Field Theory & the EW Standard Model
Part II: SM in a nutshell

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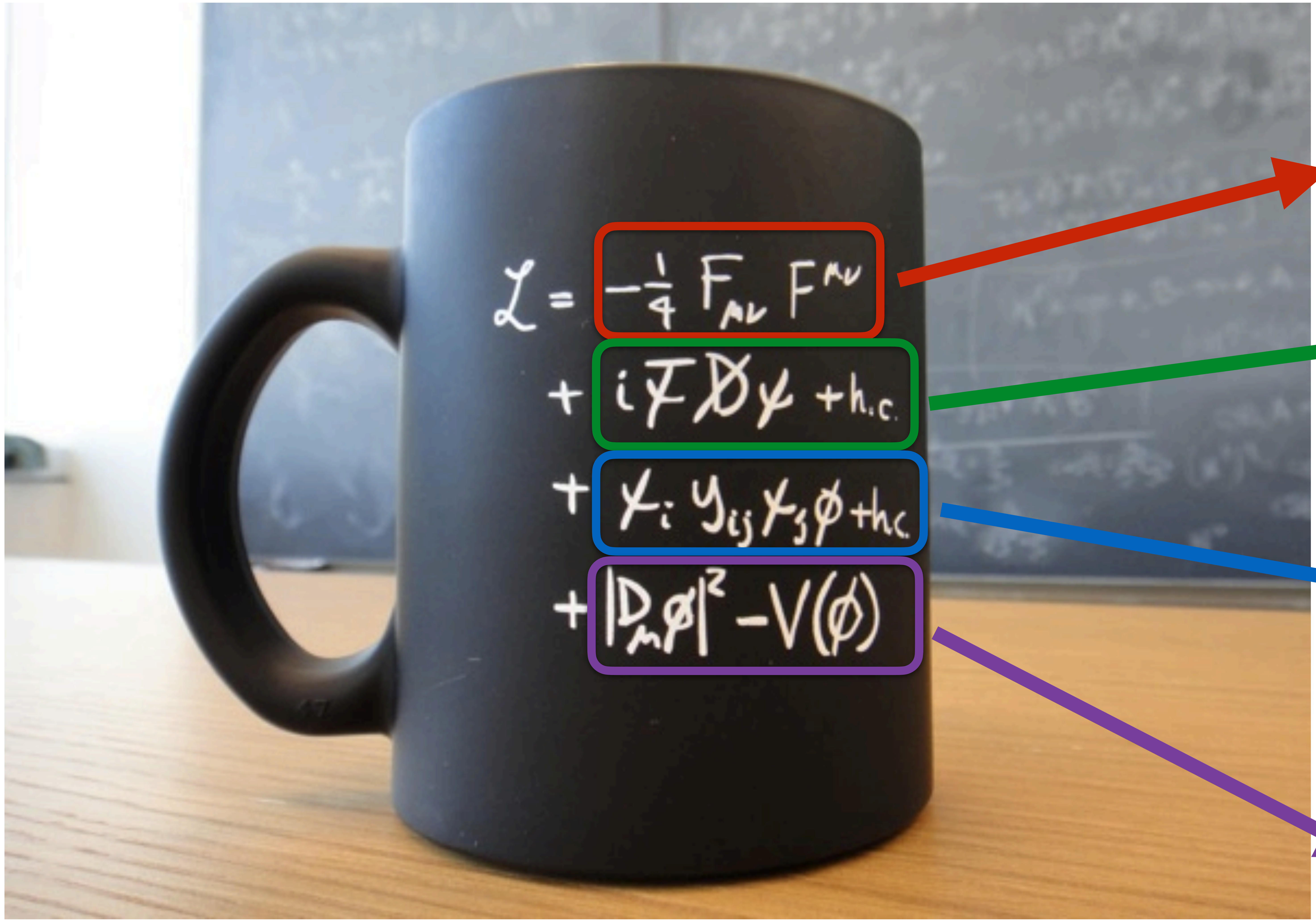
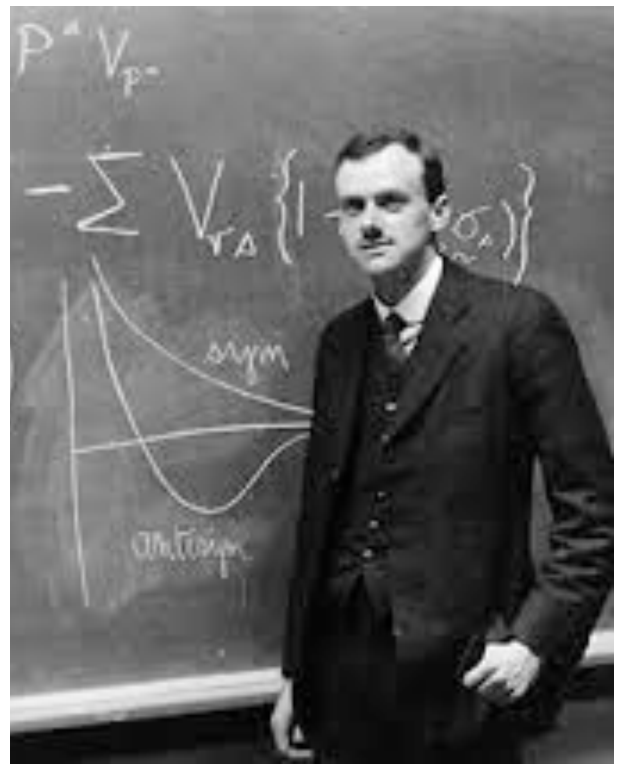
The Standard Model



$\mathcal{L}_{\text{Yang-Mills}}$

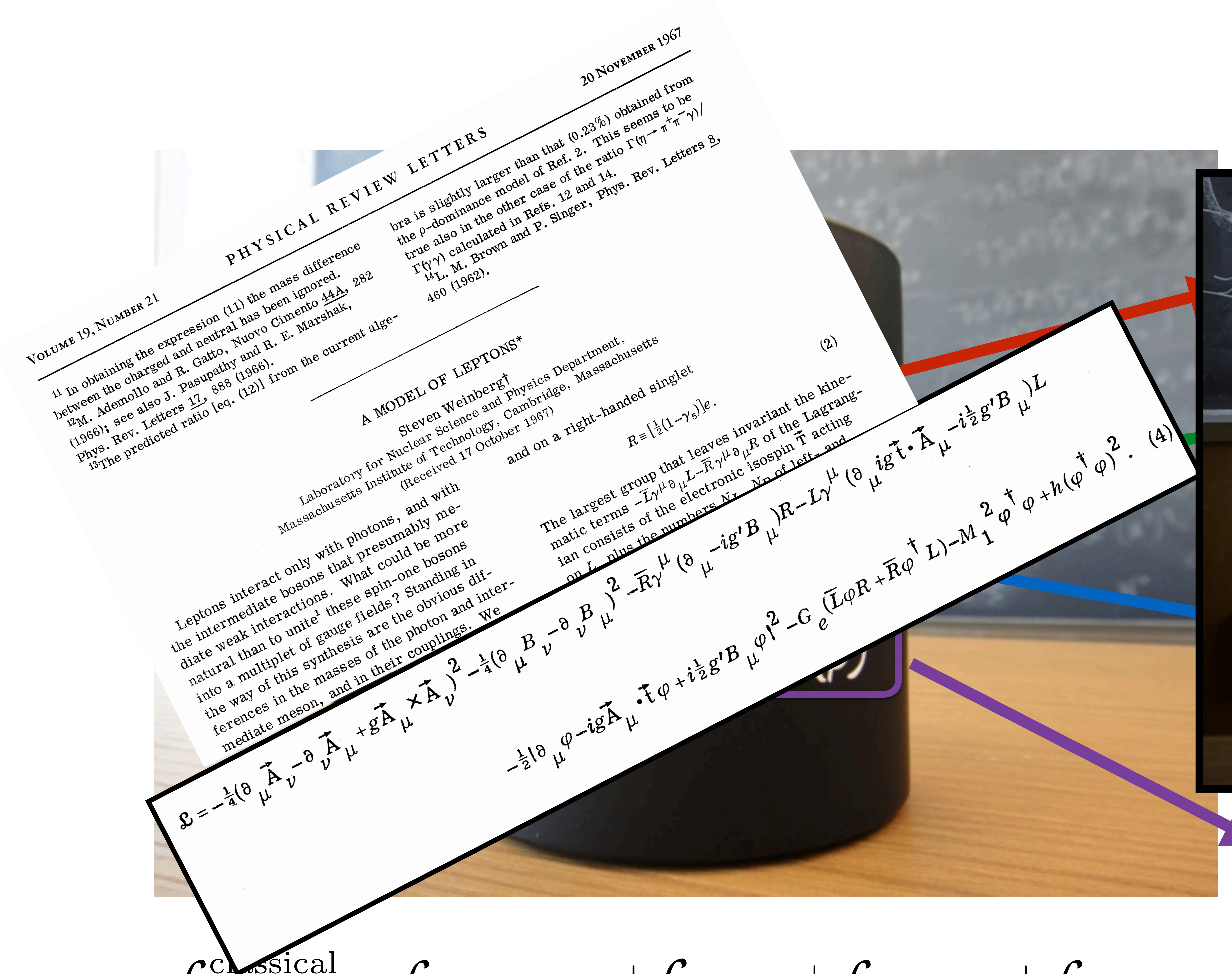
$\mathcal{L}_{\text{Dirac}}$



$\mathcal{L}_{\text{Yukawa}}$

$\mathcal{L}_{\text{Higgs}}$

$$\mathcal{L}_{\text{SM}}^{\text{classical}} = \mathcal{L}_{\text{Yang-Mills}} + \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{Higgs}}$$



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[11] In obtaining the expression (11) the mass difference between the charged and neutral has been ignored.

[12] M. Ademollo and R. Gatto, Nuovo Cimento 44A, 282 (1966); see also J. Pasupathy and R. E. Marshak, Phys. Rev. Letters 17, 888 (1966).

[13] The predicted ratio [eq. (12)] from the current algebra is slightly larger than that (0.23%) obtained from the ρ-dominance model of Ref. 2. This seems to be true also in the other case of the ratio $\Gamma(\eta\to\pi^+\pi^-\gamma)/\Gamma(\gamma\gamma)$ calculated in Refs. 12 and 14.

[14] L. M. Brown and P. Singer, Phys. Rev. Letters 8, 460 (1962).

A MODEL OF LEPTONS*

Steven Weinberg†

Laboratory for Nuclear Science and Physics Department, Massachusetts Institute of Technology, Cambridge, Massachusetts

(Received 17 October 1967)

Leptons interact only with photons, and with the intermediate bosons that presumably mediate weak interactions. What could be more natural than to unite[1] these spin-one bosons into a multiplet of gauge fields? Standing in the way of this synthesis are the obvious differences in the masses of the photon and intermediate meson, and in their couplings. We

and on a right-handed singlet

$$R\equiv[\tfrac{1}{2}(1-\gamma_5)]e. \quad (2)$$

The largest group that leaves invariant the kinematic terms $-\bar{L}\gamma^\mu\partial_\mu L-\bar{R}\gamma^\mu\partial_\mu R$ of the Lagrangian consists of the electronic isospin $\vec{T}$ acting on L, plus the numbers N_L, N_R of left- and

$$\mathcal{L}=-\tfrac{1}{4}(\partial_\mu\vec{A}_\nu-\partial_\nu\vec{A}_\mu+g\vec{A}_\mu\times\vec{A}_\nu)^2-\tfrac{1}{4}(\partial_\mu B_\nu-\partial_\nu B_\mu)^2-\bar{R}\gamma^\mu(\partial_\mu-ig'B_\mu)R-\bar{L}\gamma^\mu(\partial_\mu\, ig\vec{t}\cdot\vec{A}_\mu-i\tfrac{1}{2}g'B_\mu)L$$
$$-\tfrac{1}{2}|\partial_\mu\varphi-ig\vec{A}_\mu\cdot\vec{t}\varphi+i\tfrac{1}{2}g'B_\mu\varphi|^2-G_e(\bar{L}\varphi R+\bar{R}\varphi^\dagger L)-M_1^{\,2}\varphi^\dagger\varphi+h(\varphi^\dagger\varphi)^2. \quad (4)$$

$\mathcal{L}_{\text{Higgs}}$

$$\mathcal{L}_{\text{SM}}^{\text{classical}}=\mathcal{L}_{\text{Yang-Mills}}+\mathcal{L}_{\text{Dirac}}+\mathcal{L}_{\text{Yukawa}}+\mathcal{L}_{\text{Higgs}}$$

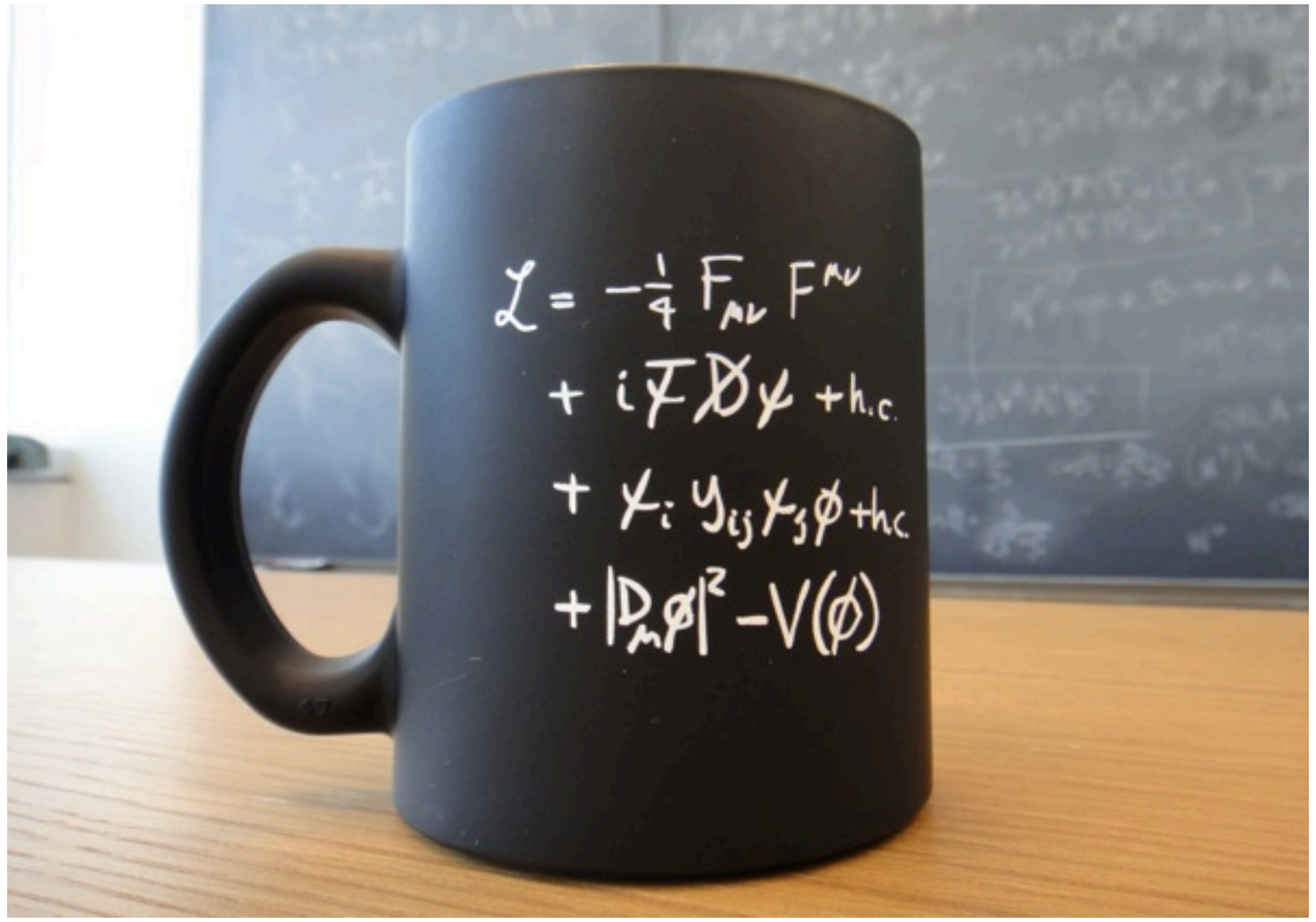
Guiding principles

- Causality
- Unitarity (conservation of probability)

- **Symmetry**
  - space-time: Lorentz invariance
  - internal: gauge invariance

- Renormalisability

- Minimality / Occam's razor



Symmetry transformations

Discrete transformations

Parity: $\phi'(t,\vec{x}) = P\phi(t,\vec{x}) = \phi(t,-\vec{x})$

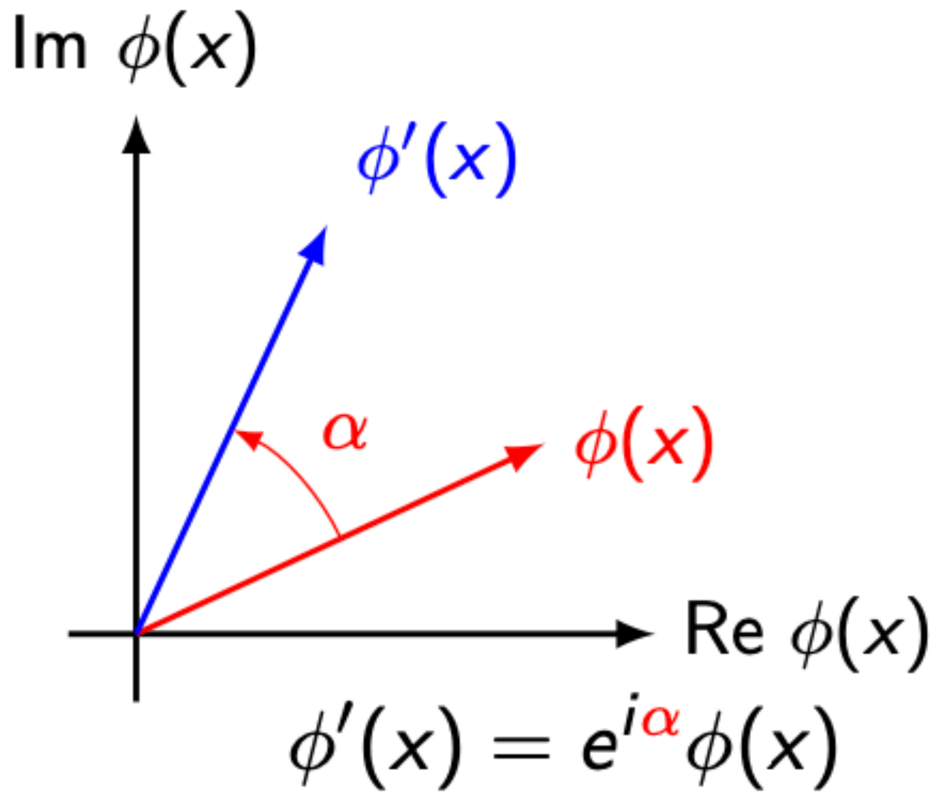
Time-reversal: $\phi'(t,\vec{x}) = T\phi(t,\vec{x}) = \phi(-t,\vec{x})$

Charge-conj.: $\phi'(t,\vec{x}) = C\phi(t,\vec{x}) = \phi^{\dagger}(t,\vec{x})$

Continuous transformations

Space-time symmetry : $\phi'(x) = \phi(x-a)$

Internal symmetry : $\phi'(x) = e^{i\alpha(x)}\phi(x)$



Given a system is invariant under such a transformation ↔ **symmetry**

Transformation of a quantum state: $|\phi'> = U|\phi>$

If symmetry: $<\phi'|\phi'> = <\phi|U^{\dagger}U|\phi> = <\phi|\phi> \quad \rightarrow \quad U^{\dagger}U = 1$ i.e. U is unitary

Group theory

- Mathematical language of symmetry transformations: **group theory**
- Group: set with operation "·", such that $g_3 = g_1 \cdot g_2$ is also element of the group, where
  - $g_1 \cdot (g_2 \cdot g_3) = (g_1 \cdot g_2) \cdot g_3$
  - unity element e is element of a group: $e \cdot g = g$
  - for every group element there is an inverse: $g \cdot g^{-1} = e$

(examples: integers with addition, rotations in 2D, modular arithmetic,…)

- **Abelian group**: $g_i \cdot g_j = g_j \cdot g_i$ for all group elements — Example: U(1) of QED
- **Non-abelian group**: $g_i \cdot g_j \neq g_j \cdot g_i$ for any two group elements — Example: SU(3) of QCD

- **Lie group**: "a group on which you can make differential calculus"

➡ any group element can be obtained as $U(x) = 1 + \alpha^a(x)T^a + \ldots = e^{i\alpha^a(x)T^a}$

T^a: generators of the group

linear combination

➡ $T^a = 1 \rightarrow$ U(1)

The Standard Model

Symmetry:

$$SU(3)_C \times SU(2)_L \times U(1)_Y \xrightarrow{<H>} SU(3)_C \times U(1)_{\mathrm{EM}}$$

Matter content:

- 3 families of matter particles (quarks and leptons) in “fundamental representations”
- 8+3+1 Gauge fields in “adjoint representations”
- 1 Higgs doublet in “fundamental representation” of SU(2) acquires vacuum expectation
  → electroweak symmetry breaking (EWSB)

Some SU(N) group theory

SU(N): NxN unitary matrices U with determinant 1: $UU^\dagger = U^\dagger U = 1, \quad \det(U) = 1\,.$

This is a Lie group, thus every group element can be obtained as $U = e^{i\alpha^a T^a}$ (summation over a)

T^a: **generators** of the group with commutator $[T^a, T^b] = i f^{abc}\, T^c$

$a = 1 \ldots N^2 - 1 \quad N \times N$ matrices

structure constants

conventional normalisation:
$\mathrm{Tr}\,(T^a T^b) = \frac{1}{2}\,\delta^{ab}$

E.g. $SU(3)_C \times SU(2)_L \times U(1)_Y$

8 + 3 + 1 = 12 generators

→ gauge bosons

Some more SU(N) group theory

All particles are embedded in a **representation** $D(U)$ of the gauge groups.

($D : G \rightarrow$ invertible matrices)

E.g. "fundamental representation”: $D(U) = U_{ij}$ for all U in $SU(N)$

- **fundmental rep**: $\psi \rightarrow \psi' = U\psi$, where ψ is N-component column vector, called “$\mathbf{N}$”
- **anti-fundmental rep**: $\overline{\psi} \rightarrow \overline{\psi}' = \overline{\psi}\, U^\dagger$, where $\overline{\psi}$ is N-component row vector, called "$\overline{\mathbf{N}}$”
- **singlet rep**: $\phi \rightarrow \phi' = \phi$, i.e. $D(U) = \mathbf{1}$, called “$\mathbf{1}$”
- **adjoint rep**: $W \rightarrow W' = U\, W\, U^\dagger$, where $W = W_{ij}$ is a matrix, $i, j = 1 \ldots N$, called "$\mathbf{N^2 - 1}$”

In the SM matter particles (fermions, Higgs) transform in the fundamental rep or as singlet.

Gauge bosons always transform in the adjoint representation.

Field content of the SM

Source: The Particle Zoo

Field content of the SM

	field			spin
quarks	$\begin{pmatrix} u \\ d \end{pmatrix}_L$	$\begin{pmatrix} c \\ s \end{pmatrix}_L$	$\begin{pmatrix} t \\ b \end{pmatrix}_L$	1/2
	u_R	c_R	t_R	1/2
	d_R	s_R	b_R	1/2
leptons	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	1/2
	e_R	μ_R	τ_R	1/2
Higgs-doublet		$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}_L$		0
gauge bosons		G_μ^a		1
		W_μ^i		1
		B_μ		1

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$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

Field content of the SM

	field			spin	$SU(3)_C$	$\times SU(2)_L$	$\times U(1)_Y$
quarks	$\begin{pmatrix} u \\ d \end{pmatrix}_L$	$\begin{pmatrix} c \\ s \end{pmatrix}_L$	$\begin{pmatrix} t \\ b \end{pmatrix}_L$	1/2	**3**	**2**	1/3
	u_R	c_R	t_R	1/2	**3**	**1**	4/3
	d_R	s_R	b_R	1/2	**3**	**1**	-2/3
leptons	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	1/2	**1**	**2**	-1
	e_R	μ_R	τ_R	1/2	**1**	**1**	-2
Higgs-doublet		$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}_L$		0	**1**	**2**	1
gauge bosons		G^a_μ		1	**8**	**1**	0
		W^i_μ		1	**1**	**3**	0
		B_μ		1	**1**	**1**	0

1 : singlet

2 : doublet

3 : triplet

8 : octet

Non-abelian gauge theories I

Consider: $\mathscr{L}_{\text{Dirac}} = \overline{\psi}\,(i\partial\!\!\!/ - m)\,\psi$ and we demand $\psi\,/\,\overline{\psi}$ to transform as $\mathbf{N}\,/\,\overline{\mathbf{N}}$: $\psi = \psi_i\,, \overline{\psi} = \overline{\psi}_j$ under $SU(N)$

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$\mathscr{L}_{\text{Dirac}} = \bar{\psi}_i\,(i\not{\partial}\,\delta_{ij} - m\delta_{ij})\,\psi_j$ this is invariant under **global** $\psi \to U\psi$, but not under **local** $U = U(x)$

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minimal coupling

$\mathscr{L} = \overline{\psi}_i\,(iD\!\!\!\!/_{ij} - m\delta_{ij})\,\psi_j$ with $\partial^\mu \to D^\mu_{ij} = \partial^\mu\delta_{ij} - ig\,\mathbf{V}^\mu_{ij}$

$$\mathbf{V}^\mu_{ij}(x) = \sum_{a=1}^{N^2-1} T^a_{ij}\,V^{\mu,a}(x)$$

vector/gauge-field

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$$\mathbf{V}^\mu_{ij}(x) = \sum_{a=1}^{N^2-1} T^a_{ij}\,V^{\mu,a}(x)$$

vector/gauge-field

This introduces a coupling between the the fermion and the vector field:

$$\mathscr{L}_{\text{Dirac}} \to \mathscr{L} = \mathscr{L}_{\text{Dirac}} + \mathscr{L}_{\text{int}} \quad \text{with} \quad \mathscr{L}_{\text{int}} = g\,\bar{\psi}\gamma^\mu \mathbf{V}_\mu \psi \;= g\,\bar{\psi}\gamma^\mu T_a \psi V^a_\mu$$

new Lagrangian is invariant under **local gauge transformation**

$$\psi \to \psi' = U\psi$$

$$\mathbf{V}_\mu \to \mathbf{V}'_\mu = U\,\mathbf{V}_\mu\,U^\dagger - \frac{i}{g}(\partial_\mu U)U^\dagger$$

Non-abelian gauge theories II

We need to add a kinetic term for the gauge field to allow it to propagate 

Generalisation of field-strength tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \frac{i}{e}[D_\mu, D_\nu] \;\rightarrow\; \mathbf{F}_{\mu\nu} = \frac{i}{g}[\mathbf{D}_\mu, \mathbf{D}_\nu]$$

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$$\mathbf{V}_\mu = T^a V^a_\mu$$

$$\rightarrow\; \mathbf{F}_{\mu\nu} = \partial_\mu \mathbf{V}_\nu - \partial_\nu \mathbf{V}_\mu - i\,g\,[\mathbf{V}_\mu, \mathbf{V}_\nu] = T^a F^a_{\mu\nu}$$

$$F^a_{\mu\nu} = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu + g f^{abc}\, V^b_\mu\, V^c_\nu$$

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$$\mathbf{V}_\mu = T^a\,V^a_\mu$$

$$F^a_{\mu\nu} = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu + g f^{abc}\,V^b_\mu\,V^c_\nu$$

Under the gauge transformation $\mathbf{V}_\mu \rightarrow U\,\mathbf{V}_\mu\,U^\dagger - \frac{i}{g}(\partial_\mu U)U^\dagger$ we have $\mathbf{F}_{\mu\nu} \rightarrow \mathbf{F}'_{\mu\nu} = U\mathbf{F}_{\mu\nu}U^\dagger$

trace cyclic

$$\mathrm{Tr}(\mathbf{F}'_{\mu\nu}\mathbf{F}'^{\mu\nu}) = \mathrm{Tr}(U\mathbf{F}_{\mu\nu}U^\dagger\,U\mathbf{F}^{\mu\nu}U^\dagger) = \mathrm{Tr}(U^\dagger U\mathbf{F}_{\mu\nu}U^\dagger\,U\mathbf{F}^{\mu\nu}) = \mathrm{Tr}(\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu})$$ is gauge invariant

Non-abelian gauge theories III

Kinetic term for non-abelian gauge bosons

$$\mathscr{L}_{\text{YM}} = -\frac{1}{2}\,\text{Tr}(\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu}) = -\frac{1}{2}\,\underbrace{\text{Tr}(T^aT^b)}_{\frac{1}{2}\delta^{ab}}F^a_{\mu\nu}\,F^{b,\mu\nu} = -\frac{1}{4}\,F^a_{\mu\nu}\,F^{a,\mu\nu}$$

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$$\frac{1}{2}\delta^{ab} \qquad F^a_{\mu\nu} = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu + g f^{abc} V^b_\mu V^c_\nu$$

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$$= -\frac{1}{4}\,(\partial_\mu V^a_\nu - \partial_\nu V^a_\mu)\,(\partial^\mu V^{a,\nu} - \partial^\nu V^{a,\mu})$$ kinetic-term

$$-\frac{g}{2}\,f_{abc}\,(\partial_\mu V^a_\nu - \partial_\nu V^a_\mu)\,V^{b,\mu}\,V^{c,\nu}$$ trilinear interactions

$$-\frac{g^2}{4}\,f_{abc}f_{ade}\,V^b_\mu\,V^c_\nu\,V^{d,\mu}\,V^{e,\nu}$$ quartic interactions

determined by gauge structure

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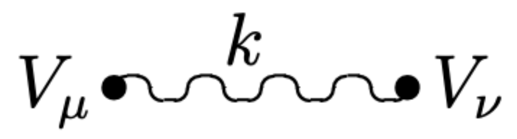
kinetic-term



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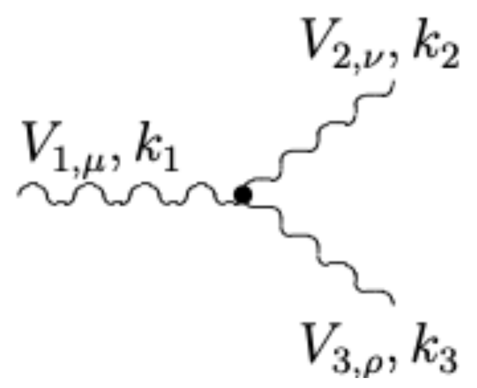
trilinear interactions



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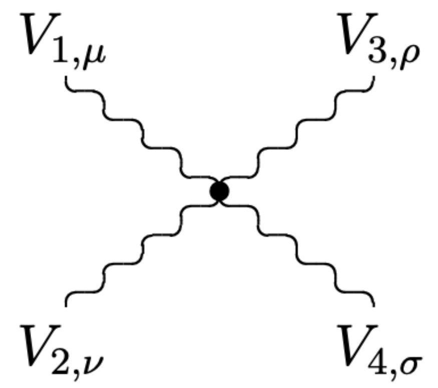
quartic interactions



determined by gauge structure

$$\mathscr{L} = \mathscr{L}_{\text{YM}} + \mathscr{L}_{\text{Dirac}} = -\frac{1}{2}\,\text{Tr}(\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu}) + \Psi\,(i\,\not{\mathbf{D}} - m\delta_{ij})\,\Psi$$

gauge-invariant fermion

$$\mathscr{L} = \mathscr{L}_{\text{YM}} + \mathscr{L}_{\text{cKG}} = -\frac{1}{2}\,\text{Tr}(\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu}) + (\mathbf{D}_\mu\Phi)^\dagger(\mathbf{D}^\mu\Phi) - m^2\,\Phi^\dagger\Phi$$

gauge-invariant complex scalar

The Standard Model

Symmetry:

$$SU(3)_C \times SU(2)_L \times U(1)_Y \xrightarrow{<H>} SU(3)_C \times U(1)_{\text{EM}}$$

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  → electroweak symmetry breaking (EWSB)

QCD

- QCD = invariance under local SU(3)
- quarks transform in fundamental rep of SU(3) $\rightarrow$ triplets, i.e. they carry an additional colour-charge index
- the corresponding gauge field (=gluons) transforms in the adjoint rep. of SU(3), i.e. as $\mathbf{8}$

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QCD Lagrangian for one quark-type of mass m:

$$\mathscr{L}_{\rm QCD} = -\frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu} + \bar{\psi}_i \left(i \not{D}_{ij} - m\, \delta_{ij} \right) \psi_j$$

gluon-colour index, a=1…8

quark-colour index i,j=1,2,3

where $$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu \,, \qquad D^\mu_{ij} = \partial^\mu \delta_{ij} + i g_s t^a_{ij} G^{a\mu}$$

structure constants of SU(3)

strong coupling "constant": $\alpha_S = \dfrac{g_s^2}{4\pi}$

generators of SU(3) in fundamental rep: 3x3 matrices

$$[t^a, t^b] = i f^{abc} t^c$$

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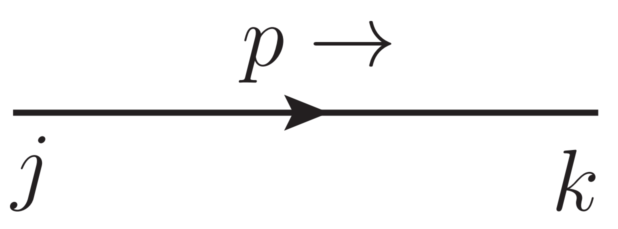
QCD Lagrangian for f={u,d,c,s,t,b} quarks:

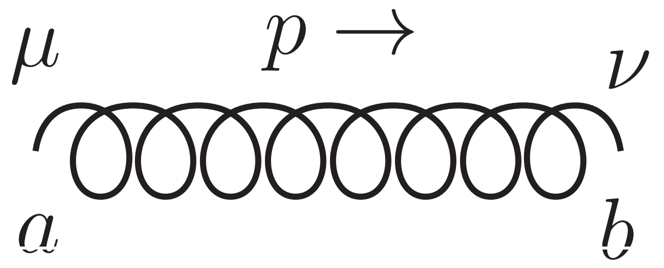
$$\mathscr{L}_{\mathrm{QCD}} = -\frac{1}{4} G^{a\mu\nu} G^{a}_{\mu\nu} + \sum_f \overline{\psi}^f_i \left(i \not{D}_{ij} - m^f \, \delta_{ij} \right) \psi^f_j$$

↪ 6 identical copies

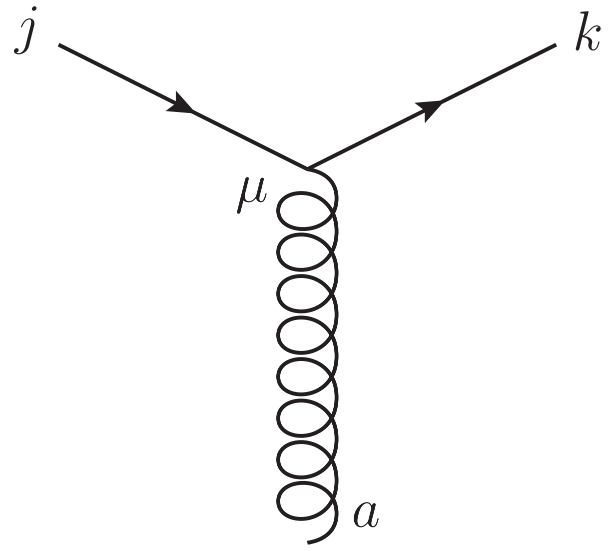
QCD Feynman rules



$$\delta_{kj} \, \frac{i(\not{p} + m)}{p^2 - m^2 + i\varepsilon}$$

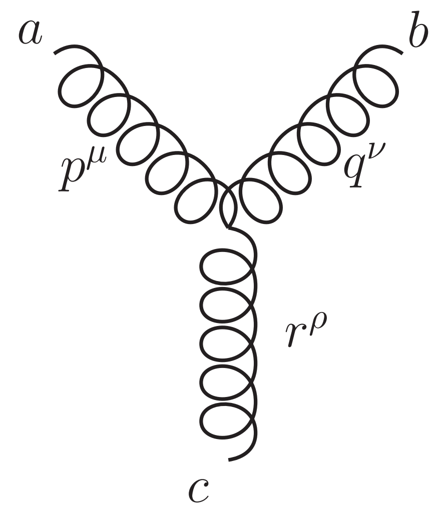


$$\delta^{ab} \, \frac{-ig^{\mu\nu}}{p^2 + i\varepsilon}$$

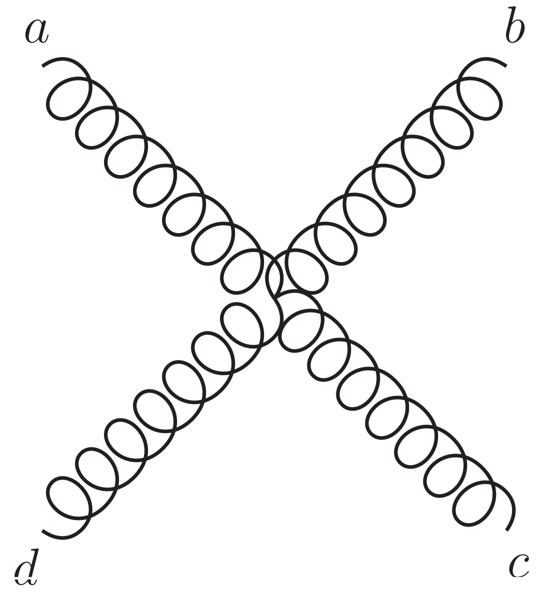


$$-ig_s \gamma^\mu t^a_{kj}$$

all the same



$$g_s f^{abc} \left(g^{\mu\nu}(p-q)^\rho + g^{\nu\rho}(q-r)^\mu + g^{\rho\mu}(r-p)^\nu \right)$$



$$\begin{aligned} -ig_s^2 \Big(& f^{abe} f^{cde} (g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) \\ & + f^{ace} f^{bde} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho}) \\ & + f^{ade} f^{bce} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma}) \Big) \end{aligned}$$

→ Gavin's QCD course after the coffee break

The Standard Model

Symmetry:

$$SU(3)_C \times SU(2)_L \times U(1)_Y \xrightarrow{<H>} SU(3)_C \times U(1)_{\mathrm{EM}}$$

Matter content:

- 3 families of matter particles (quarks and leptons) in “fundamental representations”
- 8+3+1 Gauge fields in “adjoint representations”
- 1 Higgs doublet in “fundamental representation” of SU(2) acquires vacuum expectation

  → electroweak symmetry breaking (EWSB)

The unbroken Standard Model

$SU(3)_C \times SU(2)_L \times U(1)_Y$

$$\mathscr{L}_{\rm YM} = -\frac{1}{4}G^{a\,\mu\nu}G^a_{\mu\nu} - \frac{1}{4}W^{i\,\mu\nu}W^i_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}$$

with the *field strength tensors:*

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu \,,$$

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g_2 \epsilon^{ijk} W^j_\mu W^k_\nu \,,$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

structure constants

gauge couplings

G^a_μ: $SU(3)_C$ bosons, 8 gluons
W^i_μ: $SU(2)_L$ bosons, W^0, W^1, W^2 bosons
B_μ: $U(1)_Y$ boson

Chiral fermions

Wu experiment 1957: weak interactions violate parity conservation

charged currents only involve left-handed particles (right-handed anti-particles)

under $SU(2)_L$ left-handed fermions: doublets, while right-handed fermions: singlets

	field			spin	$SU(3)_C$	$SU(2)_L$	Y
quarks	$\begin{pmatrix} u \\ d \end{pmatrix}_L$	$\begin{pmatrix} c \\ s \end{pmatrix}_L$	$\begin{pmatrix} t \\ b \end{pmatrix}_L$	1/2	**3**	**2**	1/3
	u_R	c_R	t_R	1/2	**3**	**1**	4/3
	d_R	s_R	b_R	1/2	**3**	**1**	-2/3
leptons	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	1/2	**1**	**2**	-1
	e_R	μ_R	τ_R	1/2	**1**	**1**	-2

assign hyper-charge Y such that

$$Q = I_3 + \frac{Y}{2}$$

Q=electromagnetic charge

(Gell-Mann - Nishijima relation)

Chiral fermions II

$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$

Starting from a Dirac ψ fermion we define $\psi_L = \frac{1-\gamma_5}{2}\psi, \quad \psi_R = \frac{1+\gamma_5}{2}\psi$

for massless fermions:
chirality = helicity

j = generation flavour=e, μ, τ, u, c, t

$$\psi_L^j = \begin{pmatrix} \psi_{L+}^j \\ \psi_{L-}^j \end{pmatrix}$$

L up-type = $e,\ \mu,\ \tau,\ u,\ c,\ t$

L down-type = $\nu_e, \nu_\mu, \nu_\tau, d, s, b$

ψ_{R+}^j R up-type = e, μ, τ, u, c, t

ψ_{R-}^j R down-type = d, s, b

no right-handed neutrinos

$SU(2)_L \times U(1)_Y$ - invariant covariant derivatives:

$$\mathbf{D}_\mu^L = \partial_\mu + i\,g_2\,\mathbf{I}^i\,W_\mu^i + i\,g_1\,\frac{Y}{2}\,\mathbf{1}\,B_\mu$$
$$\mathbf{D}_\mu^R = \partial_\mu + i\,g_1\,\frac{Y}{2}\,1\,B_\mu$$

$SU(2)_L$ generstor $\mathbf{I}^i = \frac{1}{2}\sigma^i$ (Pauli matrices)

$U(1)_Y$ generator

The unbroken Standard Model

$$\mathscr{L}_{\rm SM}^{\rm classical} = \mathscr{L}_{\rm YM} + \mathscr{L}_{\rm Dirac} + \dots$$

$$\mathscr{L}_{\rm YM} = -\frac{1}{4} G^{a\,\mu\nu} G^a_{\mu\nu} - \frac{1}{4} W^{i\,\mu\nu} W^i_{\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu}$$

with the *field strength tensors:*

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu \,,$$

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g_2 \epsilon^{ijk} W^j_\mu W^k_\nu \,,$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$\mathscr{L}_{\rm Dirac} = \sum_{i=1}^{3} [{q^i_L}^\dagger \bar{\sigma}^\mu D_\mu q^i_L + {u^i_R}^\dagger \sigma^\mu D_\mu u^i_R + {d^i_R}^\dagger \sigma^\mu D_\mu d^i_R + {l^i_L}^\dagger \bar{\sigma}^\mu D_\mu l^i_L + {e^i_R}^\dagger \sigma^\mu D_\mu e^i_R]$$

with the *gauge covariant derivative:*

$$D_\mu = \partial_\mu + i g_s \mathbf{T}^a G^a_\mu + i g_2 \mathbf{I}^i W^i_\mu + i g_1 \frac{Y}{2} \mathbf{1} B_\mu$$

structure constants

gauge couplings

➡ F-F-V, V-V-V (TG) and V-V-V-V (QG) **couplings are related!**

The unbroken Standard Model

$$\mathscr{L}_{\rm SM}^{\rm classical} = \mathscr{L}_{\rm YM} + \mathscr{L}_{\rm Dirac} + \ldots$$

$$\mathscr{L}_{\rm YM} = -\frac{1}{4} G^{a\,\mu\nu} G^a_{\mu\nu} - \frac{1}{4} W^{i\,\mu\nu} W^i_{\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu}$$

$$\mathscr{L}_{\rm Dirac} = \sum_{i=1}^{3} [q_L^{i\,\dagger} \bar{\sigma}^\mu D_\mu q_L^i + u_R^{i\,\dagger} \sigma^\mu D_\mu u_R^i + d_R^{i\,\dagger} \sigma^\mu D_\mu d_R^i + l_L^{i\,\dagger} \bar{\sigma}^\mu D_\mu l_L^i + e_R^{i\,\dagger} \sigma^\mu D_\mu e_R^i]$$

still: $W^i_{\mu\nu} W^{i\,\mu\nu}$ terms not allowed by gauge invariance
$\rightarrow$ **no vector-boson mass terms allowed**

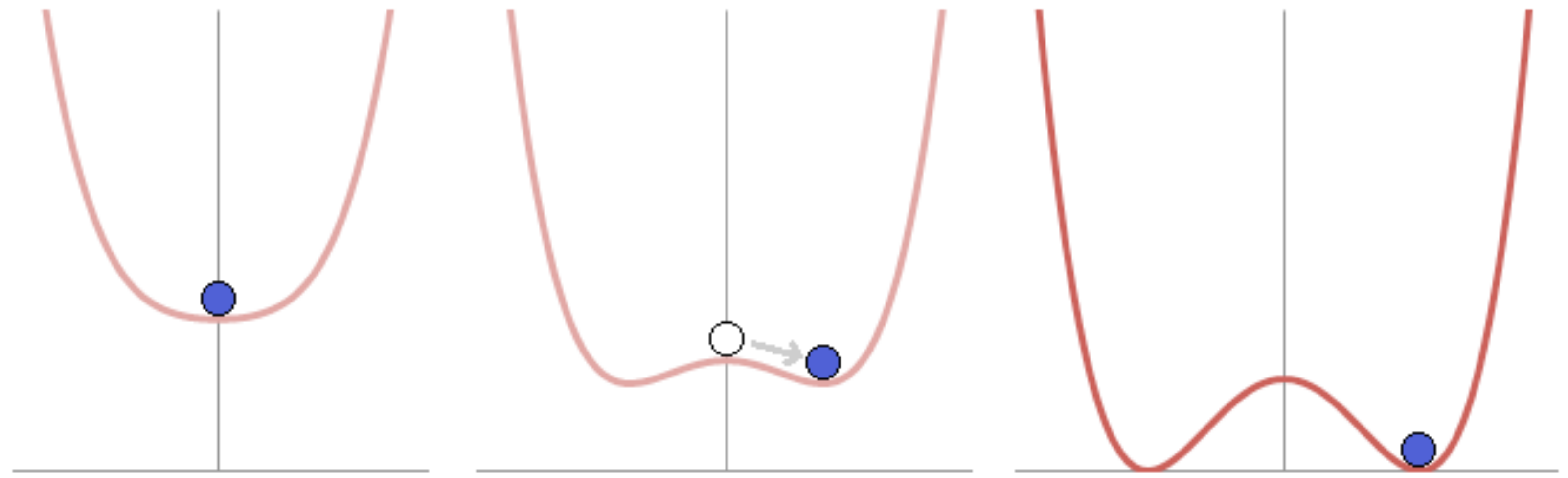
also: **no fermion mass terms allowed** as $m\overline{\psi}\psi = m\left(\overline{\psi}_L\psi_R + \overline{\psi}_R\psi_L\right)$ would mix left- and right-handed fields,

Solution: **Spontaneous Symmetry Breaking** (SSB)

Spontaneous Symmetry Breaking

Key idea: Lagrangian is invariant under gauge symmetry, but vacuum is not = vacuum breaks symmetry.

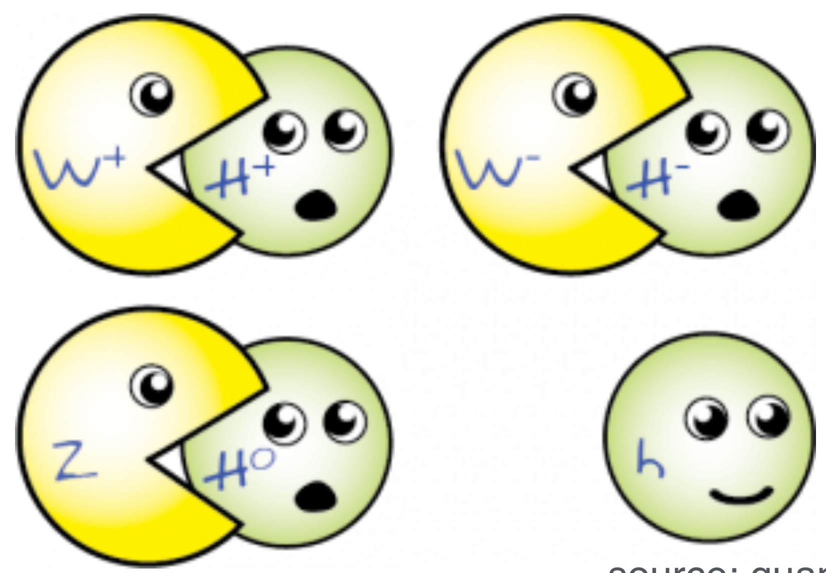
source: wikipedia



unbroken symmetry | broken symmetry

Goldstone theorem: for every broken generator there is a massless mode

Goldstone theorem combined with gauge theories: massless goldstone modes are absorbed (=eaten) to become longitudinal modes of the gauge bosons associated to the broken generators.



source: quantumdiaries.org

The Higgs mechanism

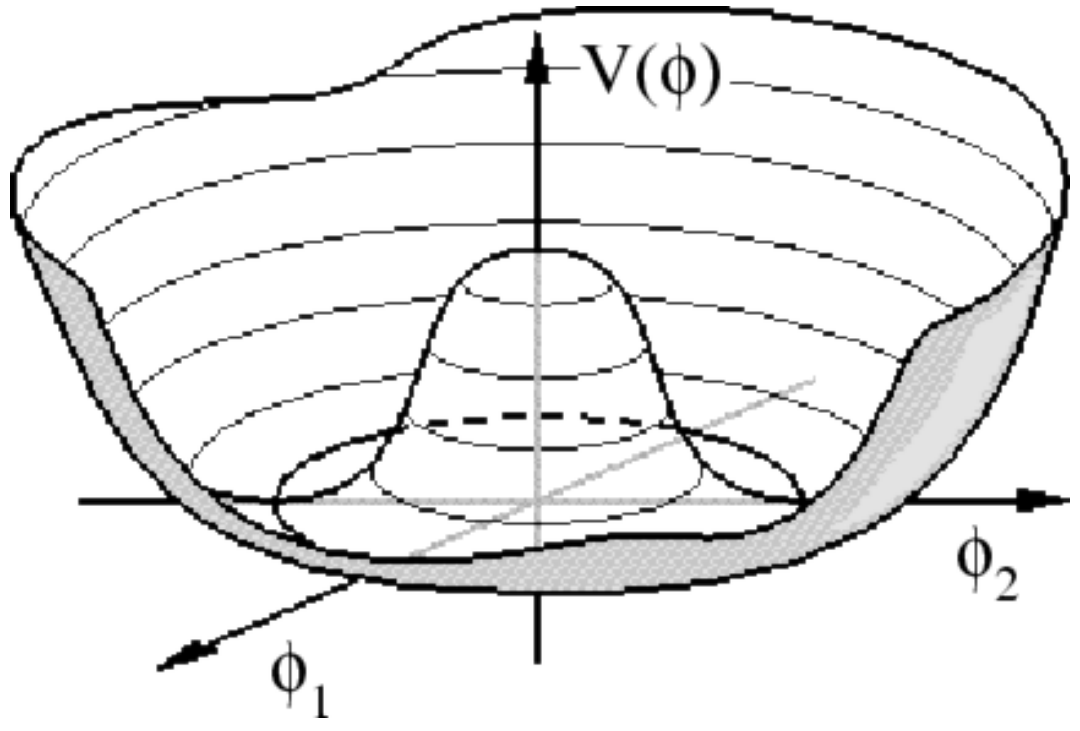
$$\mathscr{L}_{\text{SM}}^{\text{classical}} = \mathscr{L}_{\text{YM}} + \mathscr{L}_{\text{Dirac}} + \mathscr{L}_{\text{Higgs}} + \dots$$

$$\mathcal{L}_{\text{Higgs}} = (D^\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi)$$

Higgs potential: $V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \frac{\lambda}{4}(\Phi^\dagger \Phi)^2$

complex scalar SU(2)-doublet $\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix}$

$$D_\mu = \partial_\mu + i\, g_2 \frac{\sigma_a}{2} W_\mu^a + i\, \frac{g_1}{2} B_\mu$$



The Higgs mechanism

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Higgs potential: $V(\Phi) = -\mu^2\Phi^{\dagger}\Phi + \frac{\lambda}{4}(\Phi^{\dagger}\Phi)^2$

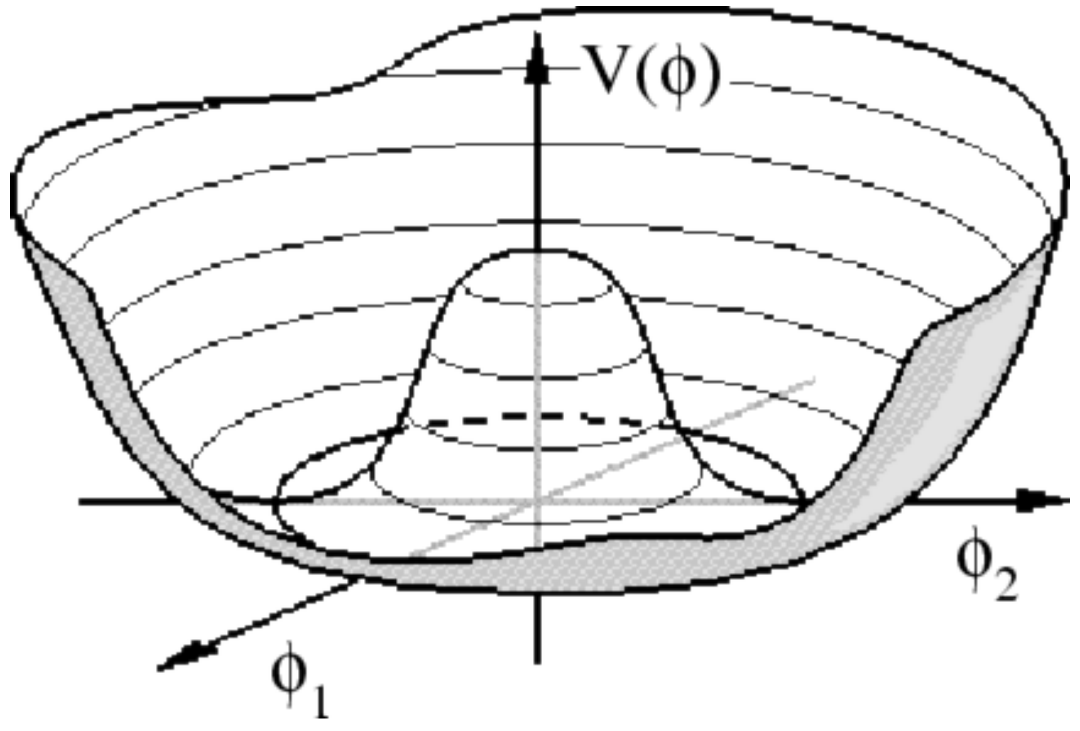
minimum at $\Phi^{\dagger}\Phi = \frac{2\mu^2}{\lambda}$ $\qquad \mu^2, \lambda > 0$

$<\Phi> = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}$ with $v = \frac{2\mu}{\sqrt{\lambda}}$

vacuum expectation value

complex scalar SU(2)-doublet $\Phi(x) = \begin{pmatrix}\phi^{+}(x)\\ \phi^{0}(x)\end{pmatrix}$

$$D_{\mu} = \partial_{\mu} + i\, g_2 \frac{\sigma_a}{2} W_{\mu}^{a} + i\, \frac{g_1}{2} B_{\mu}$$



such that $Q <\Phi> = \left(I_3 + \frac{Y}{2}\right) <\Phi> = \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}\begin{pmatrix}0\\ v\end{pmatrix} = 0$

vacuum electrically neutral / invariant under $U(1)_{\text{EM}}$

vacuum NOT invariant under $SU(2)_L \times U(1)_Y$ transformations

The Higgs mechanism

Expand $\mathbf{\Phi}$-field around minimum: $\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}(v + h^0(x) + i\chi^0(x)) \end{pmatrix}$

$<h^0> = <\chi^0> = <\phi^\pm> = 0$

would-be Goldstone bosons

The Higgs mechanism

Expand Φ-field around minimum: $\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}(v + h^0(x) + i\chi^0(x)) \end{pmatrix}$

$<h^0> = <\chi^0> = <\phi^\pm> = 0$

would-be Goldstone bosons

gauge
transformation
="unitary gauge"

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h^0(x) \end{pmatrix}$$

The Higgs mechanism

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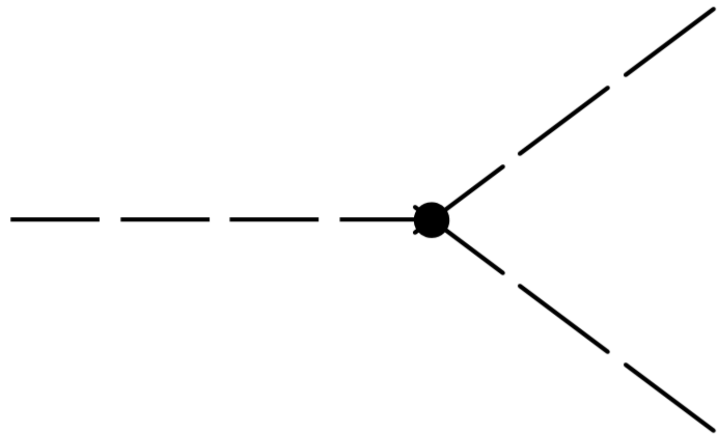
would-be Goldstone bosons

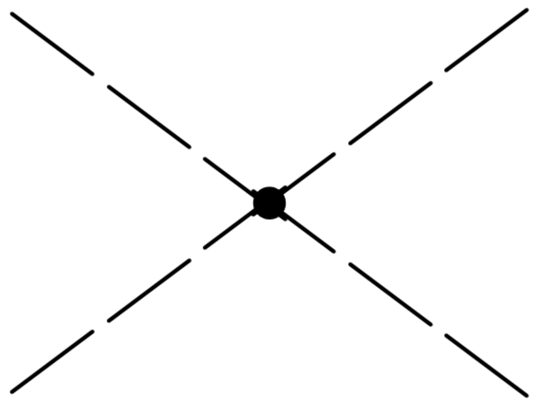
gauge transformation ="unitary gauge"

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h^0(x) \end{pmatrix}$$

I. Higgs potential $V = \mu^2 (h^0)^2 + \frac{\mu^2}{v}(h^0)^3 + \frac{\mu^2}{4v^2}(h^0)^4 = \frac{m_h}{2}(h^0)^2 + \ldots$

$m_{h^0} = \sqrt{2}\mu = \frac{v\mu}{2}$





Gauge boson masses

Expand Φ-field around minimum: $\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}(v + h^0(x) + i\chi^0(x)) \end{pmatrix}$

$<h^0> = <\chi^0> = <\phi^\pm> = 0$

would-be Goldstone bosons

gauge transformation ="unitary gauge"

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h^0(x) \end{pmatrix}$$

mass terms for W, B!!

II. kinetic term $(D^\mu \Phi)^\dagger (D_\mu \Phi) = \frac{1}{2}\left(\frac{g_2}{2} v\right)^2 (W_1^2 + W_2^2) + \frac{1}{2}\left(\frac{v}{2}\right)^2 \left(W_\mu^3, B_\mu\right) \begin{pmatrix} g_2^2 & g_1 g_2 \\ g_1 g_2 & g_1^2 \end{pmatrix} \begin{pmatrix} W^{3,\mu} \\ B^\mu \end{pmatrix} + \ldots$

redefine!

diagonalise!

Gauge boson masses

Expand Φ-field around minimum: $\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}(v + h^0(x) + i\chi^0(x)) \end{pmatrix}$

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redefine!

diagonalise!

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp i W_\mu^2)$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

physical fields

unbroken fields

Gauge boson masses

Expand Φ-field around minimum: $\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}(v + h^0(x) + i\chi^0(x)) \end{pmatrix}$

$<h^0> = <\chi^0> = <\phi^\pm> = 0$

gauge transformation ="unitary gauge" $= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h^0(x) \end{pmatrix}$

would-be Goldstone bosons

mass terms for W, B!!

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redefine!

diagonalise!

$$= M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2}(A_\mu, Z_\mu)\begin{pmatrix} 0 & 0 \\ 0 & M_Z^2 \end{pmatrix}\begin{pmatrix} A^\mu \\ Z^\mu \end{pmatrix} + \cdots$$

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp i W_\mu^2)$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix}\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

physical fields

unbroken fields

Gauge boson masses

Expand Φ-field around minimum: $\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}(v + h^0(x) + i\chi^0(x)) \end{pmatrix}$

$<h^0> = <\chi^0> = <\phi^\pm> = 0$

would-be Goldstone bosons

gauge transformation ="unitary gauge"

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h^0(x) \end{pmatrix}$$

mass terms for W, B!!

II. kinetic term $(D^\mu\Phi)^\dagger(D_\mu\Phi) = \frac{1}{2}\left(\frac{g_2}{2}v\right)^2 (W_1^2 + W_2^2) + \frac{1}{2}\left(\frac{v}{2}\right)^2 \left(W_\mu^3, B_\mu\right)\begin{pmatrix} g_2^2 & g_1 g_2 \\ g_1 g_2 & g_1^2 \end{pmatrix}\begin{pmatrix} W^{3,\mu} \\ B^\mu \end{pmatrix} + \dots$

redefine!

diagonalise!

$$= M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2}(A_\mu, Z_\mu)\begin{pmatrix} 0 & 0 \\ 0 & M_Z^2 \end{pmatrix}\begin{pmatrix} A^\mu \\ Z^\mu \end{pmatrix} + \cdots$$

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp i W_\mu^2)$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix}\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

physical fields

unbroken fields

$$M_W = \frac{1}{2} g_2 v\,, \quad M_Z = \frac{1}{2}\sqrt{g_1^2 + g_2^2}\, v$$

$$\cos\theta_W = \frac{g_2}{\sqrt{g_1^2 + g_2^2}} = \frac{M_W}{M_Z}$$

EW mixing angle

➡**couplings and gauge boson masses are related!**

Gauge-Higgs couplings

Expand Φ-field around minimum: $\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}(v + h^0(x) + i\chi^0(x)) \end{pmatrix}$

$<h^0> = <\chi^0> = <\phi^\pm> = 0$

would-be Goldstone bosons

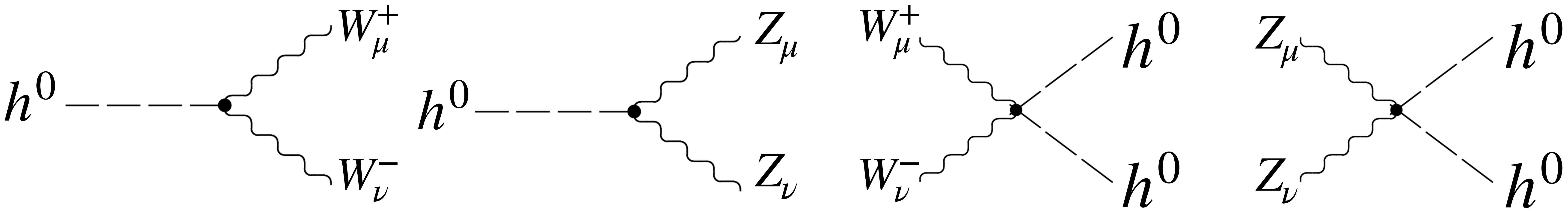
gauge transformation ="unitary gauge"

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h^0(x) \end{pmatrix}$$

II. kinetic term (diagonalised)

$$(D^\mu\Phi)^\dagger(D_\mu\Phi) = M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2}(A_\mu, Z_\mu)\begin{pmatrix} 0 & 0 \\ 0 & M_Z^2 \end{pmatrix}\begin{pmatrix} A^\mu \\ Z^\mu \end{pmatrix}$$

$$+\frac{g_2^2 v}{2} h^0 W^+ W^- + \frac{g_1^2 + g_2^2}{4} v\, h^0 ZZ + \frac{g_2^2 v^2}{4} h^0 h^0 W^+ W^- + \frac{g_1^2 + g_2^2}{8} v^2\, h^0 h^0 ZZ$$



Yukawa terms

$y\bar{\psi}\phi\psi$

$$\mathscr{L}_{\text{SM}}^{\text{classical}} = \mathscr{L}_{\text{YM}} + \mathscr{L}_{\text{Dirac}} + \mathscr{L}_{\text{Higgs}} + \mathscr{L}_{\text{Yukawa}}$$

Yukawa couplings

$$\mathcal{L}_{\text{Yukawa}} = -\sum_{i,j=1}^{3}\left[y_{ij}^{d}(q_L^i)^\dagger \Phi d_R^j \;+ y_{ij}^{u}(q_L^i)^\dagger \Phi^c u_R^j \;+ y_{ij}^{l}(l_L^i)^\dagger \Phi e_R^j \;+ \text{h.c.}\right]$$

$\Phi^c \equiv i\sigma^2\Phi^*$

Yukawa terms

$$\mathscr{L}_{\text{SM}}^{\text{classical}} = \mathscr{L}_{\text{YM}} + \mathscr{L}_{\text{Dirac}} + \mathscr{L}_{\text{Higgs}} + \mathscr{L}_{\text{Yukawa}}$$

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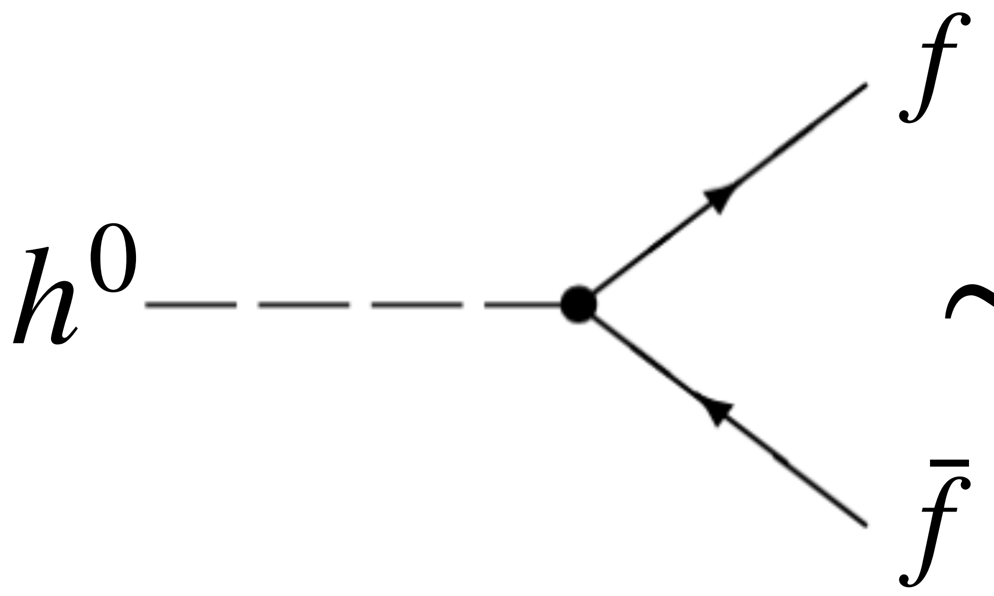
$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h^0 \end{pmatrix}$$

$$\sim -\sum_f m_f \overline{\psi}_f \psi_f - \sum_f \frac{m_f}{v} \overline{\psi}_f \psi_f \, h^0$$



$\sim m/v$

$$\Phi^c \equiv i\sigma^2 \Phi^*$$

Yukawa terms

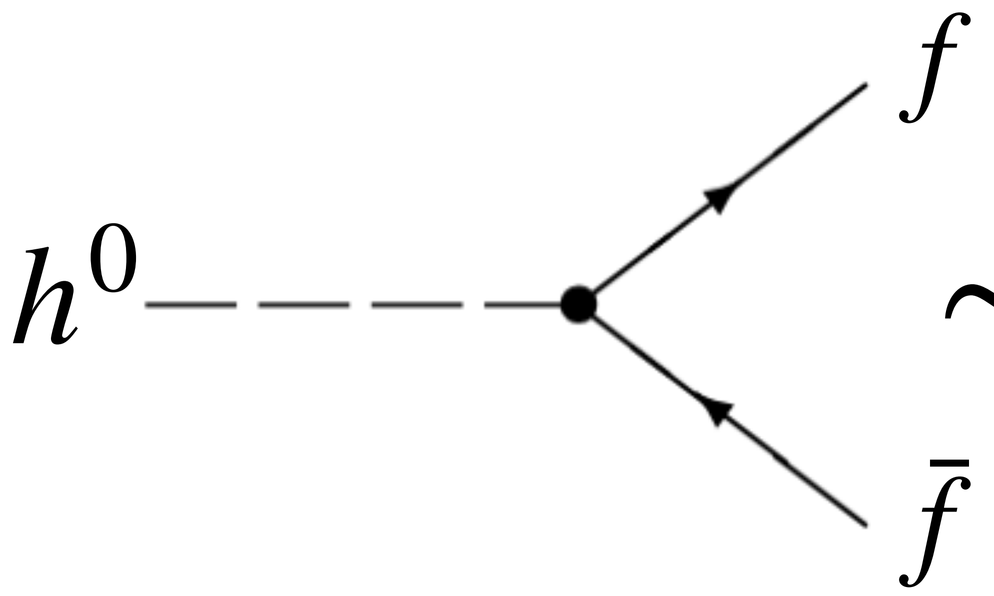
$$\mathscr{L}_{\rm SM}^{\rm classical} = \mathscr{L}_{\rm YM} + \mathscr{L}_{\rm Dirac} + \mathscr{L}_{\rm Higgs} + \mathscr{L}_{\rm Yukawa}$$

Yukawa couplings

$$\mathcal{L}_{\rm Yukawa} = -\sum_{i,j=1}^{3}\left[y_{ij}^{d}(q_L^i)^\dagger \Phi d_R^j + y_{ij}^{u}(q_L^i)^\dagger \Phi^c u_R^j + y_{ij}^{l}(l_L^i)^\dagger \Phi e_R^j + \text{h.c.}\right]$$

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h^0 \end{pmatrix}$$

$$\sim -\sum_f m_f \overline{\Psi}_f \Psi_f - \sum_f \frac{m_f}{v}\overline{\Psi}_f \Psi_f \, h^0$$



$\sim m/v$

to be precise: $$m_{ij}^f = \frac{v}{\sqrt{2}} y_{ij}^f$$

diagonalised: $$m_{f,i} = \frac{v}{\sqrt{2}} \sum_{k,m}^{3} U_{ik}^{f,L} y_{km}^f \left(U_{mi}^{f,R}\right)^\dagger \equiv \frac{v}{\sqrt{2}}\lambda_i^f$$

- due to unitarity these matrices drop out in NC interactions: no FCNCs in the SM
- a non-trivial matrix remains in CC interactions: CKM matrix

$\Phi^c \equiv i\sigma^2 \Phi^*$

→ Timothy's Flavour Physics course starting on Monday

Fermion-gauge couplings

$$\mathscr{L}_{\rm SM}^{\rm classical} = \mathscr{L}_{\rm YM} + \mathscr{L}_{\rm Dirac} + \mathscr{L}_{\rm Higgs} + \mathscr{L}_{\rm Yukawa}$$

$$\mathscr{L}_{\rm Dirac} = \sum_{i=1}^{3} [q_L^{i\,\dagger}\bar{\sigma}^\mu D_\mu q_L^i \; + u_R^{i\,\dagger}\sigma^\mu D_\mu u_R^i \; + d_R^{i\,\dagger}\sigma^\mu D_\mu d_R^i$$
$$+ l_L^{i\,\dagger}\bar{\sigma}^\mu D_\mu l_L^i \; + e_R^{i\,\dagger}\sigma^\mu D_\mu e_R^i \;]$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp i W_\mu^2)$$

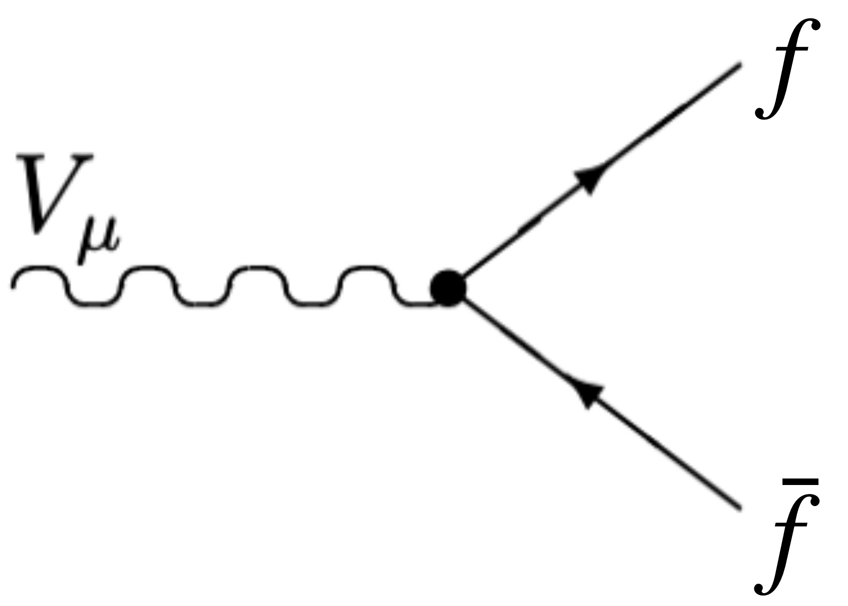
Fermion-gauge couplings

$$\mathscr{L}_{\rm SM}^{\rm classical} = \mathscr{L}_{\rm YM} + \mathscr{L}_{\rm Dirac} + \mathscr{L}_{\rm Higgs} + \mathscr{L}_{\rm Yukawa}$$

$$\mathscr{L}_{\rm Dirac} = \sum_{i=1}^{3} [q_L^{i\,\dagger}\bar{\sigma}^\mu D_\mu q_L^i \; + u_R^{i\,\dagger}\sigma^\mu D_\mu u_R^i \; + d_R^{i\,\dagger}\sigma^\mu D_\mu d_R^i$$
$$+ l_L^{i\,\dagger}\bar{\sigma}^\mu D_\mu l_L^i \; + e_R^{i\,\dagger}\sigma^\mu D_\mu e_R^i \;]$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2)$$



$$\mathscr{L}_{\rm Dirac} = \ldots + J_{\rm em}^\mu A_\mu + J_{\rm NC}^\mu Z_\mu + J_{\rm CC}^\mu W_\mu^+ + J_{\rm CC}^{\mu\,\dagger} W_\mu^- \quad = \ldots - \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}} \bar{e}\gamma_\mu e A^\mu + \ldots$$

$$e = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}}$$

gauge coupling of remaining $U(1)_{\rm EM}$

Fermion-gauge couplings

$$\mathscr{L}_{\rm SM}^{\rm classical} = \mathscr{L}_{\rm YM} + \mathscr{L}_{\rm Dirac} + \mathscr{L}_{\rm Higgs} + \mathscr{L}_{\rm Yukawa}$$

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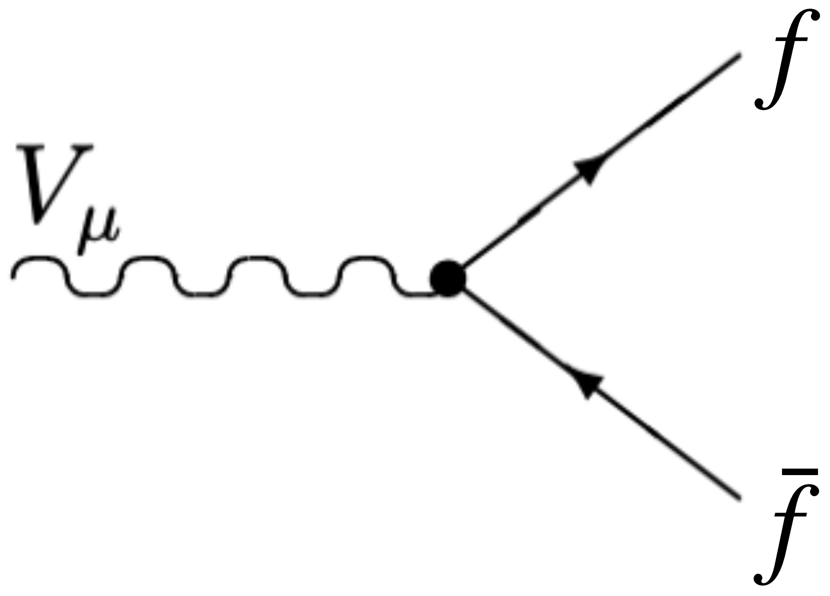

$$\mathscr{L}_{\rm Dirac} = \ldots + J^\mu_{\rm em} A_\mu + J^\mu_{\rm NC} Z_\mu + J^\mu_{\rm CC} W^+_\mu + J^{\mu\,\dagger}_{\rm CC} W^-_\mu \quad = \ldots - \frac{g_1 g_2}{\sqrt{g_1^2+g_2^2}} \bar{e}\gamma_\mu e A^\mu + \ldots$$

$$J^\mu_{\rm EM} = -e \sum_{f=l,q} Q_f \overline{\psi}_f \gamma^\mu \psi_f \,,$$

$$J^\mu_{\rm NC} = \frac{g_2}{2\cos\theta_W} \sum_{f=l,q} \overline{\psi}_f (v_f\gamma^\mu - a_f\gamma^\mu\gamma_5)\psi_f \,,$$

$$v_f = I_3^f - 2Q_f \sin^2\theta_W \,,$$
$$a_f = I_3^f$$

$$J^\mu_{\rm CC} = \frac{g_2}{\sqrt{2}} \left(\sum_{i=1,2,3} \overline{\nu}^i \gamma^\mu \frac{1-\gamma_5}{2} e^i + \sum_{i,j=1,2,3} \overline{u}^i \gamma^\mu \frac{1-\gamma_5}{2} V_{ij} d^j \right)$$

$V_{\rm CKM}$

$$e = \frac{g_1 g_2}{\sqrt{g_1^2+g_2^2}}$$

gauge coupling of remaining $U(1)_{\rm EM}$

Fermion-gauge couplings

$$\mathscr{L}_{\rm SM}^{\rm classical} = \mathscr{L}_{\rm YM} + \mathscr{L}_{\rm Dirac} + \mathscr{L}_{\rm Higgs} + \mathscr{L}_{\rm Yukawa}$$

$$\mathscr{L}_{\rm YM} = -\frac{1}{4}G^{a\,\mu\nu}G^a_{\mu\nu} - \frac{1}{4}W^{i\,\mu\nu}W^i_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix}$$

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Fermion-gauge couplings

$$\mathscr{L}_{\text{SM}}^{\text{classical}} = \mathscr{L}_{\text{YM}} + \mathscr{L}_{\text{Dirac}} + \mathscr{L}_{\text{Higgs}} + \mathscr{L}_{\text{Yukawa}}$$

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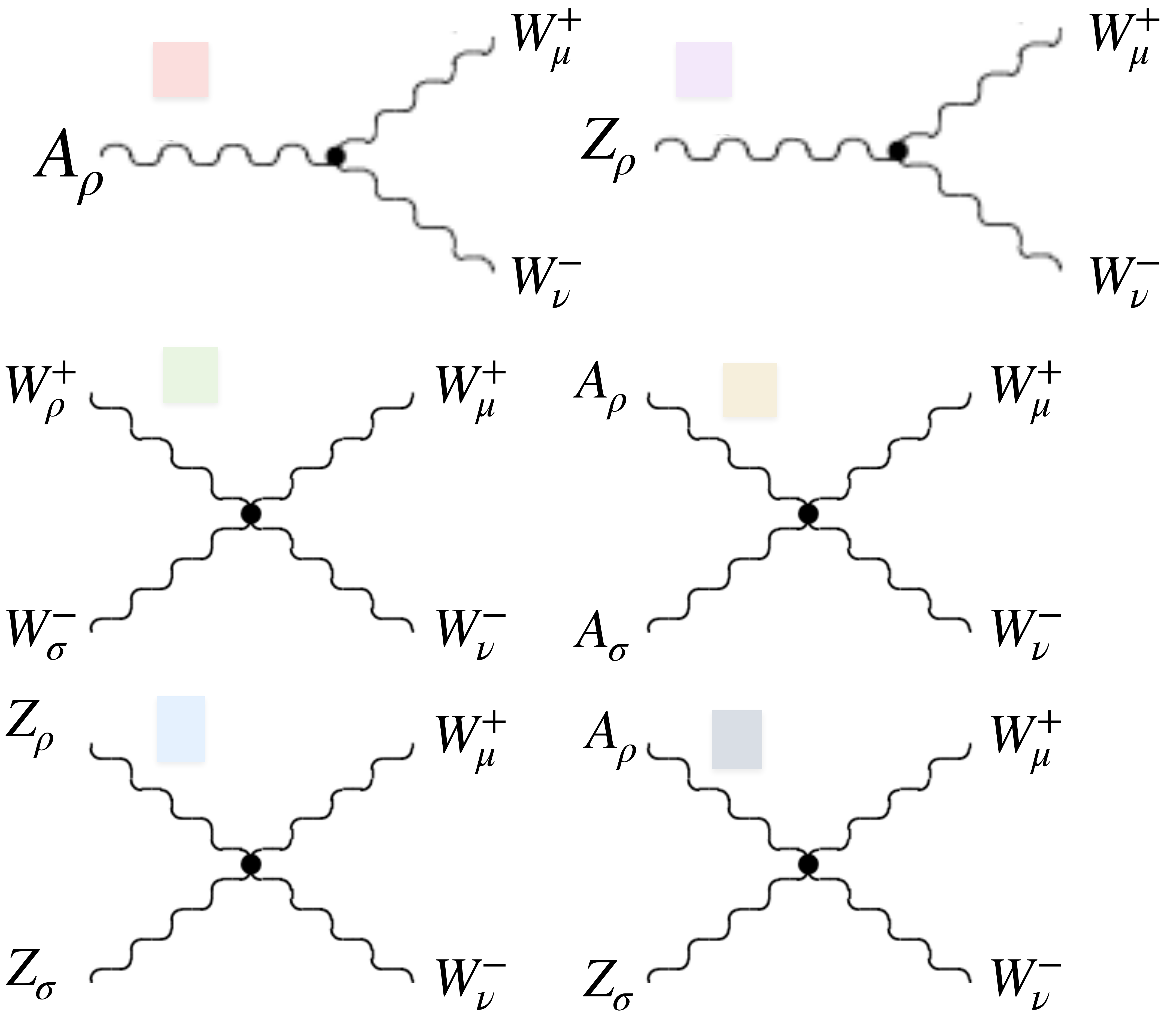
$$\begin{aligned}
\mathscr{L}_{\text{YM}} = \ldots &+ e\left[(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)\, W^{-\mu} A^\nu + W^+_\mu W^-_\nu F^{\mu\nu} + h.c.\right] \\
&+ e \cot\theta_W \left[(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)\, W^{-\mu} Z^\nu + W^+_\mu W^-_\nu Z^{\mu\nu} + h.c.\right] \\
&- e^2/(4\sin\theta_W)\, [(W^-_\mu W^+_\nu - W^-_\nu W^+_\mu) W^+_\mu W^-_\nu + h.c.] \\
&- e^2/4\, (W^+_\mu A_\nu - W^+_\nu A_\mu)(W^{-\mu}A^\nu - W^{-\nu}A^\mu) \\
&- e^2/4\, \cot^2\theta_W\, (W^+_\mu Z_\nu - W^+_\nu Z_\mu)(W^{-\mu}Z^\nu - W^{-\nu}Z^\mu) \\
&+ e^2/2\, \cot\theta_W\, (W^+_\mu A_\nu - W^+_\nu A_\mu)(W^{-\mu}Z^\nu - W^{-\nu}Z^\mu) + h.c.
\end{aligned}$$



SM input parameters

- Unbroken theory $\mathcal{L}_{\rm SM}^{\rm classical}$:
  - Couplings: g_1, g_2, g_S
  - Parameters of the Higgs potential: μ, λ
  - Yukawa couplings: y_{ij}^f

- After EWSM:
  - Couplings: g_1, g_2, g_S or $\alpha_{\rm EM}, \sin\theta_W, \alpha_S$
  - EW boson masses: m_{h^0}, m_W, m_Z, m_f
  - CKM matrix elements: $\mathbf{V}_{\rm CKM}$

➡ Important tree-level relations between input parameters: e.g.: $\cos\theta_W = \frac{m_W}{m_Z}$, …

➡ EW couplings and EW boson masses are not independent

➡ Yukawa couplings and masses are not independent

➡ These tree-level relations receive higher-order corrections: in general depend on all inputs.

EW input schemes

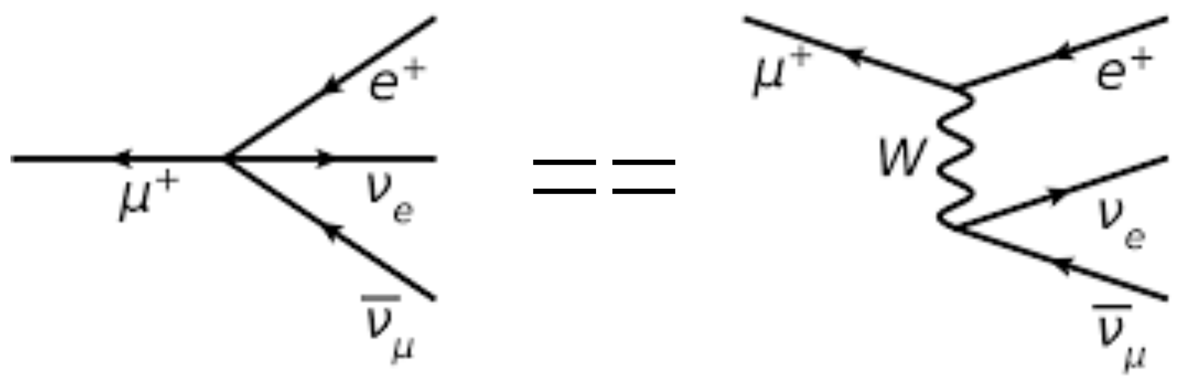
➡ Additional inputs: $\{m_h^0, m_f\}$

➡ **Common input schemes:** $e = \sqrt{4\pi\alpha},\ g_1 = e/\cos\theta_W,\ g_2 = e/\sin\theta_W$

- $\{\alpha(0), m_W, m_Z\}$-scheme: $\alpha(0) \approx 1/137 = 0.0073\ldots$ (Thomsen limit: $Q \to 0$)
- $\{G_\mu, m_W, m_Z\}$-scheme: $\alpha|_{G_\mu} = \sqrt{2}/\pi\, G_\mu m_W^2 \sin^2\theta_W \approx 1/132 = 0.0076\ldots$
- $\{\alpha(m_Z), m_W, m_Z\}$-scheme: $\alpha(m_Z) \approx 1/128 = 0.0078\ldots$

from: $|\frac{8}{\sqrt{2}} G_\mu|^2 = |\frac{g_2^2}{m_W^2}|^2 = |\mathcal{M}|^2$

(relation between squared matrix elements for the muon decay in the Fermi theory to corresponding W-exchange matrix elements in the low-energy limit)



where: $G_\mu = 1.1663710^{-5}\mathrm{GeV}^{-2}$

EW input schemes

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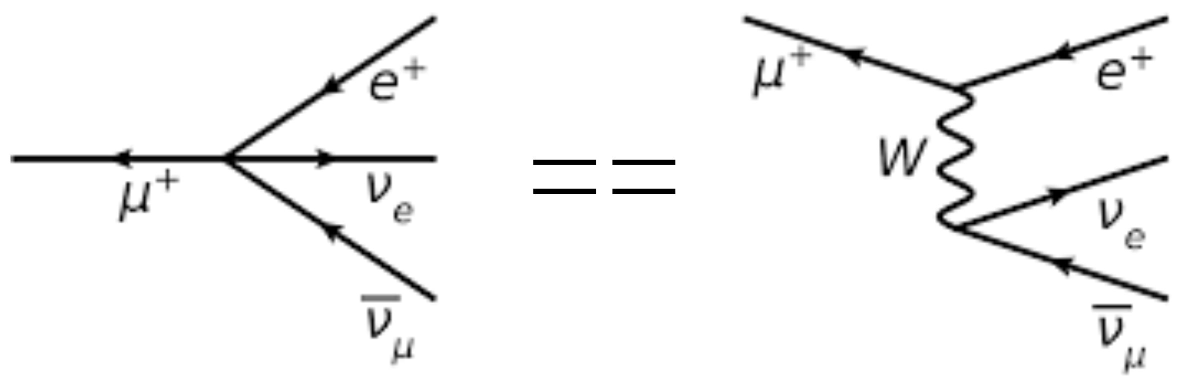

where: $G_\mu = 1.1663710^{-5}\mathrm{GeV}^{-2}$

➡ Differences between these scheme at 5-7% level (*scheme uncertainties*).
➡ Scheme dependence reduced when including higher-order corrections.
➡ One scheme might be more appropriate than others for different processes.

EW input schemes

➡ Additional inputs: $\{m_h^0,\, m_f\}$

➡ **Common input schemes:** $e = \sqrt{4\pi\alpha},\ g_1 = e/\cos\theta_W,\ g_2 = e/\sin\theta_W$

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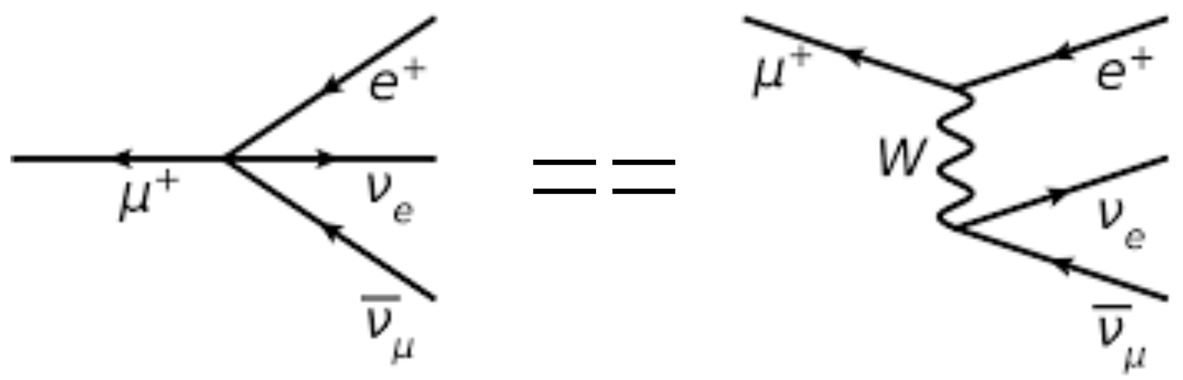


where: $G_\mu = 1.1663710^{-5}\mathrm{GeV}^{-2}$

At NLO: $$\frac{\alpha|_{G_\mu}}{|s_W^2 \mu_W^2|} = \frac{\sqrt{2}G_\mu}{\pi} = \alpha(0)\left|\frac{1+\Delta r}{s_W^2 \mu_W^2}\right|$$

$$\Delta r = \Pi^{AA}(0) - \frac{c_W^2}{s_W^2}\left(\frac{\Sigma_T^{ZZ}(M_Z^2)}{M_Z^2} - \frac{\Sigma_T^W(M_W^2)}{M_W^2}\right) + \frac{\Sigma_T^W(0) - \Sigma_T^W(M_W^2)}{M_W^2}$$
$$+2\frac{c_W}{s_W}\frac{\Sigma_T^{AZ}(0)}{M_Z^2} + \frac{\alpha}{4\pi s_W^2}\left(6 + \frac{7-4s_W^2}{2s_W^2}\log c_W^2\right)$$

(depends on all parameters of the SM)

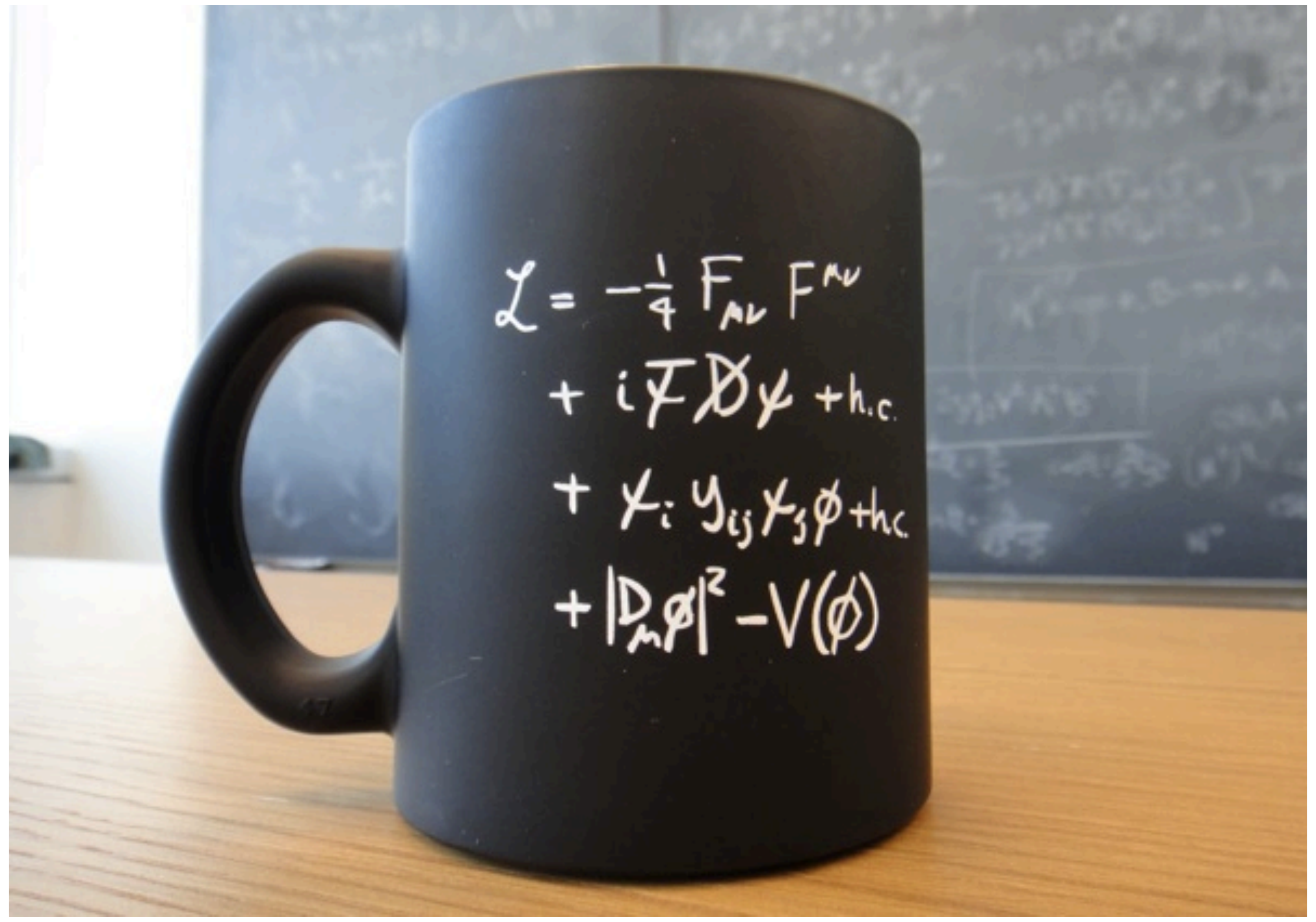
➡ G_μ-scheme incorporates these universal corrections into LO couplings

➡ improved perturbative convergence for processes dominated by SU(2) interactions at (or above) the EW scale.

Conclusions

Symmetry:

$$SU(3)_C \times SU(2)_L \times U(1)_Y \xrightarrow{<H>} SU(3)_C \times U(1)_{\mathrm{EM}}$$



Questions?

SM input parameters

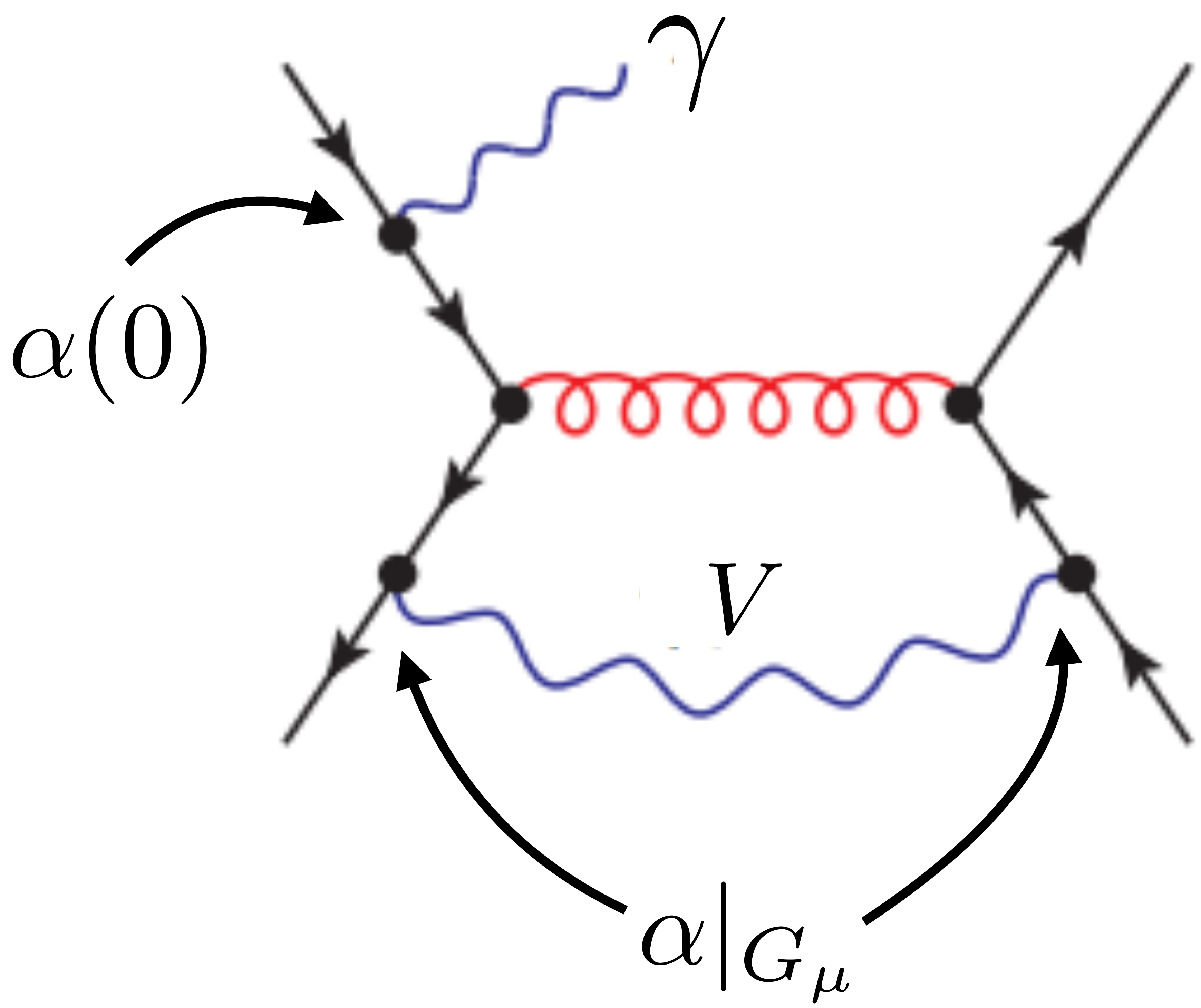
➡ Generally, only a well defined set of independent input parameters are "free" parameters of the model

➡ derived parameters are only short-hands to keep the notation tidy.

➡ when performing measurements (comparing data with theory), only input parameters of used calculation can be extracted from data.

Mixed EW input schemes

- external (on-shell) photons effectively couple with $Q \to 0$
- natural to consider a mixed scheme



$$\mathcal{O}(\alpha(0)\alpha|_{G_\mu}\alpha_S) \text{ wrt } \mathcal{O}(\alpha(0)\alpha_S)$$