Cosmology à Peebles

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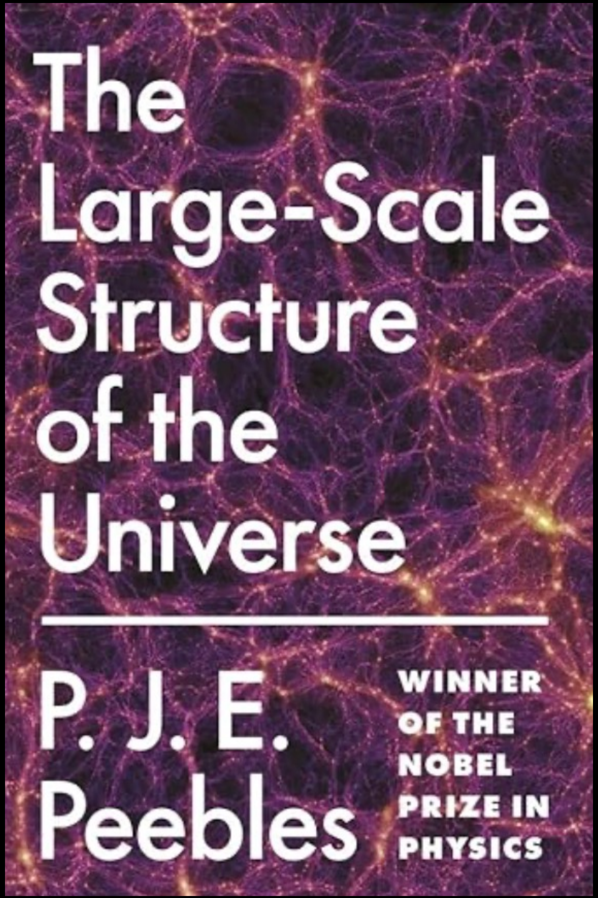
University of Edinburgh and Perimeter Institute

October 6 and 7, 2024

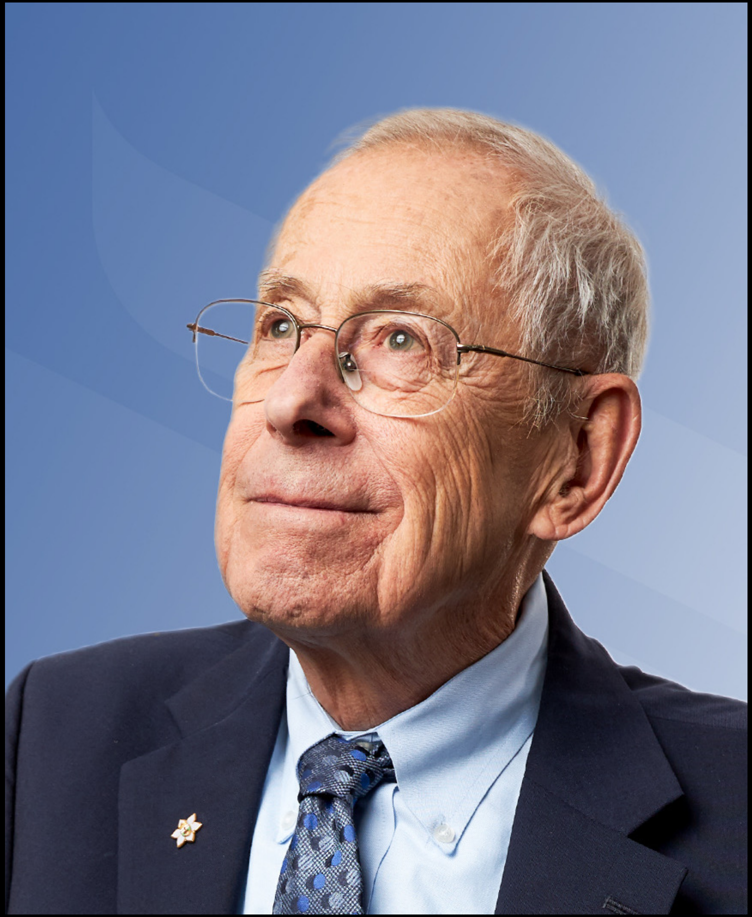
Lecture 1: overview of standard cosmology
Lecture 2: a minimal SM/LCDM cosmology

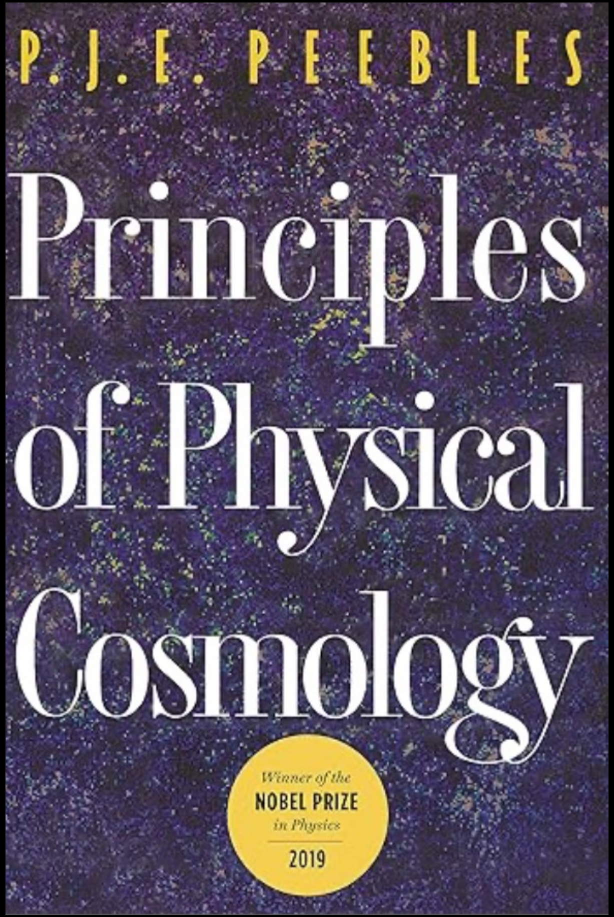


The Large-Scale Structure of the Universe

P. J. E. Peebles

WINNER OF THE NOBEL PRIZE IN PHYSICS





P. J. E. PEEBLES

Principles of Physical Cosmology

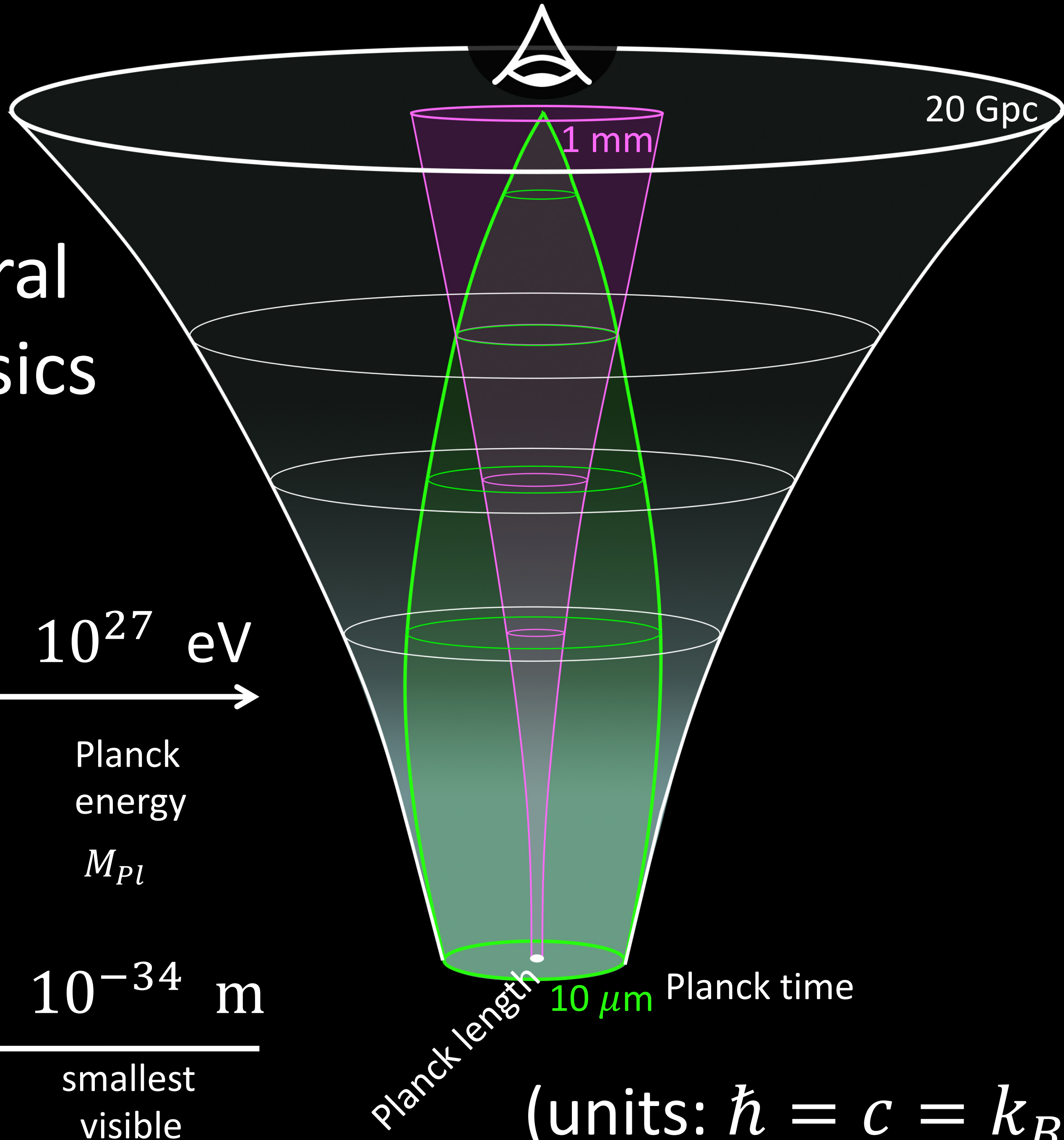
Winner of the NOBEL PRIZE in Physics 2019

the universe is an amazing natural laboratory for fundamental physics

Energy

10^{-33}	10^{-4}	10^{13}	10^{27}	eV
de Sitter expansion rate	CMB Temperature	LHC cm energy	Planck energy	
H_Λ	$T_{\text{CMB},0}$	$\sqrt{s}$	M_{Pl}	
10^{26}	10^{-3}	10^{-20}	10^{-34}	m
largest visible			smallest visible	

Length = 1/Energy



(units: $\hbar = c = k_B$

There are many profound puzzles and paradoxes:

Why is there an apparent arrow of time?

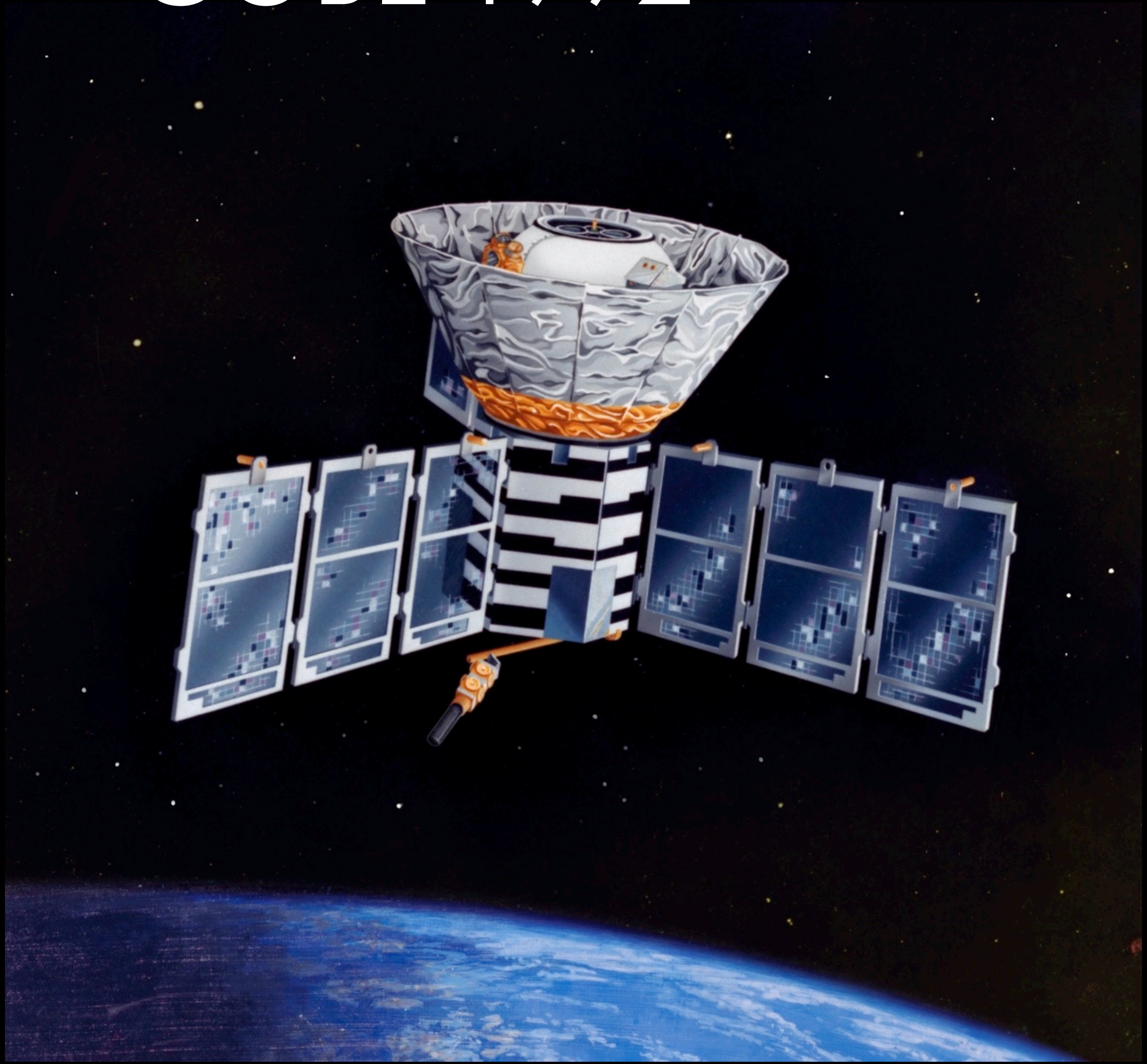
How could the universe have emerged from a single point?

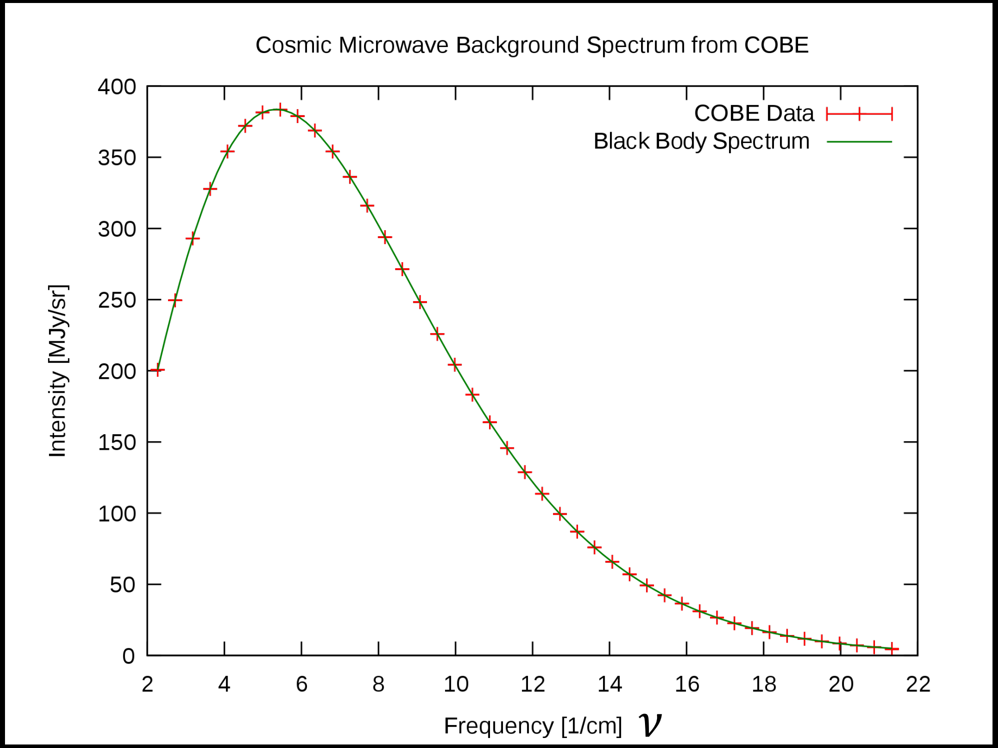
Why are we heading towards a strange “vacuous” future?

Why is the universe so incredibly simple on large scales?

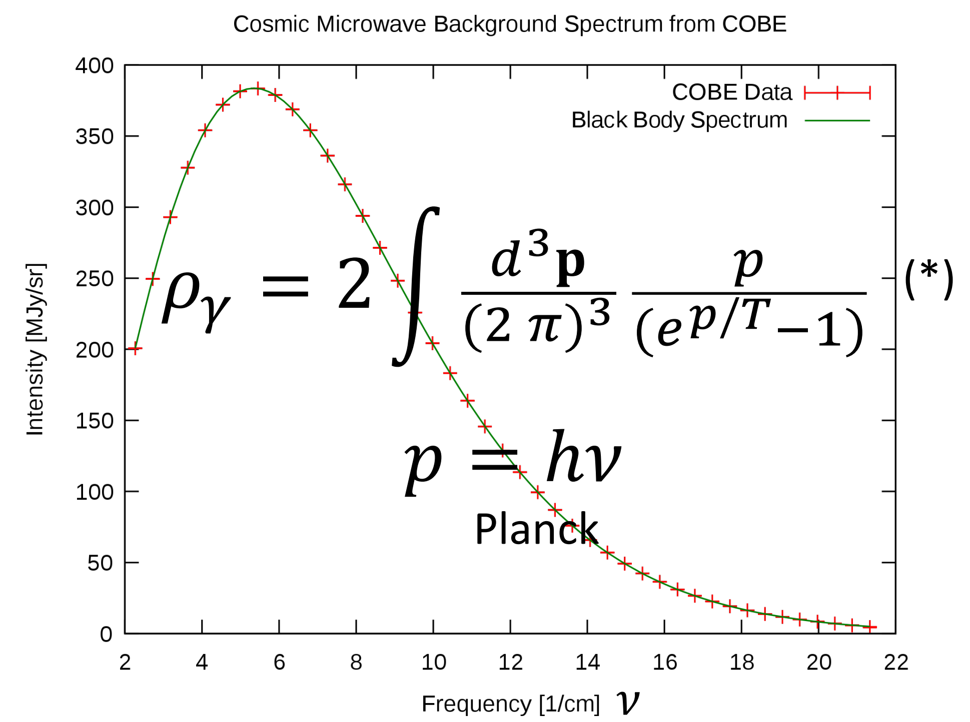
With recent data, we may be on the brink of new understanding …

COBE 1992




Cosmic Microwave Background Spectrum from COBE
COBE Data
Black Body Spectrum
Intensity [MJy/sr]
400
350
300
250
200
150
100
50
0
2
4
6
8
10
12
14
16
18
20
22
Frequency [1/cm] ν

COBE 1992



$$\rho_\gamma = 2\int \frac{d^3\mathbf{p}}{(2\,\pi)^3}\,\frac{p}{(e^{p/T}-1)} \quad (*)$$

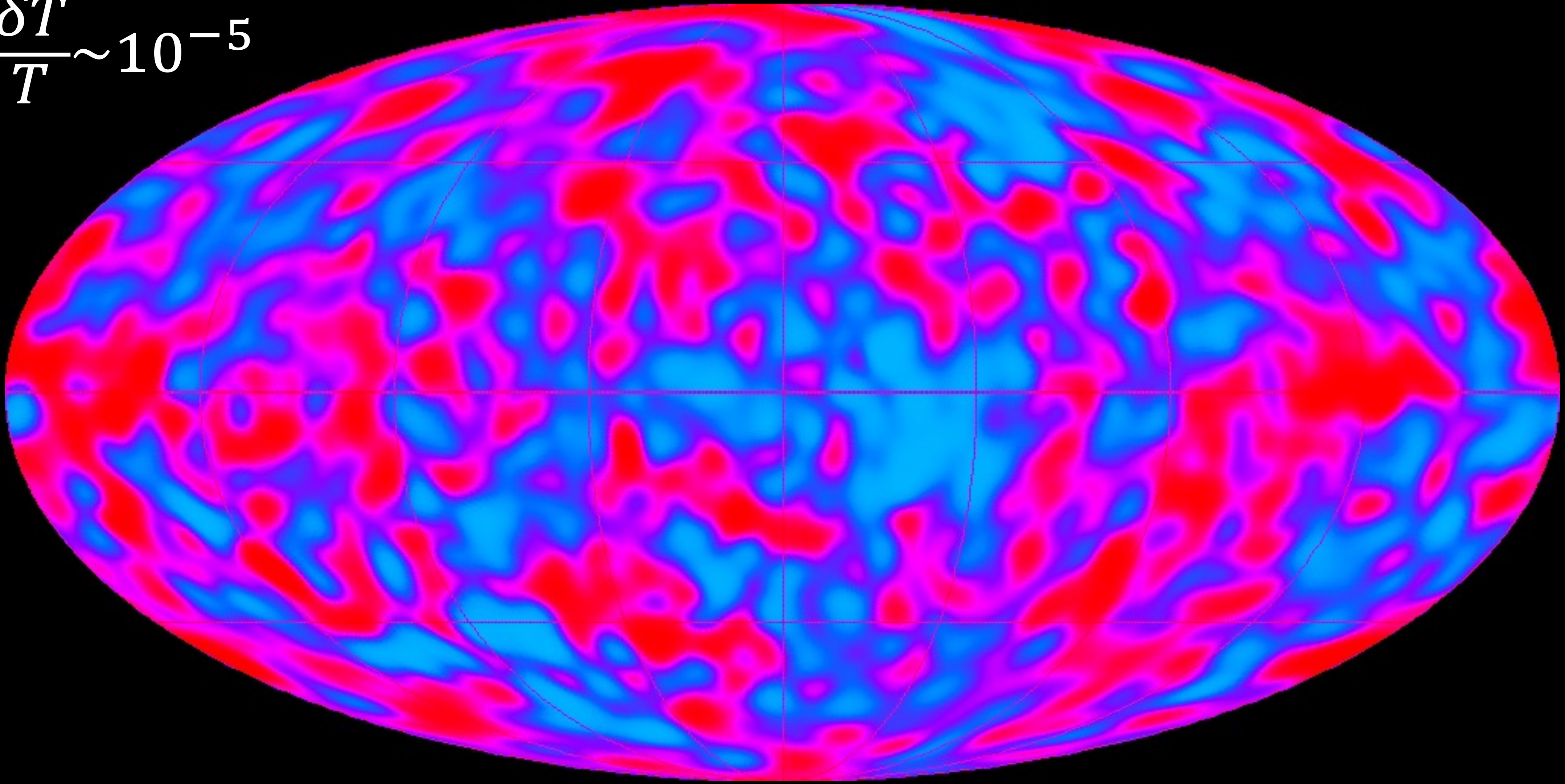
$$p = h\nu$$

Planck

quantum mechanics works on the largest accessible scales!

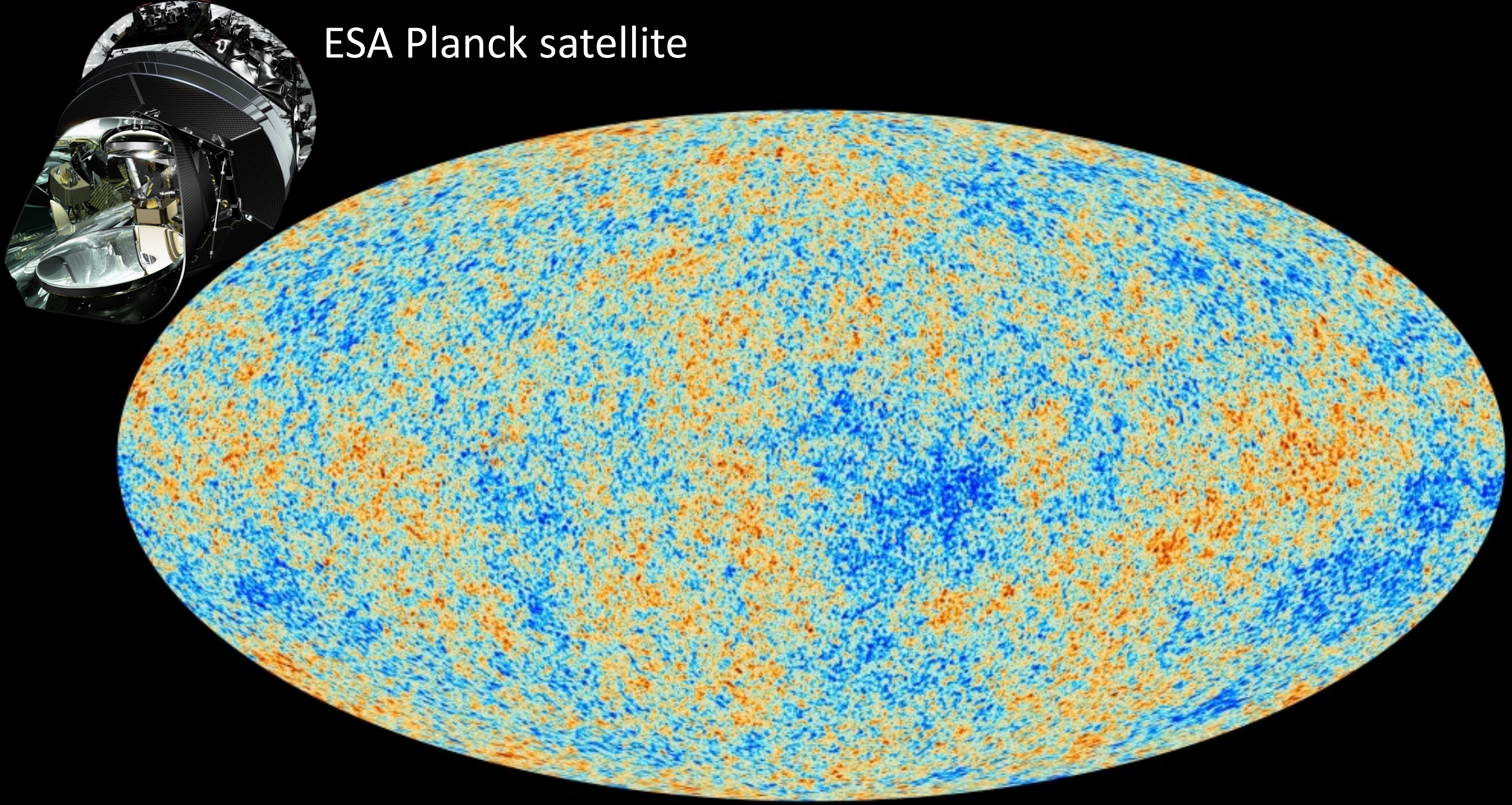
(*) follows directly from $Z[T] = Tr\left(e^{-\frac{H}{T}}\right);\;\; H = p^0 = p \equiv |\boldsymbol{p}|$

$$\frac{\delta T}{T} \sim 10^{-5}$$



Statistically, a Gaussian random field, like a quantum field in its vacuum

ESA Planck satellite



FLRW metric: homogeneous and isotropic 3-space

$$ds^2 = -dt^2 + a(t)^2\gamma_{ij}dx^i dx^j; \quad a(t_0) \equiv 1$$

proper time

comoving coordinates: for a maximally symmetric space

Purely kinematical effects

$$R^{(3)} = 6\kappa \begin{cases} > 0 \text{ sphere} \\ = 0 \text{ torus} \\ < 0 \text{ hyperboloid} \end{cases}$$

1. Redshifting of wavelengths and momenta
   e.g. photons

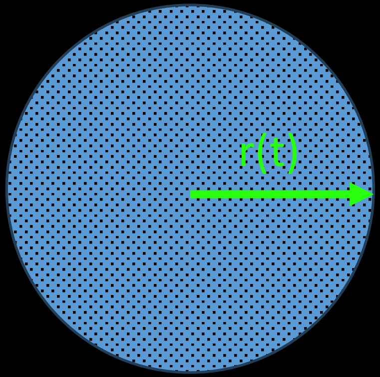
$$\lambda \propto a(t) \Rightarrow \frac{\lambda_{obs}-\lambda_{em}}{\lambda_{em}} = \frac{a_{obs}}{a_{em}} - 1 \equiv Z_{em}$$ redshift $a_{em} \propto 1/(1+Z)$

likewise the de Broglie wavelength and momentum of massive particles

2. Distances: objects which comove with expansion obey
   $d \propto a(t) \Rightarrow \frac{\dot{d}}{d} = \frac{\dot{a}}{a} = H(t)$: recession velocity $\dot{d} \propto$ distance d Hubble's Law

Friedmann-Lemaitre-Roberston-Walker Dynamics

Newtonian derivation



$$\frac{1}{2}\dot{r}^2 - \frac{GM}{r} = \frac{E}{m} = const; \;\; M = \frac{4}{3}\pi r^3 \rho = const$$

$$\Rightarrow r \propto a(t); \;\; \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho + \frac{const}{a^2}$$

Einstein’s derivation: $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$

where, for a perfect fluid at rest in comoving coords, $T_\mu^{\ \nu} = \begin{pmatrix} -\rho & 0 & 0 & 0 \\ 0 & & & \\ 0 & & P(\rho)\delta^i_j & \\ 0 & & & \end{pmatrix}$

Equation of state

G_{00} equation is a *constraint:* $$3\frac{\dot{a}^2}{a^2} - 3\frac{\kappa}{a^2} = 8\pi G\rho$$ Friedmann

(involves only first time derivatives)

Conservation of energy-momentum $\nabla_\mu T^{\mu\nu} = 0 \Rightarrow \dot{\rho} = -3\frac{\dot{a}}{a}(P+\rho)$

Types of energy:

(dE=-P dV)

1. cosmological constant $T_{\mu\nu} = -\Lambda\, g_{\mu\nu} \Rightarrow P = -\rho, \quad \rho = const\ > 0$
2. matter (nonrelativistic) $P = 0 \Rightarrow \rho \propto\ a^{-3}$ (approx.)
3. Radiation (relativistic) $P = \frac{1}{3}\rho$ (or, more fundamentally, $T^\mu_\mu = 0$)

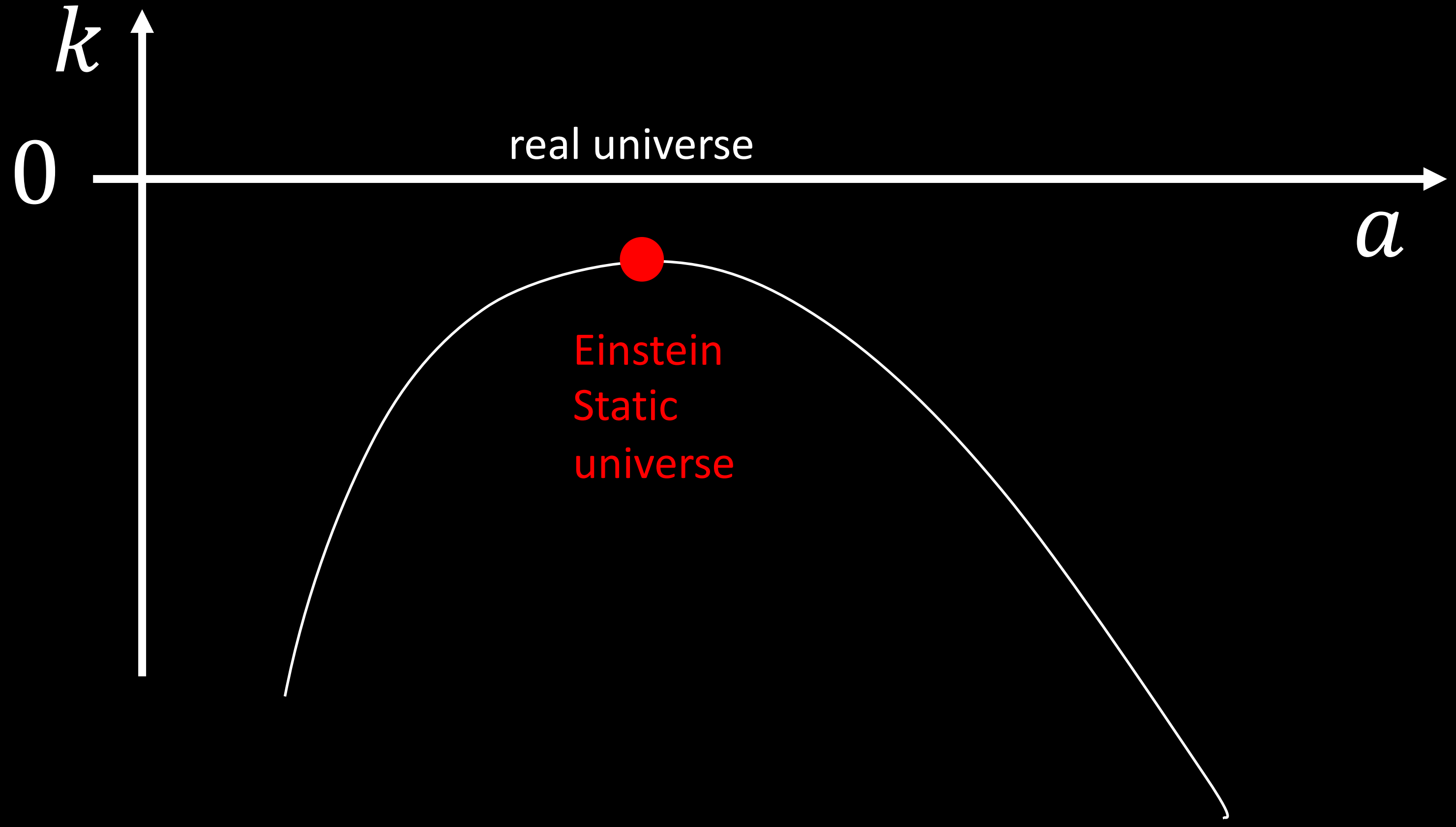
conformal symmetry

$\Rightarrow \rho \propto\ a^{-4}$

i.e., energy per photon $E_\gamma \propto \lambda^{-1} \propto a^{-1}$

So Friedmann reads $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_\Lambda - \frac{\kappa}{a^2} + \rho_m + \rho_r) = H_0^2\left(\Omega_\Lambda + \frac{\Omega_\kappa}{a^2} + \frac{\Omega_m}{a^3} + \frac{\Omega_r}{a^4}\right)$

$$i.e.,\ \dot{a}^2 - \lambda a^2 - ma^{-1} - ra^{-2} = k, \text{ with } \lambda, m, r, k\ const$$



FLRW metric: homogeneous and isotropic 3-space

$$ds^2 = -dt^2 + a(t)^2 \gamma_{ij} dx^i dx^j; \quad a(t_0) \equiv 1$$

proper time

comoving symmetric space

Friedmann equation

$$R^{(3)} = 6\kappa \begin{cases} > 0 \text{ sphere} \\ = 0 \text{ torus} \\ < 0 \text{ hyperboloid} \end{cases}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_{tot} - \frac{\kappa}{a^2}$$

$$= H_0^2\left(\Omega_\Lambda + \frac{\Omega_\kappa}{a^2} + \frac{\Omega_m}{a^3} + \frac{\Omega_r}{a^4}\right)$$

Lambda

space curvature

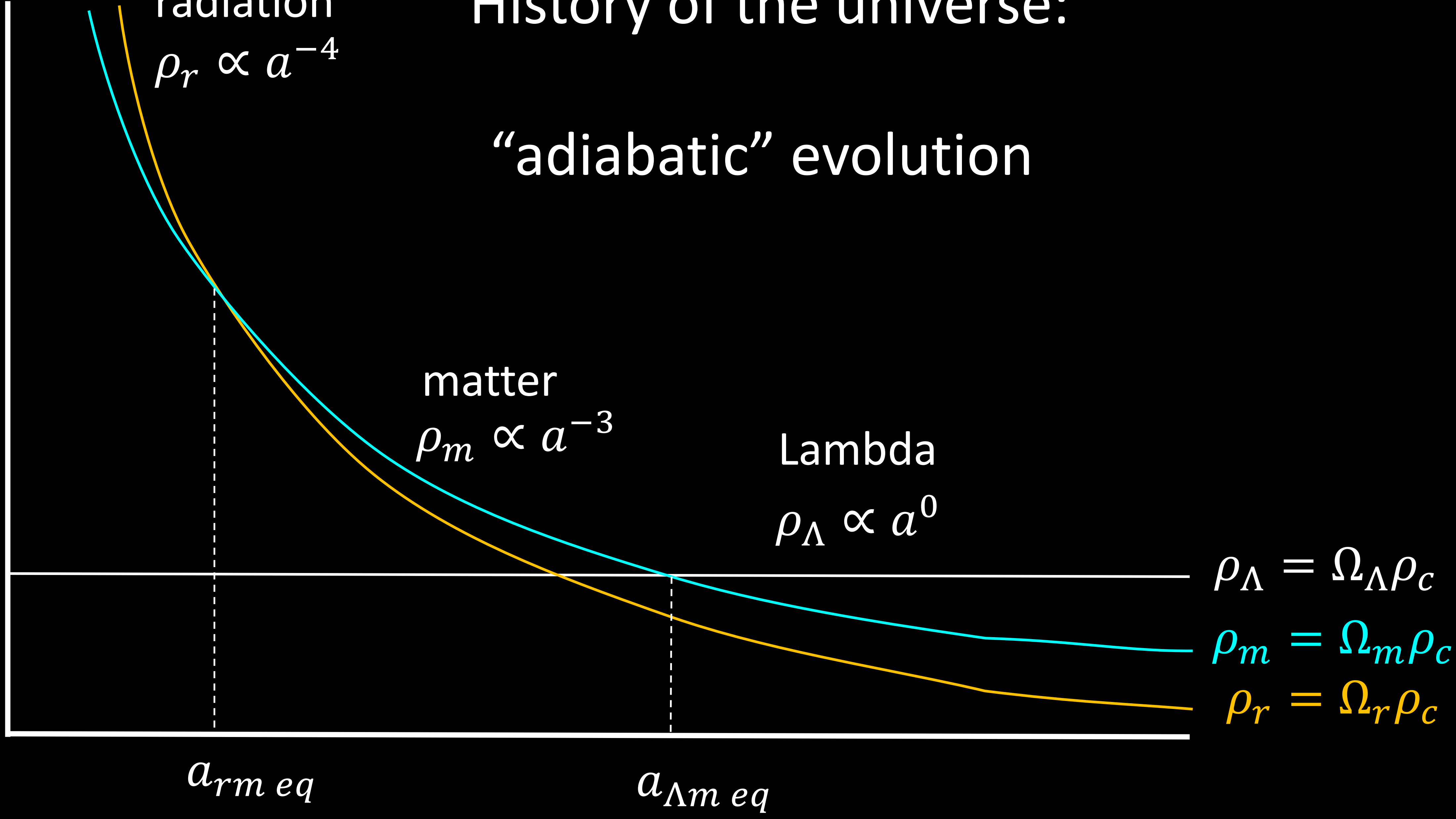
matter

radiation

$$(\Omega_\Lambda + \Omega_\kappa + \Omega_m + \Omega_r \equiv 1)$$

History of the universe:

“adiabatic” evolution



Radiation: $\rho = \frac{\pi^2}{30} N_{eff} T^4 \Rightarrow t = 2.4\, N_{eff}^{-1/2} \left(\frac{MeV}{T}\right)^2$s

	E/GeV	t/s	era	N_{eff}	
radiation	10^{18}	10^{-40}	Planck	???	
	10^{3}	10^{-12}	LHC era	106.75	
	10^{-3}	10^{0}	QCD PT		
	10^{-4}	10^{2}	Nucleosynthesis	10.75	$\gamma' s, e^{\pm\prime} s, \nu' s$
	10^{-9}	10^{12}	Recombination		$\gamma' s, \nu' s$
matter			Newtonian era	DM, atoms and ions	
Λ	10^{-12}	10^{17}	DE/CC era	Λ	

Today,

$H_0 \equiv 100h$ km s^{-1} Mpc^{-1} ; $h = 0.7 \pm 0.05$; 1pc=3.26 light years

Hubble time: $H_0^{-1} = 9.78 \times 10^9 h^{-1}$ years

Hubble radius: $H_0^{-1} c = 2998\ h^{-1}$ Mpc

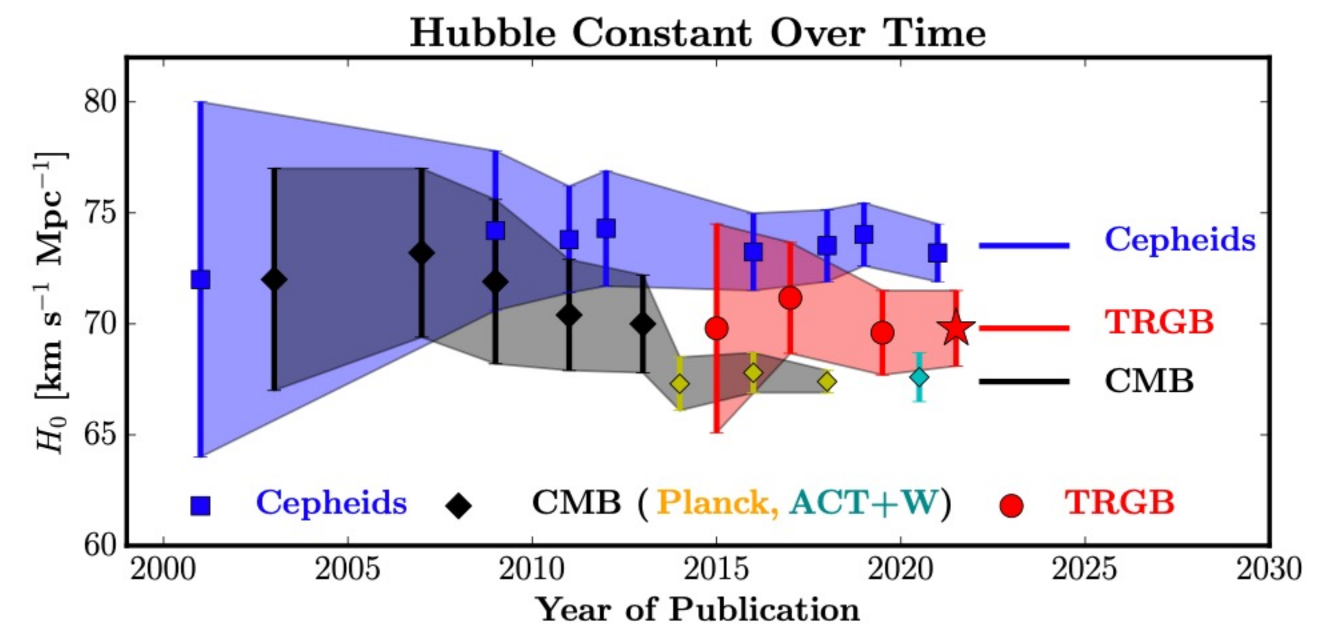
Critical density: $\rho_c = \frac{3}{8\pi G} H_0^2 = 2.8 \times 10^{11} h^2\ M_\odot$ Mpc^{-3}

Radiation density $\rho_{r,0} = 2\frac{\pi^2}{30} T_0^4 (1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_\nu) \Rightarrow \Omega_r = 8.5 \times 10^{-5} h_{.7}^{-2}$

0.68 for 3 light ν's

(reheating of photons due to $e^\pm$ annihilation

Recent tension…



SHOES 73 ± 1

Planck 67.4±0.5

A. Shahib

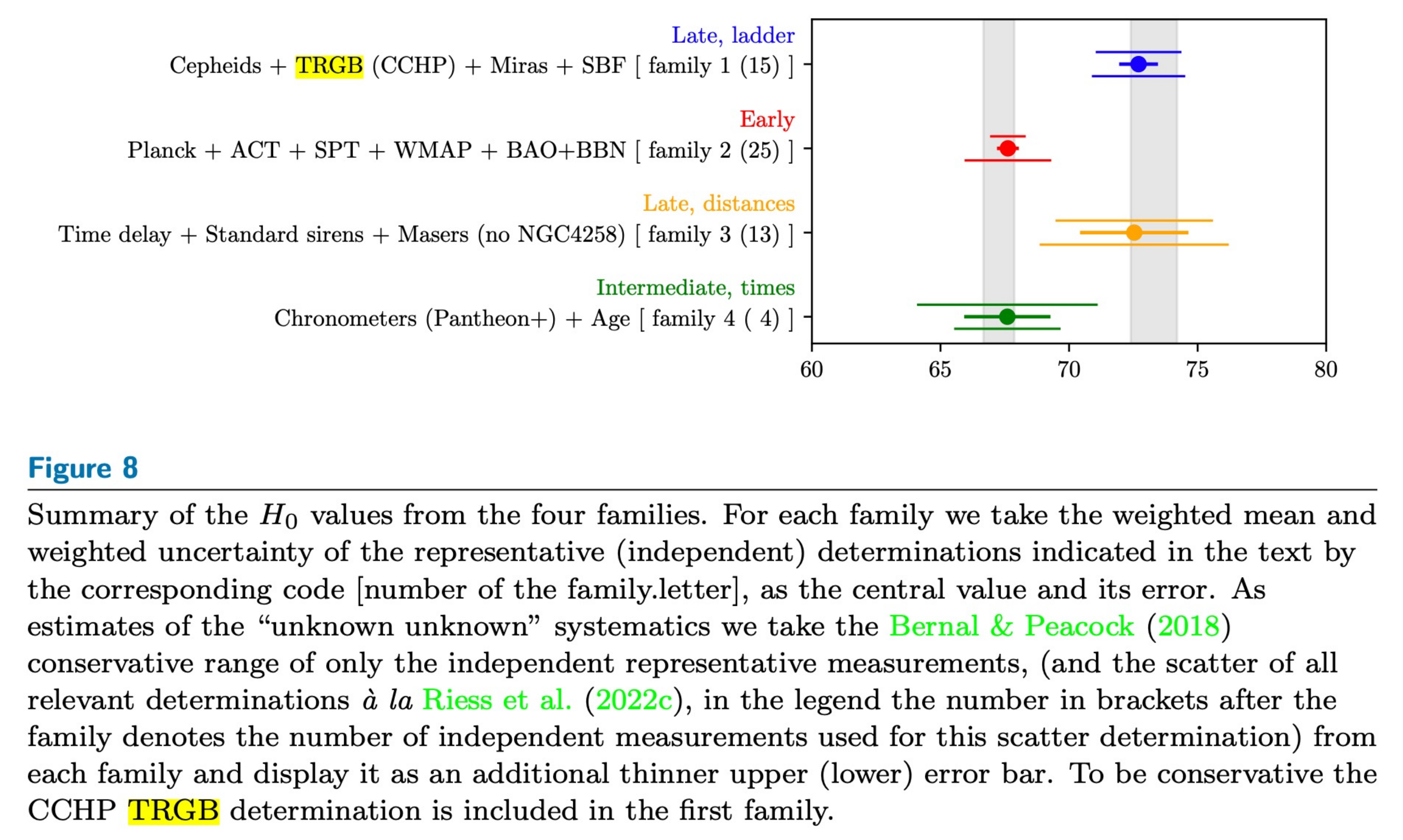
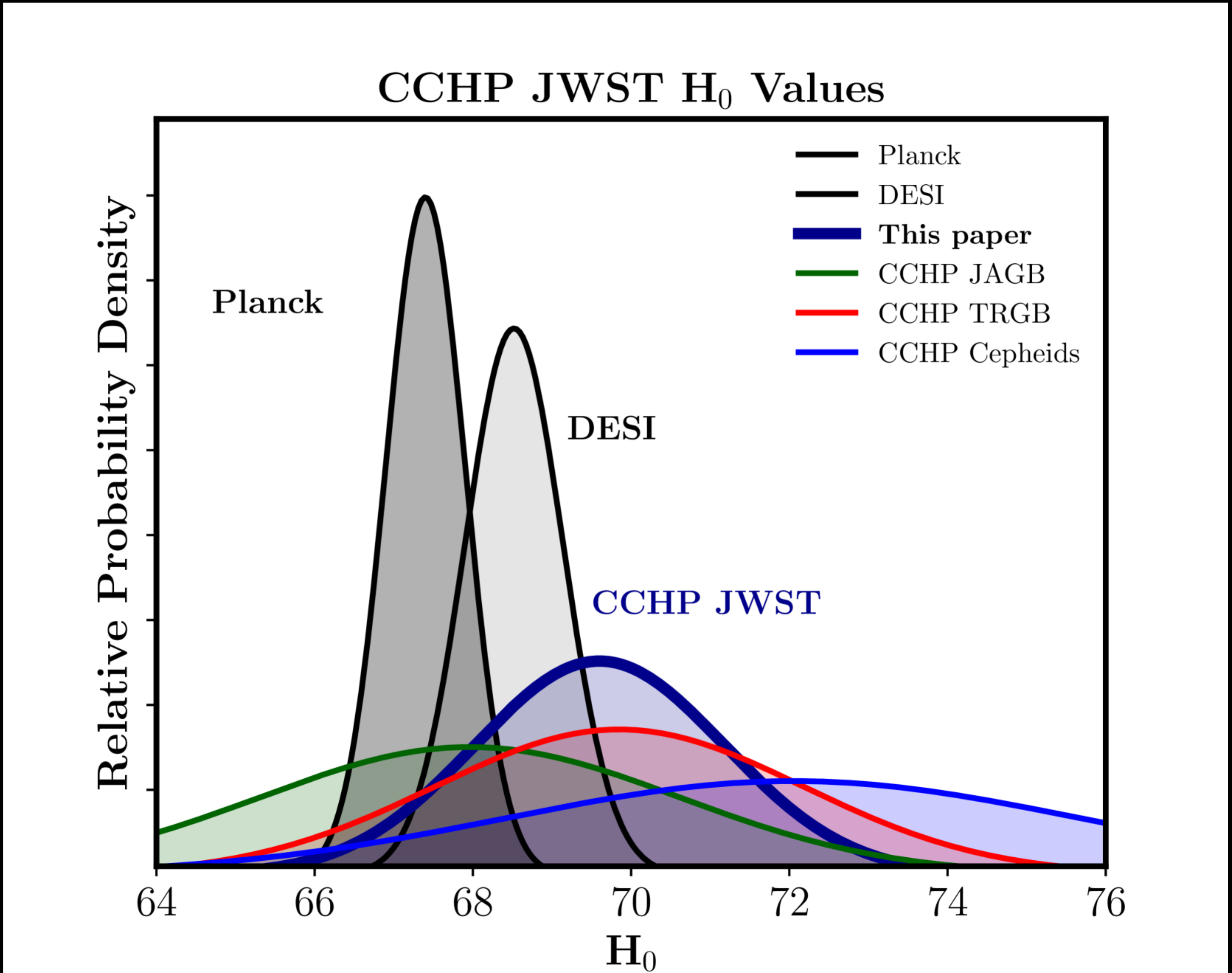


Figure 8

Summary of the H_0 values from the four families. For each family we take the weighted mean and weighted uncertainty of the representative (independent) determinations indicated in the text by the corresponding code [number of the family.letter], as the central value and its error. As estimates of the "unknown unknown" systematics we take the Bernal & Peacock (2018) conservative range of only the independent representative measurements, (and the scatter of all relevant determinations *à la* Riess et al. (2022c), in the legend the number in brackets after the family denotes the number of independent measurements used for this scatter determination) from each family and display it as an additional thinner upper (lower) error bar. To be conservative the CCHP TRGB determination is included in the first family.

Review: Verde et al. arXiv:2311.133905 (2023)

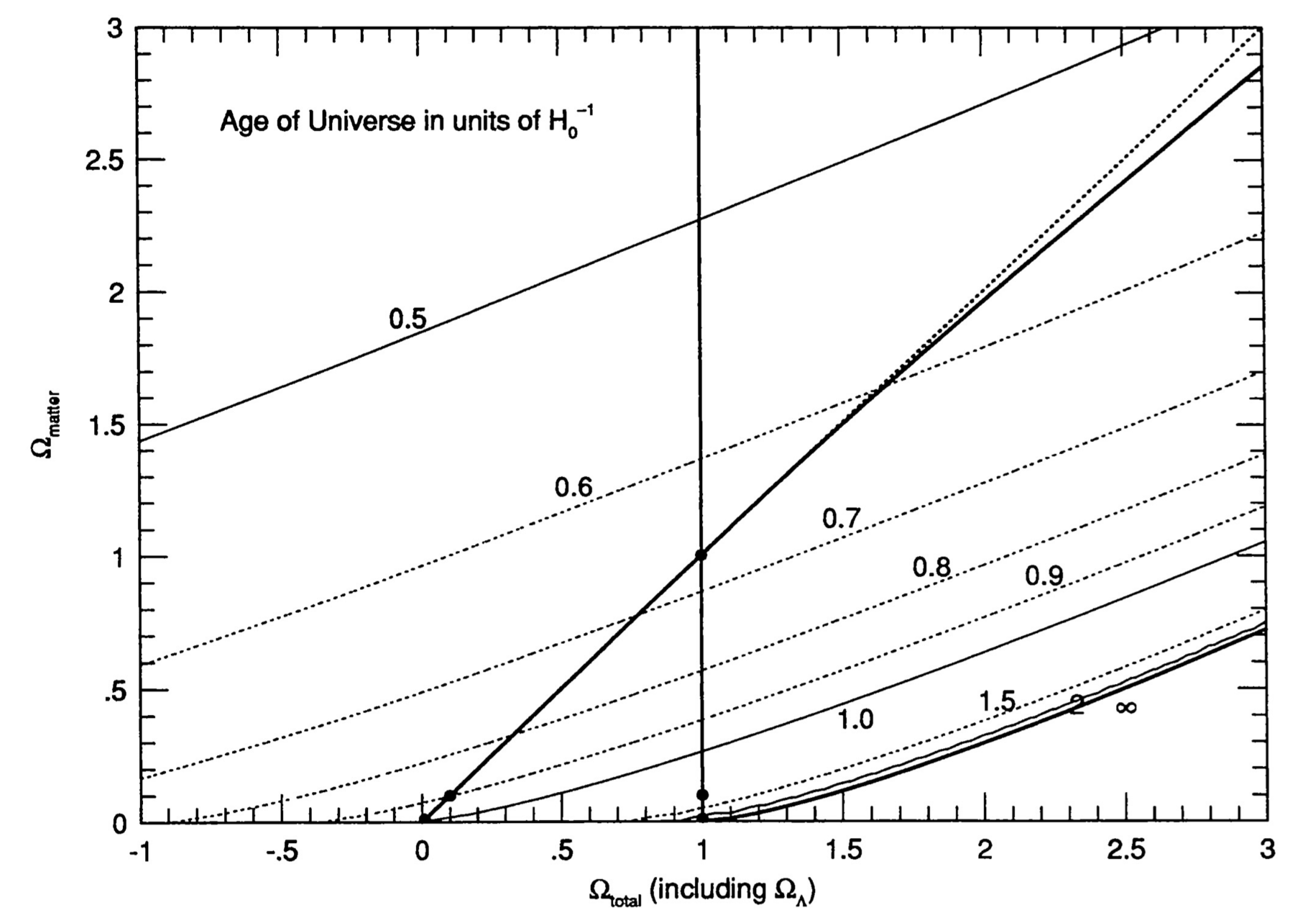


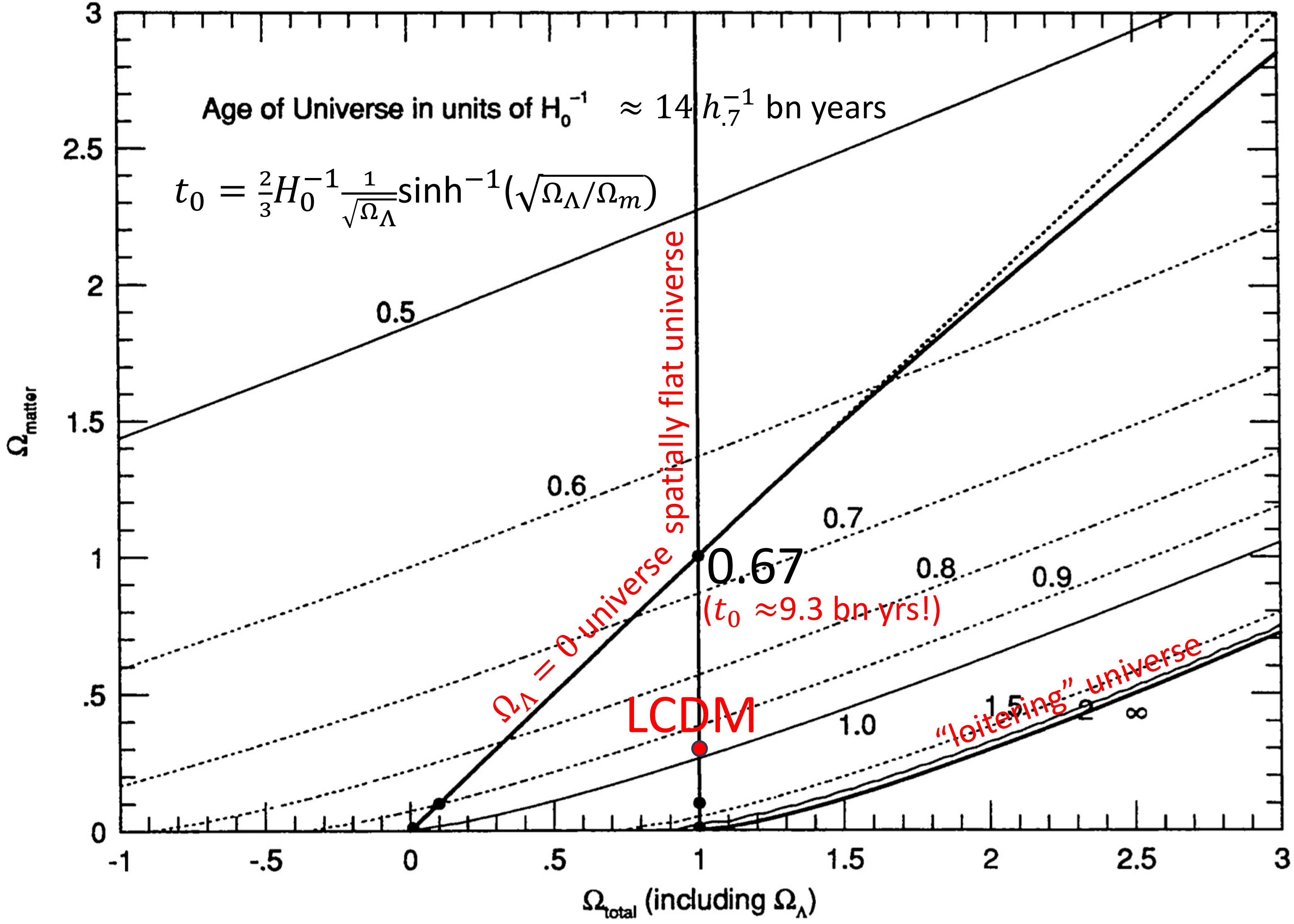
Freedman *et al.*
arXIv:2408.06153

Exact solution for LCDM model (neglecting radiation and spatial curvature)

$$a(t) = \left(\frac{1-\Omega_\Lambda}{\Omega_\Lambda}\right)^{1/3} \left(\sinh \tfrac{3}{2} H_0 \Omega_\Lambda^{1/2} t\right)^{2/3} \Rightarrow t_0 = \tfrac{2}{3} H_0^{-1} \frac{1}{\sqrt{\Omega_\Lambda}} \sinh^{-1}\left(\sqrt{\Omega_\Lambda/(1-\Omega_\Lambda)}\right)$$

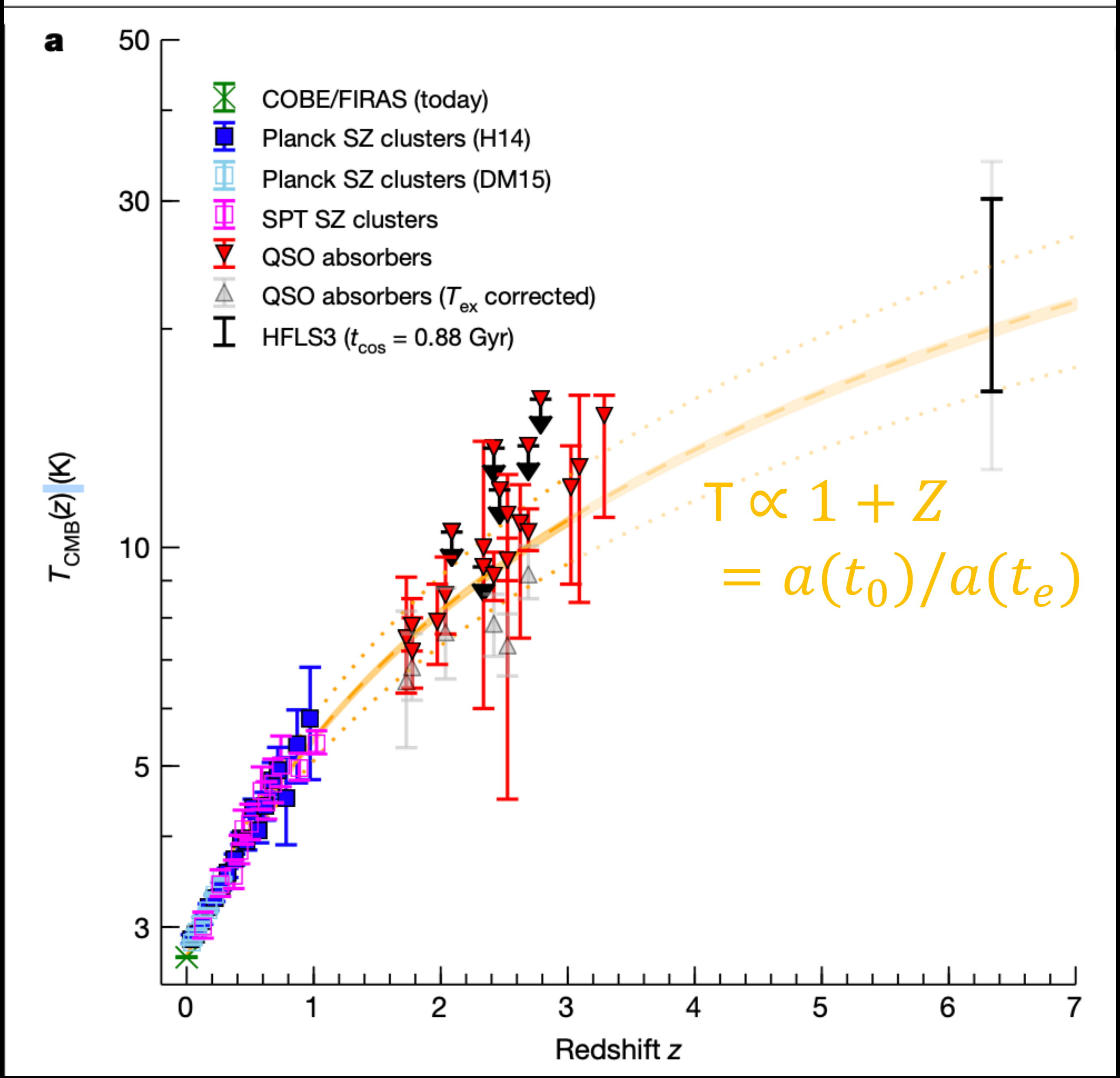
(recommended exercise!)





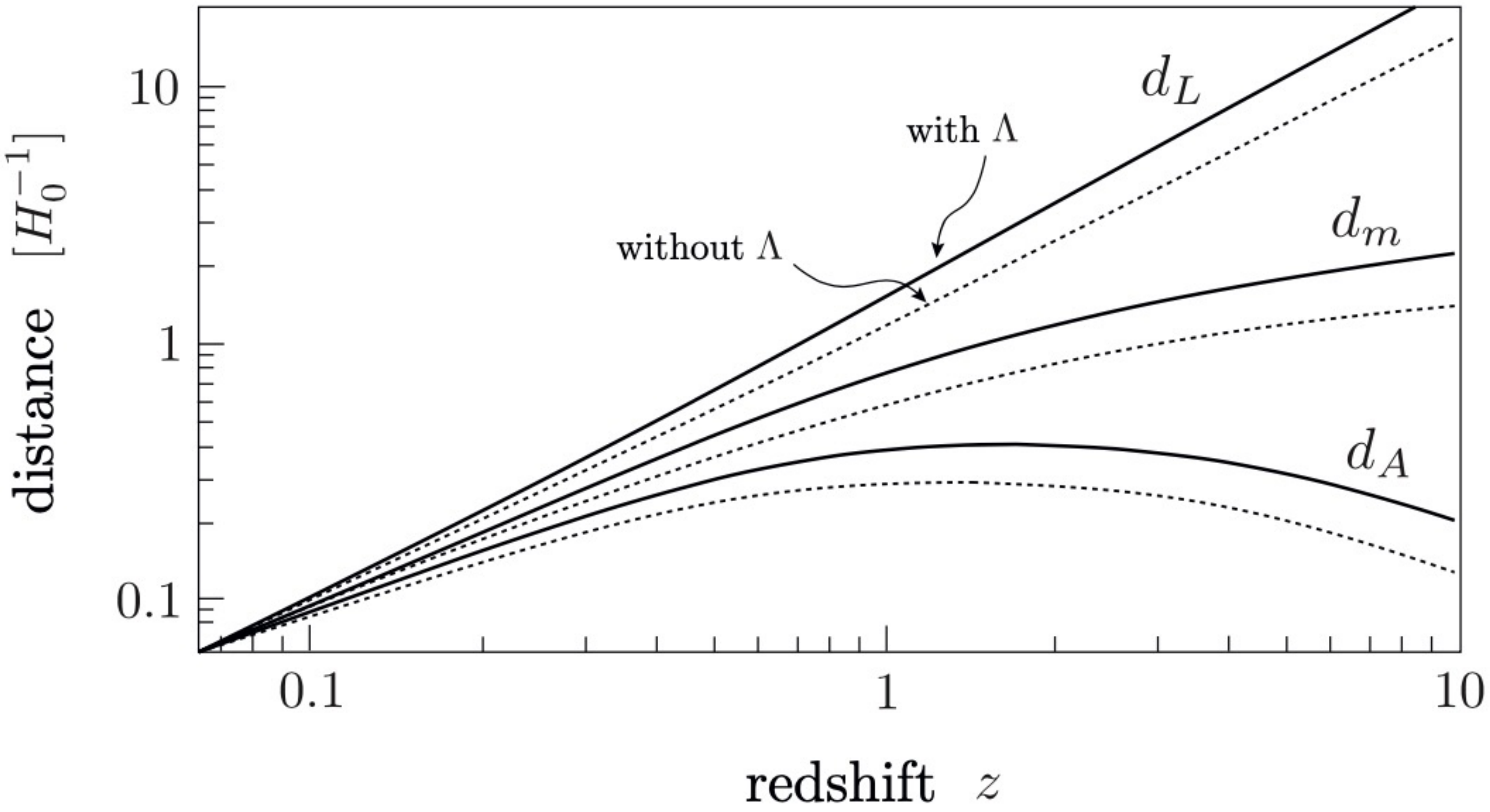
Age of Universe in units of H_0^{-1} $\approx 14\, h_{.7}^{-1}$ bn years

$$t_0 = \tfrac{2}{3} H_0^{-1} \tfrac{1}{\sqrt{\Omega_\Lambda}} \sinh^{-1}\left(\sqrt{\Omega_\Lambda/\Omega_m}\right)$$

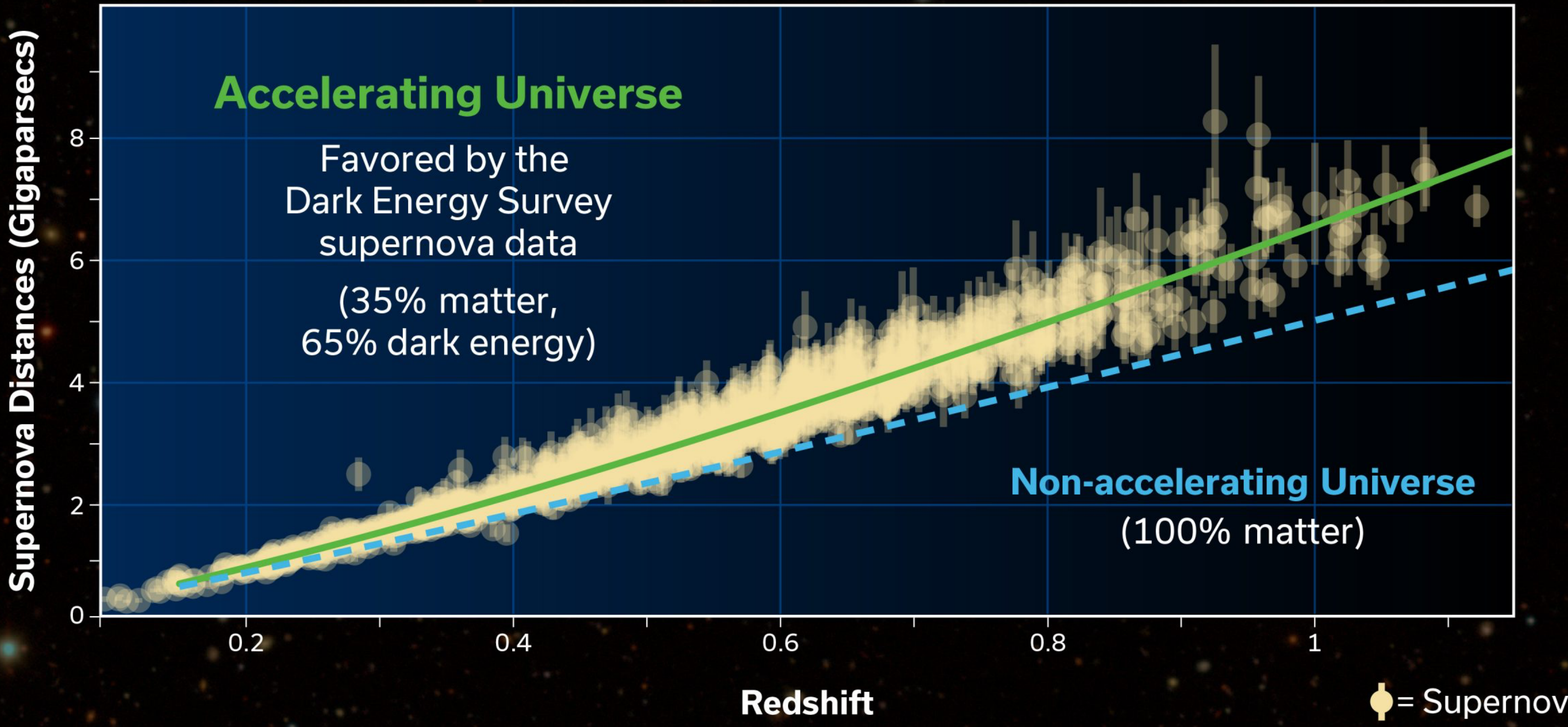


$$T \propto 1 + Z$$
$$= a(t_0)/a(t_e)$$

Riechers et al.
Nature 602, 58 (2022)
"Microwave background temperature at a redshift of 6.34 from H_2O absorption"


d_L
with Λ
without Λ
d_m
d_A
10
1
0.1
0.1
1
10
distance $[H_0^{-1}]$
redshift z

SUPERNOVA HUBBLE DIAGRAM



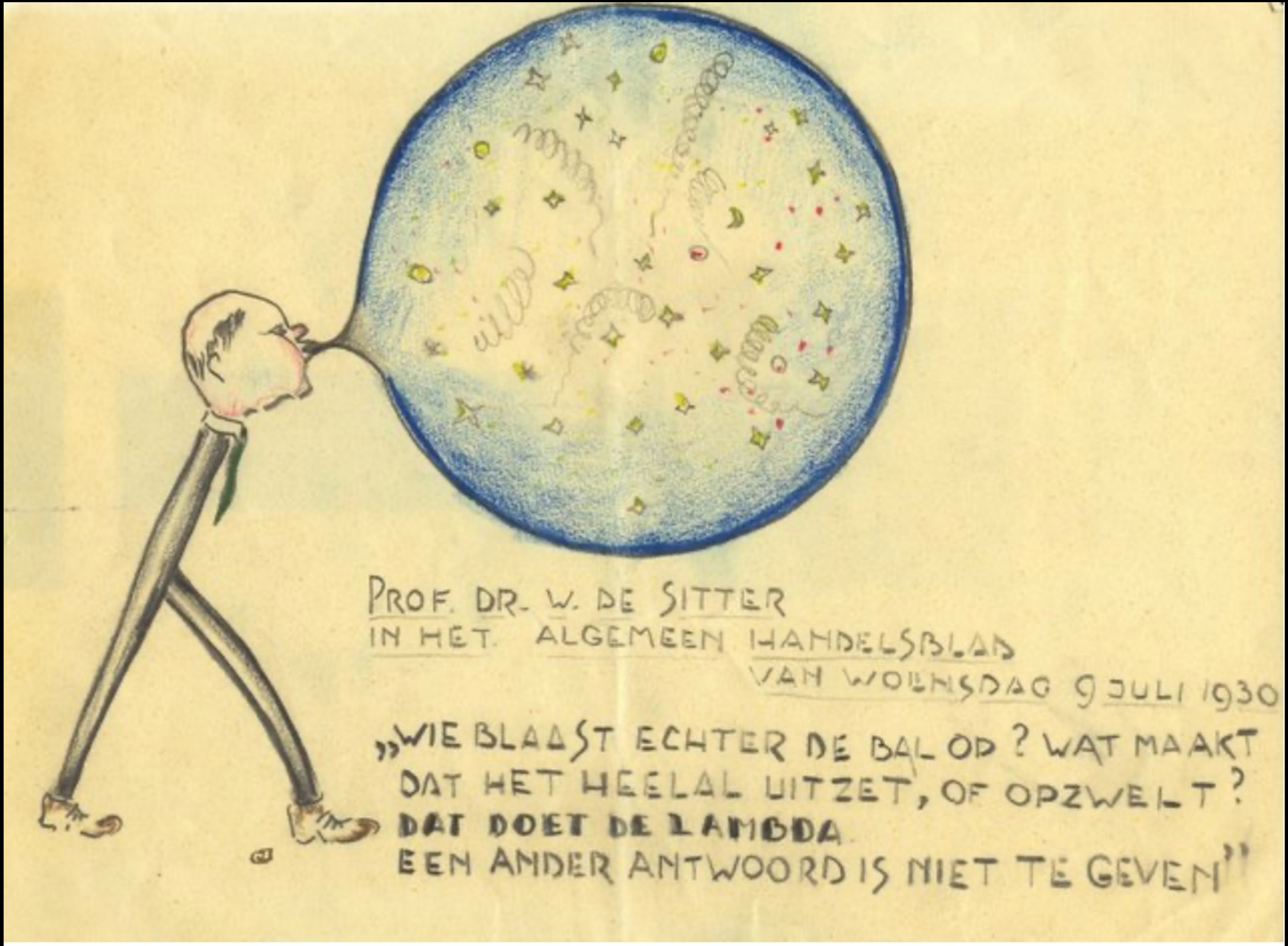
Accelerating Universe

Favored by the
Dark Energy Survey
supernova data

(35% matter,
65% dark energy)

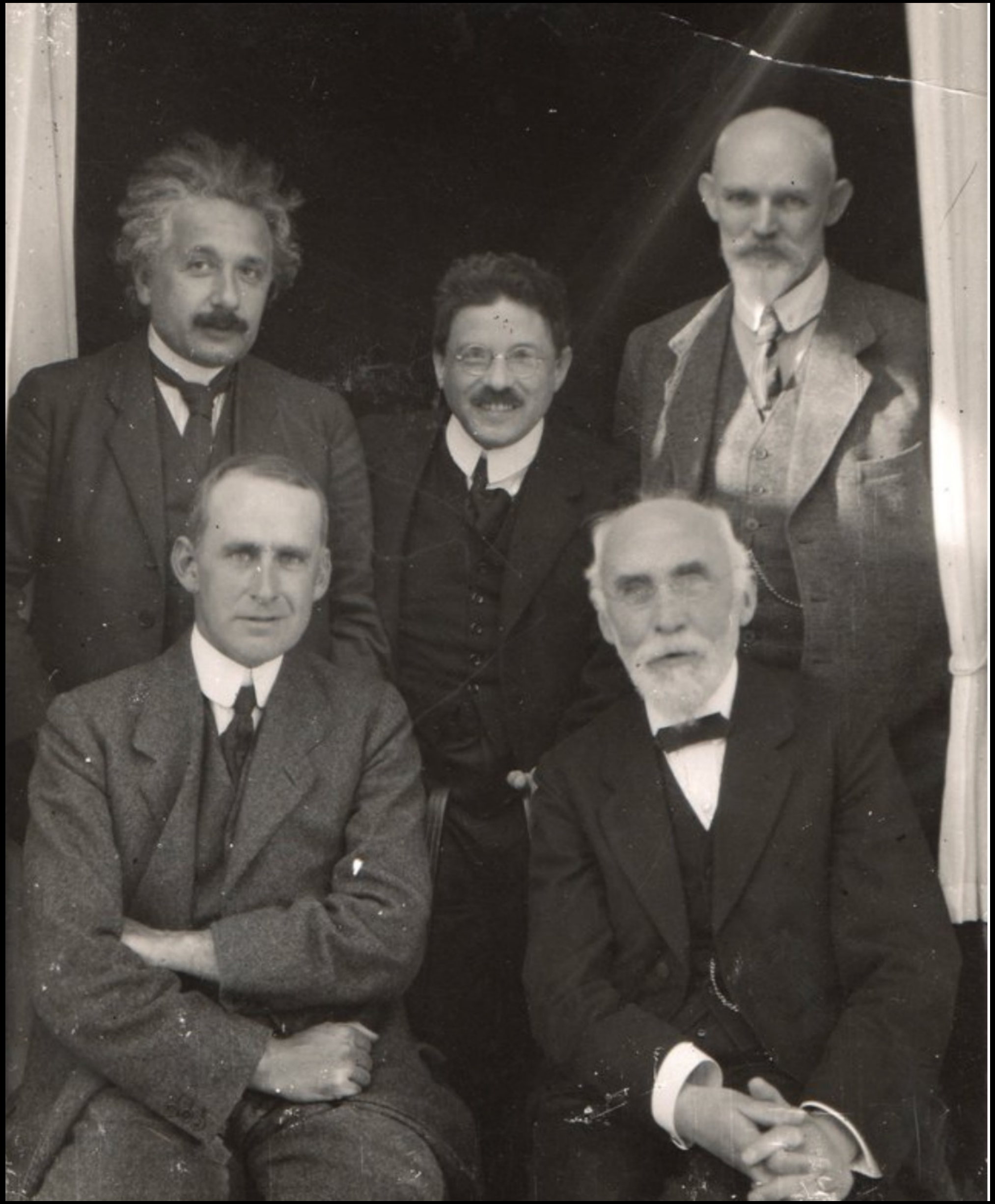
Non-accelerating Universe

(100% matter)



PROF. DR. W. DE SITTER
IN HET ALGEMEEN HANDELSBLAD
VAN WOENSDAG 9 JULI 1930

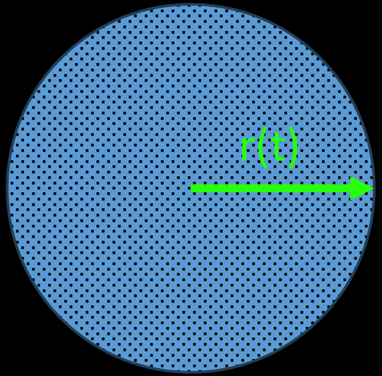
„WIE BLAAST ECHTER DE BAL OP? WAT MAAKT
DAT HET HEELAL UITZET, OF OPZWELT?
DAT DOET DE LAMBDA.
EEN ANDER ANTWOORD IS NIET TE GEVEN"



1923, Leiden

Cosmological perturbations (“Newtonian” derivation)

(recommended exercise!)



$$\frac{1}{2}\dot{r}^2 - \frac{GM}{r} - \frac{4\pi G}{3} r^2 \rho_\Lambda = const$$

1. $r \to r + \delta r \Rightarrow \dot{r}\dot{\delta r} + \frac{GM}{r^2}\delta r - \frac{8\pi G}{3} r \delta r \rho_\Lambda = \delta k$
2. $\frac{d}{dt} \Rightarrow \ddot{r} = -\frac{GM}{r^2} + \frac{8\pi G}{3}\rho_\Lambda r \Rightarrow \frac{d}{dt}\left(\frac{\delta r}{r}\right) = \frac{\ddot{r}}{\dot{r}}\frac{\delta r}{r} - \frac{\dot{r}\delta r}{r\, r} + \frac{\delta k}{r\dot{r}}$

Fractional matter density perturbation $\delta = -3\frac{\delta r}{r}$

$$\Rightarrow \dot{\delta} + \left(\frac{\dot{a}}{a} - \frac{\ddot{a}}{a}\right)\delta = \frac{const}{a\dot{a}} \Rightarrow \delta = \underbrace{A\,\frac{\dot{a}}{a}}_{\text{decaying mode}} + \underbrace{const\,\frac{\dot{a}}{a}\int_0^t \frac{dt}{\dot{a}^2}}_{\text{growing mode}};$$

(recommended exercise!)

$$\delta_m \propto \frac{5}{2}\Omega_m\sqrt{\Omega_\Lambda + \Omega_m a^{-3}}\int_0^a \frac{da}{a^3(\Omega_\Lambda + \Omega_m a^{-3})^{\frac{3}{2}}} \approx a\,, \qquad a \ll \left(\frac{\Omega_m}{\Omega_\Lambda}\right)^{1/3},$$

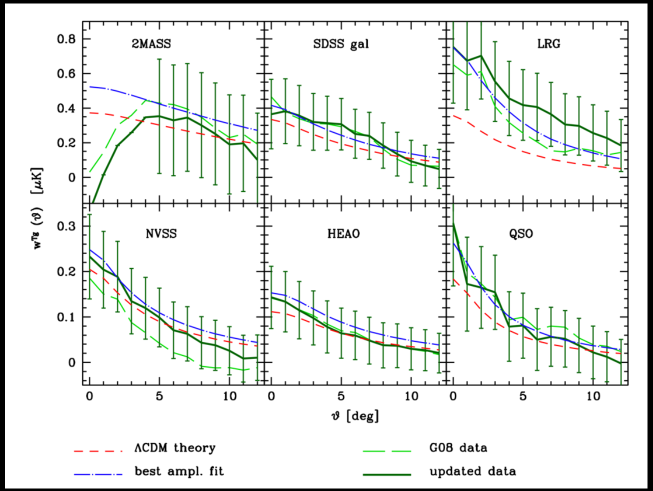
$$\approx 1.43\left(\frac{\Omega_m}{\Omega_\Lambda}\right)^{1/3}, \quad a \gg \left(\frac{\Omega_m}{\Omega_\Lambda}\right)^{1/3}$$

Newtonian potential $a^{-2}\,\nabla^2\Phi = 4\pi G\rho_m\delta_m \Rightarrow \Phi \sim const,\ a^{-1}$ respectively

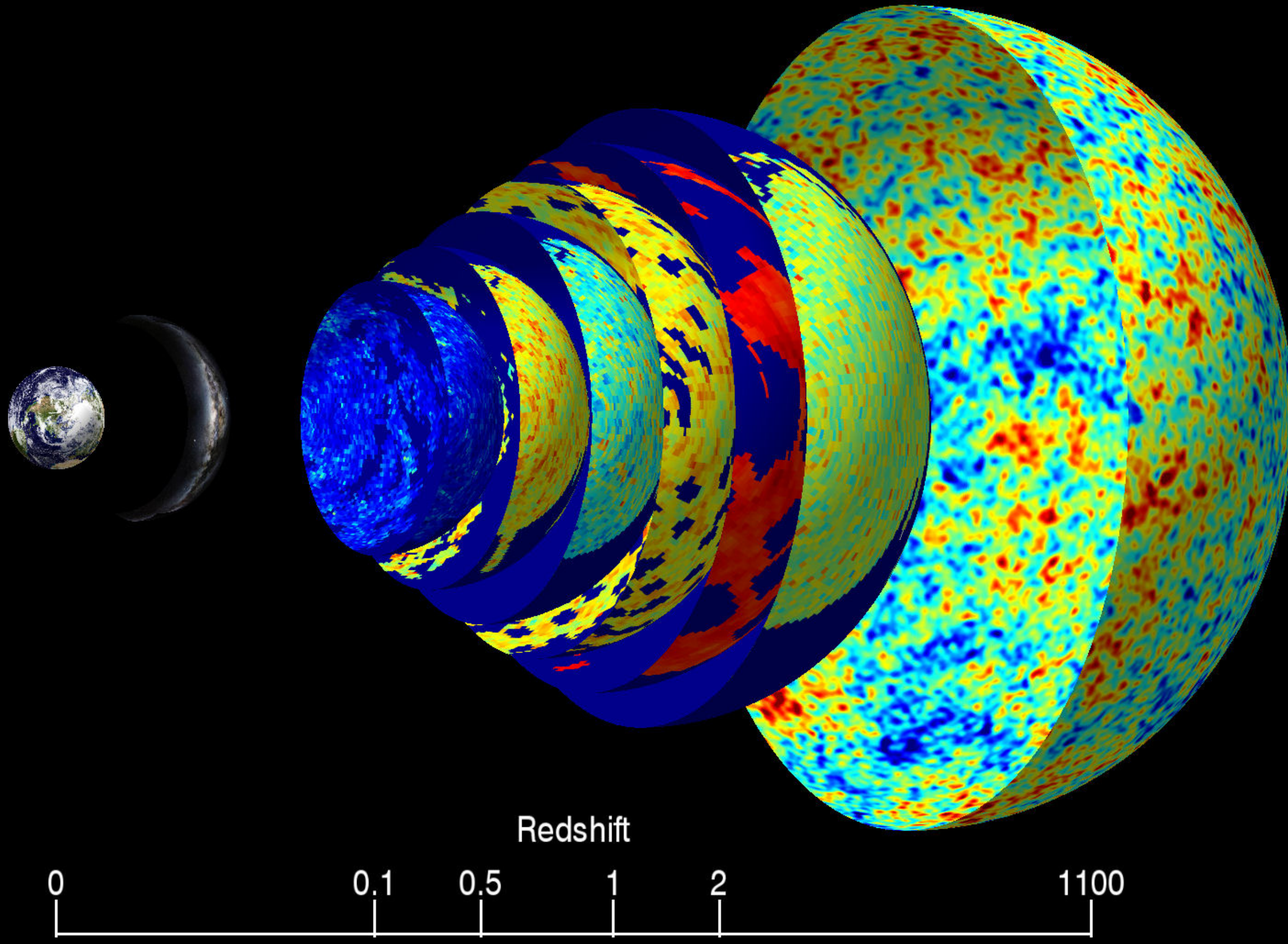
Detecting Lambda with the ISW effect

R.G. Crittenden and NT
Phys. Rev. Lett. **76**, 575 (1996) –

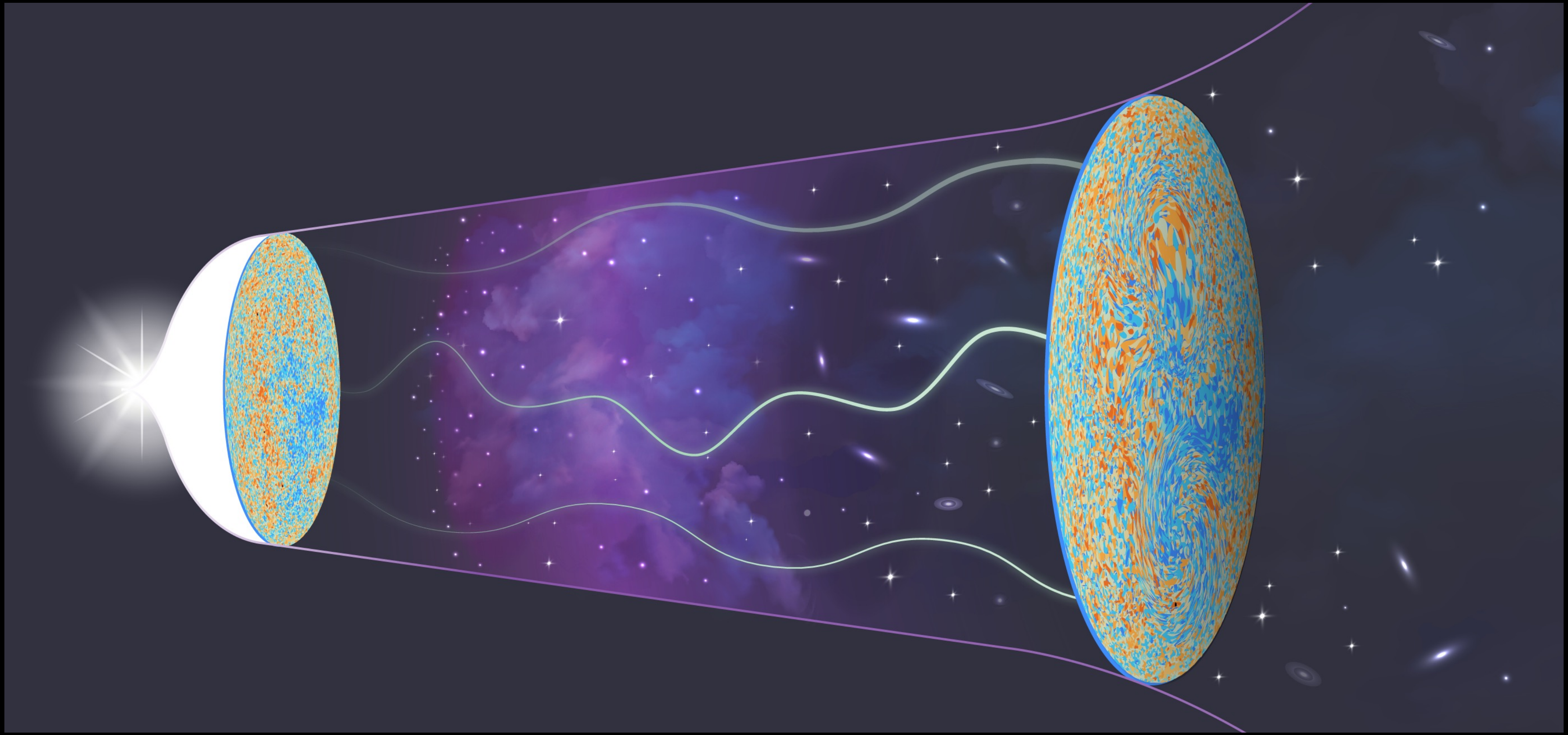
Giannantonio et al
arXiv:1209.2125 (2012)



Seraille et al.
arXiv:2401.06221 (2024)

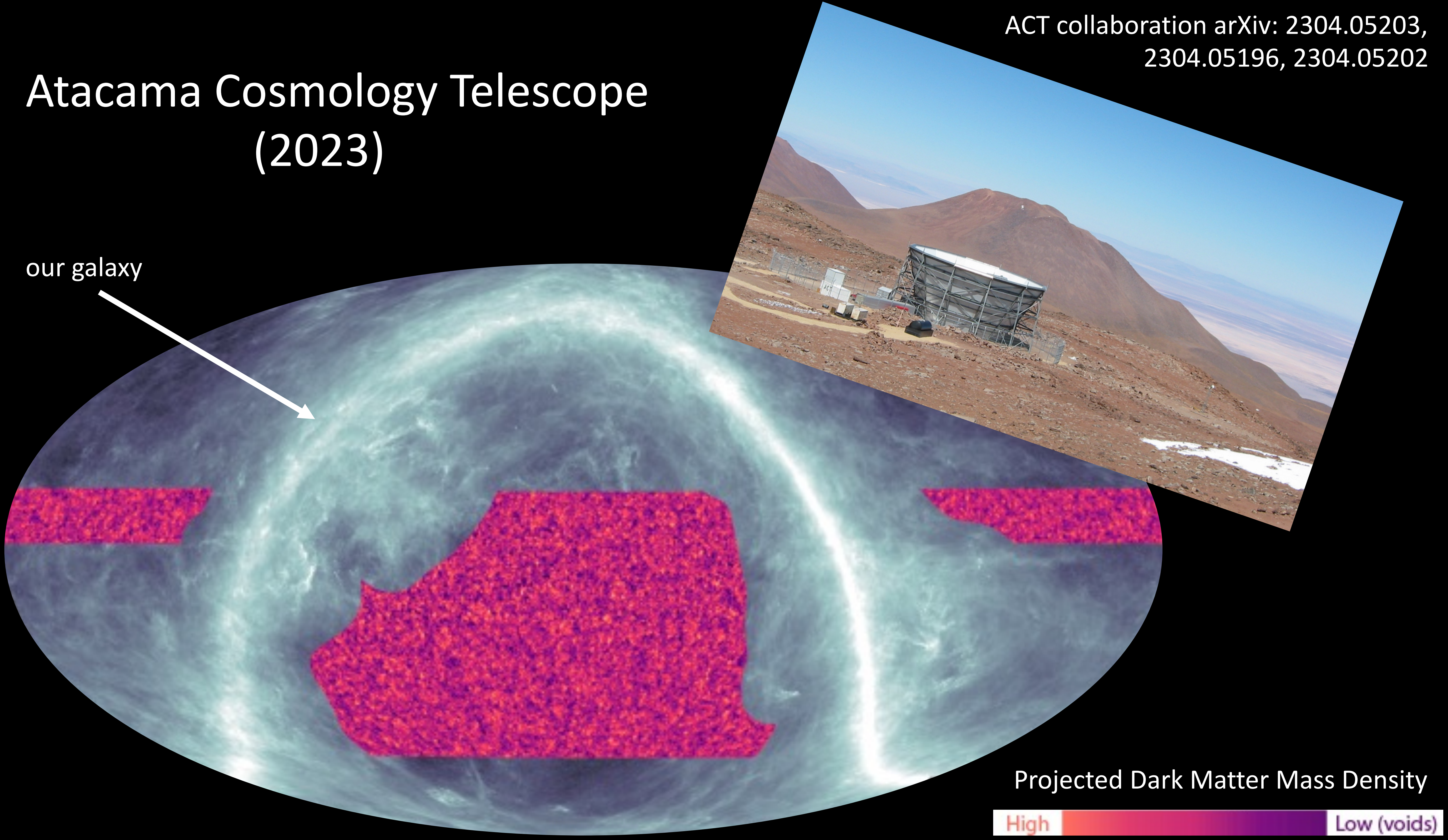


“seeing” the dark matter with gravitational lensing



Atacama Cosmology Telescope (2023)

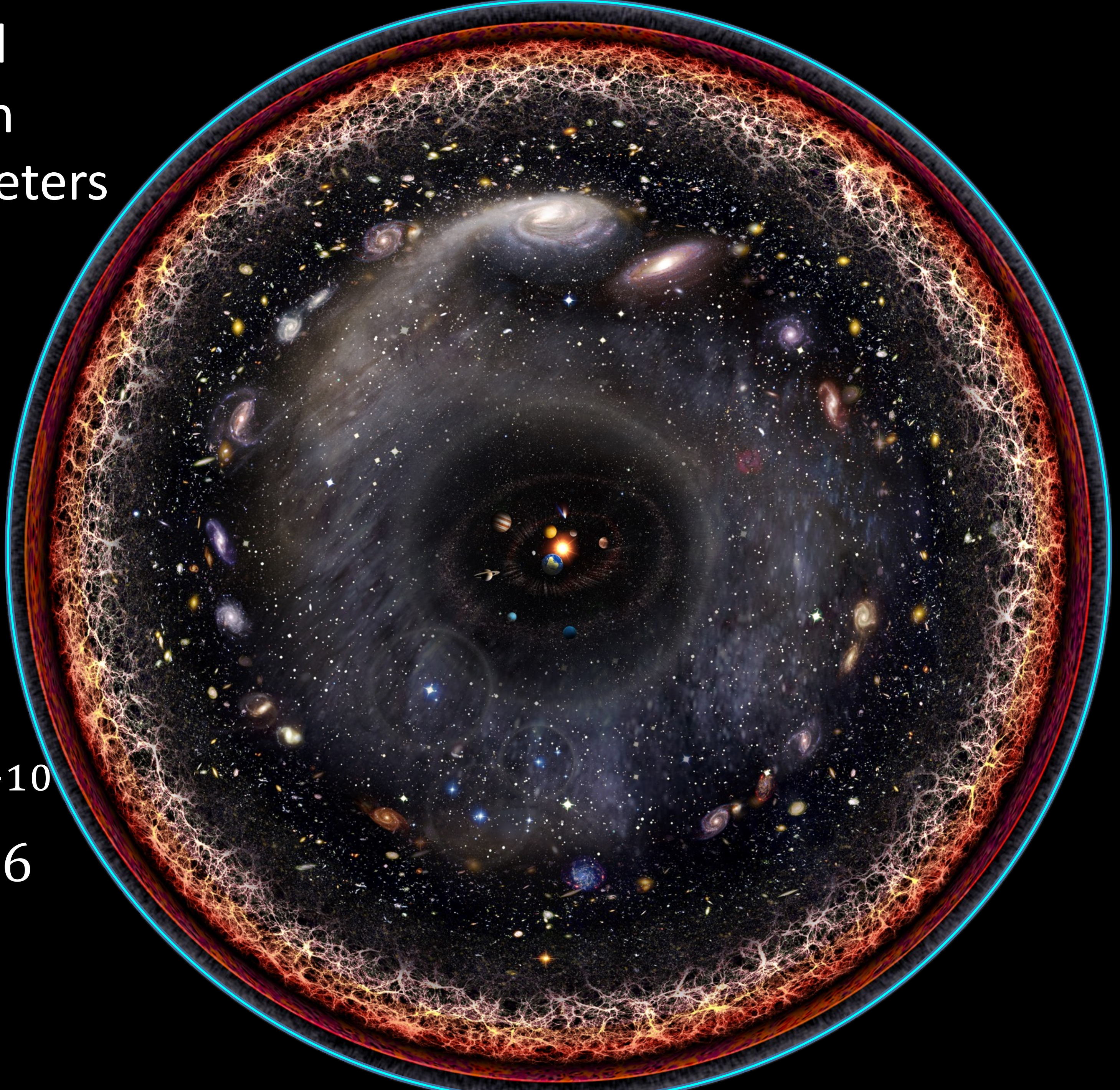
ACT collaboration arXiv: 2304.05203, 2304.05196, 2304.05202



our galaxy

Projected Dark Matter Mass Density

LCDM provides a remarkably good fit to the large-scale universe, with just 5 fundamental physics parameters



the energy content

1. $\rho_\Lambda \quad = (2.3\ \text{meV})^4$ (± 1%)
2. $\rho_{DM}/\rho_B = 5.36$ (± 1%)
3. $n_B/n_\gamma = 6\times10^{-10}$ (± 1%)

the perturbations

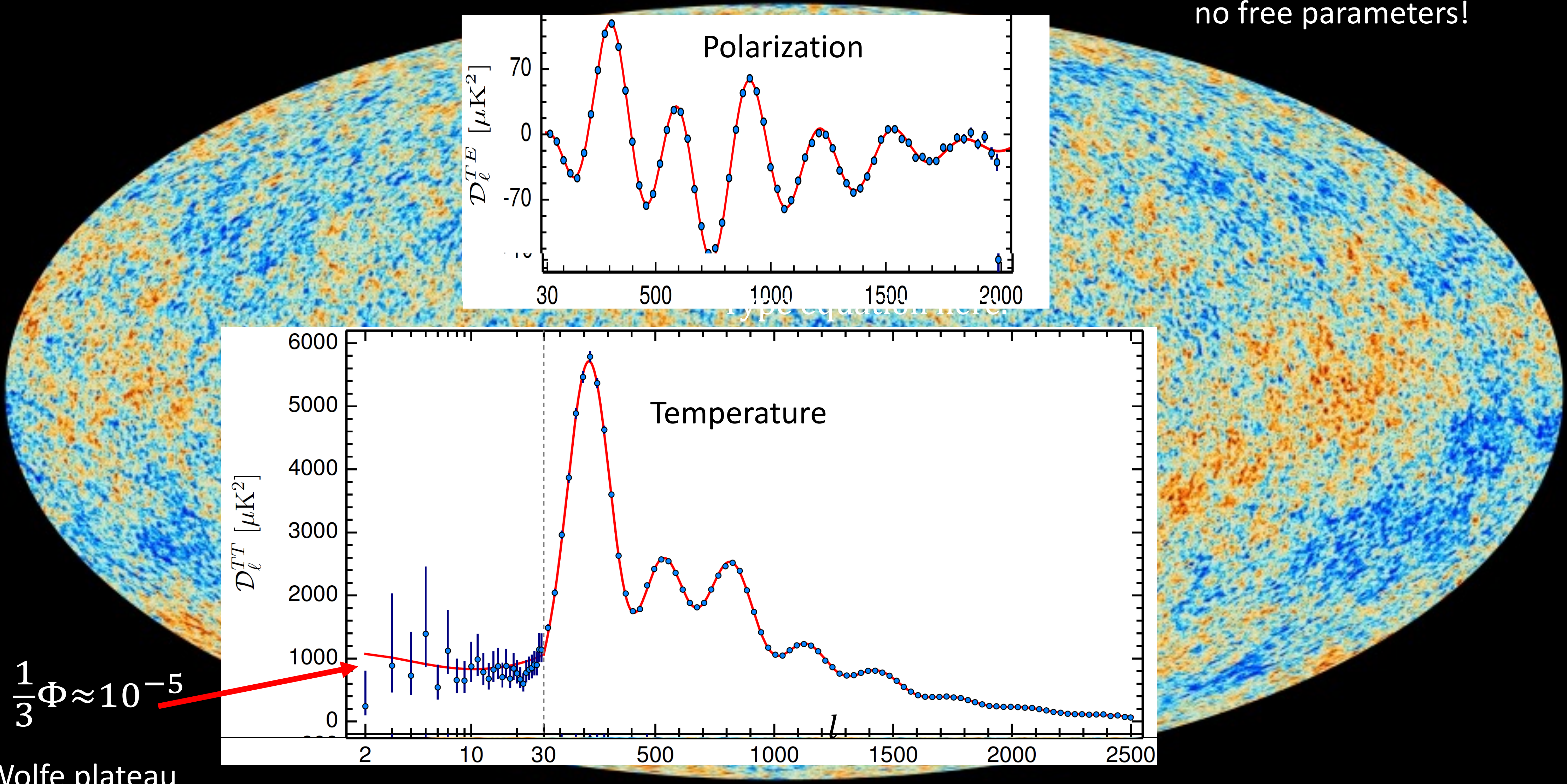
$$\langle\Phi^2\rangle = \int \frac{dk}{k} A_\Phi \left(\frac{k}{k_*}\right)^{n_s-1}$$

4. amplitude $A_\Phi \approx 7.6 \pm 0.1\times10^{-10}$
5. "tilt" $n_s - 1 \approx -0.041 \pm 0.0056$

many parameters so far consistent with zero:
tensor and "isocurvature" perturbations,
spatial curvature κ, non-Gaussianity…

LCDM is an amazingly successful fit

Coulson, Crittenden, NT (1994))
no free parameters!



$$\frac{\delta T}{T} \approx \frac{1}{3}\Phi \approx 10^{-5}$$

Sachs-Wolfe plateau

acoustic peaks (sound waves in plasma)

ESA Planck satellite