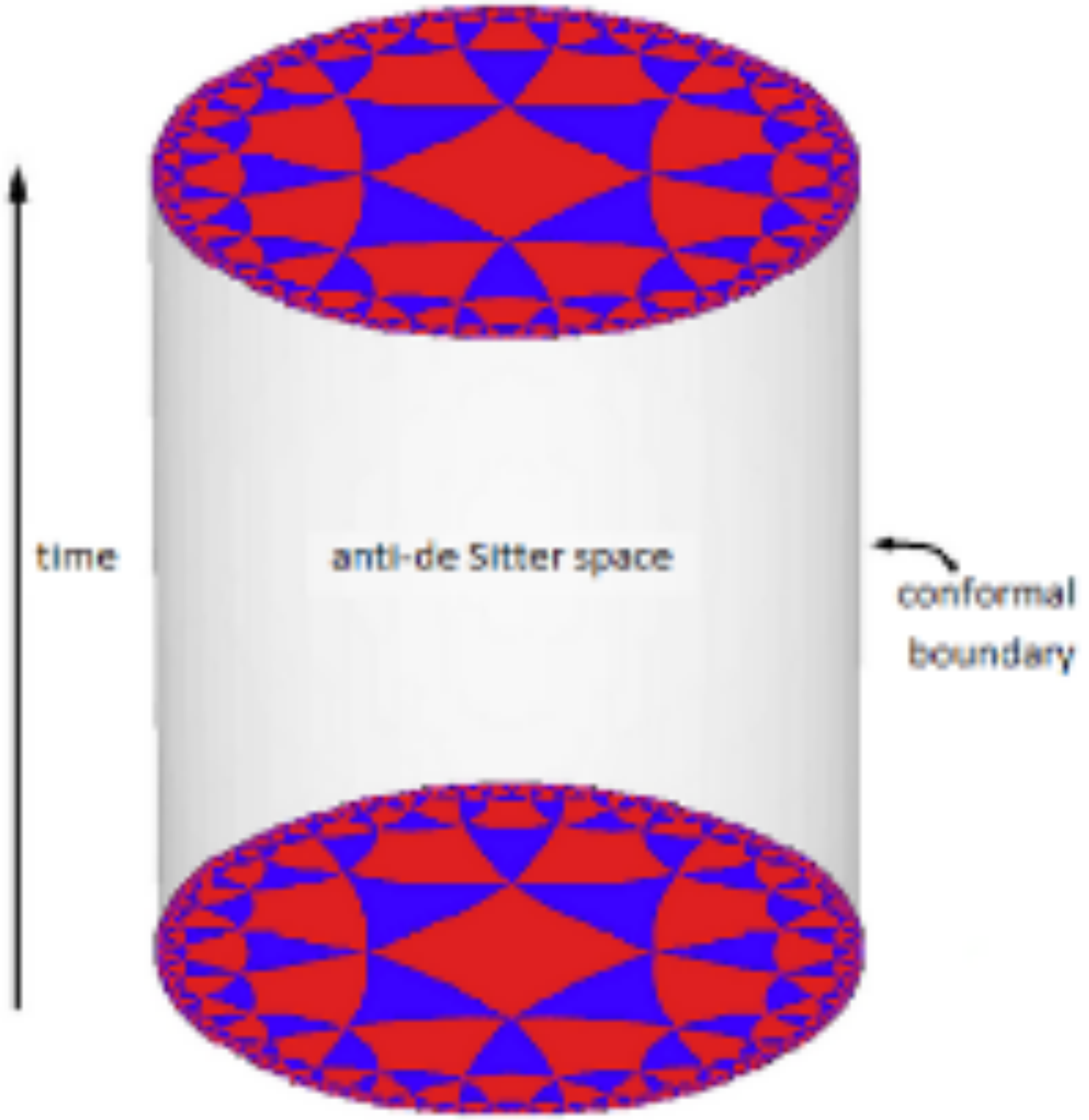
AdS/CFT

AdS/CFT is an example of 't Hooft's gauge-string duality where the emergent strings living in a higher dimensional theory of quantum gravity.

$\mathcal{N} = 4$ gauge theory in 4D $\longleftrightarrow$ IIB String theory on AdS5 x S5

$\lambda \qquad N$

$$g_s = \frac{\lambda}{N} \qquad \frac{l_s}{R_{AdS}} = \frac{1}{\lambda^{1/4}}$$



Relates bulk semi-classical gravity to strongly coupled quantum physics on the boundary

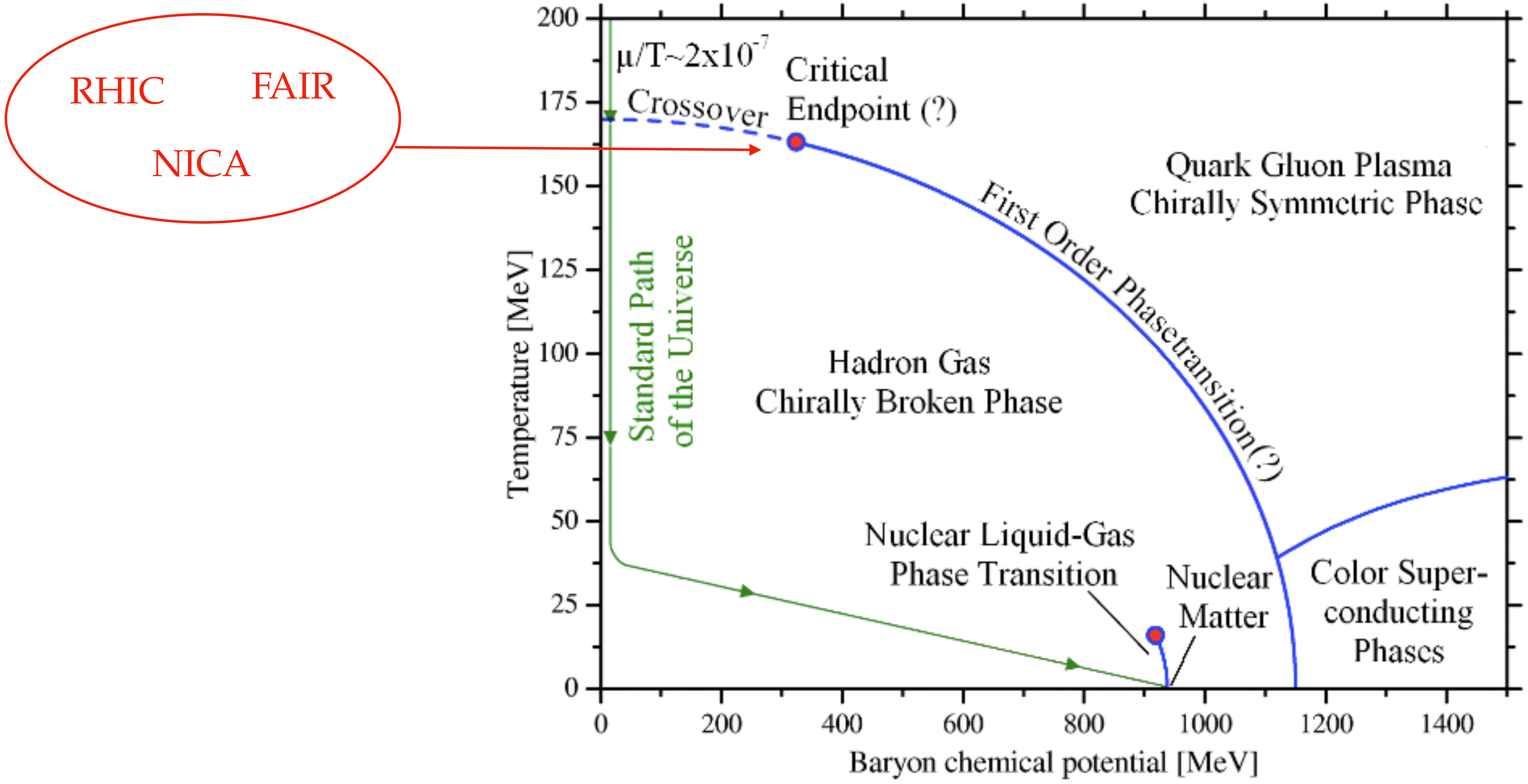
Useful for studying universal behavior in strongly coupled QFT, e.g. hydrodynamics and chaotic phenomena

Black holes and black branes play a distinguished role. They behave like thermal systems with temperature and entropy.

$$S = \frac{A(M)}{4G}$$

Holography and Hydrodynamics at strong coupling

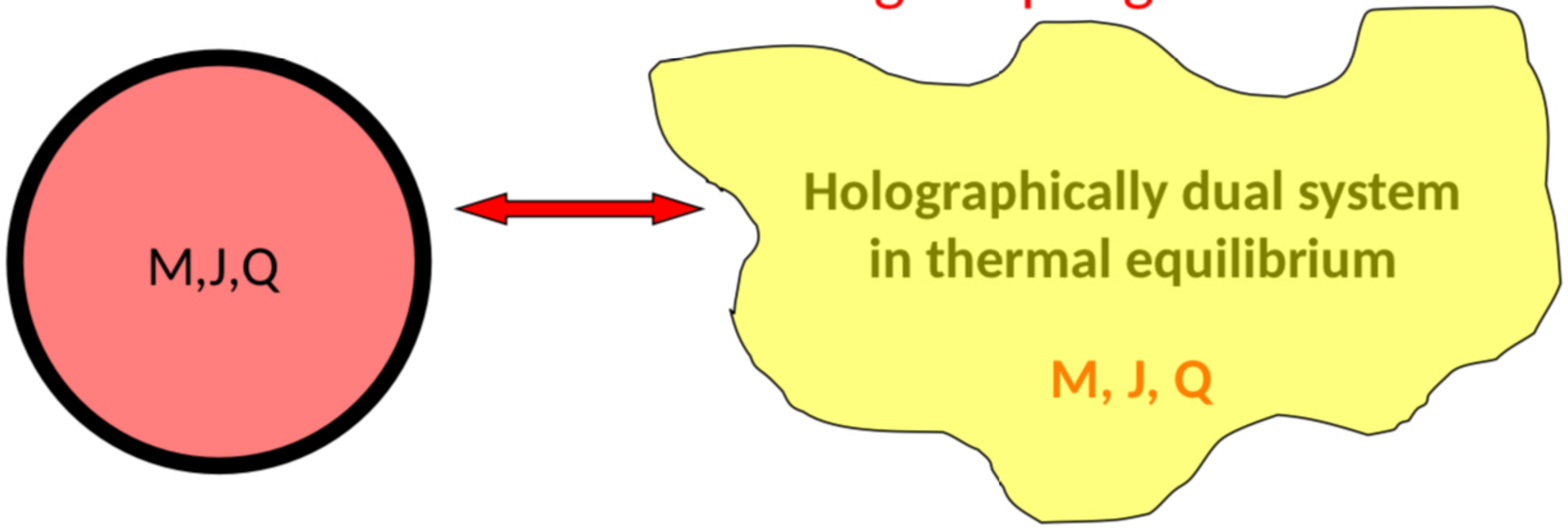
QCD Phase Diagram



Use holography to probe qualitative behavior near the critical point of non-equilibrium, strongly coupled gauge theory at finite density

10-dim gravity

4-dim gauge theory – large N, strong coupling



T_{Hawking} $S_{\text{Bekenstein-Hawking}}$ $\Longleftrightarrow$ T S

Gravitational fluctuations $\Longleftrightarrow$ Long wavelength deviations from equilibrium

$$g_{\mu\nu} = g^0_{\mu\nu} + h_{\mu\nu}$$

$$\text{“}\Box\text{”} h_{\mu\nu} = 0 \quad \Longleftrightarrow$$

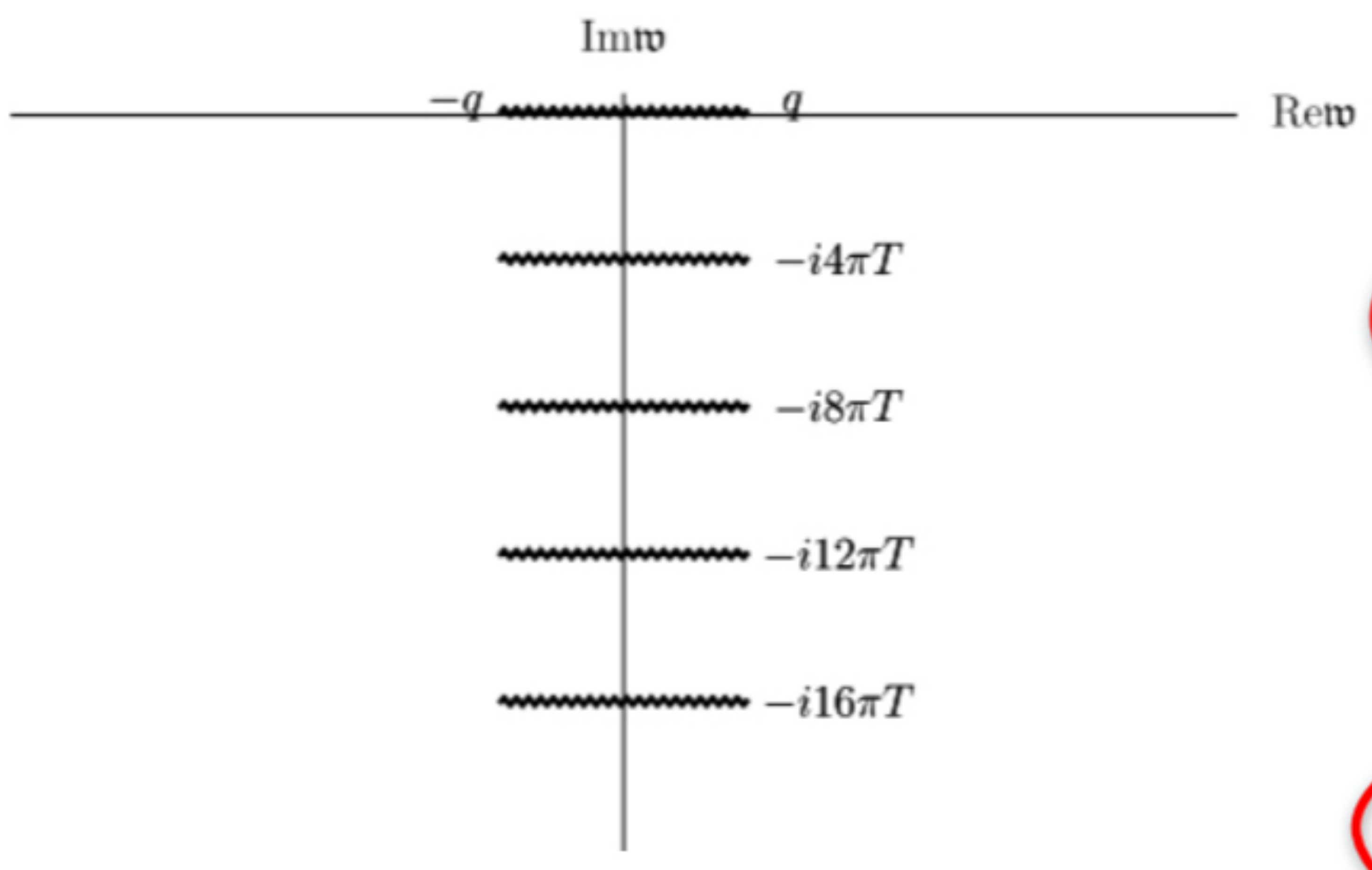
Equations of motion=conservative laws
+constitutive relations

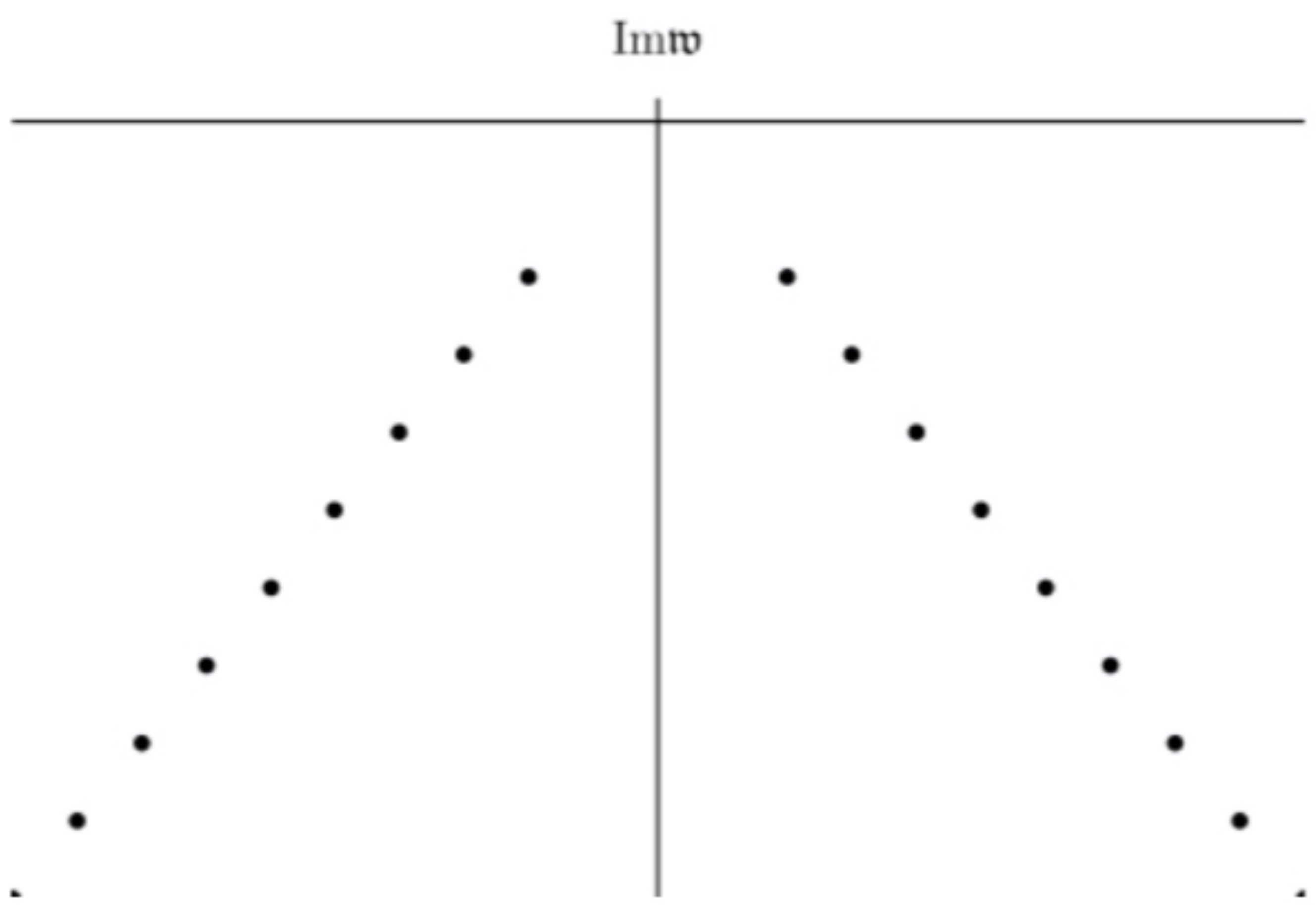
Quasi normal mode spectrum $\Longleftrightarrow$ Hydrodynamical modes (poles of the retarded Green's function

(Son,Starinets, 2002)

Cuts versus poles: a mystery

Singularities of a Green's function in the complex frequency plane





Weak (vanishing) coupling

Hartnoll, Kumar (2005)

Strong (infinite) coupling

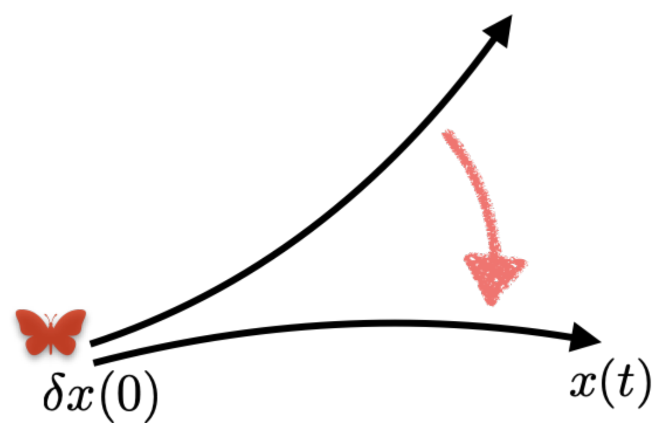
Starinets (2002)

We should be able to interpolate between the two limits…

Grozdanov, Starinets (2018) Romatschke (2019) DeWolfe, Romatschke (2019)

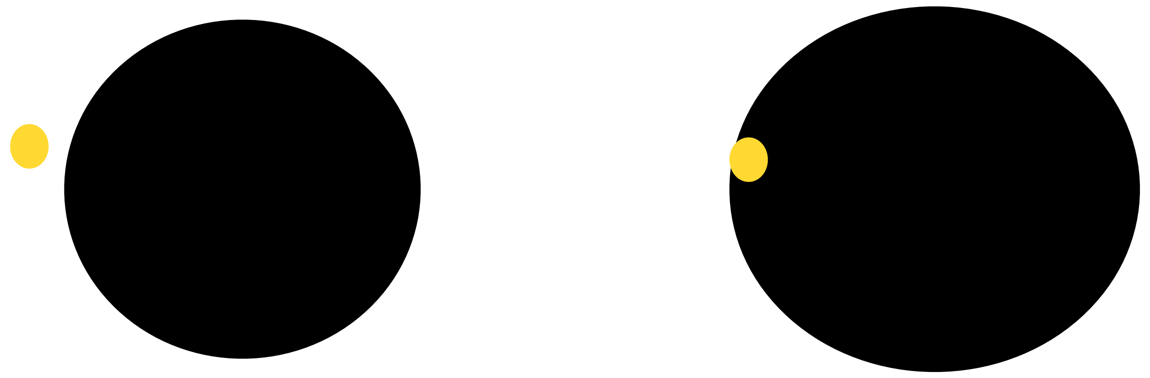
Chaotic dynamics

Classical chaos~ butterfly effect



Chaos leads to simplifications, e.g. thermalization :

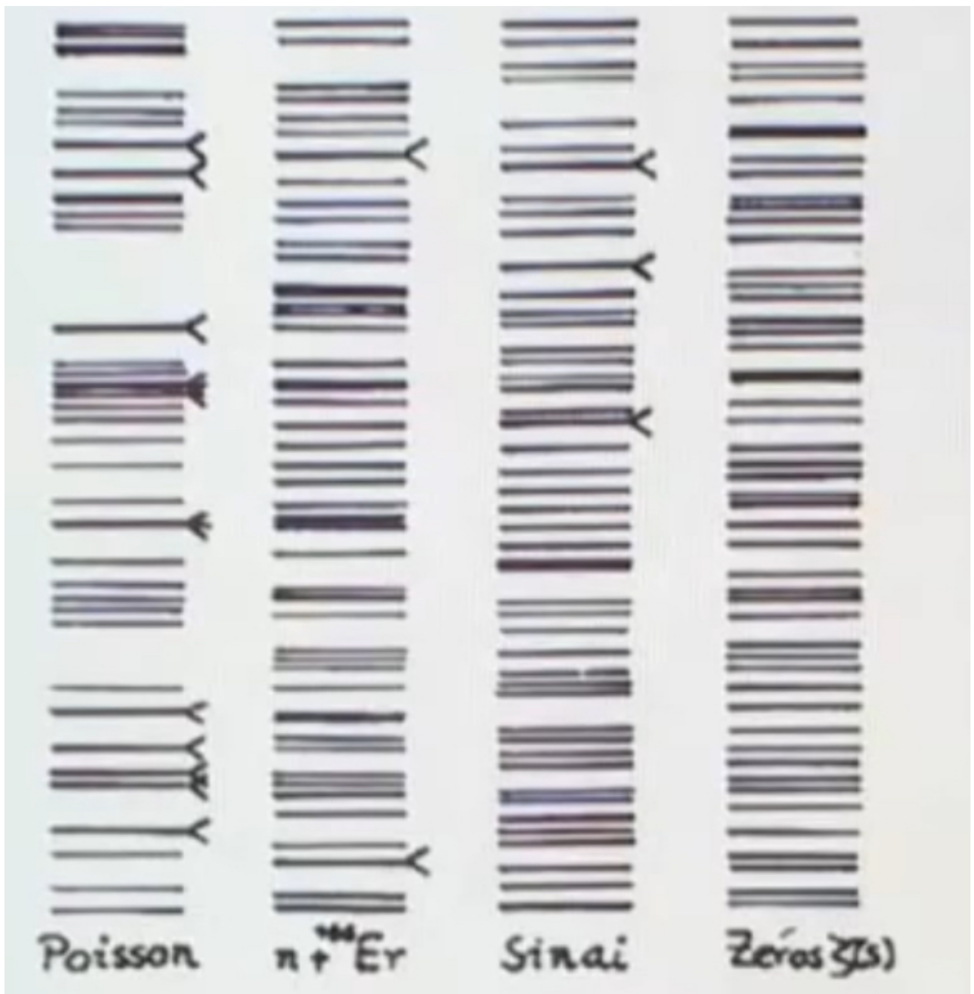
Black hole horizons exhibit the butterfly effect : small perturbation of the horizon area dramatically changes the dynamics



Quantum Chaos ~ Spectrum of a chaotic Hamiltonian ``looks like" the spectrum of a random matrix

The black hole density of states $\rho(M) = \exp \frac{A(M)}{4G}$ exhibits quantum chaos

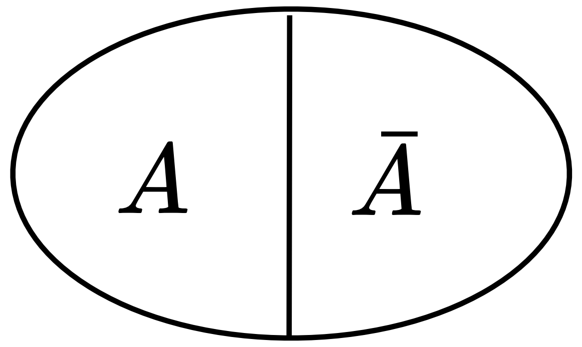
(Saad, Shenker, Stanford)



Using black holes to probe quantum chaotic dynamics

Thermalization in a quantum chaotic system is described by a black hole that forms from the collapse of matter

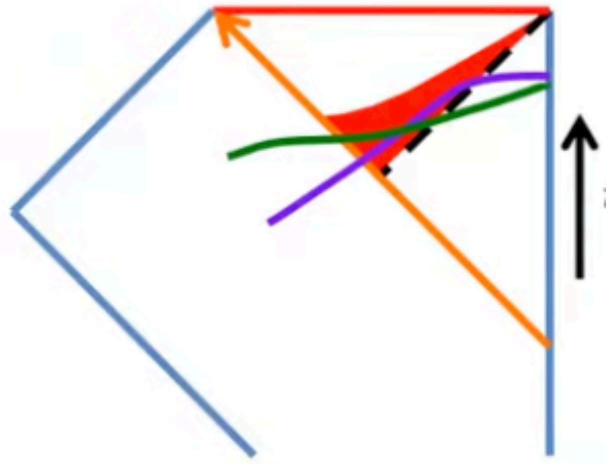
$$\Delta S_A(t) \rightarrow \Delta S_A^{(\mathrm{eq})}(\beta) = s_{\mathrm{th}}(\beta)\,\mathrm{vol}(A)$$



Entanglement dynamics of the boundary theory is described by a membrane (Mezei)

AdS/CFT

$\sim$



Black holes also captures more sophisticated chaos indicators like quantum Lyapunov exponents and butterfly velocities (Mezei, Stanford 2016, Mezei, van der Schee 2020)

Pure Gravity as a quantum system

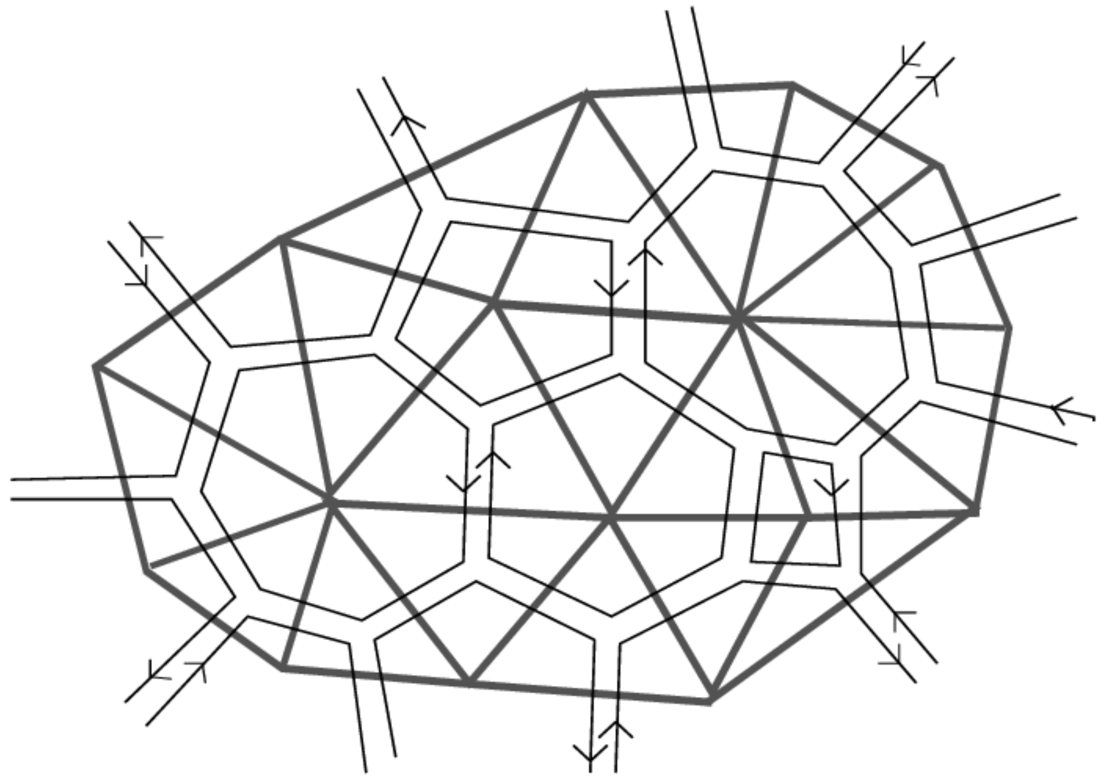
Full IIB string theory is dual to a very special quantum system (N=4 SYM). We want to focus on universal behavior on the boundary : this is related to black holes, which are solutions of pure gravity

What is its quantum mechanical system dual of pure gravity?

$$Z_{\text{grav}} = \int D[g]e^{-S[g]} \longleftrightarrow \text{???}$$

A basic problem: action is unbounded due to the conformal mode problem. Also not clear which saddles to include (wormholes?)

In 2D, the sum over geometries can be captured by summing over dense triangulations of a manifold (a la 't Hooft !)



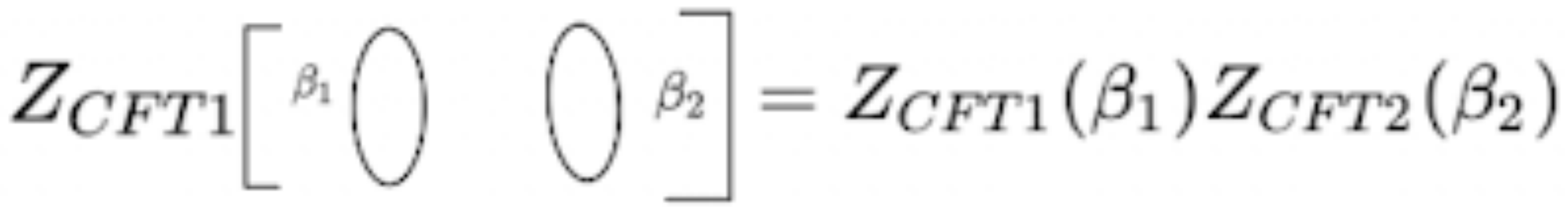
In 3D or higher the naive simplicial approach to gravity fails: the continuum limit of the triangulations give singular manifolds that don't seem to have a gravity interpretation

We develop a holographic duality for 3d gravity, which gives new type of simplicial gravity

(WIP with Dan Jafferis, Liza Rozenberg, GW)

Insights from 2D holography (SSS)

Euclidean gravity path integral should includes sum over wormhole topologies. The holographic dual is a random ensemble of Hamiltonians



In 2D, the wormhole topology is needed to recover the chaotic behavior of the spectral form factor

$$Z(\beta + iT) Z(\beta - iT)$$

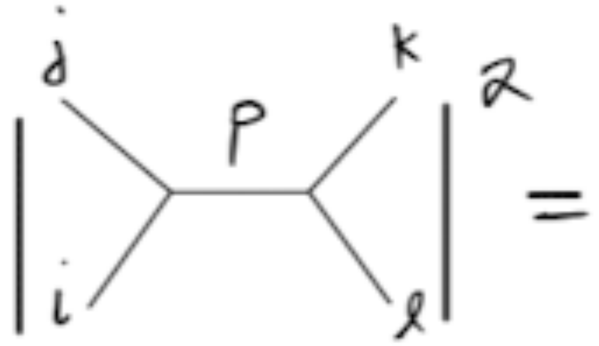
Pure AdS3 Gravity as an ensemble of CFT's

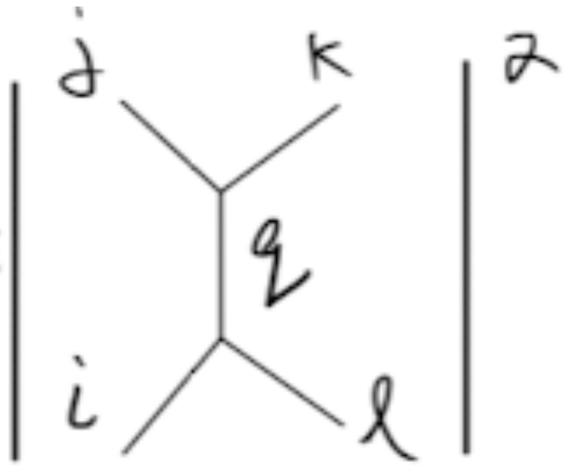
Using insights from SSS, we define an ensemble of CFT2 data, given by the conformal dimensions and OPE coefficients

$$\langle \phi_i(x) \phi_j(y) \rangle = \frac{\delta_{ij}}{|x-y|^{2\Delta_i}}$$

$$\langle \phi_i(x_1) \phi_j(x_2) \phi_k(x_3) \rangle = \frac{C_{ijk}}{|x_{12}|^{\Delta_1+\Delta_2-\Delta_3} |x_{23}|^{\Delta_2+\Delta_3-\Delta_1} |x_{13}|^{\Delta_3+\Delta_1-\Delta_2}}$$

The CFT data is constrained by crossing symmetry

$$\langle \phi_i(x_1) \phi_j(x_2) \phi_k(x_3) \phi_l(x_4) \rangle = \sum_p C_{ijp} C_{pkl}$$


$$= \sum_q C_{jkq} C_{qli}$$


In 2D there is also modular invariance: these are the equations of the conformal bootstrap.

Ensemble of approximate CFT's

The rigid constraints suggests we should consider an ensemble of approximate CFT2 data. Assume central charge c>>1, and only the vacuum as the light state. Truncate to finite N number of primaries

$$Z = \sum_{\text{spin } s} \int D\Delta^s_{ij} DC_{ijk} \ \exp(-\frac{1}{\hbar} V[\Delta^s_{ij}, C_{ijk}])$$

parameterizes violation of crossing

Dilatation operator as a random matrix

OPE coefficient as a random tensor

$$V = (\text{constraint})^2$$ is defined to be minimized on the solutions of the bootstrap .

We expand in $1/\hbar$ and let the path integral impose the constraints via the perturbative expansion

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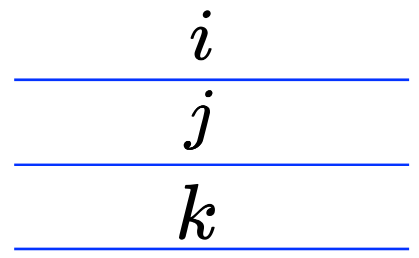
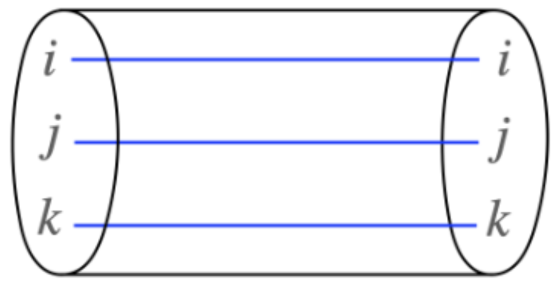
OPE coefficient as a random tensor

't Hooft diagrams for Cijk become triple line diagrams.

The perturbative sum becomes a sum over 3 manifolds, reproducing 3d gravity !

$$\overbrace{C_{ijk} C^*_{ijk}}$$

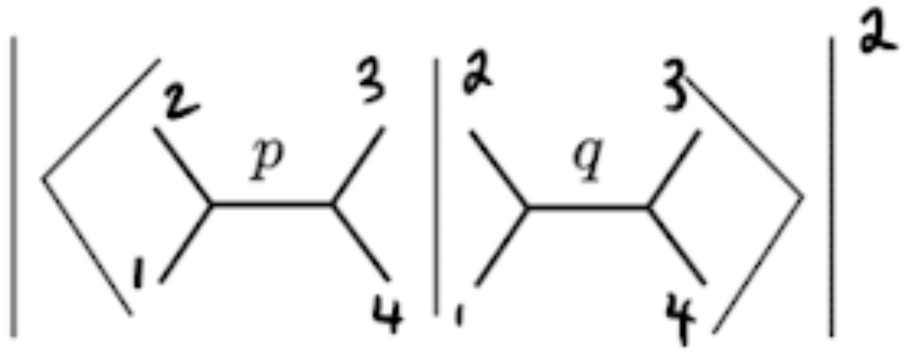




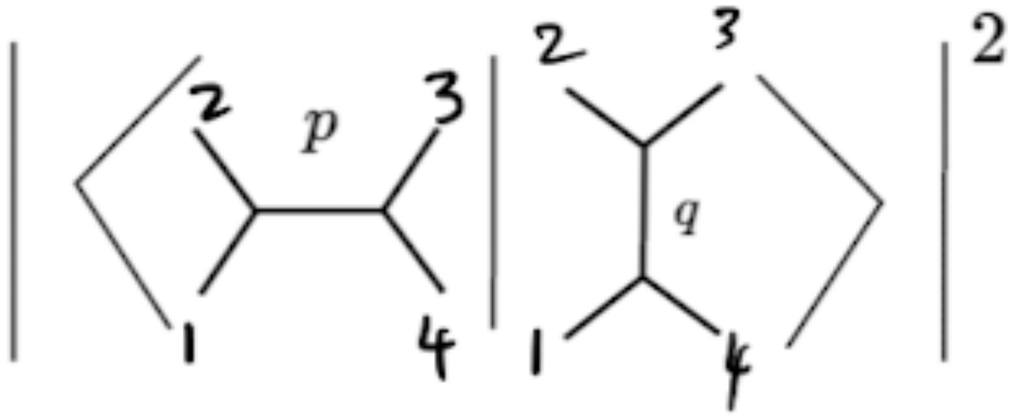
$S^2 \times I$

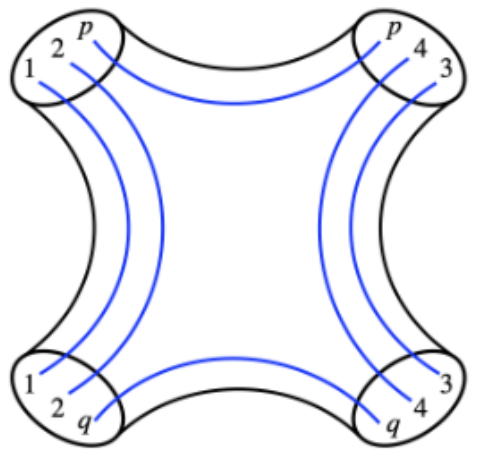
Quartic potential and 3 manifolds

$$V_4 = \sum_{i_1,i_2,i_3,i_4} \sum_{p,q} C_{i_1 i_2 p} C_{p i_3 i_4} C^*_{i_1 i_2 q} C^*_{q i_3 i_4}$$

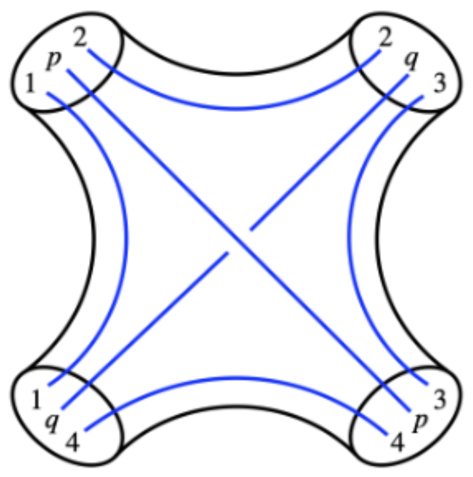


$$- \; C_{i_1 i_2 p} C_{p i_3 i_4} C^*_{q i_4 i_1} C^*_{i_2 i_3 q}$$



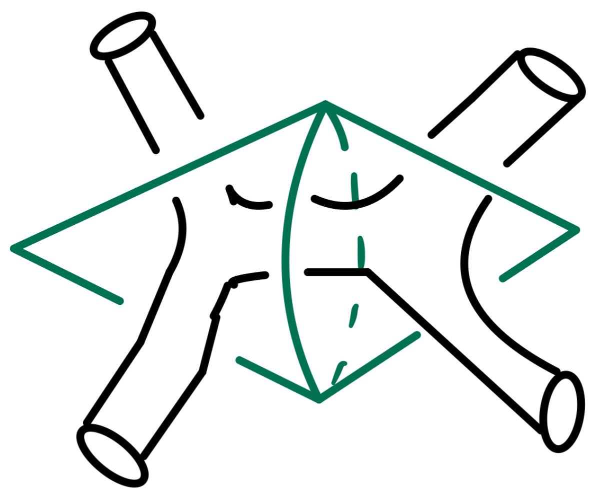


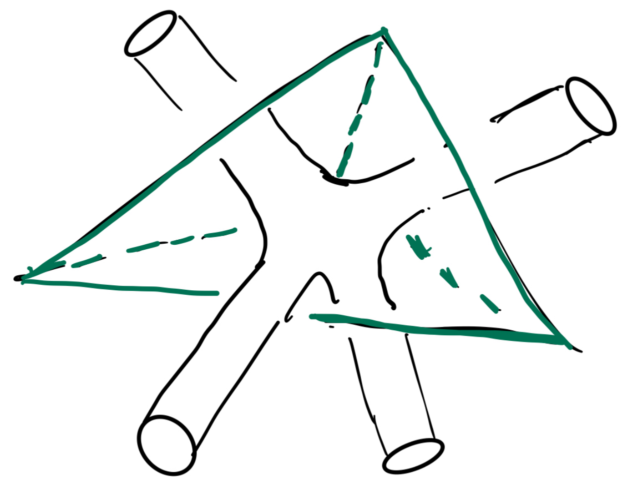
Pillow



Tetrahedral / 6J

The Feynman rules build up 3 manifolds from these wormhole geometries. Equivalently, they can be viewed as simplicial decompositions



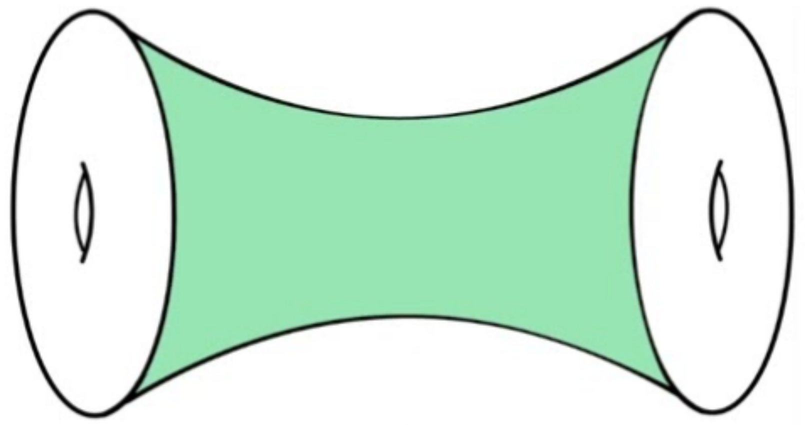


The Feynman rules build up 3 manifolds from these wormhole geometries. Equivalently, they can be viewed as simplicial decompositions

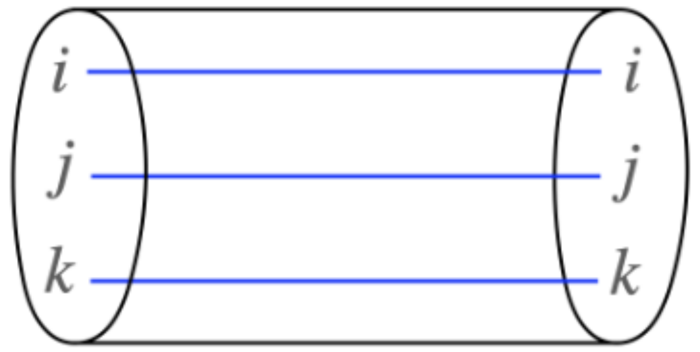
The punchline

The chaotic dynamics of 2D conformal field theories is equivalent by AdS3 gravity + wormholes

$$\overline{Z_{CFT}(\tau_1)Z_{CFT}(\tau_2)} =$$

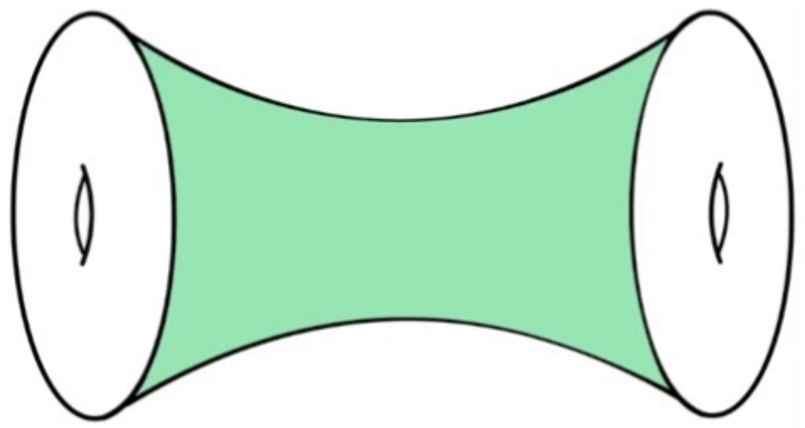


$$\overline{C_{ijk}C^*_{ijk}} =$$



In 2D the chaotic dynamics of conformal field theories is equivalent by AdS3 gravity + wormholes

$$\overline{Z_{CFT}(\tau_1) Z_{CFT}(\tau_2)} =$$



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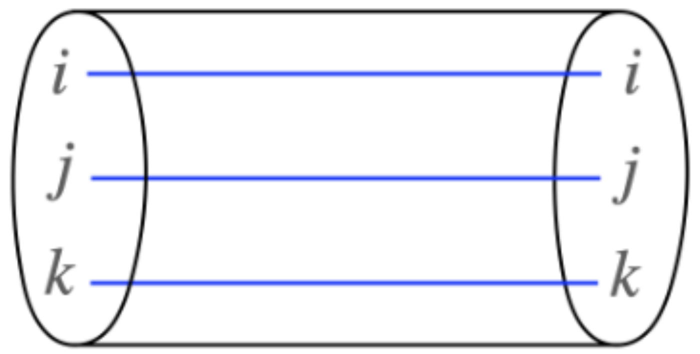
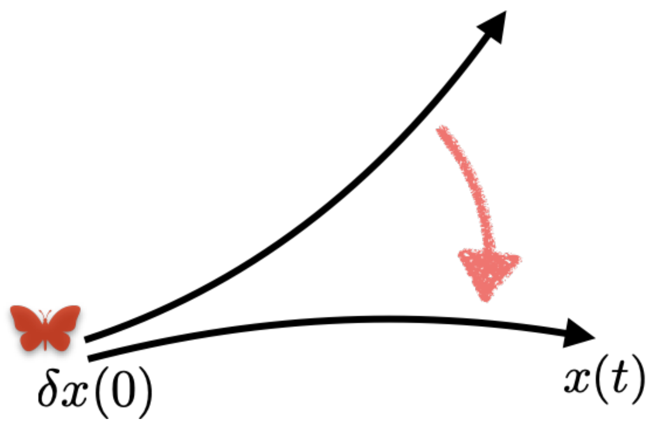


The CFT ensemble gives a new formulation and interpretation of 3D simplicial gravity

It may shed light on the conformal bootstrap and 3D TQFT involving irrational CFT's

Quantum Chaos

Classical chaos~ butterfly effect



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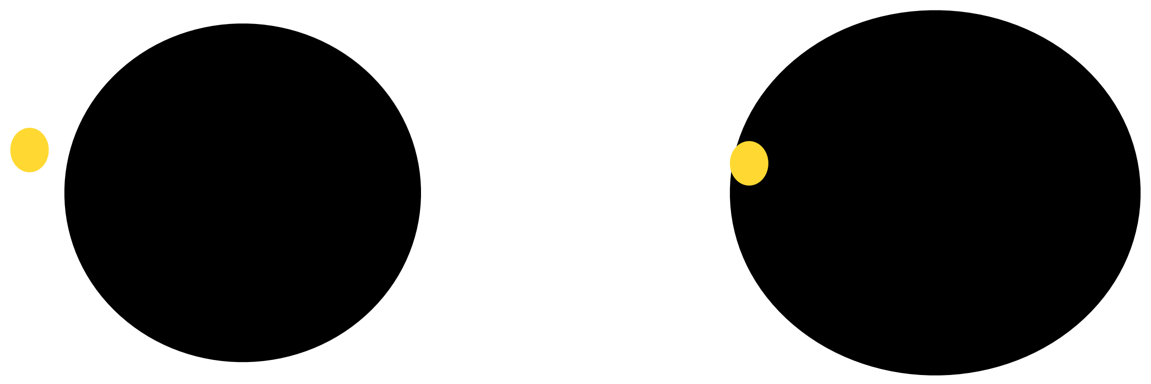

Chaos leads to simplifications, e.g. thermalization :

Quantum Chaos ~ Spectrum of a chaotic Hamiltonian ``looks like" the spectrum of a random matrix

It's much easier to find the spectrum of a RMT then to diagonalize a complicated Hamiltonian (Wigner-Dyson)

The black hole density of states $\rho(M) = \exp \frac{A(M)}{4G}$ exhibits quantum chaos

(Saad, Shankar, Stanford)