Quantum Entanglement & Bell In-equality Violation in $t\bar{t}$ Events

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TH, M. Low, A. Wu, arXiv:2310.17696; TH, K. Cheng, M. Low, arXiv: 2311.09166

Motivation

"If you think you understand quantum mechanics, you don't understand quantum mechanics."

-- Richard P. Feynman

"… it is my task to convince you not to turn a way because you don't understand it. You see my physics students don't understand it. That's because I don't understand it. Nobody does."

Test QM in the HE relativistic regime!

Quantum State

For a state vector $|\phi_i\rangle$

Density matrix

a state

$$\rho = \sum_i n_i |\phi_i\rangle \langle\phi_i|$$

an observable

$$\langle\mathcal{O}\rangle = \mathrm{Tr}(\mathcal{O}\rho)$$

For a pure state: $n_i = 1$; for a mixed state: $\boldsymbol{\Sigma}_i n_i = 1$.

For a single qubit (*i.e.*, a doublet of spin, iso-spin etc.):

$$\rho = \frac{1}{2}\left(\mathbb{I}_2 + \sum_i B_i \sigma_i\right)$$

For a bipartite system (*i.e.*, ½⨂½)

$$\rho = \frac{1}{4}\left(\mathbb{I}_4 + \sum_i \left(B_i^{\mathcal{A}} \left(\sigma_i \otimes \mathbb{I}_2\right) + B_i^{\mathcal{B}} \left(\mathbb{I}_2 \otimes \sigma_i\right)\right) + \sum_{i,j} C_{ij} \left(\sigma_i \otimes \sigma_j\right)\right)$$

$B_i^{A,B}$ the polarizations, C_{ij} the spin-correlation matrix

The 15 coefficients → quantum tomography for the bipartite.

Quantum Entanglement

For a bipartite system, *i.e.*, $\frac{1}{2}\otimes\frac{1}{2} = 1\oplus 0$:

Singlet:

entangled → $|0,0\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$

Triplet:

$$
\begin{aligned}
|1,1\rangle &= \uparrow\uparrow \\
|1,0\rangle &= \tfrac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow) \\
|1,-1\rangle &= \downarrow\downarrow
\end{aligned}
$$

← entangled

Quantum entanglement
→ inseparable

$$\rho \neq \sum_{a=1}^{N} p_a \, \rho_a^{\mathcal{A}} \otimes \rho_a^{\mathcal{B}}$$

Peres-Horodecki criterion:
a necessary condition for entanglement

A state is entangled (inseparable) if a partial transpose

$$\rho^{\mathrm{T}_2} = \sum_n p_n \rho_n^a \otimes \left(\rho_n^b\right)^{\mathrm{T}}$$

is not non-negative.

Quantum Entanglement

Peres-Horodecki criterion leads to several inequalities.

It has been a customary to introduce the concurrence, that can be written in C_i, the eigenvalues of C_{ij}:

Concurrence

$$\mathcal{C}(\rho) = \begin{cases} \frac{1}{2}\max(|C_1 + C_2| - 1 - C_3, 0), & C_3 \leq 0 \\ \frac{1}{2}\max(|C_1 - C_2| - 1 + C_3, 0), & C_3 \geq 0 \end{cases}$$

It is shown that :

$$0 \leq \mathcal{C}(\rho) \leq 1$$

No entanglement

Maximumly entangled

→ Quantum information even in space-like separation

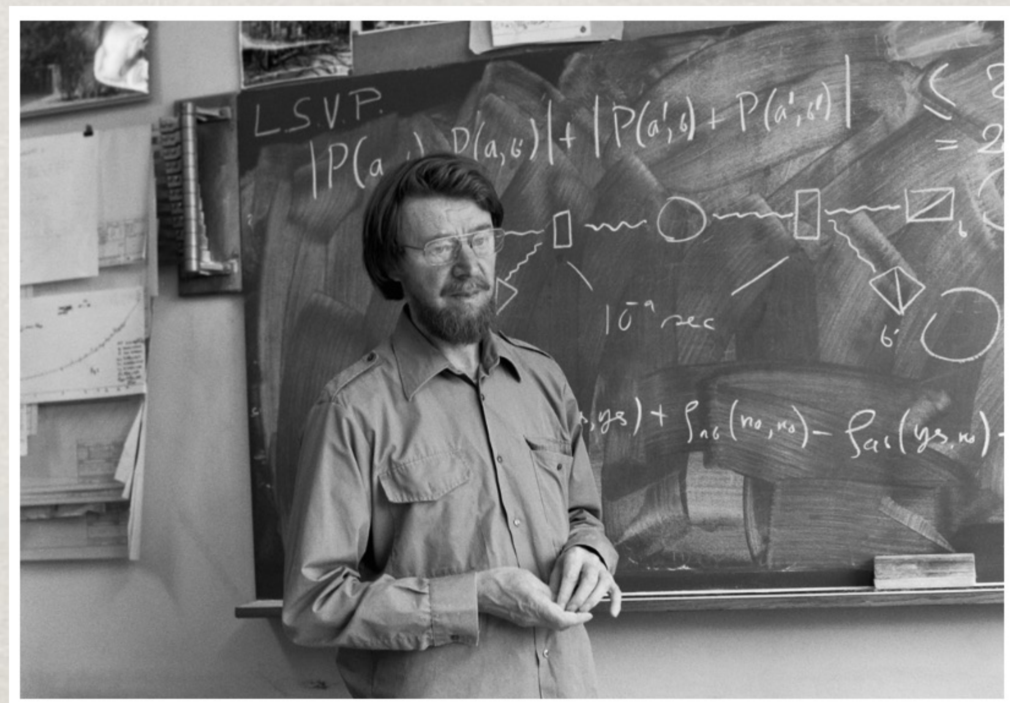
Afik and Munoz de Nova, arXiv: 2003.02280

John Bell's Inequality

Einstein-Podolsky-Rosen (Phys. Rev. 1935)

"Can quantum-mechanical description of physical reality be considered complete?"

"On the Einstein-Podolsky-Rosen paradox" 1964



Alice & Bob's individual measurements:

$$\langle A_1 B_1 \rangle - \langle A_1 B_2 \rangle + \langle A_2 B_1 \rangle + \langle A_2 B_2 \rangle \le 2$$

E.g., choosing

$$A_1 = \sigma_1, \quad A_2 = \sigma_3, \quad B_1 = \pm\frac{1}{\sqrt{2}}(\sigma_1 + \sigma_3), \quad B_2 = \pm\frac{1}{\sqrt{2}}(-\sigma_1 + \sigma_3)$$

$$\Rightarrow |C_{11} \pm C_{33}| \le \sqrt{2}$$

Top-pair & spin correlation

$\rho_{ab,\bar{a}\bar{b}}$ can be extracted from the angular distribution of decay product

$$\sigma(XY \to t\bar{t} \to (A_1A_2A_3)(B_1B_2B_3)) = \int \mathrm{d}\Omega^A \mathrm{d}\Omega^B \left(\frac{\mathrm{d}\Gamma_{ab}}{\mathrm{d}\Omega^A}\right) R_{ab,\bar{a}\bar{b}} \left(\frac{\mathrm{d}\bar{\Gamma}_{\bar{a}\bar{b}}}{\mathrm{d}\Omega^B}\right)$$

$$\frac{\mathrm{d}\Gamma_{ab}}{\mathrm{d}\Omega} \propto \delta_{ab} + \kappa\, \sigma^i_{ab}\, \Omega^i$$

Spin analyzing power

$$\Rightarrow \quad \frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega^A \mathrm{d}\Omega^B} = \frac{1}{(4\pi)^2}\left(1 + \kappa^A\, P_i^A \Omega_i^A + \kappa^B\, P_i^B \Omega_i^B + \kappa^A \kappa^B\, \Omega_i^A C_{ij} \Omega_j^B\right)$$

Direction of A, B

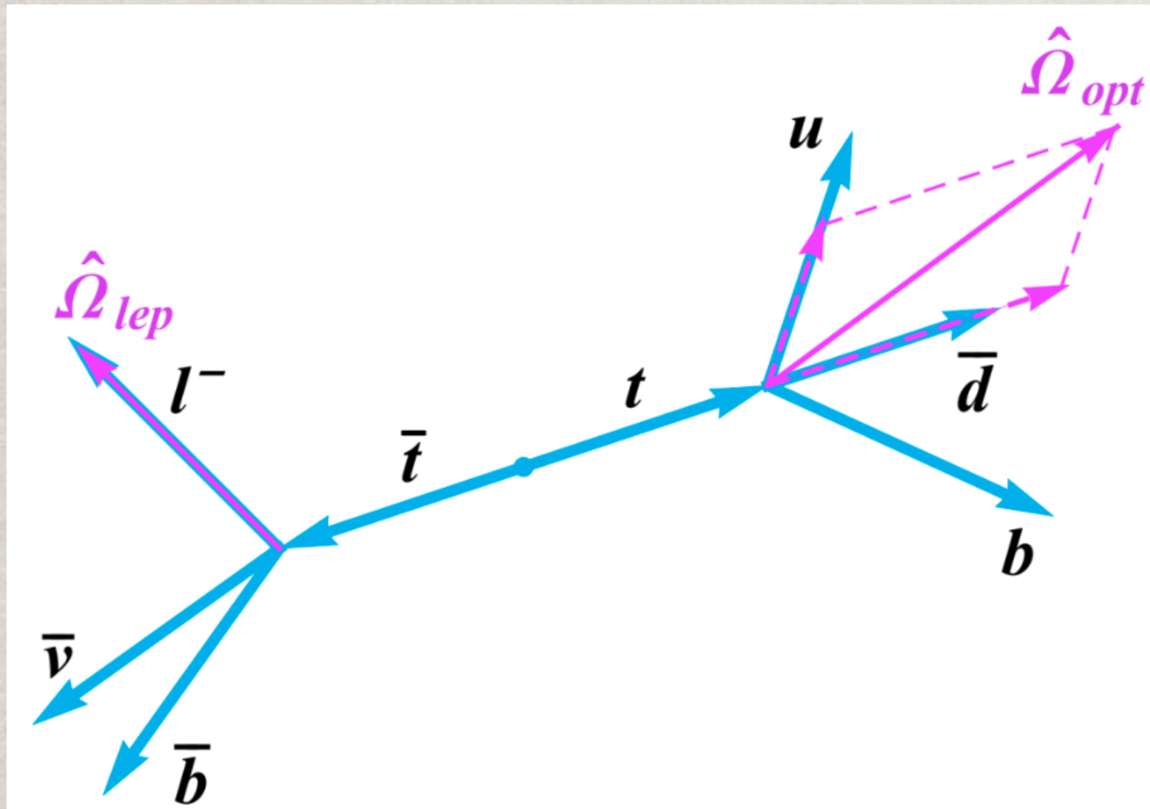
$$\Rightarrow \quad \frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}(\cos\theta_i^A \cos\theta_j^B)} = -\frac{1 + \kappa^A \kappa^B C_{ij} \cos\theta_i^A \cos\theta_j^B}{2} \log\left|\cos\theta_i^A \cos\theta_j^B\right|$$

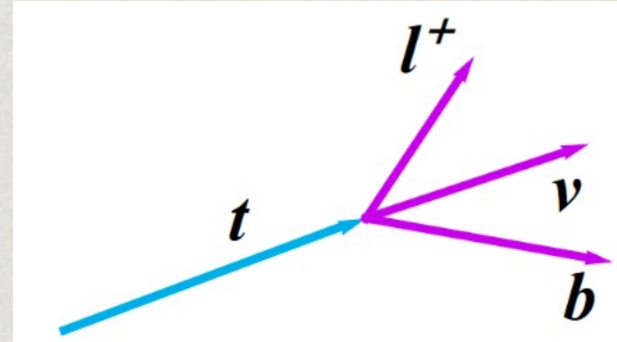
Polar angle of A with respect to the i-th axis

$$\Rightarrow \quad C_{ij} = \frac{4}{\kappa^A \kappa^B} \frac{N\left(\cos\theta_i^A \cos\theta_j^B > 0\right) - N\left(\cos\theta_i^A \cos\theta_j^B < 0\right)}{N\left(\cos\theta_i^A \cos\theta_j^B > 0\right) + N\left(\cos\theta_i^A \cos\theta_j^B < 0\right)}$$

Top-pair leptonic + hadronic decays

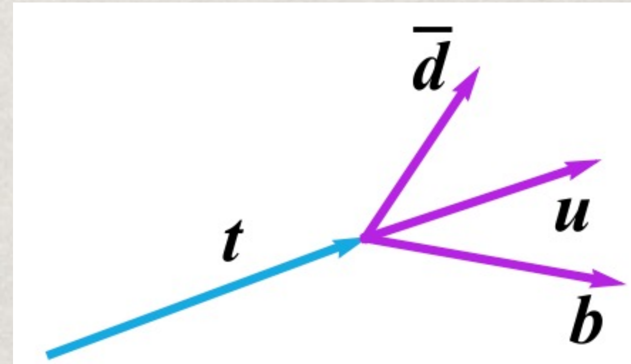
Z. Dong, Dorival Goncalves, et al., arXiv:2305.07075
TH, M. Low, A. Wu, arXiv:2310.17696





Leptonic decay:

$\kappa_{\ell^+} = 1.0$



Hadronic decay:

$\kappa_{\bar{d}} = 1.0$

$\kappa_u = 0.34$

optimized direction:

$$\vec{\Omega}_{\rm opt}(\cos\theta_W) = P_{d\to p_{\rm soft}}(\cos\theta_W)\,\hat{p}_{\rm soft} + P_{d\to p_{\rm hard}}(\cos\theta_W)\,\hat{p}_{\rm hard}$$

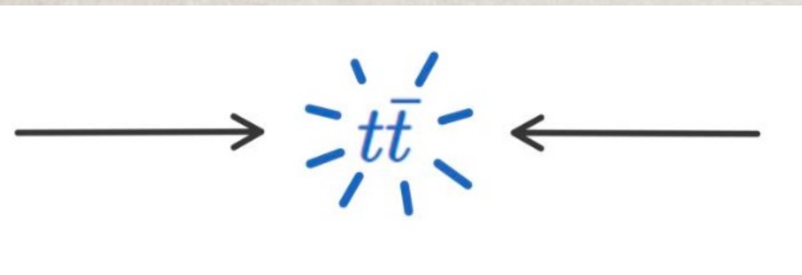
$\Rightarrow$ $\kappa_{\rm opt} = 0.64$

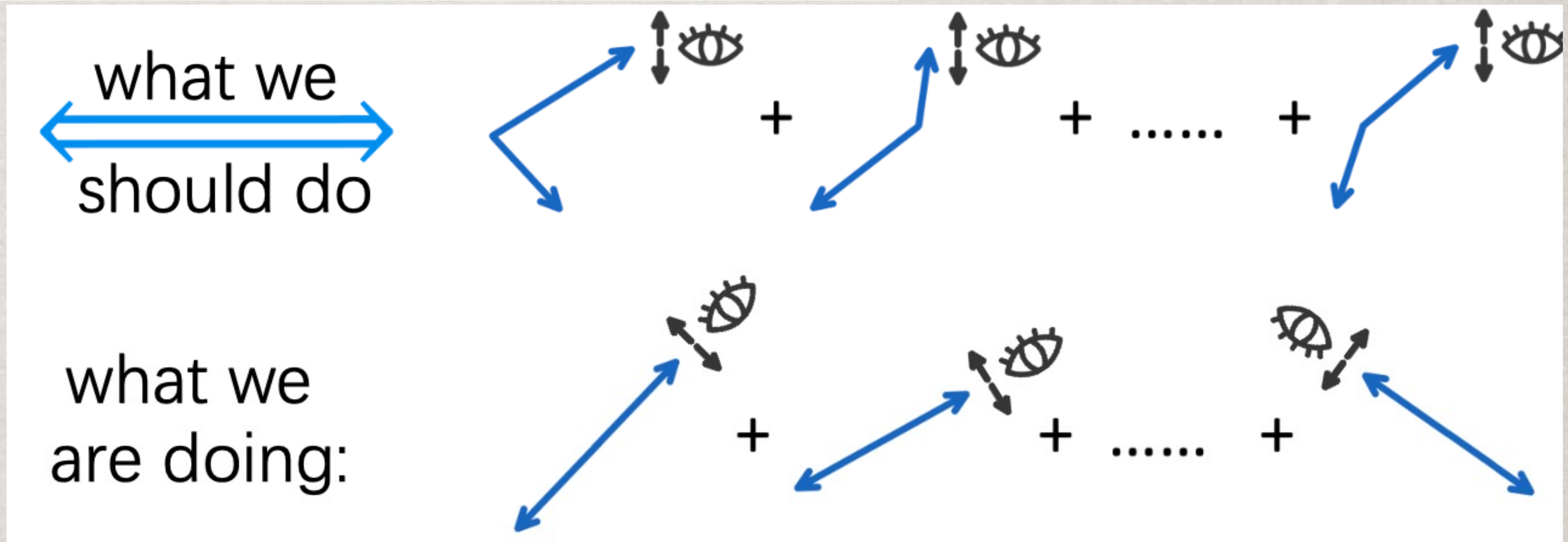
(arXiv:1401.3021)

Quantum entanglement at high energies: Fictitious states

$$\rho \;=\; \ldots\ldots + \sum_{i,j} C_{ij}(\sigma_i \otimes \sigma_j)$$

$$C_{ij} = \langle S_i^t S_j^{\bar{t}} \rangle$$





In different frame:

$$|t\rangle = |p_t\rangle \otimes |\phi\rangle = |p_t\rangle \otimes |\alpha\rangle\, \phi_\alpha, \qquad |t\rangle = |p_t'\rangle \otimes |\phi'\rangle = |p_t'\rangle \otimes |\alpha\rangle\, \phi_\alpha'.$$

$$p_t'^{\mu} = \Lambda^\mu_{\ \nu} p_t^\nu, \qquad \phi'_{\alpha'} = U(\Lambda)^\dagger_{\alpha'\alpha}\phi_\alpha$$

$$\bar{\rho} \;=\; \ldots\ldots + \sum_{\bar{i},\bar{j}} C_{\bar{i}\bar{j}}(\sigma_{\bar{i}} \otimes \sigma_{\bar{j}})$$

“fictitious”

$$C_{\bar{i}\bar{j}} = \begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix}_{\hat{n}(p_1)\,\hat{r}(p_1)\,\hat{k}(p_1)} + \begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix}_{\hat{n}(p_2)\,\hat{r}(p_2)\,\hat{k}(p_2)} + \ldots\ldots + \begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix}_{\hat{n}(p_N)\,\hat{r}(p_N)\,\hat{k}(p_N)}$$

Afik and Munoz de Nova, arXiv: 2003.02280; TH, K. Cheng, M. Low, arXiv: 2311.09166

Quantum entanglement at high energies: Fictitious states

From a well-prepared quantum state to a fictitious state:

$$\bar{\rho} \to \sum_{a \in \text{events}} U_a^\dagger \rho_a U_a \neq U^\dagger \bar{\rho} U \,.$$

Thus, a measurement on a fictitious state
depends on the frame/base choice of each measurement!

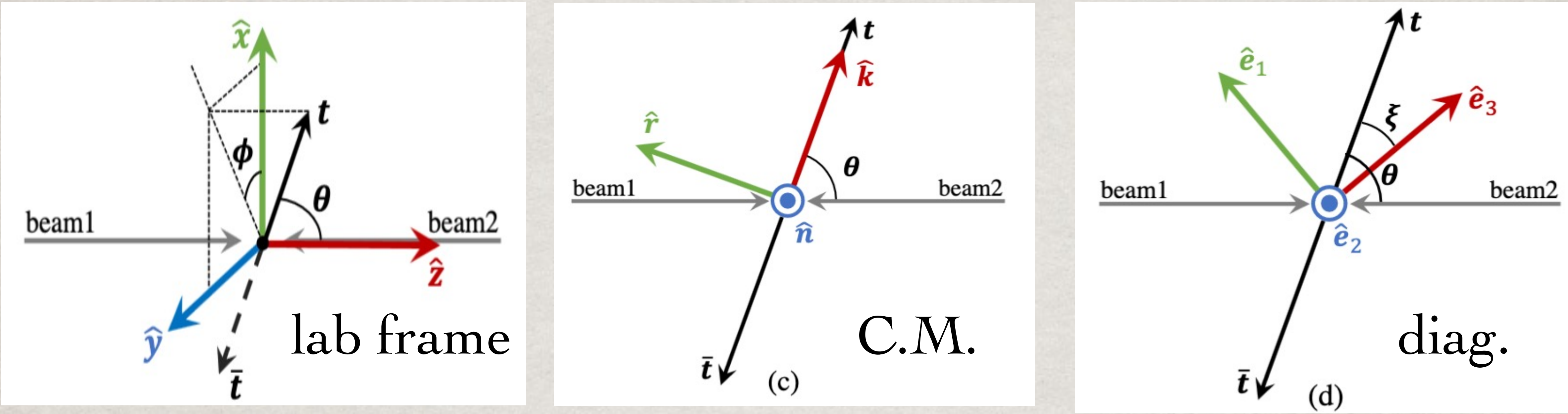
We showed: TH, K. Cheng, M. Low, arXiv: 2311.09166

$$\mathcal{C}(\rho_{\text{fictitious}}) > 0 \quad \Rightarrow \quad \mathcal{C}(\rho_{\text{sub}} \in \rho) > 0$$

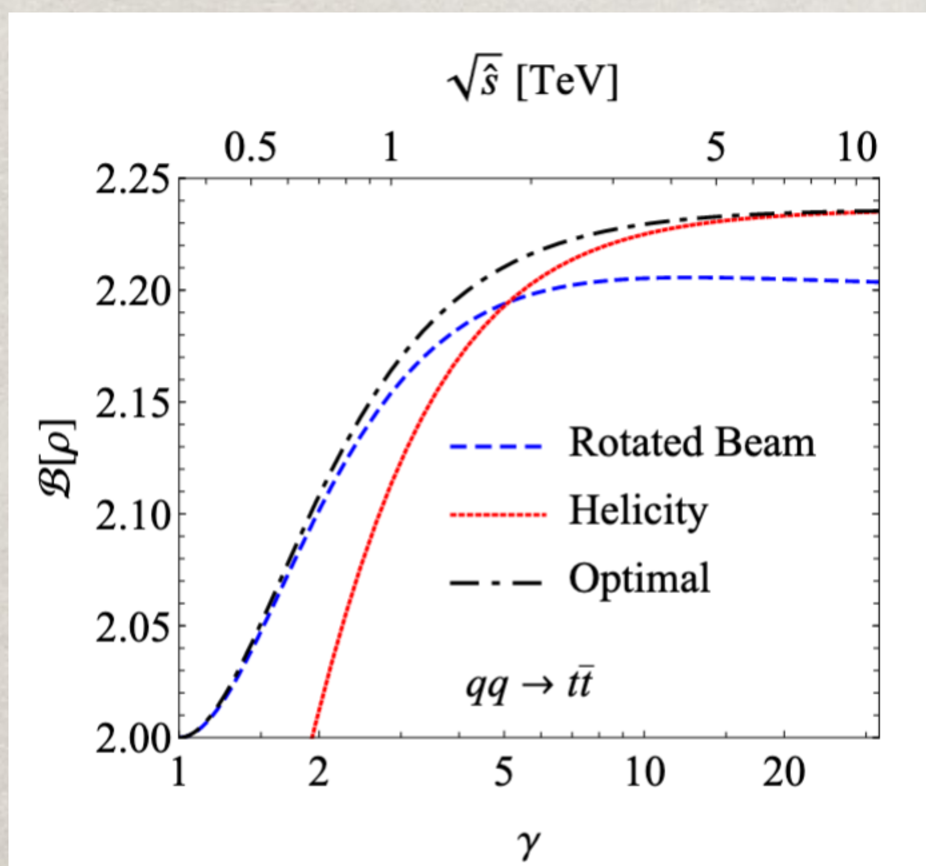
$$\text{Bell}(\rho_{\text{fictitious}}) > \sqrt{2} \quad \Rightarrow \quad \text{Bell}(\rho_{\text{sub}} \in \rho) > \sqrt{2}$$

Fictious states carry the system quantum information!

Frame optimization



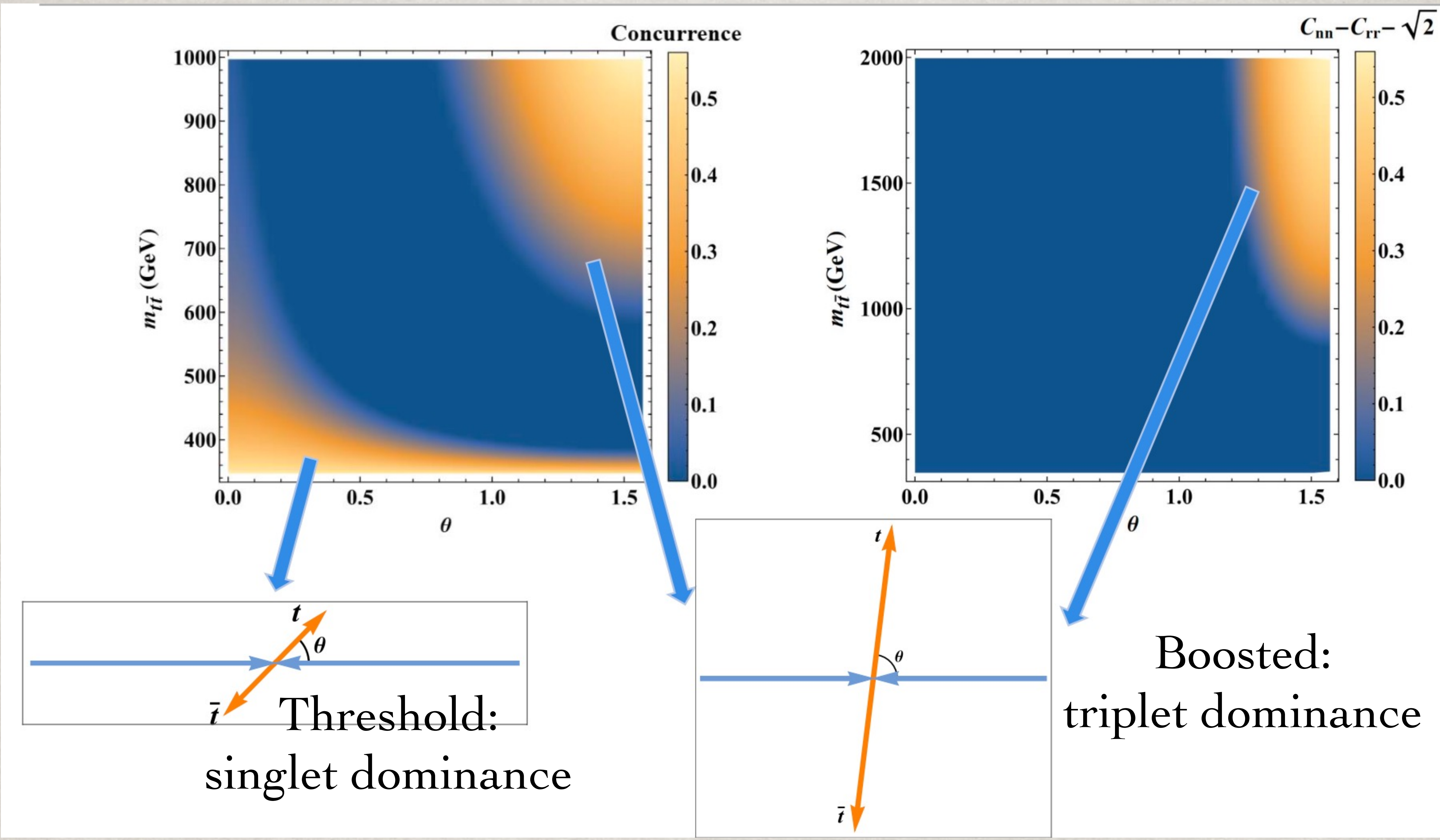
The frame that diagonalizes C_{ij}
leads to the maximum sensitivity.



$$C_\Omega^{\mathrm{diag}} = R_\Omega C_\Omega R_\Omega^T = \begin{pmatrix} \mu_{1,\Omega} & 0 & 0 \\ 0 & \mu_{2,\Omega} & 0 \\ 0 & 0 & \mu_{3,\Omega} \end{pmatrix}$$

TH, M. Low, A. Wu, arXiv:2310.17696
TH, K. Cheng, M. Low, arXiv: 2311.09166;
arXiv:2406.xxxxx to appear.

Partonic level results



Realistic simulations: MadGraph 5+Pythia 8+Delphes 3

Detector effects by “parametric fit”

Our results

Entanglement

$C > 0$

	Result(139fb^{-1})	Precision
Boosted	0.276 ± 0.026	9.5%
Threshold	0.261 ± 0.008	3.0%

Bell's inequality violation

$|C_{11} \pm C_{33}| \leq \sqrt{2}$

Result(3ab^{-1})	Significance
0.23 ± 0.06	4.1σ

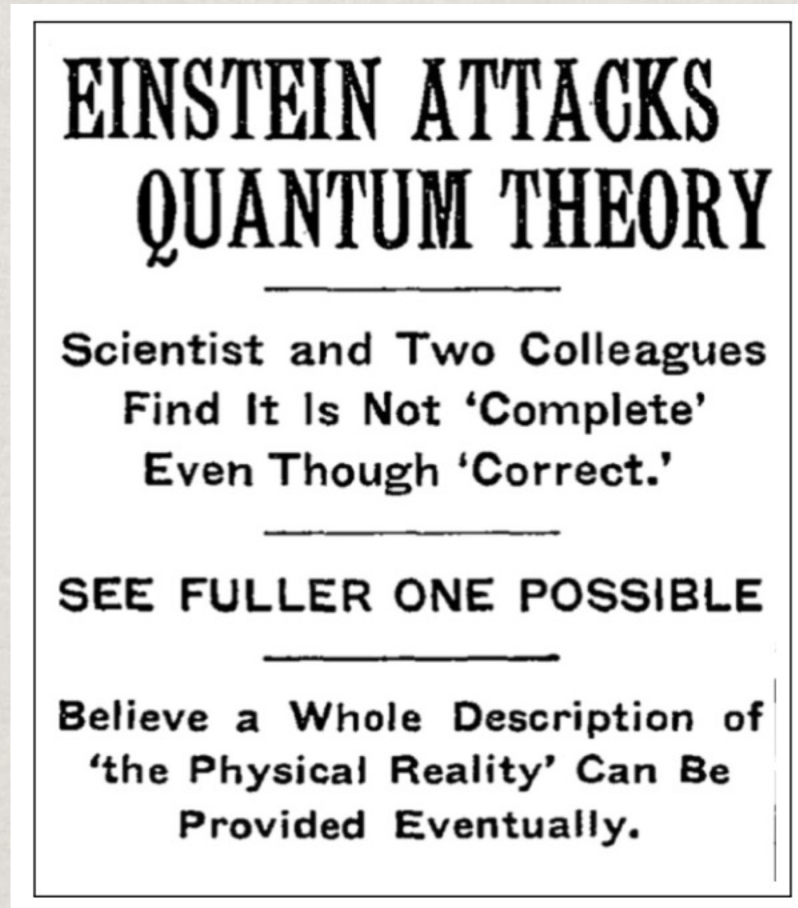
Conclusions

We propose & calculate the test of QM in tt events @ LHC.
We clarify the "fictitious states" and confirm the test method.
We identify the optimal axis choice to enhance the sensitivity.
→ encouraging results for entanglement & Bell ineq. tests.

Recent LHC studies for top leptonic decays:
ATLAS: arXiv:2311.07288; CMS: **CMS PAS TOP-23-001**

The Spooky Action

May 4, 1935



EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

"Can quantum-mechanical description of physical reality be considered complete?" Einstein-Podolsky-Rosen, Phys. Rev. 1935