Lecture 2: a minimal SM/LCDM cosmology

Neil Turok

University of Edinburgh and Perimeter Institute

with Latham Boyle

and Maegan Andersen, Sam Bateman, Kostas Tsanavaris

LCDM provides a remarkably good fit to the large-scale universe, with just 5 fundamental physics parameters

the energy content

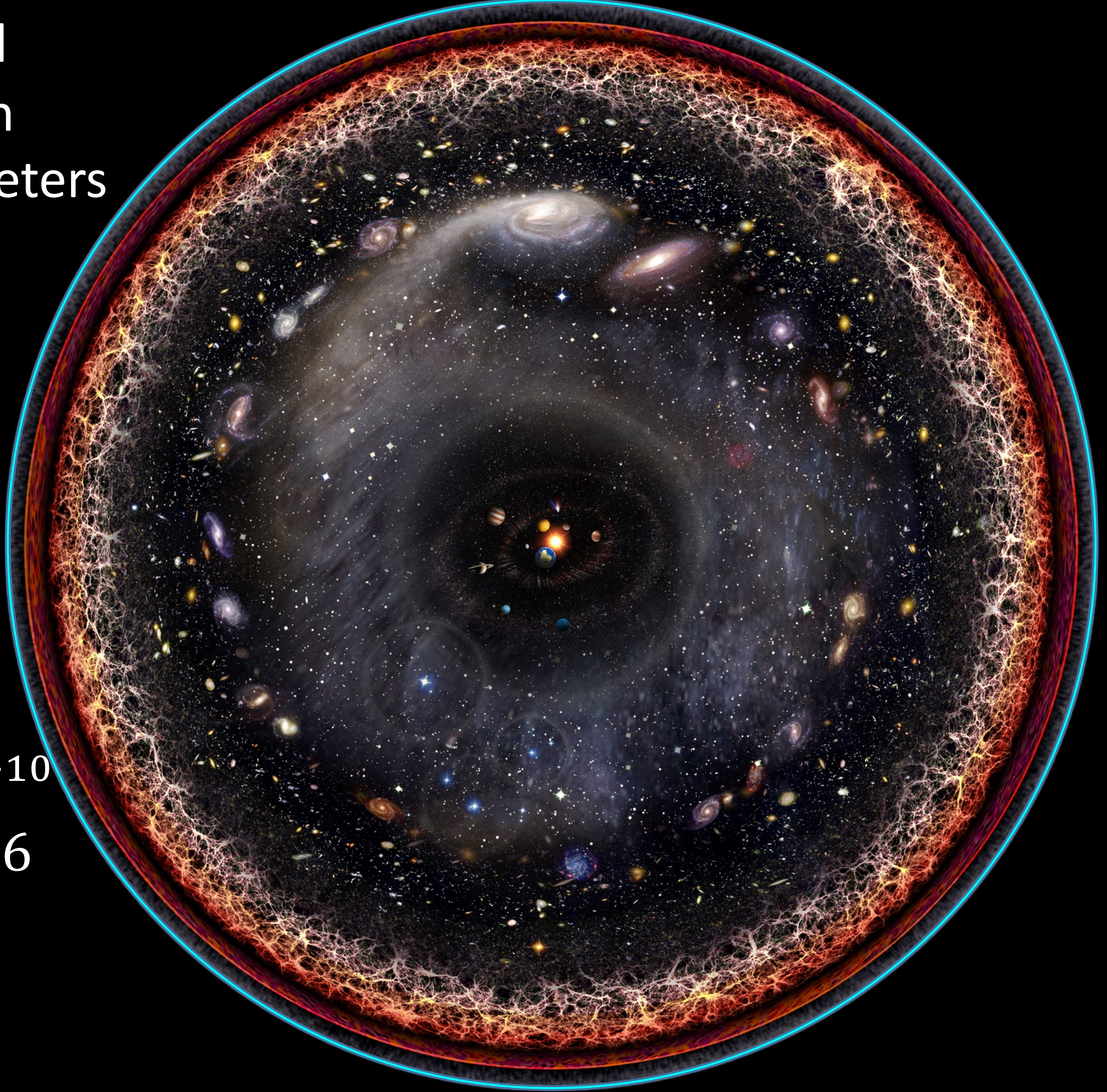
1. $\rho_\Lambda \;\; = (2.3 \text{ meV})^4$ ($\pm$ 1%)
2. $\rho_{DM}/\rho_B = 5.36$ ($\pm$ 1%)
3. $n_B/n_\gamma = 6\times 10^{-10}$ ($\pm$ 1%)

the perturbations

$$\langle \Phi^2 \rangle = \int \frac{dk}{k} A_\Phi \left(\frac{k}{k_*}\right)^{n_s - 1}$$

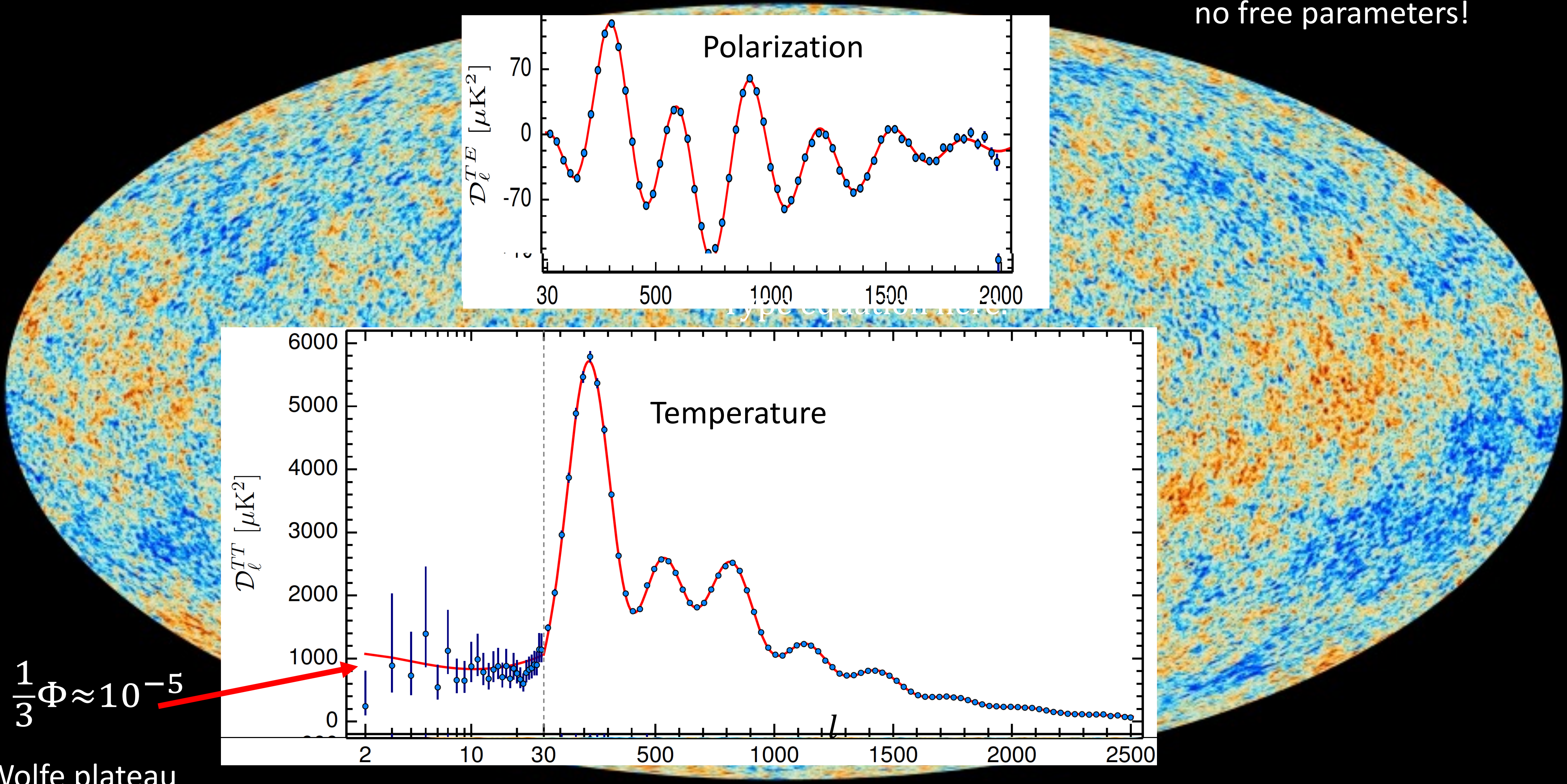
4. amplitude $A_\Phi \approx 7.6 \pm 0.1\times 10^{-10}$
5. "tilt" $n_s - 1 \approx -0.041 \pm 0.0056$

many parameters so far consistent with zero: tensor and "isocurvature" perturbations, spatial curvature κ, non-Gaussianity…



LCDM is an amazingly successful fit

Coulson, Crittenden, NT (1994))
no free parameters!



$$\frac{\delta T}{T} \approx \frac{1}{3}\Phi \approx 10^{-5}$$

Sachs-Wolfe plateau

acoustic peaks (sound waves in plasma)

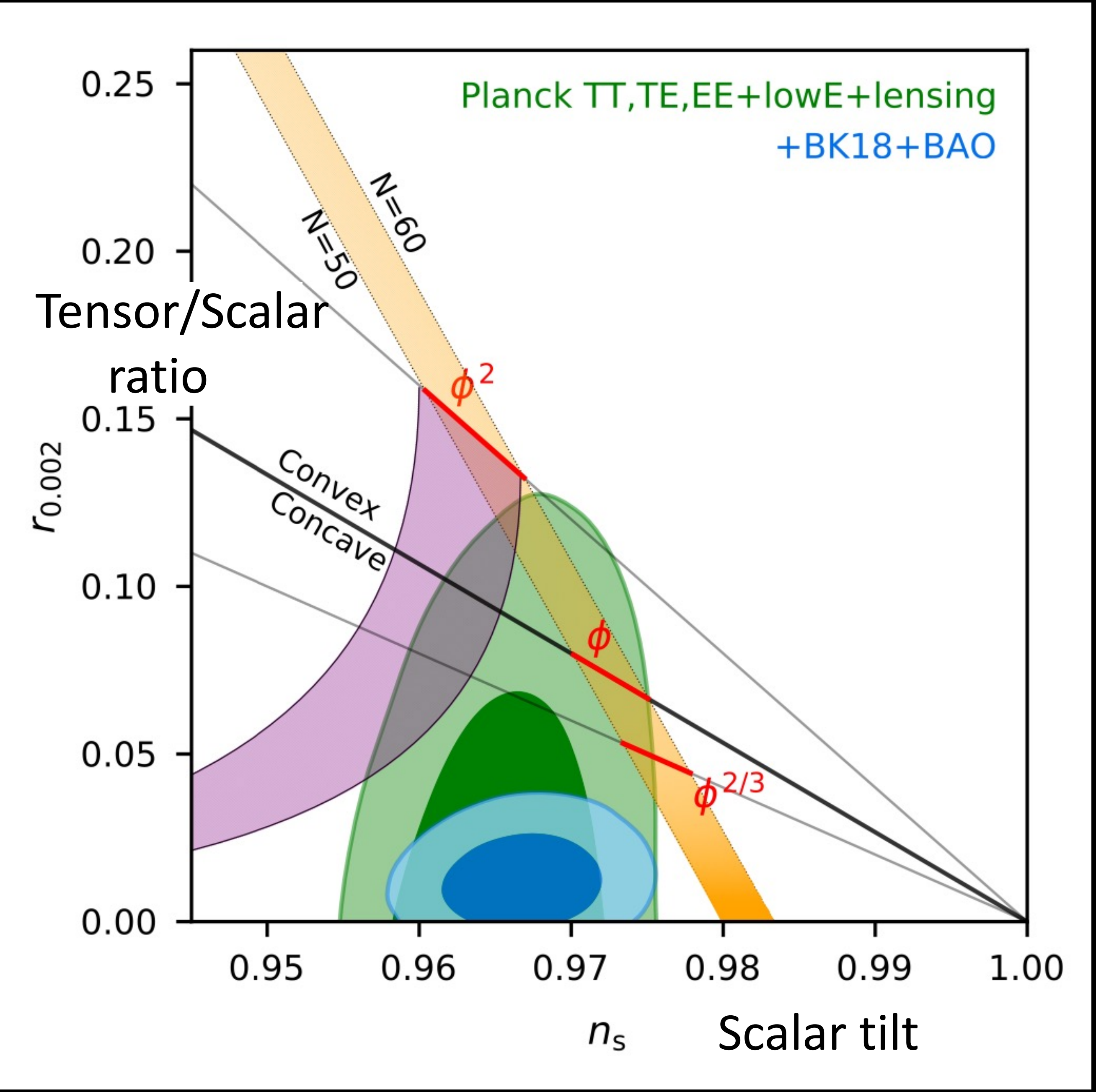
ESA Planck satellite

no sign of inflation's "smoking gun" signal, *i.e.*, long wavelength gravitational waves (tensor modes)

BICEP/Keck Collaboration
2203.16556 [astro-ph]
PRL **127**, 151301 (2021)
r< 0.036 at 95% confidence

anticipated limit
r<.003
using SPT for "delensing"
(2027)

inflation is (imho) steadily becoming less plausible



A similar story in particle physics: no deviations from the SM up to 10 TeV.

What is the unexpected simplicity on very large and very small scales telling us?

Maybe the expected complexity will be revealed at smaller (or larger?) scales…

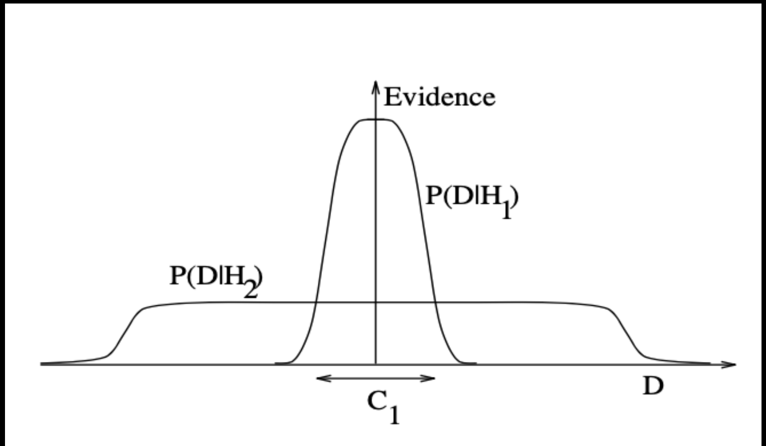
Or maybe the universe really is very simple on both very large and small scales!

The most exciting possibility is that the observations are teaching us something important and major advances in basic physics are imminent…

Occam’s Razor

‘A theory with mathematical beauty is more likely to be correct than an ugly one that fits some experimental data’ (Paul Dirac)

‘Coherent inference automatically embodies Occam’s razor, quantitatively.’
(David Mackay, Ch. 28, Information Theory, Inference and Learning Algorithms)



$$\text{Occam factor} = \frac{\sigma_{w|D}}{\sigma_w},$$

i.e., *the Occam factor is equal to the ratio of the posterior accessible volume of $\mathcal{H}_i$’s parameter space to the prior accessible volume*

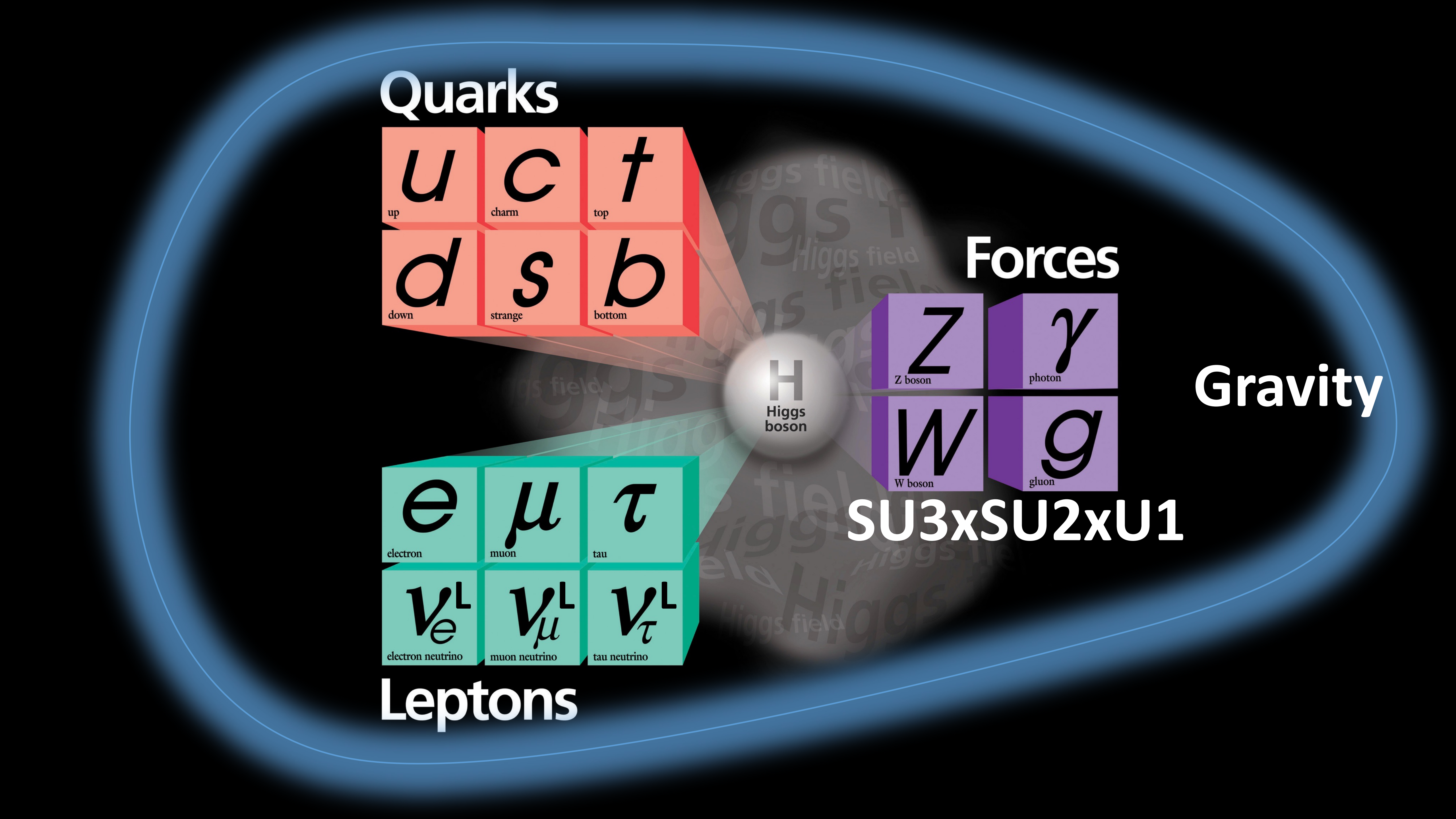
Theories or scenarios with a lot of freedom in their realization, *e.g.* multifield inflation or string compactifications, are seriously penalized by their Occam factor.

Our approach:

what is the *minimal* extension of known physics consistent with everything we know from particle physics and cosmology?

the deepest paradoxes and puzzles are our best clues

e.g. the cosmological dark matter



Quarks
u
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Leptons
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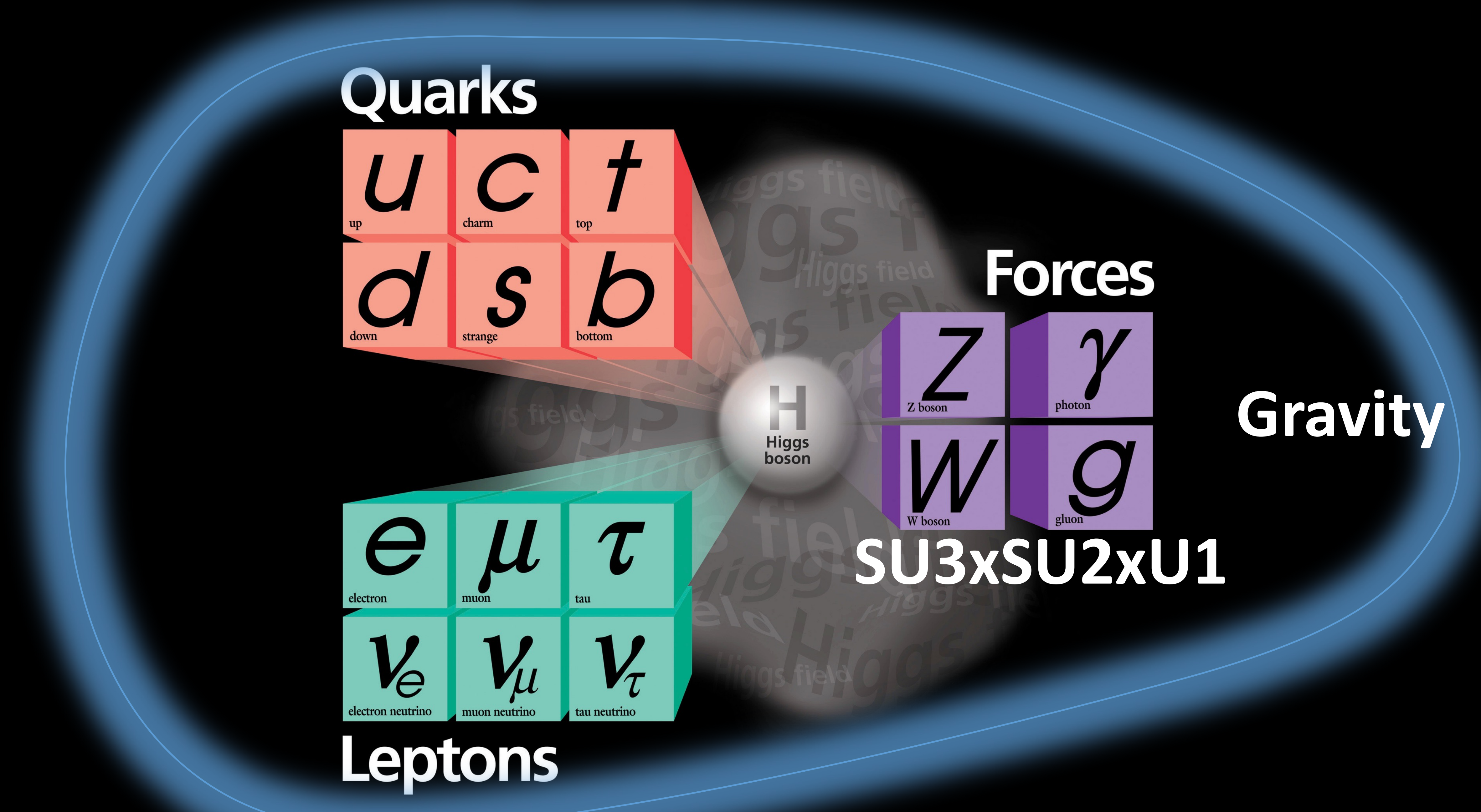
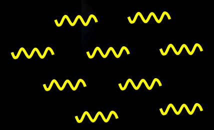
ν_e^L
electron neutrino
ν_μ^L
muon neutrino
ν_τ^L
tau neutrino
H
Higgs boson
Higgs field
Forces
Z
Z boson
γ
photon
W
W boson
g
gluon
SU3xSU2xU1
Gravity



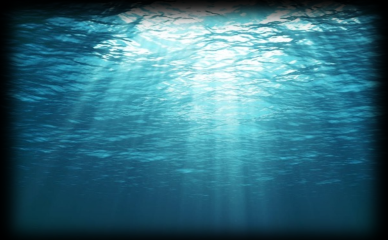
(can explain the dark matter: see later)

the physics we know is already quite unified

gauge fields

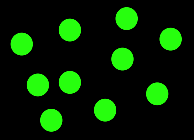


Higgs



$$\int e^{\frac{i}{\hbar}\int\left(\frac{R}{16\pi G} - \frac{1}{4}F^2 + \overline{\psi} i \not{D}\psi - \lambda H\overline{\psi}\psi + |DH|^2 - V(H) \right)}$$

gravity



particles

but there are many mysteries

Why is the large-scale geometry of the universe so simple?

- homogeneous, isotropic and spatially flat $(\kappa \approx 0)$

$$ds^2 = -\underbrace{dt^2}_{\text{proper time}} + a(t)^2 \underbrace{\gamma_{ij}(x) dx^i dx^j}_{\text{comoving coordinates: for a maximally symmetric space}}; \quad a(t_0) \equiv 1$$

$$R^{(3)} = 6\kappa \begin{cases} > 0 \text{ sphere} \\ = 0 \text{ torus} \\ < 0 \text{ hyperboloid} \end{cases}$$

to see the causal structure, its better to use conformal time: $dt = a(t)dt'$

$$ds^2 = a(t')^2 \big(\underbrace{-dt'^2 + \gamma_{ij}(x) dx^i dx^j}\big)$$

e.g. radiation dominated flat universe $a(t) \propto t^{\frac{1}{2}} \propto t'$;

matter dominated flat universe $a(t) \propto t^{\frac{2}{3}} \propto t'^2$;

Lambda dominated flat universe $a(t) \propto e^{H_\Lambda t} \propto \frac{1}{t_* - t'}$.

(recommended exercise!)

(from now on, t is the *conformal* time)

Generalize to inhomogeneous, anisotropic spacetimes

$$ds^2 = a(t)^2\left(-dt^2 + h_{ij}(t,x)dx^i dx^j\right)$$

comoving geometry (including density perts and gravitational waves)

Massless particles or fields (e.g. Maxwell/Dirac) do not “see” the scale factor $a(t)$

In conformal time $a(t)$ obeys

radiation matter space curvature Lambda

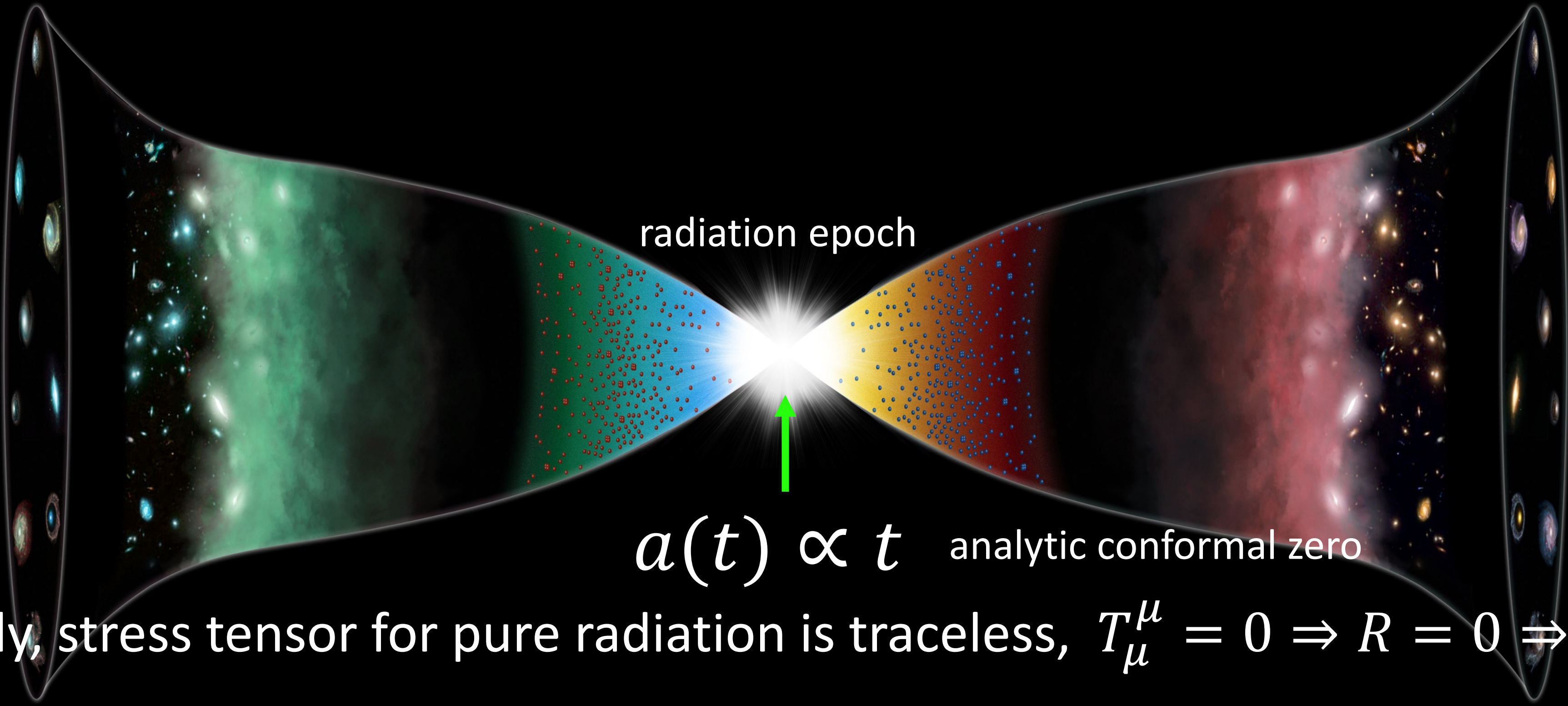
$$3\dot{a}^2 = r + \mu a - 3\kappa a^2 + \lambda a^4$$

Maximal analytic extension is two-sided (just like black holes): the two sides are *exchanged* by CPT!

Boyle & NT *Phys. Lett.* B849 (2024) 138442; *Phys. Lett.* B849 (2024) 138443

Boyle, Finn, NT (2018)

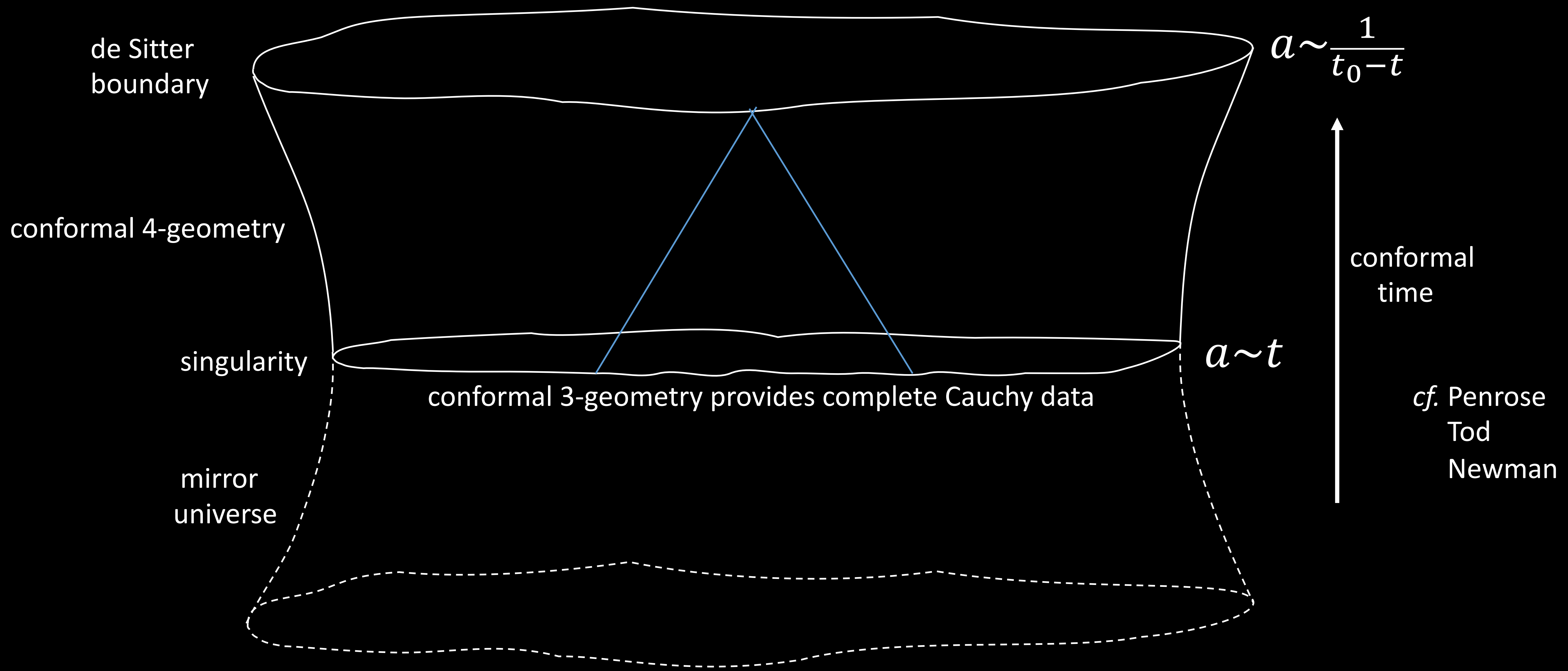
radically minimal hypothesis: the universe is CPT symmetric



(classically, stress tensor for pure radiation is traceless, $T^{\mu}_{\mu} = 0 \Rightarrow R = 0 \Rightarrow a(t) \propto t$ as $t \downarrow 0$)

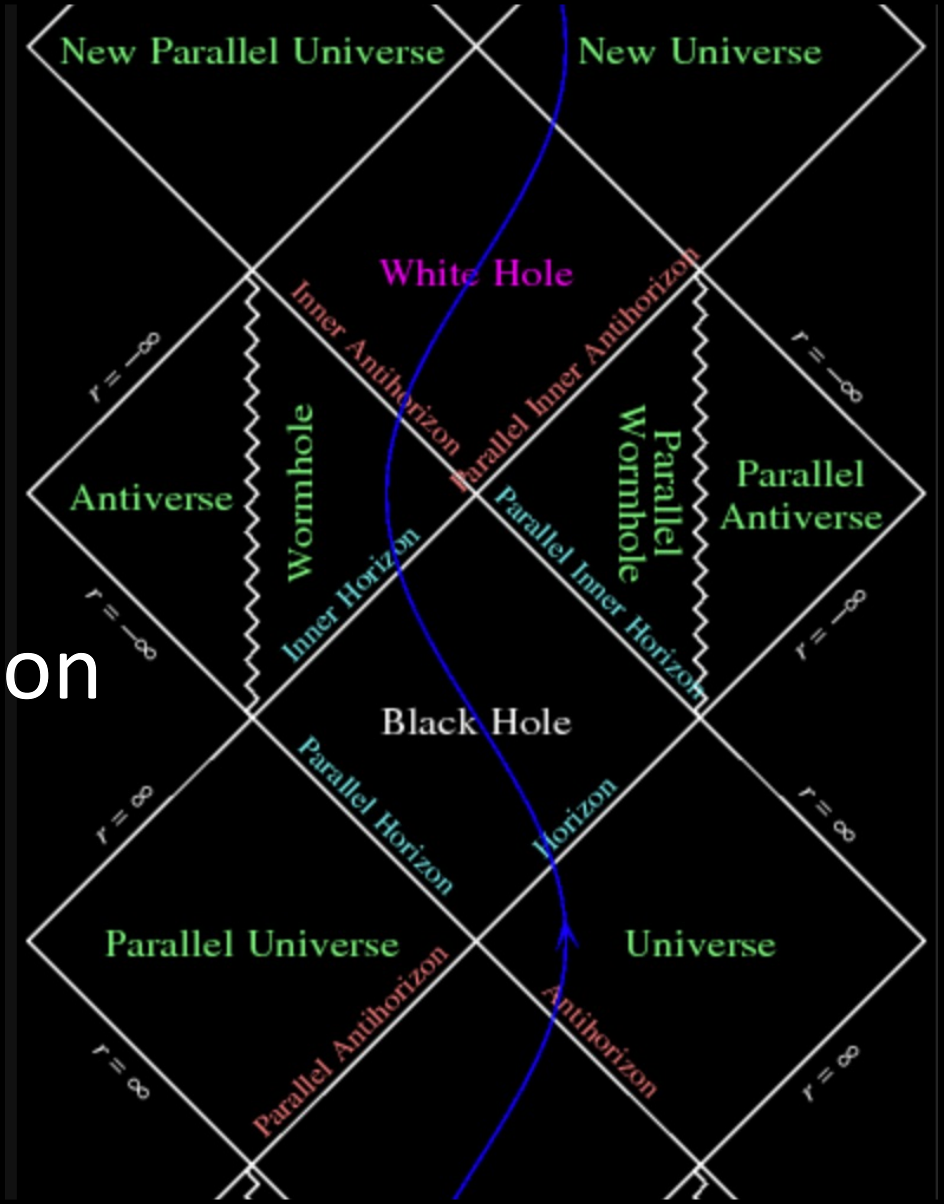
CPT-symmetric bc's imposed via the method of images – the big bang is a mirror!

Generalizes to inhomogeneous, conformal radiation-dominated cosmology. Half of the solutions are time-reversal symmetric: these correspond to a *regular conformal 4-geometry*



Full nonlinear Einstein-fluid equations can be solved in a covariant gradient expansion by matching the two asymptotic expansions: (NT 2024)

cf. black holes
maximal extension
also two-sided
(Kruskal)



Black hole thermodynamics

Hawking
Bekenstein
Bardeen
Geroch
Gibbons
Perry
Hartle
Unruh
Wald

$$\langle f | e^{-iHt} | i \rangle \Longrightarrow Tr(e^{-\beta H})$$

$$Z = e^{S_{ord} + S_g}$$

partition function

gravitational entropy

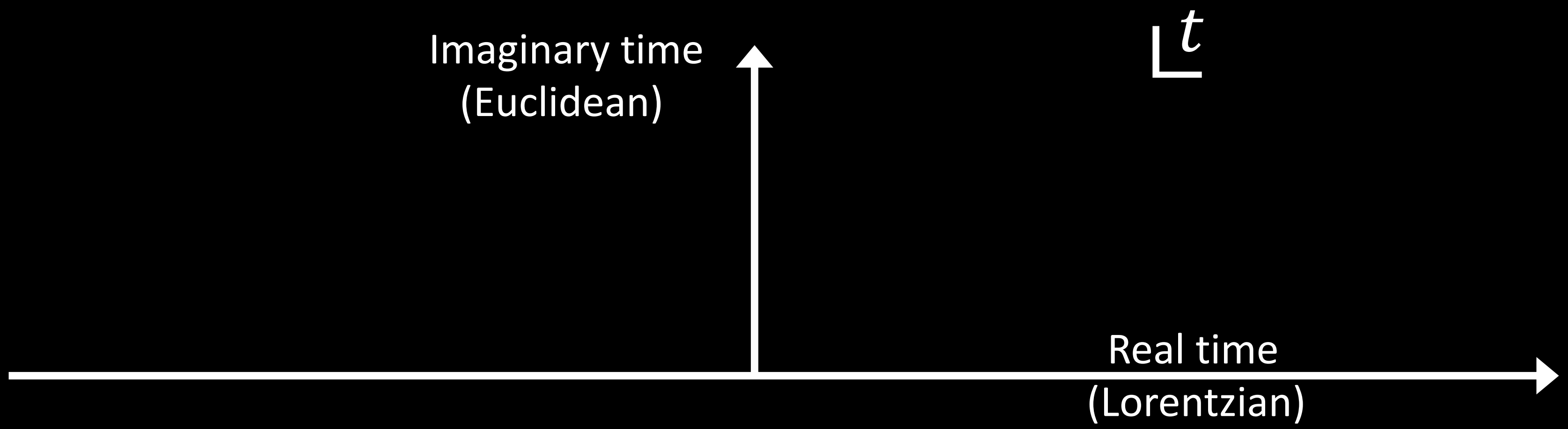
Hawking temperature T_H and gravitational entropy S_g

(imaginary time period)

(action calculated over one period)

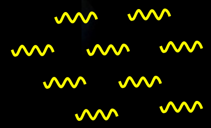
All black hole and cosmological solutions are double-sided!

Explanation: they all have nice Euclidean versions:

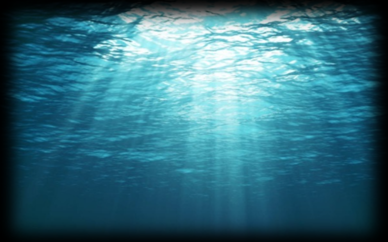


path integrals and thermodynamics

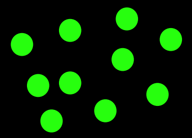
gauge fields



Higgs



$$\int e^{\frac{i}{\hbar}\int\left(\frac{R}{16\pi G}-\frac{1}{4}F^2+\overline{\psi}i\not{D}\psi-\lambda H\overline{\psi}\psi+|DH|^2-V(H)\right)}$$



particles

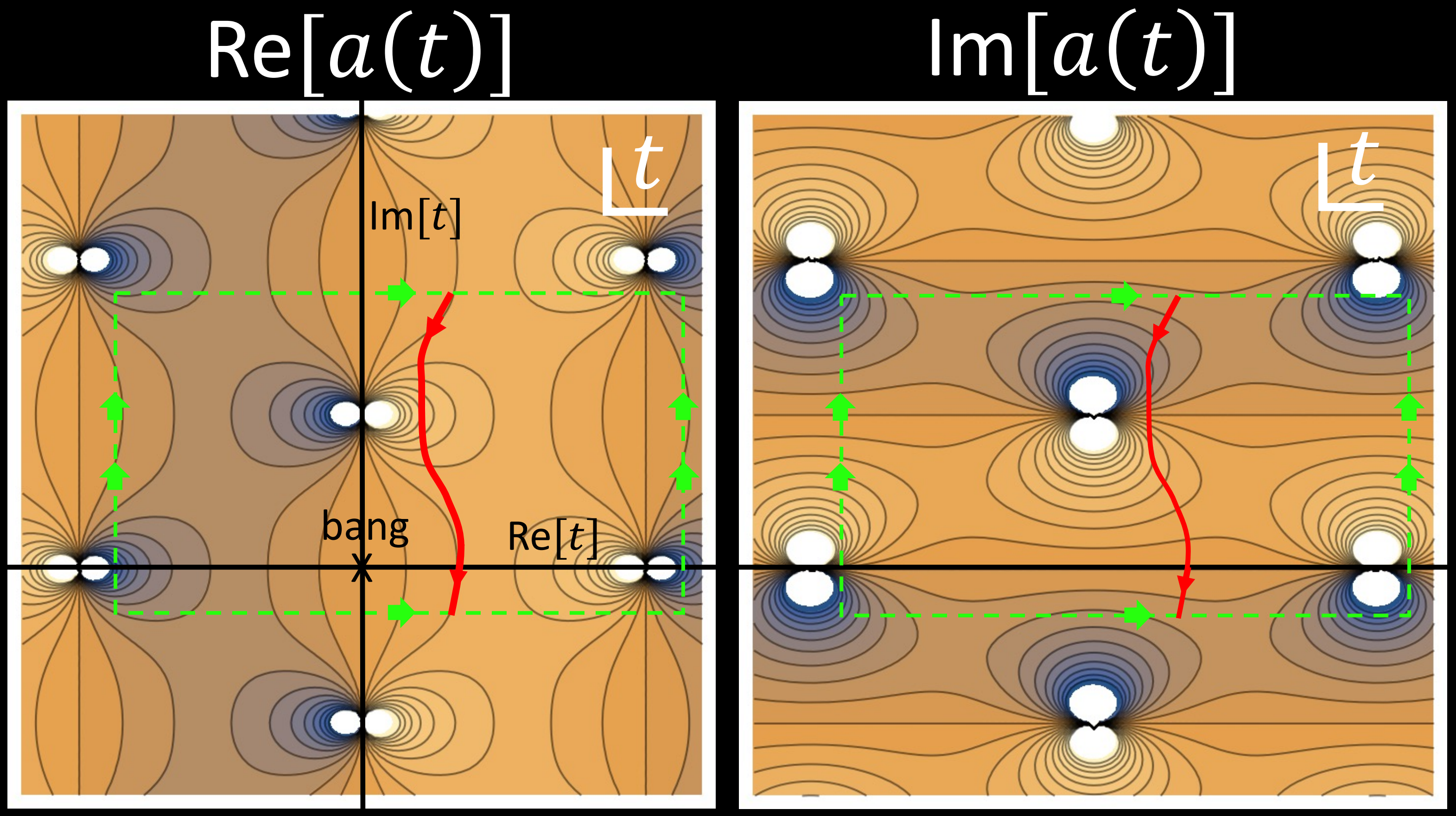
$$\langle f|e^{-iHt}|i\rangle \Longrightarrow Tr(e^{-\beta H})$$

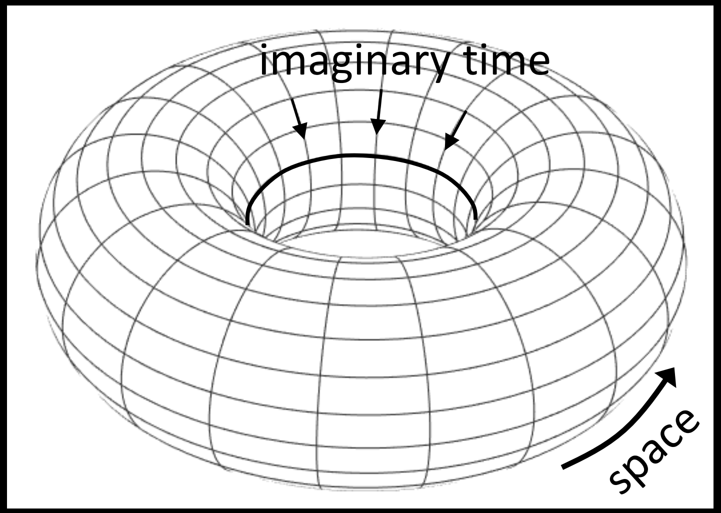
gravitational entropy

with pbcs in imaginary time, $Z = e^{S_{tot}} = e^{S_{ord}+S_g}$

partition function

$a(t)$ is single-valued and doubly periodic in the complex t-plane: its only singularities are simple poles. The imaginary time period and the action computed over a period determine T_H and the gravitational entropy S_g





Euclidean instanton for a universe w/radiation, matter, curvature, Lambda

Boyle & NT *Phys. Lett.* B849 (2024) 138442; *Phys. Lett.* B849 (2024) 138443

S_g can be calculated for realistic cosmologies with CPT-symmetric bc's, treating inhomogeneities in cosmological perturbation theory.

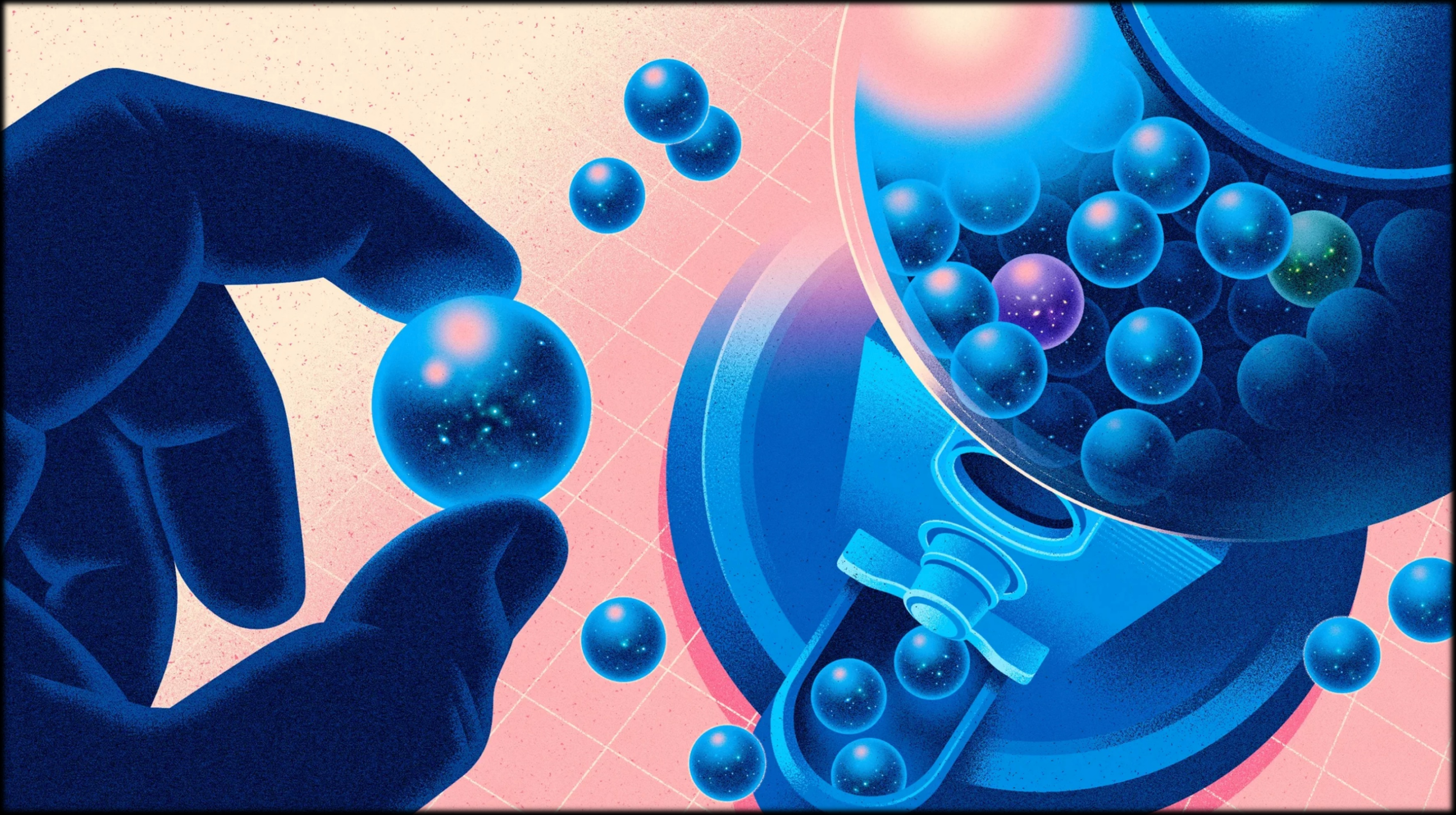
S_g provides a measure on cosmology. It is greatest for

1. homogeneous, isotropic, spatially flat universes
2. a small, positive cosmological constant (echoing old arguments by Baum, Hawking, Coleman…)

This is a thermodynamic explanation of the large-scale geometry of the cosmos.

Inflation is no longer needed.

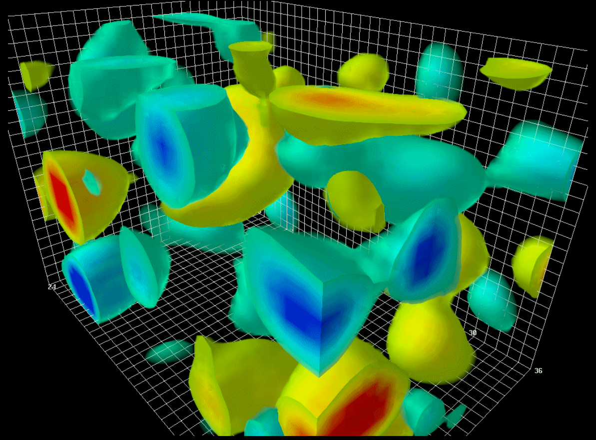
Note: the gravitational entropy is, in general, a global property of a *spacetime, not a configuration at a moment of time.*



Quanta Magazine, Nov 17, 2022; *WIRED*, Jan 22, 2023

What about the density fluctuations?

Quantum fields in curved spacetime



Vacuum “zero point” energy and pressure are divergent.

Can be renormalized away but their infinite gravity is concerning.

Likewise, divergences spoil the local scale symmetry of Maxwell and massless Dirac fields

$$\langle T^{\mu}{}_{\mu}\rangle = a\,E + c\,C^2; \;\; E = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} - 4R_{\alpha\beta}R^{\alpha\beta} + R^2; C^2 = C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta}$$

These “trace anomalies” *cannot* be renormalized away.

SM-type fields contribute positively to a and c, so cancellations are not possible.

A minimal resolution: dimension zero scalars

A *four*-derivative, locally scale-invariant action

Boyle+NT
Andersen, Bateman, Herzog, NT

$$S_4 = -\frac{1}{2}\int d^4x\sqrt{-g}\left((\Box\varphi)^2 + .. \right) \quad (*)$$

Canonical quantization leads to negative norm states. However, as in a gauge theory, these can be removed using an infinite-dimensional symmetry.

The only remaining physical state is the vacuum, with scale-invariant fluctuations:

$\langle\varphi(0,\boldsymbol{x})\varphi(0,\boldsymbol{y})\rangle = \int \frac{d^3\boldsymbol{k}}{(2\,\pi)^3}\frac{e^{i\boldsymbol{k}.(\boldsymbol{x}-\boldsymbol{y})}}{4\,k^3}$ *cf.* Φ fluctuations observed in cosmology

Exercise: show that a scalar field ϕ with a two-derivative action $\int d^4x\sqrt{-g}^2\left(\partial_\mu\phi\,\partial^\mu\phi\right)+..$ has dimension one and hence

That, up to numerical factors, its two point correlation function $\langle\boldsymbol{\phi}(0,\boldsymbol{x})\phi(0,\boldsymbol{y})\rangle\sim\int\frac{d^3\boldsymbol{k}}{k}\,e^{i\boldsymbol{k}.(\boldsymbol{x}-\boldsymbol{y})}$

L. Boyle+NT
2110.06258 [hep-th]

dim zero scalars can cancel the anomalies in coupling the SM to gravity (at zeroth, free field order)

number of dim one scalars | chiral fermions | gauge bosons | dim zero scalars

1. Vacuum energy $\propto n_{s,1} - 2n_F + 2n_A + 2n_{s,0}$
2. Conformal anomaly (Euler) $\propto n_{s,1} + \frac{11}{2} n_F + 62\, n_A - 28\, n_{s,0}$
3. Conformal anomaly (Weyl^2) $\propto n_{s,1} + 3\, n_F + 12\, n_A - 8\, n_{s,0}$

(Recommended exercise:)

1) Vanishing of all three implies $n_{s,1} = 0 \Longrightarrow$ *no* fundamental dim one scalars
(the Higgs must be composite: opportunity to explain the gauge-gravity hierarchy)

2) Any two equations then give $n_F = 4n_A$; for $SU3\times SU2\times U1$, $n_A = 12$
Hence, *we predict* n_F =48, *i.e., 3 fermion generations each including a RH* ν

3) $n_{s,0} = 3n_A = 36$, suggestive of a new underlying symmetry Sp(8) – related to twistors

Density perturbations

primordial perturbations from dimension 0 fields

Boyle+NT
2302.00344 [hep-ph]

Running couplings violate scale symmetry: at high temperature,

$$T_\beta^{SM} \equiv \langle T^{SM\mu}{}_{\mu} \rangle_\beta = 3P - \rho \approx \sum c_i \alpha_i^2 T^4 \equiv c_\beta^{SM} T^4; \quad \text{in SM, } c_\beta^{SM} \equiv \frac{125}{108}\alpha_Y^2 - \frac{95}{72}\alpha_2^2 - \frac{49}{6}\alpha_3^2$$

This anomalous trace can be cancelled by introducing a linear coupling in the effective action,

$$\Gamma^\varphi = \sum_{j=1}^{n_{s,0}} \frac{1}{2} \int -a\varphi_j \Delta_4 \varphi_j + \left[a\left(E - \tfrac{2}{3}\Box R\right) + cC^2 - n_{s,0}^{-1} T_\beta^{SM}\right]\varphi_j$$

Note: the linear term is consistent with the infinite dimensional symmetry (and generalizes a trick used in string theory to preserve conformal symmetry on the world sheet.)

The final term corrects the Friedmann-fluid equations, converting quantum correlations in the dim-0 fields into large scale curvature fluctuations:

$$\dot{a}^2 = \frac{8\pi G}{3}\rho_r a^4 (1 + c_\varphi \bar{\varphi}(x)) \text{ with } \bar{\varphi}(x) = n_{s,0}^{-1} \sum \varphi_j(x), \ c_\varphi = c_\beta^{SM} / \left(\tfrac{\pi^2}{30}\mathcal{N}_{eff}\right), \mathcal{N}_{eff} \approx 106\tfrac{1}{4}$$

This creates a "comoving curvature perturbation" $\mathcal{R}(x) = \frac{1}{4} c_\varphi \bar{\varphi}(x)$

(adiabatic, Gaussian, scalar: no primordial long-wavelength gravitational waves)

primordial perturbations

Under some technical assumptions, we found conformal symmetry implies

$\mathcal{P}_{\mathcal{R}}(k) = \frac{3^2 5^2}{7(2\pi)^4}\left(\frac{c_\beta^{SM}}{\mathcal{N}_{eff}}\right)^2\left(\frac{k}{k_{Pl}}\right)^{-\frac{7\alpha_3}{\pi}}$; k_{Pl}= comoving Planck wavenumber

with $c_\beta^{SM} \equiv \frac{125}{108}\alpha_Y^2 - \frac{95}{72}\alpha_2^2 - \frac{49}{6}\alpha_3^2$ and $\mathcal{N}_{eff} = 106\frac{1}{4}$

(SM gauge couplings extrapolated to the Planck scale)

Using $(k_{Pl}/k_*)^{1-n_s} = 14.8 \pm 5.1$, $k_* \equiv$0.05 Mpc^{-1}

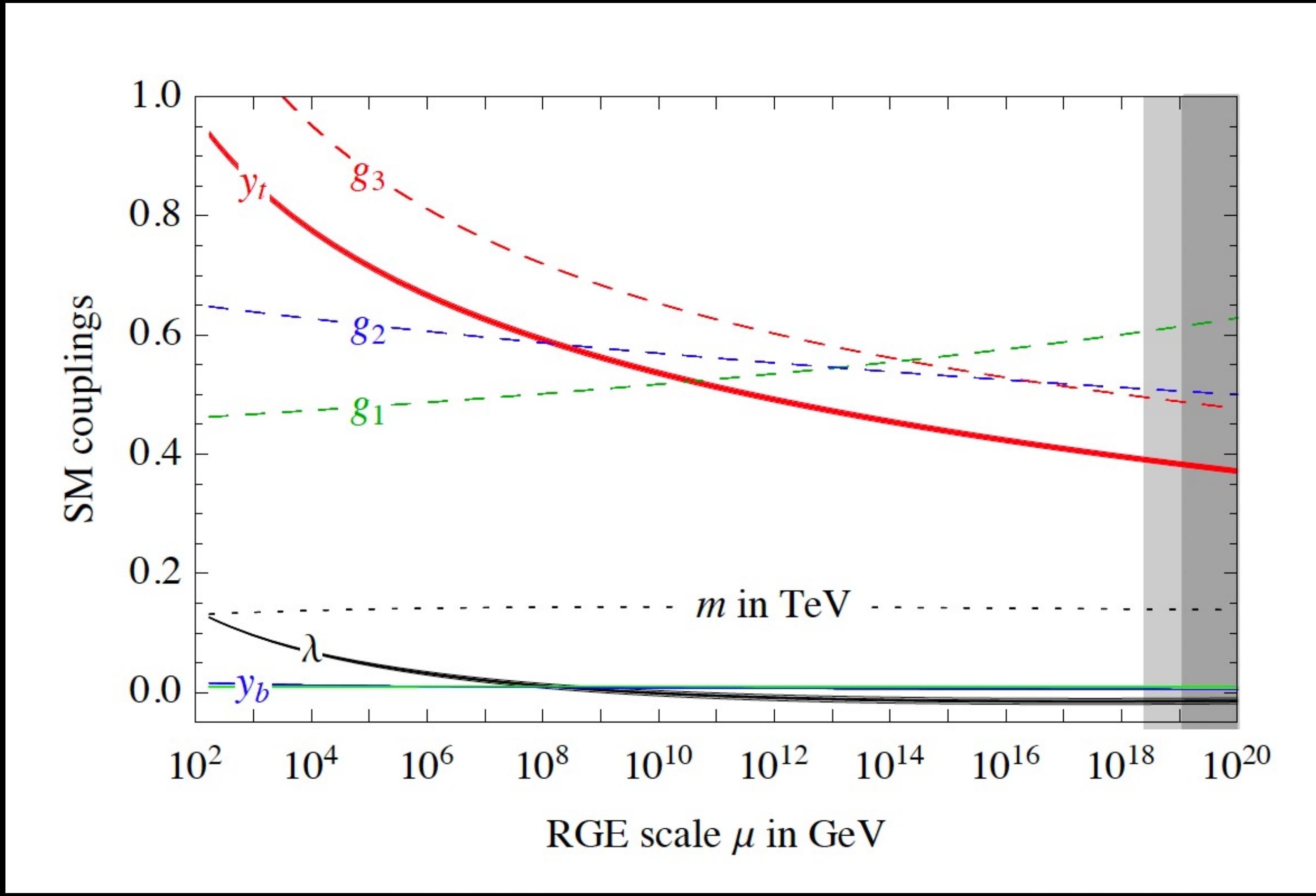
Thus, we predict $\mathcal{P}_{\mathcal{R}} = A\left(\frac{k}{k_*}\right)^{n_s-1}$, $A = (13 \pm 5)\times 10^{-10}$; $n_s = 0.958$

cf. Planck satellite: $A = (21 \pm 0.3)\times 10^{-10}$; $n_s = 0.959 \pm 0.006$

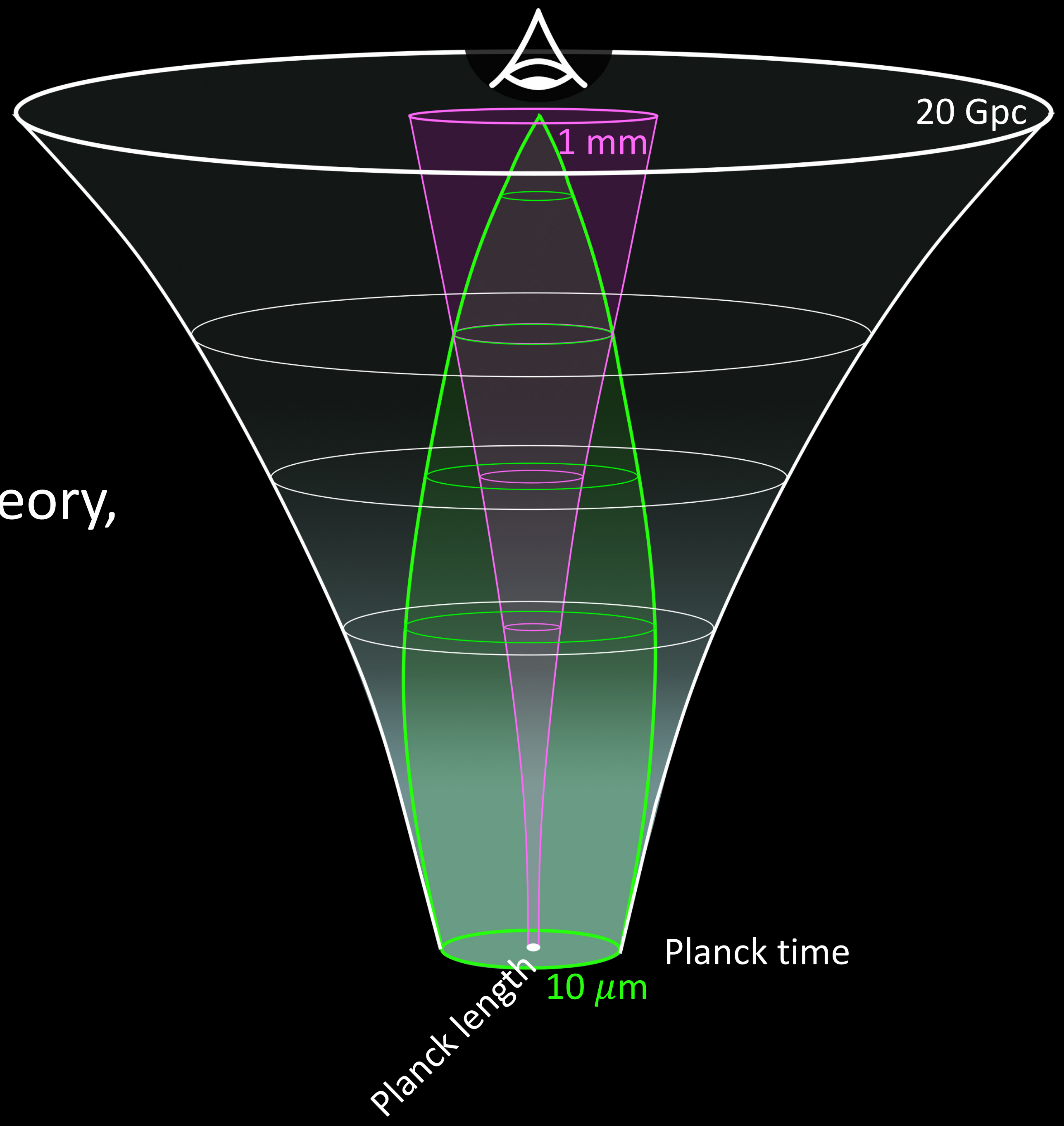
Red tilt

Dominated by QCD: asymptotic freedom/infrared slavery

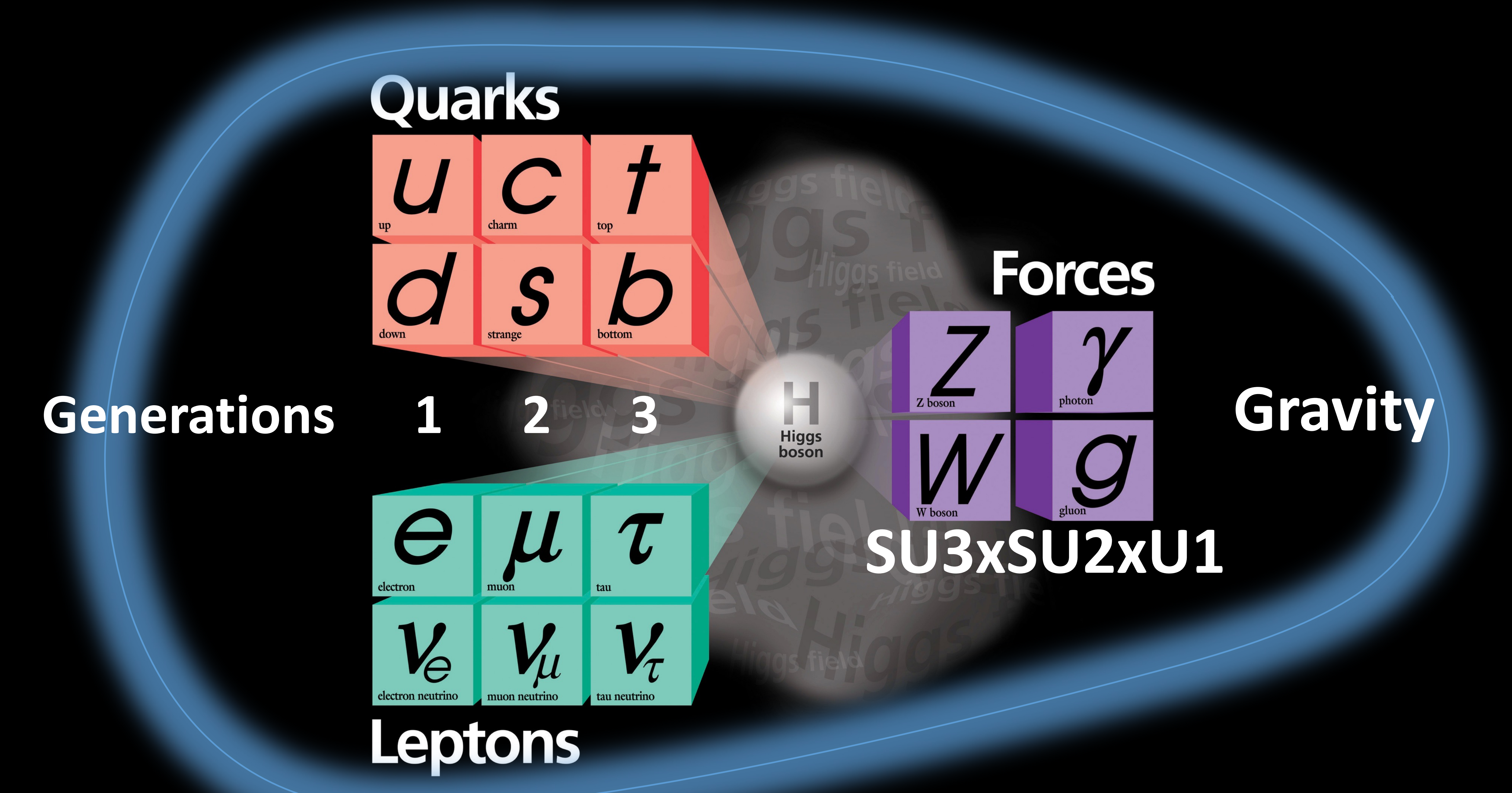
Buttazzo et al
1307.3536
[hep-ph]



Potentially an extremely predictive theory, connecting the smallest and largest observable scales in the universe.



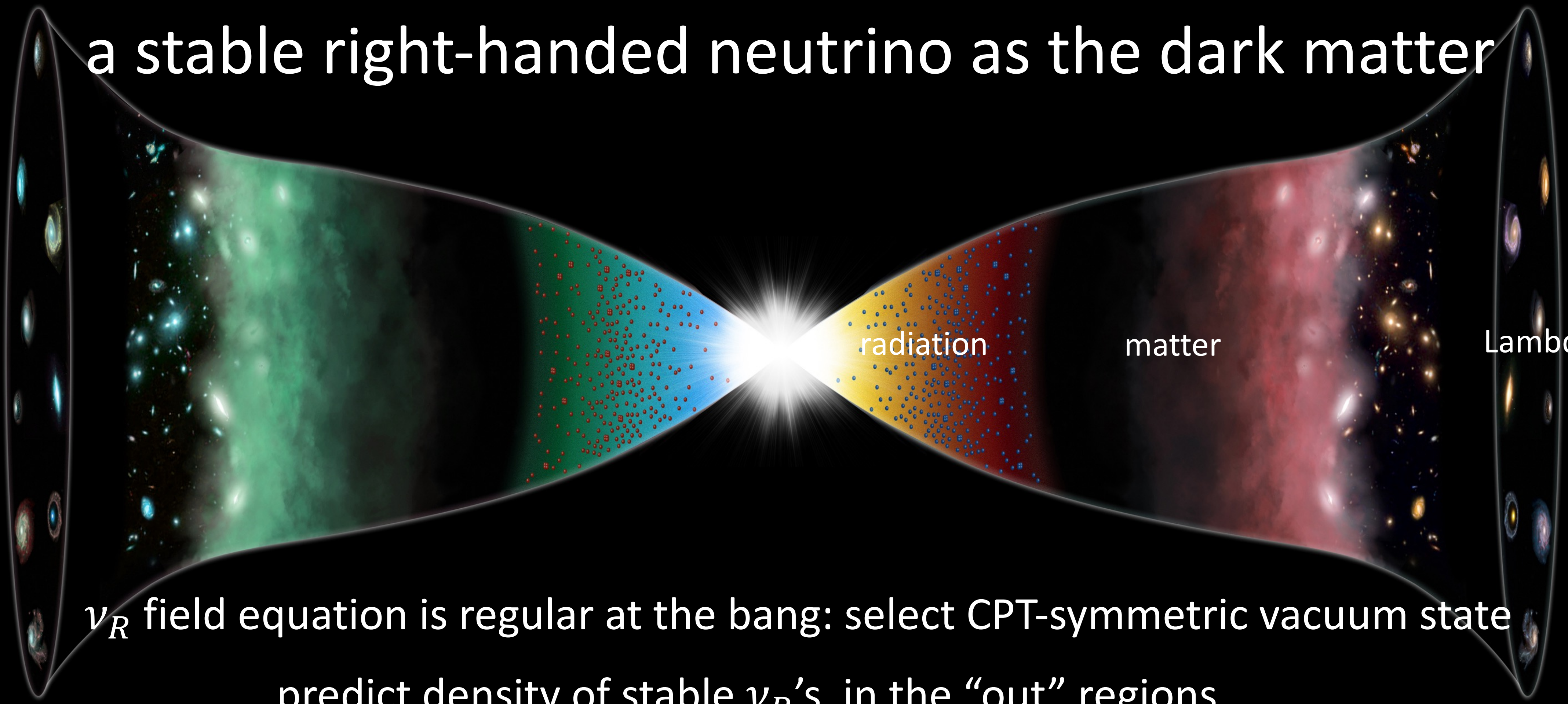
Dark matter



(16 chiral fermions per generation: 48 in total)

Boyle, Finn & NT, *Phys. Rev. Lett.* 121 (2018) 251301; *Annals of Physics* 438 (2022) 168767

a stable right-handed neutrino as the dark matter

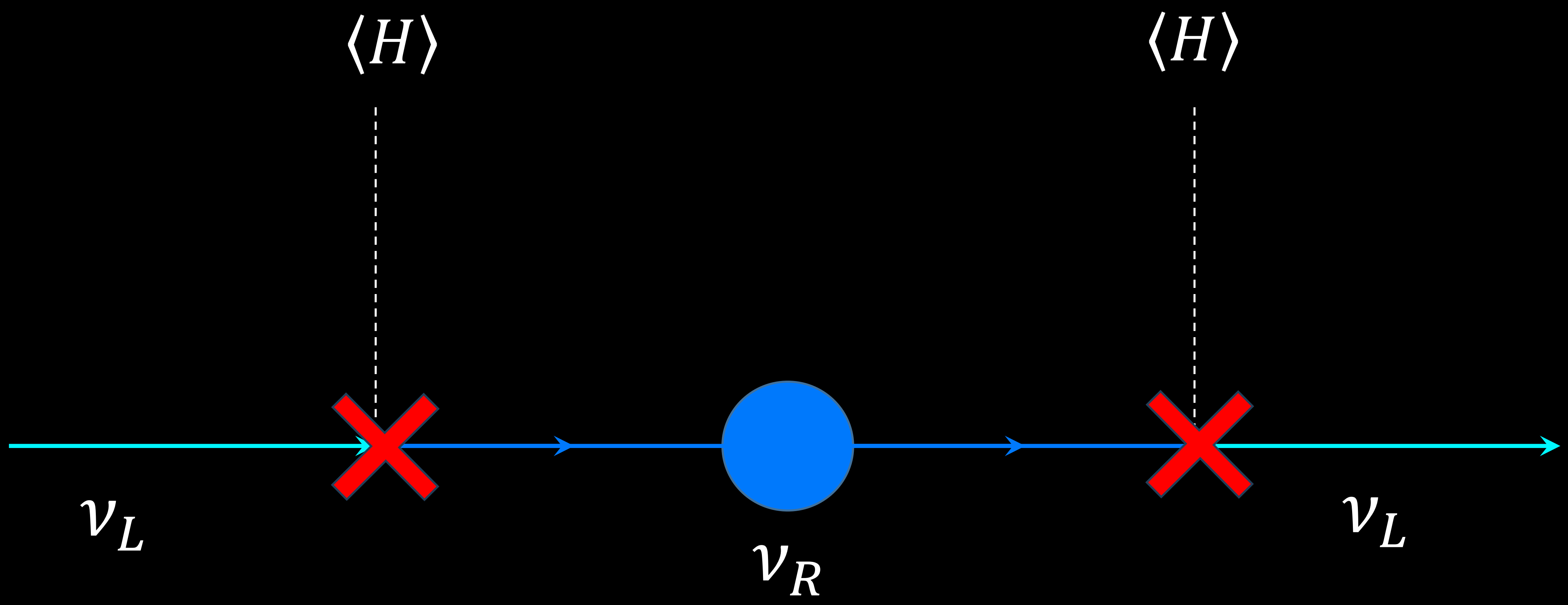


ν_R field equation is regular at the bang: select CPT-symmetric vacuum state

predict density of stable ν_R's in the "out" regions

if one ν_R is stable, its density matches Ω_{DM} if its mass $M \approx 5\times10^8 GeV$

$\mathbb{Z}_2$ symm $\Rightarrow$ one RH neutrino stable $\Rightarrow$ lightest ν massless

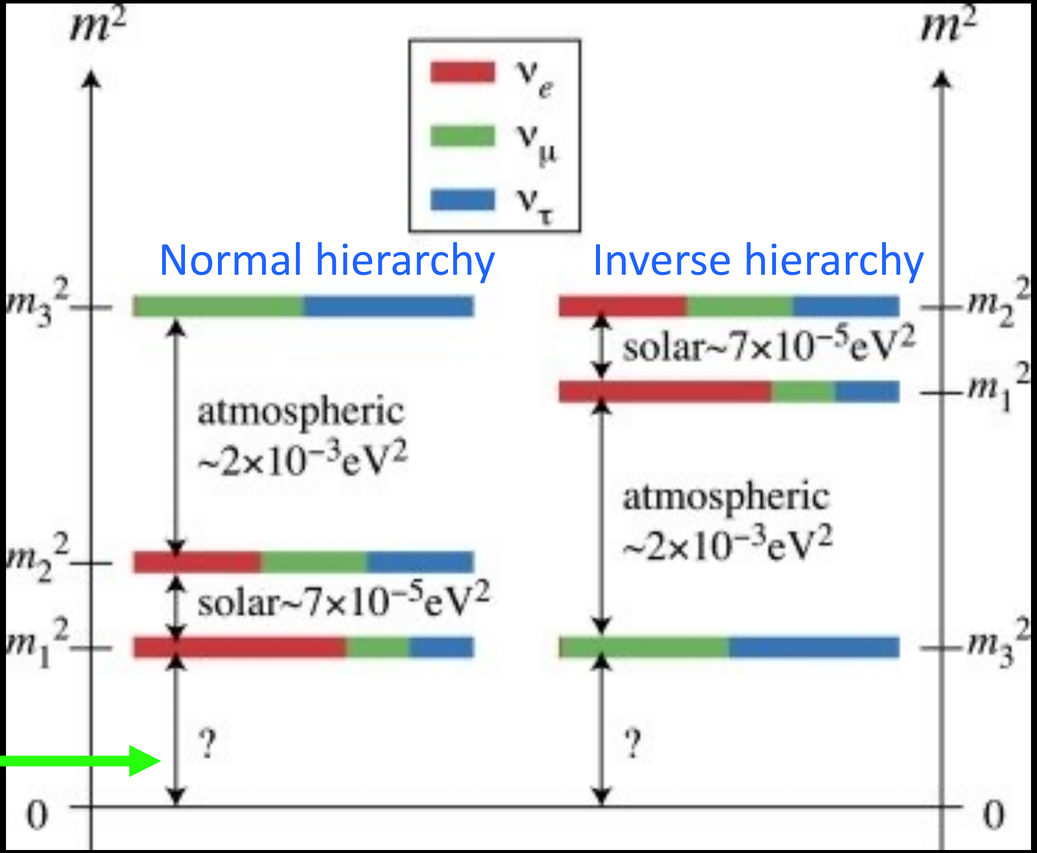


about to be tested using EUCLID, LSST and S4

Light neutrinos: observations

current data

eBOSS 2007.08991



we predict zero

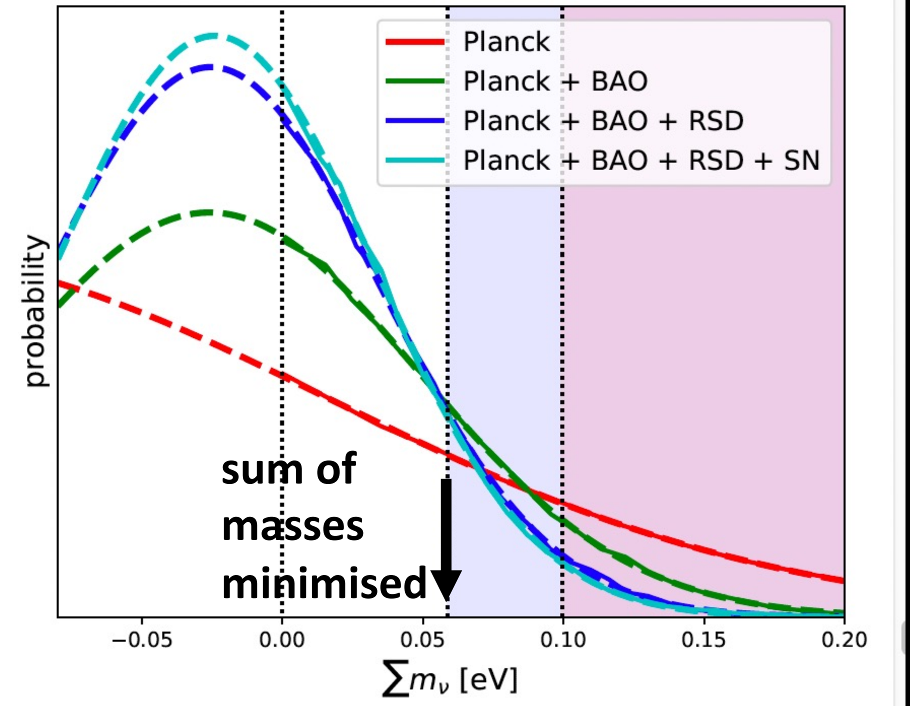


FIG. 13.— Posterior for sum of neutrino masses for selected con binations of data with a $\nu\Lambda$CDM cosmology. Dashed curves sho the implied Gaussian fits. Shaded regions correspond to lower lin its on normal and inverted hiearchies. Likelihood curves are no malized to have the same area under the curve for $\sum m_\nu > 0$.

Normal hierarchy: $M_\nu \equiv \sum m_\nu \approx 0.06\ eV$

Inverted hierarchy: $M_\nu \approx 0.1\ eV$

Summary

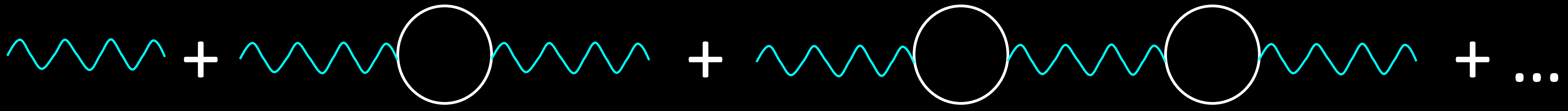
- a thermodynamic explanation for the observed large scale geometry
- a new way to cancel the SM’s vacuum energy and trace anomalies
- predicts large scale density perturbations
- predicts 3 generations of fermions, each with a RH neutrino
- a minimal, testable explanation for the cosmic dark matter
- baryon asymmetry explained via leptogenesis

this is (clearly) only the beginning … much remains to be done

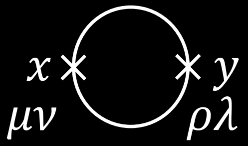
Thank you for listening!

Graviton propagator with 1 loop SM corrections

Tomboulis 70's
Han Willenbrook
Donoghue Menezes



Loop is given by the Fourier transform of the stress-energy correlator: for a CFT,



$$= \langle T^{\mu\nu}(x) T^{\rho\lambda}(y) \rangle = C^T \frac{1}{4\,\pi^4 x^8} I^{\mu\nu,\rho\lambda}(x-y)$$

where $I^{\mu\nu,\rho\lambda}(x) = \frac{1}{2}\left(I^{\mu\rho}(x) I^{\nu\lambda}(x) + I^{\mu\lambda}(x) I^{\rho\nu}(x)\right) - \frac{1}{4}\eta^{\mu\nu}\eta^{\rho\lambda}$ and $I^{\mu\nu}(x) = \eta^{\mu\nu} - 2\frac{x^\mu x^\nu}{x^2}$

$C^T = \frac{4}{3}[n_{s,1} + 3\, n_F + 12\, n_A - 8\, n_{s,0}] \equiv \frac{4}{3} n_{eff}$ ($\propto$ coefft c of Weyl squared trace anomaly)

Projector onto spin 2 component
- gauge invariant

Dim reg and min sub $\Rightarrow$ $$D^{\alpha\beta,\mu\nu}(k) = \frac{P^{\alpha\beta,\mu\nu}(k)}{k^2\left((1 - \frac{n_{eff}}{240\pi} G k^2 \ln(-\frac{k^2}{\mu^2})\right)}$$

SM corrections to the graviton propagator:

1. Inconsistent with Källén-Lehmann repn. $D(k) = \int_0^\infty dm^2\, \rho(m^2) \frac{1}{k^2 - m^2 + i\varepsilon}$

(follows from Poincare invariance and positivity of the physical Hilbert space)

2. Specifically, resummed $D(k)$ (i) falls off as $|k|^{-4}$ at large $|k|$

(ii) has complex (acausal) poles on physical sheet

Similarly, dim-0 scalar loops alone violate K-L: (i) $|k|^{-4}$ fall off; (ii) a tachyonic pole

BUT:

SM + dim-0 combination is consistent with Poincaré, positivity and microcausality (at one loop in SM gauge+fermion fields: we are now examining higher orders)

Anderson, Bateman, Herzog, NT in prep

We have recently shown that the interacting theory is in fact unitary.
(Adamo, Nakach & Tseytlin 2018 and Tseytlin 2023 reached the opposite conclusion but ignored some of the states. See also Holdom 2023)

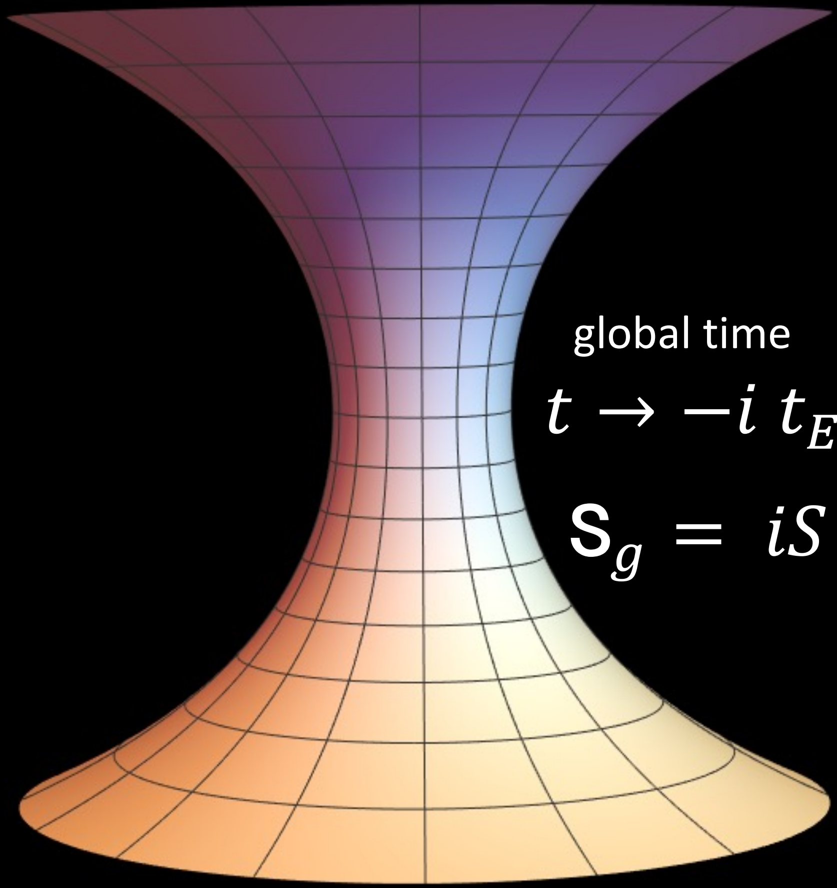
However, the theory has negative norm states. Anselmi (2022, 20233) has developed an elegant diagrammatic procedure to remove them at every order of perturbation theory. In this minimal theory, the (interacting) vacuum is then the only physical state. Only vacuum diagrams remain, which contribute to the vacuum energy and conformal anomalies as above (but with interactions).

The theory is asymptotically free. Its strong coupling phase offers a new approach to the Higgs as a composite field, with the potential to explain the gauge-gravity hierarchy without supersymmetry or extra dimensions.

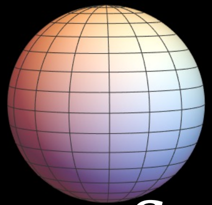
(*cf.* O(N) vector model at large N: Romatchke 2024 based on Abbott *et al.* 1976).

de Sitter

gravitational entropy from the Euclidean path integral



S^4



global time

$t \to -i\, t_E$

trace of Einstein

$R = \frac{4\rho_\Lambda}{M_P^2}$

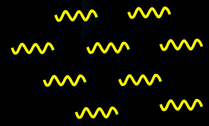
$$\mathbf{S}_g = iS = -S_E = \int\left(\tfrac{1}{2}M_P^2 R - \rho_\Lambda\right) = \rho_\Lambda \,\mathrm{Vol} = \frac{24\pi^2 M_P^4}{\rho_\Lambda}$$

$\equiv \mathbf{S}_\lambda \approx 3.26\times10^{122}$ for measured ρ_Λ

de Sitter Entropy

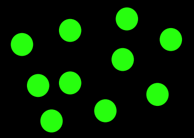
path integrals and thermodynamics

gauge fields



Higgs

$$\int e^{\frac{i}{\hbar}\int\left(\frac{R}{16\pi G}-\frac{1}{4}F^2+\overline{\psi}i\not{D}\psi-\lambda H\overline{\psi}\psi+|DH|^2-V(H)\right)}$$



particles

$$\langle f|e^{-iHt}|i\rangle \Longrightarrow Tr(e^{-\beta H})$$

gravitational entropy

with pbcs in imaginary time, $Z = e^{S_{tot}} = e^{S_{ord}+S_g}$

partition function

Bezrukov & Shaposhnikov 2007

Inflation's minimal version – Higgs inflation

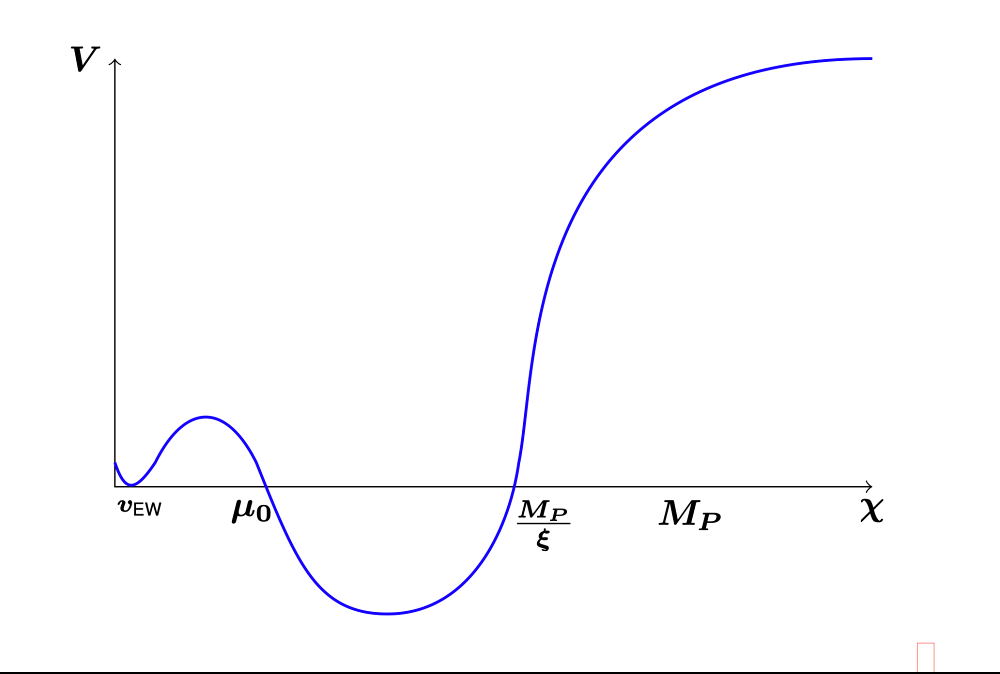
Standard Model Higgs (renormalizable)

$$S_{G-Higgs} = \int \frac{1}{2}(M_{Pl}^2+\xi h^2)R - \frac{1}{2}(\partial h)^2 - V(h); \qquad \xi \gg 1$$

With a large ξ, transforming to Einstein frame yields a minimally coupled scalar χ with an exponentially flat potential at large χ.

The SM's parameters lie surprisingly close to the boundary of vacuum metastability (perhaps pointing to some selection principle).

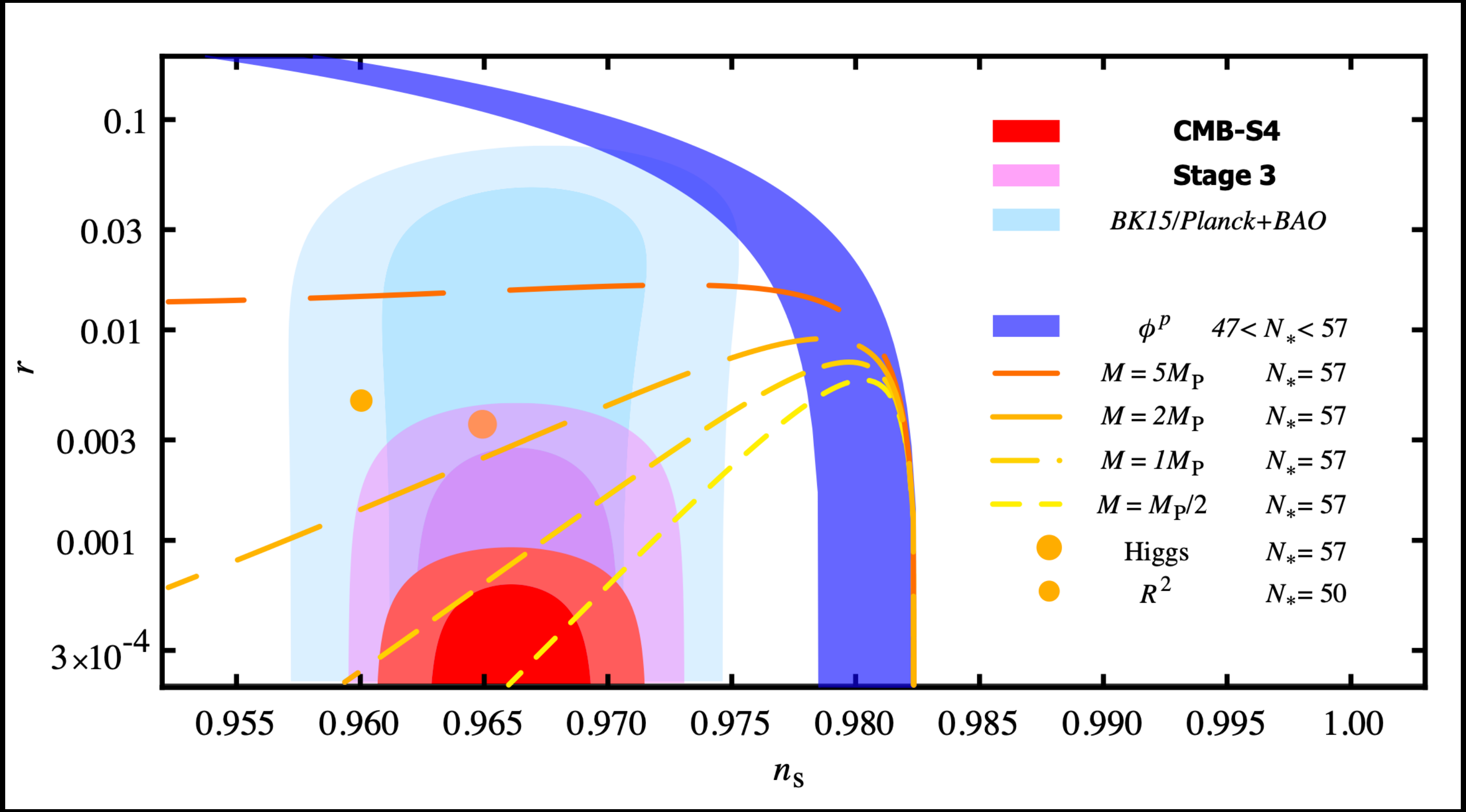
Achilles heel: gravity isn't renormalizable so at best this is an EFT with many free parameters.



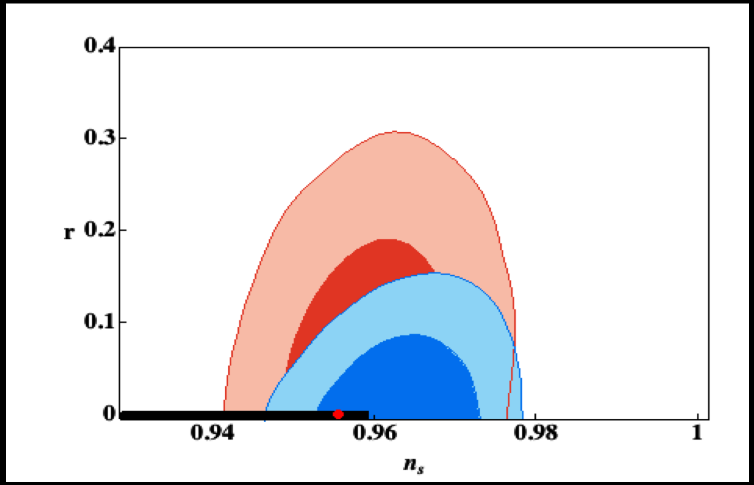
Including a “minimal” set of operators and assuming $N_I \approx 60$ e-folds of inflation,

perturbation amplitude is fit with $\xi \approx 5{\times}10^4$; then predict

$n_s = 1 - \frac{2}{N_I} \approx 0.97, \quad r = \frac{12}{N_I^2} \approx 0.003$: target for expts including CMB-S4, Lite Bird



More general set of operators (Burgess, Patil, Trott 2015)



also possible to rule out

Flauger 2021

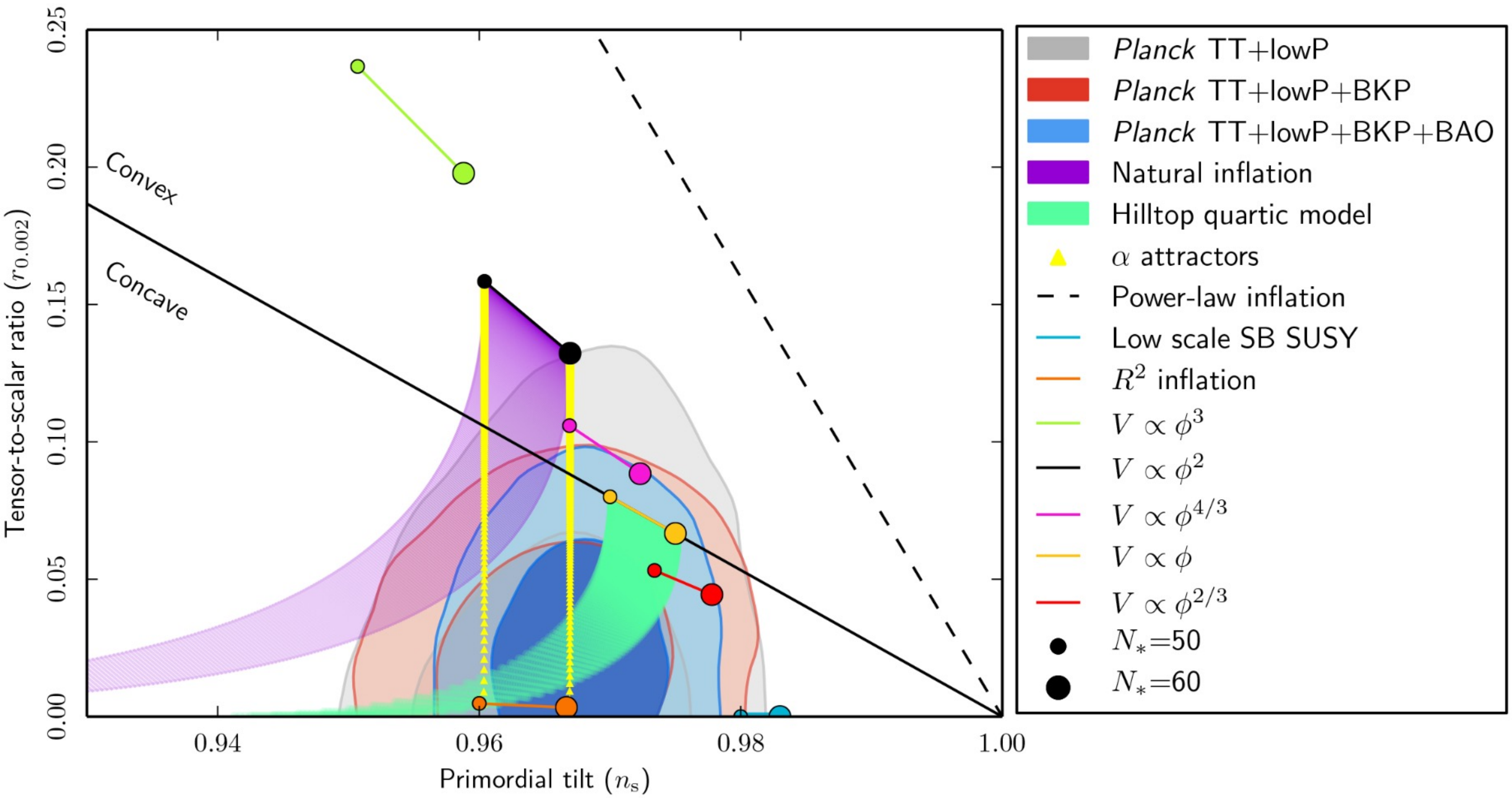


Fig. 54. Marginalized joint 68 % and 95 % CL regions for n_s and $r_{0.002}$ from *Planck* alone and in combination with its cross-correlation with BICEP2/Keck Array and/or BAO data compared with the theoretical predictions of selected inflationary models.