Neil Turok Lecture 1

the universe is an amazing natural laboratory for fundamental physics

Energy

10^{-33}	10^{-4}	10^{13}	10^{27}	eV
de Sitter	CMB Temperature	LHC cm energy	Planck	
H_Λ	$T_{CMB,0}$	s	M_{Pl}	
10^{26}	10^{-3}	10^{-20}	10^{-34}	m
largest visible			smallest visible	

Length = 1/Energy

FLRW metric: homogeneous and isotropic 3-spac

$$ds^2 = -dt^2 + a(t)^2 \gamma_{ij} dx^i dx^j; \quad a(t_0) \equiv 1$$

proper time — comoving coordinates: for a maximally symmetric space

Purely kinematical effects

$$R^{(3)} = 6\kappa \begin{cases} > 0 \text{ sphere} \\ = 0 \text{ torus} \\ < 0 \text{ hyperboloid} \end{cases}$$

1. Redshifting of wavelengths and momenta

   e.g. photons

$$\lambda \propto a(t) \Rightarrow \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}} = \frac{a_{obs}}{a_{em}} - 1 \equiv Z_{em} \quad \text{redshift} \quad a_{em} \propto 1/(1+Z)$$

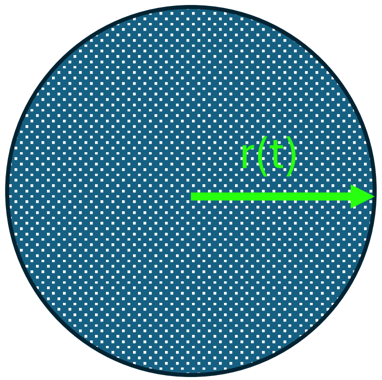
likewise the de Broglie wavelength and momentum of massive particles

2. Distances: objects which comove with expansion obey

$d \propto a(t) \Rightarrow \frac{\dot{d}}{d} = \frac{\dot{a}}{a} = H(t)$: recession velocity $\dot{d} \propto$ distance d Hubble's Law

Friedmann-Lemaitre-Roberston-Walker Dynamics

Newtonian derivation



$$\frac{1}{2}\dot{r}^2 - \frac{GM}{r} = \frac{E}{m} = const; \; M = \frac{4}{3}\pi r^3 \rho = const$$

$$\Rightarrow r \propto a(t); \;\; \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho + \frac{const}{a^2}$$

Einstein's derivation: $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$

where, for a perfect fluid at rest in comoving coords, $T_\mu{}^\nu = \begin{pmatrix} -\rho & 0 & 0 & 0 \\ 0 & & & \\ 0 & & P(\rho)\delta^i_j & \\ 0 & & & \end{pmatrix}$

Equation of state

G_{00} equation is a *constraint:* $3\frac{\dot{a}^2}{a^2} - 3\frac{\kappa}{a^2} = 8\pi G\rho$ Friedmann

(involves only first time derivatives)

Conservation of energy-momentum $\nabla_\mu T^{\mu\nu} = 0 \Rightarrow \dot{\rho} = -3\frac{\dot{a}}{a}(P+\rho)$

(dE=-P dV)

Types of energy:

1. cosmological constant $T_{\mu\nu} = -\Lambda\, g_{\mu\nu} \Rightarrow P = -\rho$
2. matter (nonrelativistic) $P = 0 \Rightarrow \rho \propto a^{-3}$ (approx.)
3. Radiation (relativistic) $P = \frac{1}{3}\rho$ (or, more fundamentally, $T^\mu_\mu = 0$)

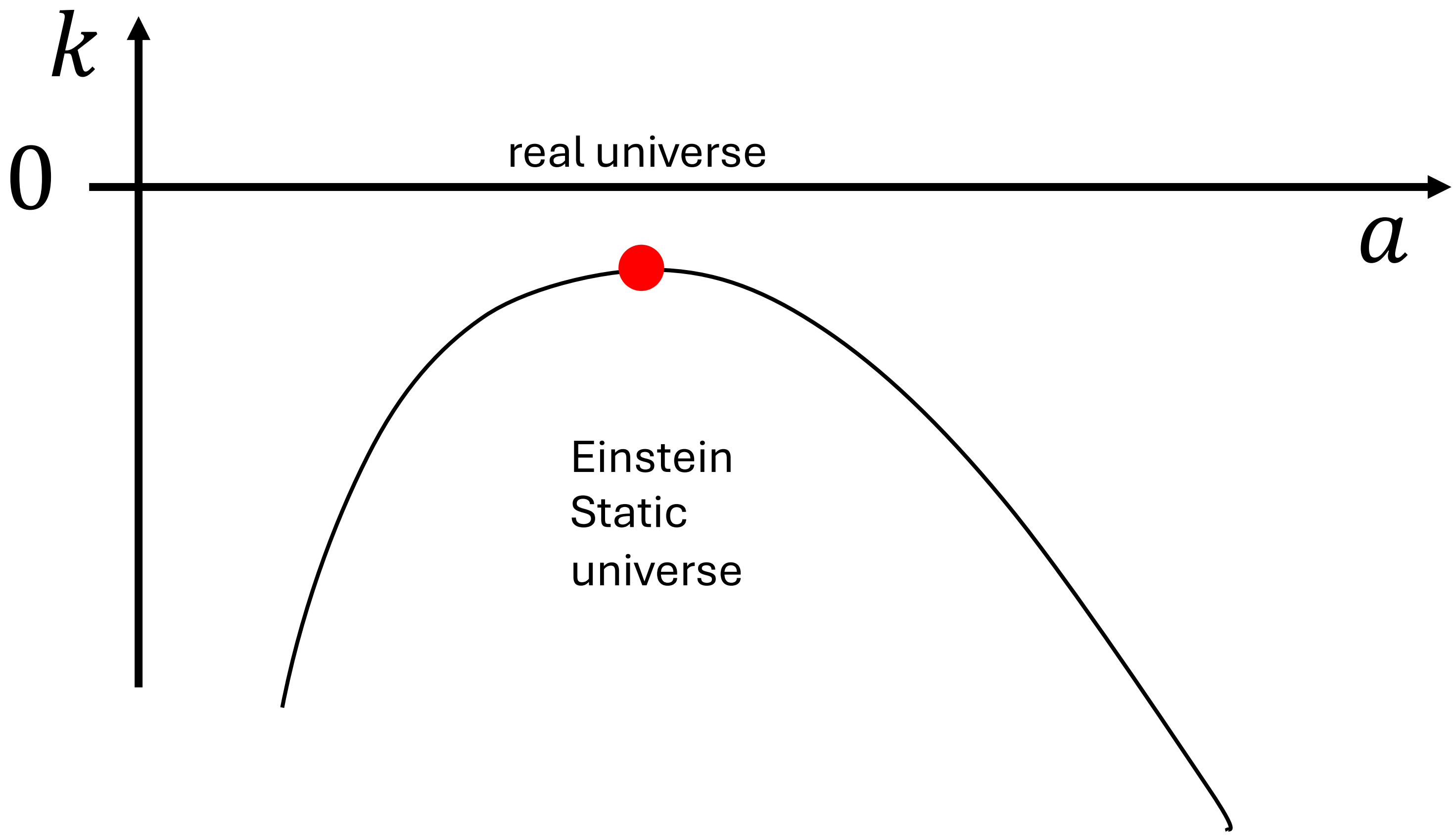
conformal symmetry

$\Rightarrow \rho \propto a^{-4}$

i.e., energy per photon $E_\gamma \propto \lambda^{-1} \propto a^{-1}$

So Friedmann reads $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_\Lambda - \frac{\kappa}{a^2} + \rho_m + \rho_r) = H_0^2\left(\Omega_\Lambda + \frac{\Omega_\kappa}{a^2} + \frac{\Omega_m}{a^3} + \frac{\Omega_r}{a^4}\right)$

i.e., $\dot{a}^2 - \lambda a^2 - m a^{-1} - r a^{-2} = k$, with λ, m, r, k *const*



FLRW metric: homogeneous and isotropic 3-spac

$$ds^2 = -dt^2 + a(t)^2 \gamma_{ij} dx^i dx^j; \quad a(t_0) \equiv 1$$

proper time

comoving symmetric space

$$R^{(3)} = 6\kappa \begin{cases} > 0 \text{ sphere} \\ = 0 \text{ torus} \\ < 0 \text{ hyperboloid} \end{cases}$$

Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_{tot} - \frac{\kappa}{a^2}$$

$$= H_0^2\left(\Omega_\Lambda + \frac{\Omega_\kappa}{a^2} + \frac{\Omega_m}{a^3} + \frac{\Omega_r}{a^4}\right)$$

Lambda space curvature matter radiation

$$(\Omega_\Lambda + \Omega_\kappa + \Omega_m + \Omega_r \equiv 1)$$

Today,

$H_0 \equiv 100h$ km s^{-1} Mpc^{-1} ; $h = 0.7 \pm 0.05$; 1pc=3.26 light years

Hubble time: $H_0^{-1} = 9.78\times10^9 h^{-1}$ years

Hubble radius: $H_0^{-1} c = 2998\ h^{-1}$ Mpc

Critical density: $\rho_c = \frac{3}{8\pi G} H_0^2 = 2.8\times10^{11} h^2\ M_\odot$ Mpc^{-3}

Radiation density $\rho_{r,0} = 2\frac{\pi^2}{30} T_0^4 (1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_\nu) \Rightarrow \Omega_r = 8.5\times10^{-5} h_{.7}^{-2}$

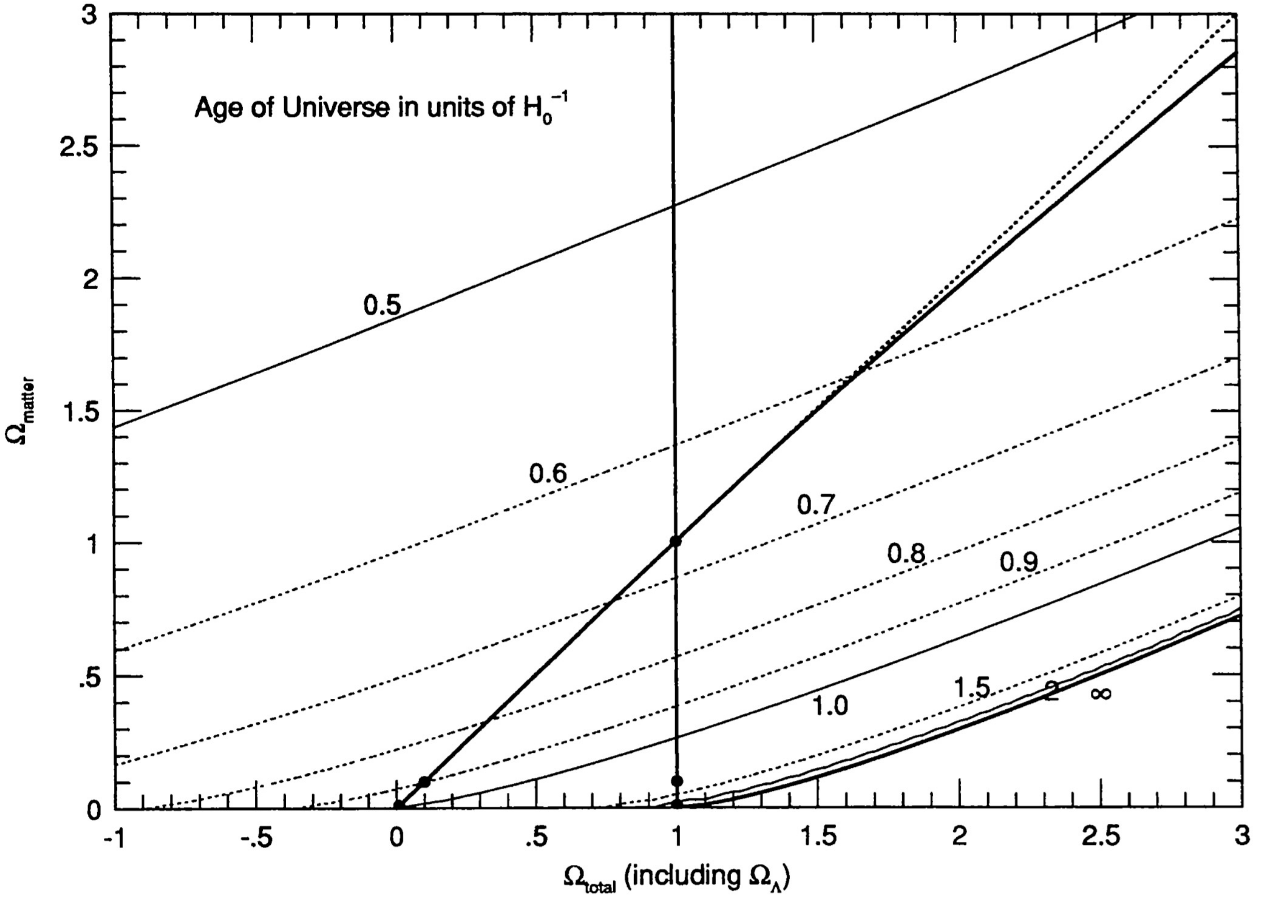
0.68 for 3 light ν's

(reheating of photons due to $e^\pm$ annihilation

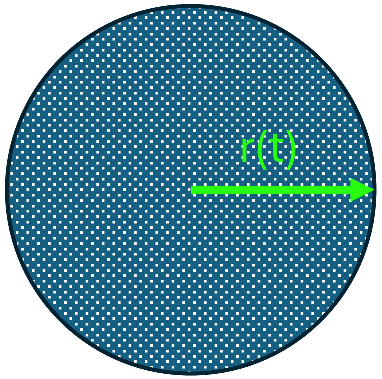
Exact solution for LCDM model (neglecting radiation and spatial curvature)

$$a(t) = \left(\frac{1-\Omega_\Lambda}{\Omega_\Lambda}\right)^{1/3} \left(\sinh \tfrac{3}{2} H_0 \Omega_\Lambda^{1/2} t\right)^{2/3} \Rightarrow t_0 = \tfrac{2}{3} H_0^{-1} \frac{1}{\sqrt{\Omega_\Lambda}} \sinh^{-1}\left(\sqrt{\Omega_\Lambda/(1-\Omega_\Lambda)}\right)$$

(recommended exercise!)



Cosmological perturbations ("Newtonian" derivation)



$$\frac{1}{2}\dot{r}^2 - \frac{GM}{r} - \frac{4\pi G}{3} r^2 \rho_\Lambda = const$$

1. $r \to r + \delta r \Rightarrow \dot{r}\dot{\delta r} + \frac{GM}{r^2}\delta r - \frac{8\pi G}{3} r \delta r \rho_\Lambda = \delta k$

2. $\frac{d}{dt} \Rightarrow \ddot{r} = -\frac{GM}{r^2} + \frac{8\pi G}{3}\rho_\Lambda r \Rightarrow \frac{d}{dt}\left(\frac{\delta r}{r}\right) = \frac{\ddot{r}}{\dot{r}}\frac{\delta r}{r} - \frac{\dot{r}\delta r}{r\, r} + \frac{\delta k}{r\dot{r}}$

Fractional matter density perturbation $\delta = -3\frac{\delta r}{r} \quad \Rightarrow \dot{\delta} + \left(\frac{\dot{a}}{a} - \frac{\ddot{a}}{\dot{a}}\right)\delta = \frac{const}{a\dot{a}} \quad \Rightarrow \delta = A\,\frac{\dot{a}}{a} + const\,\frac{\dot{a}}{a}\int_0^t \frac{dt}{\dot{a}^2};$

decaying mode — growing mode

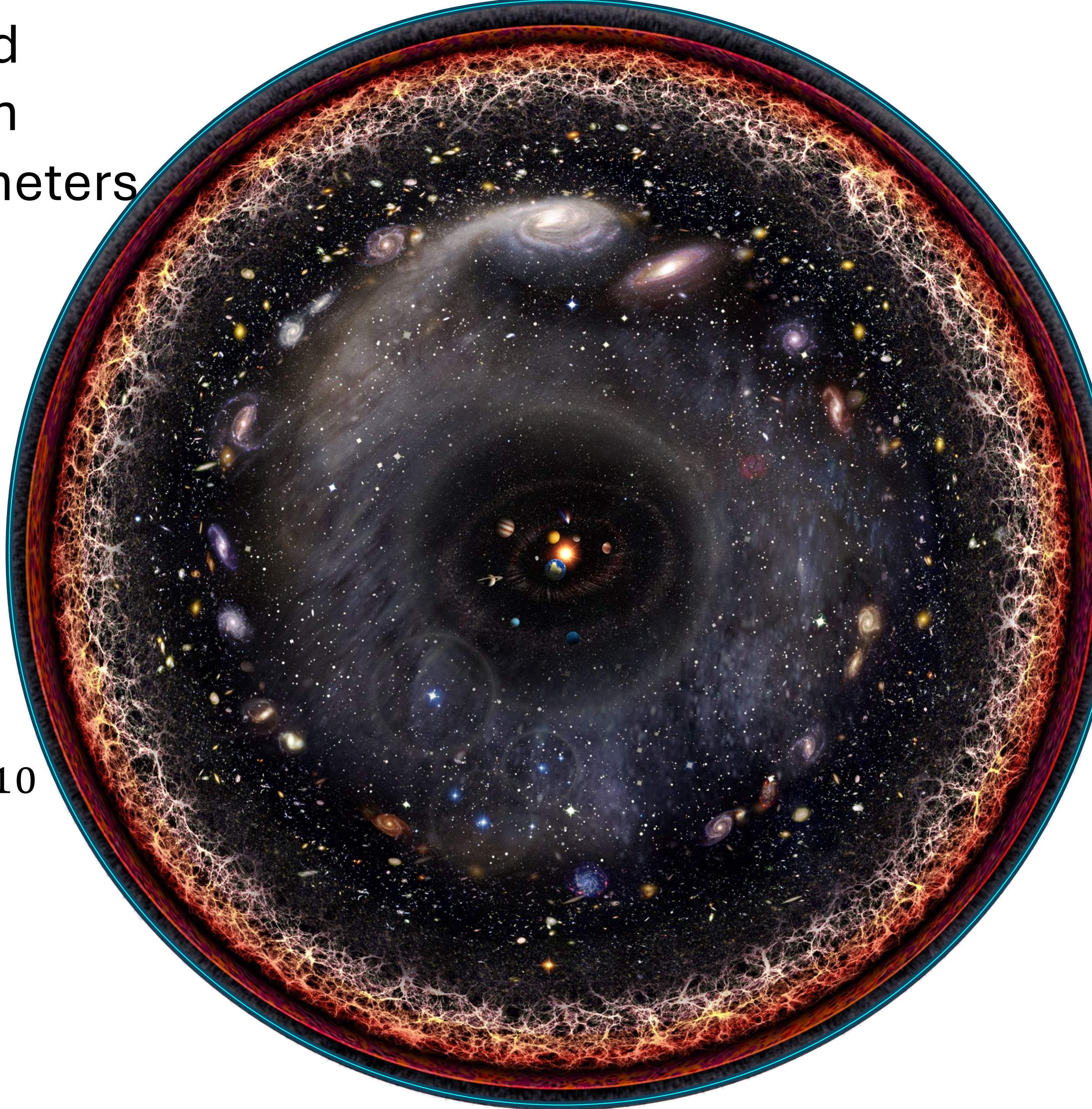
(recommended exercise!)

$$\delta_m \propto \frac{5}{2}\Omega_m\sqrt{\Omega_\Lambda + \Omega_m a^{-3}}\int_0^a \frac{da}{a^3(\Omega_\Lambda + \Omega_m a^{-3})^{\frac{3}{2}}} \approx a\,, \qquad a \ll \left(\frac{\Omega_m}{\Omega_\Lambda}\right)^{1/3},$$

$$\approx 1.43\left(\frac{\Omega_m}{\Omega_\Lambda}\right)^{1/3}, \quad a \gg \left(\frac{\Omega_m}{\Omega_\Lambda}\right)^{1/3}$$

Newtonian potential $a^{-2}\,\nabla^2\Phi = 4\pi G\rho_m\delta_m \Rightarrow \Phi \sim const,\ a^{-1}$ respectively

LCDM provides a remarkably good fit to the large-scale universe, with just 5 fundamental physics parameters



the energy content

1. $\rho_\Lambda \;\; = (2.3\ \text{meV})^4$ ($\pm$ 1%)
2. $\rho_{DM}/\rho_B = 5.36$ ($\pm$ 1%)
3. $n_B/n_\gamma = 6{\times}10^{-10}$ ($\pm$ 1%)

the perturbations

$$\langle\Phi^2\rangle = \int \frac{dk}{k} A_\Phi \left(\frac{k}{k_*}\right)^{n_s-1}$$

4. amplitude $A_\Phi \approx 7.6 \pm 0.1{\times}10^{-10}$
5. “tilt” $n_s{-}1 \approx -0.041 \pm 0.0056$

many parameters so far consistent with zero: tensor and “isocurvature” perturbations, spatial curvature κ, non-Gaussianity…