QCD (for Colliders) Lecture 3

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the real world?



GLUON EMISSION FROM A QUARK



Consider an emission with

- ► energy $\mathbf{E} \ll \sqrt{\mathbf{s}}$ ("soft")
- angle θ < 1
 ("collinear" wrt quark)

Examine correction to some hard process with cross section σ_0

$$d\sigma \simeq \sigma_0 \times \frac{2\alpha_s C_F}{\pi} \frac{dE}{E} \frac{d\theta}{\theta}$$

This has a divergence when $E \rightarrow 0$ or $\theta \rightarrow 0$ [in some sense because of quark propagator going on-shell] Probability P_g of emitting gluon from a quark with energy Q:

$$P_g \simeq \frac{2\alpha_s C_F}{\pi} \int^Q \frac{dE}{E} \int^1 \frac{d\theta}{\theta} \Theta(E\theta > Q_0)$$

We cut off the integral for transverse momenta ($p_T \simeq E \theta$) below some non-perturbative threshold Q_0 .

> On the grounds that perturbation theory doesn't apply for $p_T \sim \Lambda_{QCD}$ i.e. language of quarks and gluons becomes meaningless

With this cutoff, the result is

$$P_g \simeq \frac{\alpha_s C_F}{\pi} \ln^2 \frac{Q}{Q_0} + \mathcal{O}\left(\alpha_s \ln Q\right)$$

this is called a "double logarithm" [it crops up all over the place in QCD]

Suppose we're not inclusive — e.g. calculate probability of emitting a gluon

Probability P_g of emitting gluon from a quark with energy Q:

$$P_g \simeq \frac{2\alpha_s C_F}{\pi} \int_{Q_0}^Q \frac{dE}{E} \int_{\frac{Q_0}{E}}^1 \frac{d\theta}{\theta}$$

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Suppose we're not inclusive — e.g. calculate probability of emitting a gluon

Suppose we take $Q_0 \sim \Lambda_{QCD}$, what do we get?

Let's use $a_s = a_s(Q) = 1/(2b \ln Q/\Lambda)$ [Actually over most of integration range this is optimistically small]

$$P_g \simeq \frac{\alpha_s C_F}{\pi} \ln^2 \frac{Q}{Q_0} \to \frac{C_F}{2b\pi} \ln \frac{Q}{\Lambda_{QCD}} \to \frac{C_F}{4b^2 \pi \alpha_s}$$

Put in some numbers:

Q = 100 GeV, $\Lambda_{QCD} \simeq 0.2$ GeV, $C_F = 4/3$, $b(=b_0) \simeq 0.6$

$$P_g \simeq 2.2$$

This is supposed to be an O(α_s) correction. But the final result ~ 1/α_s QCD hates to not emit gluons!

correct way of doing it: with running coupling inside the integral

Adding running coupling is straightforward: just use $\alpha_s(p_t)$ with $p_t = E\theta$, in the integrand:

$$P_g = \frac{2C_F}{\pi} \int_{Q_0}^Q \frac{dp_t}{p_t} \alpha_s(p_t) \int_{p_t/Q}^1 \frac{dz}{z} = \frac{C_F}{\pi b_0} \left(\ln \frac{Q}{\Lambda} \ln \ln \frac{Q}{\Lambda} + \cdots \right)$$

Structure of answer changes a bit: it's larger than $1/a_s(Q)$, by a factor ln ln Q/Λ .

But to keep expressions simple in these lectures we'll often restrict ourselves to a fixed-coupling approximation.

Picturing a QCD event



Start off with a qqbar system a gluon gets emitted at small angles it radiates a further gluon and so forth

Picturing a QCD event



then a non-perturbative transition occurs

giving a pattern of hadrons that "remembers" the gluon branching (hadrons mostly produced at small angles wrt qqbar directions — two **"jets"**)

resummation and parton showers

the previous slides applied in practice

Resummation

Analytical, or semi-numerical, calculation of dominant logarithmically enhanced terms, to all orders in the strong coupling. **Applies when you place a strong constraint on an observable.**

Calculations are often specific to a single observable.

Parton shower Monte Carlo

Simulation of emission of arbitrary number of particles, usually ordered in angle or p_t .

Underlying algorithm should reproduce many of the singular limits of multi-particle QCD amplitudes, including virtual corrections.

Can be used to calculate arbitrary observables.

Resummation: one way of seeing the underlying key idea

Calculate cross section for some observable $v(p_1,...,p_m)$, a function of the event momenta, to be less than some cut V.

Illustrate structure in soft limit, fixed coupling, ignore secondary emissions from soft gluons. *Any number of real gluons*



Resummation example result

- It's common to ask questions like "what is the probability that a Z boson is produced with transverse momentum < p_T"
- Answer is given (~) by a "Sudakov form factor", i.e. the probability of not emitting any gluons with transverse momentum > p_T.

$$P(Z \text{ trans.mom.} < p_T) \simeq \exp\left[-\frac{2\alpha_s C_F}{\pi} \ln^2 \frac{M_Z}{p_T}\right]$$

when p_T is small, the logarithm is large and compensates for the smallness of a_s — so you need to resum log-enhanced terms to all orders in a_s.

What do we know about resummation?

- ➤ You'll sometimes see mention of "NNLL" or similar
- This means next-next-to-leading logarithmic
- ➤ Most common definition of Leading logarithmic (LL): you sum all terms with p=n+1 (for n=1...∞) in

$$\exp\left[-\sum_{n,p} \alpha_s^n \ln^p \frac{M_H}{p_T}\right]$$

- > NLL: include all terms with p=n (for $n=1...\infty$)
- > NNLL: include all terms with p=n-1 (for $n=1...\infty$)

In real life, the function that appears in the resummation is sometimes instead a Fourier or Mellin transform of an exponential

Resummation of Higgs p_T spectrum (same formula, with $C_F \rightarrow C_A$)



Resummation of Z p_T spectrum v. data







Chen et al <u>2203.01565</u>

resummation v. parton showers (the basic idea, ignoring secondary emsn. from gluons)

- ► a resummation predicts **one observable** to high accuracy
- a parton shower takes the same idea of a Sudakov form factor and uses it to generate emissions
- From probability of not emitting gluons above a certain p_T, you can deduce p_T distribution of first emission
- 1. use a random number generator (r) to sample that $p_{\rm T}$ distribution

deduce
$$p_T$$
 by solving $r = \exp\left[-\frac{2\alpha_s C_A}{\pi}\ln^2\frac{p_{T,\max}^2}{p_T^2}\right]$

2. repeat for next emission, etc., until p_T falls below some non-perturbative cutoff

very similar to radioactive decay, with time t ~ 1/p_T and a decay rate ~ (log t) / t

A toy shower

(fixed coupling, primary branching only, only p_T , no energy conservation, no PDFs, etc.)

```
#!/usr/bin/env python3
# an oversimplified (QED-like) parton shower
# for Zuoz lectures (2016) by Gavin P. Salam
from random import random
from math import pi, exp, log, sqrt
ptHigh = 100.0
ptCut = 1.0
alphas = 0.12
CA=3
def main():
    for iev in range(0,10):
        print ("\nEvent", iev)
        event()
def event():
    # start with maximum possible value of Sudakov
    sudakov = 1
    while (True):
        # scale it by a random number
        sudakov *= random()
        # deduce the corresponding pt
        pt = ptFromSudakov(sudakov)
        # if pt falls below the cutoff, event is finished
        if (pt < ptCut): break</pre>
        print (" primary emission with pt = ", pt)
def ptFromSudakov(sudakovValue):
    """Returns the pt value that solves the relation
       Sudakov = sudakovValue (for 0 < sudakovValue < 1)
    .....
    norm = (2*CA/pi)
    \# r = Sudakov = exp(-alphas * norm * L^2)
    \# \rightarrow \log(r) = - \operatorname{alphas} * \operatorname{norm} * L^2
    \# \longrightarrow L^2 = \log(r)/(-alphas*norm)
    L2 = log(sudakovValue)/(-alphas * norm)
    pt = ptHigh * exp(-sqrt(L2))
    return pt
if __name__ == "__main__": main()
```

```
% python ./toy-shower.py
Event 0
 primary emission with pt = 58.4041962726
 primary emission with pt = 3.61999582015
 primary emission with pt = 2.31198814996
Event 1
 primary emission with pt = 32.1881228375
 primary emission with pt = 10.1818306204
 primary emission with pt = 10.1383134201
 primary emission with pt = 7.24482350383
 primary emission with pt = 2.35709074796
 primary emission with pt = 1.0829758034
Event 2
 primary emission with pt = 64.934992001
 primary emission with pt = 16.4122436094
 primary emission with pt = 2.53473253194
```

If you want to play: replace $C_A=3$ (emission from gluons) with $C_F=4/3$ (emission from quarks) and see how pattern of emissions changes (multiplicity, p_T of hardest emission, etc.)

Secondary, tertiary gluons: many showers use colour dipoles (Pythia, Sherpa & option in Herwig)



Original dipole MC: Ariadne (90's)

- Use large-N_C idea of colour structure
- ► Initial $q\bar{q}$ event = 1 colour dipole.
- ➤ Radiated gluon turns 1
 dipole → 2 dipoles
- ► Each dipole then

 radiates independently
 (different colour = no
 interference), creating
 new colour dipoles at
 each step

Event record from a real–world shower (Herwig6 — old shower with compact record)

IHEP ID 9 UQRK 10 CONE 11 GLUON 12 GLUON 13 GLUON 14 GLUON 15 GLUON 16 UD 17 GLUON 18 CONE 19 GLUON 20 UD 21 UORK	IDPDG IST 94 141 0 100 21 2 21 2 21 2 21 2 21 2 21 2 21	MO1 M 4 9 9 9 9 9 9 9 5 17 17	102 DA1 6 11 5 0 12 32 13 34 14 36 15 38 16 40 25 42 6 19 8 0 20 43 21 45 32 46	DA2 16 33 35 37 39 41 21 0 44 44 45	P-X 2.64 -0.27 -1.02 0.25 -0.87 -0.81 -0.19 0.00 -2.23 0.77 1.60 0.00 0.63	P-Y -9.83 0.96 3.59 1.46 1.62 4.17 -1.01 0.00 0.44 0.64 0.58 0.00 -1.02	P-Z 592.2 0.1 5.6 3.6 4.7 3611.7 1727.7 1054.6 -233.5 0.2 -2.1 -2687.6 -4076.9	ENERGY 590.2 1.0 6.7 4.0 5.1 3611.7 1727.7 1054.6 232.8 1.0 2.8 2687.6 4076.9	MASS -49.07 0.00 0.75- 0.75- 0.75- 0.75- 0.75- 0.32- -18.36 0.00 0.75 0.32 0.32	INITIAL STATE SHOWER
22 ZO/GAMA* 23 UQRK 24 CONE 25 UQRK 26 GLUON 27 GLUON 28 GLUON 30 GLUON 31 GLUON	23 195 94 144 0 100 2 2 21 2 21 2 21 2 21 2 21 2	7 8 23 23 23 23 23 23	22 251 6 25 5 0 26 47 27 48 28 50 29 52 30 54 31 56 43 58	252 - 31 42 49 51 53 55 57 59	-257,66 258,06 0,21 26,82 8,50 73,27 73,66 67,58 6,98 1,24	-219.68 210.29 0.17 24.33 8.18 61.24 58.54 52.13 4.60 1.26	324.8 33.9 -1.0 23.7 6.0 12.0 -6.3 -7.3 2.3 3.6	477.5 345.5 1.0 43.3 13.3 96.2 94.3 85.7 8.7 4.1	88,56 86,10 0,00 0,32 0,75 0,75 0,75 0,75 0,75 0,75	FINAL STATE SHOWER

L

proton

U

proton

simulations use General Purpose Monte Carlo event generators THE BIG 3



Herwig 7

Pythia 8

Sherpa 3

used in ~95% of ATLAS/CMS publications they do an amazing job of simulation vast swathes of data; collider physics would be unrecognisable without them

combining showers & fixed order

essential for accurate cross sections & multijet states $\stackrel{\wedge}{=} \stackrel{\circ}{E.g.}$ jet multiplicity in events with a W $\stackrel{\rightarrow}{V.Pythia} \stackrel{\circ}{GeV}$



shower MCs on their own cannot reproduce pattern of hard multijet states

(there are topologies that are almost inaccessible via showering)





shower Z+parton

.















- ► Hard jets above scale Qmerge have distributions given by tree-level ME
- Rejection procedure eliminates "double-counted" jets from parton shower
- Rejection generates Sudakov form factors between individual jet scales

An alternative approach is called **CKKW** (similar in spirit, Sudakov put in manually)

Combining NLO accuracy with parton showers (1)

MC@NLO ideas

Frixione & Webber '02

Expand your Monte Carlo branching to first order in α_s

Rather non-trivial – requires deep understanding of MC

- Calculate differences wrt true $\mathcal{O}(\alpha_s)$ both in real and virtual pieces
- If your Monte Carlo gives correct soft and/or collinear limits, those differences are finite
- Generate extra partonic configurations with phase-space distributions proportional to those differences and shower them

$$\mathsf{MC@NLO} = \mathsf{MC} \times \left(1 + \alpha_{\mathsf{s}}(\sigma_{1V} - \sigma_{1V}^{MC}) + \alpha_{\mathsf{s}} \int dE(\sigma_{1R}(E) - \sigma_{1R}^{MC}(E)) \right)$$

All weights finite, but can be ± 1

almost any process can be generated automatically in MadGraph5_aMCatNLO (+ Pythia); also in Sherpa & Herwig

POWHEG ideas

Aims to work around MC@NLO limitations

- the (small fraction of) negative weights
- the tight interconnection with a specific MC

Principle

Write a simplified Monte Carlo that generates just one emission (the hardest one) which alone gives the correct NLO result.

Essentially uses special Sudakov

 $\Delta(k_t) = \exp(-\int \text{exact real-radition probability above } k_t)$

• Lets your default parton-shower do branchings below that k_t .

most processes available in the POWHEGBox (+Pythia or Herwig; or natively in Herwig)

Nason '04

- (Much) more efficient ways of combining tree-level and showers: Vincia
- Getting shower samples that are simultaneously NLO accurate at different multiplicities (FxFx, Sherpa NLO matching)
- Showers with NNLO fixed order: MiNNLO, Geneva, [UNNLOPS]
- ➤ Showers that are NLL accurate: PanScales, Alaric, Apollo, FHP
- Steps towards NNLL accurate showers: PanScales
- Understanding interplay of matching & log accuracy, subleading colour accuracy in showers, etc.



- Modern tools give
 good predictions for
 multijet rates with
 vector bosons
- (up to ~ 4 jets, sometimes beyond)



hadronisation & MPI

essential models for realistic events i.e. events with hadrons

two main models for the parton-hadron transition ("hadronisation")



String Fragmentation (Pythia and friends)

Cluster Fragmentation (Herwig) (& Sherpa)

Pictures from ESW book

multi-parton interactions (MPI, a.k.a. underlying event)

(a) Z (b) \mathbf{Z} (c)

figures taken from <u>2307.05693</u>

Additional 2→2 scatterings of multiple other partons in the incoming protons

Models such as Pythia have ~ 10 MPI scatterings per hard pp collision

Underlying event properties v. MCs



 N_T = charged particle multiplicity in the transverse region







While you can see jets with your eyes, to do quantitative physics, you need an algorithmic procedure that defines what exactly a jet is

make a choice, specify a Jet Definition



- Which particles do you put together into a same jet?
- How do you recombine their momenta (4-momentum sum is the obvious choice, right?)

"Jet [definitions] are legal contracts between theorists and experimentalists" -- MJ Tannenbaum

They're also a way of organising the information in an event 1000's of particles per events, up to 40.000,000 events per second



projection to jets should be resilient to QCD effects





2 clear jets

3 jets?



2 clear jets

3 jets? or 4 jets?

Jet definition ingredients

Jet algorithm

A set of rules that you apply to combine particles into jets

Jet algorithm parameters

Thresholds that help specify when two particles belong to the same jet or not.

Most hadron collider jet algorithms have two threshold parameters:

► Jet angular radius parameter R:

particles closer in angle than R get recombined (NB: usually implemented as a condition on the distance parameter on the standard hadron collider rapidity-azimuth $[y, \phi]$ cylinder)

Transverse momentum threshold:

jets should have $p_T > p_{T,min}$

$$\Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$
$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}$$

A Sequential recombination algorithm

Involves calculating "clustering distance" between pairs of particles

$$d_{ij} = \frac{1}{\max(p_{ti}^2, p_{tj}^2)} \frac{\Delta R_{ij}^2}{R^2}, \qquad d_{iB} = \frac{1}{p_{ti}^2}$$

- 1. Find smallest of d_{ij} , d_{iB}
- 2. If *ij*, recombine them
- 3. If *iB*, call i a jet and remove from list of particles
- 4. repeat from step 1 until no particles left
 - Only use jets with $p_t > p_{t,min}$

anti-k_t algorithm Cacciari, GPS & Soyez, 0802.1189

Anti-kt jet clustering example

Anti-k_t gradually makes its way through the "blob" at rapidity 2.5–3 Dominant hierarchy in clustering is distance from the jet core

$$d_{ij} = \frac{1}{\max(p_{ti}^2, p_{tj}^2)} \frac{\Delta R_{ij}^2}{R^2}$$

$$d_{iB} = \frac{1}{p_{ti}^2} \frac{[here \ R = 2.0]}{p_{T,min} = 20 \ GeV}$$

$$\Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2 \quad y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}$$

anti-kt in action [full simulated event]

Clustering grows around hard cores

$$d_{ij} = \frac{1}{\max(p_{ti}^2, p_{tj}^2)} \frac{\Delta R_{ij}^2}{R^2}, \quad d_{iB} = \frac{1}{p_{ti}^2}$$



Anti-kt gives circular jets ("cone-like") in a way that's infrared safe

Seeing W's and tops in a single high-pT jet

W's in a single jet



tops in a single jet





using full jet/event information for H/W/Z-boson tagging

F. Dreyer & H. Qu, <u>2012.08526</u>



Gavin Salam (Oxford)

conclusions

ATLAS H \rightarrow WW* ANALYSIS [1604.02997]

3 Signal and background models

The ggF and VBF production modes for $H \rightarrow WW^*$ are modelled at next-to-leading order (NLO) in the strong coupling α_S with the Powneg MC generator [22–25], interfaced with PYTHIA8 [26] (version 8.165) for the parton shower, hadronisation, and underlying event. The CT10 [27] PDF set is used and the parameters of the Pythia8 generator controlling the modelling of the parton shower and the underlying event are those corresponding to the AU2 set [28]. The Higgs boson mass set in the generation is 125.0 GeV, which is close to the measured value. The Powneg ggF model takes into account finite quark masses and a running-width Breit–Wigner distribution that includes electroweak corrections at NLO [29]. To improve the modelling of the Higgs boson $p_{\rm T}$ distribution, a reweighting scheme is applied to reproduce the prediction of the next-to-next-to-leading-order (NNLO) and next-to-next-to-leading-logarithm (NNLL) dynamic-scale calculation given by the HREs 2.1 program [30] Events with ≥ 2 jets are further reweighted to reproduce the p_T^H spectrum predicted by the NLO Powneg simulation of Higgs boson production in association with two jets (H + 2 jets) [31]. Interference with continuum WW production [32, 33] has a negligible impact on this analysis due to the transverse-mass selection criteria described in Section 4 and is not included in the signal model.

Jets are reconstructed from topological clusters of calorimeter cells [50–52] using the anti- k_t algorithm with a radius parameter of R = 0.4 [53]. Jet energies are corrected for the effects of calorimeter non-

WHAT DO ATLAS & CMS USE MOST FREQUENTLY?



WHAT DO ATLAS & CMS USE MOST FREQUENTLY?

in the last 2 lectures we've seen a good number of the tools used at LHC



CONCLUSIONS

- A huge number of ingredients goes into hadron-collider predictions and studies (a_s, PDFs, matrix elements, resummation, parton showers, non-perturbative models, jet algorithms, etc.)
- ➤ a key idea is the separation of (time) scales, "factorisation"
 - short timescales: the hard process
 - Iong timescales: hadronic physics
 - ► in between: parton showers, resummation, DGLAP
- as long as you ask the right questions (e.g. look at jets, not individual hadrons), you can exploit this separation for quantitative, accurate, collider physics
- maximising accuracy and information extracted is today's research frontier

Introductory level

QCD lecture notes from CERN schools, e.g.

- ► Peter Skands, <u>arXiv:1207.2389</u>
- ► GPS, <u>arXiv:1011.5131</u> (getting increasingly dated!)

More advanced

Slides from QCD and Monte Carlo specific schools

- CTEQ schools: <u>https://www.physics.smu.edu/scalise/cteq/#Summer</u>
- MCNet schools: <u>https://www.montecarlonet.org/schools/</u>
 <u>Books</u>
- QCD and collider physics, Ellis, Stirling & Webber
- The Black Book of Quantum Chromodynamics, Campbell, Huston & Krauss

intentionally blank