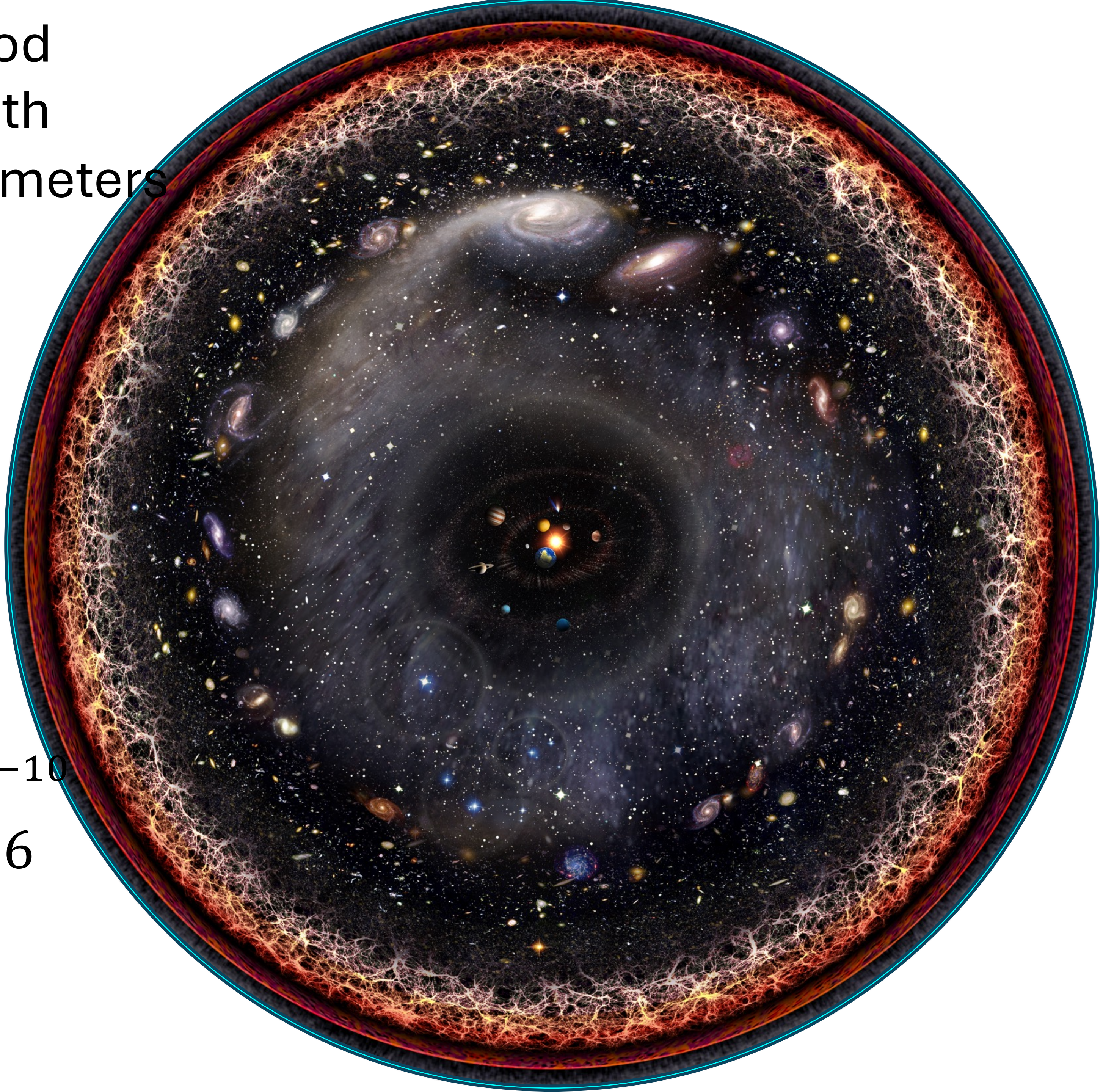
N Turok Lecture 2
(notes and exercises)

LCDM provides a remarkably good fit to the large-scale universe, with just 5 fundamental physics parameters



the energy content

1. $\rho_\Lambda = (2.3\ \text{meV})^4$ (± 1%)
2. $\rho_{DM}/\rho_B = 5.36$ (± 1%)
3. $n_B/n_\gamma = 6\times10^{-10}$ (± 1%)

the perturbations

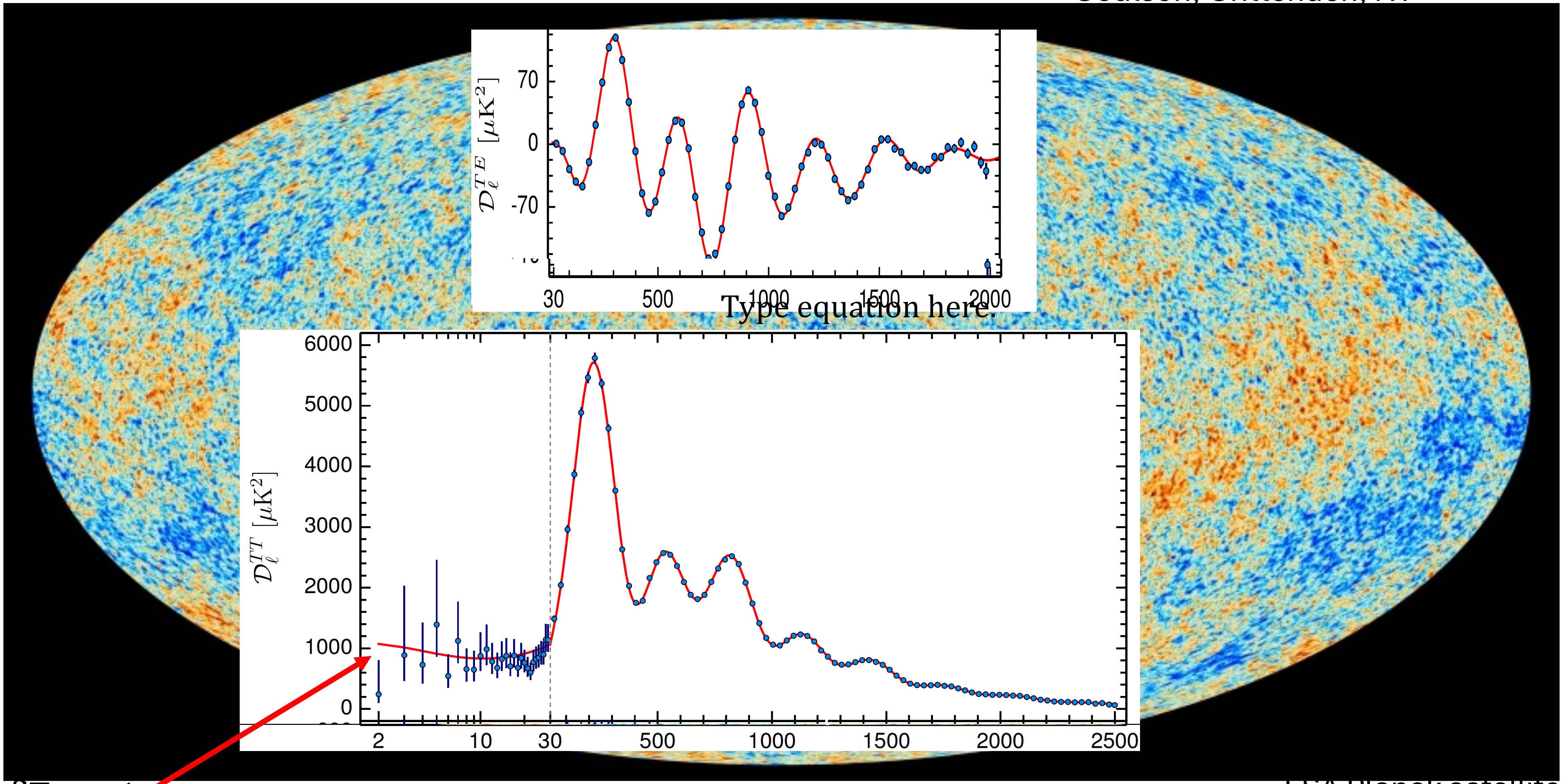
$$\langle\Phi^2\rangle = \int \frac{dk}{k} A_\Phi \left(\frac{k}{k_*}\right)^{n_s-1}$$

4. amplitude $A_\Phi \approx 7.6 \pm 0.1\times10^{-10}$
5. “tilt” $n_s-1 \approx -0.041 \pm 0.0056$

many parameters so far consistent with zero: tensor and “isocurvature” perturbations, spatial curvature κ, non-Gaussianity...

LCDM is an amazingly successful fit

Coulson, Crittenden, NT



ESA Planck satellite

$\frac{\delta T}{T} \approx \frac{1}{3}\Phi \approx 10^{-5}$ Sachs-Wolfe plateau

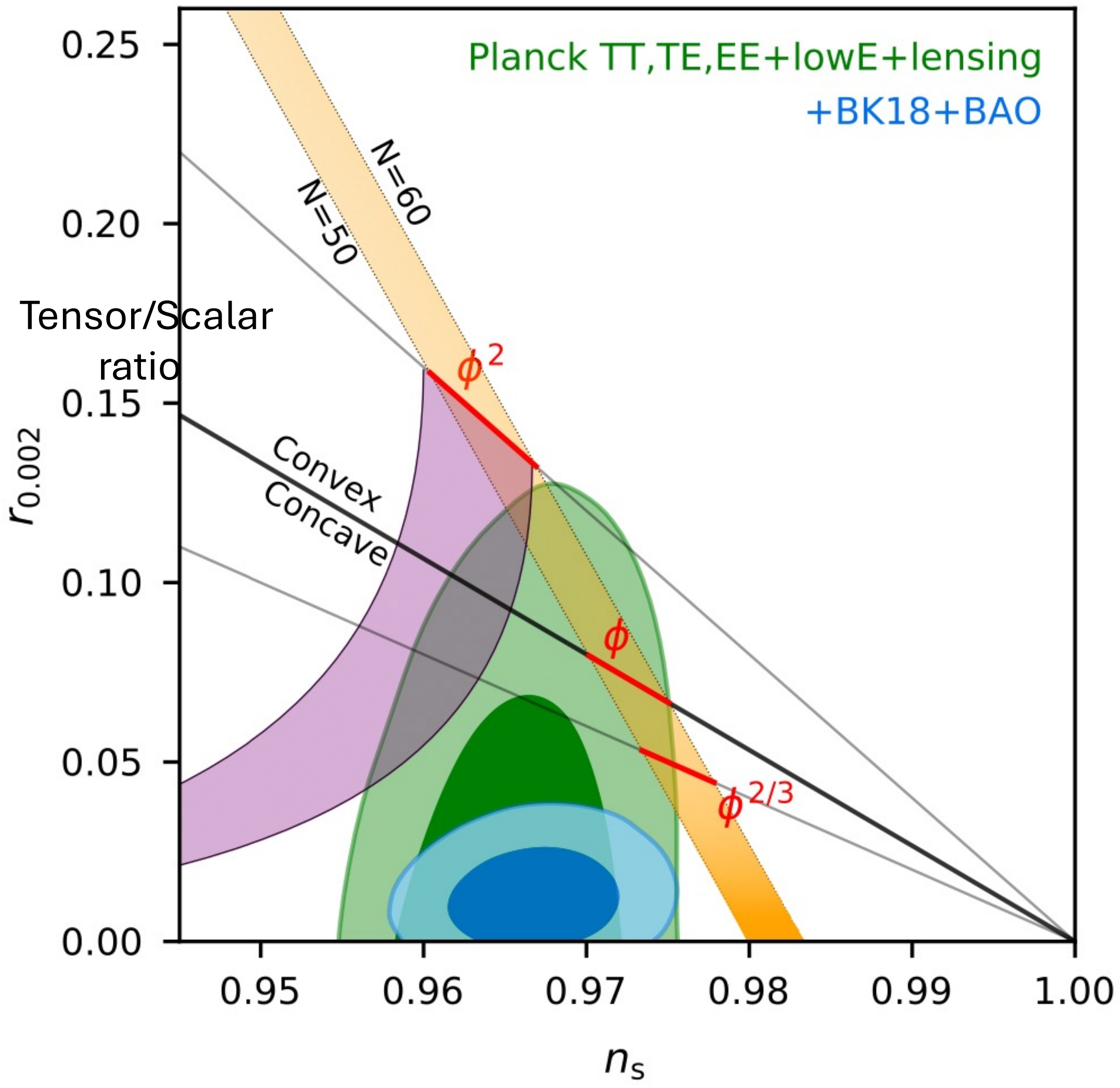
acoustic peaks (sound waves in plasma)

no sign of
inflation's "smoking gun"
signal, *i.e.*, long
wavelength gravitational
waves (tensor modes)

BICEP/Keck
Collaboration
2203.16556 [astro-ph]
PRL **127**, 151301 (2021)
r< 0.036 at 95% confidence

anticipated limit
r<.003
using SPT for
"delensing"
(2027)

inflation is (imho)
steadily becoming
less plausible



A similar story in particle physics: no deviations from the SM up to 10 TeV.

What is the unexpected simplicity on very large and very small scales telling us

Maybe the expected complexity will be revealed at smaller (or larger?) scales…

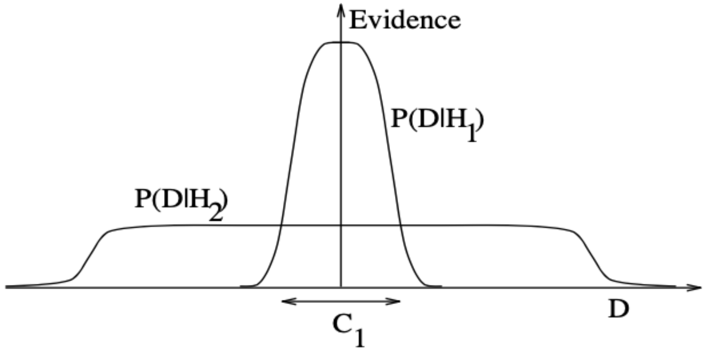
Or maybe the universe really is very simple on both very large and small scales

The most exciting possibility is that the observations are teaching us something important and major new advances in basic physics are imminent

Occam's Razor

'A theory with mathematical beauty is more likely to be correct than an ugly one that fits some experimental data' (Paul Dirac)

'Coherent inference automatically embodies Occam's razor, quantitatively.'
(David Mackay, Ch. 28, Information Theory, Inference and Learning Algorithms)



$$\text{Occam factor} = \frac{\sigma_{w|D}}{\sigma_w},$$

i.e., *the Occam factor is equal to the ratio of the posterior accessible volume of $\mathcal{H}_i$'s parameter space to the prior accessible volume*

Theories or scenarios with a lot of freedom in their realization, *e.g.* multifield inflation or string compactifications, are seriously penalized by their Occam factor.

Our approach:

what is the *minimal* extension of known physics consistent with everything we know from particle physics and cosmology?

the deepest paradoxes and puzzles are our best clues

e.g. the cosmological dark matter

Why is the large-scale geometry of the universe so simple?

homogeneous, isotropic and spatially flat ($\kappa \approx 0$)

$$ds^2 = -dt^2 + a(t)^2\gamma_{ij}(x)dx^i dx^j; \quad a(t_0) \equiv 1$$

proper time

comoving coordinates: for a maximally symmetric space

$$R^{(3)} = 6\kappa \begin{cases} > 0 \text{ sphere} \\ = 0 \text{ torus} \\ < 0 \text{ hyperboloid} \end{cases}$$

to see the causal structure, its better to use conformal time: $dt = a(t)dt'$

$$ds^2 = a(t')^2\underbrace{\left(-dt'^2 + \gamma_{ij}(x)dx^i dx^j\right)}$$

e.g. radiation dominated flat universe $a(t) \propto t^{\frac{1}{2}} \propto t'$;

matter dominated flat universe $a(t) \propto t^{\frac{2}{3}} \propto t'^2$;

Lambda dominated flat universe $a(t) \propto e^{H_\Lambda t} \propto \frac{1}{t_* - t'}$.

(recommended exercise!)

(from now on, t is the *conformal* time)

$$ds^2 = a(t)^2\left(-dt^2 + h_{ij}(t,x)dx^i dx^j\right)$$

comoving geometry
(including gravirational waves)

Massless particles or fields (e.g. Maxwell/Dirac) do not "see" the scale factor a

Friedmann $$3\dot{a}^2 = r + \mu a - 3\kappa a^2 + \lambda a^4$$

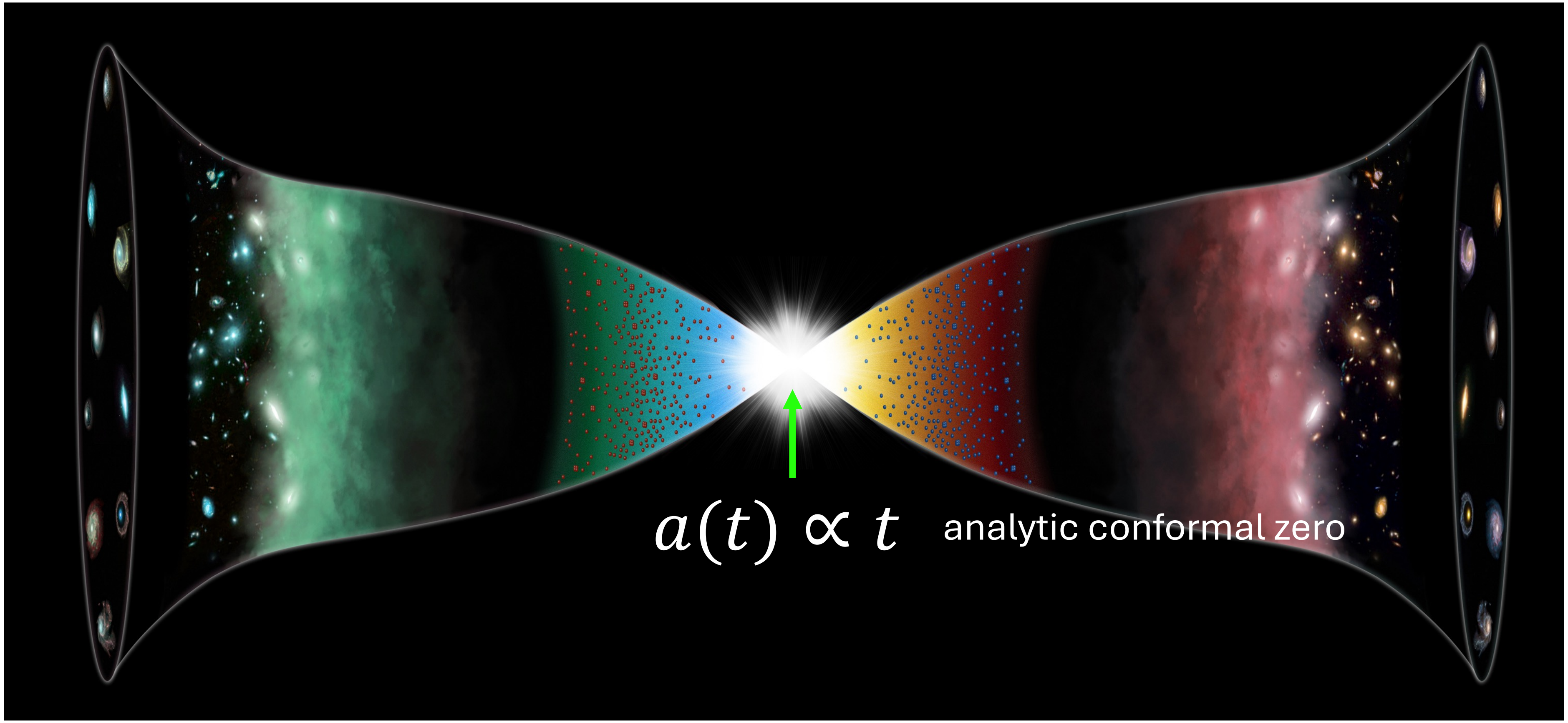
radiation matter space curvature Lambda

Maximal analytic extension is two-sided (just like black holes)

Boyle & NT *Phys. Lett.* B849 (2024) 138442; *Phys. Lett.* B849 (2024) 138443

Boyle, Finn, NT (2018)

radically minimal hypothesis: the universe is CPT symmetric



de Sitter epoch

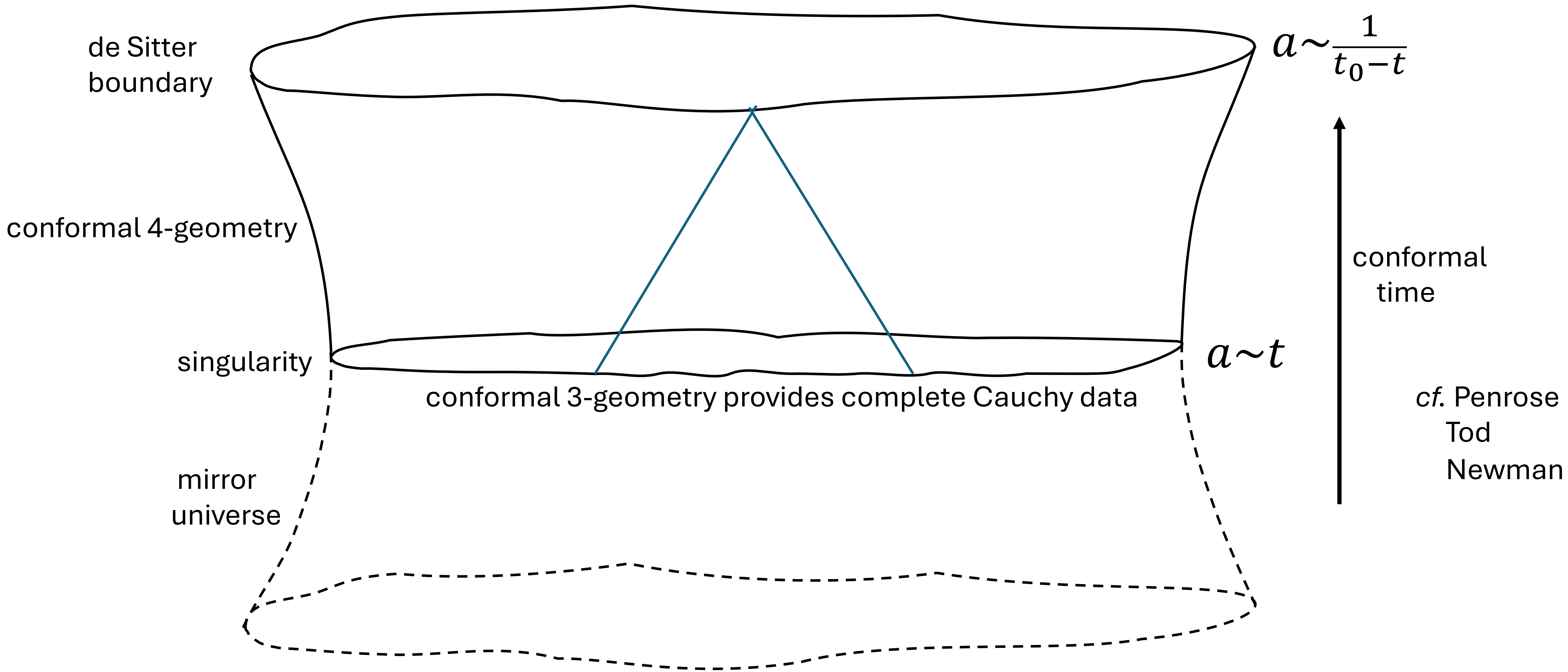
$$a(t) \propto \frac{1}{t_* - t}$$

simple pole

(classically, stress tensor for pure radiation is traceless, $T^\mu_\mu = 0 \Rightarrow R = 0 \Rightarrow a(t) \propto t$ as $t \downarrow 0$

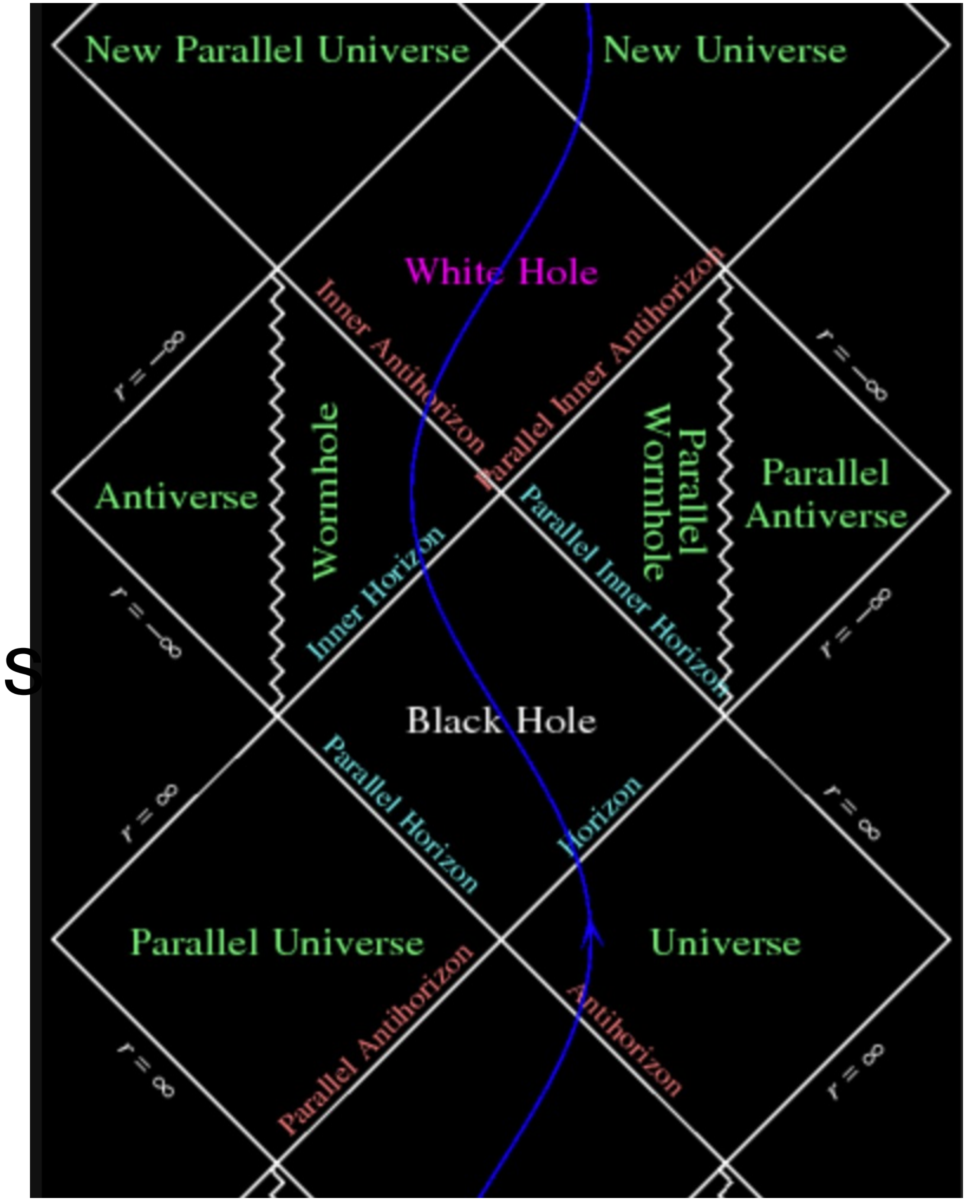
CPT-symmetric bc's imposed via the method of images – the big bang is a mirror!

Generalizes to inhomogeneous, conformal radiation-dominated cosmology. Half of the solutions are time-reversal symmetric: these correspond to a *regular conformal 4-geometry*



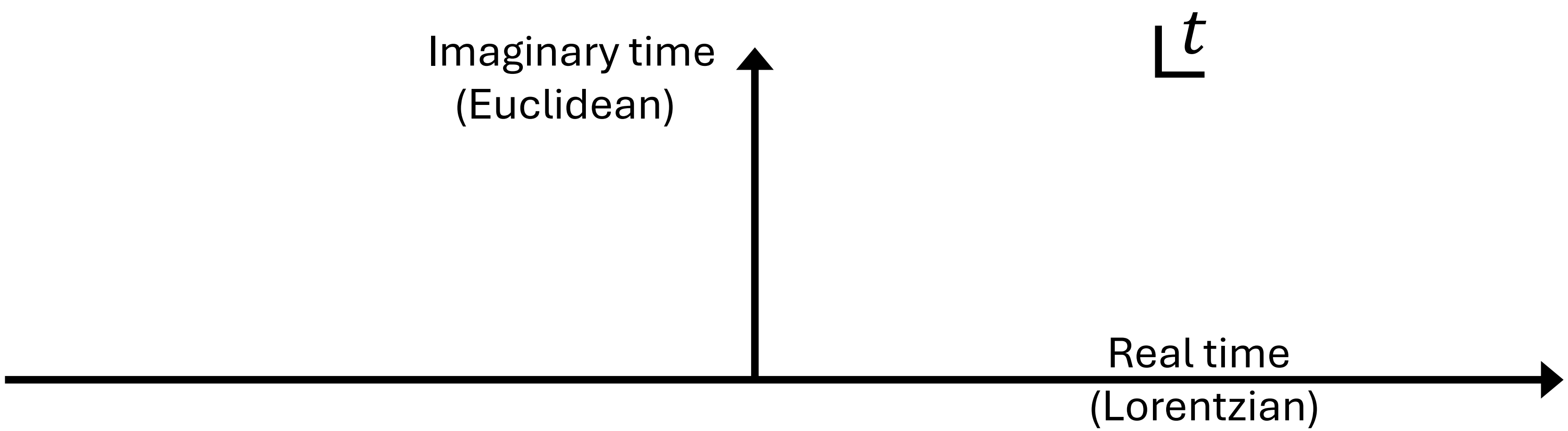
Full nonlinear Einstein-fluid equations can be solved in a covariant gradient expansion by matching the two asymptotic expansions: (NT 2024)

cf. black holes
maximal extens
also two-sided
(Kruskal)

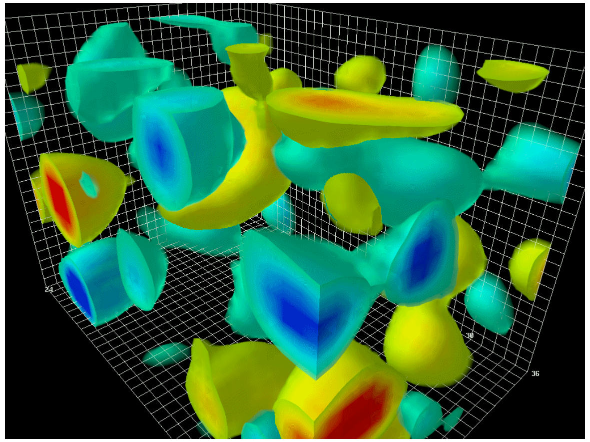


All black hole and cosmological solutions are periodic in imaginary time!

Explanation: they all have nice Euclidean versions:



Quantum fields in curved spacetime



Vacuum “zero point” energy and pressure are divergent.

Can be renormalized away but their infinite gravity is concerning.

Likewise, divergences spoil the local scale symmetry of Maxwell and massless Dirac fields

$$\langle T^{\mu}{}_{\mu} \rangle = a\, E + c\, C^2; \;\; E = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} - 4R_{\alpha\beta}R^{\alpha\beta} + R^2; C^2 = C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta}$$

These “trace anomalies” *cannot* be renormalized away.

SM-type fields contribute positively to a and c, so cancellations not possible.

A minimal resolution: dimension zero scalars

A *four*-derivative, locally scale-invariant action

Boyle+NT 2110.06258 [hep-th]
Andersen, Bateman,
Herzog, NT (2024)

$$S_4 = -\frac{1}{2}\int d^4x\sqrt{-g}\left(\,(\Box\varphi)^2 + ..\right) \qquad (*)$$

Canonical quantization leads to negative norm states. However, as in a gauge theory, these can be removed using an infinite-dimensional symmetry.

The only remaining physical state is the vacuum, with scale-invariant fluctuations:

$$\langle\varphi(0,\boldsymbol{x})\varphi(0,\boldsymbol{y})\rangle = \int \frac{d^3\boldsymbol{k}}{(2\,\pi)^3}\frac{e^{i\boldsymbol{k}.(\boldsymbol{x}-\boldsymbol{y})}}{4\,k^3}$$ *cf.* Φ fluctuations observed in cosmology

Exercise: show that a scalar field ϕ with a two-derivative action $\int d^4x\sqrt{-g}^2\left(\partial_\mu\phi\,\partial^\mu\phi\right)+..$ has dimension one and hence

That, up to numerical factors, its two point correlation function $\langle\boldsymbol{\phi}(0,\boldsymbol{x})\phi(0,\boldsymbol{y})\rangle \sim \int \frac{d^3\boldsymbol{k}}{\boldsymbol{k}}\,e^{i\boldsymbol{k}.(\boldsymbol{x}-\boldsymbol{y})}$

dim zero scalars can cancel the anomalies in coupling the SM to gravity (at zeroth, free field order)

L. Boyle+NT
2110.06258 [hep-th]

number of dim one scalars; chiral fermions; gauge bosons; dim zero scalars

1. Vacuum energy $\propto n_{s,1} - 2n_F + 2n_A + 2n_{s,0}$
2. Conformal anomaly (Euler) $\propto n_{s,1} + \frac{11}{2} n_F + 62\, n_A - 28\, n_{s,0}$
3. Conformal anomaly (Weyl^2) $\propto n_{s,1} + 3\, n_F + 12\, n_A - 8\, n_{s,0}$

(Recommended exercise:)

1) Vanishing of all three implies $n_{s,1} = 0 \Longrightarrow$ *no* fundamental dim one scalars (the Higgs must be composite: might explain the gauge-gravity hierarchy)

2) Any two equations then give $n_F = 4n_A$; for $SU3\times SU2\times U1$, $n_A = 12$
Hence, *we predict* n_F =48, *i.e., 3 fermion generations each including a RH ν*

3) $n_{s,0} = 3n_A = 36$, suggestive of new underlying symmetry Sp(8) – related to twistors