The background of the slide is a complex, abstract pattern of thin, overlapping lines and small dots in various colors, including green, blue, orange, and purple, set against a light grey background. The lines and dots are scattered across the entire frame, creating a sense of dynamic energy and complexity.

# Field Theory & the EW Standard Model

## Part III: EW tests and phenomenology

Jonas M. Lindert

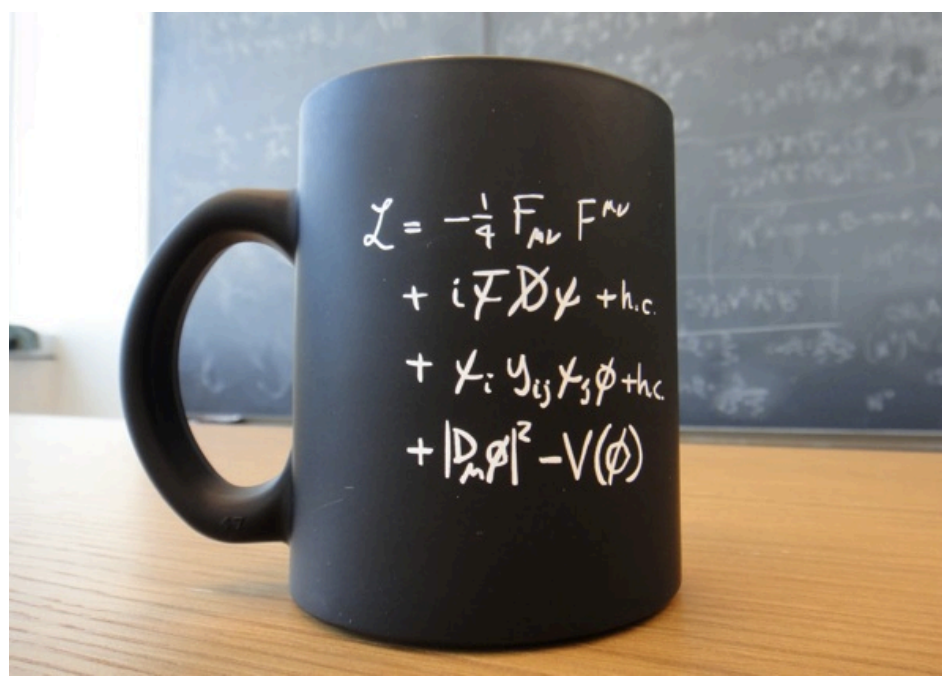
**US**

University of Sussex

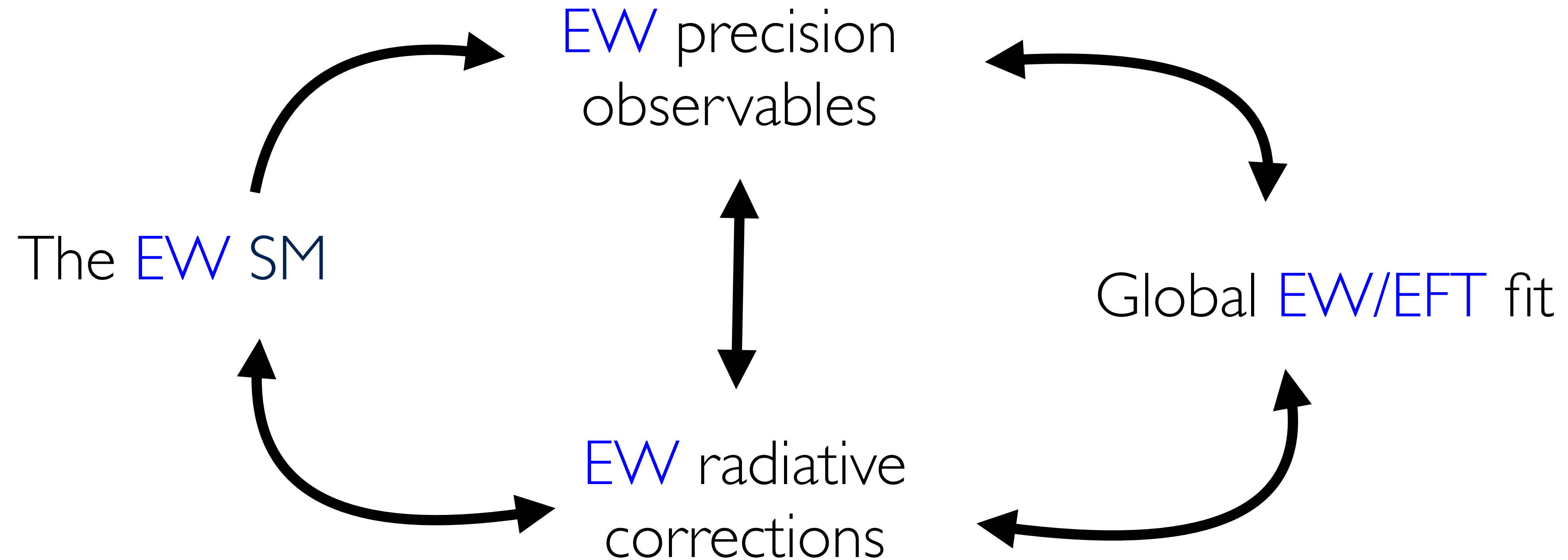
2024 EUROPEAN SCHOOL OF HIGH-ENERGY PHYSICS

Peebles, Scotland, UK

September 2024



@

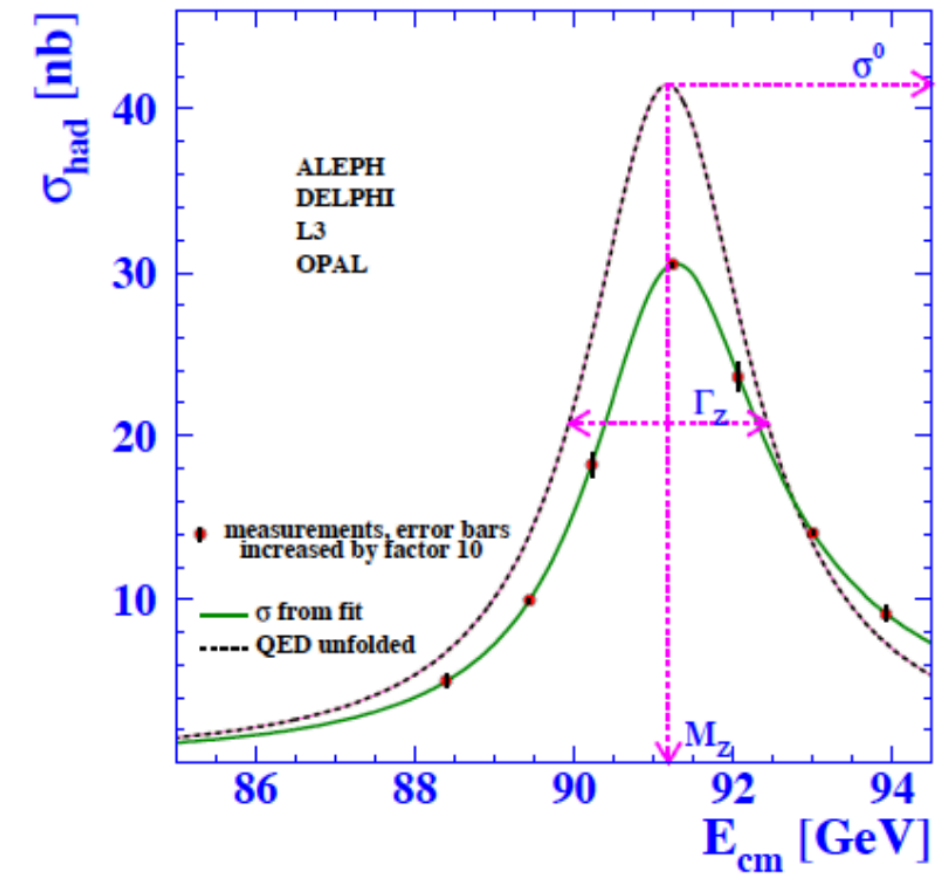
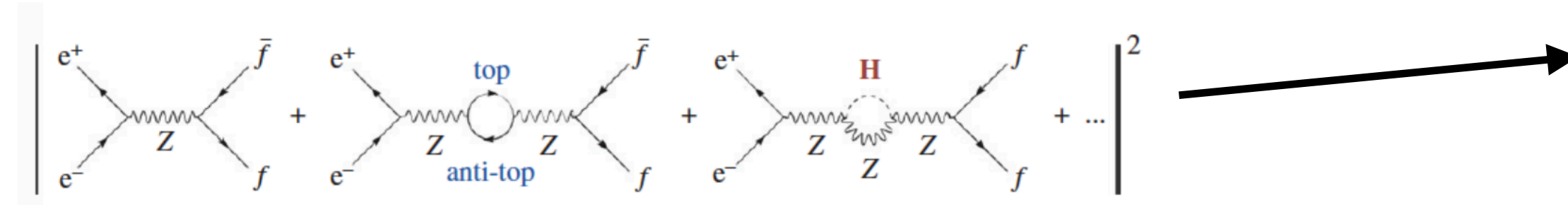


- ➡ Study dynamics of the EW SM at the TeV scale
- ➡ Test BSM via indirect EW probes
- ➡ Constrain backgrounds in direct searches for New Physics

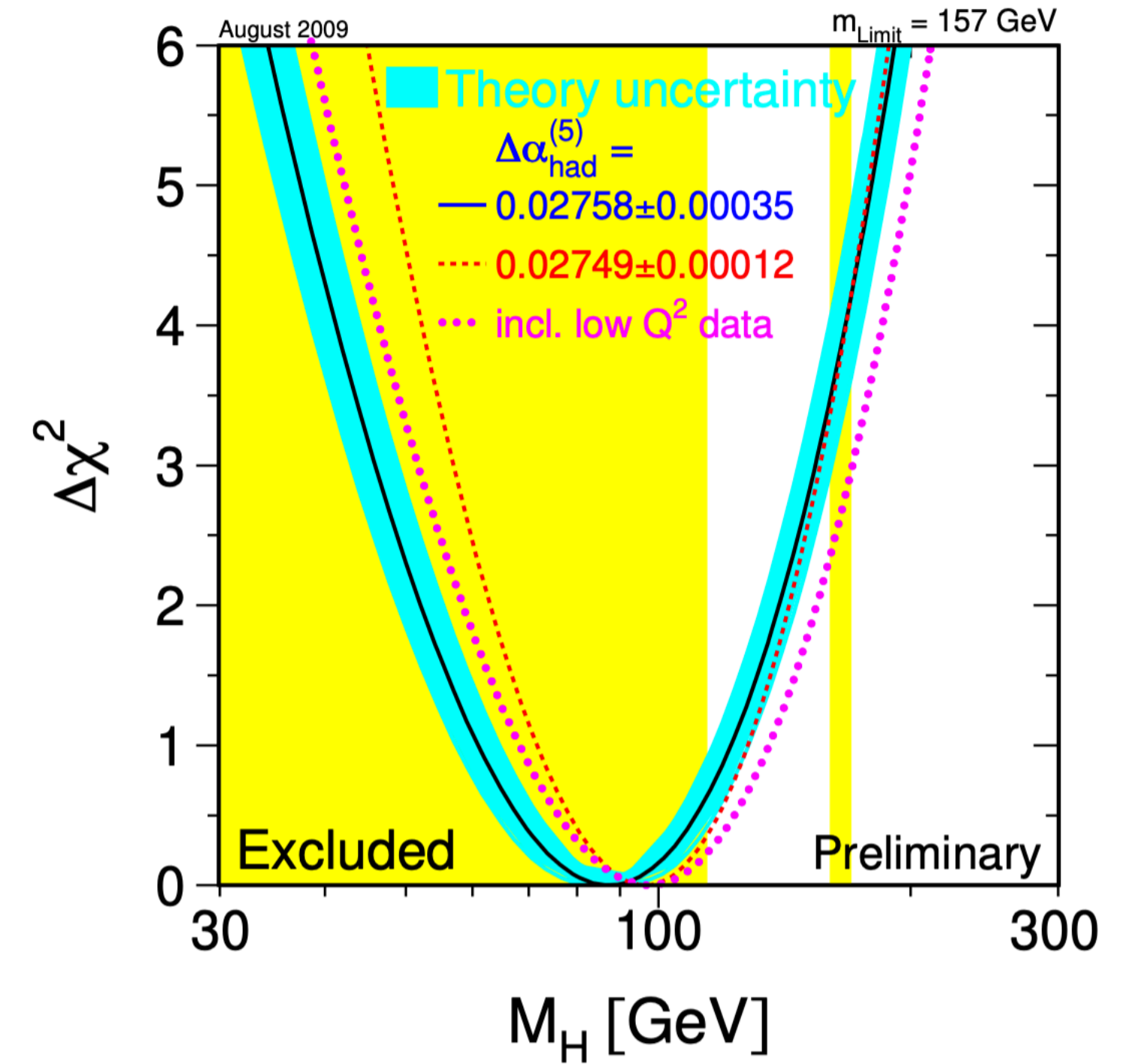
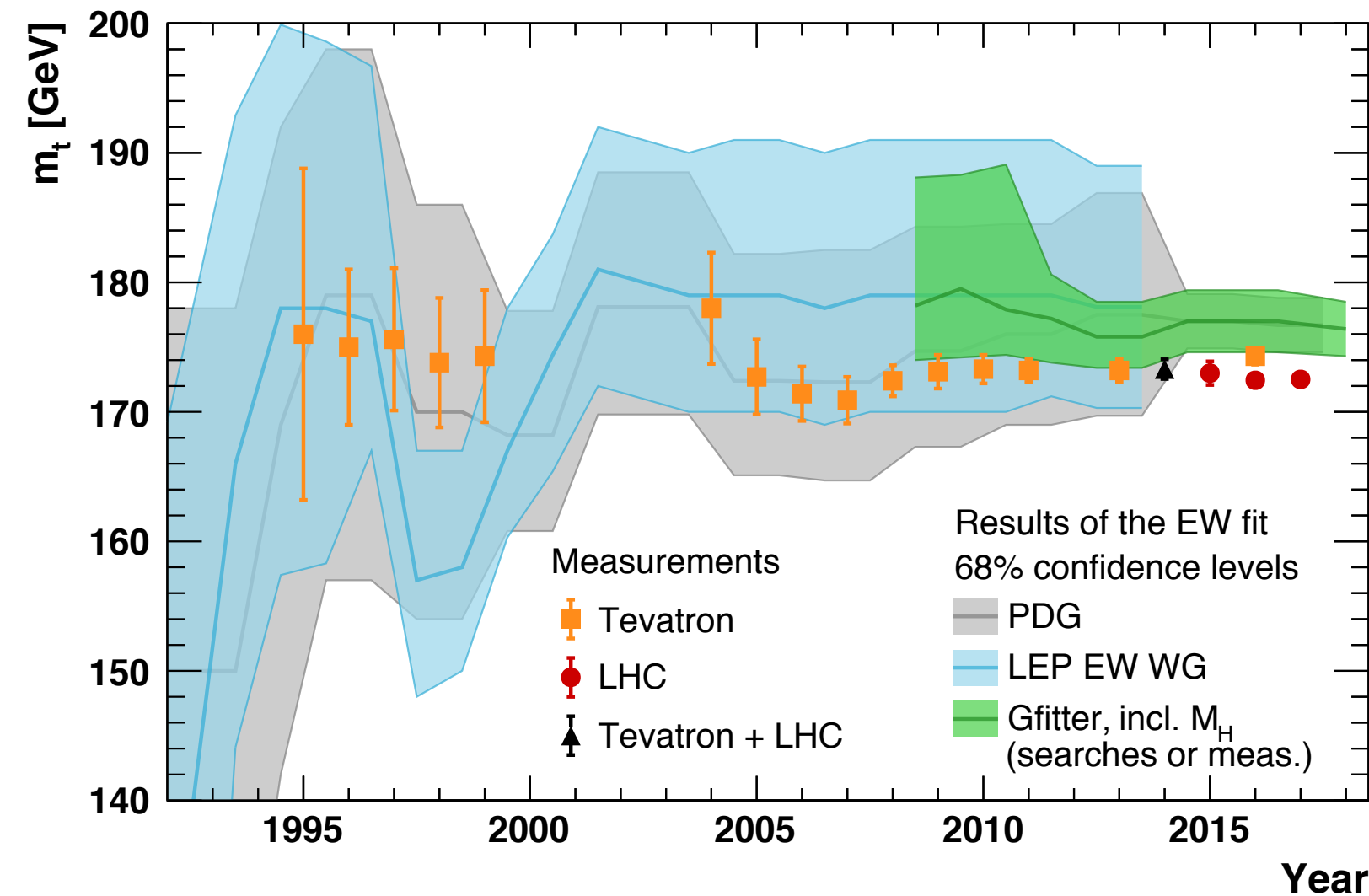
# Classical tests I of EW theory @ LEP

• Correlation between different EW parameters (at higher-order)

- ▶  $m_W = \frac{1}{2} g_2 v$
- ▶  $\sin \theta_W = m_W / m_Z$
- ▶ Z-pole observables



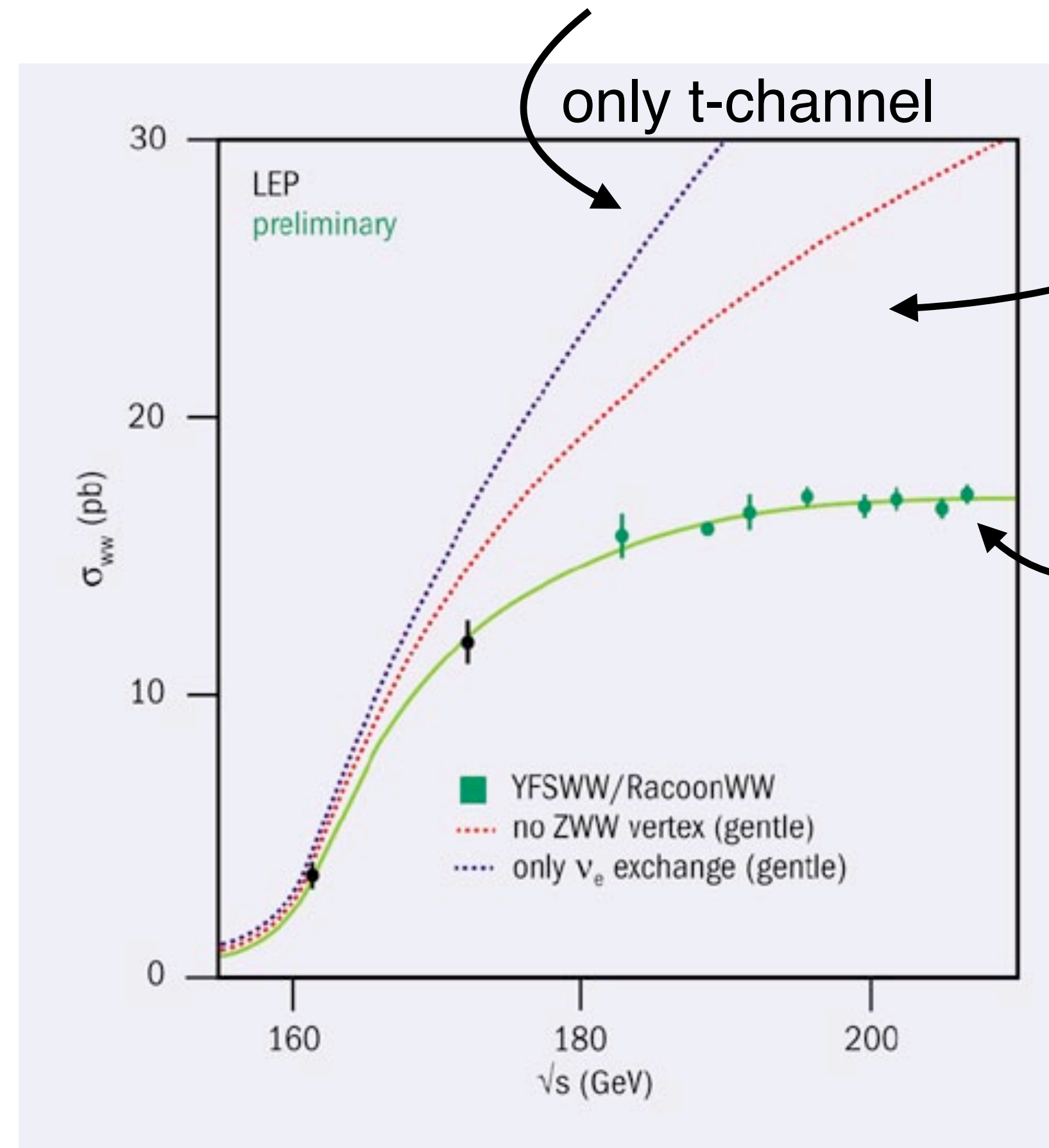
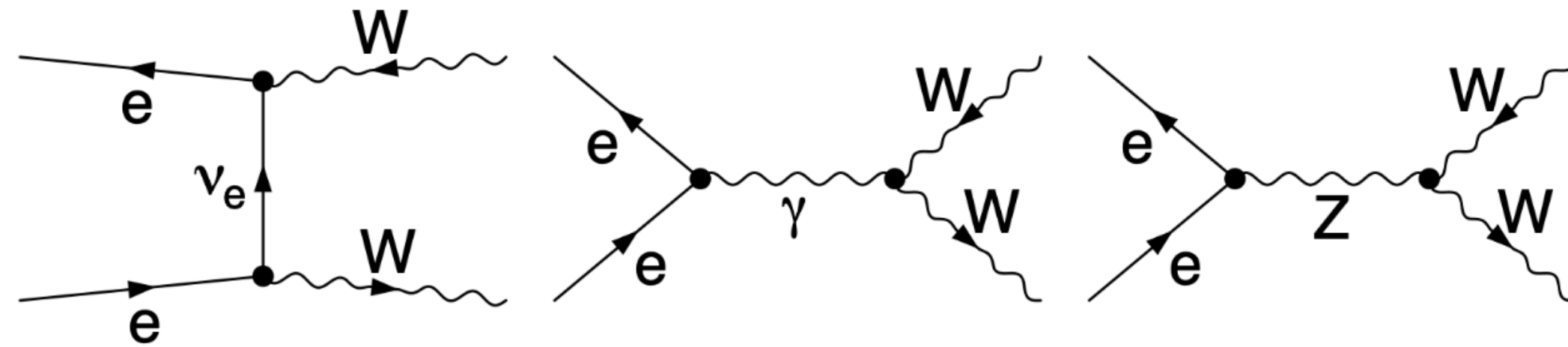
	Measurement	Fit	$(O^{\text{meas}} - O^{\text{fit}}) / \sigma^{\text{meas}}$
$\Delta\alpha_{\text{had}}^{(5)}(m_Z)$	$0.02758 \pm 0.00035$	0.02768	0.1
$m_Z$ [GeV]	$91.1875 \pm 0.0021$	91.1874	-0.001
$\Gamma_Z$ [GeV]	$2.4952 \pm 0.0023$	2.4959	0.003
$\sigma_{\text{had}}^0$ [nb]	$41.540 \pm 0.037$	41.478	-0.15
$R_l$	$20.767 \pm 0.025$	20.742	-0.10
$A_{\text{fb}}^{0,l}$	$0.01714 \pm 0.00095$	0.01645	-0.07
$A_l(P_\gamma)$	$0.1465 \pm 0.0032$	0.1481	0.12
$R_b$	$0.21629 \pm 0.00066$	0.21579	-0.005
$R_c$	$0.1721 \pm 0.0030$	0.1723	0.002
$A_{\text{fb}}^{0,b}$	$0.0992 \pm 0.0016$	0.1038	0.46
$A_{\text{fb}}^{0,c}$	$0.0707 \pm 0.0035$	0.0742	0.50
$A_b$	$0.923 \pm 0.020$	0.935	0.13
$A_c$	$0.670 \pm 0.027$	0.668	-0.03
$A_l(\text{SLD})$	$0.1513 \pm 0.0021$	0.1481	-0.21
$\sin^2 \theta_{\text{eff}}^{\text{lep}}(Q_{\text{fb}})$	$0.2324 \pm 0.0012$	0.2314	-0.08
$m_W$ [GeV]	$80.399 \pm 0.023$	80.379	-0.20
$\Gamma_W$ [GeV]	$2.098 \pm 0.048$	2.092	-0.06
$m_t$ [GeV]	$173.1 \pm 1.3$	173.2	0.1



→ EW precision observables

# Classical tests II of EW theory @ LEP

$$e^+e^- \rightarrow W^+W^-$$

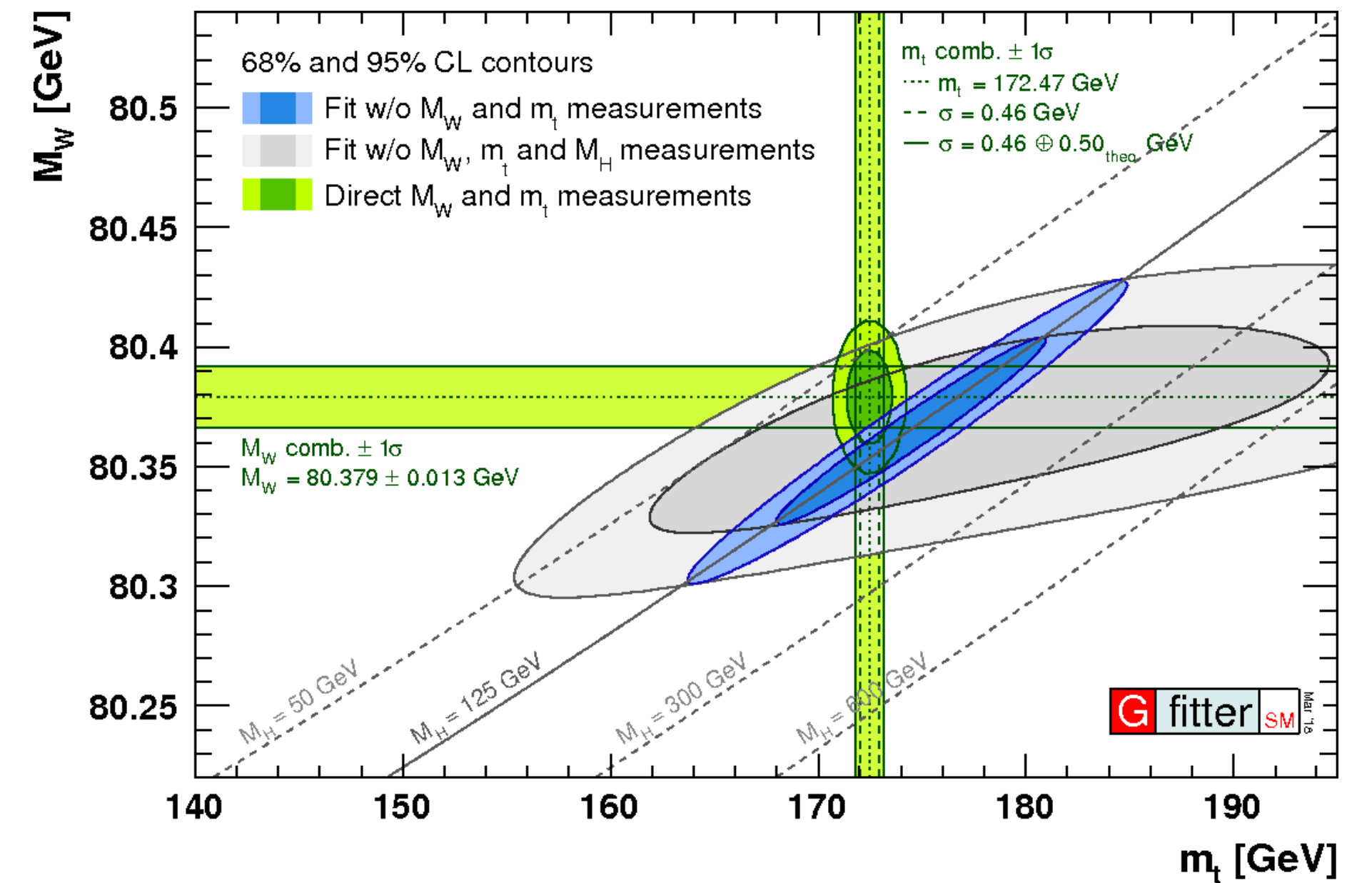
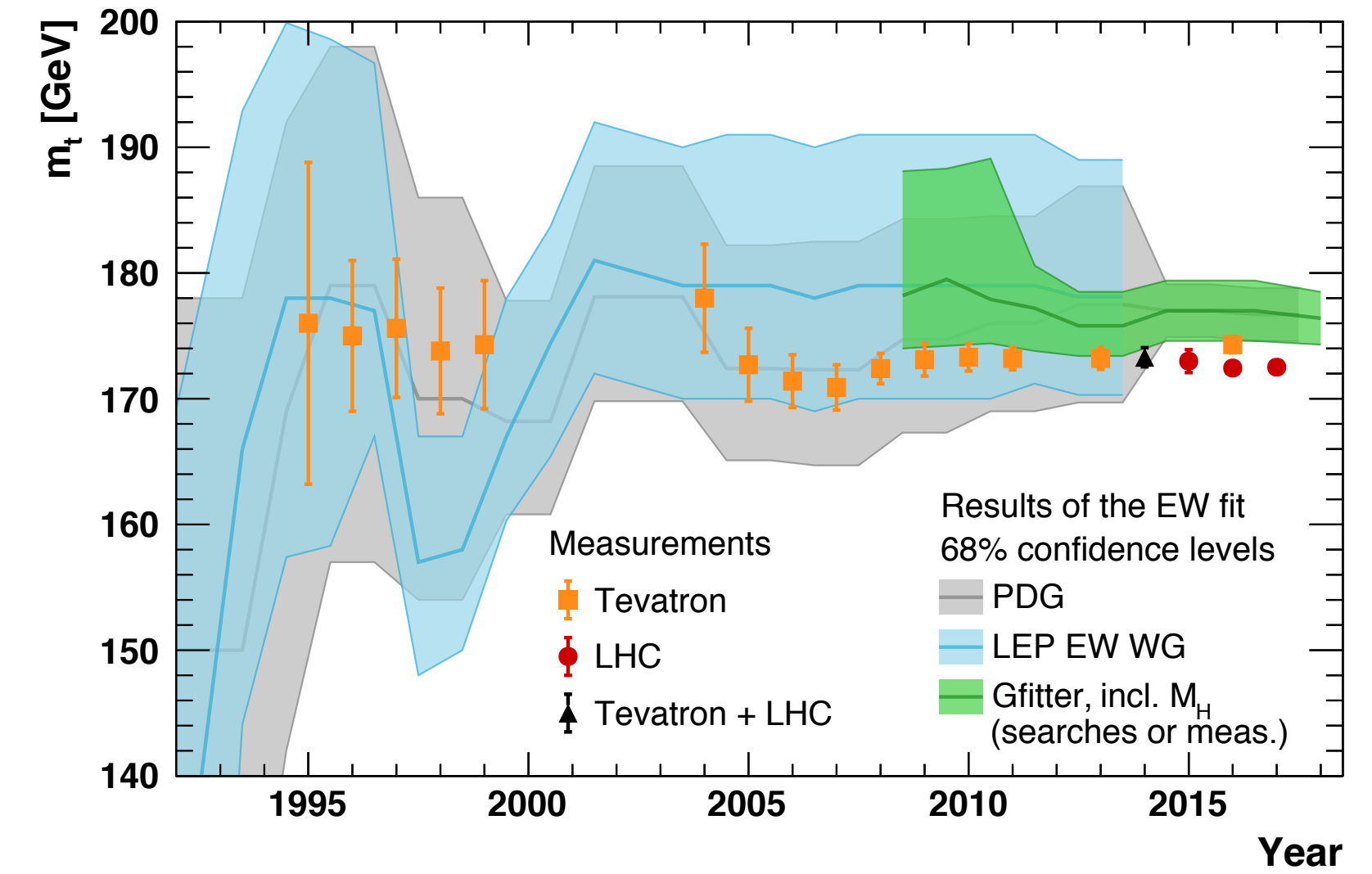


→ tails of kinematic distributions

# The global EW fit

Parameter	Input value	Free in fit	Fit Result	w/o exp. input in line	w/o exp. input in line, no theo. unc
$M_H$ [GeV]	$125.1 \pm 0.2$	yes	$125.1^{+0.2}_{-0.2}$	$100.2^{+24.4}_{-20.6}$	$100.3^{+23.5}_{-19.9}$
$M_W$ [GeV]	$80.379 \pm 0.013$	–	$80.363 \pm 0.007$	$80.356 \pm 0.008$	$80.356 \pm 0.007$
$\Gamma_W$ [GeV]	$2.085 \pm 0.042$	–	$2.091 \pm 0.001$	$2.091 \pm 0.001$	$2.091 \pm 0.001$
$M_Z$ [GeV]	$91.1875 \pm 0.0021$	yes	$91.1879 \pm 0.0020$	$91.1967 \pm 0.0099$	$91.1969 \pm 0.0096$
$\Gamma_Z$ [GeV]	$2.4952 \pm 0.0023$	–	$2.4950 \pm 0.0014$	$2.4945 \pm 0.0016$	$2.4945 \pm 0.0016$
$\sigma_{\text{had}}^0$ [nb]	$41.540 \pm 0.037$	–	$41.483 \pm 0.015$	$41.474 \pm 0.016$	$41.474 \pm 0.015$
$R_\ell^0$	$20.767 \pm 0.025$	–	$20.744 \pm 0.017$	$20.725 \pm 0.026$	$20.724 \pm 0.026$
$A_{\text{FB}}^{0,\ell}$	$0.0171 \pm 0.0010$	–	$0.01623 \pm 0.0001$	$0.01622 \pm 0.0001$	$0.01624 \pm 0.0001$
$A_\ell$ (*)	$0.1499 \pm 0.0018$	–	$0.1471 \pm 0.0005$	$0.1471 \pm 0.0005$	$0.1472 \pm 0.0004$
$\sin^2\theta_{\text{eff}}^\ell(Q_{\text{FB}})$	$0.2324 \pm 0.0012$	–	$0.23151 \pm 0.00006$	$0.23151 \pm 0.00006$	$0.23150 \pm 0.00005$
$\sin^2\theta_{\text{eff}}^\ell(\text{TEV})$	$0.2318 \pm 0.0003$	–	$0.23151 \pm 0.00006$	$0.23150 \pm 0.00006$	$0.23150 \pm 0.00005$
$A_c$	$0.670 \pm 0.027$	–	$0.6679 \pm 0.00022$	$0.6679 \pm 0.00022$	$0.6680 \pm 0.00016$
$A_b$	$0.923 \pm 0.020$	–	$0.93475 \pm 0.00004$	$0.93475 \pm 0.00004$	$0.93475 \pm 0.00003$
$A_{\text{FB}}^{0,c}$	$0.0707 \pm 0.0035$	–	$0.0737 \pm 0.0003$	$0.0737 \pm 0.0003$	$0.0737 \pm 0.0002$
$A_{\text{FB}}^{0,b}$	$0.0992 \pm 0.0016$	–	$0.1031 \pm 0.0003$	$0.1033 \pm 0.0004$	$0.1033 \pm 0.0003$
$R_c^0$	$0.1721 \pm 0.0030$	–	$0.17226^{+0.00009}_{-0.00008}$	$0.17226 \pm 0.00008$	$0.17226 \pm 0.00006$
$R_b^0$	$0.21629 \pm 0.00066$	–	$0.21579 \pm 0.00011$	$0.21578 \pm 0.00012$	$0.21577 \pm 0.00004$
$\bar{m}_c$ [GeV]	$1.27^{+0.07}_{-0.11}$	yes	$1.27^{+0.07}_{-0.11}$	–	–
$\bar{m}_b$ [GeV]	$4.20^{+0.17}_{-0.07}$	yes	$4.20^{+0.17}_{-0.07}$	–	–
$m_t$ [GeV]( $\nabla$ )	$173.06 \pm 0.94$	yes	$173.54 \pm 0.86$	$175.97^{+2.11}_{-2.12}$	$176.00^{+2.03}_{-2.04}$
$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$ ( $\dagger\Delta$ )	$2758 \pm 10$	yes	$2756 \pm 10$	$2738 \pm 41$	$2739 \pm 39$
$\alpha_s(M_Z^2)$	–	yes	$0.1197^{+0.00030}_{-0.00029}$	$0.1197 \pm 0.0030$	$0.1198 \pm 0.0028$

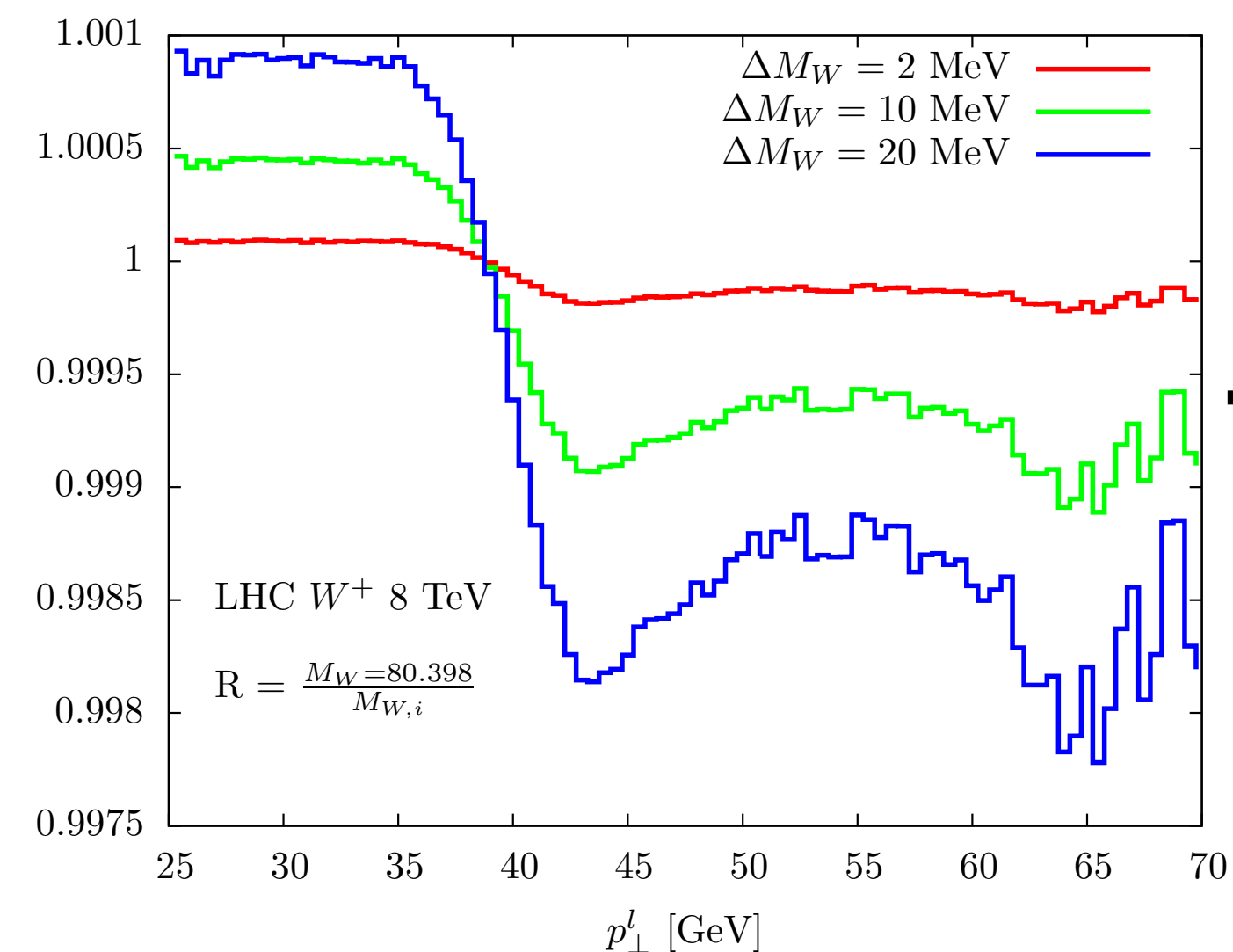
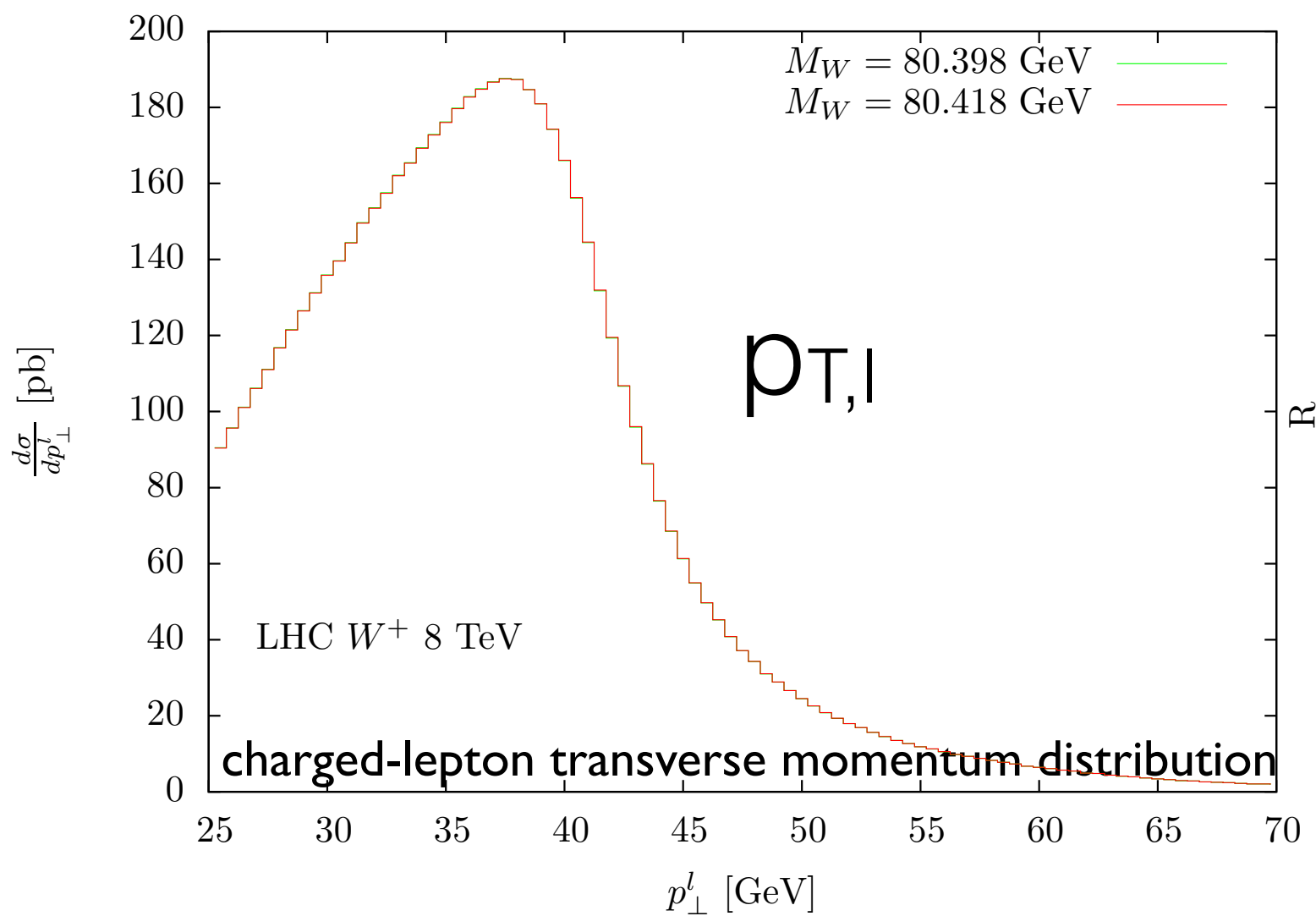
(\*) Average of LEP ( $A_\ell = 0.1465 \pm 0.0033$ ) and SLD ( $A_\ell = 0.1513 \pm 0.0021$ ) measurements, used as two measurements in the fit. The fit w/o the LEP (SLD) measurement gives  $A_\ell = 0.1471 \pm 0.0005$  ( $A_\ell = 0.1469 \pm 0.0005$ ). ( $\nabla$ ) Combination of experimental (0.8 GeV) and theory uncertainty (0.5 GeV). ( $\dagger$ ) In units of  $10^{-5}$ . ( $\Delta$ ) Rescaled due to  $\alpha_s$  dependency.



# Drell-Yan: $M_W$ measurements

- Motivation: precise measurement is a stringent test of SM!
- Method: **template fits** of sensitive CC DY distributions ( $p_{T,l}$ ,  $M_T$ ,  $E_{\text{miss}}$ )

$$M_W = 80.385 \pm 0.015 \text{ GeV}$$



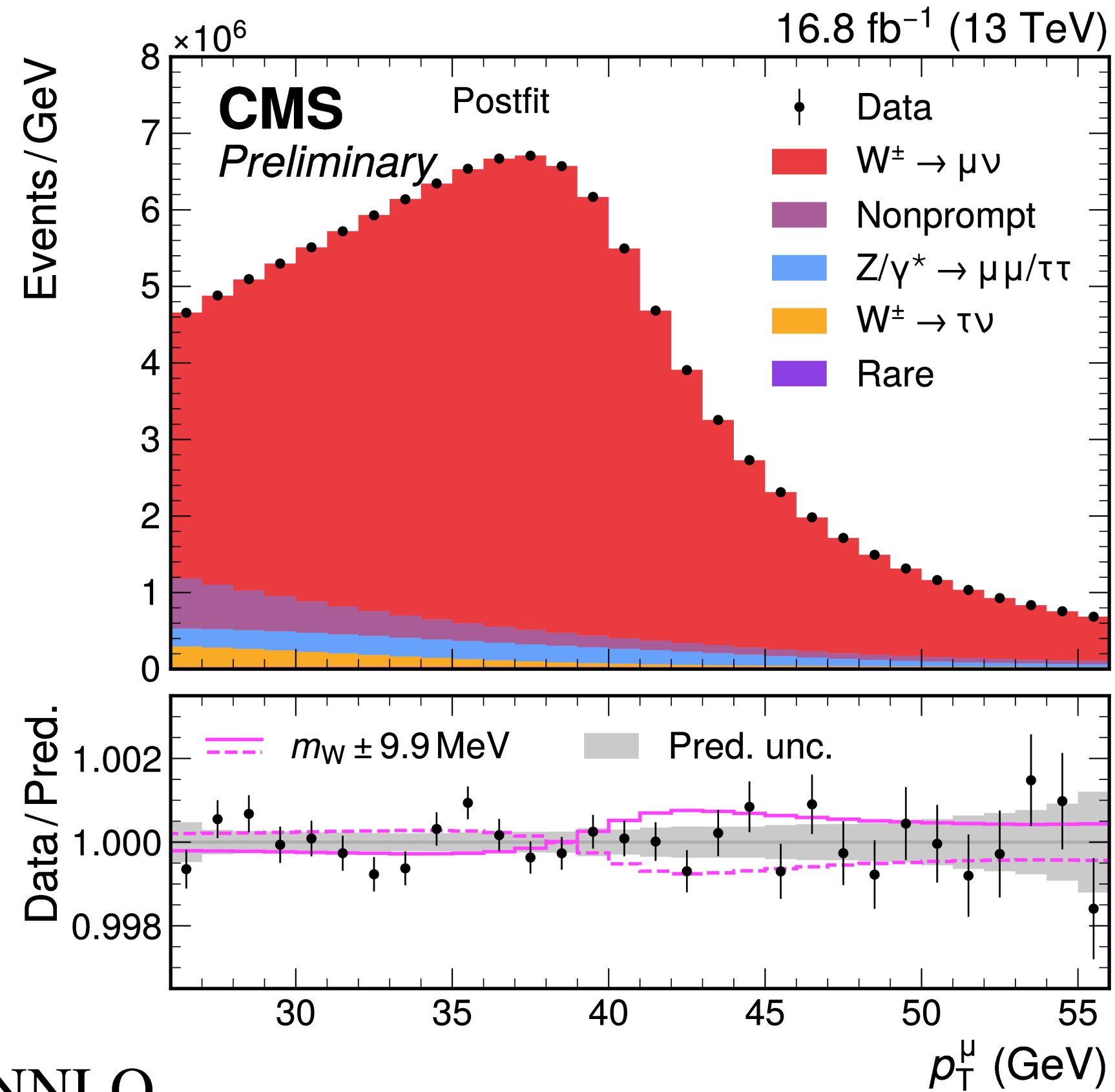
- Need to control shape effects at the sub-1% level!
- Dominant effects: **QCD** ISR and **QED** FSR

[Calame, Chiesa, Martinez, Montagna, Nicrosini, Piccinini, Vicini; 16]

→ Theory precision essential for improvements in  $m_W$  determination!

# Drell-Yan: $M_W$ measurements

CMS-PAS-SMP-23-002



LEP combination  
Phys. Rep. 532 (2013) 119

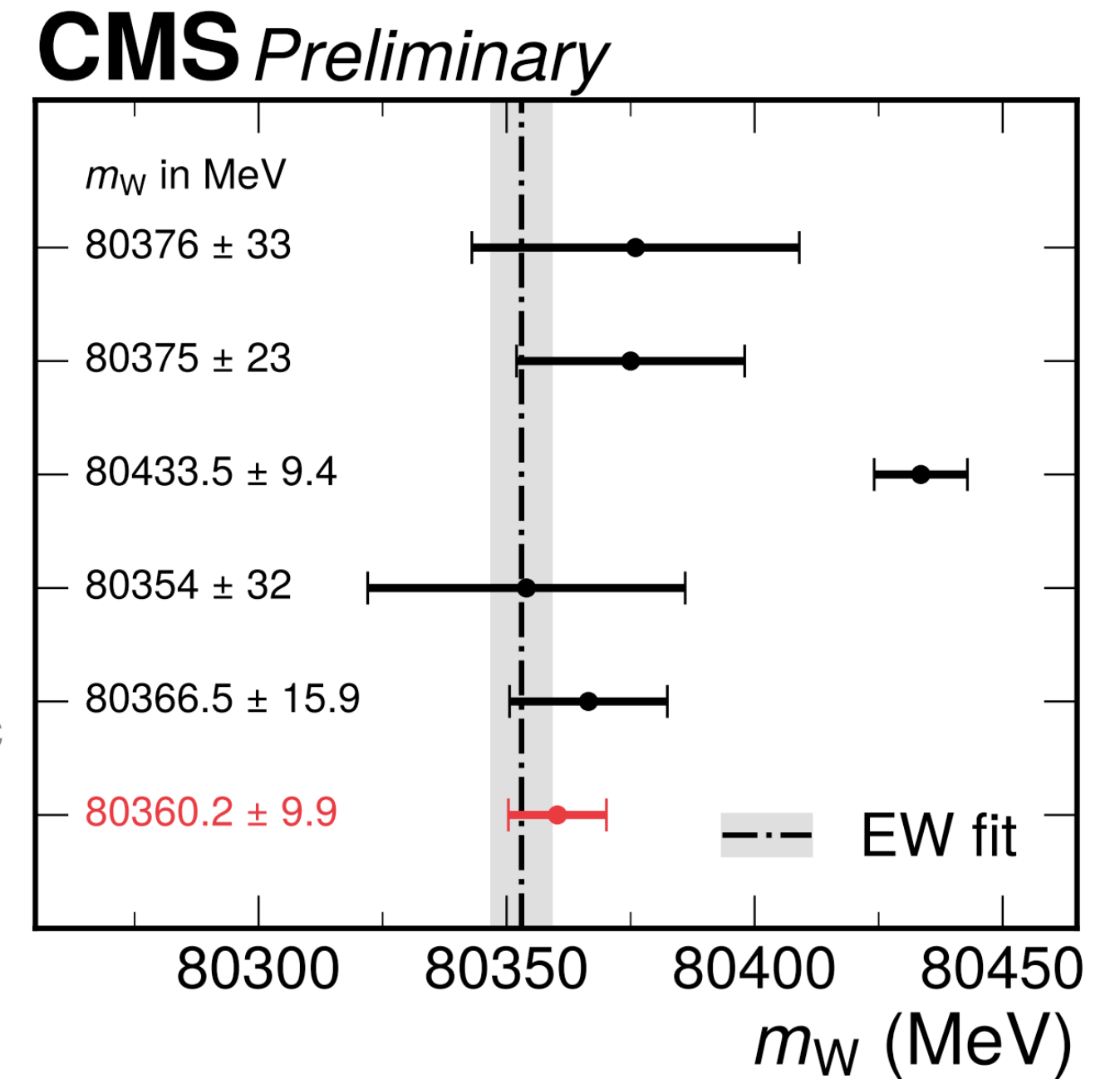
D0  
PRL 108 (2012) 151804

CDF  
Science 376 (2022) 6589

LHCb  
JHEP 01 (2022) 036

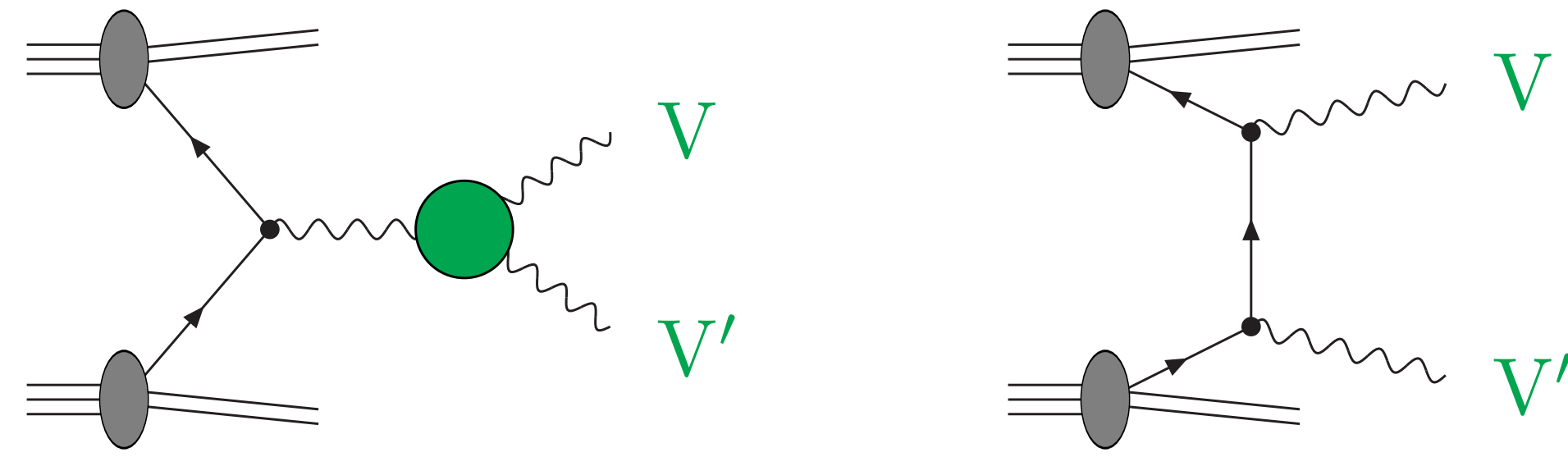
ATLAS  
arxiv:2403.15085, subm. to EPJC

**CMS**  
This Work



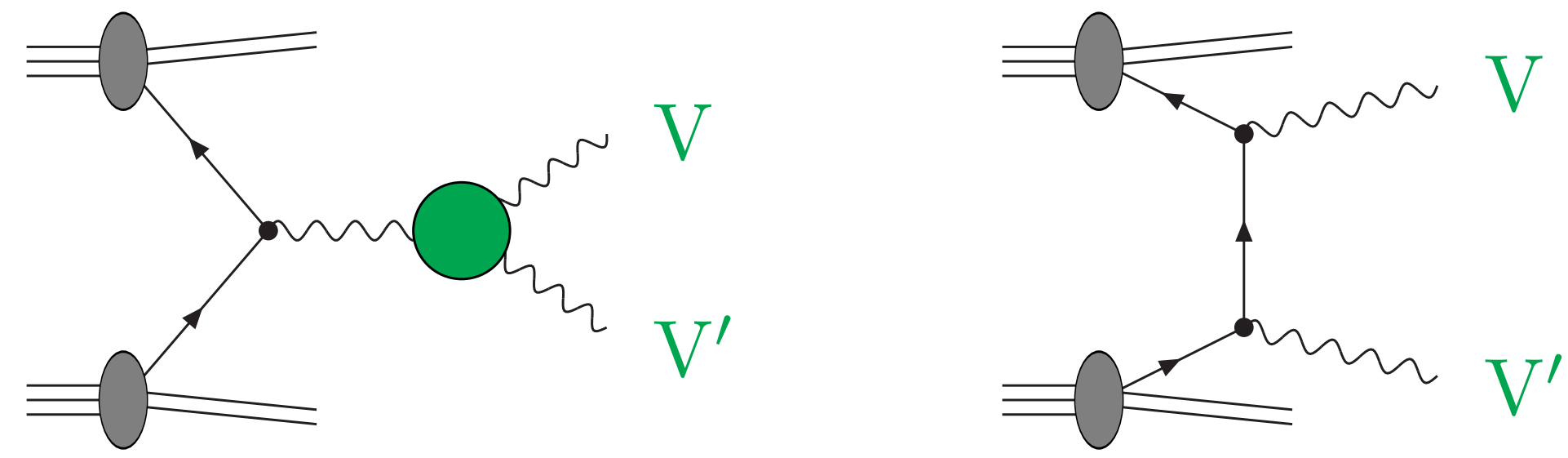
→ The LHC is an EW precision machine!

# Diboson production at the LHC

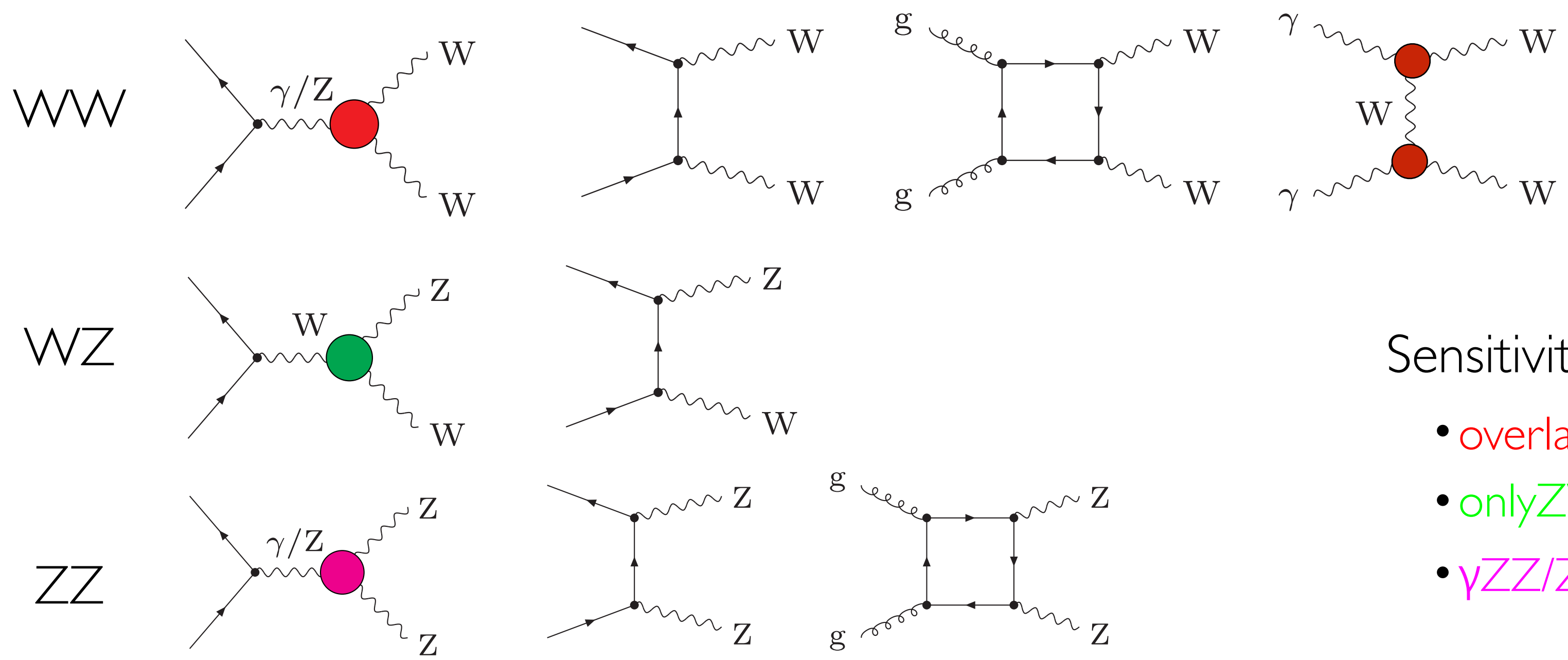




# Diboson production at the LHC



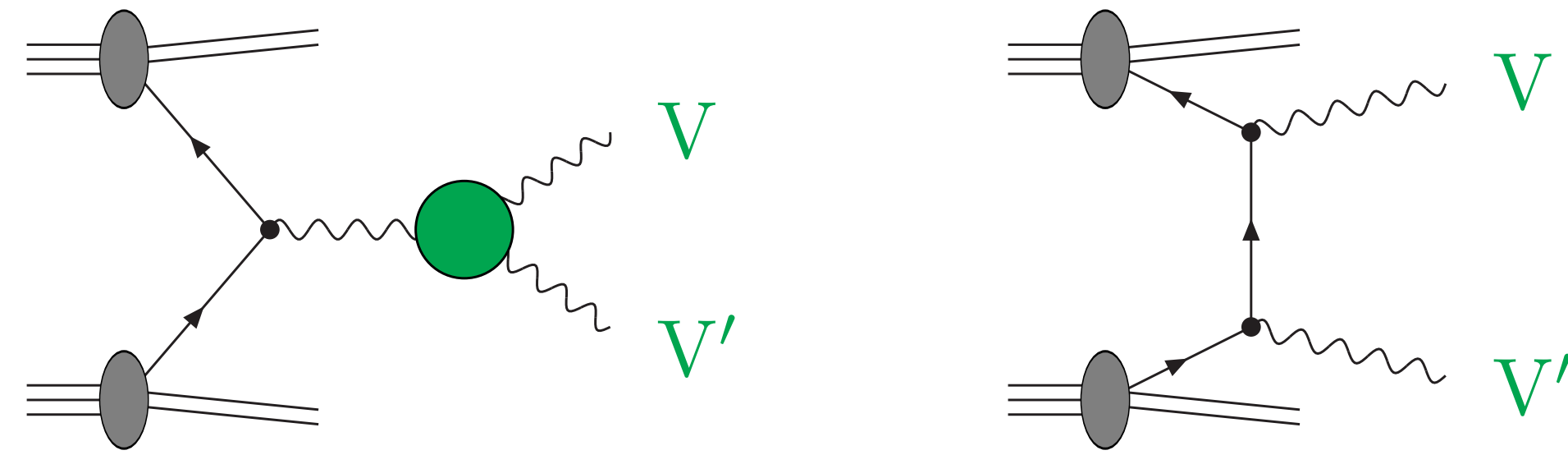
## Complementarity in WW / WZ / ZZ production



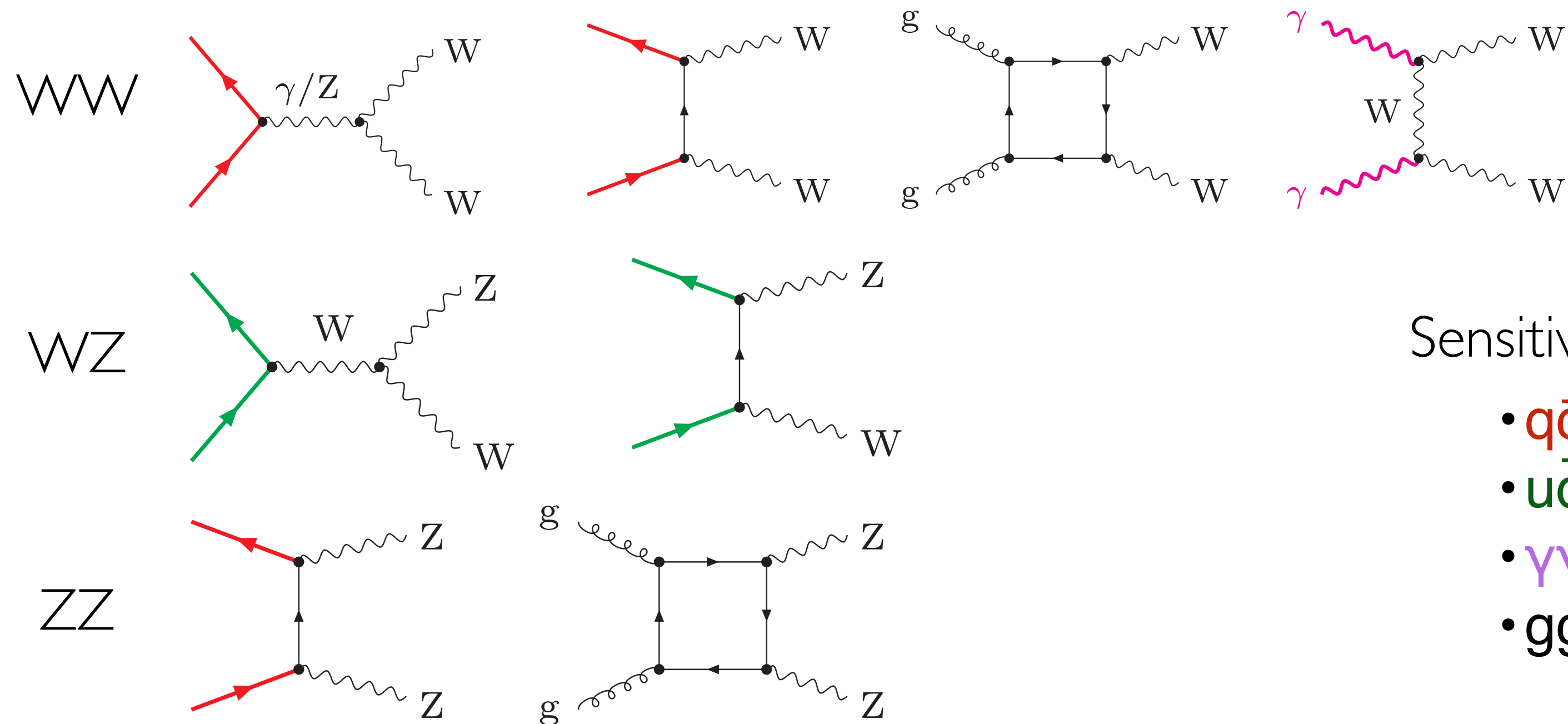
### Sensitivity to different aTGCs:

- overlay of  $\gamma WWW / Z WWW$  in WW
- only  $Z WW$  in WZ
- $\gamma ZZ / ZZZ$  in ZZ

# Diboson production at the LHC



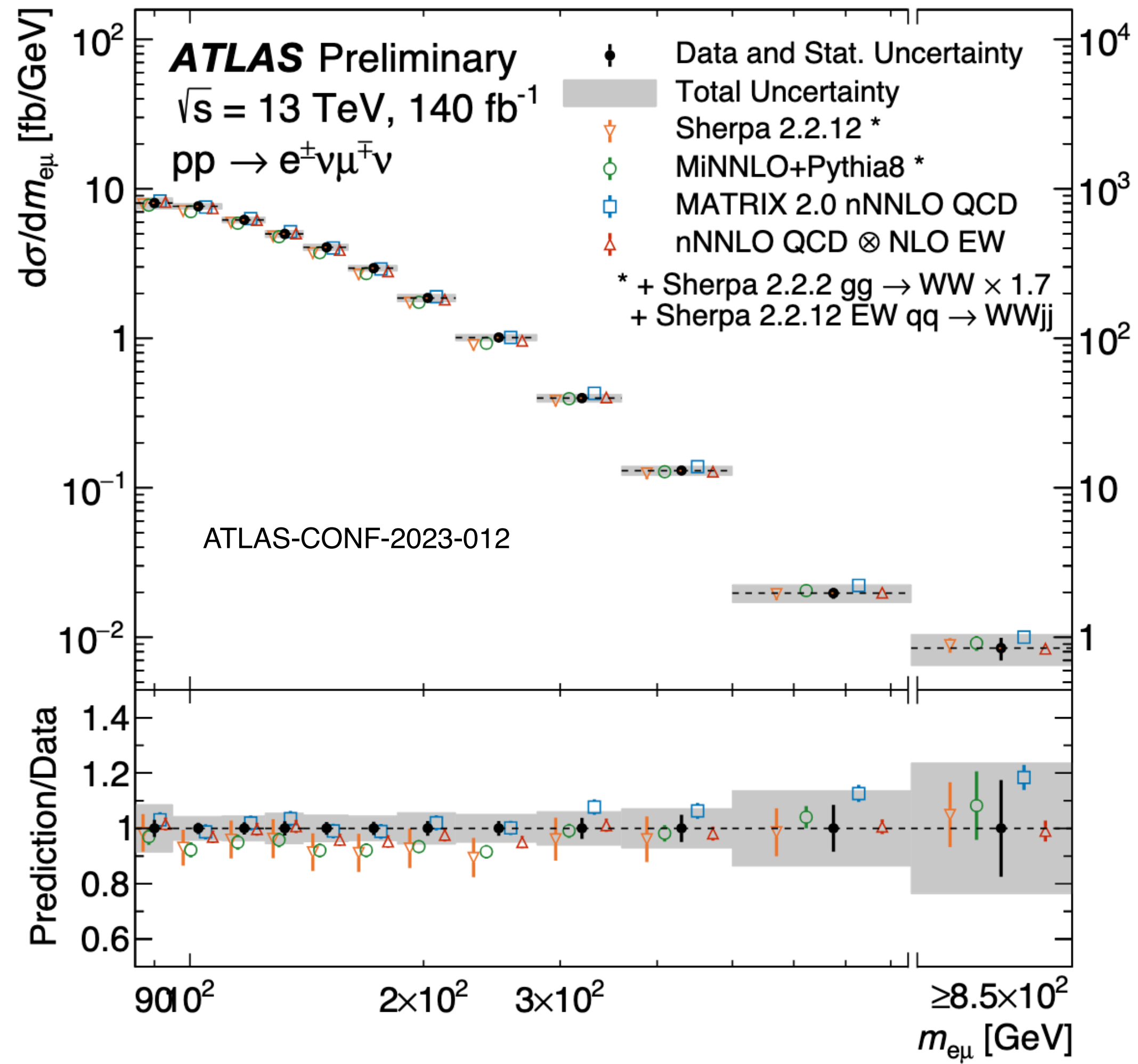
## Complementarity in WW / WZ / ZZ production



Sensitivity to different PDF combinations:

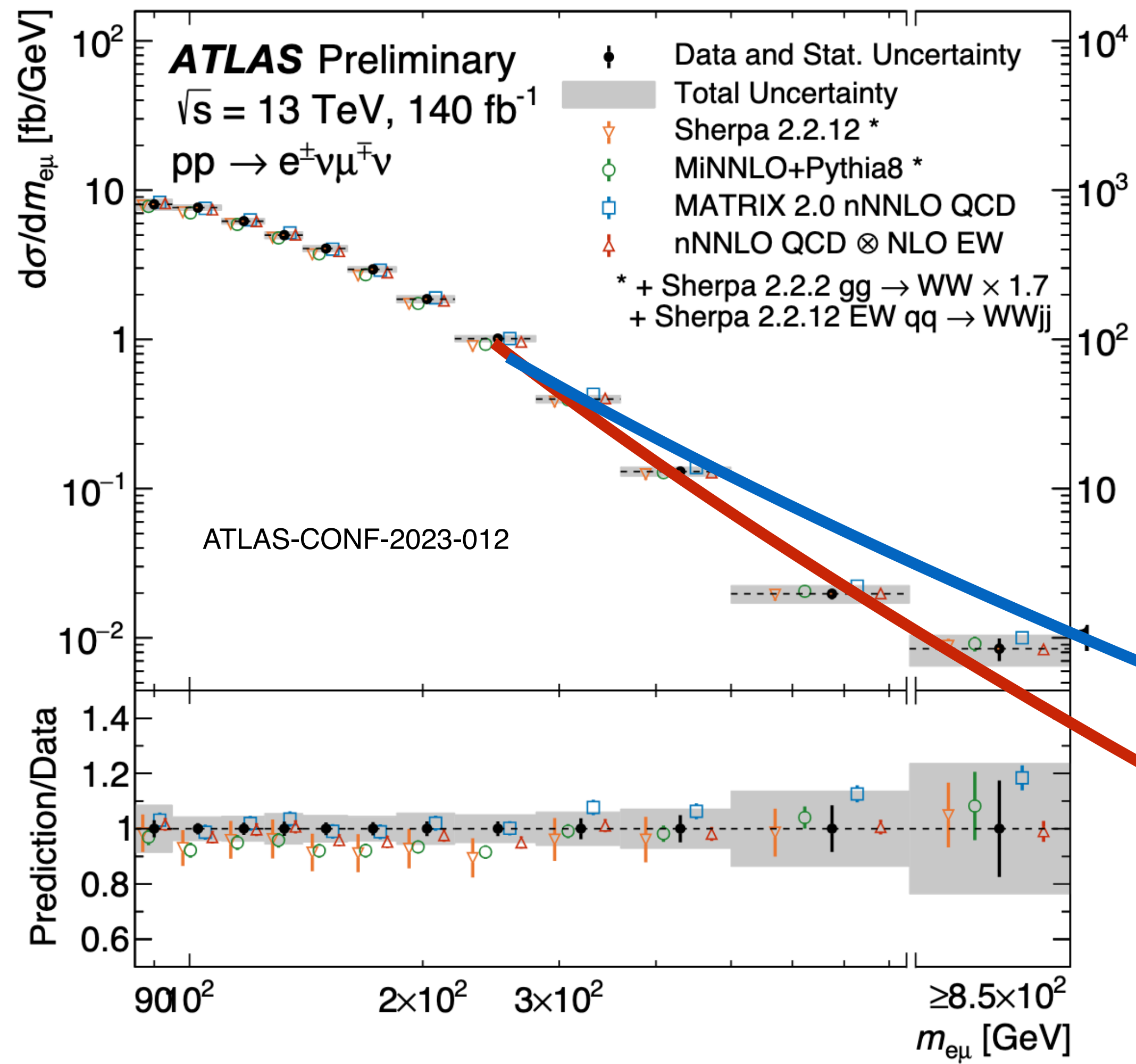
- $q\bar{q}$  in WW/ZZ
- $u\bar{d}/d\bar{u}$  in WZ
- $\gamma\gamma$  in WW
- $gg$  in WW/ZZ

# Kinematic tails



→ tails of kinematic distributions

# Kinematic tails

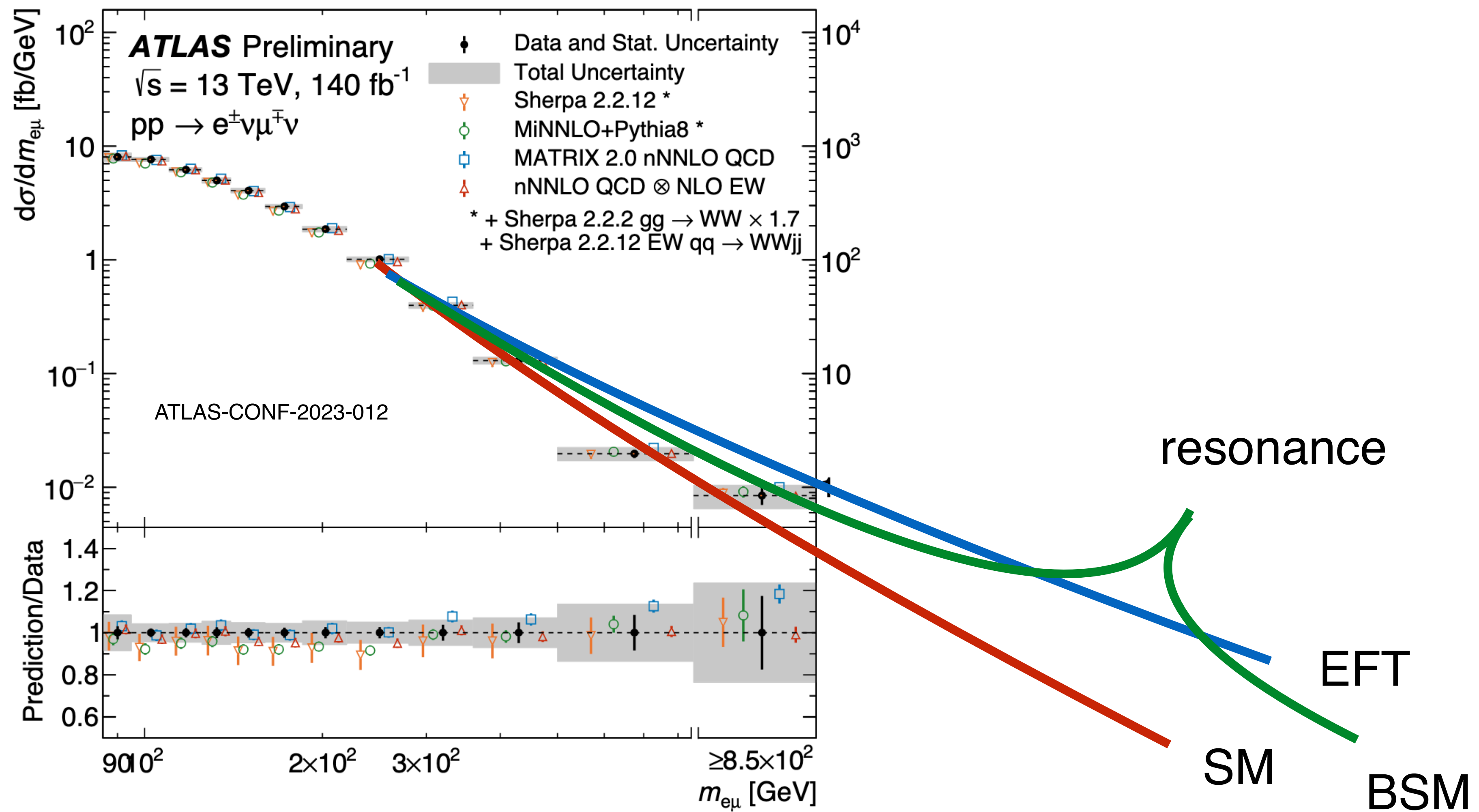


$\sim E^2/\Lambda^2$   
 EFT  
 SM

→ tails of kinematic distributions

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{i=1}^{2499} \frac{C_i^{D6}}{\Lambda^2} \mathcal{O}_i^{D6}$$

# Kinematic tails



→ tails of kinematic distributions

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{i=1}^{2499} \frac{C_i^{D6}}{\Lambda^2} \mathcal{O}_i^{D6}$$

# The global EFT/SMEFT fit

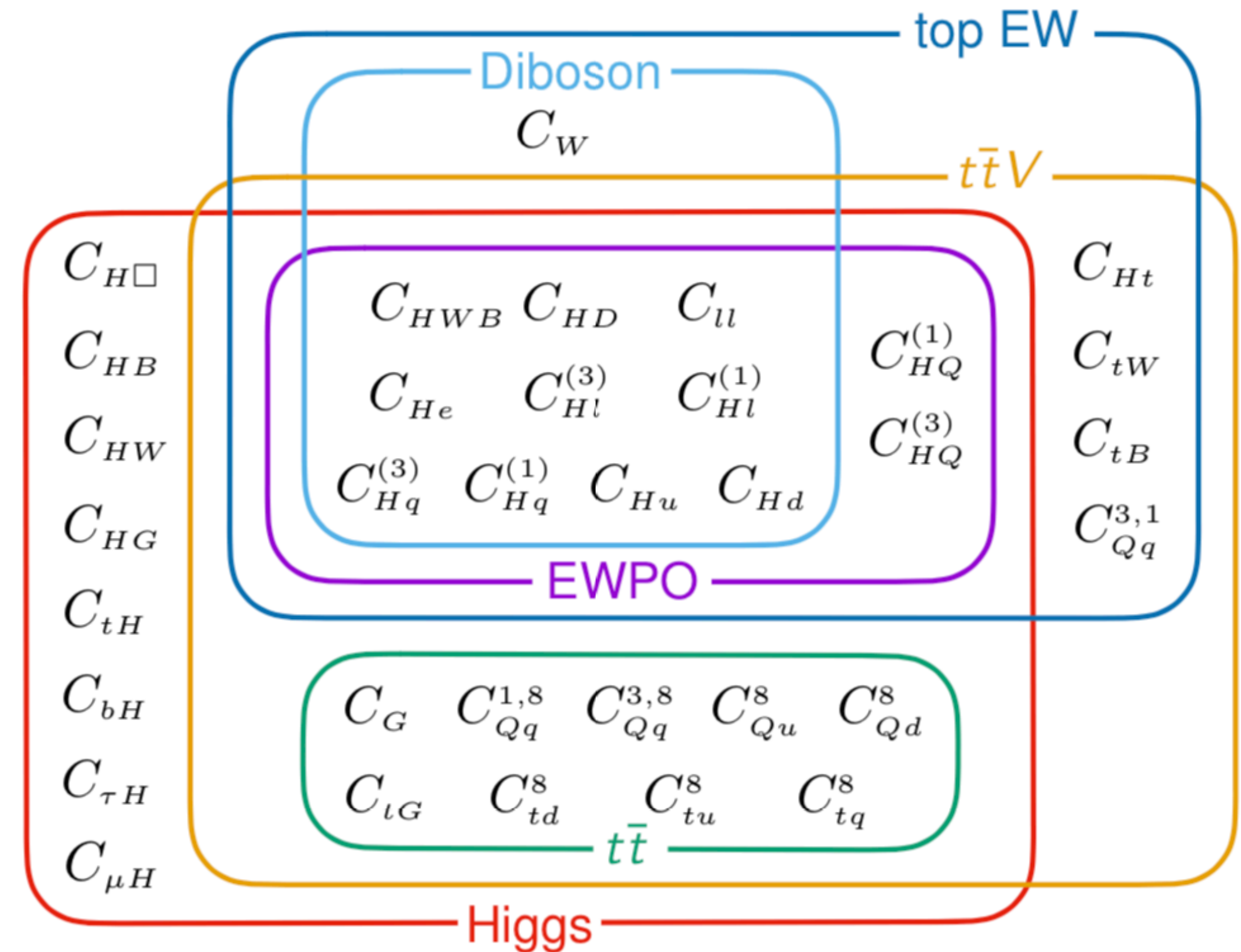
$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{i=1}^{2499} \frac{C_i^{D6}}{\Lambda^2} \mathcal{O}_i^{D6}$$

↖ Wilson coefficients
↖ dimensional scale

[Ellis, Madigan, Mimasu, Sanze, You]

$X^3$		$H^6$ and $H^4 D^2$		$\psi^2 H^3$	
$\mathcal{O}_G$	$f^{ABC} G_{\mu\nu}^A G_{\nu\rho}^B G_{\rho\mu}^C$	$\mathcal{O}_H$	$(H^\dagger H)^3$	$\mathcal{O}_{eH}$	$(H^\dagger H)(\bar{l}_p e_r H)$
$\mathcal{O}_{\bar{G}}$	$f^{ABC} \tilde{G}_{\mu\nu}^A G_{\nu\rho}^B G_{\rho\mu}^C$	$\mathcal{O}_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$	$\mathcal{O}_{uH}$	$(H^\dagger H)(\bar{q}_p u_r \tilde{H})$
$\mathcal{O}_W$	$\varepsilon^{IJK} W_{\mu\nu}^I W_{\nu\rho}^J W_{\rho\mu}^K$	$\mathcal{O}_{HD}$	$(H^\dagger D^\mu H)^* (H^\dagger D_\mu H)$	$\mathcal{O}_{dH}$	$(H^\dagger H)(\bar{q}_p d_r H)$
$\mathcal{O}_{\bar{W}}$	$\varepsilon^{IJK} \tilde{W}_{\mu\nu}^I W_{\nu\rho}^J W_{\rho\mu}^K$				
$X^2 H^2$		$\psi^2 XH$		$\psi^2 H^2 D$	
$\mathcal{O}_{HG}$	$H^\dagger H G_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{eW}$	$(\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I H W_{\mu\nu}^I$	$\mathcal{O}_{Hl}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{l}_p \gamma^\mu l_r)$
$\mathcal{O}_{H\bar{G}}$	$H^\dagger H \tilde{G}_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{eB}$	$(\bar{l}_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$	$\mathcal{O}_{Hl}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{l}_p \tau^I \gamma^\mu l_r)$
$\mathcal{O}_{HW}$	$H^\dagger H W_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{uG}$	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r) \tilde{H} G_{\mu\nu}^A$	$\mathcal{O}_{He}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r)$
$\mathcal{O}_{H\bar{W}}$	$H^\dagger H \tilde{W}_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{uW}$	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \tilde{H} W_{\mu\nu}^I$	$\mathcal{O}_{Hq}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}_p \gamma^\mu q_r)$
$\mathcal{O}_{HB}$	$H^\dagger H B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{uB}$	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tilde{H} B_{\mu\nu}$	$\mathcal{O}_{Hq}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{q}_p \tau^I \gamma^\mu q_r)$
$\mathcal{O}_{H\bar{B}}$	$H^\dagger H \tilde{B}_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{dG}$	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) H G_{\mu\nu}^A$	$\mathcal{O}_{Hu}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}_p \gamma^\mu u_r)$
$\mathcal{O}_{HWB}$	$H^\dagger \tau^I H W_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{dW}$	$(\bar{q}_p \sigma^{\mu\nu} d_r) \tau^I H W_{\mu\nu}^I$	$\mathcal{O}_{Hd}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{d}_p \gamma^\mu d_r)$
$\mathcal{O}_{H\bar{W}B}$	$H^\dagger \tau^I H \tilde{W}_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{dB}$	$(\bar{q}_p \sigma^{\mu\nu} d_r) H B_{\mu\nu}$	$\mathcal{O}_{Hud}$	$i(\tilde{H}^\dagger D_\mu H)(\bar{u}_p \gamma^\mu d_r)$
$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
$\mathcal{O}_{ll}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$	$\mathcal{O}_{ee}$	$(\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$	$\mathcal{O}_{le}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)$
$\mathcal{O}_{qq}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t)$	$\mathcal{O}_{uu}$	$(\bar{u}_p \gamma_\mu u_r)(\bar{u}_s \gamma^\mu u_t)$	$\mathcal{O}_{lu}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{u}_s \gamma^\mu u_t)$
$\mathcal{O}_{qq}^{(3)}$	$(\bar{q}_p \gamma_\mu \tau^I q_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	$\mathcal{O}_{dd}$	$(\bar{d}_p \gamma_\mu d_r)(\bar{d}_s \gamma^\mu d_t)$	$\mathcal{O}_{ld}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{d}_s \gamma^\mu d_t)$
$\mathcal{O}_{lq}^{(1)}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)$	$\mathcal{O}_{eu}$	$(\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$	$\mathcal{O}_{qe}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$
$\mathcal{O}_{lq}^{(3)}$	$(\bar{l}_p \gamma_\mu \tau^I l_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	$\mathcal{O}_{ed}$	$(\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)$	$\mathcal{O}_{qu}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{u}_s \gamma^\mu u_t)$
		$\mathcal{O}_{ud}^{(1)}$	$(\bar{u}_p \gamma_\mu u_r)(\bar{d}_s \gamma^\mu d_t)$	$\mathcal{O}_{qu}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{u}_s \gamma^\mu T^A u_t)$
		$\mathcal{O}_{ud}^{(8)}$	$(\bar{u}_p \gamma_\mu T^A u_r)(\bar{d}_s \gamma^\mu T^A d_t)$	$\mathcal{O}_{qd}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{d}_s \gamma^\mu d_t)$
		$\mathcal{O}_{qd}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{d}_s \gamma^\mu T^A d_t)$		
$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		$B$ -violating			
$\mathcal{O}_{ledq}$	$(\bar{l}_p^j e_r)(\bar{d}_s^k q_t^l)$	$\mathcal{O}_{duq}$	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jkl} [(d_p^\alpha)^T C u_r^\beta] [(q_s^j)^T C l_t^k]$		
$\mathcal{O}_{quqd}^{(1)}$	$(\bar{q}_p^j u_r) \varepsilon_{jk} (\bar{q}_s^k d_t)$	$\mathcal{O}_{qqu}$	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jkl} [(q_p^\alpha)^T C q_r^\beta] [(u_s^j)^T C e_t]$		
$\mathcal{O}_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r) \varepsilon_{jk} (\bar{q}_s^k T^A d_t)$	$\mathcal{O}_{qqq}$	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jnl} \varepsilon_{km} [(q_p^\alpha)^T C q_r^\beta] [(q_s^m)^T C l_t^n]$		
$\mathcal{O}_{lequ}^{(1)}$	$(\bar{l}_p^j e_r) \varepsilon_{jk} (\bar{q}_s^k u_t)$	$\mathcal{O}_{duu}$	$\varepsilon^{\alpha\beta\gamma} [(d_p^\alpha)^T C u_r^\beta] [(u_s^j)^T C e_t]$		
$\mathcal{O}_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r) \varepsilon_{jk} (\bar{q}_s^k \sigma^{\mu\nu} u_t)$				

flavour universality



→ John's course on Higgs, Top & Beyond starting next Thursday

# The global EFT/SMEFT fit

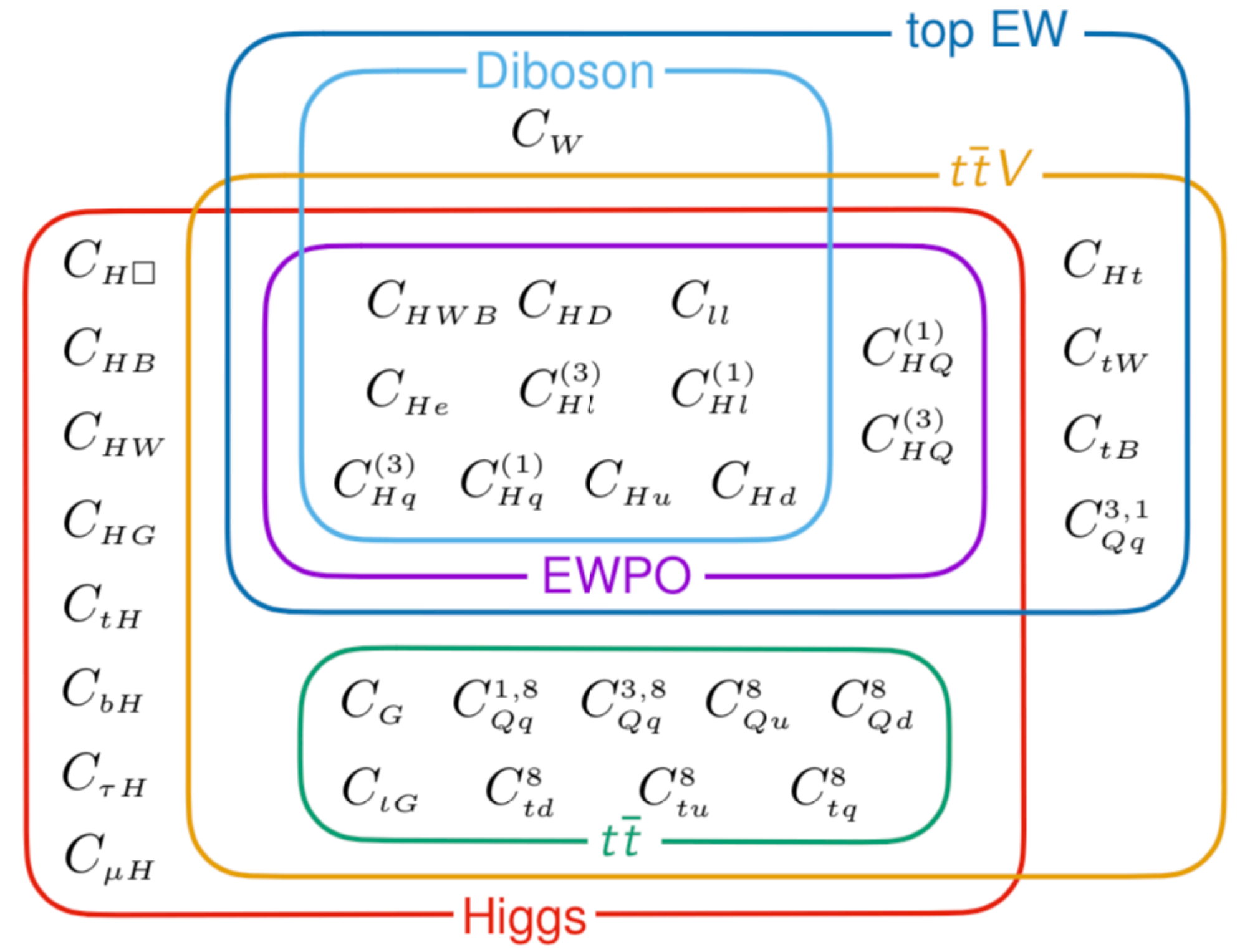
$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{i=1}^{2499} \frac{C_i^{D6}}{\Lambda^2} \mathcal{O}_i^{D6} + \sum_{i=1}^{\sim 50000} \frac{C_i^{D8}}{\Lambda^4} \mathcal{O}_i^{D8} + \dots$$

↖ Wilson coefficients  
↖ dimensional scale

[Ellis, Madigan, Mimasu, Sanze, You]

$X^3$		$H^6$ and $H^4 D^2$		$\psi^2 H^3$	
$\mathcal{O}_G$	$f^{ABC} G_{\mu\nu}^A G_{\nu\rho}^{B\rho} G_{\rho\mu}^{C\mu}$	$\mathcal{O}_H$	$(H^\dagger H)^3$	$\mathcal{O}_{eH}$	$(H^\dagger H)(\bar{l}_p e_r H)$
$\mathcal{O}_{\bar{G}}$	$f^{ABC} \tilde{G}_{\mu\nu}^A G_{\nu\rho}^{B\rho} G_{\rho\mu}^{C\mu}$	$\mathcal{O}_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$	$\mathcal{O}_{uH}$	$(H^\dagger H)(\bar{q}_p u_r \tilde{H})$
$\mathcal{O}_W$	$\varepsilon^{IJK} W_{\mu\nu}^I W_{\nu\rho}^J W_{\rho\mu}^K$	$\mathcal{O}_{HD}$	$(H^\dagger D^\mu H)^\dagger (H^\dagger D_\mu H)$	$\mathcal{O}_{dH}$	$(H^\dagger H)(\bar{q}_p d_r H)$
$\mathcal{O}_{\bar{W}}$	$\varepsilon^{IJK} \tilde{W}_{\mu\nu}^I W_{\nu\rho}^J W_{\rho\mu}^K$				
$X^2 H^2$		$\psi^2 XH$		$\psi^2 H^2 D$	
$\mathcal{O}_{HG}$	$H^\dagger H G_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{eW}$	$(\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I H W_{\mu\nu}^I$	$\mathcal{O}_{Hl}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{l}_p \gamma^\mu l_r)$
$\mathcal{O}_{H\bar{G}}$	$H^\dagger H \tilde{G}_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{eB}$	$(\bar{l}_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$	$\mathcal{O}_{Hl}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{l}_p \tau^I \gamma^\mu l_r)$
$\mathcal{O}_{HW}$	$H^\dagger H W_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{uG}$	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r) \tilde{H} G_{\mu\nu}^A$	$\mathcal{O}_{He}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r)$
$\mathcal{O}_{H\bar{W}}$	$H^\dagger H \tilde{W}_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{uW}$	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \tilde{H} W_{\mu\nu}^I$	$\mathcal{O}_{Hq}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}_p \gamma^\mu q_r)$
$\mathcal{O}_{HB}$	$H^\dagger H B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{uB}$	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tilde{H} B_{\mu\nu}$	$\mathcal{O}_{Hq}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{q}_p \tau^I \gamma^\mu q_r)$
$\mathcal{O}_{H\bar{B}}$	$H^\dagger H \tilde{B}_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{dG}$	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) H G_{\mu\nu}^A$	$\mathcal{O}_{Hu}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}_p \gamma^\mu u_r)$
$\mathcal{O}_{HWB}$	$H^\dagger \tau^I H W_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{dW}$	$(\bar{q}_p \sigma^{\mu\nu} d_r) \tau^I H W_{\mu\nu}^I$	$\mathcal{O}_{Hd}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{d}_p \gamma^\mu d_r)$
$\mathcal{O}_{H\bar{W}B}$	$H^\dagger \tau^I H \tilde{W}_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{dB}$	$(\bar{q}_p \sigma^{\mu\nu} d_r) H B_{\mu\nu}$	$\mathcal{O}_{Hud}$	$i(\tilde{H}^\dagger D_\mu H)(\bar{u}_p \gamma^\mu d_r)$
$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
$\mathcal{O}_{ll}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$	$\mathcal{O}_{ee}$	$(\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$	$\mathcal{O}_{le}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)$
$\mathcal{O}_{qq}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t)$	$\mathcal{O}_{uu}$	$(\bar{u}_p \gamma_\mu u_r)(\bar{u}_s \gamma^\mu u_t)$	$\mathcal{O}_{lu}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{u}_s \gamma^\mu u_t)$
$\mathcal{O}_{qq}^{(3)}$	$(\bar{q}_p \gamma_\mu \tau^I q_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	$\mathcal{O}_{dd}$	$(\bar{d}_p \gamma_\mu d_r)(\bar{d}_s \gamma^\mu d_t)$	$\mathcal{O}_{ld}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{d}_s \gamma^\mu d_t)$
$\mathcal{O}_{lq}^{(1)}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)$	$\mathcal{O}_{eu}$	$(\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$	$\mathcal{O}_{qe}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$
$\mathcal{O}_{lq}^{(3)}$	$(\bar{l}_p \gamma_\mu \tau^I l_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	$\mathcal{O}_{ed}$	$(\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)$	$\mathcal{O}_{qu}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{u}_s \gamma^\mu u_t)$
		$\mathcal{O}_{ud}^{(1)}$	$(\bar{u}_p \gamma_\mu u_r)(\bar{d}_s \gamma^\mu d_t)$	$\mathcal{O}_{qu}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{u}_s \gamma^\mu T^A u_t)$
		$\mathcal{O}_{ud}^{(8)}$	$(\bar{u}_p \gamma_\mu T^A u_r)(\bar{d}_s \gamma^\mu T^A d_t)$	$\mathcal{O}_{qd}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{d}_s \gamma^\mu d_t)$
		$\mathcal{O}_{qd}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{d}_s \gamma^\mu T^A d_t)$		
$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		$B$ -violating			
$\mathcal{O}_{ledq}$	$(\bar{l}_p^j e_r)(\bar{d}_s^k q_t^l)$	$\mathcal{O}_{duq}$	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jkl} [(d_p^\alpha)^T C u_r^\beta] [(q_s^j)^T C l_t^k]$		
$\mathcal{O}_{quqd}^{(1)}$	$(\bar{q}_p^j u_r) \varepsilon_{jk} (\bar{q}_s^k d_t)$	$\mathcal{O}_{qqu}$	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jkl} [(q_p^\alpha)^T C q_r^\beta] [(u_s^j)^T C e_t]$		
$\mathcal{O}_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r) \varepsilon_{jk} (\bar{q}_s^k T^A d_t)$	$\mathcal{O}_{qqq}$	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jnl} \varepsilon_{km} [(q_p^\alpha)^T C q_r^\beta] [(q_s^m)^T C l_t^n]$		
$\mathcal{O}_{lequ}^{(1)}$	$(\bar{l}_p^j e_r) \varepsilon_{jk} (\bar{q}_s^k u_t)$	$\mathcal{O}_{duu}$	$\varepsilon^{\alpha\beta\gamma} [(d_p^\alpha)^T C u_r^\beta] [(u_s^j)^T C e_t]$		
$\mathcal{O}_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r) \varepsilon_{jk} (\bar{q}_s^k \sigma^{\mu\nu} u_t)$				

flavour universality



→ John's course on Higgs, Top & Beyond starting next Thursday

# EW standard candles at the LHC

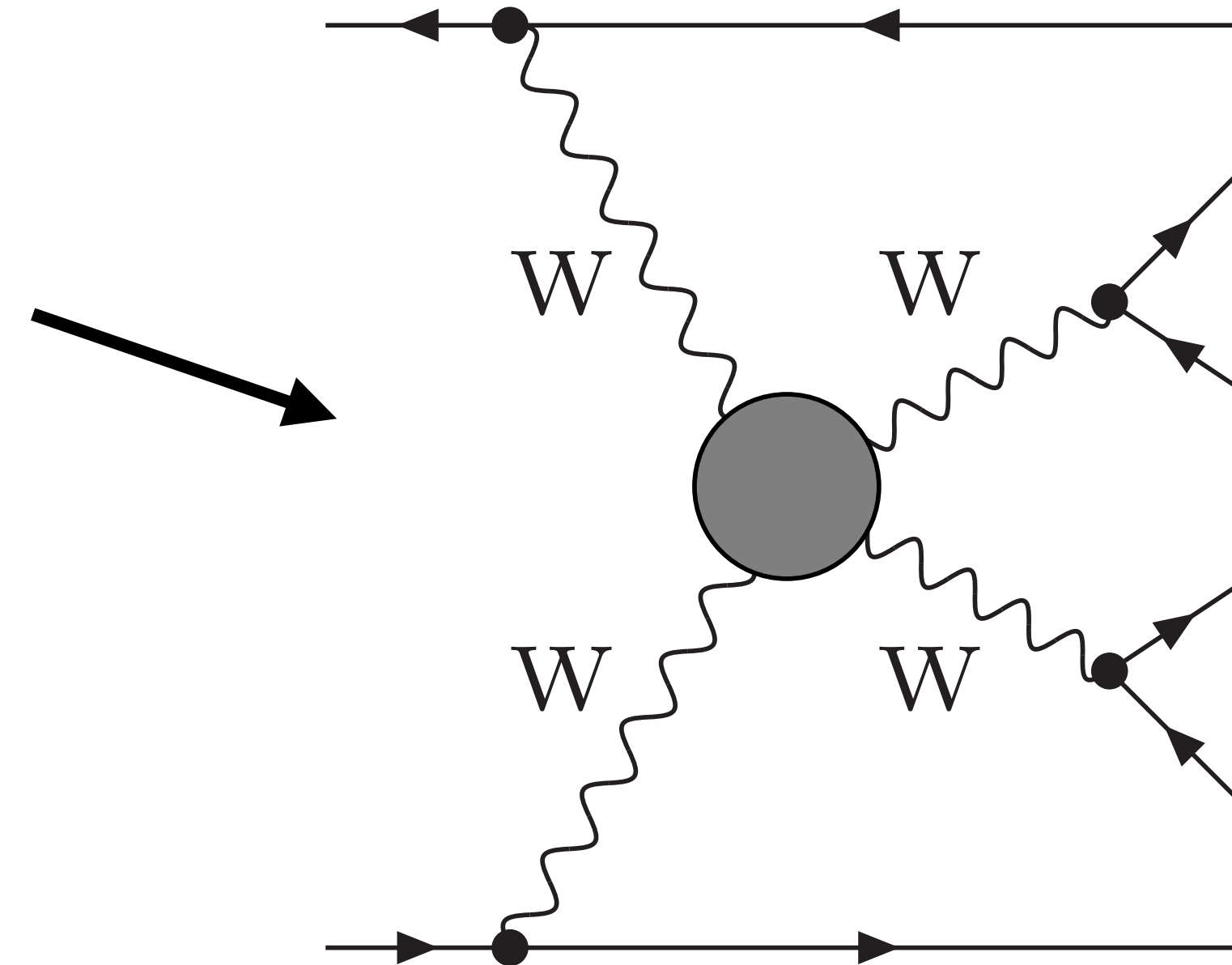
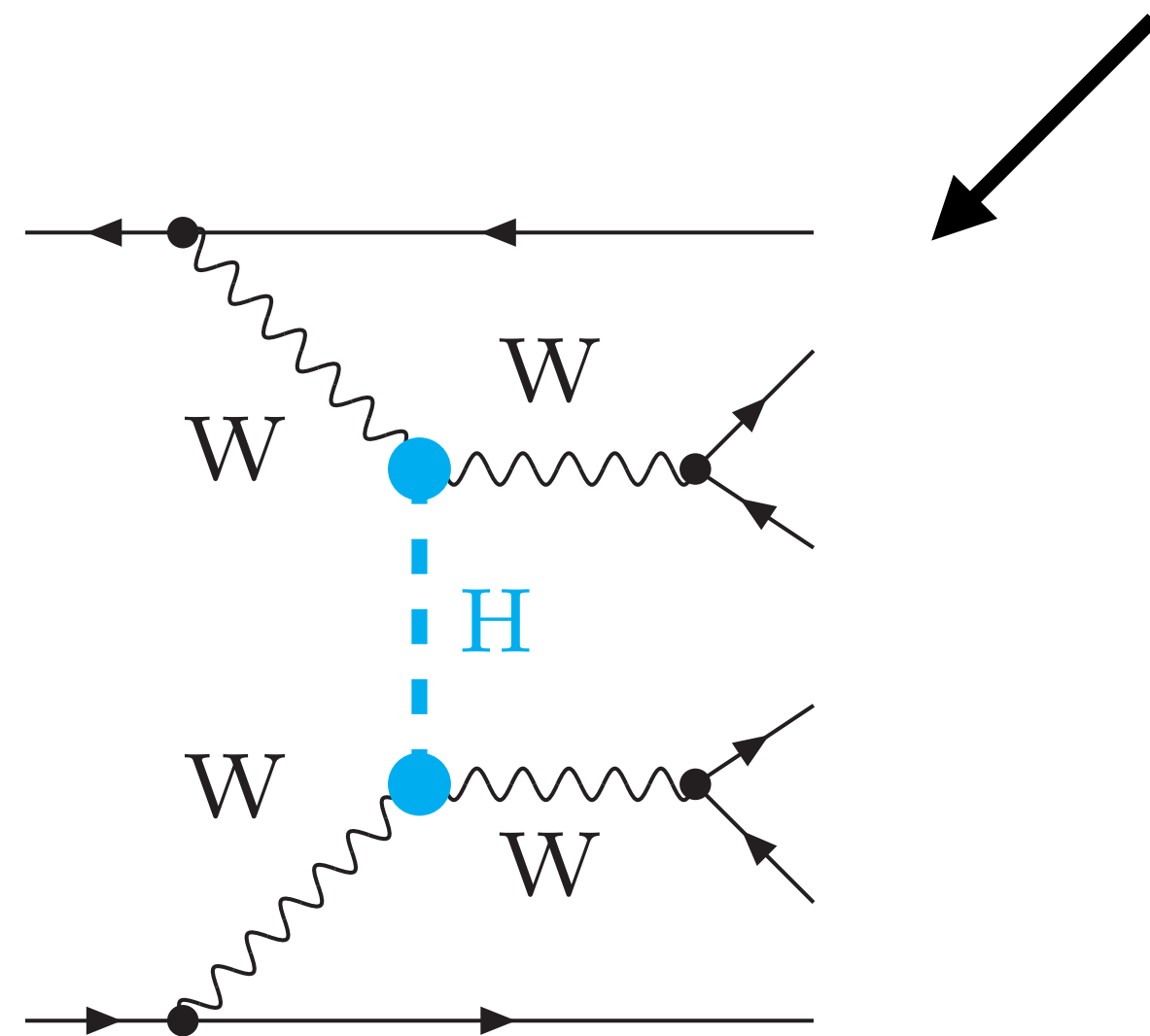


→ John's course on Higgs, Top & Beyond starting next Thursday



# Vector-boson scattering at LHC

- direct access to quartic EW gauge couplings
- VBS: longitudinal gauge bosons at high energies
- window to electroweak symmetry breaking via off-shell Higgs exchange (ensures unitarity)



Signatures:

$ssWW$ -VBS:  $l^+l^+ + \bar{\nu}\bar{\nu} + 2\text{jets}$

$WWW$ -VBS:  $l^+l^- + \bar{\nu}\nu + 2\text{jets}$

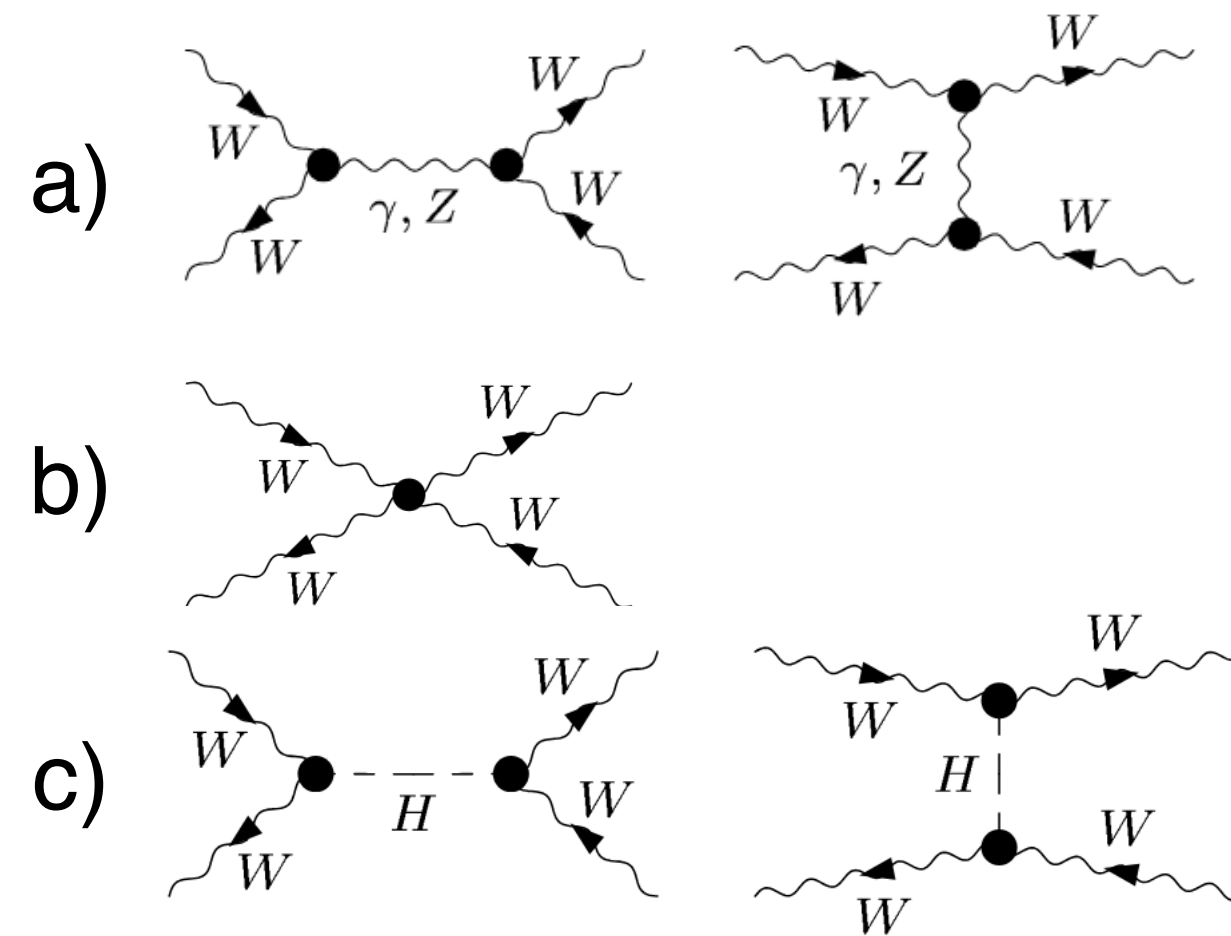
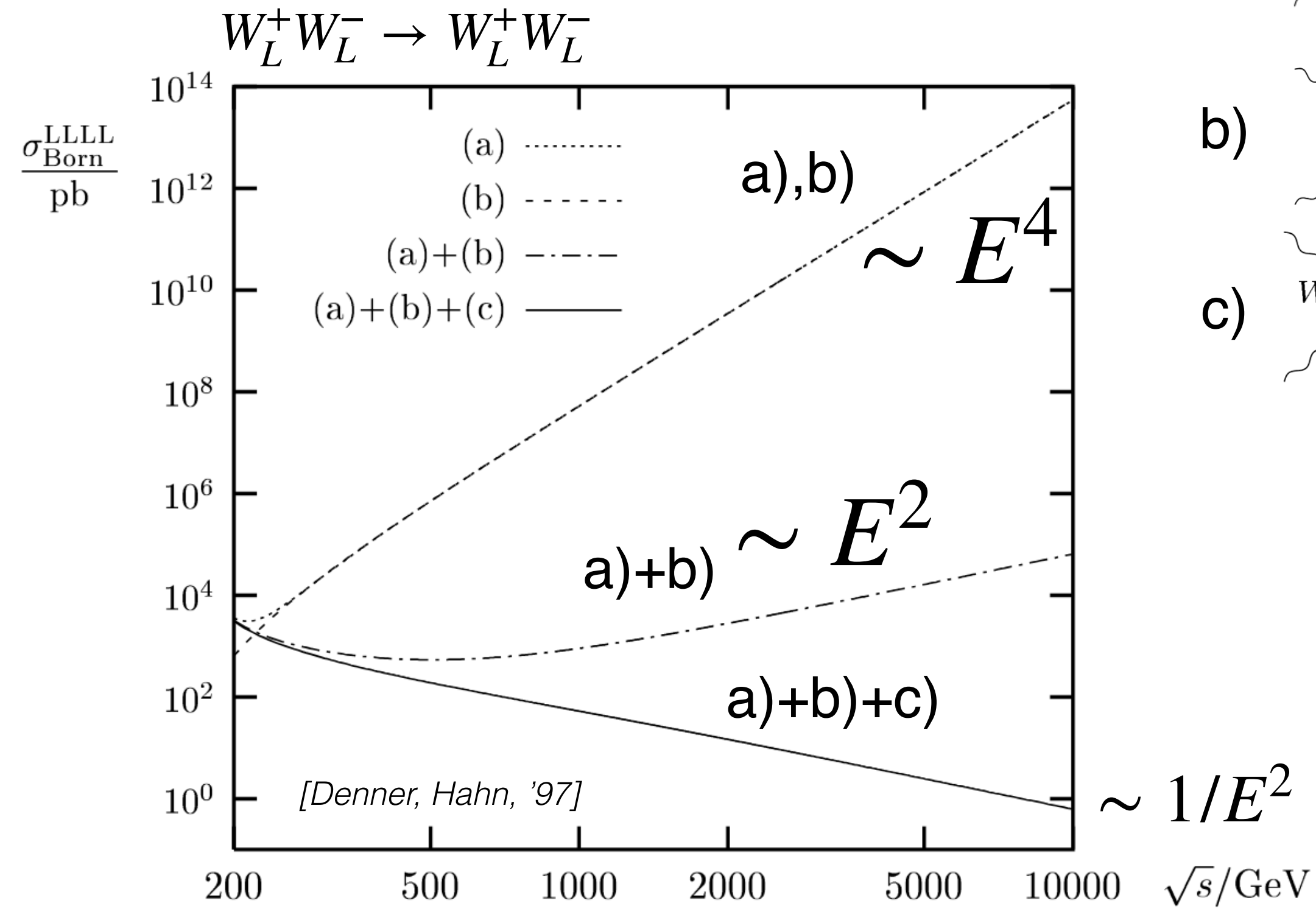
$ZZ$ -VBS:  $l^+l^-l^+l^- + 2\text{jets}$

$WZ$ -VBS:  $l^+l^-l^+\bar{\nu} + 2\text{jets}$

$W\gamma$ -VBS:  $l^\pm\nu + \gamma + 2\text{jets}$

$Z\gamma$ -VBS:  $l^+l^- + \gamma + 2\text{jets}$

# Polarisation measurements

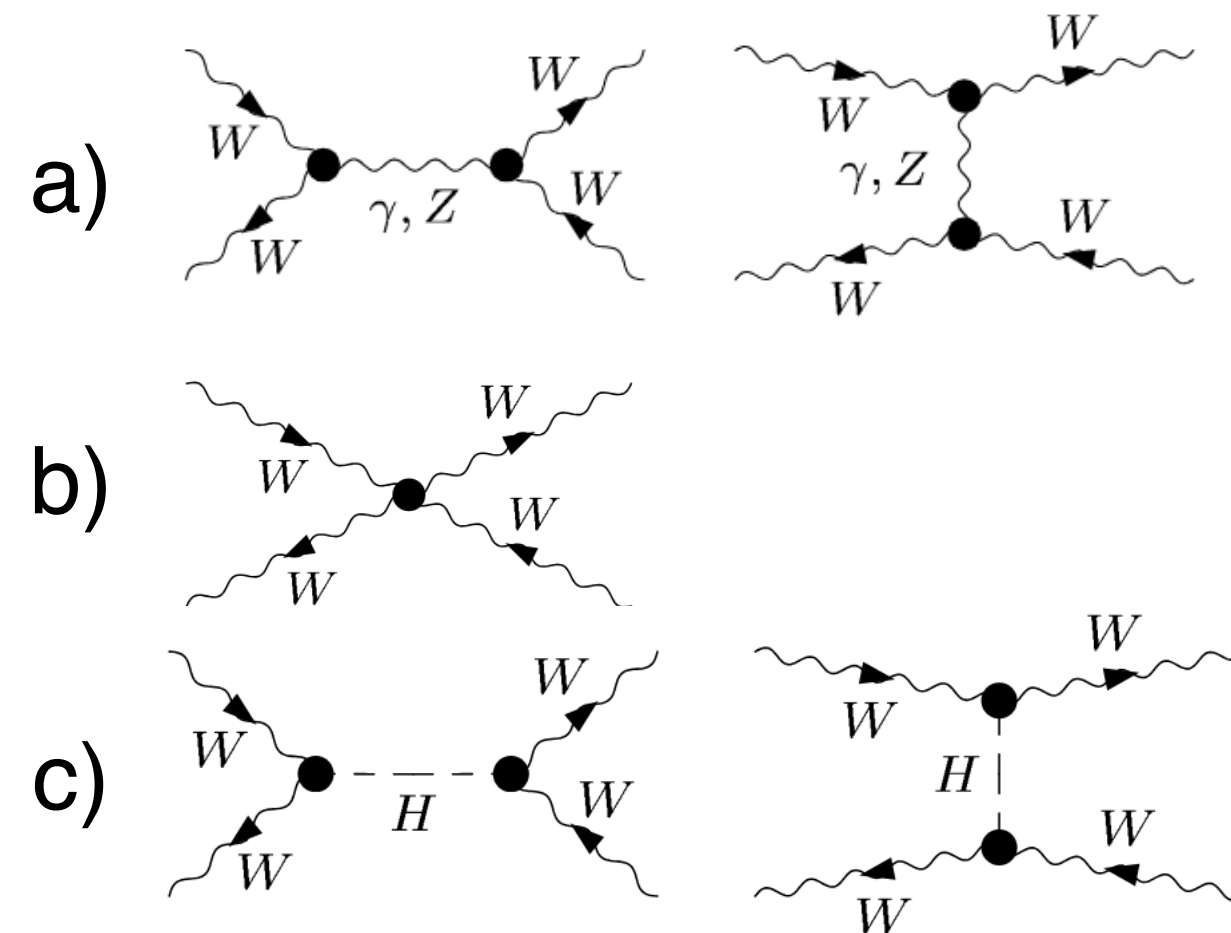
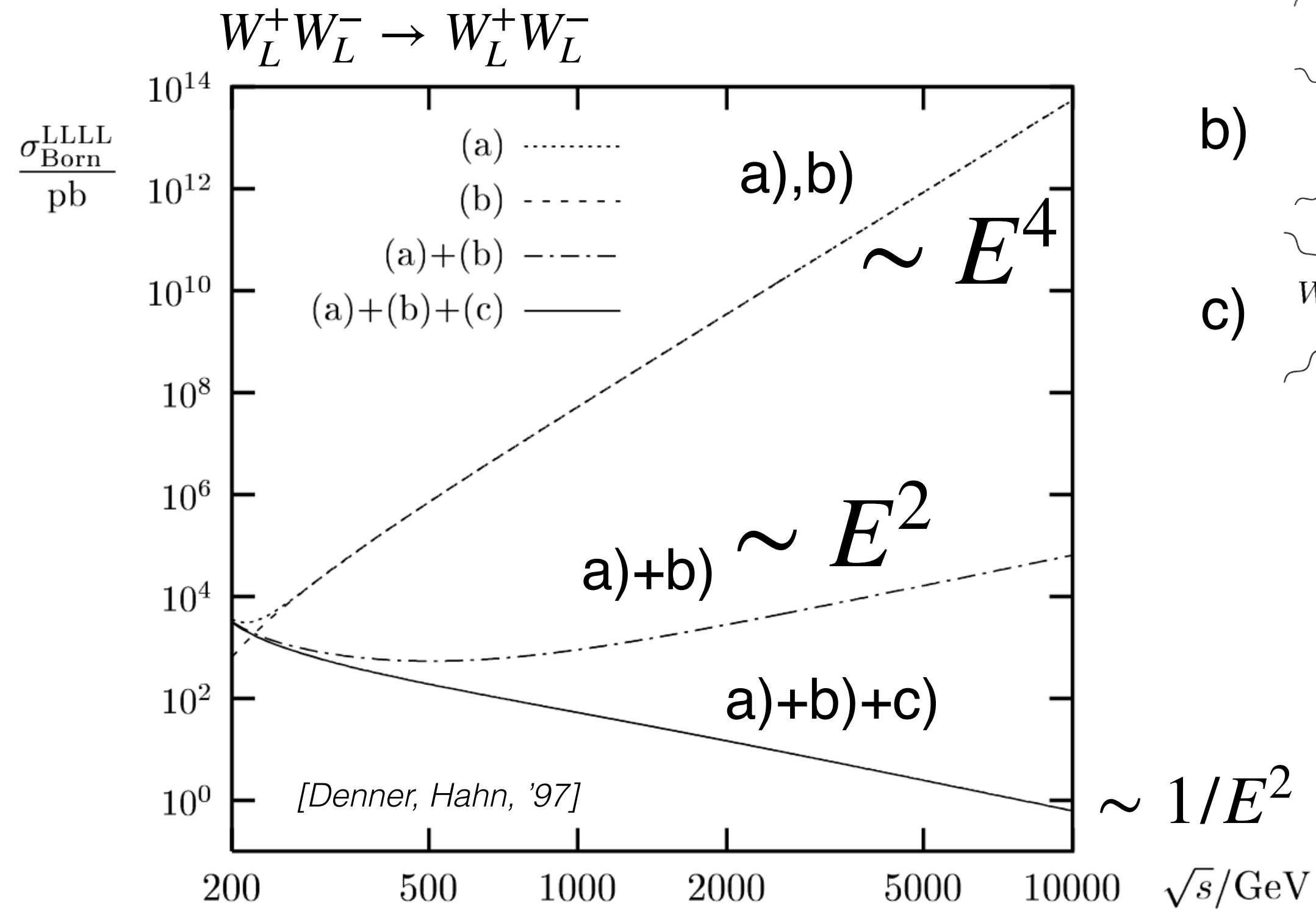


cancellation ensures unitarity!

→ crucial test of EWSB

→ problem: we can't directly observe longitudinal vector-bosons

# Polarisation measurements



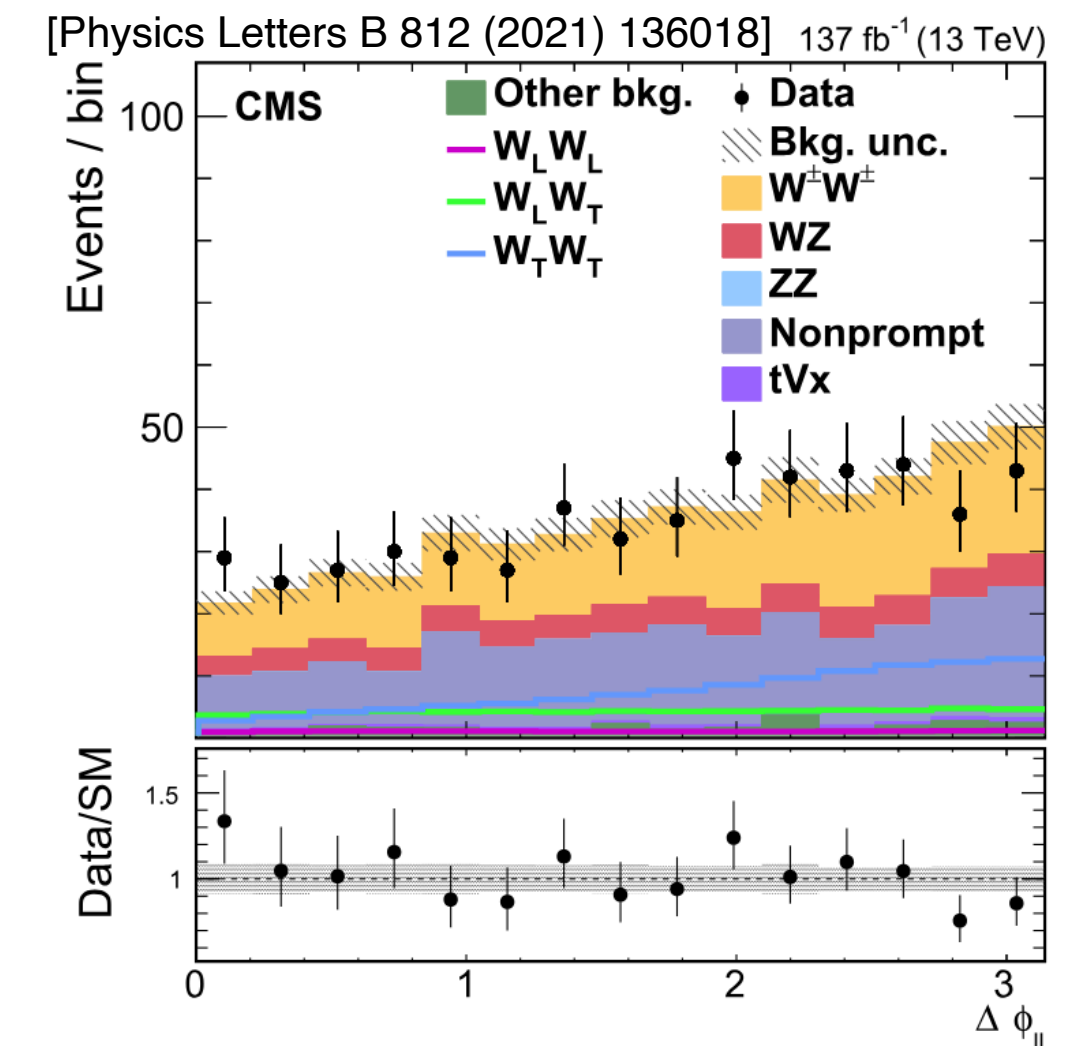
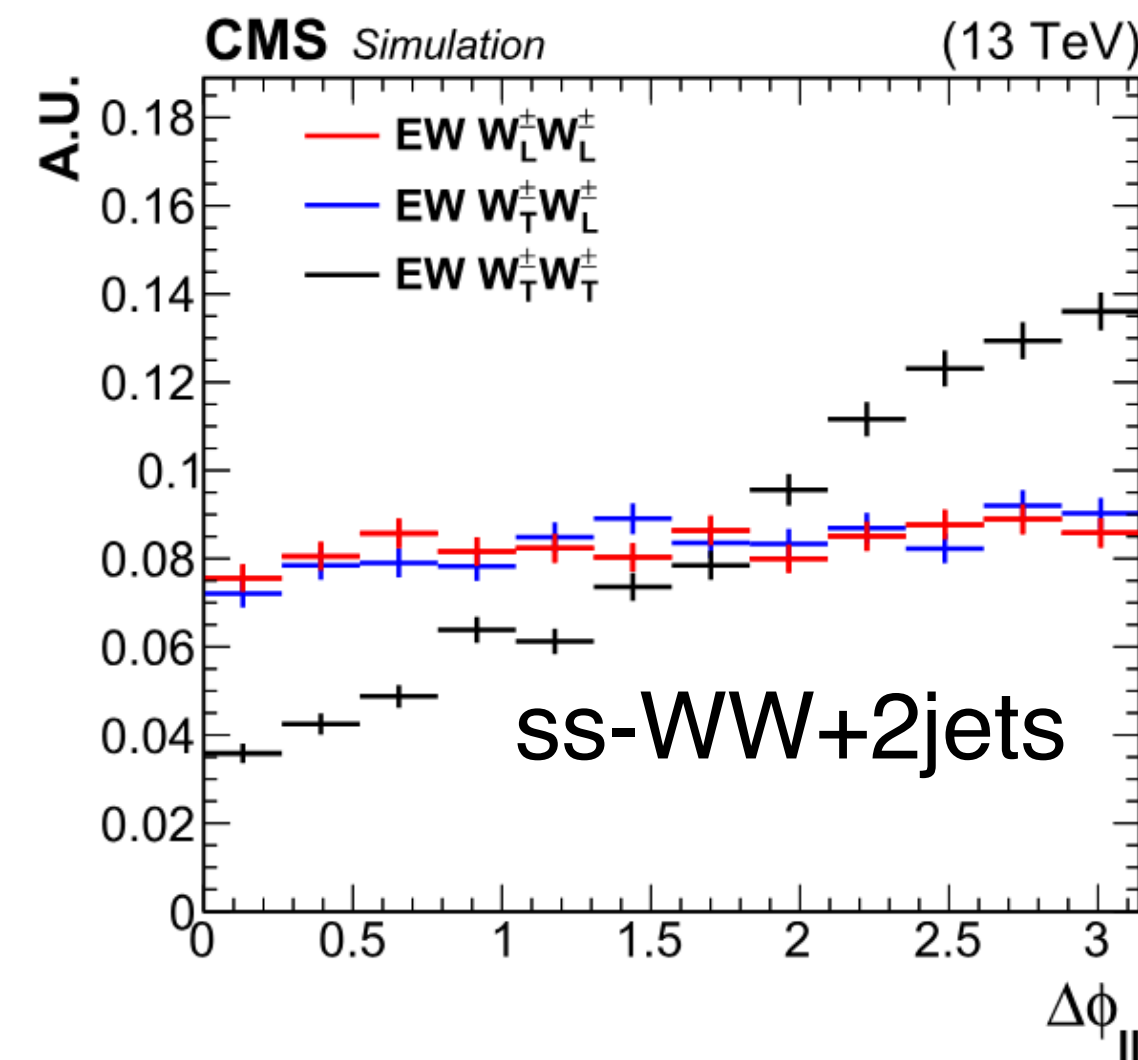
cancellation ensures unitarity!

→ crucial test of EWSB

→ problem: we can't directly observe longitudinal vector-bosons

→ exploit angular information

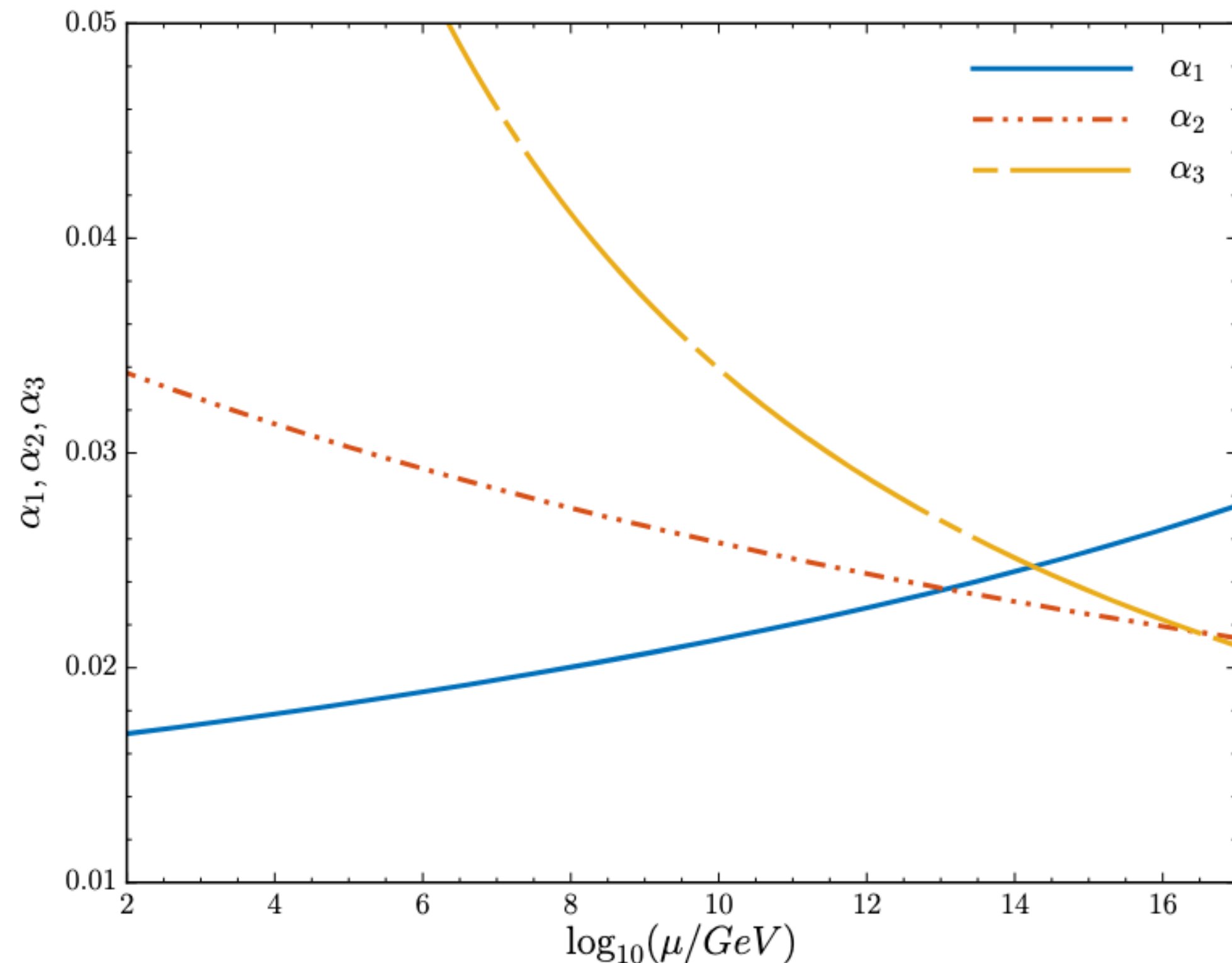
→ currently limited by stats



# Running couplings

Also the  $g_1, g_2$  couplings run!

→ Gavin's lecture yesterday



Hint of gauge unification?

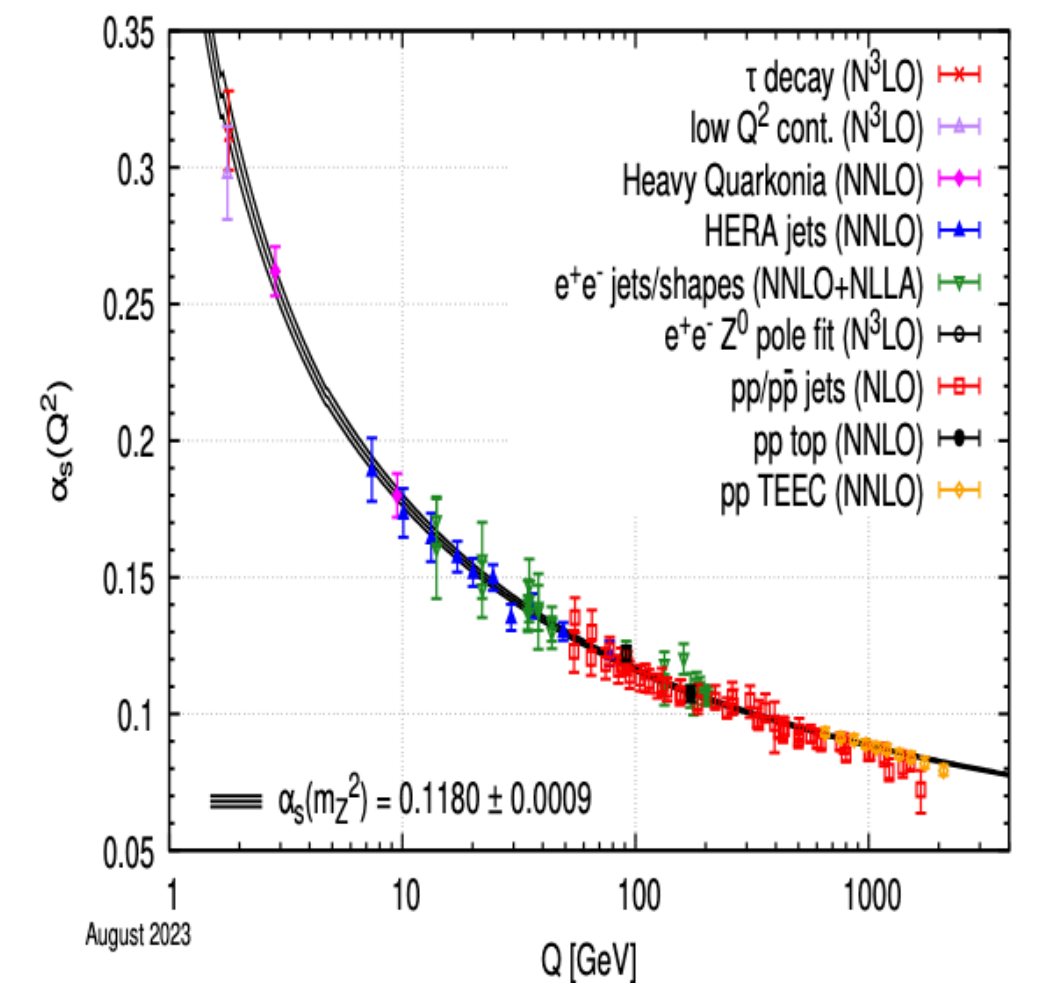
QCD lecture 1 (p. 20)  
 ↳ Basic methods  
 ↳ Perturbation theory

## Running coupling (cont.)

$$\text{Solve } Q^2 \frac{\partial \alpha_s}{\partial Q^2} = -b_0 \alpha_s^2 \Rightarrow \alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 + b_0 \alpha_s(Q_0^2) \ln \frac{Q^2}{Q_0^2}} = \frac{1}{b_0 \ln \frac{Q^2}{\Lambda^2}}$$

$\Lambda \simeq 0.2$  GeV (aka  $\Lambda_{QCD}$ ) is the fundamental scale of QCD, at which coupling blows up.

- ▶  $\Lambda$  sets the scale for hadron masses (NB:  $\Lambda$  not unambiguously defined wrt higher orders)
- ▶ Perturbative calculations valid for scales  $Q \gg \Lambda$ .

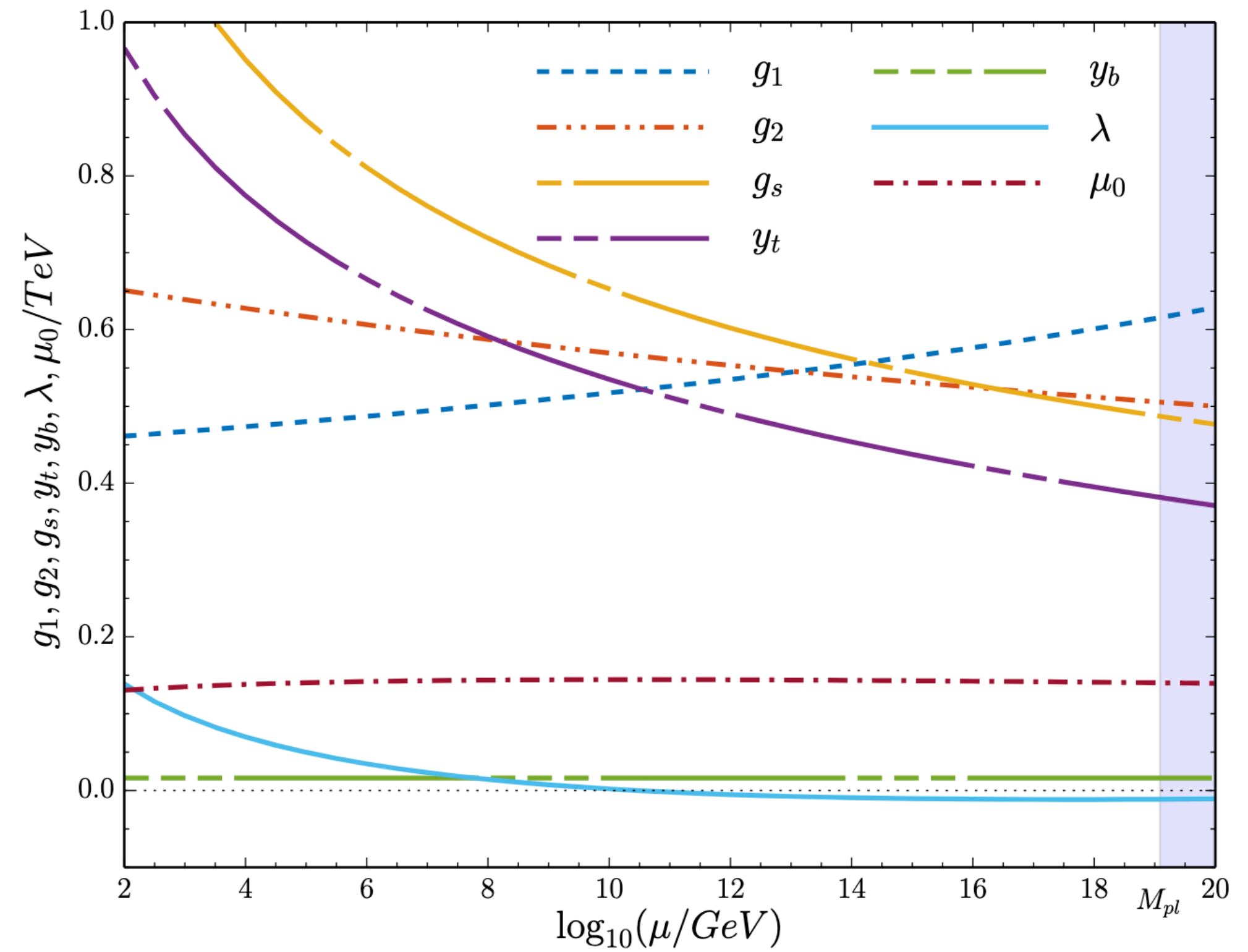
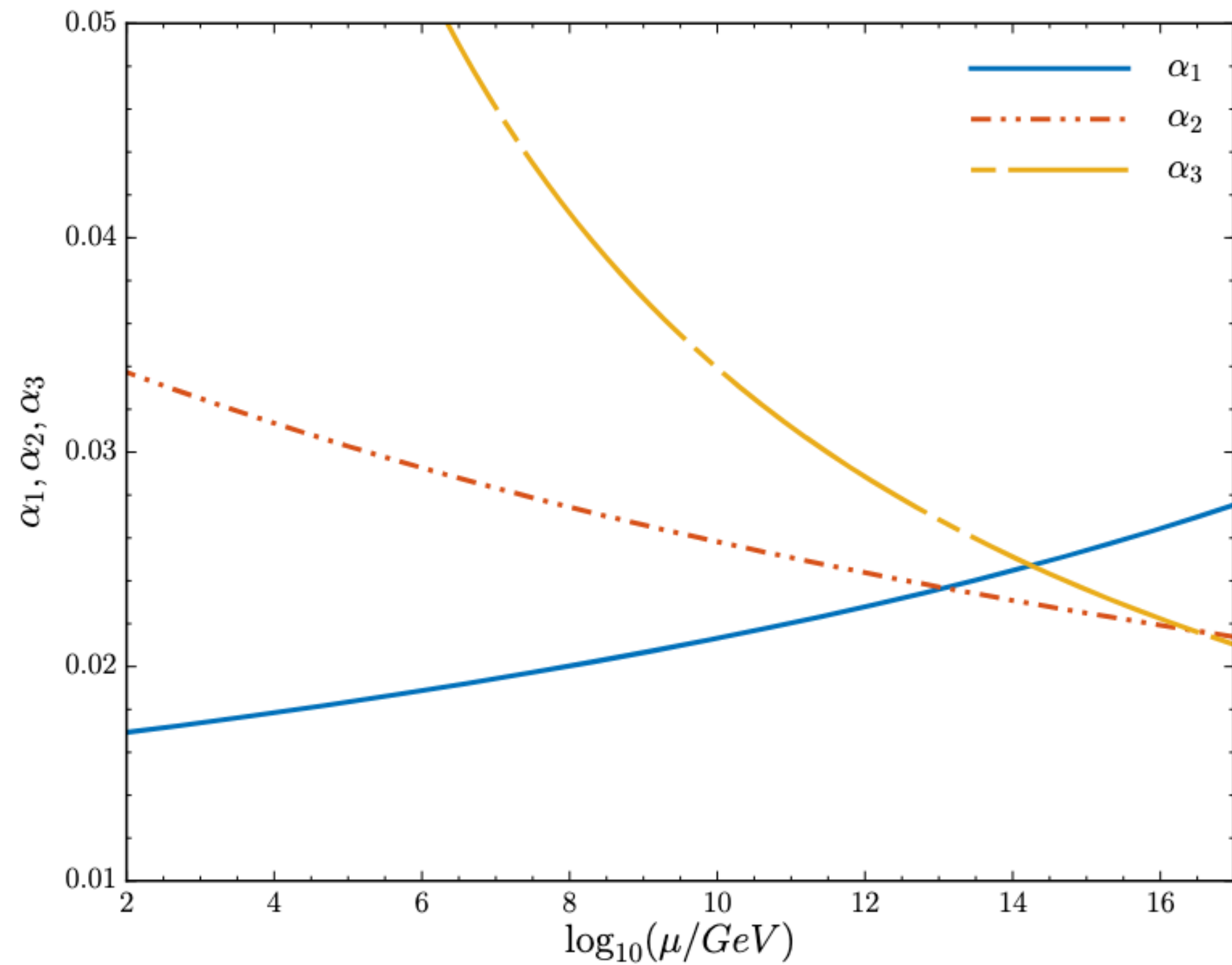


2023 PDG, QCD chapter

# Running couplings

Also the  $g_1, g_2$  couplings run!

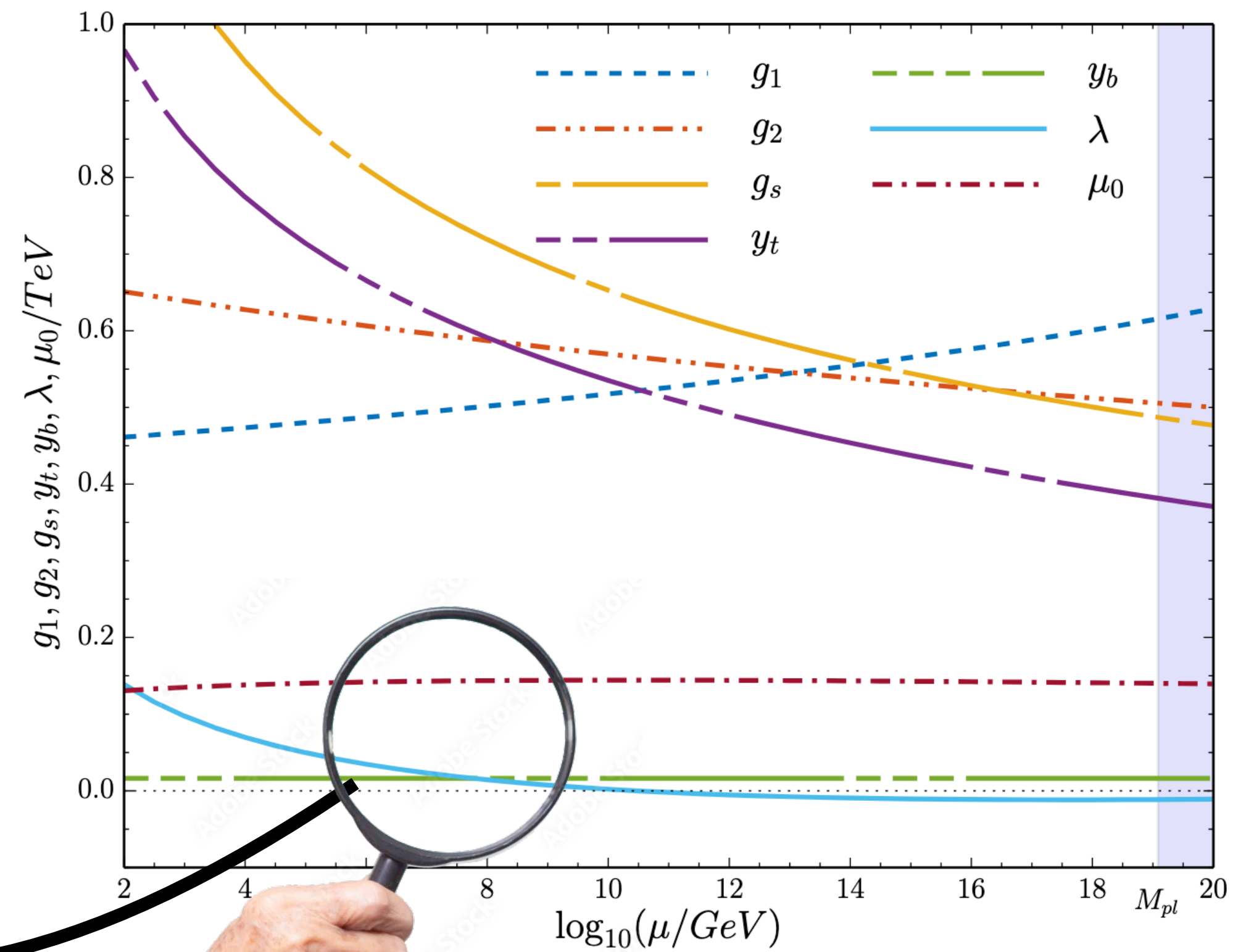
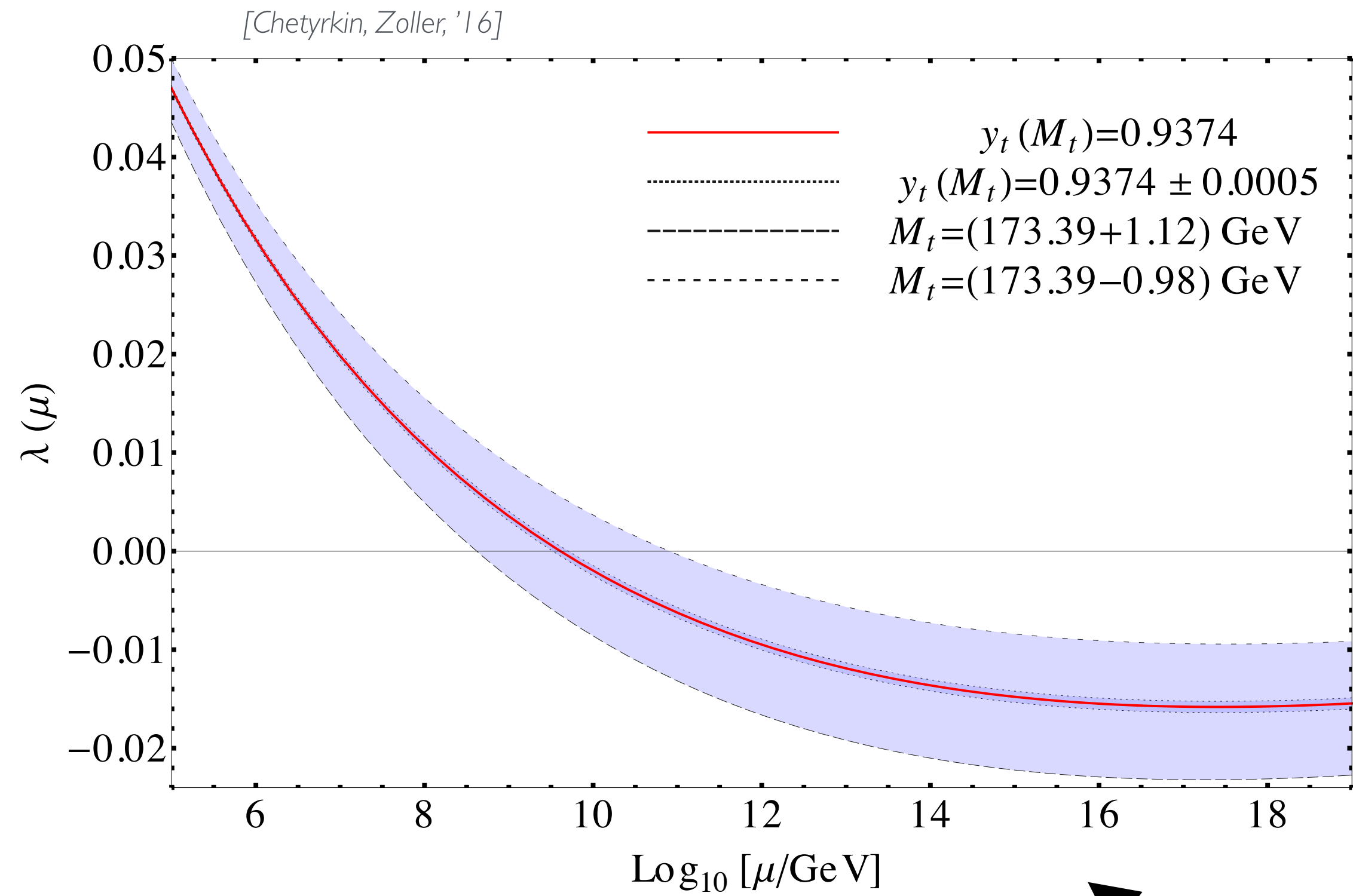
All couplings run! Incl. yukawas, trilinear



Hint of gauge unification?

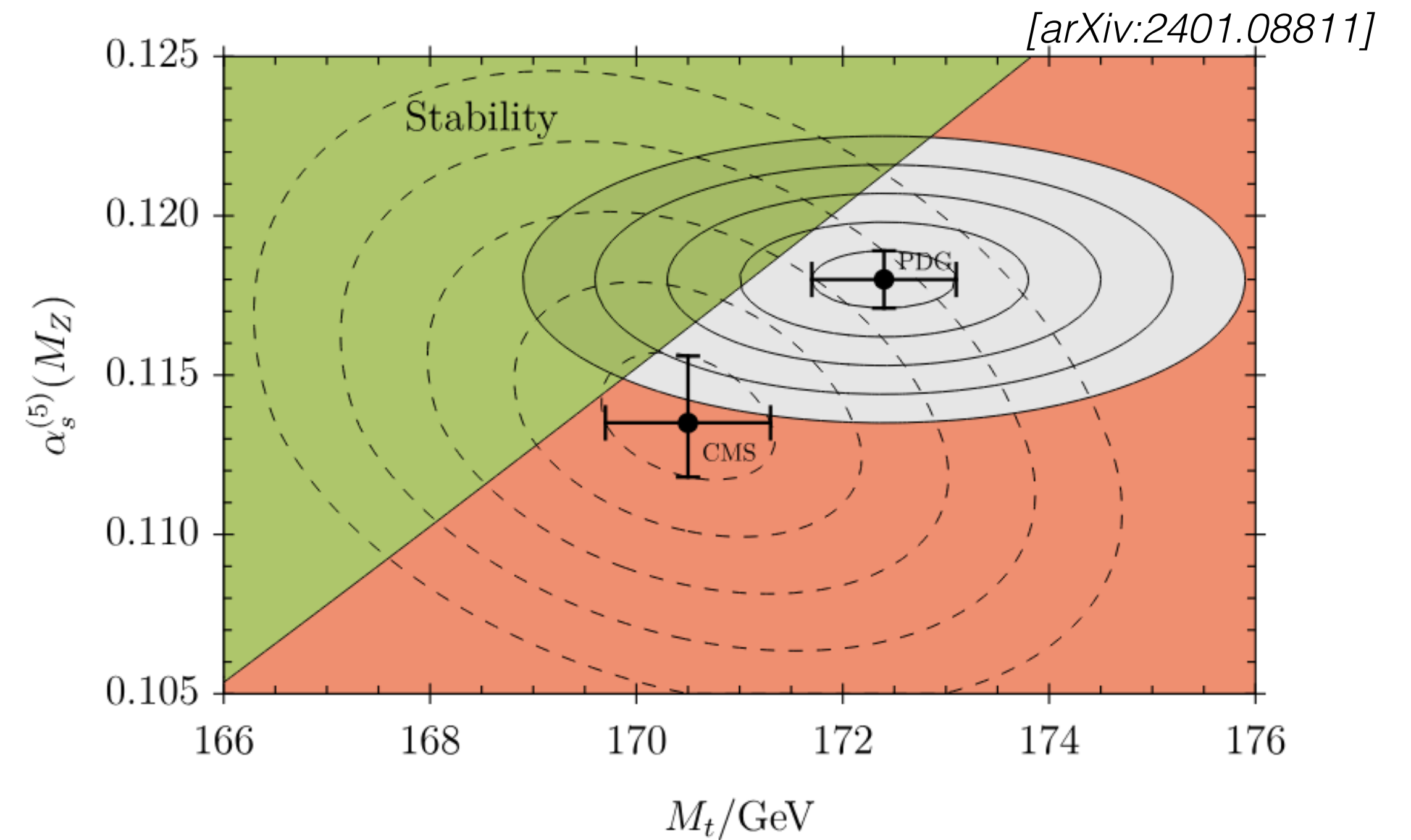
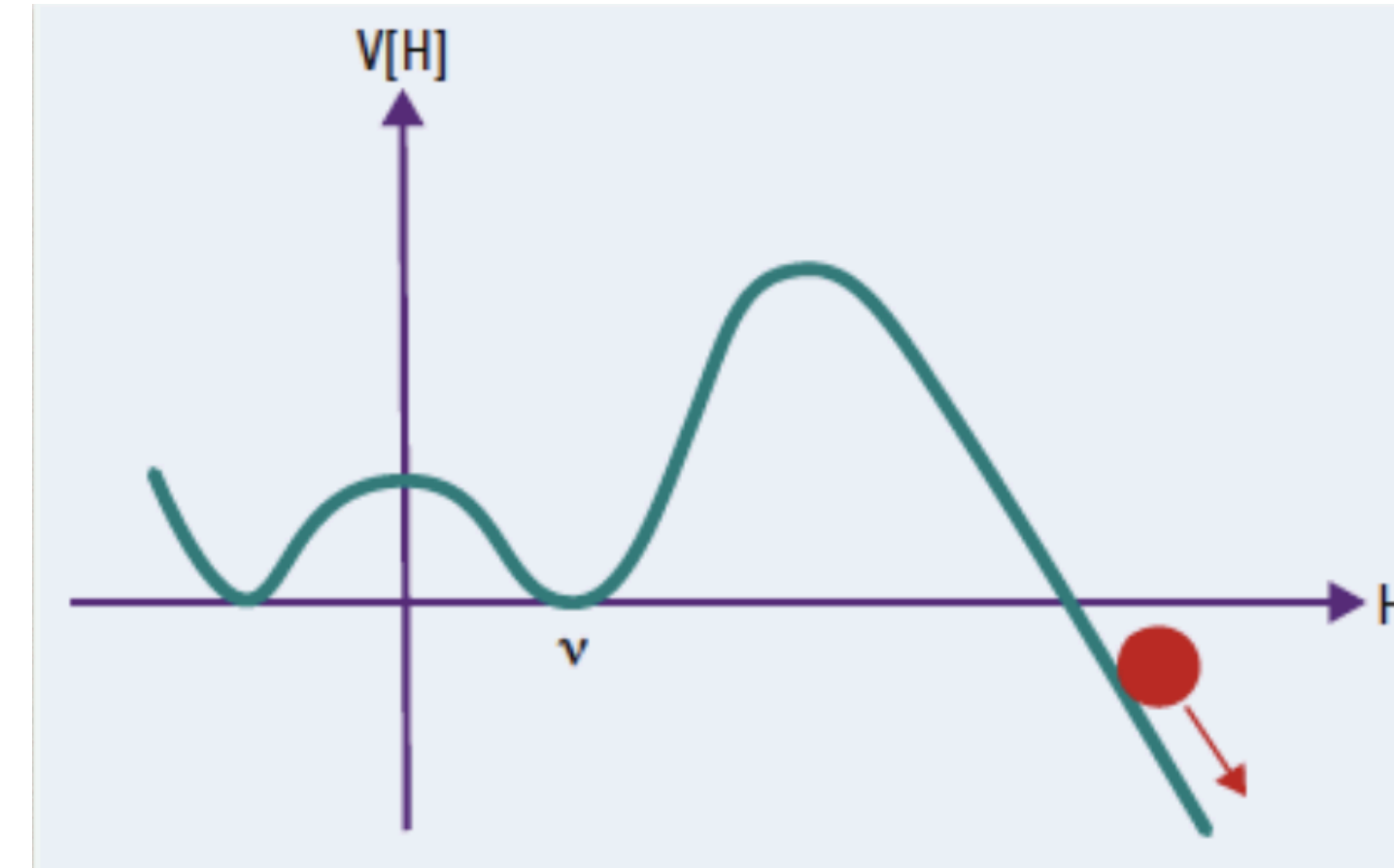
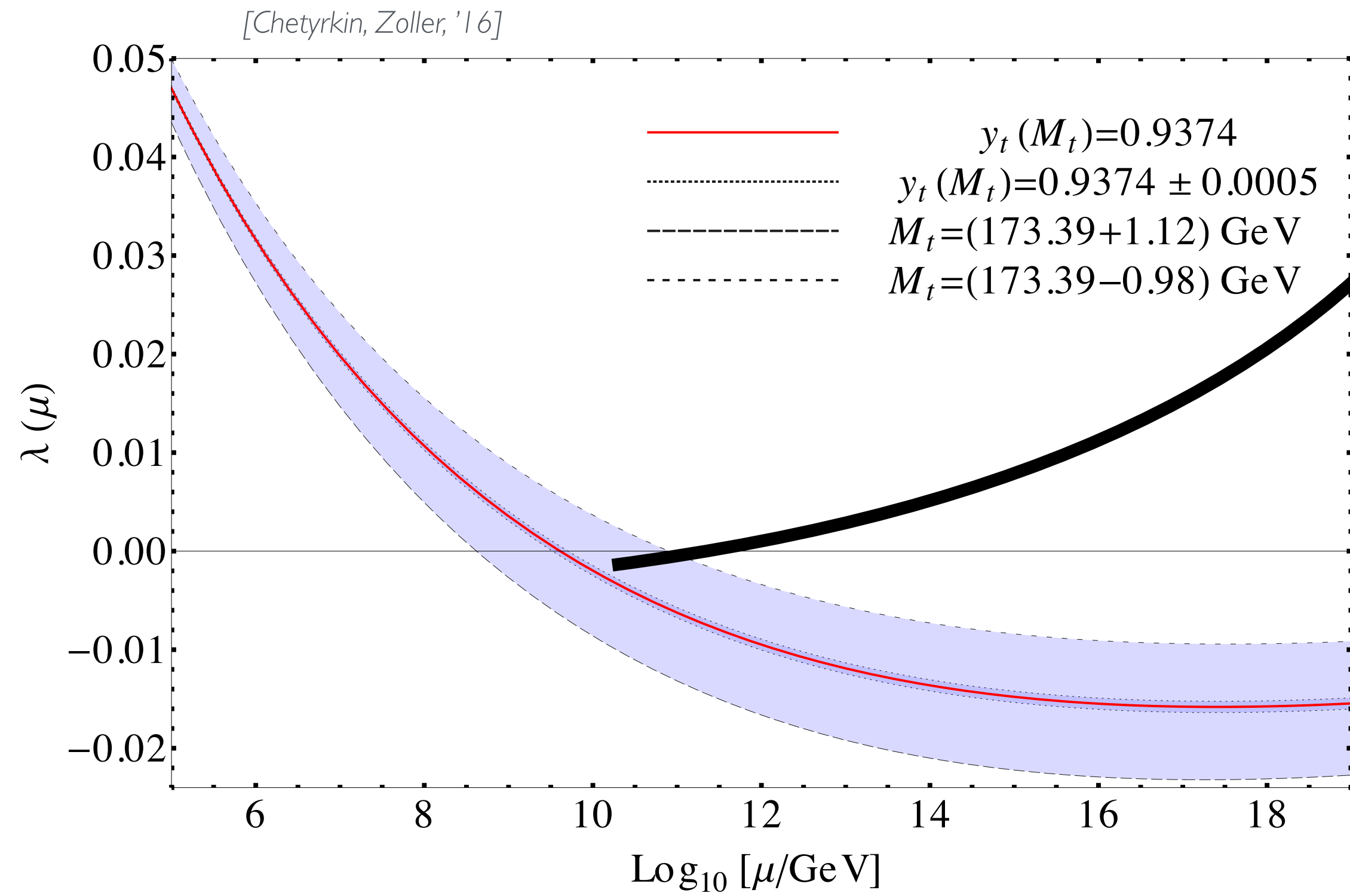
# Running couplings

All couplings run! Incl. yukawas, trilinear



# Running couplings

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \frac{\lambda}{4} (\Phi^\dagger \Phi)^2$$



➡ Precise knowledge of top mass and  $\alpha_s$  crucial!

# The need for precision

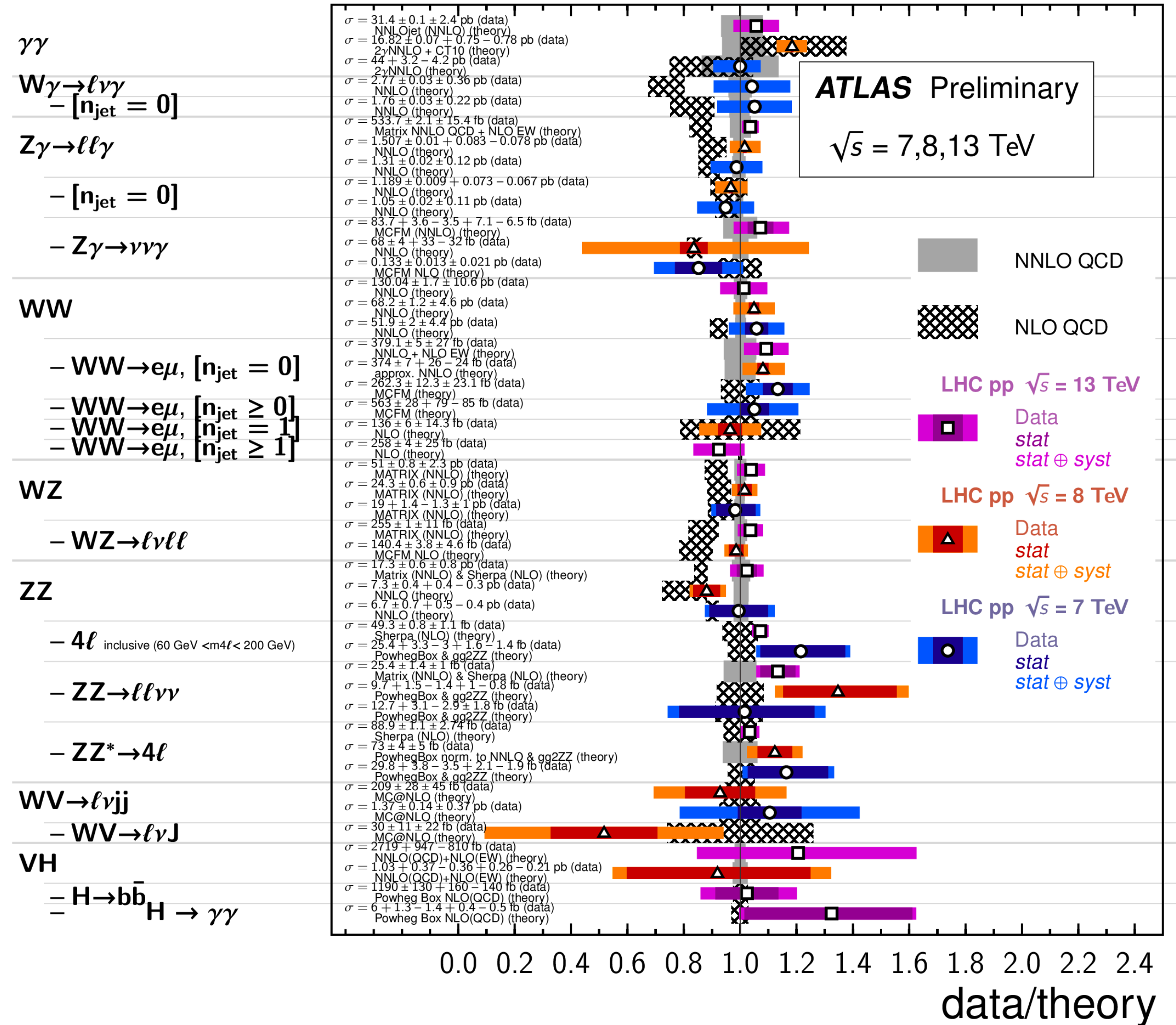
ATLAS Standard Model Summary Plots February 2022

$$\begin{aligned}
 d\sigma &= d\sigma_{\text{LO}} + \alpha_S d\sigma_{\text{NLO}} \\
 &\quad \text{NLO QCD} \quad O(100\%) \\
 &+ \alpha_S^2 d\sigma_{\text{NNLO}} \\
 &\quad \text{NNLO QCD} \quad O(10\%) \\
 &+ \alpha_S^3 d\sigma_{\text{NNLO}} + \dots \\
 &\quad \text{N3LO QCD} \quad O(1\%)
 \end{aligned}$$

$$\alpha_S \sim 0.1$$

## Diboson Cross Section Measurements

Status: February 2022



➔ Higher-order predictions mandatory for reliable predictions



# The need for precision

$$\begin{aligned} d\sigma = & d\sigma_{\text{LO}} + \alpha_S d\sigma_{\text{NLO}} + \alpha_{\text{EW}} d\sigma_{\text{NLO EW}} \\ & \text{NLO QCD} \qquad \text{NLO EW} \\ & + \alpha_S^2 d\sigma_{\text{NNLO}} \\ & \text{NNLO QCD} \\ & + \alpha_S^3 d\sigma_{\text{NNLO}} + \dots \\ & \text{N3LO QCD} \end{aligned}$$

$$\alpha_S \sim 0.1$$

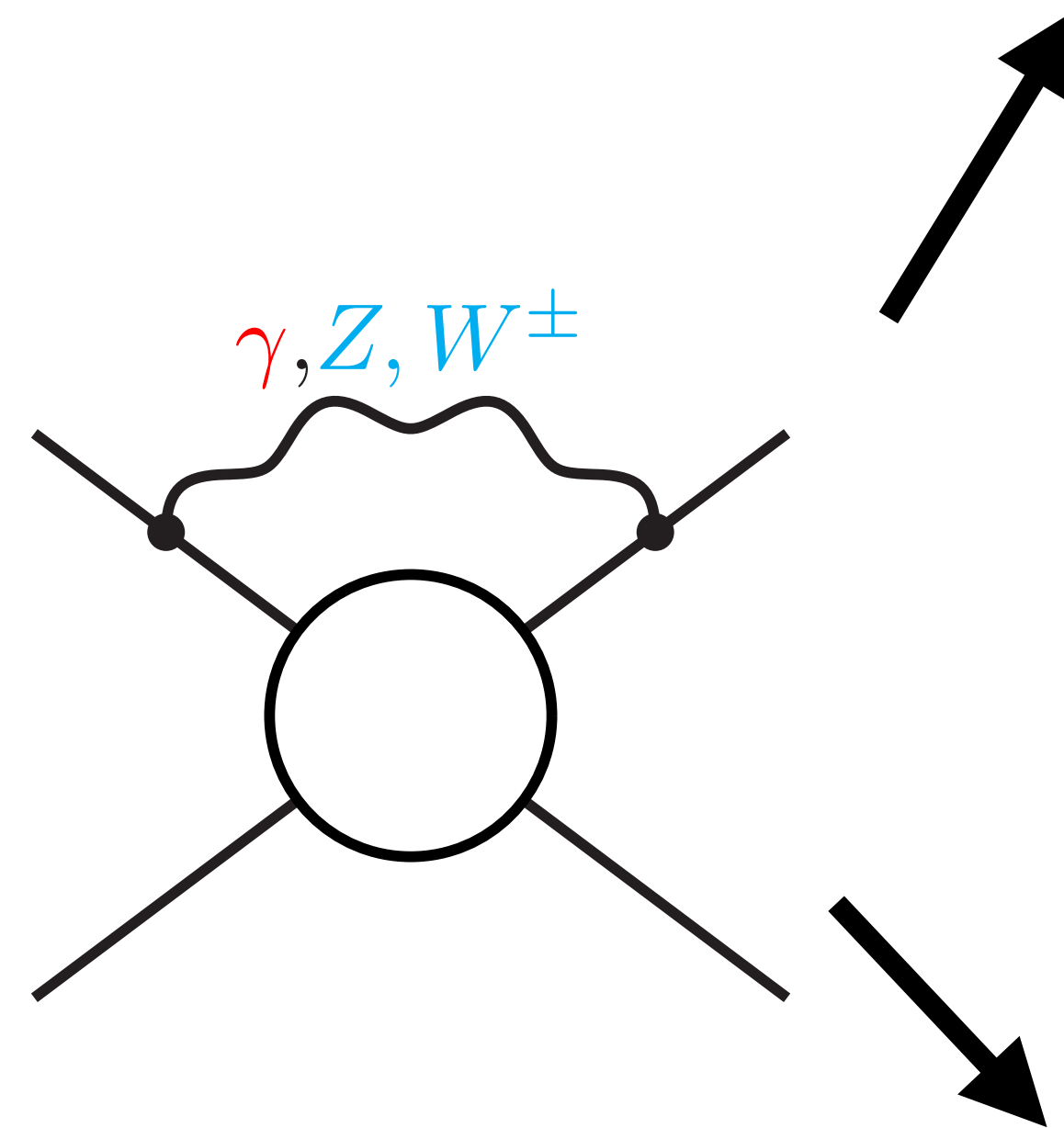
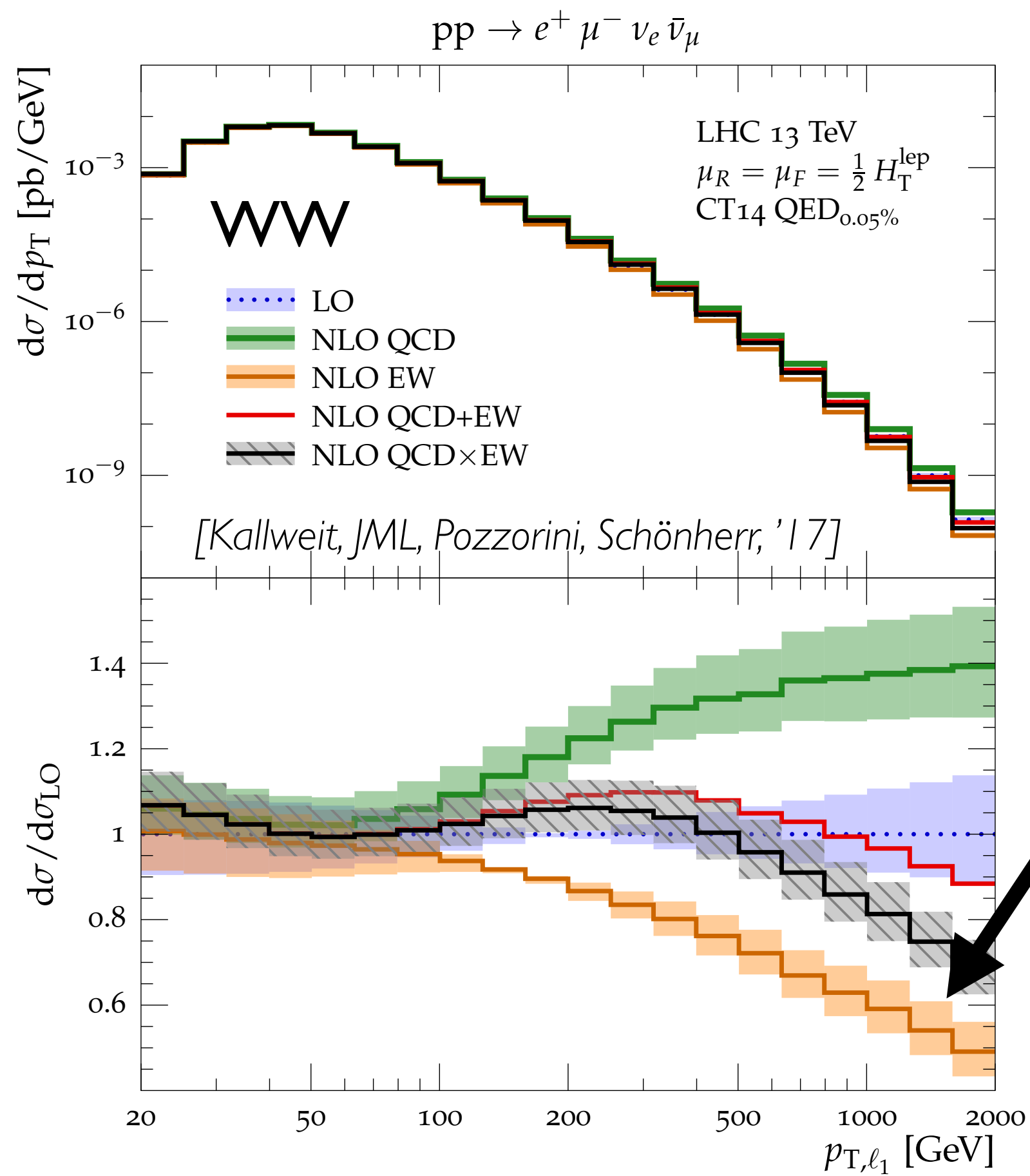
$$\alpha_{\text{EW}} \sim 0.01$$

$$\mathcal{O}(\alpha) \sim \mathcal{O}(\alpha_S^2) \Rightarrow \text{NLO EW} \sim \text{NNLO QCD}$$

➔ Higher-order predictions mandatory for reliable predictions

# Relevance of EW higher-order corrections: **virtual** Sudakov logs in the tails

I. Possible large (negative) enhancement due to soft/collinear **logs** from virtual EW gauge bosons:



[Ciafaloni, Comelli, '98;  
 Lipatov, Fadin, Martin, Melles, '99;  
 Kuehen, Penin, Smirnov, '99;  
 Denner, Pozzorini, '00]

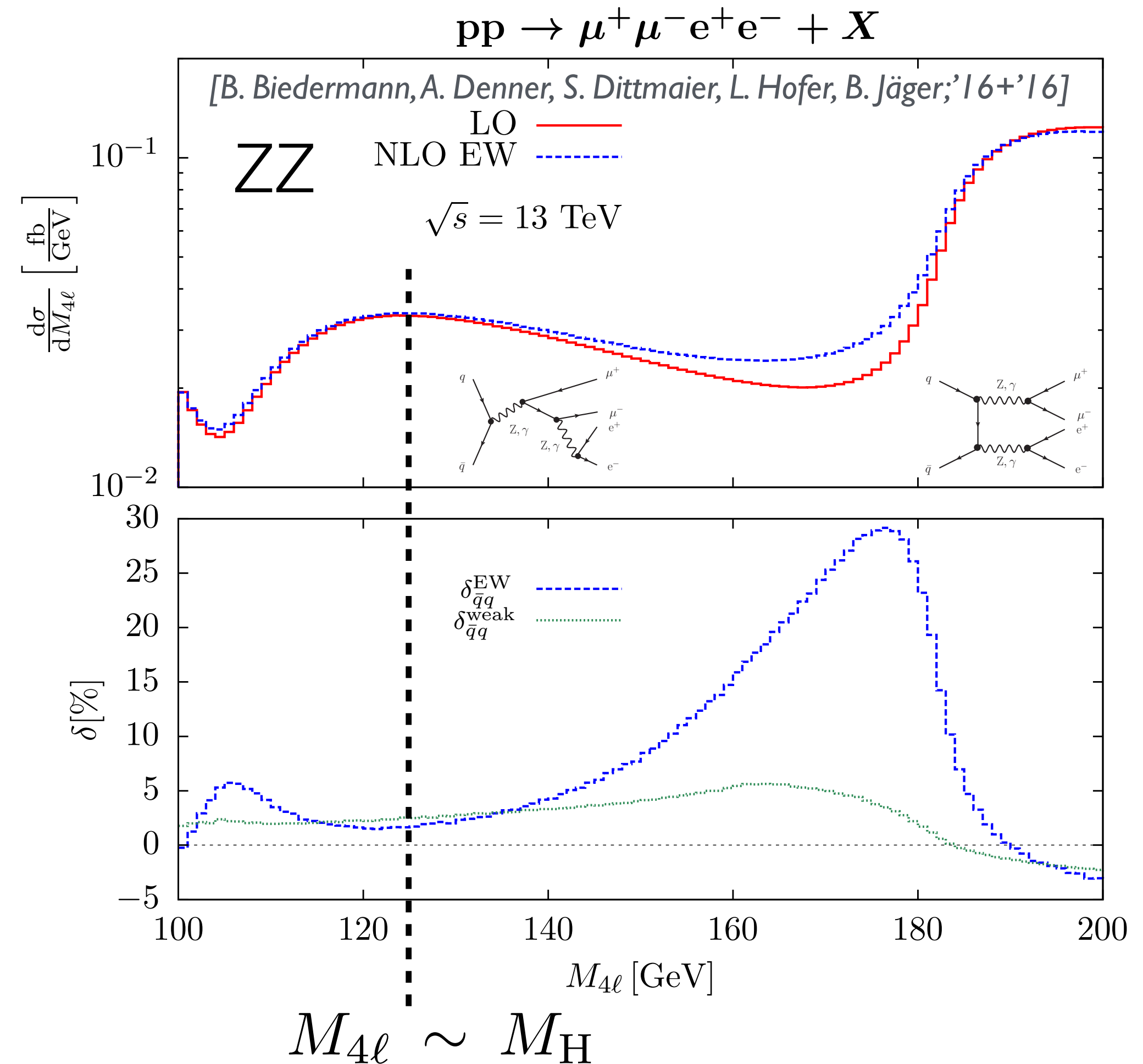
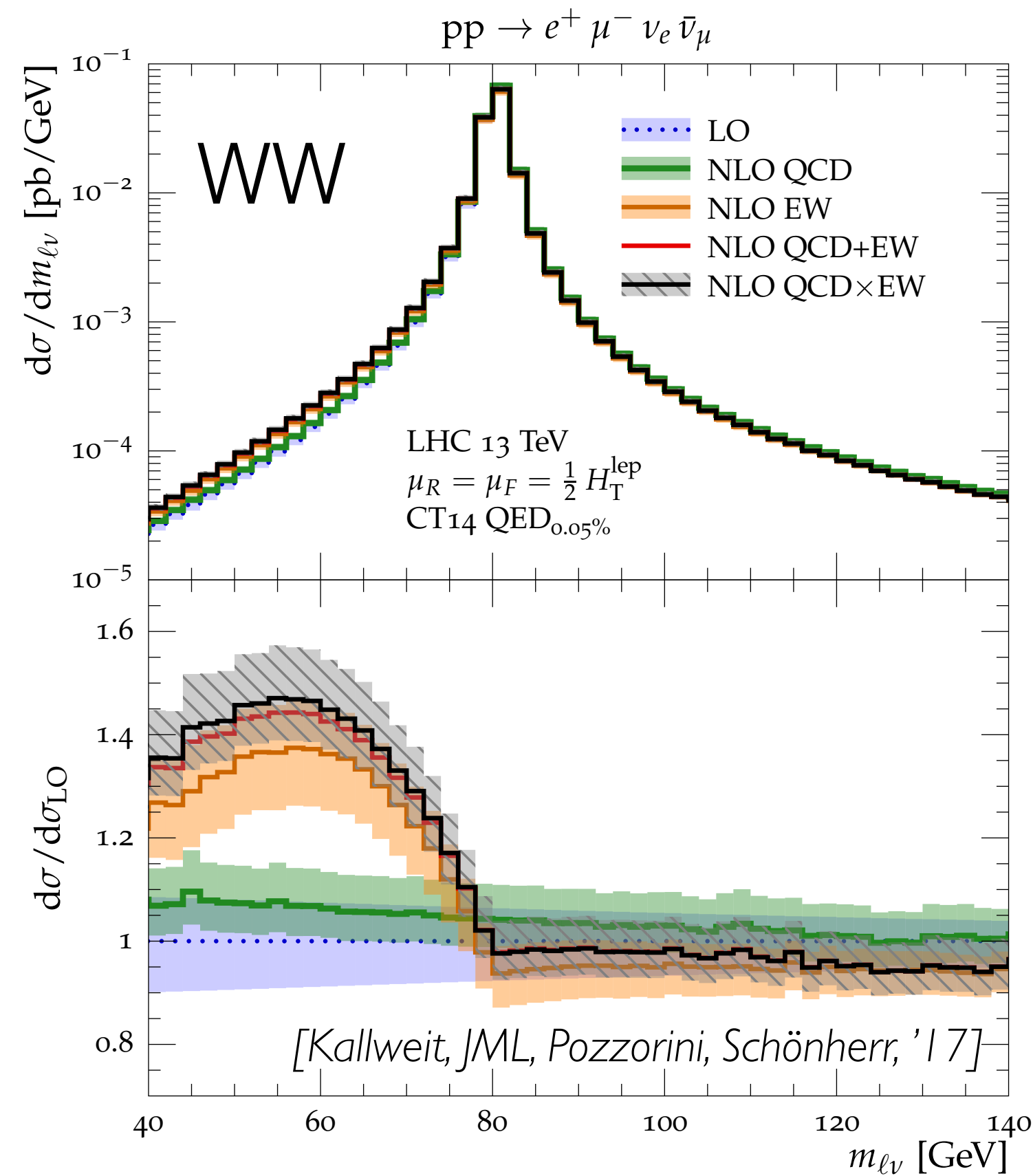
Universality and factorisation: [Denner, Pozzorini, '01]

$$\delta \mathcal{M}_{\text{LL+NLL}}^{1\text{-loop}} = \frac{\alpha}{4\pi} \sum_{k=1}^n \left\{ \frac{1}{2} \sum_{l \neq k} \sum_{a=\gamma, Z, W^\pm} I^a(k) I^{\bar{a}}(l) \ln^2 \frac{\hat{s}_{kl}}{M^2} + \gamma^{\text{ew}}(k) \ln \frac{\hat{s}}{M^2} \right\} \mathcal{M}_0$$

→ overall large (negative) effect in the tails of distributions:  
 $p_T, m_{\text{inv}}, H_T, \dots$  (relevant for BSM searches!)

# Relevance of EW higher-order corrections: collinear QED radiation

- II. Possible large enhancement due to soft/collinear **logs** from photon radiation  $\sim \alpha \log\left(\frac{m_f^2}{Q^2}\right)$  in sufficiently exclusive observables.



→ important for radiative tails, Higgs backgrounds etc.

→ typically considered via QED PS (PHOTOS / YFS)

# Relevance of EW higher-order corrections: photon-induced channels

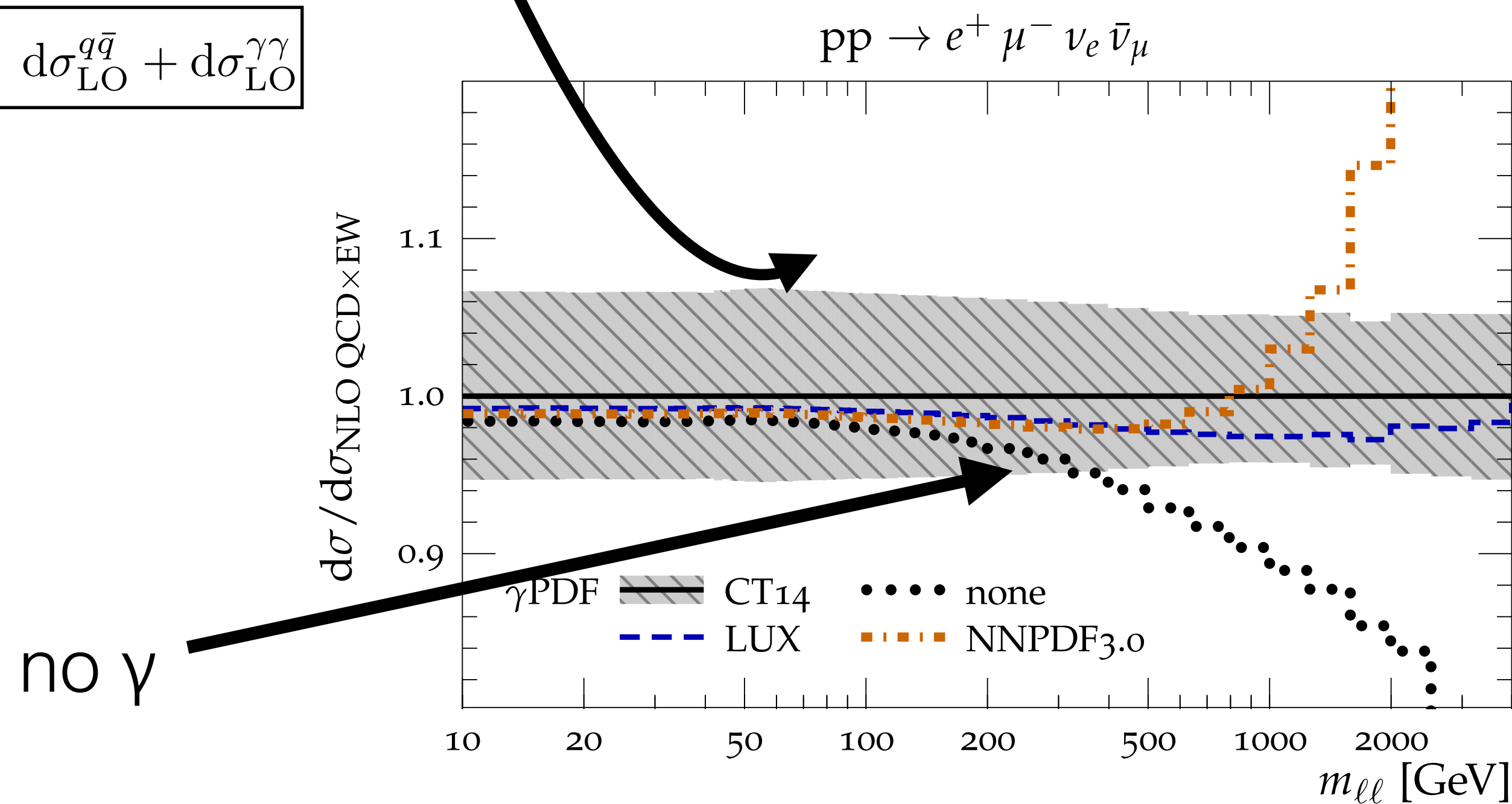
III. QED factorisation and thus photon luminosities needed to absorb IS photon singularities.

→ Possible large enhancement due to photon-induced channels in the tails of kinematic distributions,

in particular in WW:



$$d\sigma_{\text{LO}} = d\sigma_{\text{LO}}^{q\bar{q}} + d\sigma_{\text{LO}}^{\gamma\gamma}$$



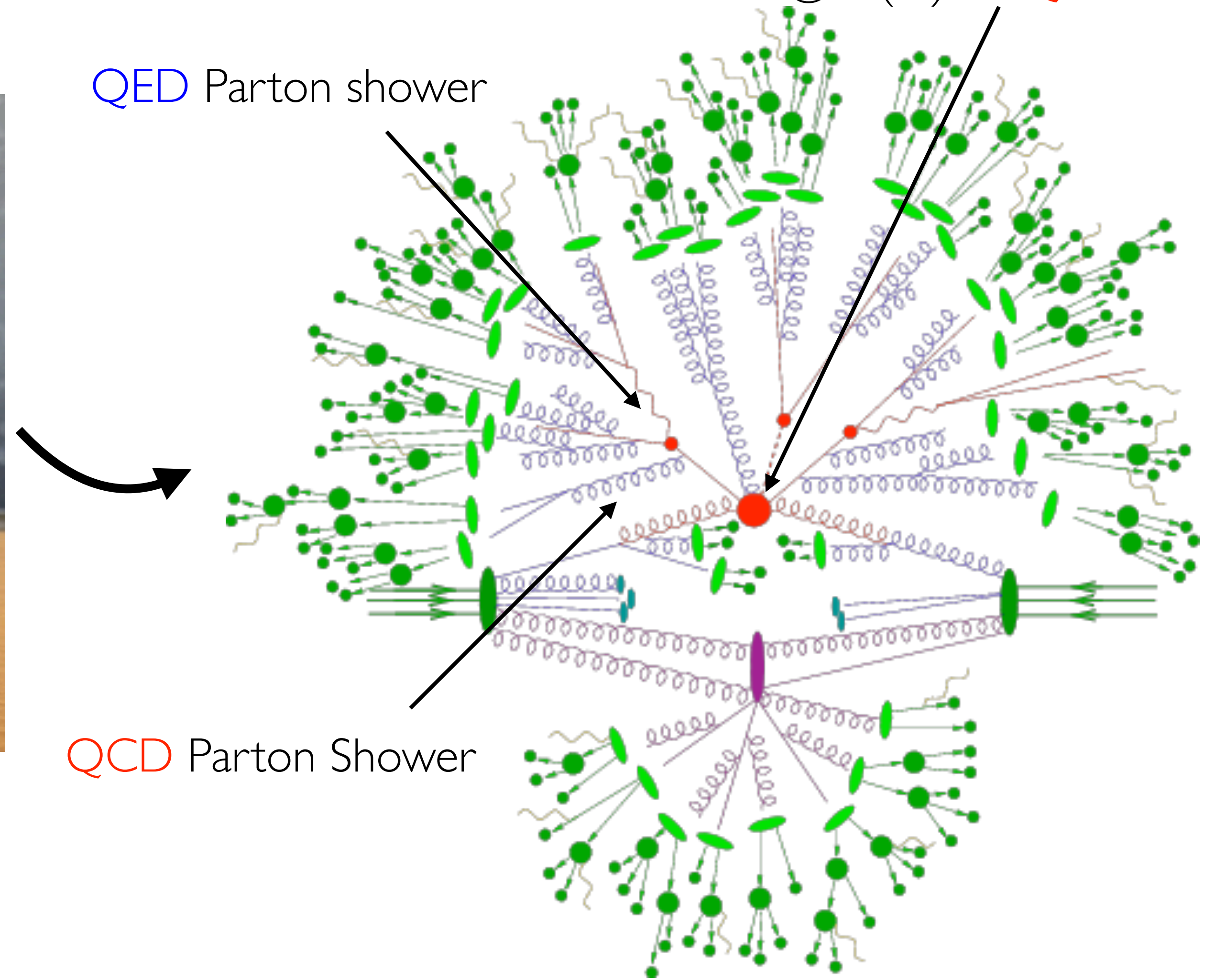
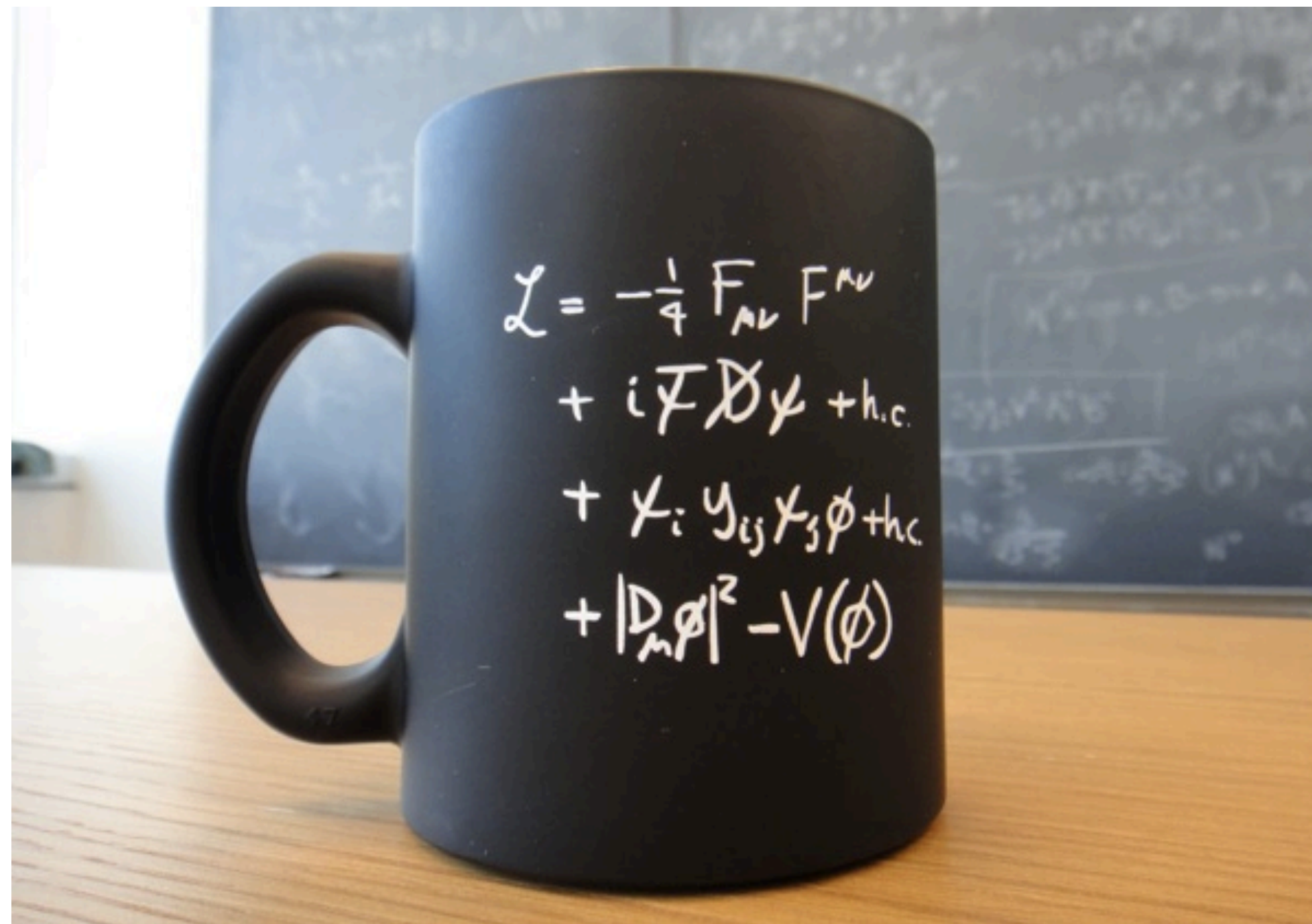
→ ask Gavin!

→ large differences between different photon descriptions. Now settled: LUXqed superior

→ up to O(10%) contributions from photon-induced channels

# EW Theoretical Predictions for the LHC

Hard (perturbative)  
scattering process  
@ N(N)LO QCD + EW



# The EW SM at quantum level in a nutshell

$$\mathcal{L}_{\text{SM}}^{\text{classical}} = \mathcal{L}_{\text{YM}} + \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}$$

At quantum level:

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{SM}}^{\text{classical}} + \mathcal{L}_{\text{gauge-fix}} + \mathcal{L}_{\text{ghost}}$$

(unitary gauge unfeasible at higher-orders in EW)

$$\mathcal{L}_{\text{gauge-fix}} = -\frac{1}{2} (F_A^2 + F_Z^2 + 2F_+ F_- + F_{G^a}^2),$$

$$\mathcal{L}_{\text{ghost}} = \bar{u}^\alpha(x) \frac{\delta F^\alpha}{\delta \theta^\beta(x)} u^\beta(x)$$

$$F_A = \frac{1}{\xi^A} \partial^\mu A_\mu,$$

$$F_{G^a} = \frac{1}{\xi^G} \partial^\mu G_\mu^a,$$

$$F_Z = \frac{1}{\xi^Z} (\partial^\mu Z_\mu^0 - m_Z \xi^Z \chi^0),$$

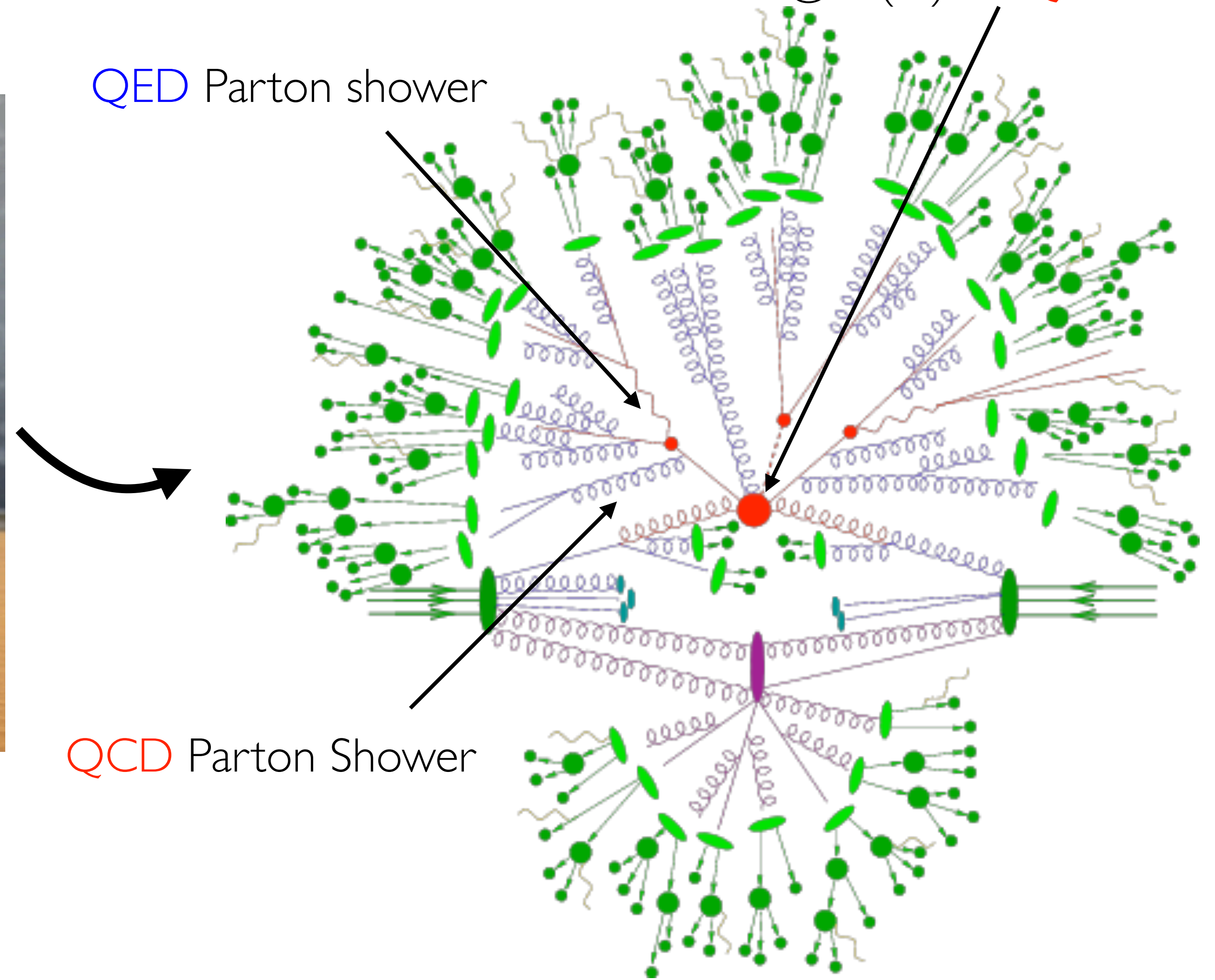
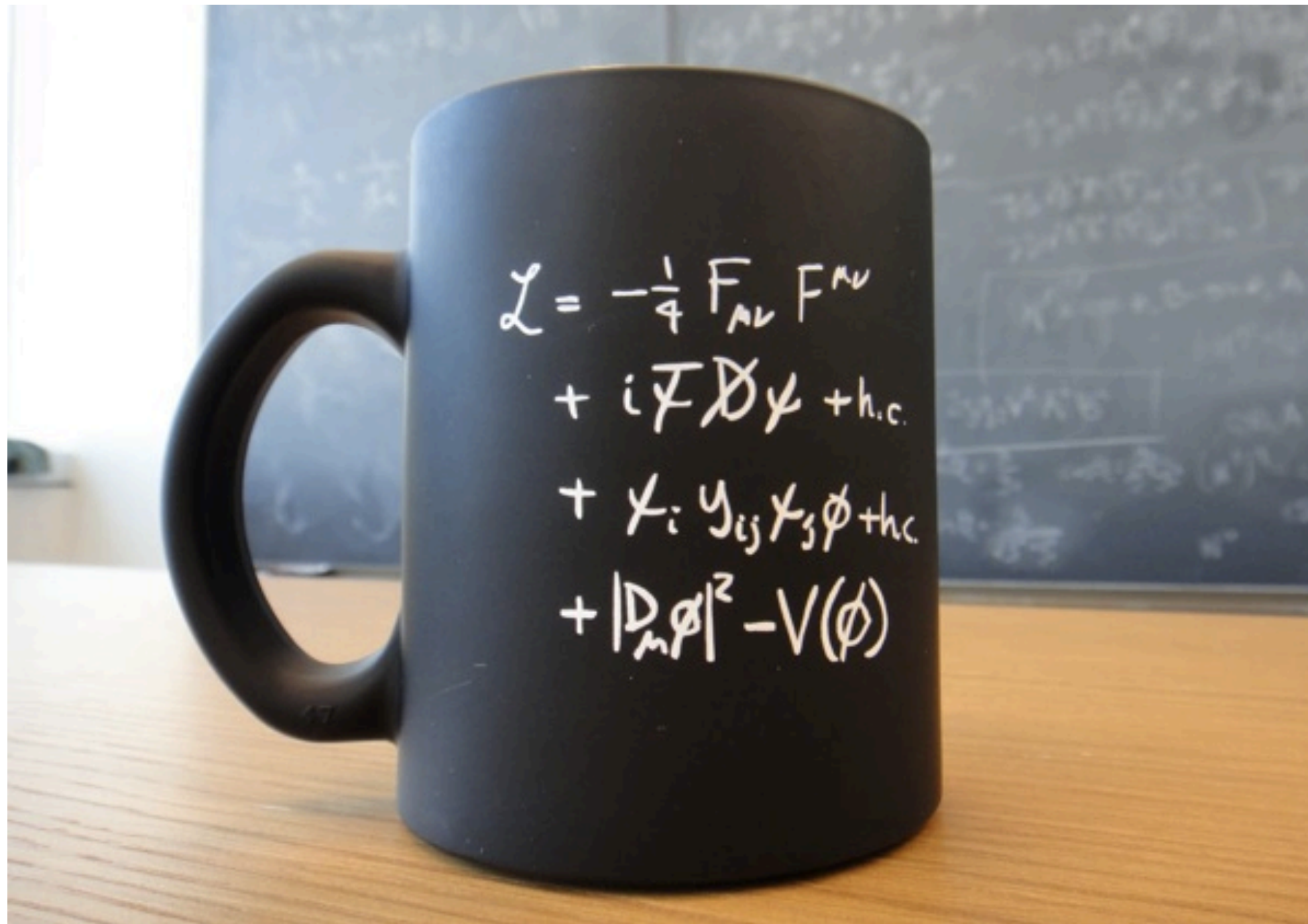
$$F_\pm = \frac{1}{\xi^W} (\partial^\mu W_\mu^\pm \mp im_W \xi^W \phi^\pm)$$

Gauge fixing parameter



# EW Theoretical Predictions for the LHC

Hard (perturbative)  
scattering process  
@ N(N)LO QCD + EW



# EW Theoretical Predictions for the LHC

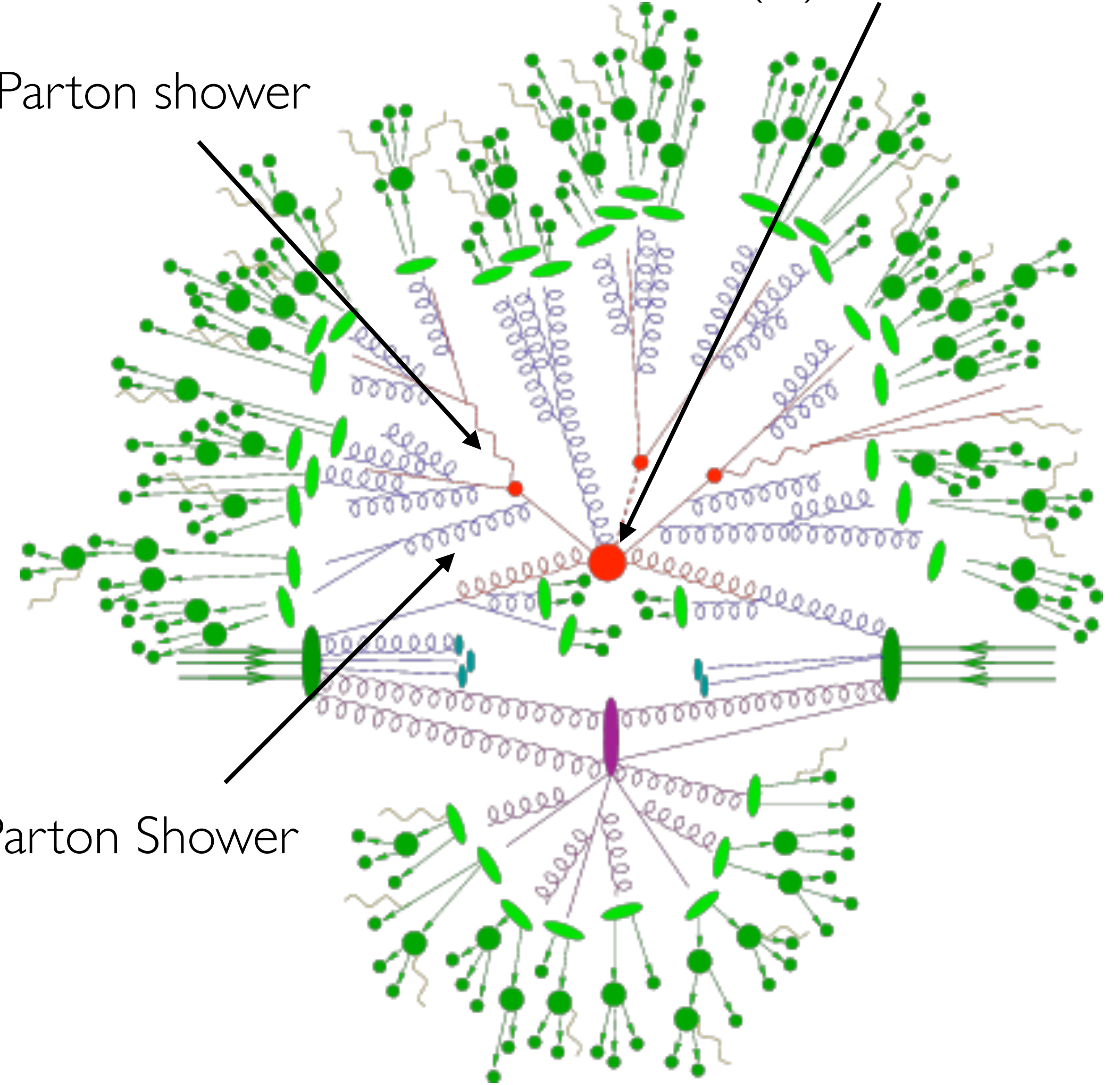
Hard (perturbative) scattering process  
@ N(N)LO QCD + EW

$$\begin{aligned}
 \mathcal{L}_{SM} = & -\frac{1}{2}\partial_\nu g_\mu^a \partial_\nu g_\mu^a - g_s f^{abc} \partial_\mu g_\nu^a g_\mu^b g_\nu^c - \frac{1}{4}g_s^2 f^{abc} f^{ade} g_\mu^b g_\nu^c g_\mu^d g_\nu^e - \partial_\nu W_\mu^+ \partial_\nu W_\mu^- \\
 & - M^2 W_\mu^+ W_\mu^- - \frac{1}{2}\partial_\nu Z_\mu^0 \partial_\nu Z_\mu^0 - \frac{1}{2c_w^2} M^2 Z_\mu^0 Z_\mu^0 - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - igc_w (\partial_\nu Z_\mu^0 (W_\mu^+ W_\nu^- - \\
 & W_\nu^+ W_\mu^-) - Z_\nu^0 (W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+) + Z_\nu^0 (W_\nu^+ \partial_\mu W_\mu^- - W_\mu^- \partial_\nu W_\nu^+)) - \\
 & ig s_w (\partial_\nu A_\mu (W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - A_\nu (W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+) + A_\nu (W_\nu^+ \partial_\mu W_\mu^- - \\
 & W_\mu^- \partial_\nu W_\nu^+)) - \frac{1}{2}g^2 W_\mu^+ W_\nu^- W_\nu^+ W_\mu^- + \frac{1}{2}g^2 W_\mu^+ W_\nu^- W_\mu^+ W_\nu^- + g^2 c_w^2 (Z_\mu^0 W_\nu^+ Z_\nu^0 W_\mu^- - \\
 & Z_\nu^0 Z_\mu^0 W_\nu^+ W_\mu^-) + g^2 s_w^2 (A_\mu W_\nu^+ A_\nu W_\mu^- - A_\mu A_\nu W_\nu^+ W_\mu^-) + g^2 s_w c_w (A_\mu Z_\nu^0 (W_\mu^+ W_\nu^- - \\
 & W_\nu^+ W_\mu^-) - 2A_\mu Z_\mu^0 W_\nu^+ W_\nu^-) - \frac{1}{2}\partial_\mu H \partial_\mu H - 2M^2 \alpha_h H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 \\
 & \beta_h \left( \frac{2M_h^2}{g^2} + \frac{2M_h}{g} H + \frac{1}{2}(H^2 + \phi^0 \phi^0 + 2\phi^+ \phi^-) \right) + \frac{2M_h^4}{g^4} \alpha_h - \\
 & g \alpha_h M (H^3 + H \phi^0 \phi^0 + 2H \phi^+ \phi^-) - \\
 & \frac{1}{8}g^2 \alpha_h (H^4 + (\phi^0)^4 + 4(\phi^+ \phi^-)^2 + 4(\phi^0)^2 \phi^+ \phi^- + 4H^2 \phi^+ \phi^- + 2(\phi^0)^2 H^2) - \\
 & g M W_\mu^+ W_\mu^- H - \frac{1}{2}g \frac{M}{c_w^2} Z_\mu^0 Z_\mu^0 H - \\
 & \frac{1}{2}ig (W_\mu^+ (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - W_\mu^- (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)) + \\
 & \frac{1}{2}g (W_\mu^+ (H \partial_\mu \phi^- - \phi^- \partial_\mu H) + W_\mu^- (H \partial_\mu \phi^+ - \phi^+ \partial_\mu H)) + \frac{1}{2}g \frac{1}{c_w} (Z_\mu^0 (H \partial_\mu \phi^0 - \phi^0 \partial_\mu H) + \\
 & M (\frac{1}{c_w} Z_\mu^0 \partial_\mu \phi^0 + W_\mu^+ \partial_\mu \phi^- + W_\mu^- \partial_\mu \phi^+)) - ig \frac{s_w^2}{c_w} M Z_\mu^0 (W_\mu^+ \phi^- - W_\mu^- \phi^+) + ig s_w M A_\mu (W_\mu^+ \phi^- - \\
 & W_\mu^- \phi^+) - ig \frac{1-2c_w^2}{2c_w} Z_\mu^0 (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) + ig s_w A_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - \\
 & \frac{1}{4}g^2 W_\mu^+ W_\mu^- (H^2 + (\phi^0)^2 + 2\phi^+ \phi^-) - \frac{1}{8}g^2 \frac{1}{c_w^2} Z_\mu^0 Z_\mu^0 (H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2 \phi^+ \phi^-) - \\
 & \frac{1}{2}g^2 \frac{s_w^2}{c_w} Z_\mu^0 \phi^0 (W_\mu^+ \phi^- + W_\mu^- \phi^+) - \frac{1}{2}ig^2 \frac{2s_w^2}{c_w} Z_\mu^0 H (W_\mu^+ \phi^- - W_\mu^- \phi^+) + \frac{1}{2}g^2 s_w A_\mu \phi^0 (W_\mu^+ \phi^- + \\
 & W_\mu^- \phi^+) + \frac{1}{2}ig^2 s_w A_\mu H (W_\mu^+ \phi^- - W_\mu^- \phi^+) - g^2 \frac{2s_w}{c_w} (2c_w^2 - 1) Z_\mu^0 A_\mu \phi^+ \phi^- - \\
 & g^2 s_w^2 A_\mu A_\mu \phi^+ \phi^- + \frac{1}{2}ig_s \lambda_{ij}^a (\bar{q}_i^\alpha \gamma^\mu q_j^\alpha) g_\mu^a - \bar{e}^\lambda (\gamma \partial + m_e) e^\lambda - \bar{\nu}^\lambda (\gamma \partial + m_\nu) \nu^\lambda - \bar{u}_j^\lambda (\gamma \partial + \\
 & m_u) u_j^\lambda - \bar{d}_j^\lambda (\gamma \partial + m_d) d_j^\lambda + ig s_w A_\mu (-\bar{e}^\lambda \gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}_j^\lambda \gamma^\mu u_j^\lambda) - \frac{1}{3}(\bar{d}_j^\lambda \gamma^\mu d_j^\lambda) + \\
 & \frac{ig}{4c_w} Z_\mu^0 \{ (\bar{\nu}^\lambda \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{e}^\lambda \gamma^\mu (4s_w^2 - 1 - \gamma^5) e^\lambda) + (\bar{d}_j^\lambda \gamma^\mu (\frac{4}{3}s_w^2 - 1 - \gamma^5) d_j^\lambda) + \\
 & (\bar{u}_j^\lambda \gamma^\mu (1 - \frac{8}{3}s_w^2 + \gamma^5) u_j^\lambda) \} + \frac{ig}{2\sqrt{2}} W_\mu^+ ((\bar{\nu}^\lambda \gamma^\mu (1 + \gamma^5) U^{lep}{}_{\lambda\kappa} e^\kappa) + (\bar{u}_j^\lambda \gamma^\mu (1 + \gamma^5) C_{\lambda\kappa} d_j^\kappa)) + \\
 & \frac{ig}{2\sqrt{2}} W_\mu^- ((\bar{e}^\kappa U^{lep}{}_{\kappa\lambda} \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{d}_j^\kappa C_{\kappa\lambda}^\dagger \gamma^\mu (1 + \gamma^5) u_j^\lambda)) + \\
 & \frac{ig}{2M\sqrt{2}} \phi^+ (-m_e^\kappa (\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa} (1 - \gamma^5) e^\kappa) + m_\nu^\kappa (\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa} (1 + \gamma^5) e^\kappa) + \\
 & \frac{ig}{2M\sqrt{2}} \phi^- (m_e^\lambda (\bar{e}^\lambda U^{lep}{}_{\lambda\kappa}^\dagger (1 + \gamma^5) \nu^\kappa) - m_\nu^\kappa (\bar{e}^\lambda U^{lep}{}_{\lambda\kappa}^\dagger (1 - \gamma^5) \nu^\kappa)) - \frac{g}{2} \frac{m_\lambda}{M} H (\bar{\nu}^\lambda \nu^\lambda) - \\
 & \frac{g}{2} \frac{m_\lambda}{M} H (\bar{e}^\lambda e^\lambda) + \frac{ig}{2} \frac{m_\lambda}{M} \phi^0 (\bar{\nu}^\lambda \gamma^5 \nu^\lambda) - \frac{ig}{2} \frac{m_\lambda}{M} \phi^0 (\bar{e}^\lambda \gamma^5 e^\lambda) - \frac{1}{4} \bar{\nu}_\lambda M_{\lambda\kappa}^R (1 - \gamma_5) \bar{\nu}_\kappa - \\
 & \frac{1}{4} \bar{\nu}_\lambda M_{\lambda\kappa}^R (1 - \gamma_5) \bar{\nu}_\kappa + \frac{ig}{2M\sqrt{2}} \phi^+ (-m_d^\lambda (\bar{u}_j^\lambda C_{\lambda\kappa} (1 - \gamma^5) d_j^\kappa) + m_u^\lambda (\bar{u}_j^\lambda C_{\lambda\kappa} (1 + \gamma^5) d_j^\kappa)) + \\
 & \frac{ig}{2M\sqrt{2}} \phi^- (m_d^\lambda (\bar{d}_j^\lambda C_{\lambda\kappa}^\dagger (1 + \gamma^5) u_j^\kappa) - m_u^\kappa (\bar{d}_j^\lambda C_{\lambda\kappa}^\dagger (1 - \gamma^5) u_j^\kappa)) - \frac{g}{2} \frac{m_\lambda}{M} H (\bar{u}_j^\lambda u_j^\lambda) - \\
 & \frac{g}{2} \frac{m_\lambda}{M} H (\bar{d}_j^\lambda d_j^\lambda) + \frac{ig}{2} \frac{m_\lambda}{M} \phi^0 (\bar{u}_j^\lambda \gamma^5 u_j^\lambda) - \frac{ig}{2} \frac{m_\lambda}{M} \phi^0 (\bar{d}_j^\lambda \gamma^5 d_j^\lambda) + \bar{G}^a \partial^2 G^a + g_s f^{abc} \partial_\mu \bar{G}^a G^b g_\mu^c + \\
 & \bar{X}^+ (\partial^2 - M^2) X^+ + \bar{X}^- (\partial^2 - M^2) X^- + \bar{X}^0 (\partial^2 - \frac{M^2}{c_w^2}) X^0 + \bar{Y} \partial^2 Y + igc_w W_\mu^+ (\partial_\mu \bar{X}^0 X^- - \\
 & \partial_\mu \bar{X}^+ X^0) + ig s_w W_\mu^+ (\partial_\mu \bar{Y} X^- - \partial_\mu \bar{X}^+ Y) + igc_w W_\mu^- (\partial_\mu \bar{X}^- X^0 - \\
 & \partial_\mu \bar{X}^0 X^+) + ig s_w W_\mu^- (\partial_\mu \bar{X}^- Y - \partial_\mu \bar{Y} X^+) + igc_w Z_\mu^0 (\partial_\mu \bar{X}^- X^+ - \\
 & \partial_\mu \bar{X}^+ X^-) + ig s_w A_\mu (\partial_\mu \bar{X}^+ X^+ - \\
 & \partial_\mu \bar{X}^- X^-) - \frac{1}{2}gM (\bar{X}^+ X^+ H + \bar{X}^- X^- H + \frac{1}{c_w} \bar{X}^0 X^0 H) + \frac{1-2c_w^2}{2c_w} igM (\bar{X}^+ X^0 \phi^+ - \bar{X}^- X^0 \phi^-) + \\
 & \frac{1}{2c_w} igM (\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-) + igM s_w (\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-) + \\
 & \frac{1}{2}igM (\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0) .
 \end{aligned}$$

QED Parton shower



QCD Parton Shower



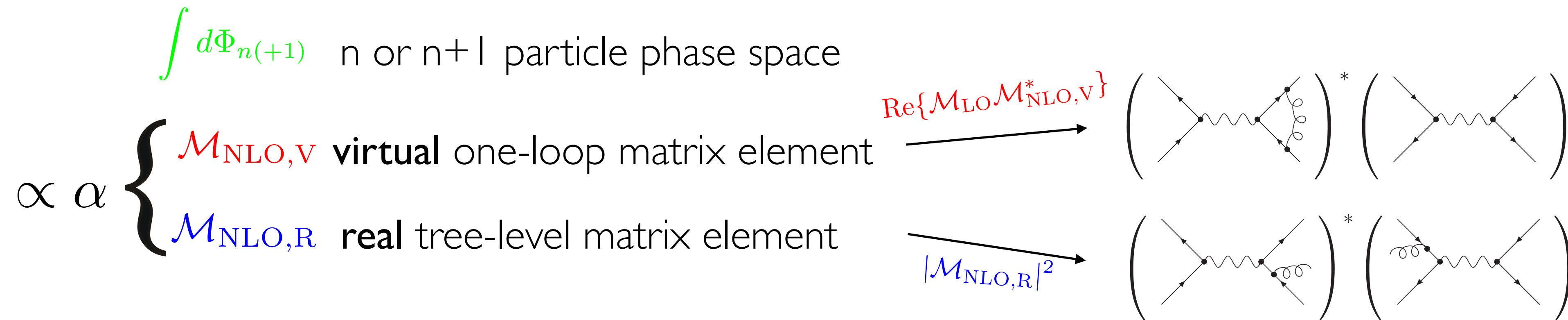


# NLO Ingredients

- NLO partonic cross section for a  $2 \rightarrow n$  process can be written as

$$d\hat{\sigma}_{\text{NLO}} = \frac{1}{2s} \int d\Phi_n [|\mathcal{M}_{\text{LO}}|^2 + 2\text{Re}\{\mathcal{M}_{\text{LO}}\mathcal{M}_{\text{NLO,V}}^*\}] + \frac{1}{2s} \int d\Phi_{n+1} |\mathcal{M}_{\text{NLO,R}}|^2$$

$$\text{NLO} = \text{B} + \text{V} + \text{R}$$



Note: real radiation might open up new partonic channels!

# NLO Tools: automation of NLO EW

- Add local subtraction terms  $S$ , and corresponding integrated subtraction term  $I$

$$d\hat{\sigma}_{\text{NLO}} = \frac{1}{2s} \int d\Phi_n [|\mathcal{M}_{\text{LO}}|^2 + 2\text{Re}\{\mathcal{M}_{\text{LO}}\mathcal{M}_{\text{NLO,V}}^*\} + I] + \frac{1}{2s} \int d\Phi_{n+1} |\mathcal{M}_{\text{NLO,R}}|^2 - S$$

- NLO Monte-Carlo integrators (+subtraction):

- [MadGraph\\_aMC@NLO](#) (FKS)
- [Sherpa](#) (CS)
- [POWHEG-BOX](#) (FKS)

- NLO fixed-order integrators:

- [MUNICH/Matrix](#) (CS)
- ...

- one-loop (& tree) amplitude provider:

- [MadLoop](#) (OpenLoops)
- [GoSam](#) (Unitarity & OPP)
- [OpenLoops](#) (OpenLoops)
- [Recola](#) (NLO Recursion)
- ...

- integral reduction libraries:

- [CutTools](#)
- [Golem95](#)
- [COLLIER](#)
- [Ninja](#)
- ...

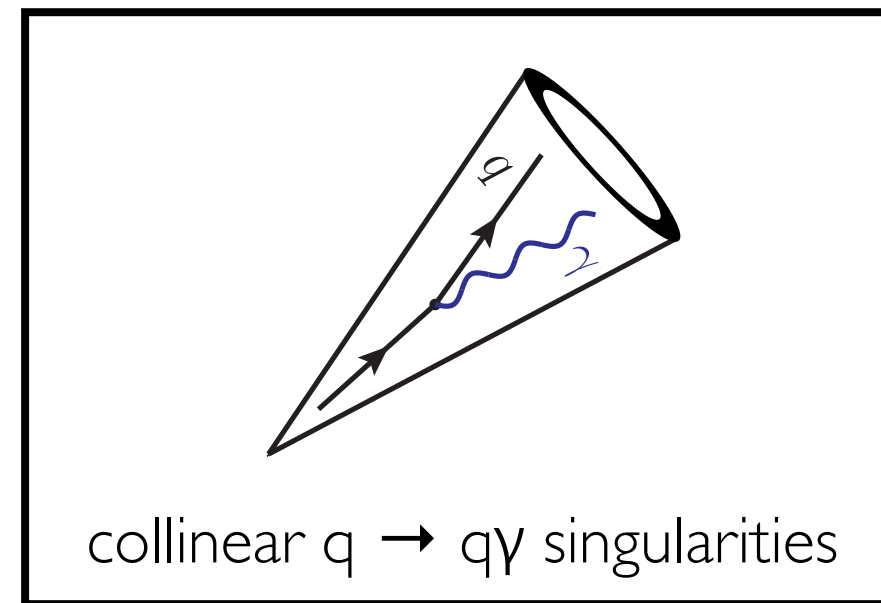
- scalar one-loop libraries

- [QCDLoop](#)
- [OneLoop](#)
- [COLLIER](#)
- ...

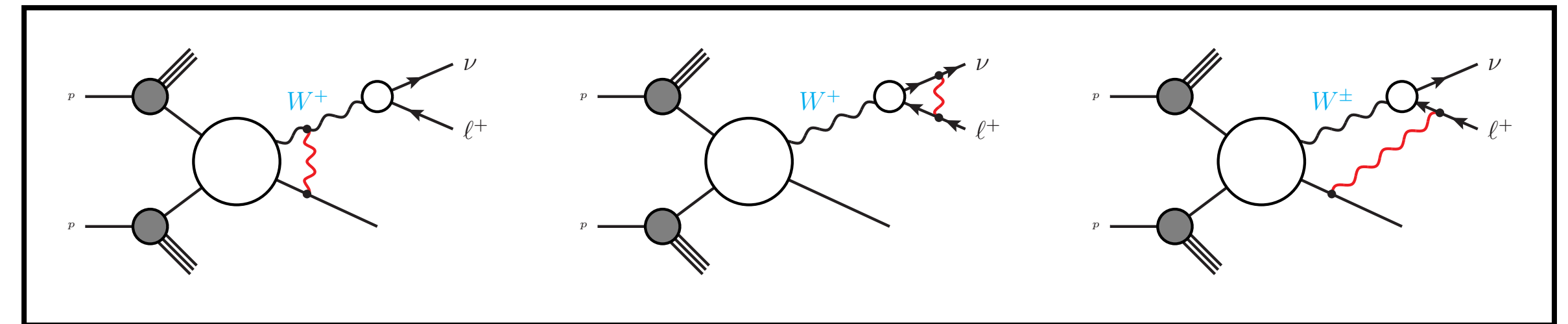


# Nontrivial features in NLO QCD $\rightarrow$ NLO EW

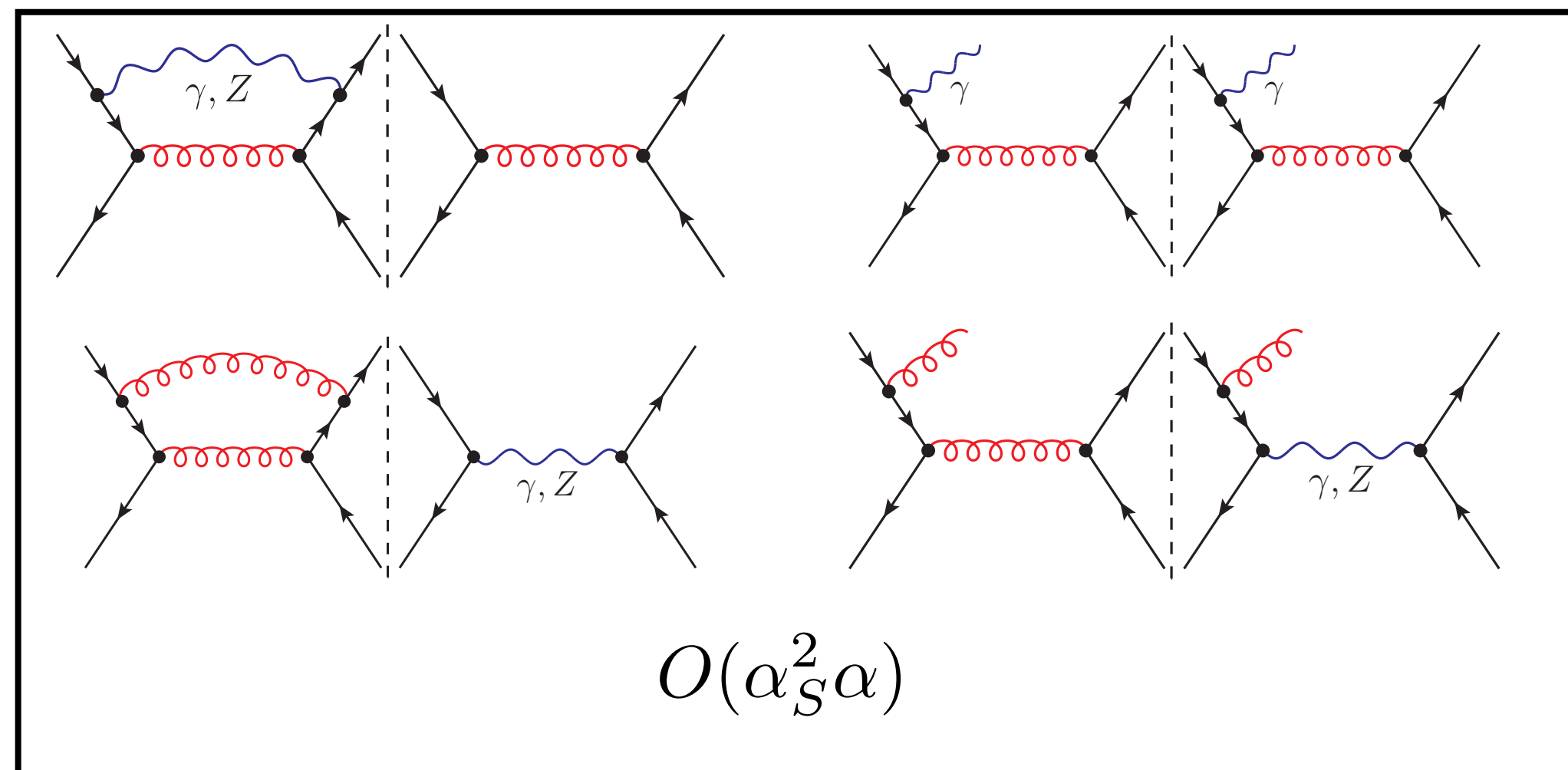
1. photon contributions in jets and proton  
 $\rightarrow$  **photon-jet separation,  $\gamma$ PDF**



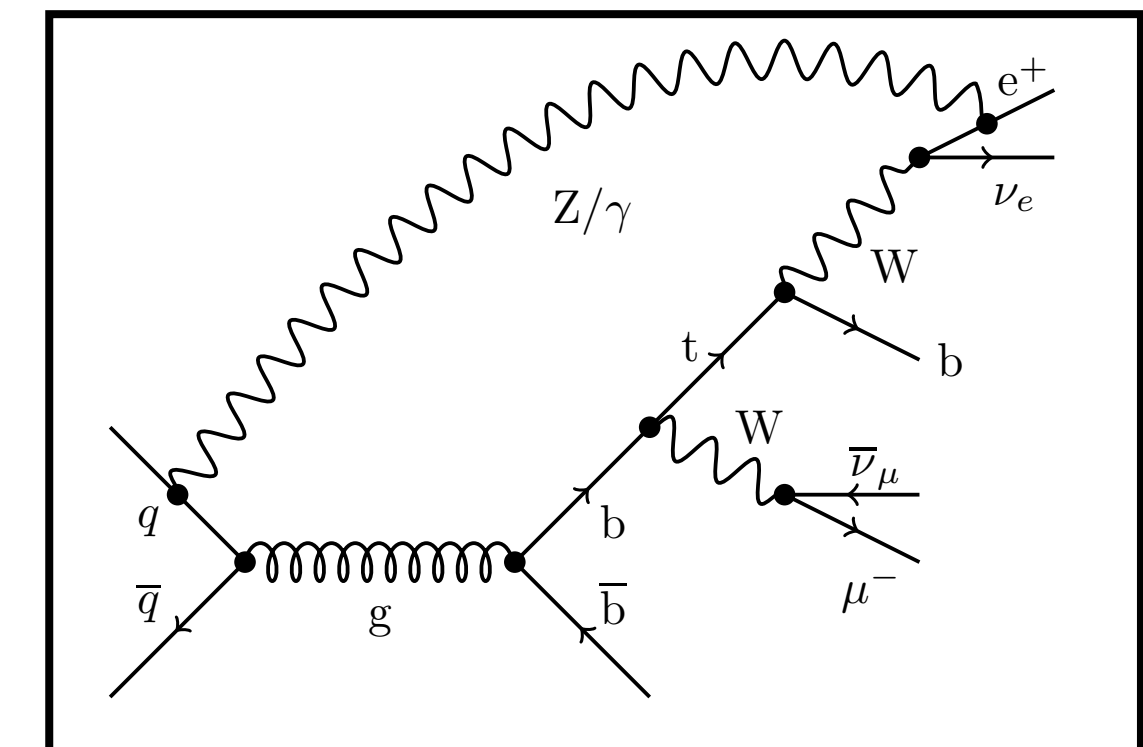
2. At NLO EW corrections in production, decay and non-factorizable contributions for  $V$  decays  
 $\rightarrow$  **complex-mass-scheme**



## 3. QCD-EW interplay

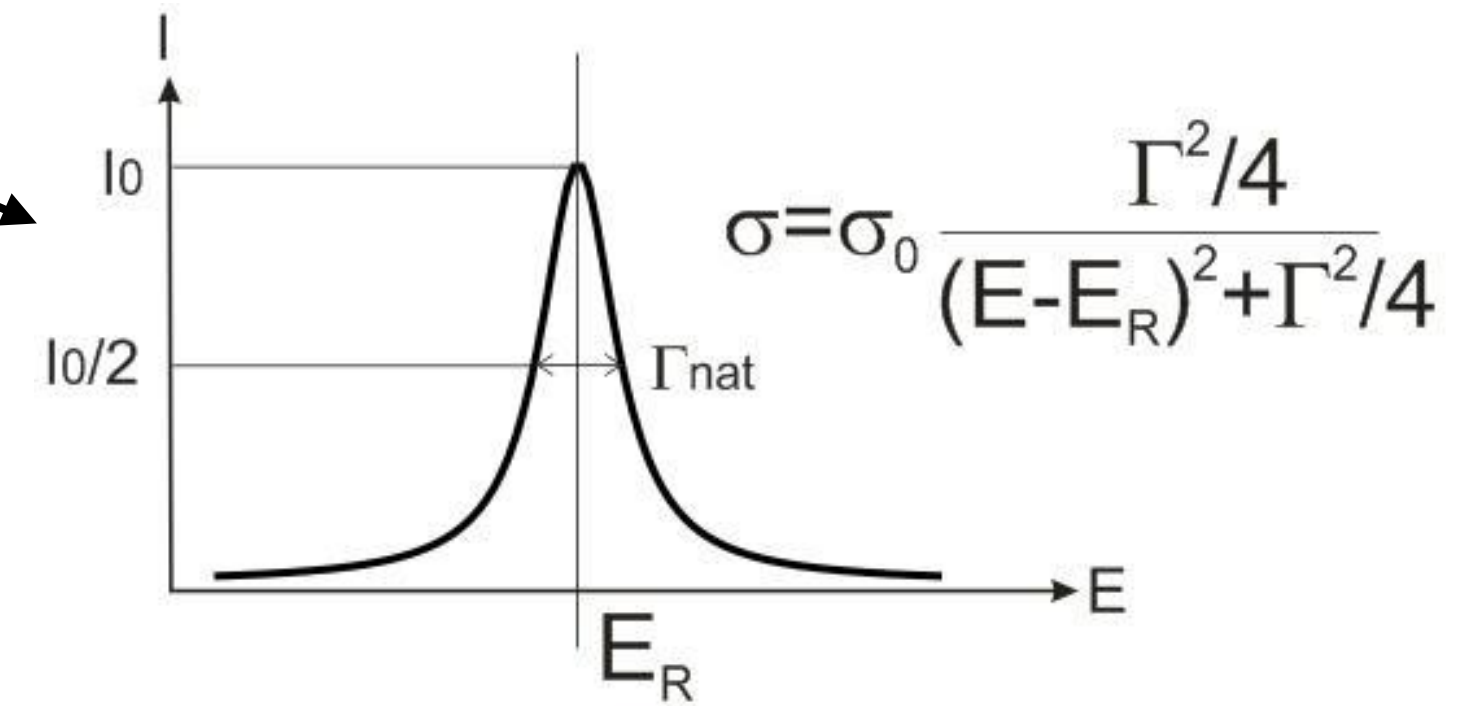
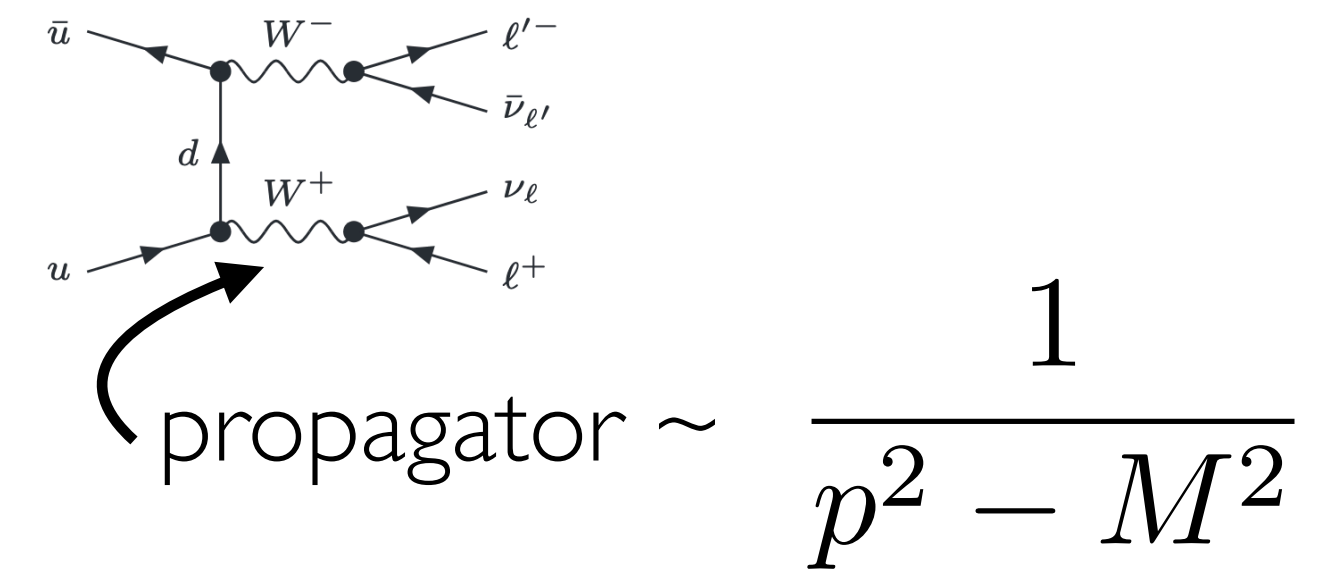


4. **virtual EW corrections** more involved than QCD  
 (many internal masses)



# Decays of heavy particles

- Naively processes with a massive s-channel propagator diverge when  $p^2 = M^2$
- Experimentally we know resonances follow **Breit-Wigner** (BW) shape



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- Naively processes with a massive s-channel propagator diverge when  $p^2 = M^2$
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- origin: all-order summation of 1PI corrections to propagator of massive particles

$$\text{propagator} \sim \frac{1}{p^2 - M^2}$$

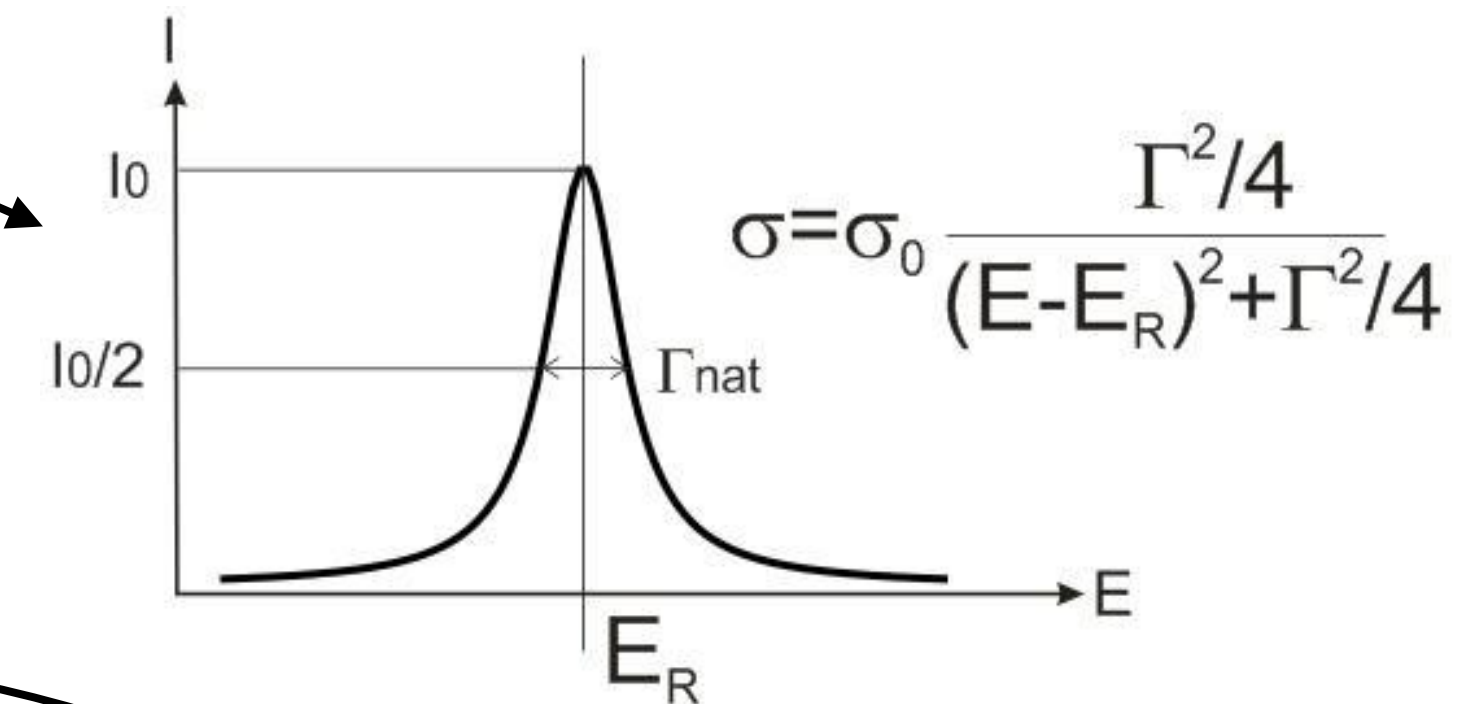
$$\text{propagator} \sim \text{---} + \text{---} \circlearrowleft \text{1PI} \text{---} + \text{---} \circlearrowleft \text{1PI} \circlearrowleft \text{1PI} \text{---} + \dots$$

$$= \frac{1}{p^2 - M_0} + \frac{1}{p^2 - M_0} (-i\Sigma) \frac{1}{p^2 - M_0} + \dots$$

Geometric series

$$= \frac{1}{k^2 - M_0^2} \sum_{n=0}^{\infty} \left( \frac{-\Sigma(k^2)}{k^2 - M_0^2} \right)^n = \frac{1}{k^2 - M_0^2 + \Sigma(k^2)} = \frac{1}{k^2 - M_0^2 - iM_0\Gamma}$$

$$\int dk^2 |M|^2 \sim \int_{-\infty}^{\infty} \frac{dk^2}{(k^2 - m^2)^2 + m^2\Gamma} \sim \text{BW}$$



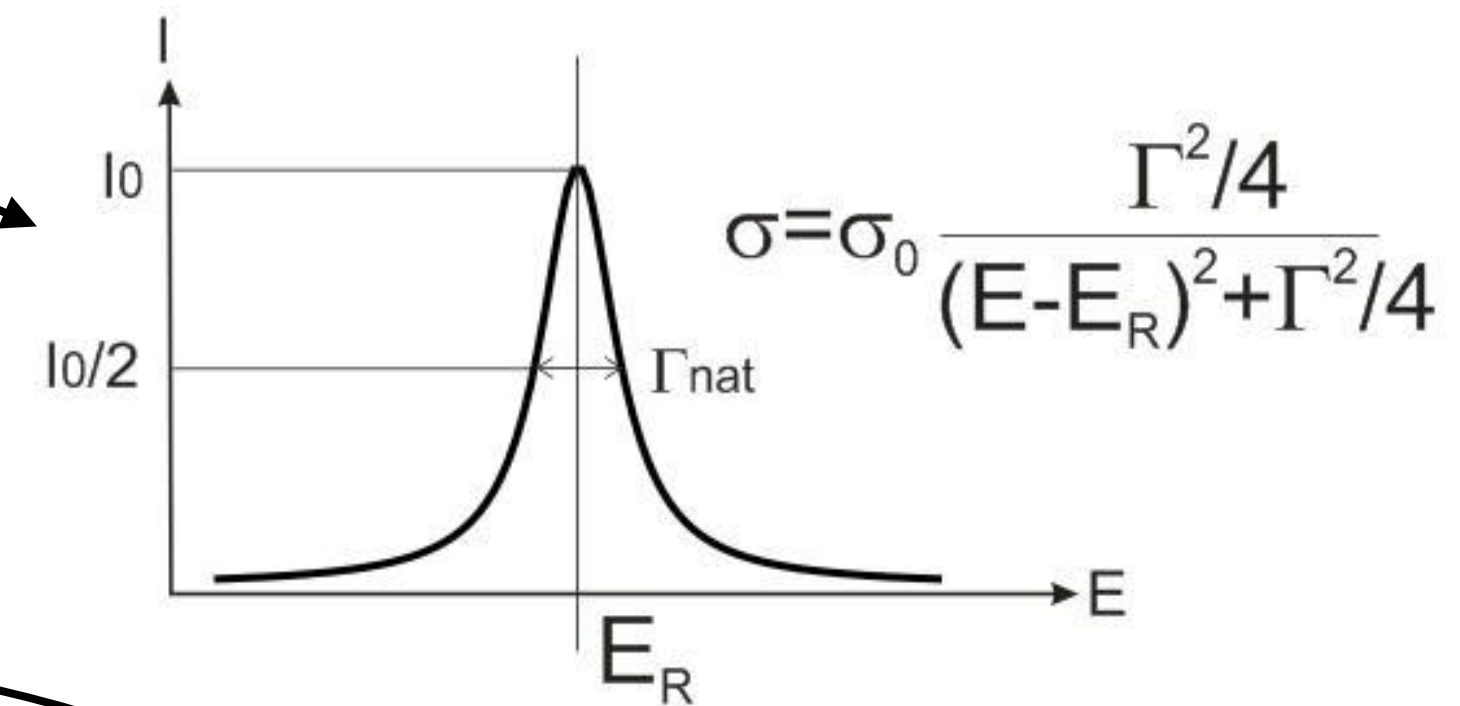
Optical theorem  $\rightarrow$

$$\Gamma \sim \frac{1}{M} \text{Im} \Sigma(M^2)$$

# Decays of heavy particles

- Naively processes with a massive s-channel propagator diverge when  $p^2 = M^2$
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$$\text{propagator} \sim \frac{1}{p^2 - M^2}$$



$$\begin{aligned} \text{propagator} &\sim \text{---} + \text{---} \circlearrowleft \text{1PI} \text{---} + \text{---} \circlearrowleft \text{1PI} \circlearrowleft \text{1PI} \text{---} + \dots \\ &= \frac{1}{p^2 - M_0} + \frac{1}{p^2 - M_0} (-i\Sigma) \frac{1}{p^2 - M_0} + \dots \end{aligned}$$

Geometric series

$$= \frac{1}{k^2 - M_0^2} \sum_{n=0}^{\infty} \left( \frac{-\Sigma(k^2)}{k^2 - M_0^2} \right)^n = \frac{1}{k^2 - M_0^2 + \Sigma(k^2)} = \frac{1}{k^2 - M_0^2 - iM_0\Gamma}$$

$$\int dk^2 |M|^2 \sim \int_{-\infty}^{\infty} \frac{dk^2}{(k^2 - m^2)^2 + m^2\Gamma} \sim \text{BW}$$

Optical theorem  $\rightarrow$

$$\Gamma \sim \frac{1}{M} \text{Im} \Sigma(M^2)$$

- However: this summation mixes different orders of perturbation theory.  
Thus, in general it might (and will) **break gauge invariance when applied naively.**
- (Usually) not a problem at LO, also not at NLO **QCD** e.g. for vector boson decays into leptons
- However: possibly severe problems at NLO **EW**

# Decays of heavy particles

• **Narrow-width approximation (NWA):**  $\Gamma/M \rightarrow 0: \int_{-\infty}^{\infty} \frac{dk^2}{(k^2 - m^2)^2 + m^2\Gamma} = \frac{\pi}{m\Gamma} \delta(k^2 - m^2)$

$\hookrightarrow d\sigma = d\sigma_{\text{prod}} \frac{d\Gamma_{\text{dec}}}{\Gamma}$

- ➔ Advantage: reduces complexity in NLO computation
- ➔ However: unable to capture off-shell effects

• **Complex Mass Scheme (CMS):**  $M \rightarrow \mu = M - i\Gamma M$  analytical continuation at Lagrangian level

➔ regularises propagators (effects also propagator numerators)

➔ effects all derived couplings, incl. weak mixing angle:

$$\sin^2 \theta_W = 1 - \frac{\mu_W^2}{\mu_Z^2}$$

➔ position of the pole in the renormalisation

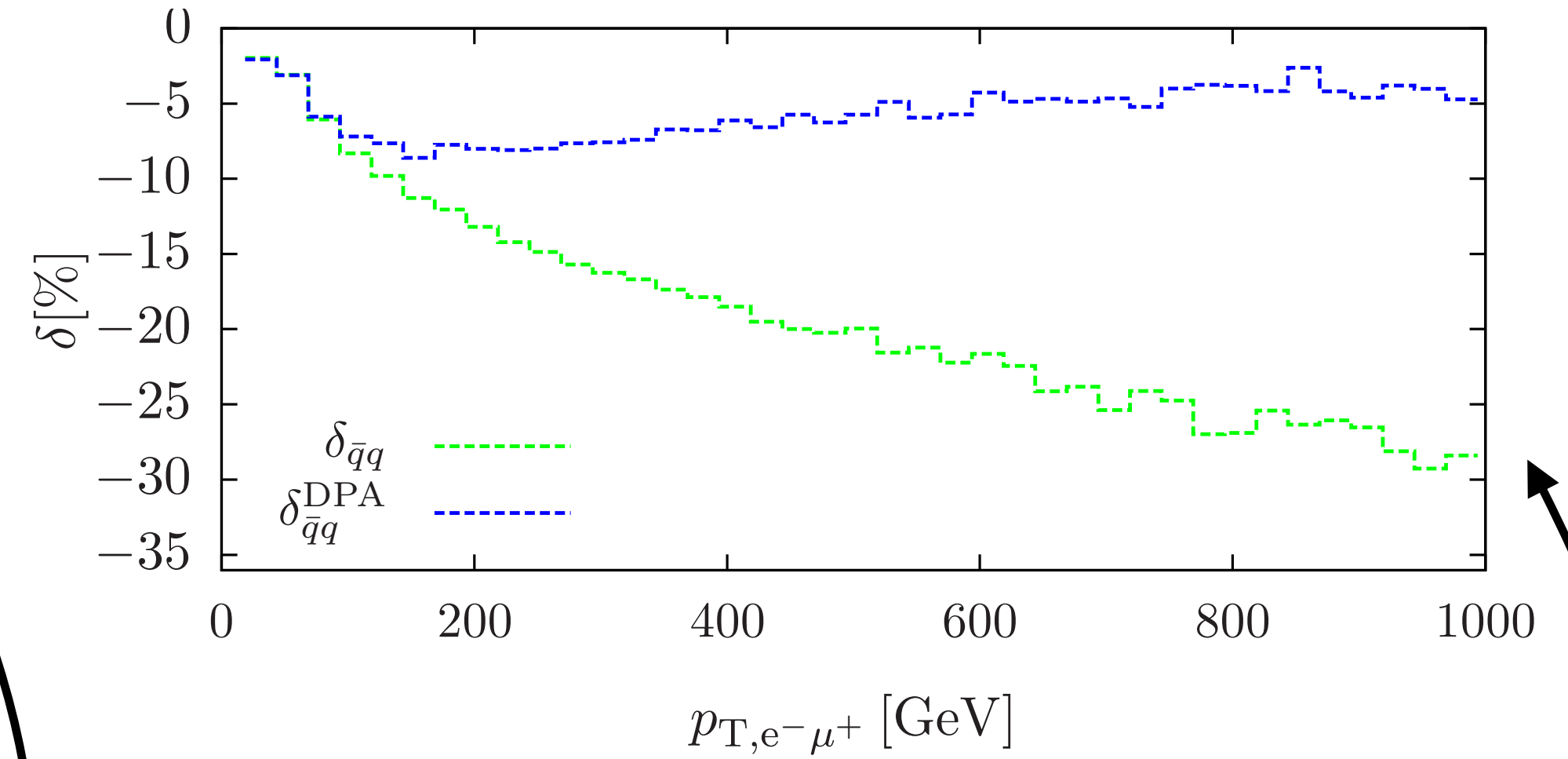
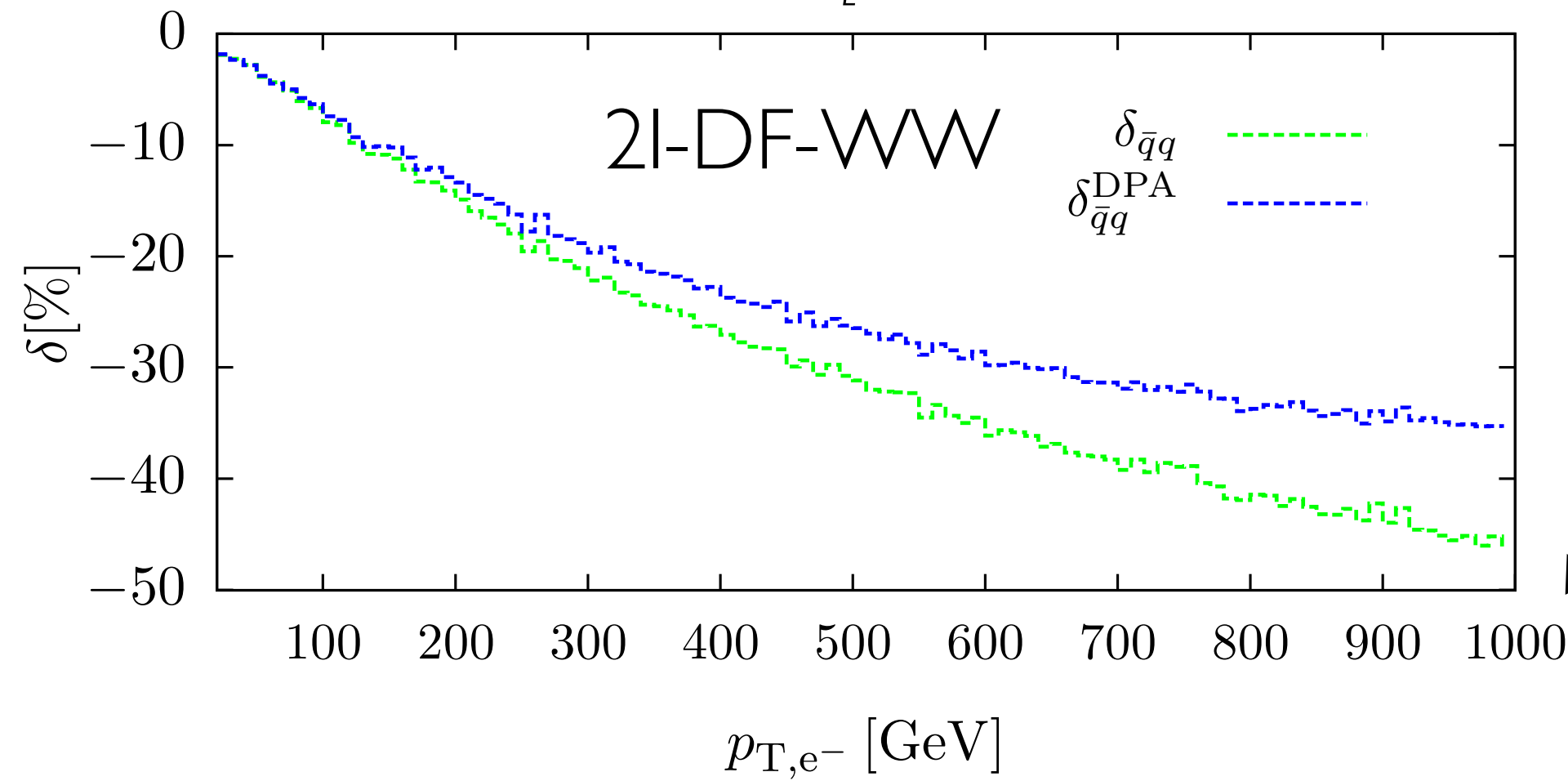
Renormalised self-energy:

$$\hat{\Sigma}^i(p^2) = \Sigma^i(p^2) - \delta\mu_i^2$$

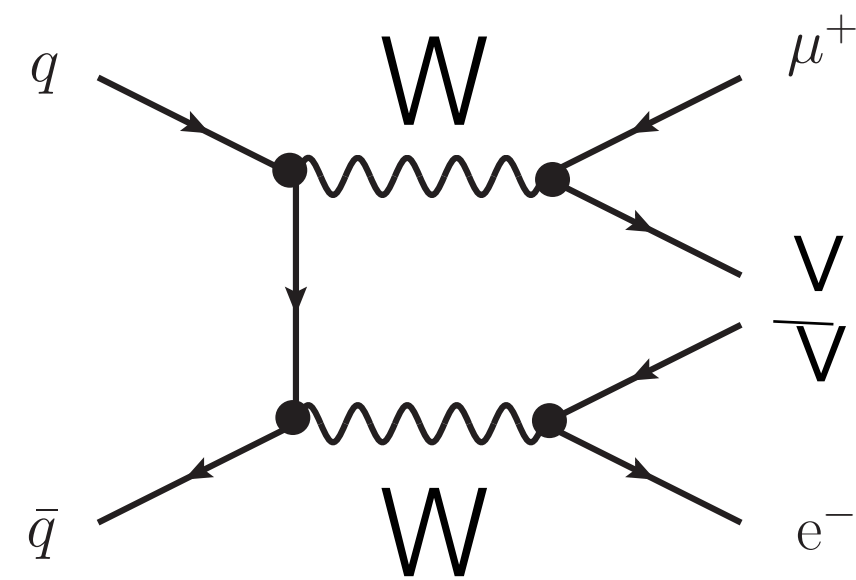
with  $\delta\mu_i^2 = \Sigma^i(p^2) \Big|_{p^2=\mu_i^2}$

# The need for off-shell computations: VV

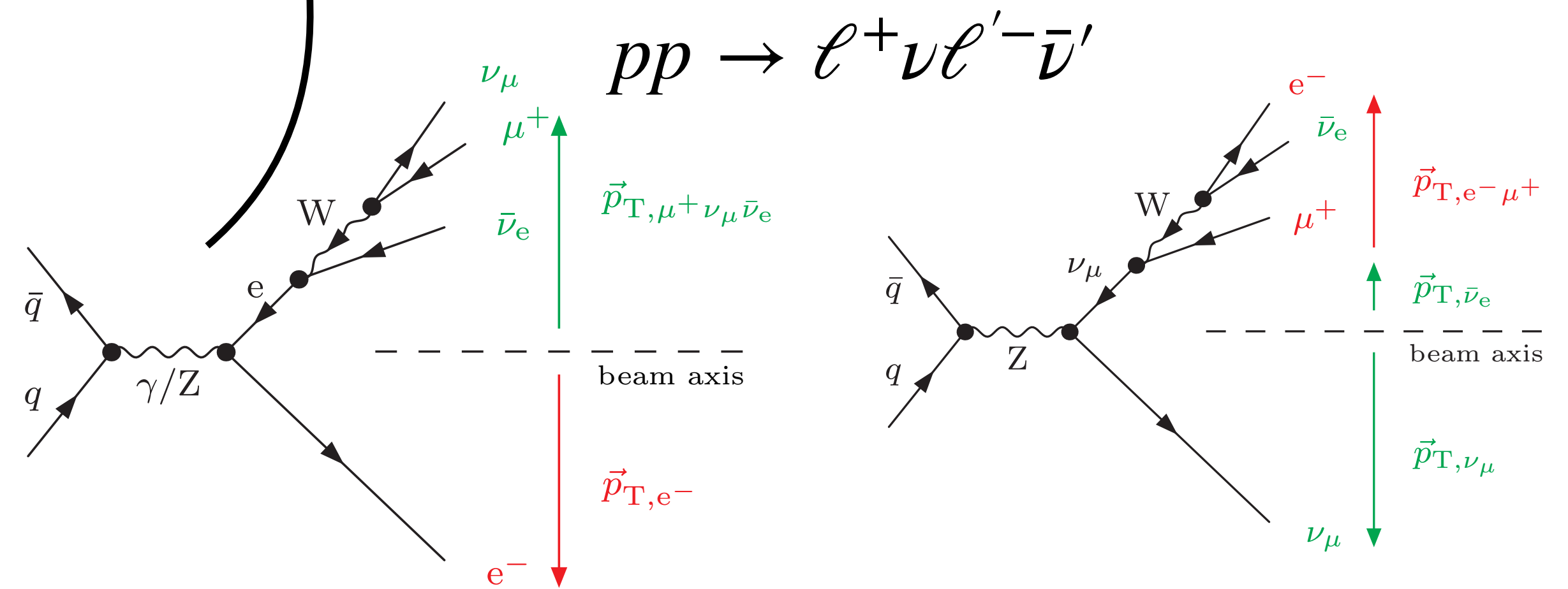
[Biedermann, M. Billoni, A. Denner, S. Dittmaier, L. Hofer, B. Jäger, L. Salfelder ;'16]



$$pp \rightarrow W(\ell^+\nu)W(\ell'^-\bar{\nu}')$$



VS.



➡ sizeable differences in fully off-shell vs. double-pole approximation in tails



# Perturbative expansion: revised

$$d\sigma = d\sigma_{\text{LO}} + \alpha_S d\sigma_{\text{NLO}} + \alpha_{\text{EW}} d\sigma_{\text{NLO EW}} + \alpha_S^2 d\sigma_{\text{NNLO}}$$

NLO QCD                      NLO EW

NNLO QCD

aMC@NLO, Sherpa, Herwig... & Recola, Madloop, Gosam, OpenLoops

dedicated MC's: Matrix, ...

# Perturbative expansion: revised

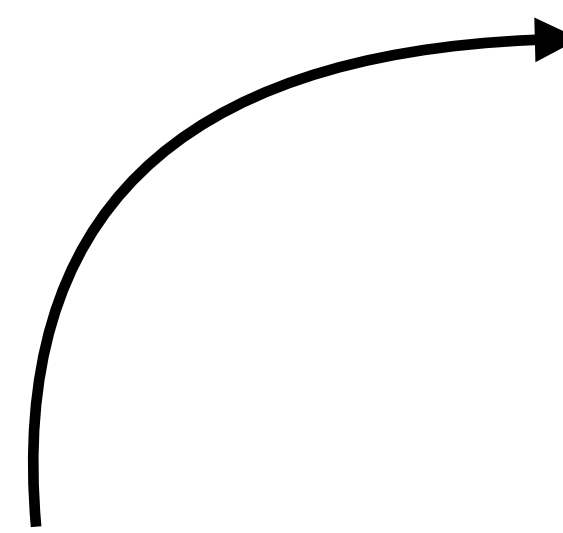
aMC@NLO, Sherpa, Herwig... & Recola, Madloop, Gosam, OpenLoops

dedicated MC's: Matrix, ...

$$\begin{aligned}
 d\sigma = & \underbrace{d\sigma_{\text{LO}}}_{\text{LO}} + \underbrace{\alpha_S}_{\text{NLO QCD}} d\sigma_{\text{NLO}} + \underbrace{\alpha_{\text{EW}}}_{\text{NLO EW}} d\sigma_{\text{NLO EW}} \\
 & + \underbrace{\alpha_S^2}_{\text{NNLO QCD}} d\sigma_{\text{NNLO}} + \underbrace{\alpha_{\text{EW}}^2}_{\text{NNLO EW}} d\sigma_{\text{NNLO EW}} + \underbrace{\alpha_S \alpha_{\text{EW}}}_{\text{NNLO QCD-EW}} d\sigma_{\text{NNLO QCDxEW}} \\
 & + \underbrace{\alpha_S^3}_{\text{N3LO QCD}} d\sigma_{\text{NNLO}} + \dots
 \end{aligned}$$

(Note: The terms  $\alpha_S^2 d\sigma_{\text{NNLO}}$ ,  $\alpha_{\text{EW}}^2 d\sigma_{\text{NNLO EW}}$ ,  $\alpha_S \alpha_{\text{EW}} d\sigma_{\text{NNLO QCDxEW}}$ , and  $\alpha_S^3 d\sigma_{\text{NNLO}}$  in the original image have question marks above them, indicating they are not yet fully implemented or are under development.)

scale variation at NNLO



# Perturbative expansion: revised

aMC@NLO, Sherpa, Herwig... & Recola, Madloop, Gosam, OpenLoops

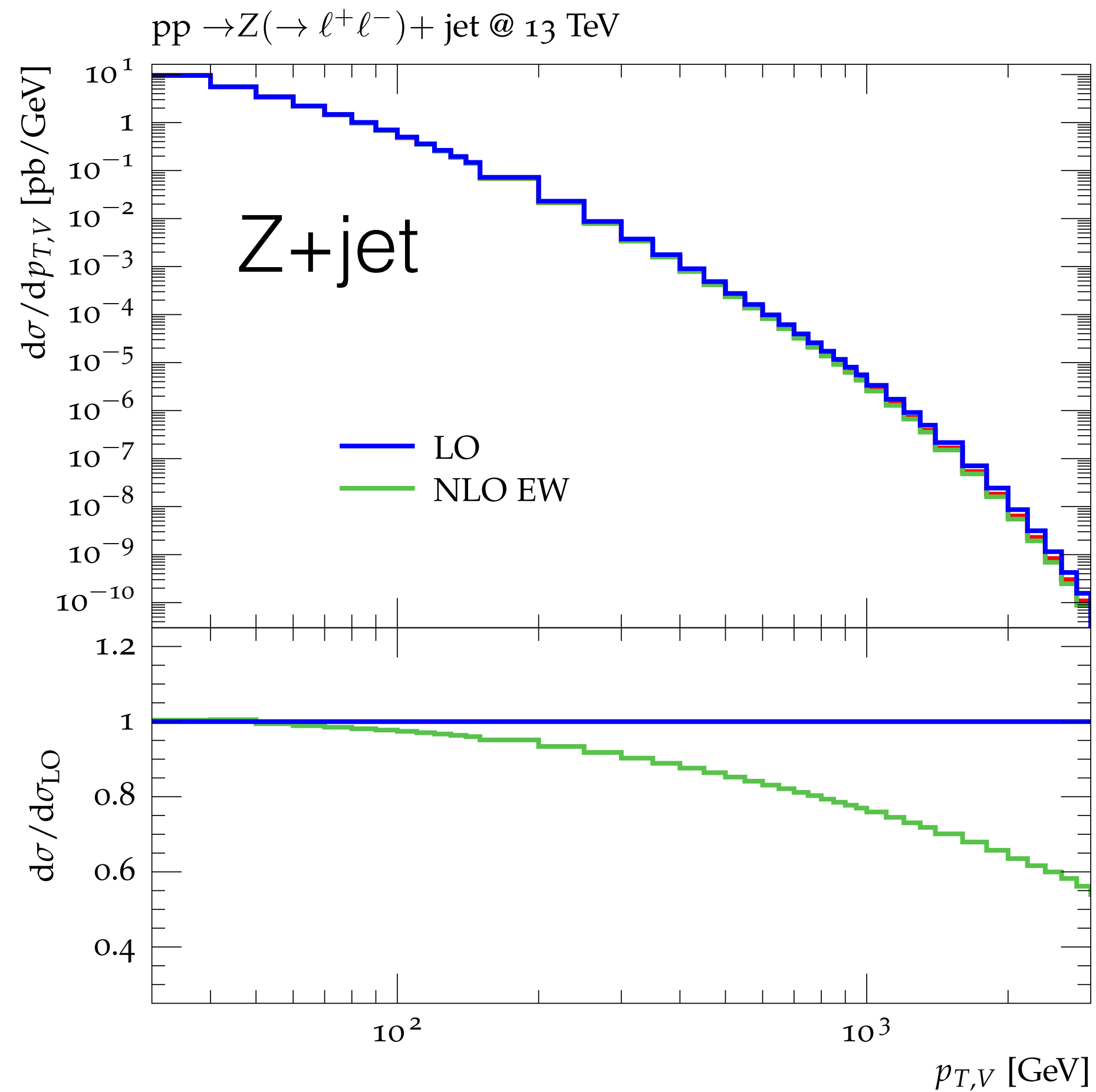
dedicated MC's: Matrix, ...

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 d\sigma = & \underbrace{d\sigma_{\text{LO}}}_{\text{LO}} + \underbrace{\alpha_S}_{\text{NLO QCD}} d\sigma_{\text{NLO}} + \underbrace{\alpha_{\text{EW}}}_{\text{NLO EW}} d\sigma_{\text{NLO EW}} \\
 & + \underbrace{\alpha_S^2}_{\text{NNLO QCD}} d\sigma_{\text{NNLO}} + \underbrace{\alpha_{\text{EW}}^2}_{\text{NNLO EW}} d\sigma_{\text{NNLO EW}} + \underbrace{\alpha_S \alpha_{\text{EW}}}_{\text{NNLO QCD-EW}} d\sigma_{\text{NNLO QCD} \times \text{EW}} + \dots \\
 & + \underbrace{\alpha_S^3}_{\text{N3LO QCD}} d\sigma_{\text{NNLO}} + \dots
 \end{aligned}$$

scale variation at NNLO

scheme variation, e.g. Gmu vs. a(mZ)  
sufficient?

# EW uncertainties: Sudakov

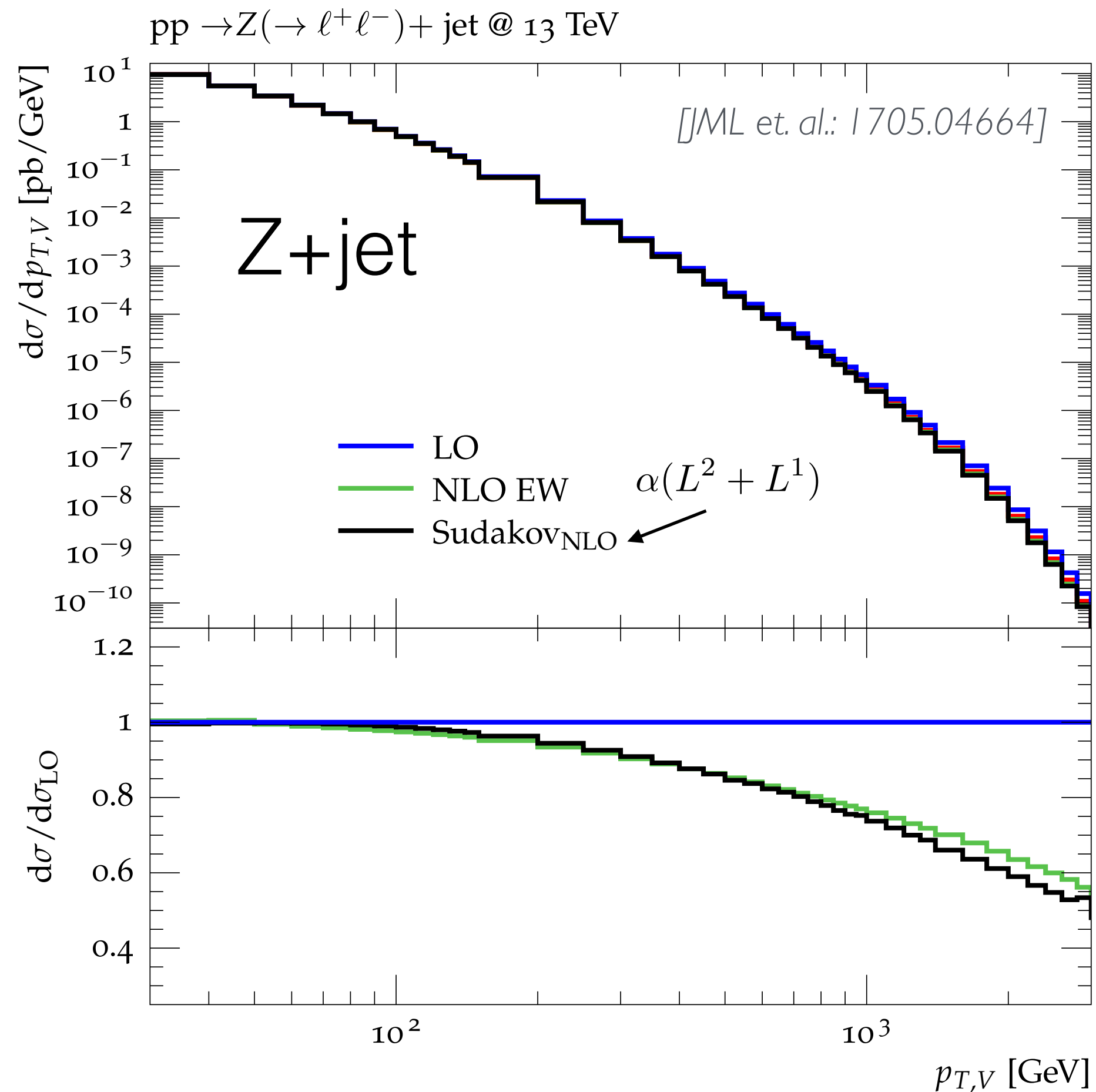


EW corrections become sizeable  
at large  $p_{T,V}$ : -30% @ 1 TeV

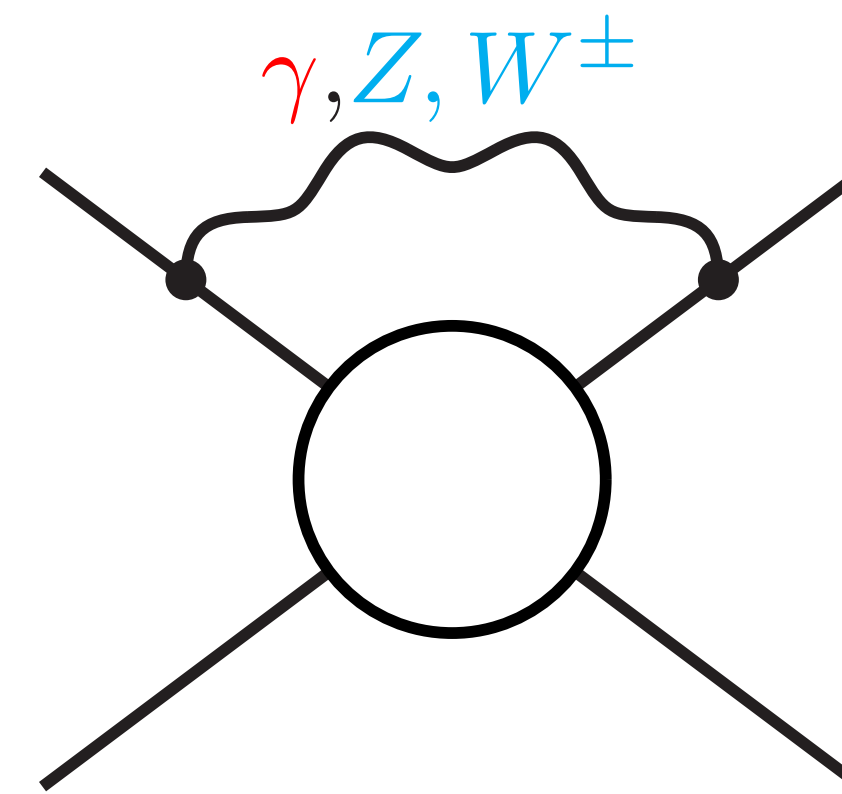
Origin: virtual EW Sudakov logarithms

How to estimate corresponding pure EW uncertainties  
of relative  $\mathcal{O}(\alpha^2)$ ?

# EW uncertainties: Sudakov



Large EW corrections dominated by Sudakov logs

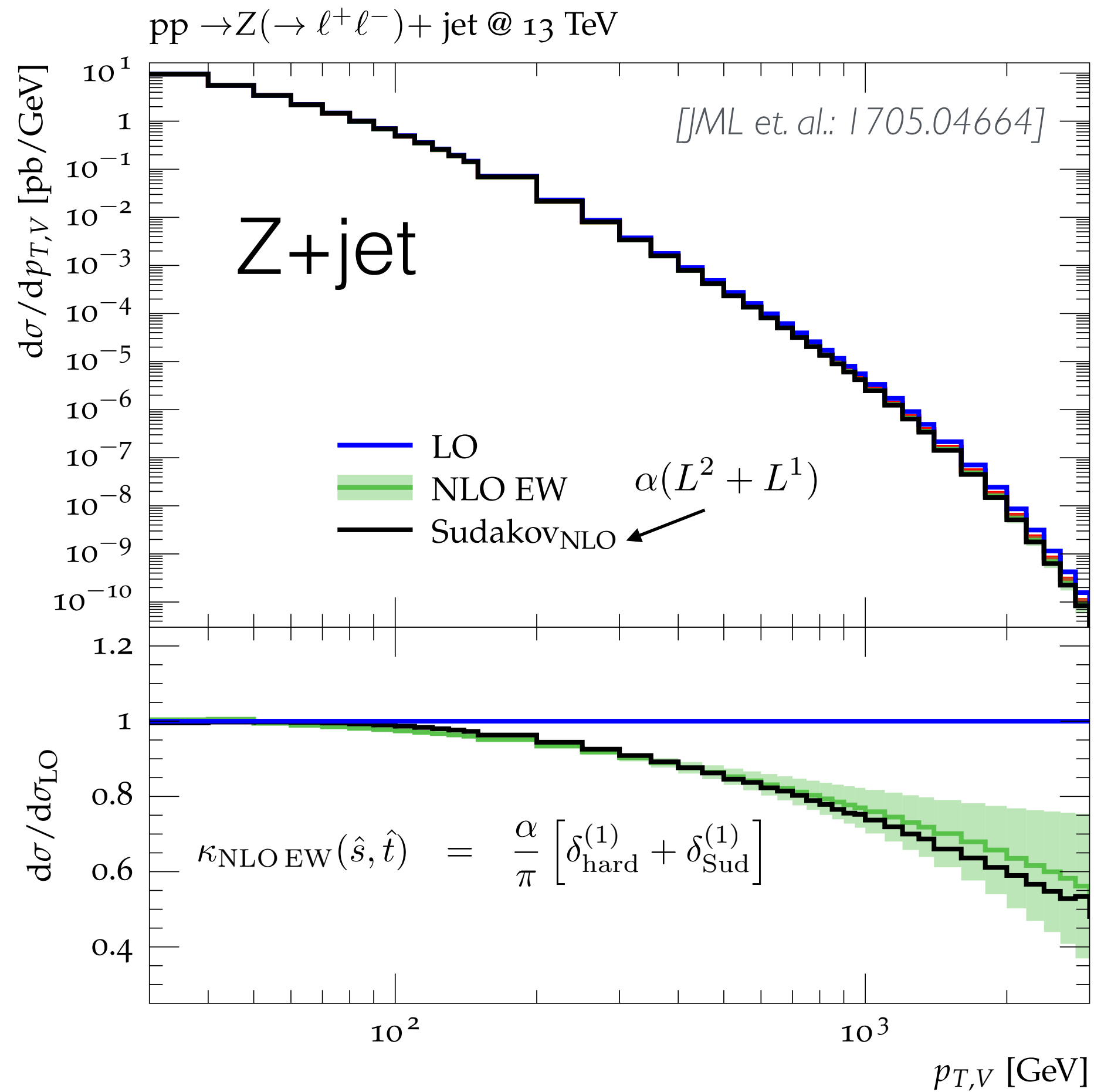


[Ciafaloni, Comelli, '98;  
 Lipatov, Fadin, Martin, Melles, '99;  
 Kuehen, Penin, Smirnov, '99;  
 Denner, Pozzorini, '00]

Universality and factorisation: [Denner, Pozzorini; '01]

$$\delta\mathcal{M}_{\text{LL+NLL}}^{1\text{-loop}} = \frac{\alpha}{4\pi} \sum_{k=1}^n \left\{ \frac{1}{2} \sum_{l \neq k} \sum_{a=\gamma, Z, W^\pm} I^a(k) I^{\bar{a}}(l) \ln^2 \frac{\hat{s}_{kl}}{M^2} + \gamma^{\text{ew}}(k) \ln \frac{\hat{s}}{M^2} \right\} \mathcal{M}_0$$

# EW uncertainties: Sudakov



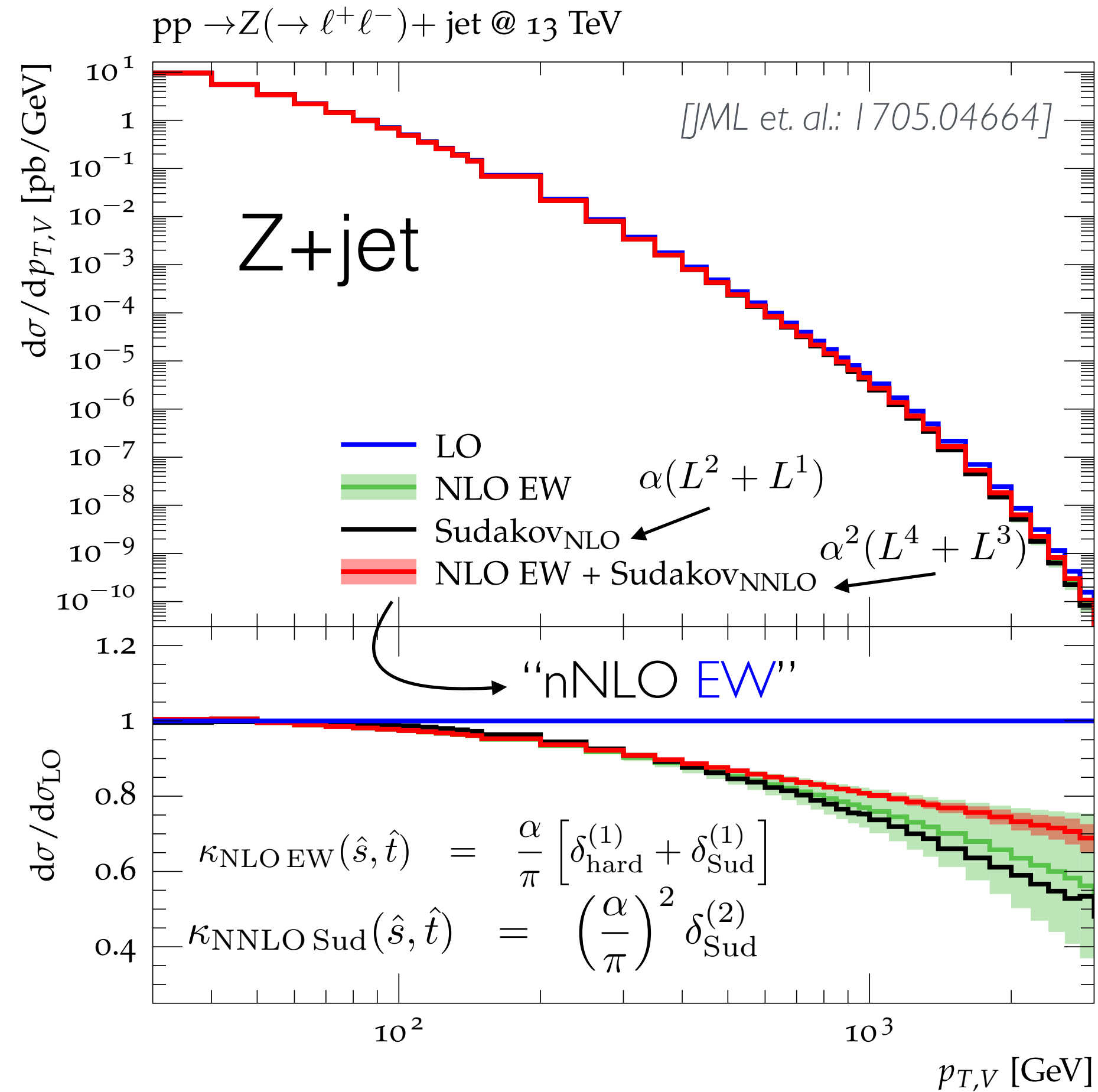
Large EW corrections dominated by Sudakov logs



Uncertainty estimate of (N)NLO EW from naive exponentiation  $\times 2$ :

$$\Delta_{\text{EW}}^{\text{Sud}} \approx (k_{\text{NLOEW}})^2$$

# EW uncertainties: Sudakov



Large EW corrections dominated by Sudakov logs

↓

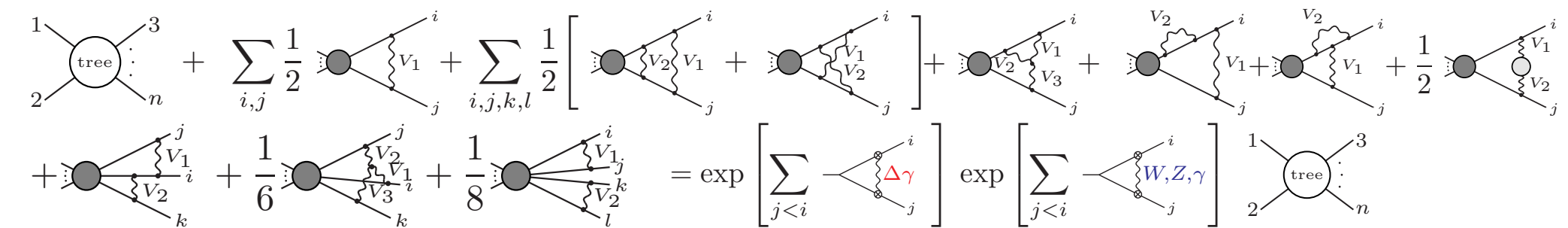
Uncertainty estimate of (N)NLO EW from naive exponentiation  $\times 2$ :

$$\Delta_{\text{EW}}^{\text{Sud}} \approx (k_{\text{NLOEW}})^2$$

↓

check against two-loop Sudakov logs

[Kühn, Kulesza, Pozzorini, Schulze; 05-07]



$$\Delta_{\text{EW}}^{\text{hard}} \approx O(1\%)$$

e.g. from scheme variation, e.g. Gmu vs. a(mZ)

# Perturbative expansion: revised

aMC@NLO, Sherpa, Herwig... & Recola, Madloop, Gosam, OpenLoops

dedicated MC's: Matrix, ...

$$\begin{aligned}
 d\sigma = & \underbrace{d\sigma_{\text{LO}}}_{\text{LO}} + \underbrace{\alpha_S}_{\text{NLO QCD}} d\sigma_{\text{NLO}} + \underbrace{\alpha_{\text{EW}}}_{\text{NLO EW}} d\sigma_{\text{NLO EW}} \\
 & + \underbrace{\alpha_S^2}_{\text{NNLO QCD}} d\sigma_{\text{NNLO}} + \underbrace{\alpha_{\text{EW}}^2}_{\text{NNLO EW}} d\sigma_{\text{NNLO EW}} + \underbrace{\alpha_S \alpha_{\text{EW}}}_{\text{NNLO QCD-EW}} d\sigma_{\text{NNLO QCDxEW}} + \dots \\
 & + \underbrace{\alpha_S^3}_{\text{N3LO QCD}} d\sigma_{\text{NNLO}} + \dots
 \end{aligned}$$

only known for DY (so far)

→ ask Luca!

scale variation at NNLO

scheme variation, e.g. Gmu vs. a(mZ)

sufficient?



# Mixed QCD-EW uncertainties

Bold estimate:

Consider real  $\mathcal{O}(\alpha\alpha_s)$  correction to  $X$  production  $\simeq$  NLO EW to  $X + \text{jets}$

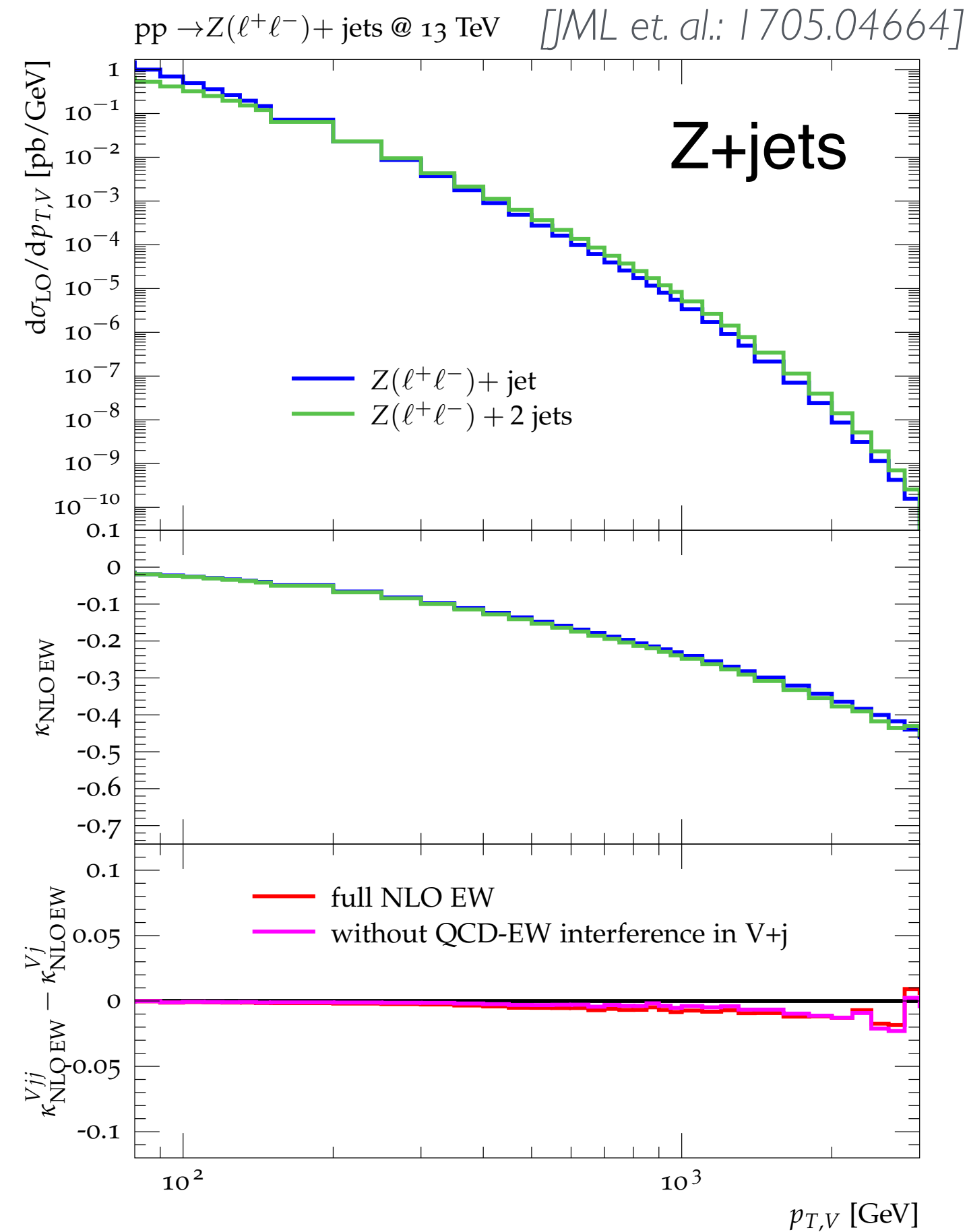
and we often observe

$$\left. \frac{d\sigma_{\text{NLO EW}}}{d\sigma_{\text{LO}}} \right|_{X + \text{jet}} - \left. \frac{d\sigma_{\text{NLO EW}}}{d\sigma_{\text{LO}}} \right|_X \simeq 1\%$$

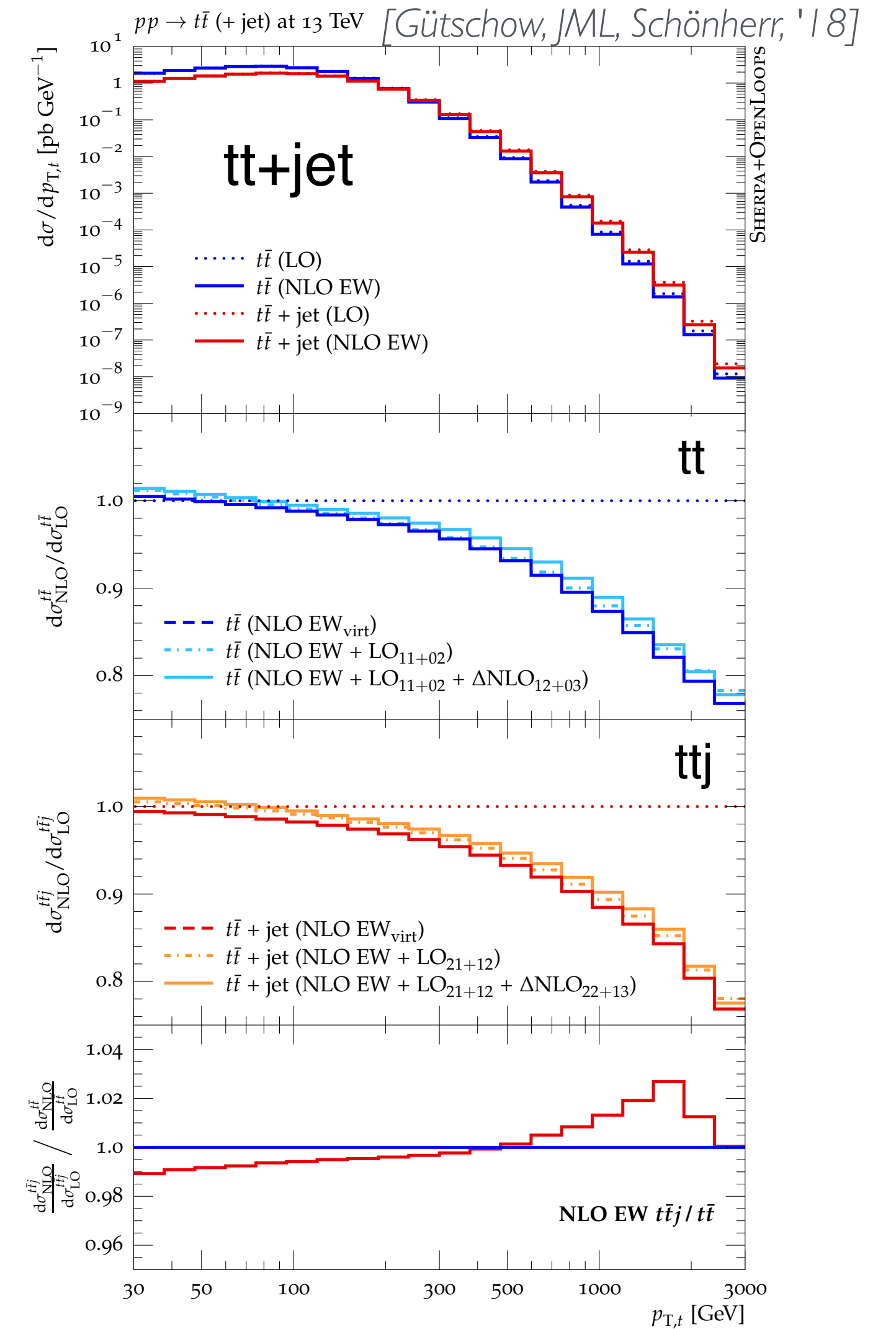
In these cases strong support for

- factorisation
- multiplicative QCD  $\times$  EW combination

$$d\sigma_{\text{NNLO QCD} \times \text{EW}} = d\sigma_{\text{LO}} (1 + \delta_{\text{QCD}}) (1 + \delta_{\text{EW}})$$



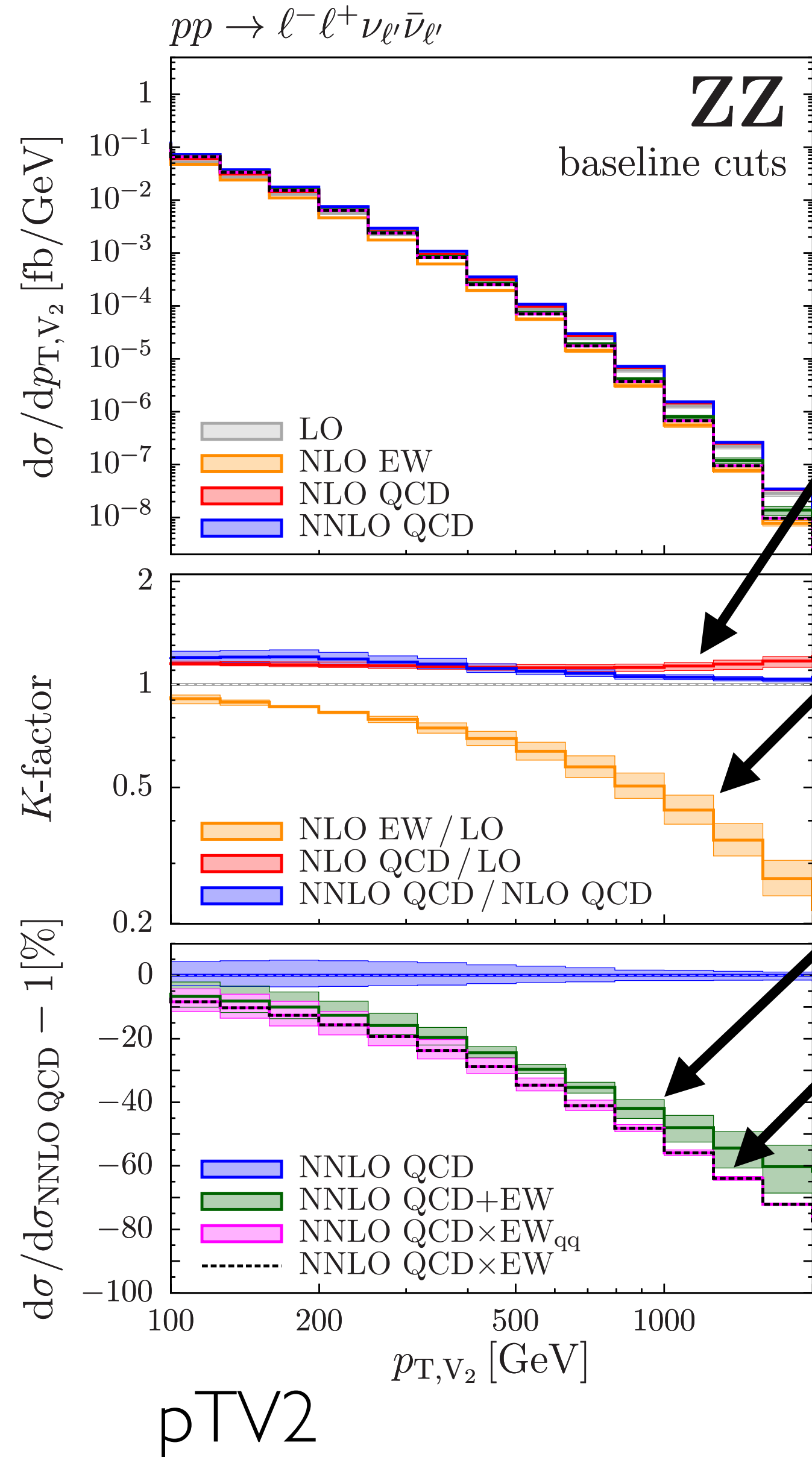
$p_{T,j,2} > 30$  GeV



$p_{T,j} > 30$  GeV

# EW uncertainties: QCD-EW interplay

[M. Grazzini, S. Kallweit, JML, S. Pozzorini, M. Wiesemann; 1912.00068]



- moderate QCD corrections

- ▶ NNLO/NLO QCD very small at large  $p_{T,V_2}$

- ▶ NNLO QCD uncertainty: few percent

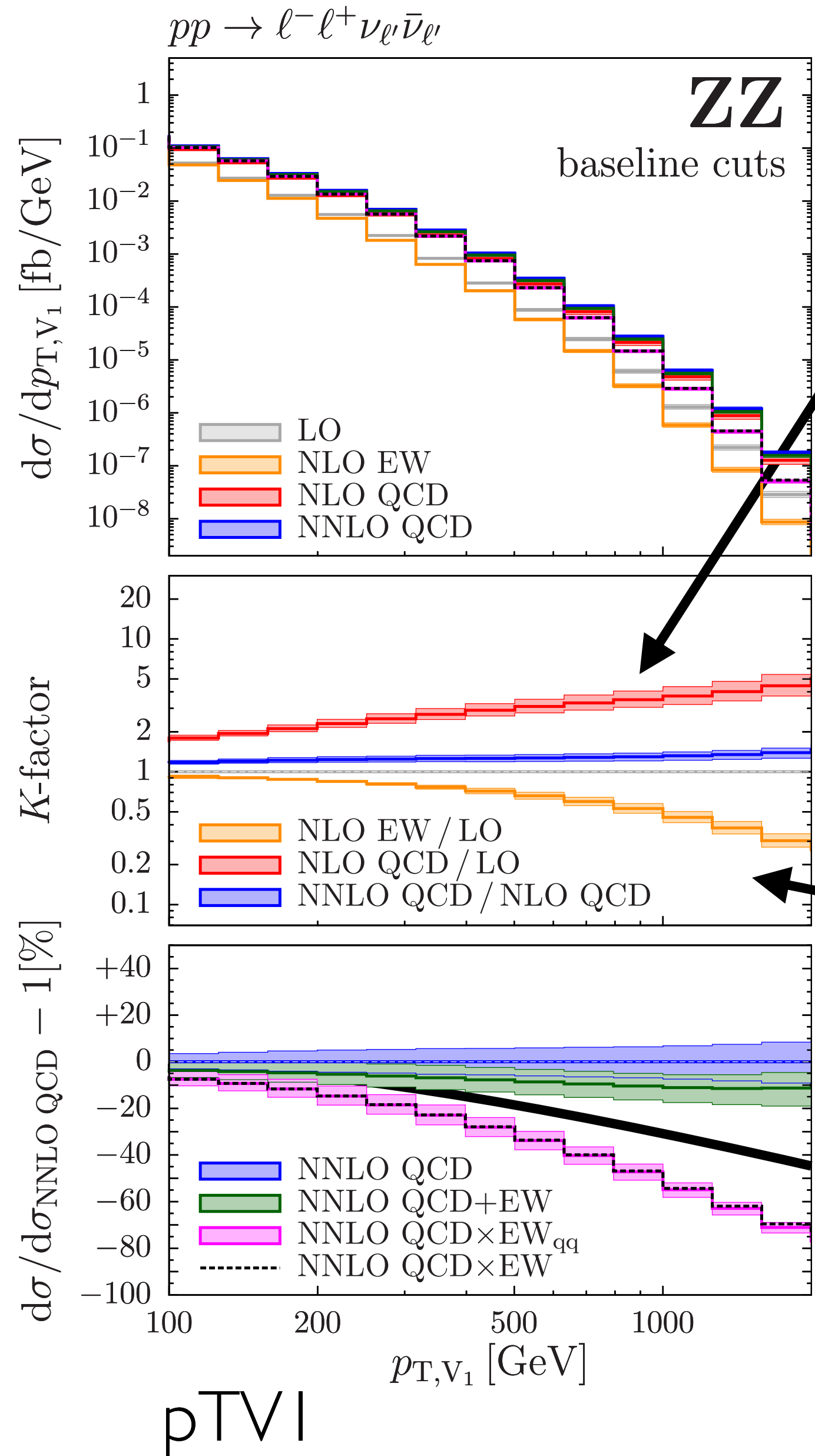
- NLO EW/LO = -(50-60)% @ 1 TeV

$$d\sigma_{\text{NNLO QCD+EW}}$$

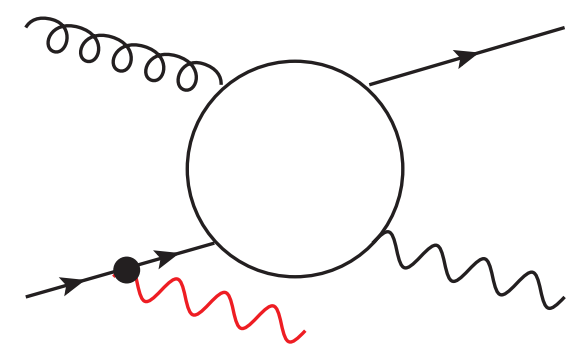
$$d\sigma_{\text{NNLO QCD} \times \text{EW}}$$

- multiplicative/factorised combination superior (EW Sudakov logs  $\times$  soft QCD)

# Combination of QCD and EW corrections



- NLO QCD/LO=2-5! (“giant K-factor”)
- at large  $p_{TVI}$ : VV phase-space is dominated by V+jet (w/ soft V radiation)



$$\frac{d\sigma^{V(V)j}}{d\sigma_{VV}^{\text{LO}}} \propto \alpha_S \log^2 \left( \frac{Q^2}{M_W^2} \right) \simeq 3 \quad \text{at } Q = 1 \text{ TeV}$$

- NNLO / NLO QCD moderate and NNLO uncert. 5-10%

- NLO EW/LO=-(40-50)%

- Very large difference  $d\sigma_{\text{NNLO QCD+EW}}$  vs.  $d\sigma_{\text{NNLO QCD}\times\text{EW}}$

- Problems:

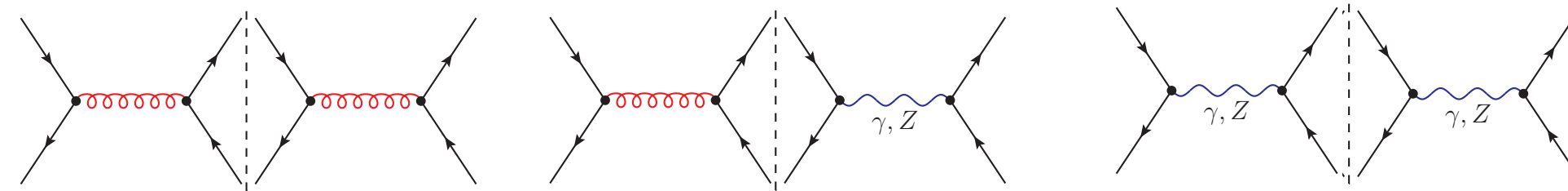
1. In additive combination dominant Vj topology does not receive any EW corrections
2. In multiplicative combination EW correction for VV is applied to Vj hard process

# Perturbative expansion: revised II

- In general combined expansion in  $\alpha_s$  and  $\alpha$  necessary:

$$d\sigma = d\sigma(\alpha_s^n \alpha^m) + d\sigma(\alpha_s^{n-1} \alpha^{m+1}) + d\sigma(\alpha_s^{n-2} \alpha^{m+2}) + \dots$$

LO “subleading Born contributions”: LO2, LO3

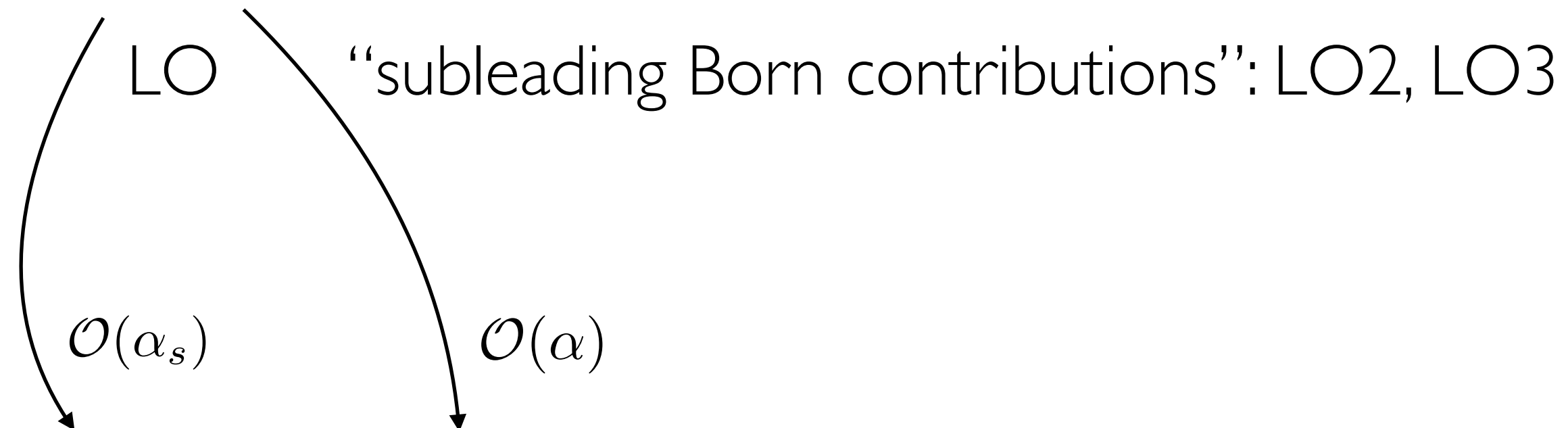


Example:  $q\bar{q} \rightarrow q\bar{q}$

# Perturbative expansion: revised II

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$$\dots + \sigma(\alpha_s^{n+1} \alpha^m) + d\sigma(\alpha_s^n \alpha^{m+1}) + \sigma(\alpha_s^{n-1} \alpha^{m+2}) + \sigma(\alpha_s^{n-2} \alpha^{m+3}) + \dots$$

“NLO QCD”

“NLO EW”

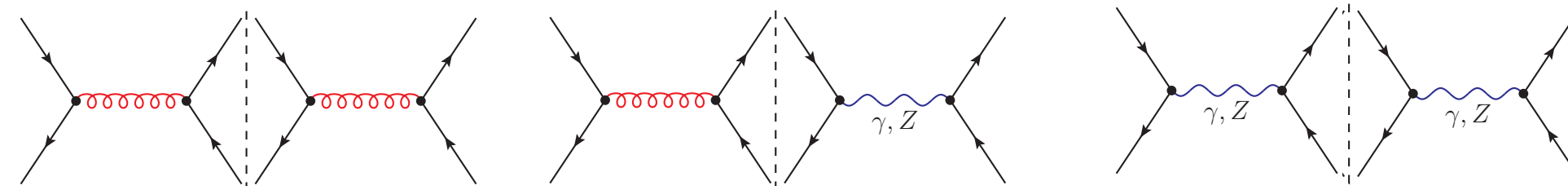
“subleading one-loop contributions”: NLO3, NLO4

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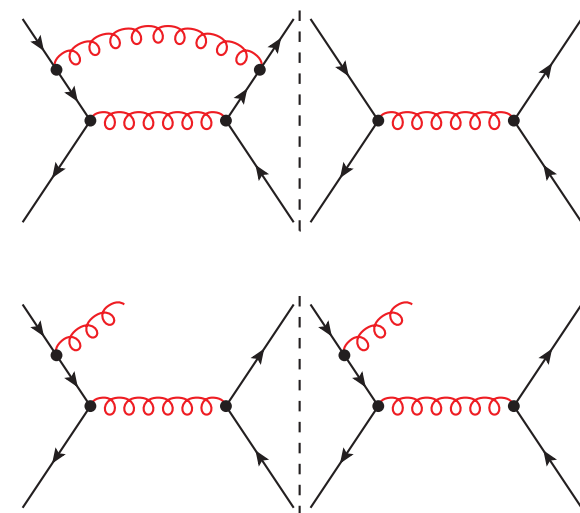
- also at NLO:

$$\dots + \sigma(\alpha_s^{n+1} \alpha^m) + d\sigma(\alpha_s^n \alpha^{m+1}) + \sigma(\alpha_s^{n-1} \alpha^{m+2}) + \sigma(\alpha_s^{n-2} \alpha^{m+3}) + \dots$$

“NLO QCD”

“NLO EW”

“subleading one-loop contributions”: NLO3, NLO4

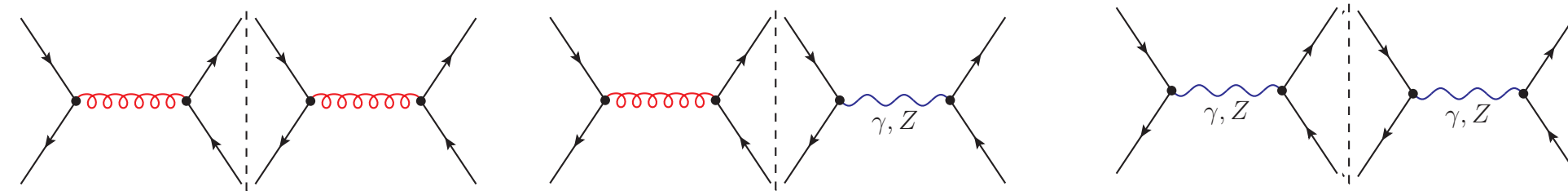


# Perturbative expansion: revised II

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$$d\sigma = d\sigma(\alpha_s^n \alpha^m) + d\sigma(\alpha_s^{n-1} \alpha^{m+1}) + \sigma(\alpha_s^{n-2} \alpha^{m+2}) + \dots$$

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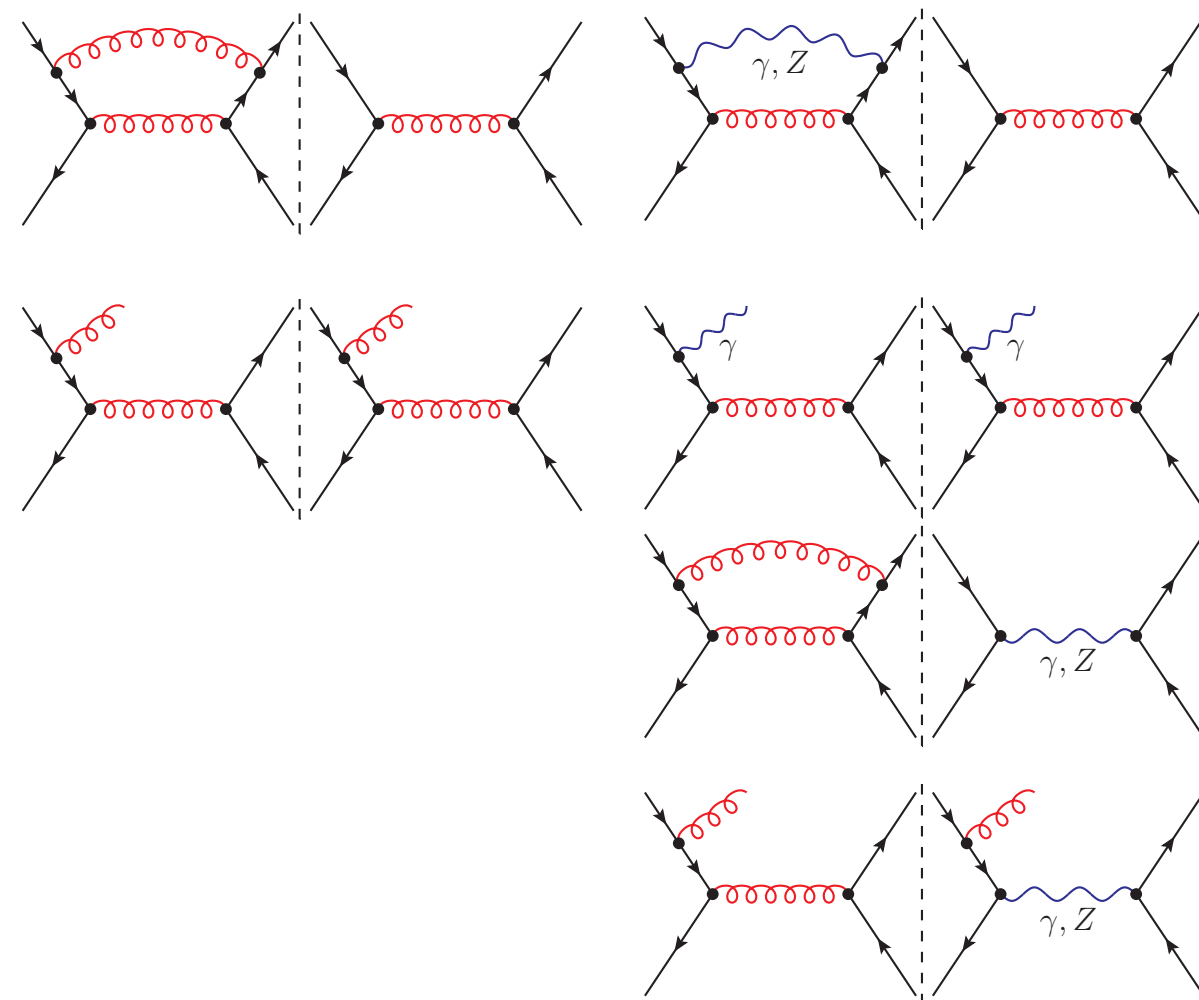
- also at NLO:

$$\dots + \sigma(\alpha_s^{n+1} \alpha^m) + d\sigma(\alpha_s^n \alpha^{m+1}) + \sigma(\alpha_s^{n-1} \alpha^{m+2}) + \sigma(\alpha_s^{n-2} \alpha^{m+3}) + \dots$$

“NLO QCD”

“NLO EW”

“subleading one-loop contributions”: NLO3, NLO4

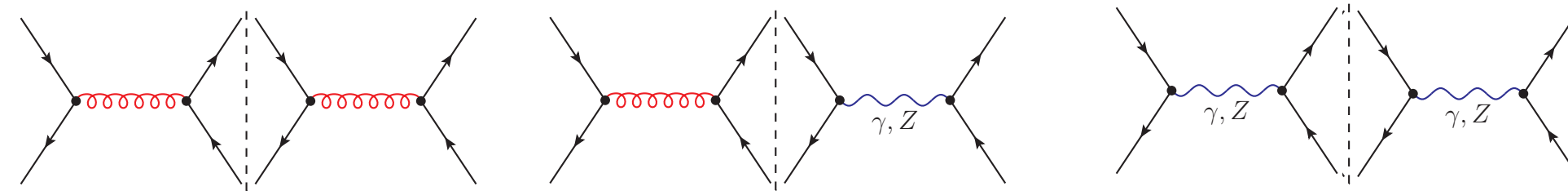


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Example:  $q\bar{q} \rightarrow q\bar{q}$

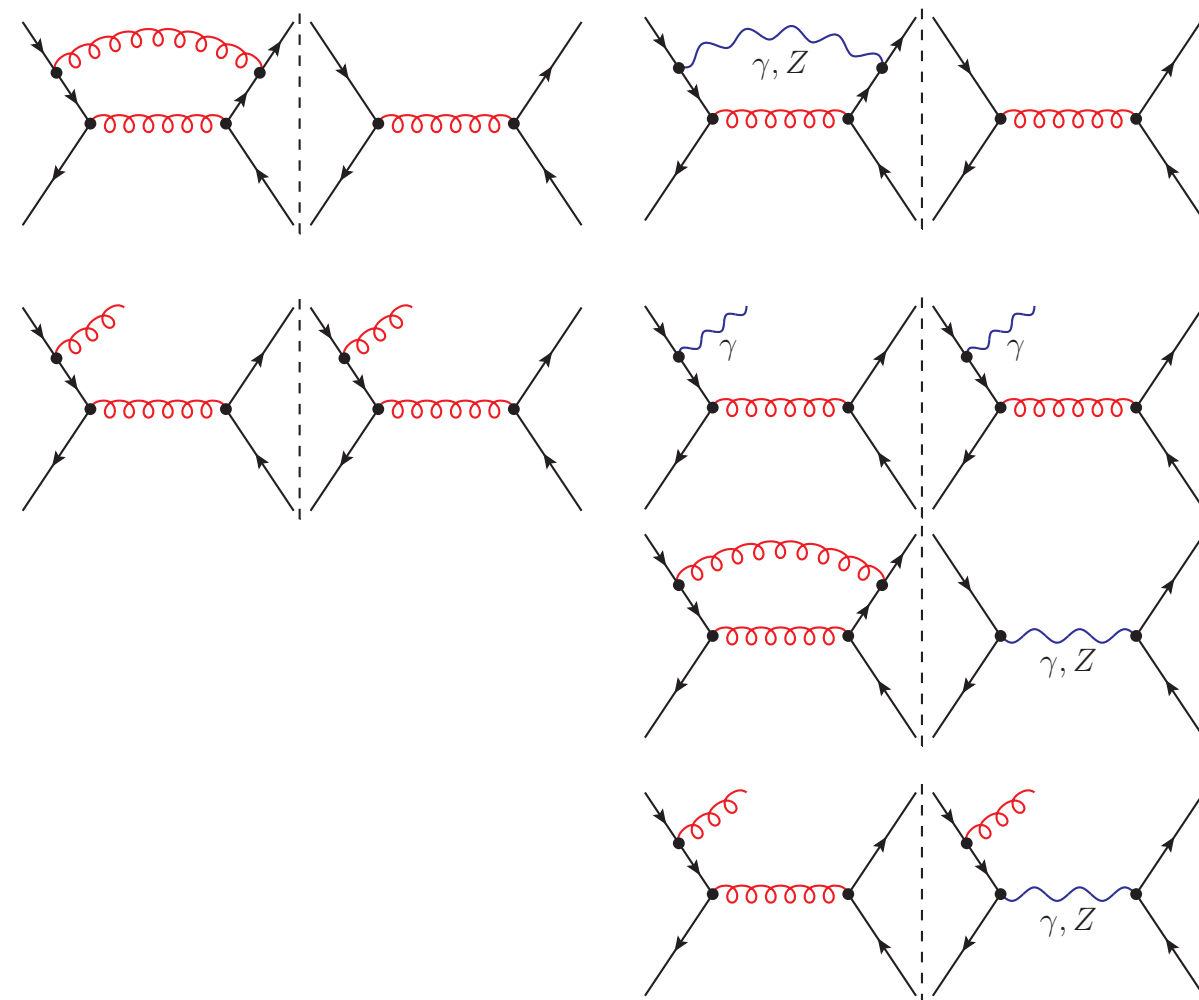
- also at NLO:

$$\dots + \sigma(\alpha_s^{n+1} \alpha^m) + d\sigma(\alpha_s^n \alpha^{m+1}) + \sigma(\alpha_s^{n-1} \alpha^{m+2}) + \sigma(\alpha_s^{n-2} \alpha^{m+3}) + \dots$$

“NLO QCD”

“NLO EW”

“subleading one-loop contributions”: NLO3, NLO4

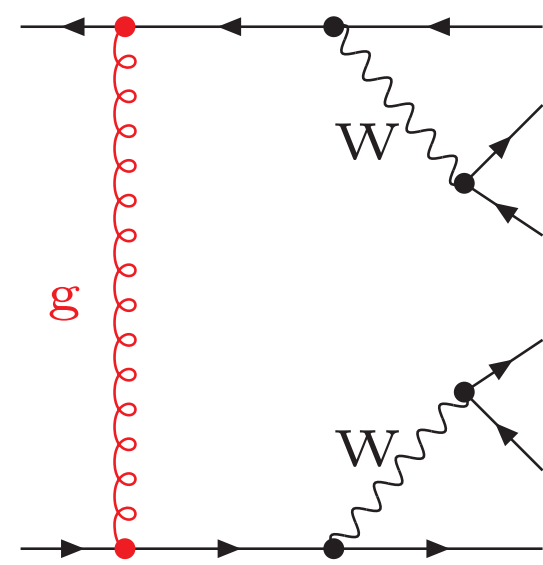


## Note:

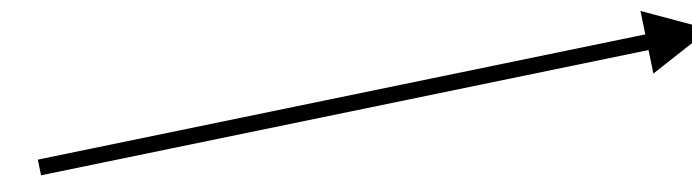
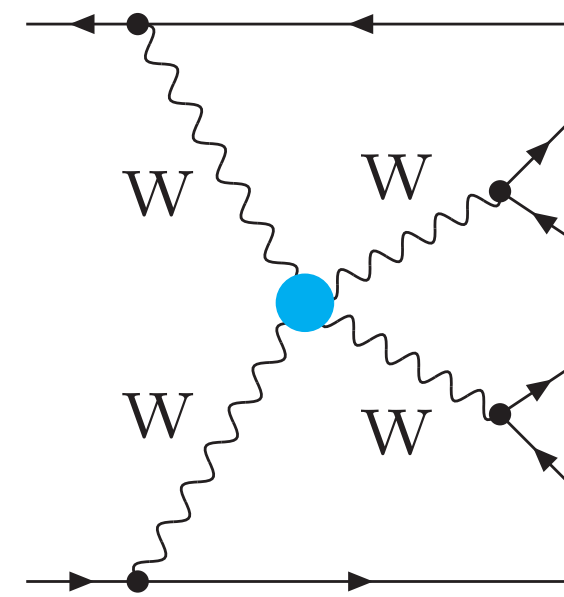
- No diagrammatic separation in NLO QCD and EW
- An IR finite & gauge invariant result is only obtained including all virtual and real contributions of a given perturbative order.



# Example: VV+2jets production



VS.



QCD-background interference VBS-signal

- direct access to quartic EW gauge couplings
- VBS: longitudinal gauge bosons at high energies
- window to electroweak symmetry breaking via off-shell Higgs exchange (ensures unitarity)

LO  $d\sigma = d\sigma(\alpha_S^2\alpha^4) + d\sigma(\alpha_S\alpha^5) + d\sigma(\alpha^6) + \dots$

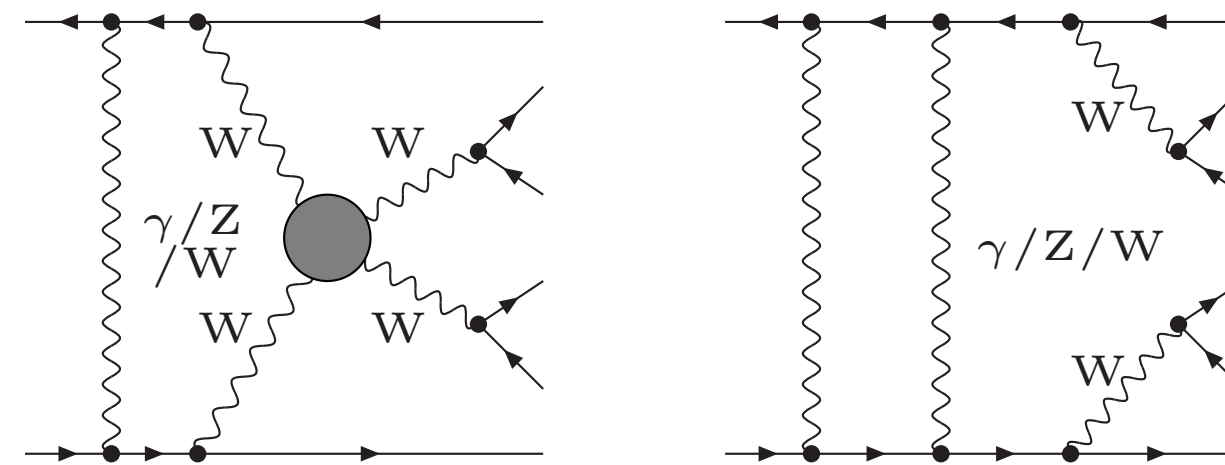
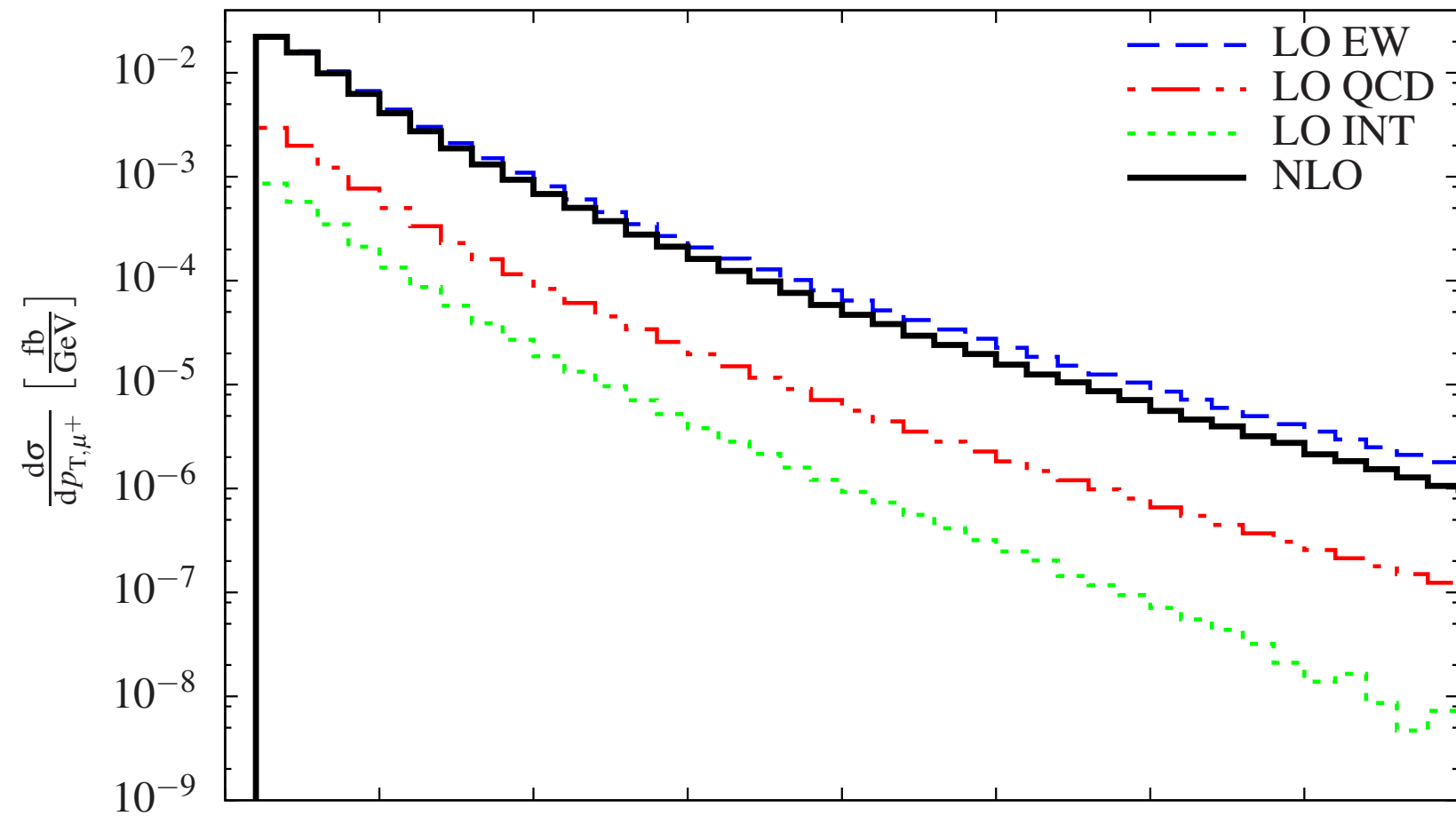
NLO  $\dots + d\sigma(\alpha_S^3\alpha^4) + d\sigma(\alpha_S^2\alpha^5) + d\sigma(\alpha_S\alpha^6) + \sigma(\alpha^7)$

“NLO QCD” “NLO EW” “NLO QCD” “NLO EW”

- ➡ separation formally meaningless at NLO
- ➡ always also consider measurements: fiducial cross sections without QCD subtraction

# VBS- $W^+W^+$ @ full NLO

[Biedermann, Denner, Pellen '16+'17]



- $2 \rightarrow 6$  particles at NLO EW!
- highly challenging computation!

- NLO corrections dominated by  $\alpha^7$ :

Order	$\mathcal{O}(\alpha^7)$	$\mathcal{O}(\alpha_s \alpha^6)$	$\mathcal{O}(\alpha_s^2 \alpha^5)$	$\mathcal{O}(\alpha_s^3 \alpha^4)$	Sum
$\delta\sigma_{\text{NLO}}$ [fb]	-0.2169(3)	-0.0568(5)	-0.00032(13)	-0.0063(4)	-0.2804(7)
$\delta\sigma_{\text{NLO}}/\sigma_{\text{LO}}$ [%]	-13.2	-3.5	0.0	-0.4	-17.1

with  $M_{jj} > 500$  GeV,  $p_{T,j} > 30$  GeV,  $p_{T,\ell} > 20$  GeV,

LO: $\mathcal{O}(\alpha^6)$	$\sigma^{\text{LO}}$ [fb]	$\sigma_{\text{EW}}^{\text{NLO}}$ [fb]	$\delta_{\text{EW}}$ [%]
NLO: $\mathcal{O}(\alpha^7)$	1.5348(2)	1.2895(6)	-16.0

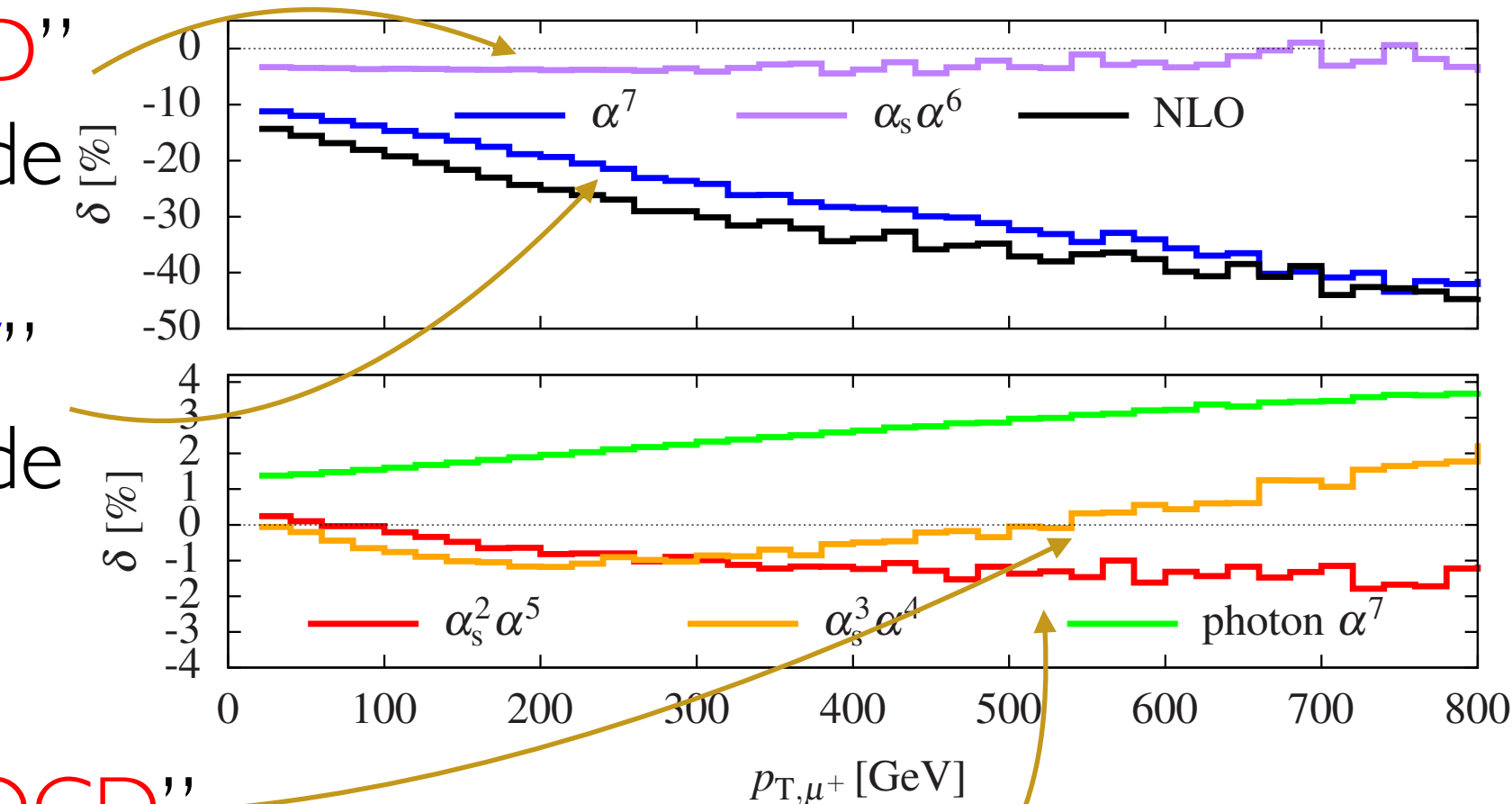
- VERY large inclusive EW corrections (dominated by Sudakov logs)

“NLO QCD”  
to EW mode

“NLO EW”  
to EW mode

“NLO QCD”  
to QCD mode

“NLO EW”  
to QCD mode



# Conclusions

- ▶ LHC is turning into a precision  $EW$  machine
- ▶ ..and precision is key for SM probes, global EFT fits, as well as for searches.
- ▶  $EW$  corrections become large at the TeV scale
- ▶ Fixed-order NLO  $EW$  largely automated
- ▶ Higher-order  $EW$  and mixed  $QCD$ - $EW$  uncertainties are becoming relevant.



Questions?

# References

These Lectures are partly based on:

Various previous ESHEP school lectures, in particular

- Wolfgang Hollik, European School of High-Energy Physics, 2009
- Anna Kulesza, European School of High-Energy Physics, 2023

and

- Ansgar Denner, DESY Monte Carlo school, 2014
- Gavin Salam, Basics of QCD, ICTP–SAIFR school on QCD and LHC physics, 2015

Backup

# LO Ingredients

- LO partonic cross section for a  $2 \rightarrow n$  process can be written as

$$\boxed{d\hat{\sigma}_{\text{LO}} = \frac{1}{2s} \int d\Phi_n |\mathcal{M}_{\text{LO}}|^2}$$

$$\int d\Phi_n = (2\pi)^4 \delta^{(4)} \left( P - \sum_{i=1}^n q_i \right) \prod_{i=1}^n \frac{d^3 q_i}{(2\pi)^3 2E_i} \quad \text{n-particle phase-space}$$

$$\mathcal{M}_{\text{LO}} \quad \text{LO matrix element: tree-level} \xrightarrow{|\mathcal{M}_{\text{LO}}|^2} \left( \text{diagram} \right)^* \left( \text{diagram} \right)$$


$$s = P^2 = (\hat{p}_1 + \hat{p}_2)^2 \quad \text{squared centre-of-mass energy of hard process}$$

- Integration over phase space by Monte Carlo methods
  - ➔ any distribution/histogram can be determined simultaneously
  - ➔ Monte Carlo events can be unweighted
- Integration over phase space analytically
  - ➔ very fast evaluation
  - ➔ analytical structure of the result can be investigated

# Perturbative expansion

- Expansion in a small coupling  $\alpha$ :

$$d\sigma = \underbrace{d\sigma(\alpha^n)}_{\text{LO}} + \underbrace{d\sigma(\alpha^{n+1})}_{\text{NLO}} + \underbrace{d\sigma(\alpha^{n+2})}_{\text{NNLO}} + \underbrace{d\sigma(\alpha^{n+3})}_{\text{N3LO}} + \dots$$

- at the LHC consider in particular  $\alpha = \alpha_s$  (QCD coupling), but also  $\alpha = \alpha_{\text{EW}}$  (EW coupling) relevant  $\rightarrow$  later!

- In QCD running strong coupling:  $\alpha_s = \alpha_s(\mu) = \frac{1}{b_0 \ln \frac{\mu^2}{\Lambda^2}} + \dots$

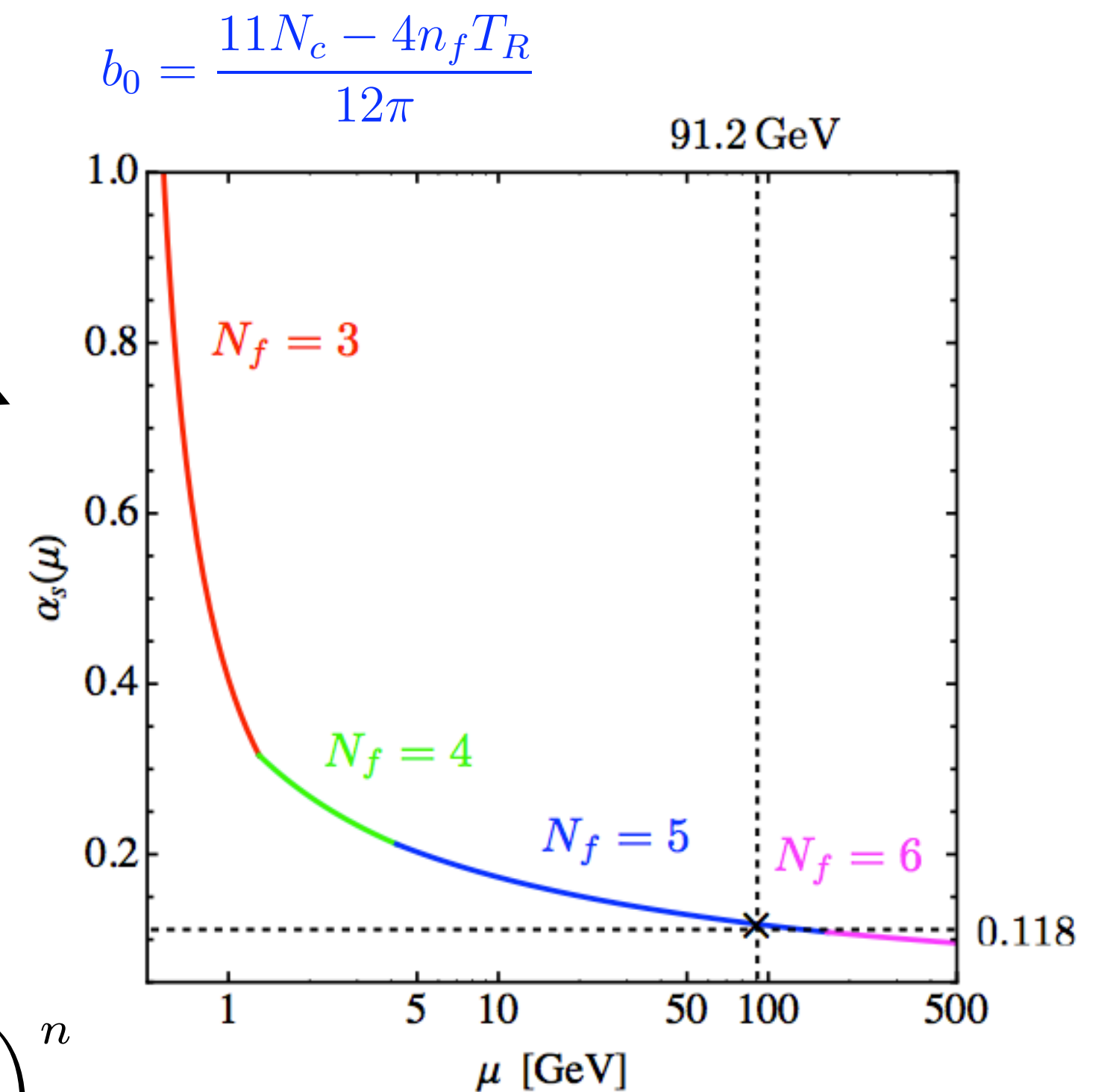
$$d\sigma^{\text{LO}}(\mu) = \alpha_s(\mu)^n A^{\text{LO}}$$

$$\rightarrow d\sigma^{\text{LO}}(\mu') = \alpha_s(\mu')^n A^{\text{LO}} = \alpha_s(\mu)^n \left( 1 + nb_0 \alpha_s(\mu) \ln \frac{\mu^2}{\mu'^2} + \dots \right) A^{\text{LO}}$$

- So the change of scale is an NLO effect ( $\propto \alpha_s$ ).

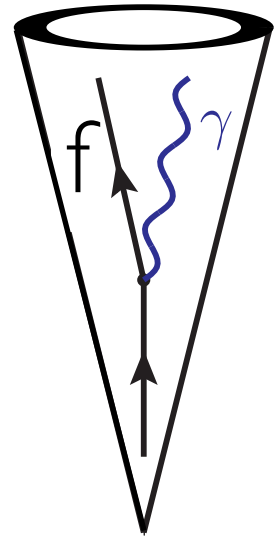
- At LO the normalisation is not under control:

$$\frac{d\sigma^{\text{LO}}(\mu)}{d\sigma^{\text{LO}}(\mu')} = \left( \frac{\alpha_s(\mu)}{\alpha_s(\mu')} \right)^n$$



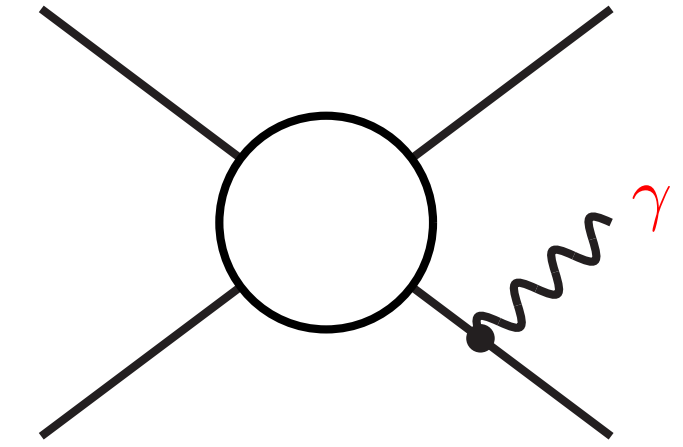


# QED radiation: IR safety



## ► collinear $f \rightarrow f\gamma$ singularities

- cancelled clustering  $f$  and  $\gamma$ , within cone of  $\Delta R_{f\gamma}$ , typically  $\Delta R_{f\gamma} = 0.1$
- or regularised via fermion masses (at LHC only relevant for  $f = \mu$ )

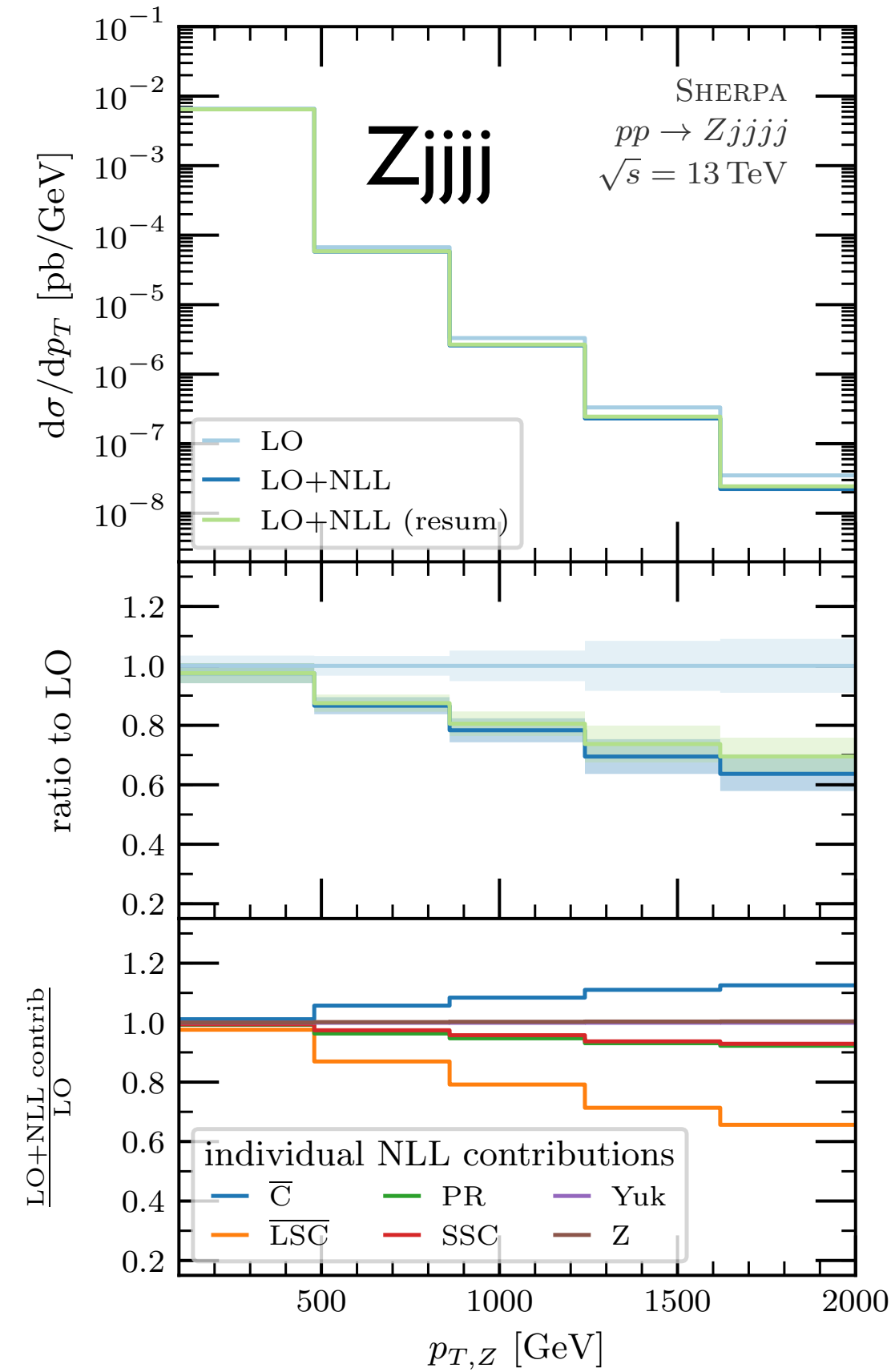


- However: for processes with jets at LO this spoils universality between quarks and gluons!
  - problematic for QCD IR safety
- Solution: *democratic jet-algorithm approach*, partonic jets  $\equiv \{q, g, \gamma, l\}$

# Tools for EW Sudakov corrections

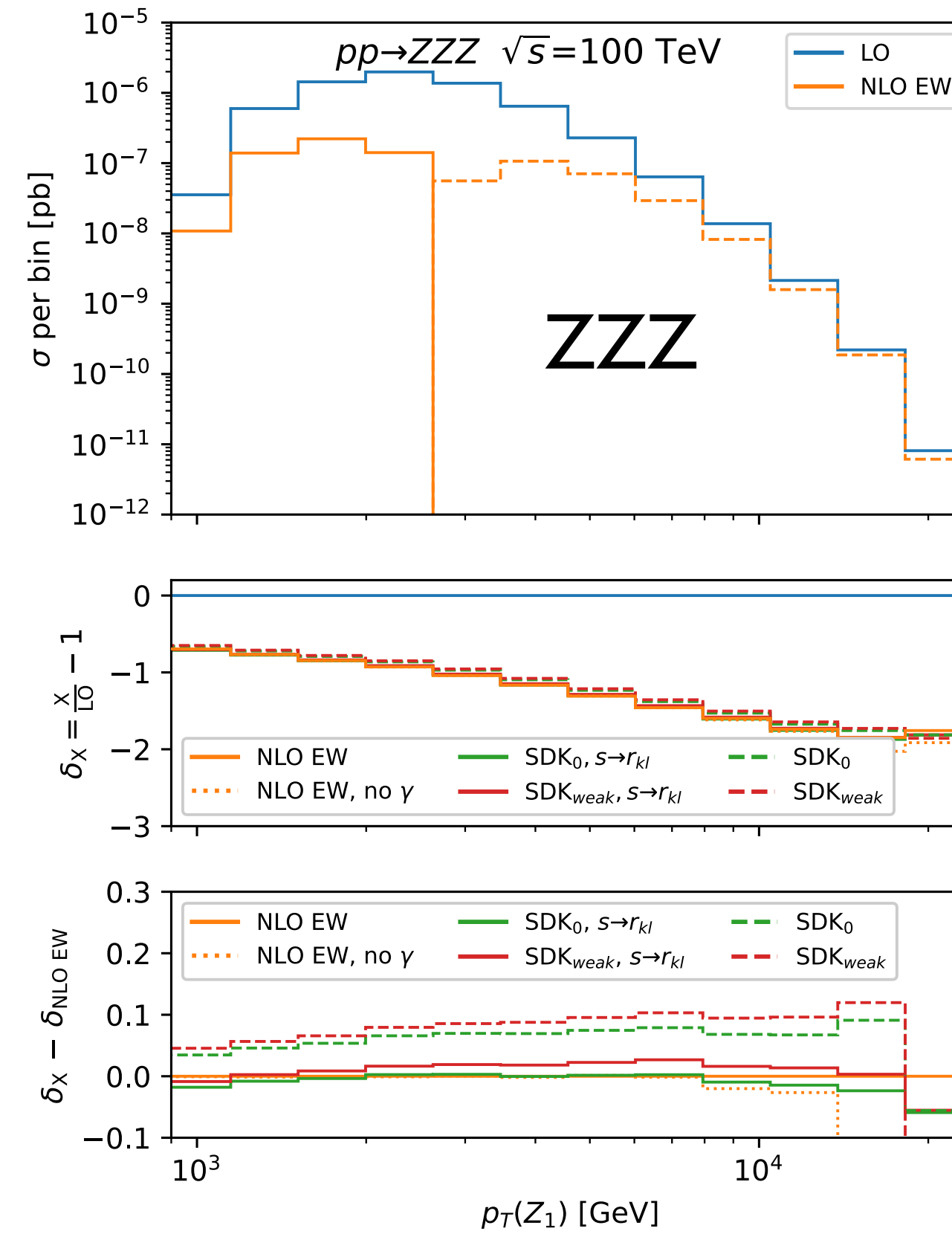
## Sherpa

[Bothmann, Napoletano, '20]



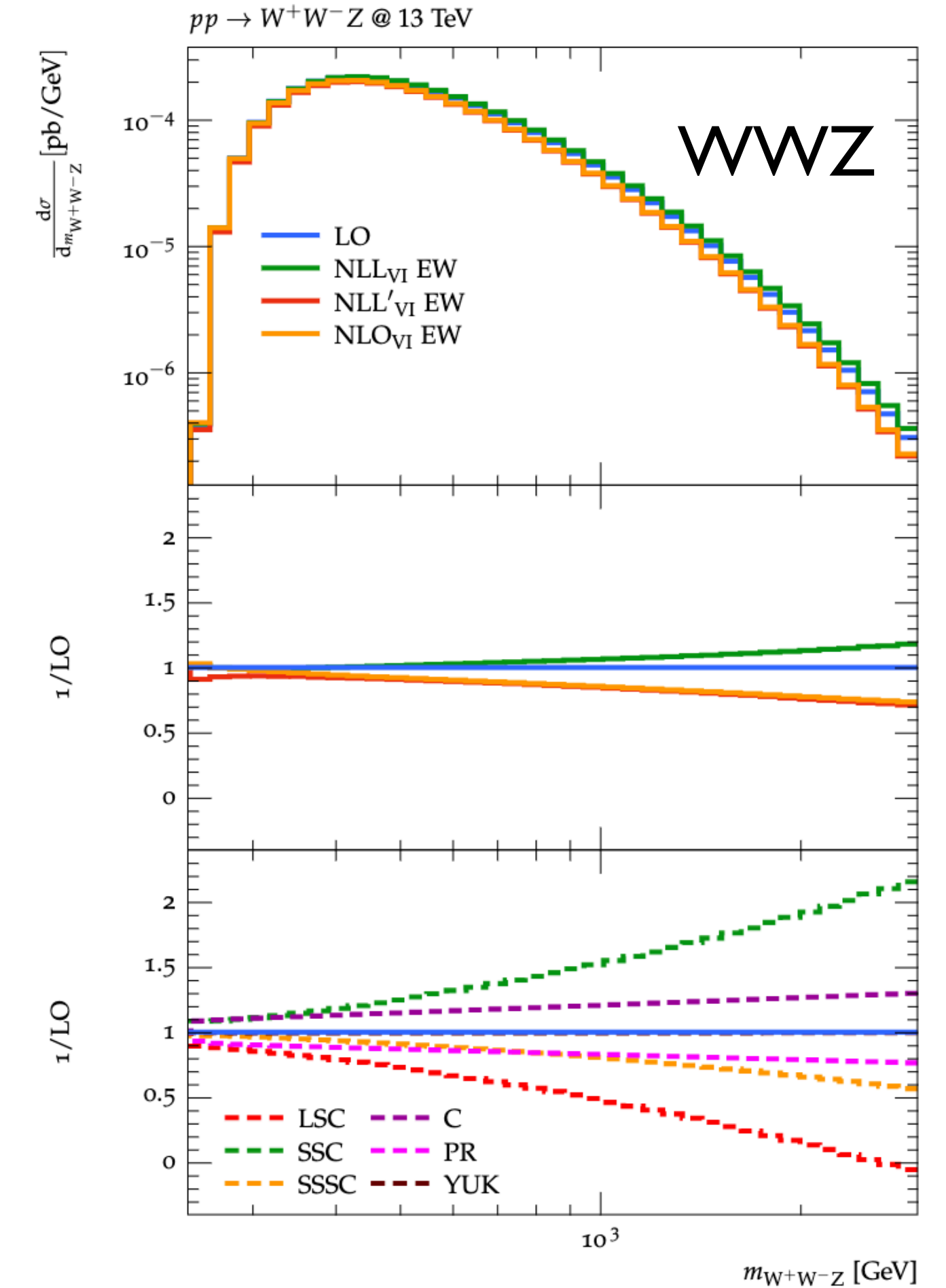
## MadGraph5\_aMC@NLO

[Pagani, Zaro, '21]



## OpenLoops

[JML, Mai, '23]



- all based on [Denner, Pozzorini, '00, '01]

# NNLO Ingredients

- NNLO partonic cross section for a  $2 \rightarrow n$  process can be written as

$$d\hat{\sigma}_{\text{NNLO}} = \frac{1}{2s} \int d\Phi_n \left[ |\mathcal{M}_{\text{LO}}|^2 + 2\text{Re}\{\mathcal{M}_{\text{LO}}\mathcal{M}_{\text{NLO},V}^*\} + 2\text{Re}\{\mathcal{M}_{\text{LO}}\mathcal{M}_{\text{NNLO},V}^*\} \right]$$

$\text{NNLO} = \text{B} + \text{V} + \text{V}^2 + \dots$

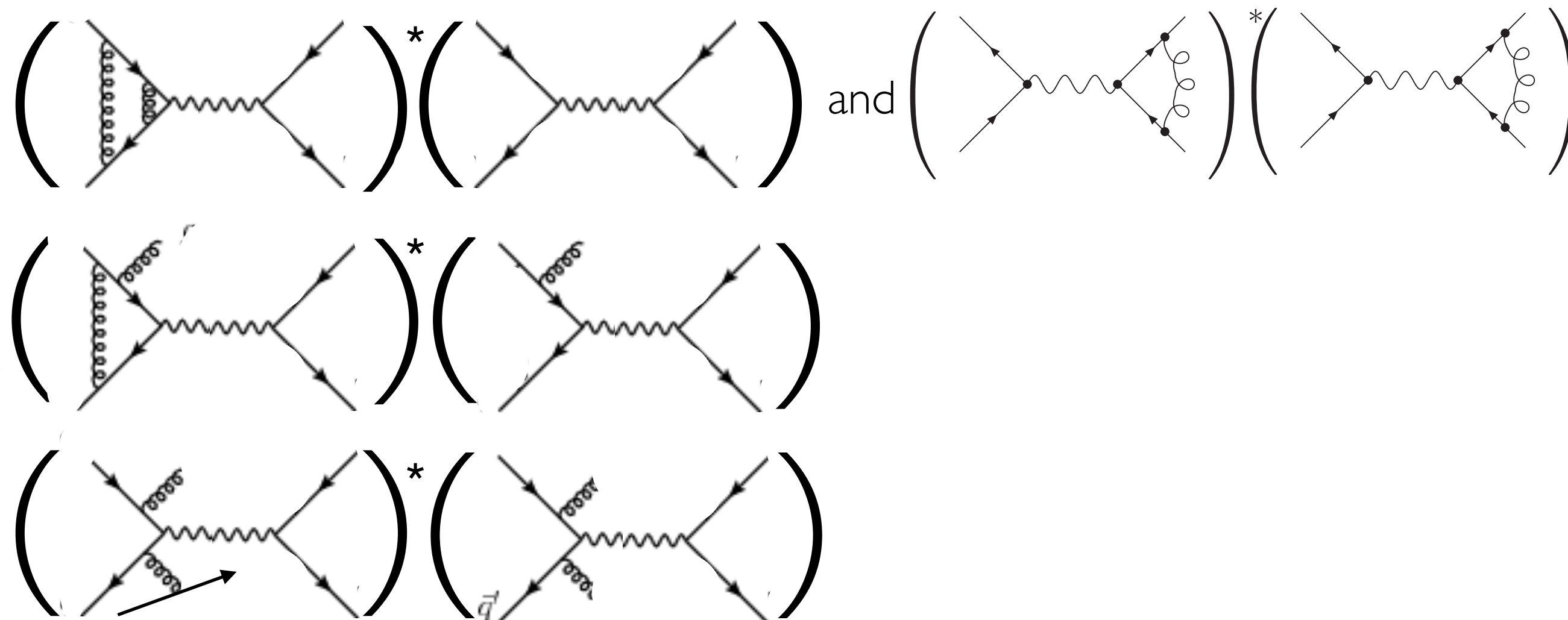
$$+ \frac{1}{2s} \int d\Phi_{n+1} \left[ |\mathcal{M}_{\text{NLO},R}|^2 + 2\text{Re}\{\mathcal{M}_{\text{NLO},R}\mathcal{M}_{\text{NNLO},RV}^*\} \right] + \frac{1}{2s} \int d\Phi_{n+2} |\mathcal{M}_{\text{NNLO},RR}|^2$$

$+ \text{R} + \text{RV} + \text{RR}$

$\int d\Phi_{n(+1)}$   $n, n+1, n+2$  particle phase space

$\Delta_{\text{NLO}} \propto \alpha$  {  $\mathcal{M}_{\text{NLO},V}$  **virtual** one-loop matrix element  
 $\mathcal{M}_{\text{NLO},R}$  **real** tree-level matrix element

$\Delta_{\text{NNLO}} \propto \alpha^2$  {  $\mathcal{M}_{\text{NNLO},V}$  **double-virtual** two-loop matrix element  
 $\mathcal{M}_{\text{NNLO},RV}$  **real-virtual** one-loop matrix element  
 $\mathcal{M}_{\text{NNLO},RR}$  **double-real** tree-level matrix element



# EW uncertainties: QED radiation

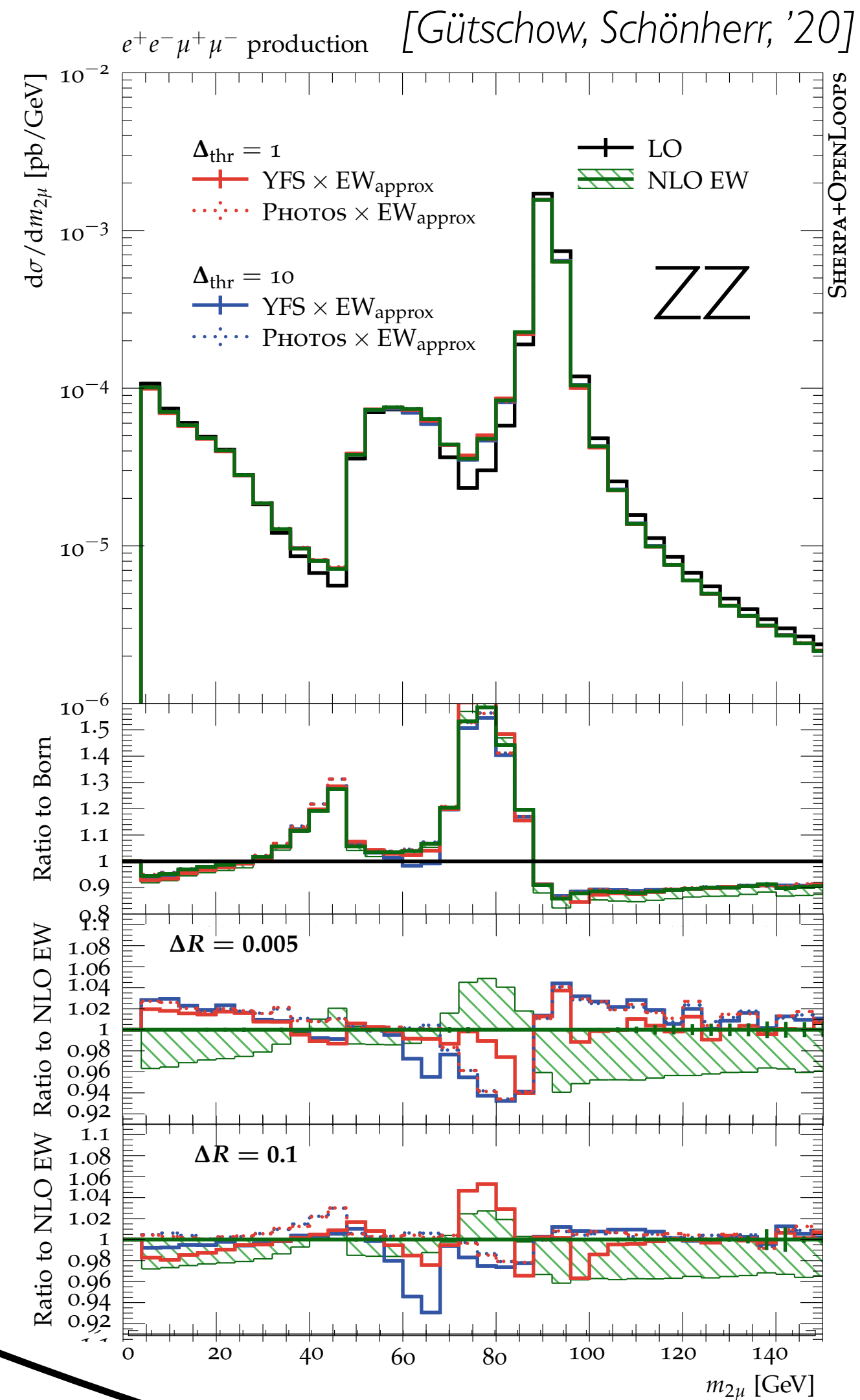
Conservative estimate of higher-order QED radiation:

NLO EW

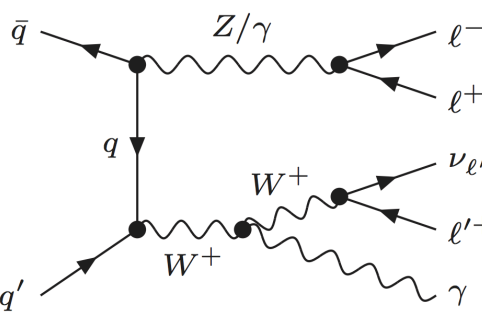
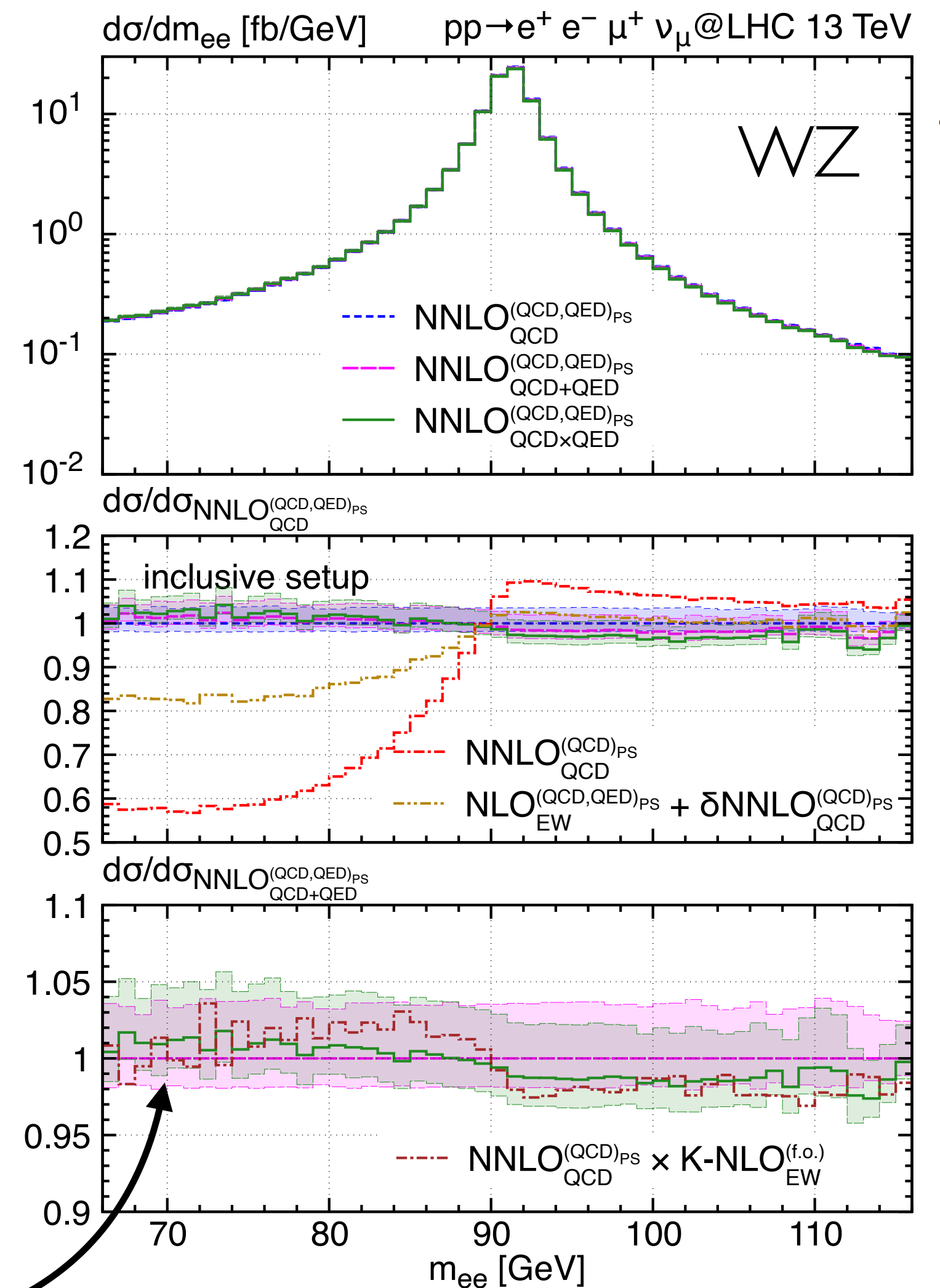
vs.

multi-photon radiation (YFS)  
or  
QED-PS

$$\Delta_{EW}^{QED} = |\delta_{EW} - \delta_{EW+PS/YFS}|$$



[JML, Lombardi, Wiesemann, Zanderighi, Zanolini, '22]



# Combination of QCD and EW corrections

- full calculations of  $\mathcal{O}(\alpha\alpha_s)$  out of reach
- Approximate combination: MEPS@NLO including (approximate) EW corrections
- key: QCD radiation receives EW corrections!
- strategy: modify MC@NLO B-function to include NLO EW virtual corrections and integrated approx. real corrections = VI

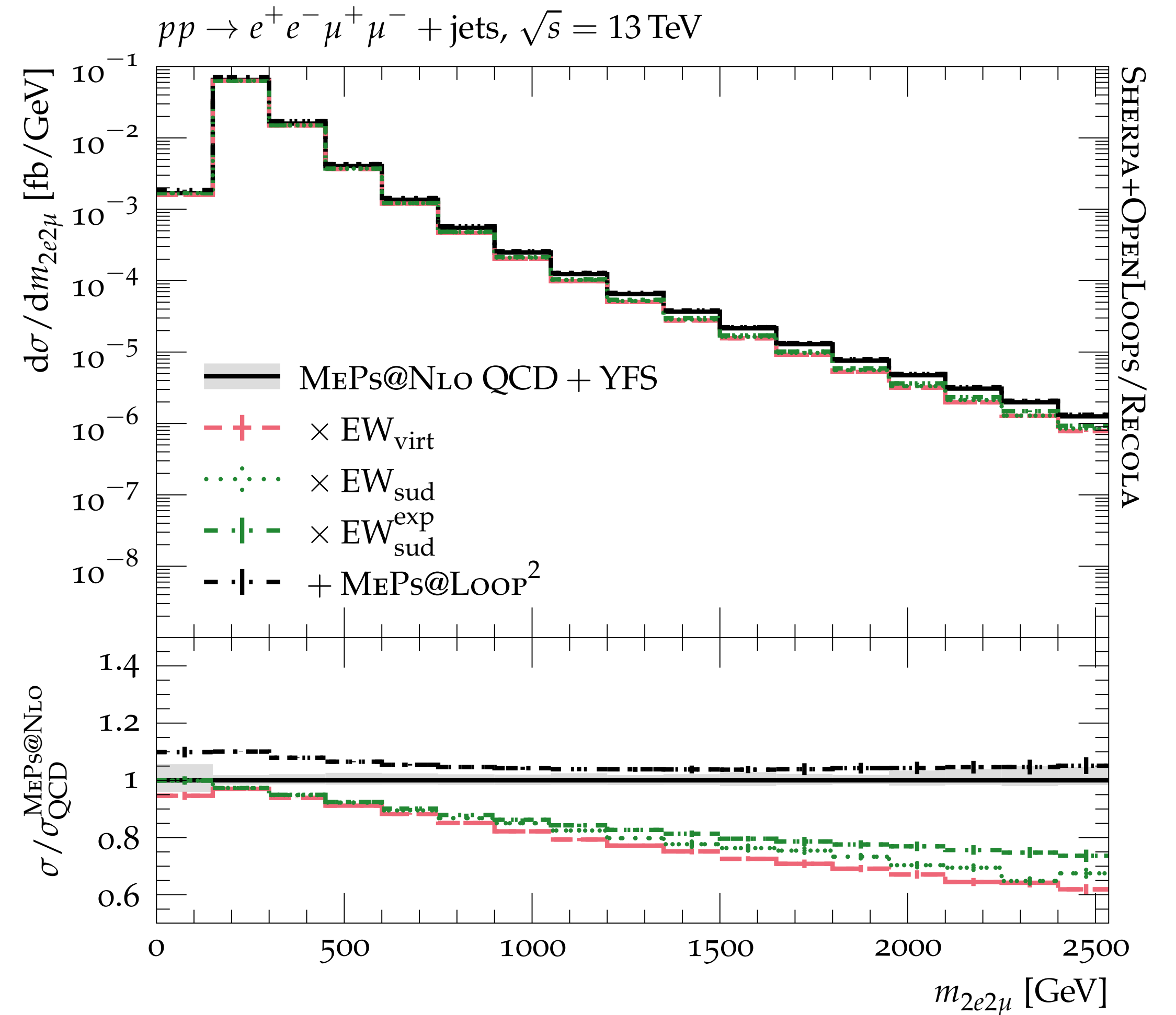
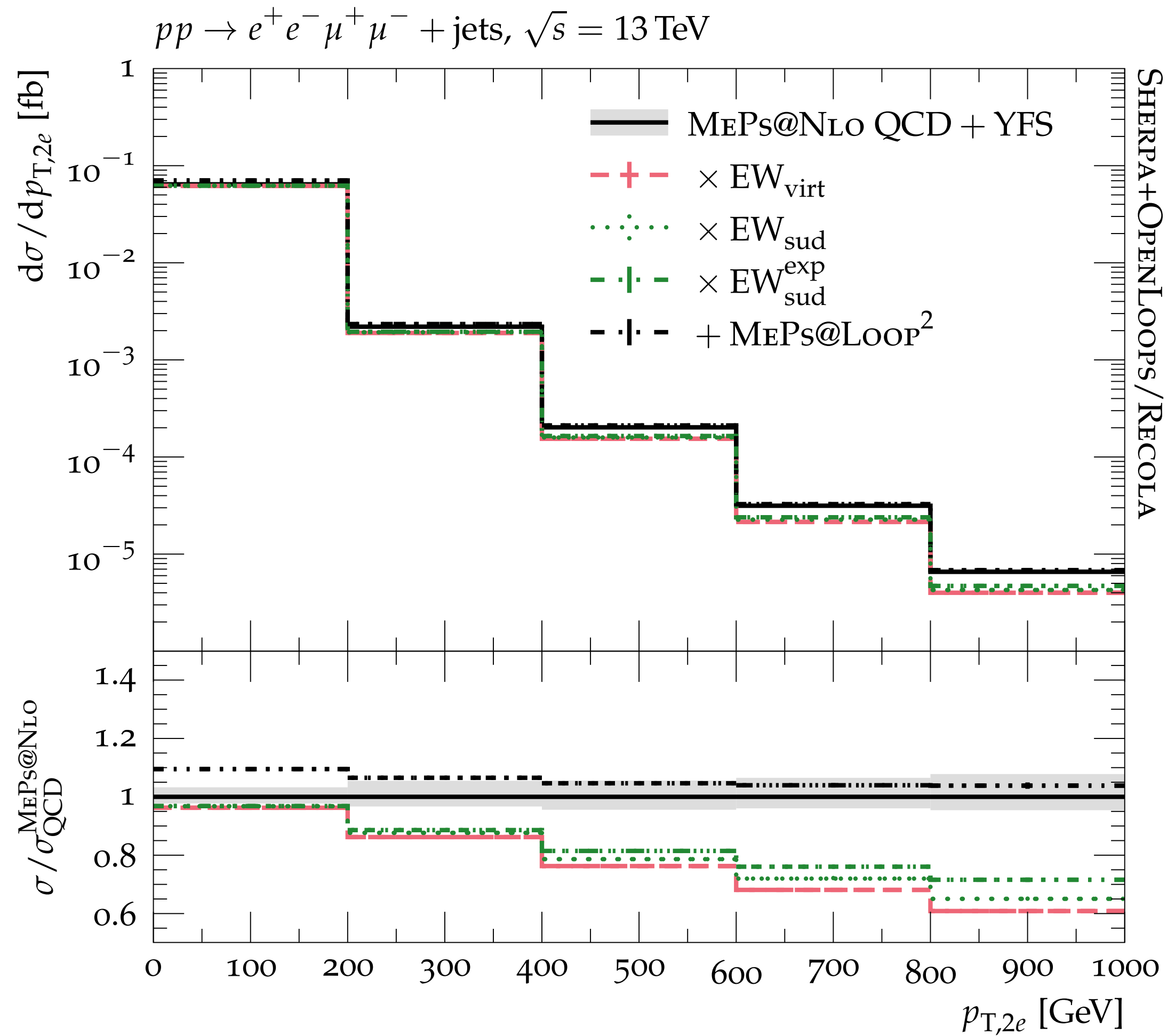
$$\bar{B}_{n,\text{QCD}+\text{EW}_{\text{virt}}}(\Phi_n) = \bar{B}_{n,\text{QCD}}(\Phi_n) + V_{n,\text{EW}}(\Phi_n) + I_{n,\text{EW}}(\Phi_n)$$

exact virtual contribution

approximate integrated real contribution

# MEPS @ NLO QCD + EW: ZZ(+jet)

[Bothmann, Napoletano, Schönherr, Schumann, Villani; '21]



ASSOCIATED CONTRIBUTIONS VARIATIONS EW;