

Stasis, Stasis, Stasis

Keith R. Dienes
University of Arizona



© QT Luong / terragalleria.com

Tucson, Arizona

Work with Lucien Heurtier, Fei Huang,
Doojin Kim, Tim M.P. Tait, and Brooks Thomas

- [arXiv:2111.04753](https://arxiv.org/abs/2111.04753)
- [arXiv:2212.01369](https://arxiv.org/abs/2212.01369)

Work with Lucien Heurtier, Fei Huang,
Tim M.P. Tait, and Brooks Thomas

- [arXiv:2309.10345](https://arxiv.org/abs/2309.10345)
- [arXiv:2405.nnnnn](https://arxiv.org/abs/2405.nnnnn)
- [arXiv:2406.nnnnn](https://arxiv.org/abs/2406.nnnnn) also w/ D. Hoover, A. Paulsen

Work with Jonah Barber and Brooks Thomas

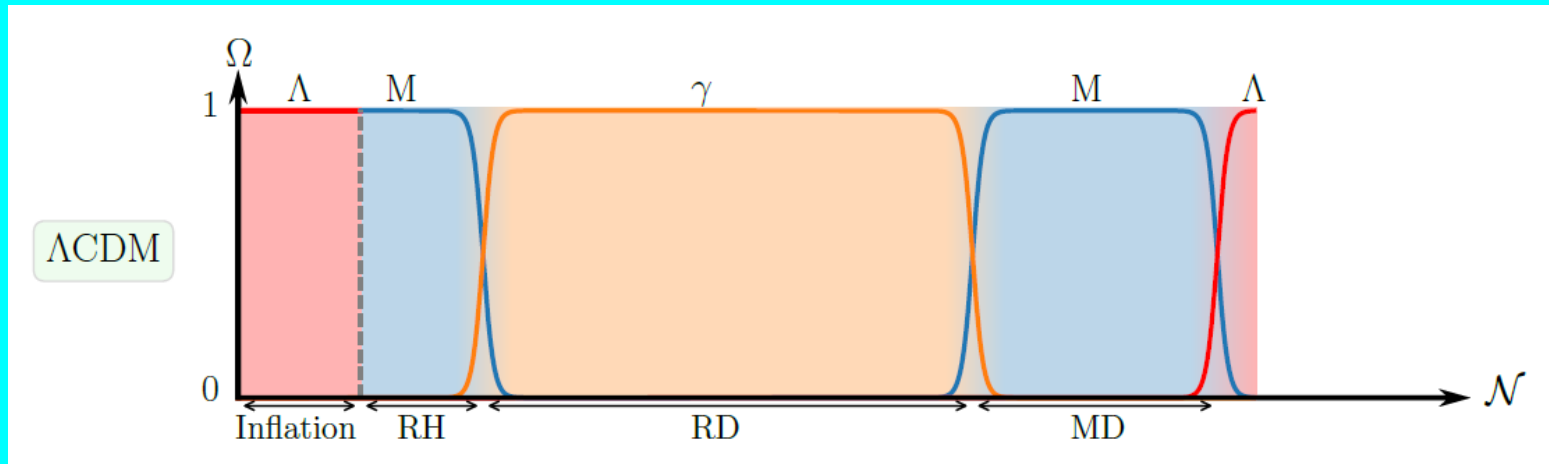
- [arXiv:2406.nnnnn](https://arxiv.org/abs/2406.nnnnn)
- [arXiv:2406.nnnnn](https://arxiv.org/abs/2406.nnnnn)

This work was supported in part by the National Science Foundation through its employee IR/D program. The opinions and conclusions expressed herein are those of the speaker, and do not necessarily represent the National Science Foundation.

Mitchell Conference
Texas A&M University
24 May 2024

I'm going to start this talk by giving away the punchline.

We are all used to the standard Λ CDM timeline...



My main message: If you believe that physics at higher energies is governed by some form of BSM physics ---

(pick your favorite theory: extra spacetime dimensions, strongly coupled sectors, string theory, etc.),
---- then this picture is very likely wrong. Not just “modified” or generalized in some corrective way, but *with a big piece missing*.

Instead, a whole new type of epoch must open up in the early universe...

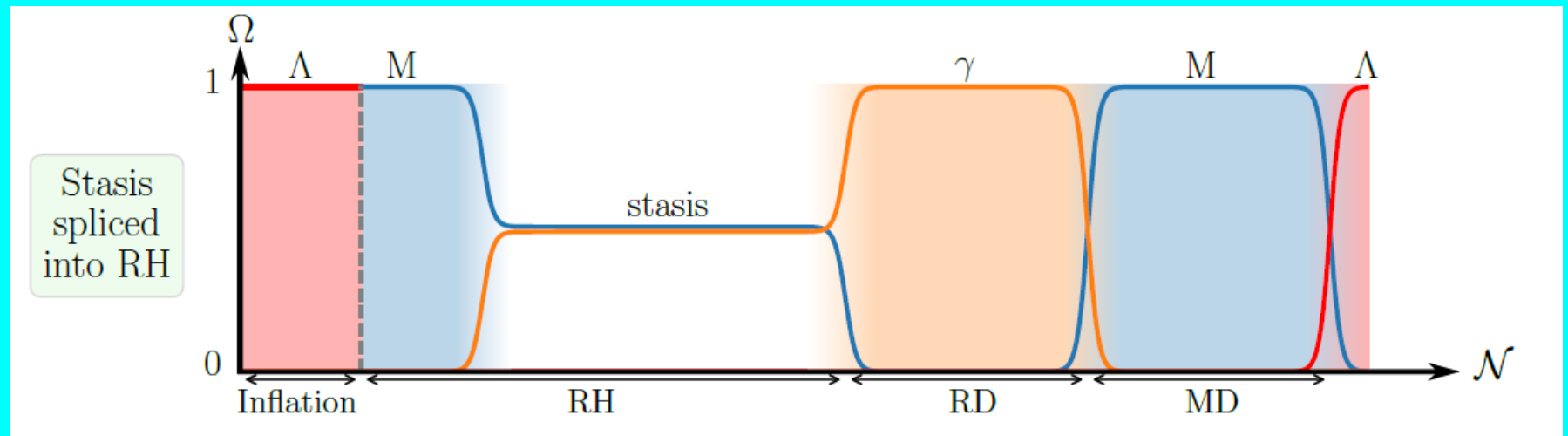
... an epoch of ***cosmological stasis***.

Such an epoch has been missed until now in most discussions of Λ CDM physics and early-universe cosmology, but it's there and must be dealt with.

Stasis

A cosmological epoch during which the abundances of different energy components (matter, radiation, vacuum energy, etc.) remain constant despite cosmological expansion.

For example,



The universe continues to expand, but the abundances stay fixed.
Time passes as measured in e -folds, but not as measured by abundances.

Already your alarm bells should be ringing.

Energy densities for each component scale differently. Cannot keep their ratios constant.

Even if you could arrange this somehow, it must be freakishly fine-tuned.

Even if it can be done without fine-tuning, why should the universe happen to fall into such a stasis state?

Even if it's an attractor for one model of BSM physics, what happens for a different model of BSM physics?

Ah, but we can!
Stasis is a fundamental epoch which need not be dominated by any one component over many e -folds.

No, it's not fine-tuned at all!

It's an attractor!
It cannot be avoided.

It's fairly general. The attractor behavior survives across a wide swath of BSM models, and for a wide range of parameters within each model.

Stasis sounds like an eternal thing.
How can it form the basis of a
cosmological epoch?

Stasis comes with its own mechanisms
for starting at one time and ending later on,
with a potentially large number of e -folds
in between. No problem either entering
stasis or exiting from it.

So if high-energy physics is governed
by a BSM theory such as extra dimensions,
strongly coupled sectors, or string theory,
are you saying that the early universe will
necessarily experience a stasis epoch?

Well, yes. There are, of course, various important
caveats and provisos. However, the critical point is
that such stasis epochs are a rather generic feature
of such BSM cosmologies, and one would need
to understand why they might not arise in certain
circumstances if one doesn't take them into account.

You sound excited
about this.

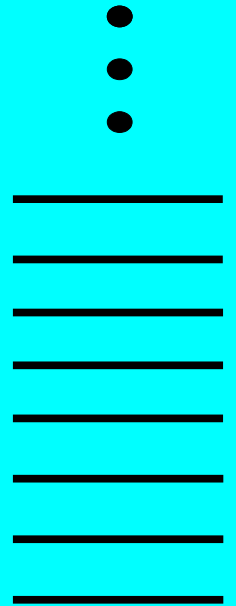
You bet.

The basic idea

So what's so special about BSM physics?

A wide variety of scenarios for BSM physics predict towers of massive, unstable states --- *e.g.*,

- Theories with extra compactified spacetime dimensions (KK towers)
- Scenarios with confining dark/ hidden-sector gauge groups (towers of bound-state resonances)
- String theory (infinite towers of KK/winding states, string resonances -- especially for bulk fields such as string moduli, axions, etc.)
- Scenarios which lead to the production of primordial black holes with an extended mass spectrum (towers of massive PBHs)



In general, such states share certain general properties

- Towers of states are potentially infinite (or bounded by a relevant cutoff) --- generally stretch across many orders of magnitude in mass.
- Such states are generally unstable and decay.

Moreover, two features tend to govern these decays

- Heavy states at top of tower tend to have largest decay widths and decay first, then lighter ones. Decays thus proceed “down the tower”.
- For any state, the dominant decay mode is to the lightest states available. Such decay products are therefore produced with huge amounts of kinetic energy (relativistic), and are effectively radiation.

So what is the effect of such infinite towers of states on early-universe cosmology?

These decays establish a sequential process working its way down the tower which continually converts matter into radiation.

This may seem rather trivial, but there is actually a competing effect which pushes the other way: ***cosmological expansion!***

- radiation scales as a^{-4} (a = FRW scale factor)
- matter scales as a^{-3}

Thus, *even if nothing else happens*, cosmological expansion causes the relative fractional energy densities (“abundances”) of matter and radiation to change

- abundance of radiation Ω_γ *drops*
- abundance of matter Ω_M *rises*

(Total remains fixed at 1 for a radiation/matter universe.)

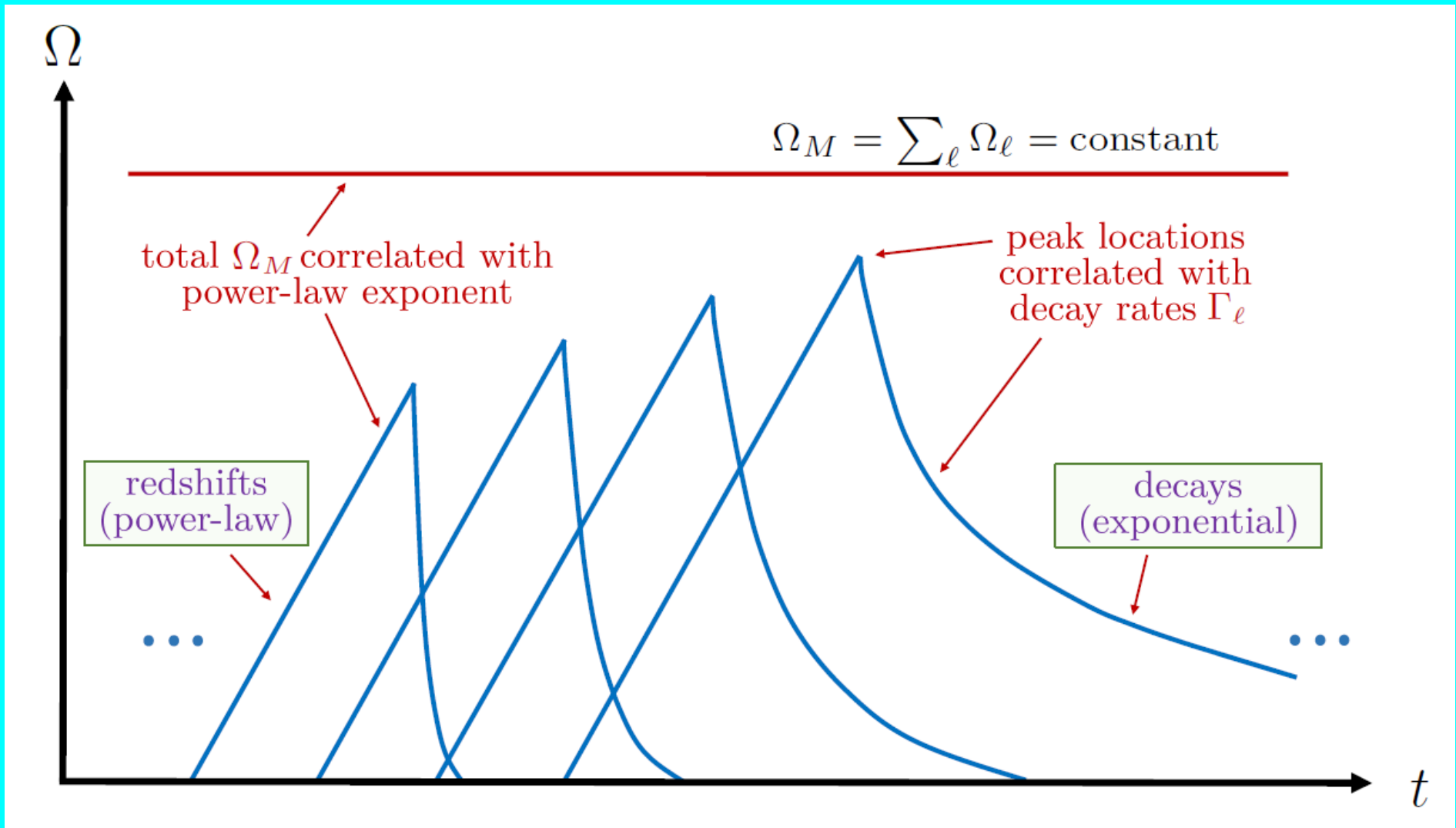
Indeed, this is how a radiation-dominated universe becomes matter-dominated universe *simply as the result of cosmic expansion.*

We thus see that

- decays along tower: convert $\Omega_M \rightarrow \Omega_\gamma$
- cosmic expansion: converts $\Omega_\gamma \rightarrow \Omega_M$

Can these two effects cancel?

This would be a way of keeping the matter and radiation abundances fixed --- at least through the time interval (which may stretch across many e -folds) during which the decays are proceeding sequentially down the tower.



Seems like too much to ask for!

But....

... they **CAN** balance

... they **DO** balance

... even if they don't start out by balancing,
the balanced solution is an **attractor**
and the system will quickly come into balance all by itself!

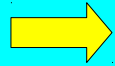
Especially remarkable because particle decay and cosmological expansion are very different things ---
one is particle physics, the other cosmology!

To understand this how this can happen,
let us analyze this system mathematically....

Very simple ingredients from Cosmology 101!

$$\Omega_i \equiv \frac{8\pi G}{3H^2} \rho_i$$

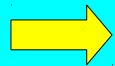
abundances Ω_i in terms of energy densities ρ_i
H = Hubble parameter (falls with time)



$$\frac{d\Omega_i}{dt} = \frac{8\pi G}{3} \left(\frac{1}{H^2} \frac{d\rho_i}{dt} - 2 \frac{\rho_i}{H^3} \frac{dH}{dt} \right)$$

Friedmann
“acceleration”
equation

$$\begin{aligned} \frac{dH}{dt} &= -H^2 - \frac{4\pi G}{3} \left(\sum_i \rho_i + 3 \sum_i p_i \right) \\ &= -\frac{1}{2} H^2 (2 + \Omega_M + 2\Omega_\gamma) \\ &= -\frac{1}{2} H^2 (4 - \Omega_M) . \end{aligned}$$



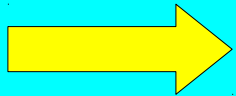
$$\frac{d\Omega_i}{dt} = \frac{8\pi G}{3H^2} \frac{d\rho_i}{dt} + H\Omega_i (4 - \Omega_M)$$

Now insert “EOMs” for ρ_i :

$$\begin{aligned} \frac{d\rho_\ell}{dt} &= -3H\rho_\ell - \Gamma_\ell\rho_\ell \\ \frac{d\rho_\gamma}{dt} &= -4H\rho_\gamma + \sum_\ell \Gamma_\ell\rho_\ell \end{aligned}$$

cosmological expansion

decay sources and sinks



$$\frac{d\Omega_M}{dt} = - \sum_\ell \Gamma_\ell \Omega_\ell + H (\Omega_M - \Omega_M^2)$$

General dynamical evolution of Ω_M

Setting $d\Omega_M/dt = 0$ then yields

$$\sum_\ell \Gamma_\ell \Omega_\ell = H (\Omega_M - \Omega_M^2) .$$

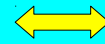
A minimum condition for stasis!

But for actual stasis, need Ω_M to stay fixed over an extended period!

- $d^n \Omega_M / dt^n = 0$ for all $n > 1$
- Instead, demand that $d\Omega_M/dt = 0$ for all t!

During stasis

$$H(t) = \left(\frac{2}{4 - \bar{\Omega}_M} \right) \frac{1}{t}$$



$$\bar{\kappa} = \frac{6}{4 - \bar{\Omega}_M}$$

$$H(t) = \kappa/(3t)$$

$$\begin{aligned} \sum_{\ell} \Gamma_{\ell} \Omega_{\ell}(t) &= \frac{\bar{\kappa}}{3} \bar{\Omega}_M (1 - \bar{\Omega}_M) \frac{1}{t} \\ &= \left(2 - \bar{\kappa} \right) \bar{\Omega}_M \frac{1}{t} . \end{aligned}$$

For stasis, this relation must be true as a function of t.

where each individual $\Omega_{\ell}(t)$ is given by

$$\Omega_{\ell}(t) = \Omega_{\ell}^{(0)} h(t^{(0)}, t) e^{-\Gamma_{\ell}(t-t^{(0)})}$$

with

$$\begin{aligned} h(t^{(0)}, t) &= h(t^{(0)}, t_*) h(t_*, t) \\ &= h(t^{(0)}, t_*) \left(\frac{t}{t_*} \right)^{2-6/(4-\bar{\Omega}_M)} \end{aligned}$$

So what does BSM physics tell us about the Ω_ℓ and Γ_ℓ across the tower?

Let us parametrize these quantities in a general way in order to encapsulate a wide variety of different BSM scenarios...

mass
spectrum

$$m_\ell = m_0 + (\Delta m)\ell^\delta$$

e.g., $(m_0, \Delta m, \delta) =$

- $(m, (2mR^2)^{-1}, 2)$ for KK on circle, $mR \gg 1$
- $(m, R^{-1}, 1)$ for KK on circle, $mR \ll 1$
- $\delta = 1/2$ for string/ strong-coupling resonances ($\alpha' M_\ell^2 = \ell$)

decay
widths

$$\Gamma_\ell = \Gamma_0 \left(\frac{m_\ell}{m_0} \right)^\gamma$$

Exponent γ determined by dominant ϕ_ℓ decay mode, *e.g.*, if ϕ_ℓ decays to photons via dimension- d contact operator $c_\ell \phi_\ell F/\Lambda^{d-4}$, then $\gamma = 2d-7$.

abundances
at production

$$\Omega_\ell^{(0)} = \Omega_0^{(0)} \left(\frac{m_\ell}{m_0} \right)^\alpha$$

Exponent α determined by production mechanism. Typically $\alpha < 0$ for misalignment production, but either sign for thermal freeze-out.

So for what scaling exponents (α, γ, δ) can we achieve stasis?

Given our constraint equation,
we need to evaluate the sum $\sum_{\ell} \Gamma_{\ell} \Omega_{\ell}(t)$!

$$\begin{aligned} \sum_{\ell} \Gamma_{\ell} \Omega_{\ell}(t) &= \frac{\bar{\kappa}}{3} \bar{\Omega}_M (1 - \bar{\Omega}_M) \frac{1}{t} \\ &= \left(2 - \bar{\kappa}\right) \bar{\Omega}_M \frac{1}{t}. \end{aligned}$$

Convenient to take continuum limit $\Delta m \rightarrow 0, N \rightarrow \infty$

Sum becomes integral

$$\sum_{\ell} \rightarrow \int d\tau n_{\tau}(\tau) \quad \text{where} \quad n_{\tau}(t) \equiv \left| \frac{d\ell}{d\tau} \right|_{\tau=t} = \frac{1}{\gamma\delta} \left(\frac{m_0}{\Delta m} \right)^{1/\delta} \Gamma_0^{-1/\gamma\delta} t^{-1-1/\gamma\delta} \quad \text{density of states}$$

Easy to integrate. Consider situation far from “edge” effects – *i.e.*, $t \gg t^{(0)}$.

Constraints to impose for stasis:

1. Realize proper $1/t$ scaling
2. Avoid potential *logarithmic time-dependence* in abundances
3. Match *prefactors* as well.



We find that all constraints are subsumed into a single relation:

$$\frac{1}{\gamma} \left(\alpha + \frac{1}{\delta} \right) = 2 - \bar{\kappa},$$

$$\frac{1}{\gamma} \left(\alpha + \frac{1}{\delta} \right) = 2 - \bar{\kappa} ,$$

So what do we learn?

This is not a constraint on (α, γ, δ) so much as
a *prediction for κ* during stasis!

Thus, so long as

$$\gamma > 0 , \quad 0 < \alpha + \frac{1}{\delta} \leq \frac{\gamma}{2}$$

← so that
 $3/2 < \kappa < 2$
radiation domination matter domination

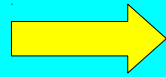
we will *always* have stasis!

The corresponding stasis abundance will be given by

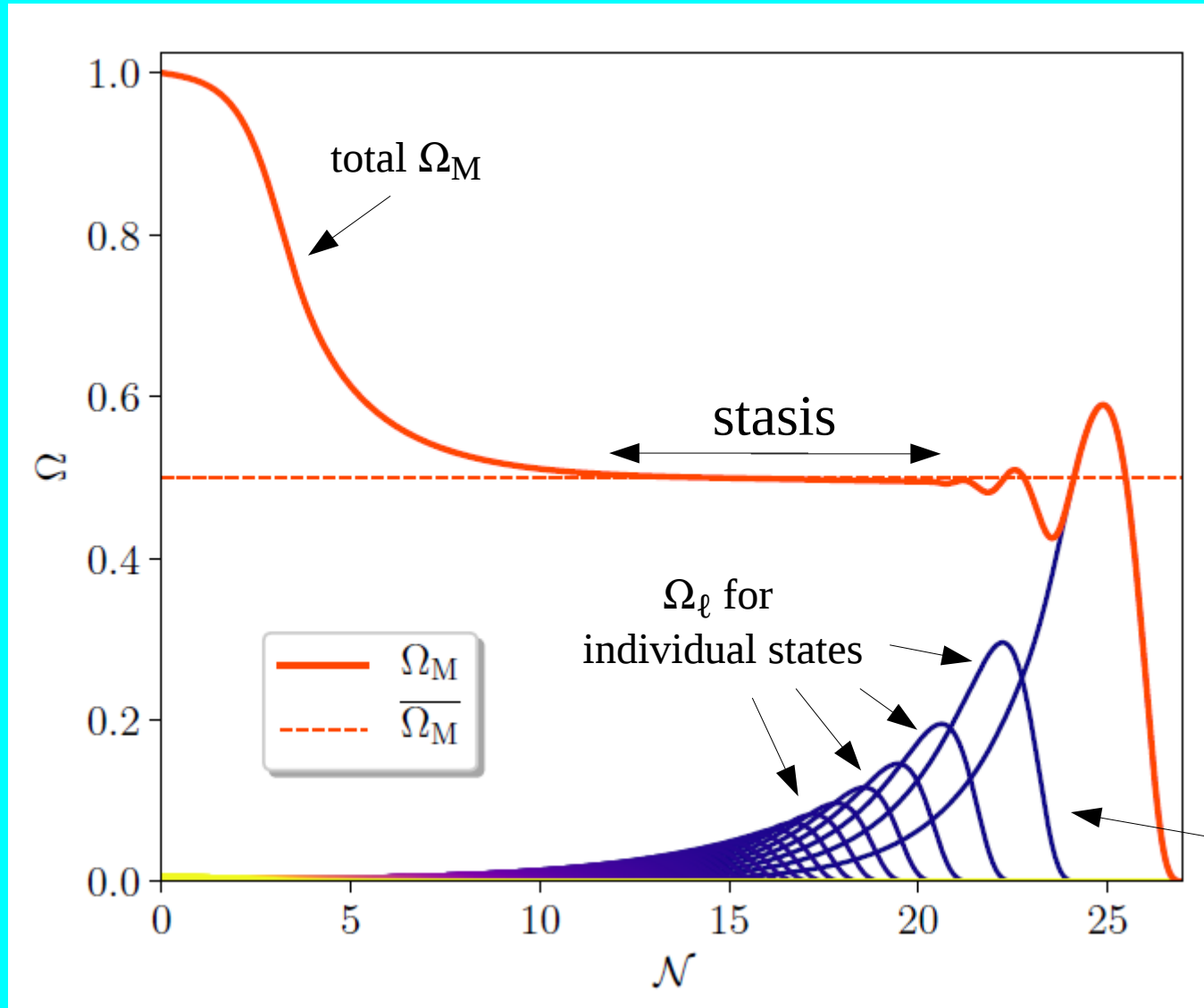
$$\bar{\Omega}_M = \frac{2\gamma\delta - 4(1 + \alpha\delta)}{2\gamma\delta - (1 + \alpha\delta)} .$$

For example, if

$$\frac{1 + \alpha\delta}{\gamma\delta} = \frac{2}{7}$$



An extended stasis epoch with
matter-radiation equality!



$$(\alpha, \gamma, \delta) = (1, 7, 1)$$

$$N = 300$$

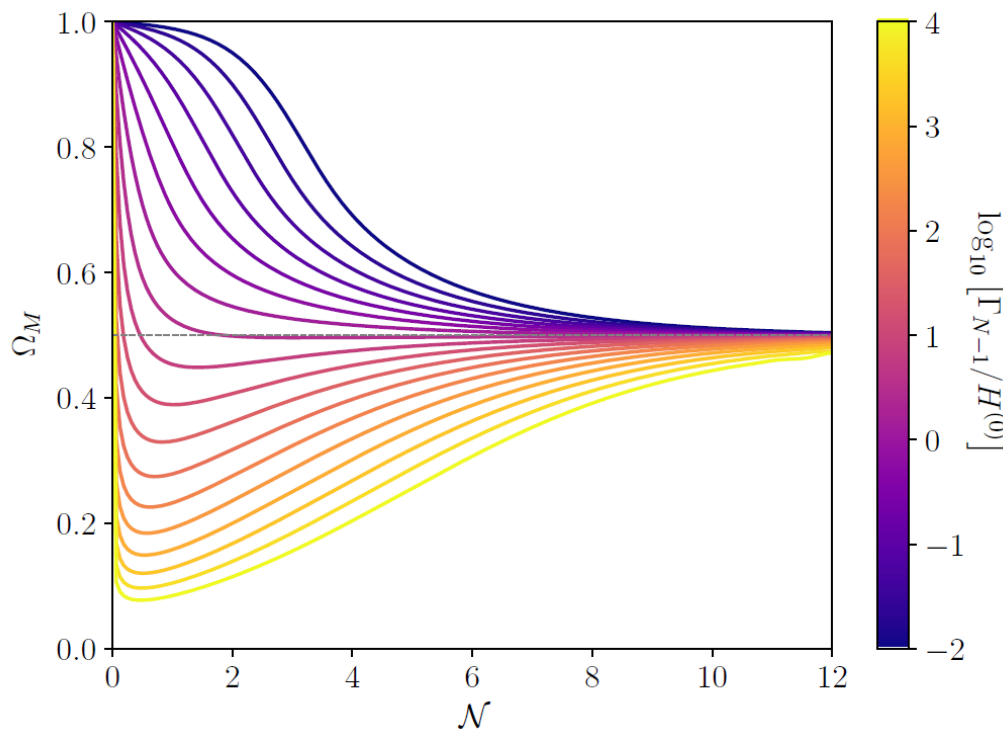
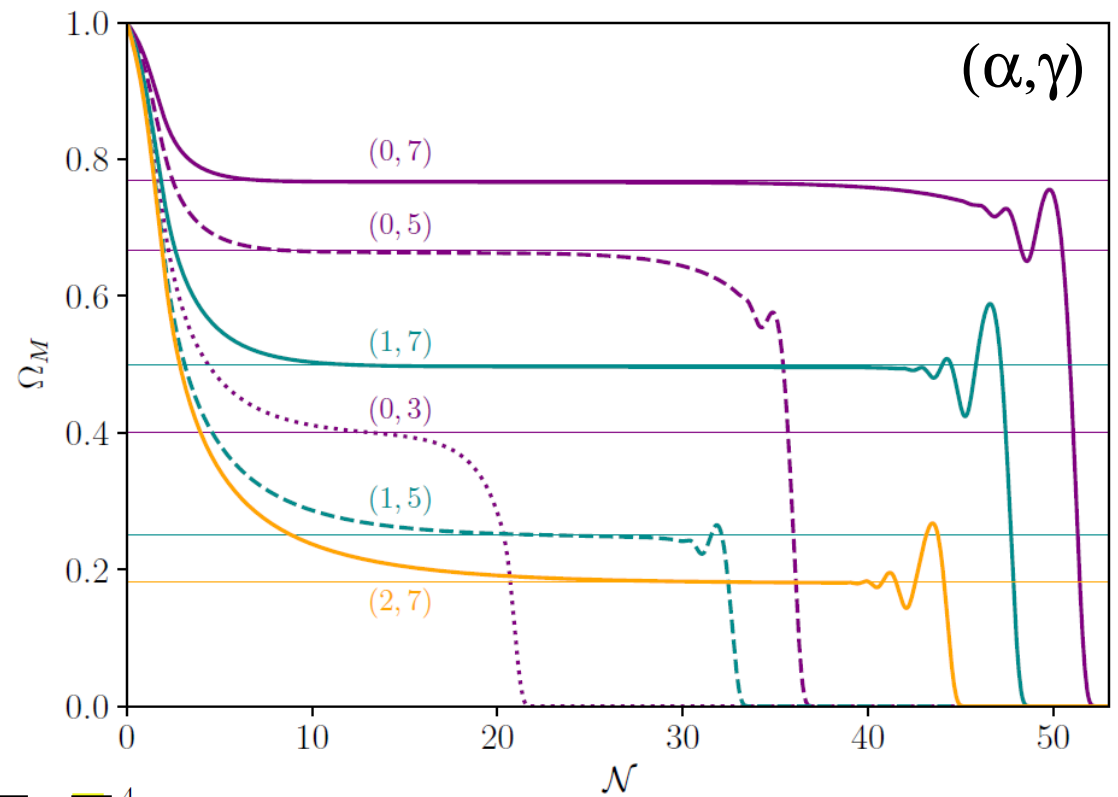
$$m_0 / \Delta m = 1$$

$$\Gamma_{N-1} / H^{(0)} = 0.1$$

Exact numerical
solution using
Boltzmann code,
no approximations.

Note interleaving
of energies of
dominant
components.

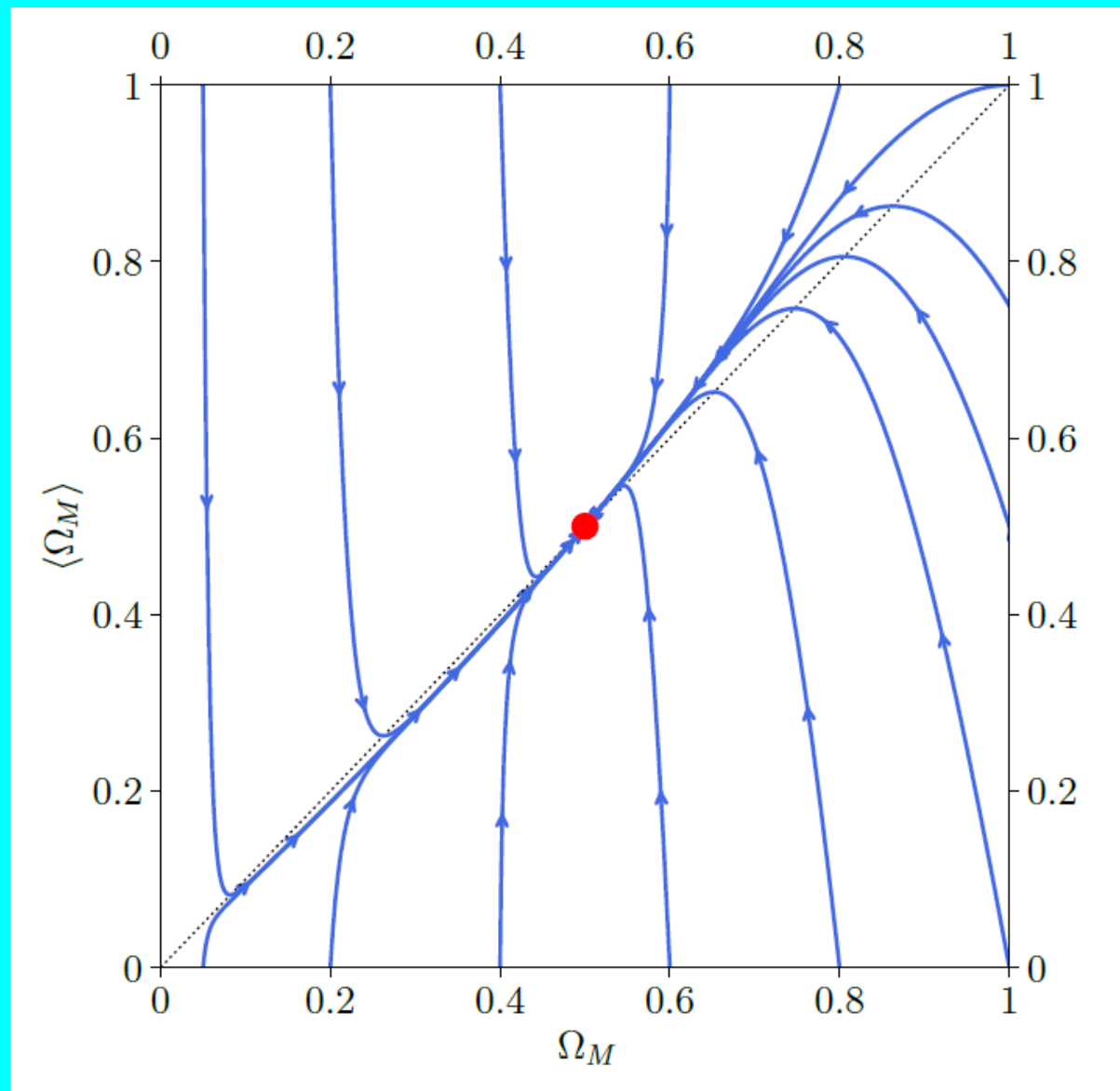
Indeed, we obtain stasis *regardless* of values of (α, γ, δ) within the allowed range! For $\delta=1$ and different (α, γ) we find



Can also vary $\Gamma_{N-1}/H^{(0)} =$ rate of decays relative to cosmological expansion. Affects initial behavior but stasis always emerges!

Indeed, matter/radiation stasis is a *global* attractor within such cosmologies...

time-averaged *history*
of abundance



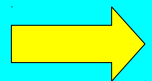
instantaneous abundance

Once we're in stasis, how long does it last?

Lasts until the last (lightest) tower component decays.

Then our stasis runs out of fuel and the universe exits from stasis.

$$\begin{aligned}\mathcal{N}_s &\equiv \log \left[\frac{a(t = \tau_0)}{a(t = t_{N-1})} \right] = \frac{2}{4 - \bar{\Omega}_M} \log \left(\frac{\Gamma_{N-1}}{\Gamma_0} \right) \\ &= \frac{2\gamma}{4 - \bar{\Omega}_M} \log \left(\frac{m_{N-1}}{m_0} \right) \\ &= \frac{2\gamma}{4 - \bar{\Omega}_M} \log \left[1 + \frac{\Delta m}{m_0} (N - 1)^\delta \right] \\ &\approx \frac{2\gamma\delta}{4 - \bar{\Omega}_M} \log N\end{aligned}$$



Number of e-folds of stasis goes as $\log(N)$.

Thus far we've focused on stasis between matter (M) and radiation (γ).
What about vacuum energy (Λ)?

$w = -1$

$w = 0$

$w = 1/3$

Vacuum energy density ρ_Λ scales as $a^0 = \text{constant}$.
Therefore cosmological expansion tends to push

Radiation \longrightarrow Matter
Matter \longrightarrow Vacuum energy

Counterbalancing “pumps” transfer abundance back again...

\longleftarrow
decay

\longleftarrow
??

**How can vacuum energy
convert to matter?**

Consider the coherent state consisting of the zero-momentum modes of a scalar field φ of mass m .

- At early times, when the Hubble parameter is large (with $3H > 2m$), this field is severely overdamped and thus has no kinetic energy. The energy of the field is pure potential energy (vacuum energy), with $w = -1$.
- However, as the universe expands, the Hubble parameter generally drops. As a result, the field eventually becomes underdamped (with $3H < 2m$) and begins to experience damped oscillations. These quickly virialize, with field energy split equally between potential and kinetic energy. We then have $w = 0$, with energy density behaving as matter as far as cosmic expansion is concerned.



**Underdamping (“turn-on”) transition at $3H(t)=2m$:
Converts vacuum energy \rightarrow matter!**

So, just as before, let us begin with a tower of $\phi_\ell =$ vacuum-energy components/species, and wait until they sequentially become underdamped and behave as matter.

Problem: Nothing ever happens!

Since this universe is initially vacuum-dominated,
Hubble = constant.

Therefore all fields remain overdamped and there is no dynamics!

How to get around this problem?

Several ideas ---

- Introduce additional non-vacuum energy component. This then introduces a non-trivial time-dependence for Hubble which in turn eventually triggers the cascading overdamped/ underdamped transitions that we require.
- Introduce a *regulator* for vacuum energy: consider w slightly bigger than -1 . Now Hubble evolves and dynamics emerges naturally. Then consider $w \rightarrow -1$ limit as the vacuum-energy limit.
- Consider full scalar-field dynamics: damped driven harmonic oscillator in which the Hubble damping terms carry a non-trivial time dependence.
- Is arbitrary and non-minimal, with new parameters to govern extra components.
- Minimal. System can be solved analytically. Can even study stasis as function of w .
- True scalar field \rightarrow has UV completion. But if universe not yet in stasis, EOMs lack analytical solutions \rightarrow must study system numerically.

How to get around this problem?

Several ideas ---

- Introduce additional non-vacuum energy component. This then introduces a non-trivial time-dependence for Hubble which in turn eventually triggers the cascading overdamped/ underdamped transitions that we require.
- Introduce a *regulator* for vacuum energy: consider w slightly bigger than -1 . Now Hubble evolves and dynamics emerges naturally. Then consider $w \rightarrow -1$ limit as the vacuum-energy limit.
- Consider full scalar-field dynamics: damped driven harmonic oscillator in which the Hubble damping terms carry a non-trivial time dependence.
- Is arbitrary and non-minimal, with new parameters to govern extra components.
- Minimal. System can be solved analytically. Can even study stasis as function of w .
- True scalar field \rightarrow has UV completion. But if universe not yet in stasis, EOMs lack analytical solutions \rightarrow must study system numerically.

How to get around this problem?

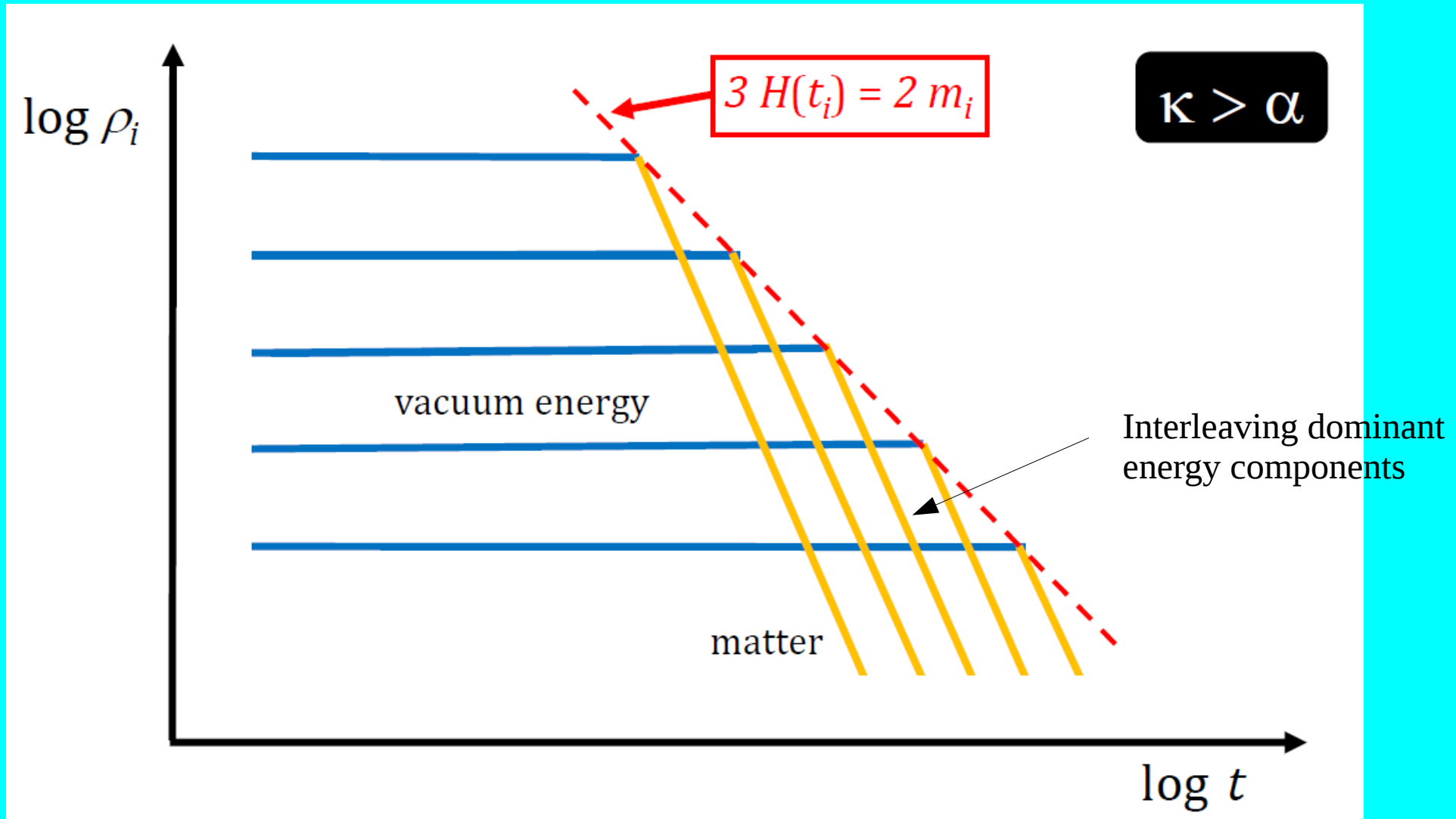
Several ideas ---

- Introduce additional non-vacuum energy component. This then introduces a non-trivial time-dependence for Hubble which in turn eventually triggers the cascading overdamped/ underdamped transitions that we require.
- Introduce a *regulator* for vacuum energy: consider w slightly bigger than -1 . Now Hubble evolves and dynamics emerges naturally. Then consider $w \rightarrow -1$ limit as the vacuum-energy limit.
- Consider full scalar-field dynamics: damped driven harmonic oscillator in which the Hubble damping terms carry a non-trivial time dependence.
- Is arbitrary and non-minimal, with new parameters to govern extra components.
- Minimal. System can be solved analytically. Can even study stasis as function of w .
- True scalar field \rightarrow has UV completion. But if universe not yet in stasis, EOMs lack analytical solutions \rightarrow must study system numerically.



Brooks's talk!

In the $w \rightarrow -1$ limit, our system has an interesting dynamics



But does this give rise to a stasis between vacuum energy and matter?

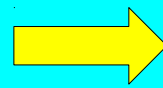
Perform similar analysis as for matter/radiation stasis, and find

Density of states per unit turn-on time

Abundance turning on at time t .

$$\begin{aligned} n_{\hat{t}}(t) \Omega(t) &= \left[-w \bar{\kappa} \bar{\Omega}_{\Lambda} (1 - \bar{\Omega}_{\Lambda}) \right] \frac{1}{t} \\ &= \left[2 - (1 + w) \bar{\kappa} \right] \bar{\Omega}_{\Lambda} \frac{1}{t} . \end{aligned}$$

1. Realize proper $1/t$ scaling
2. Avoid potential *logarithmic time dependence* in abundances
3. Match *prefactors* as well.



$$\alpha + \frac{1}{\delta} = 2 - (1 + w) \bar{\kappa} ,$$

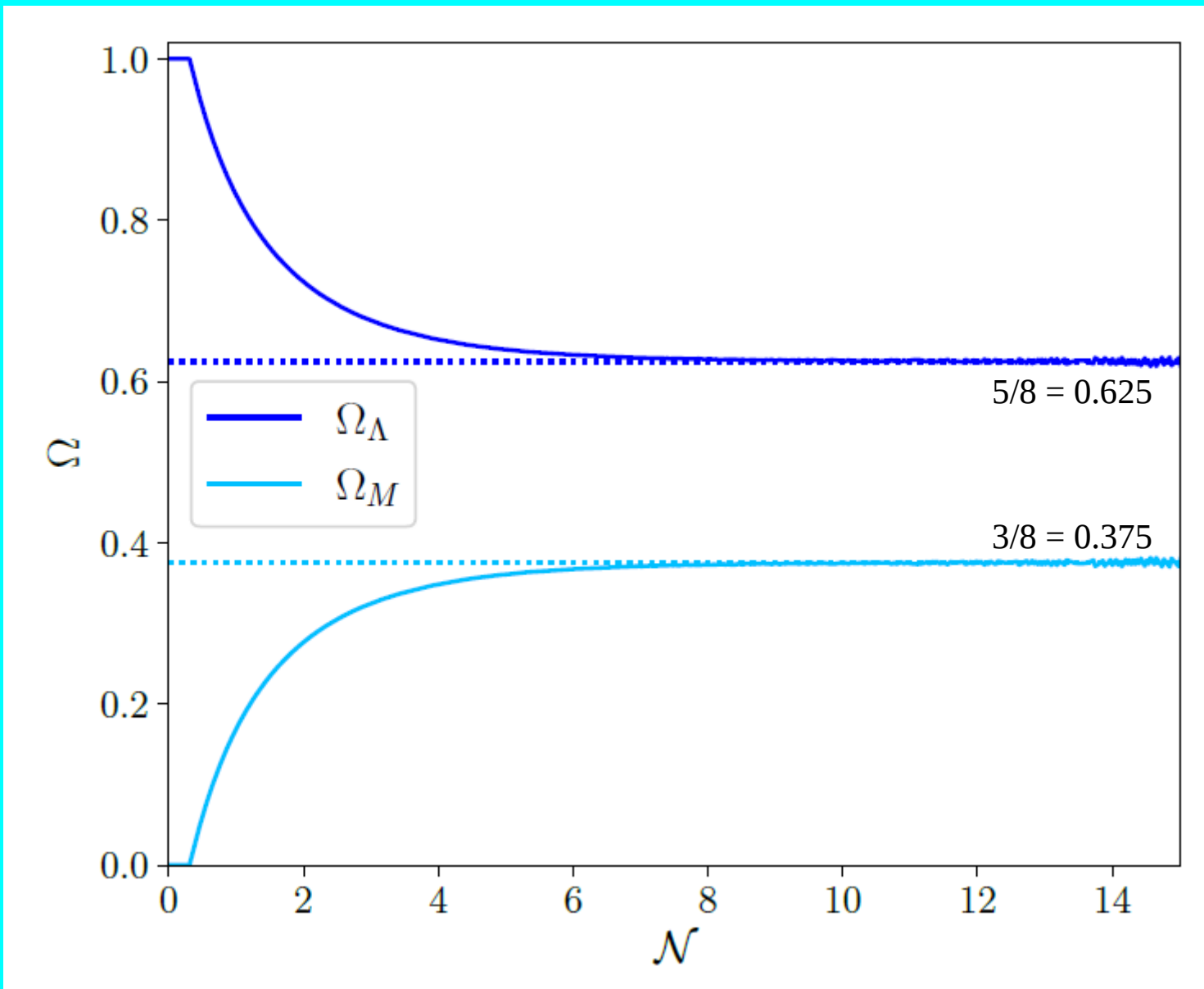
Once again, this is not a constraint on (α, δ)
so much as a *prediction for κ* during stasis!

Thus, so long as $0 < \alpha + \frac{1}{\delta} < -2w$

we will *always have stasis (!)*,
and indeed the corresponding stasis abundance will be given by

$$\bar{\Omega}_{\Lambda} = - \frac{\alpha + 1/\delta + 2w}{(\alpha + 1/\delta - 2)w}$$

Example: Vacuum-energy/matter stasis

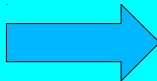


$$\delta = 2, \alpha = 0.7, \bar{\kappa} = 4, w = -0.8.$$

Vacuum-energy/radiation stasis.

Perform similar analysis as for matter/radiation stasis, find

1. Realize proper $1/t$ scaling
2. Avoid potential *logarithmic time dependence* in abundances
3. Match *prefactors* as well.



$$\begin{aligned}\sum_{\ell} \Gamma_{\ell} \Omega_{\ell}(t) &= \left[\left(\frac{1}{3} - w \right) \bar{\kappa} \bar{\Omega}_{\Lambda} (1 - \bar{\Omega}_{\Lambda}) \right] \frac{1}{t} \\ &= \left[2 - (1 + w) \bar{\kappa} \right] \bar{\Omega}_{\Lambda} \frac{1}{t} .\end{aligned}$$

$$\frac{1}{\gamma} \left(\alpha + \frac{1}{\delta} \right) = 2 - (1 + w) \bar{\kappa} .$$

This too is not a constraint on (α, γ, δ)

so much as a *prediction for κ* during stasis!

Thus, so long as $0 < \frac{1}{\gamma} \left(\alpha + \frac{1}{\delta} \right) < \frac{1 - 3w}{2}$

we will *always have stasis*.

The corresponding stasis abundance will be given by

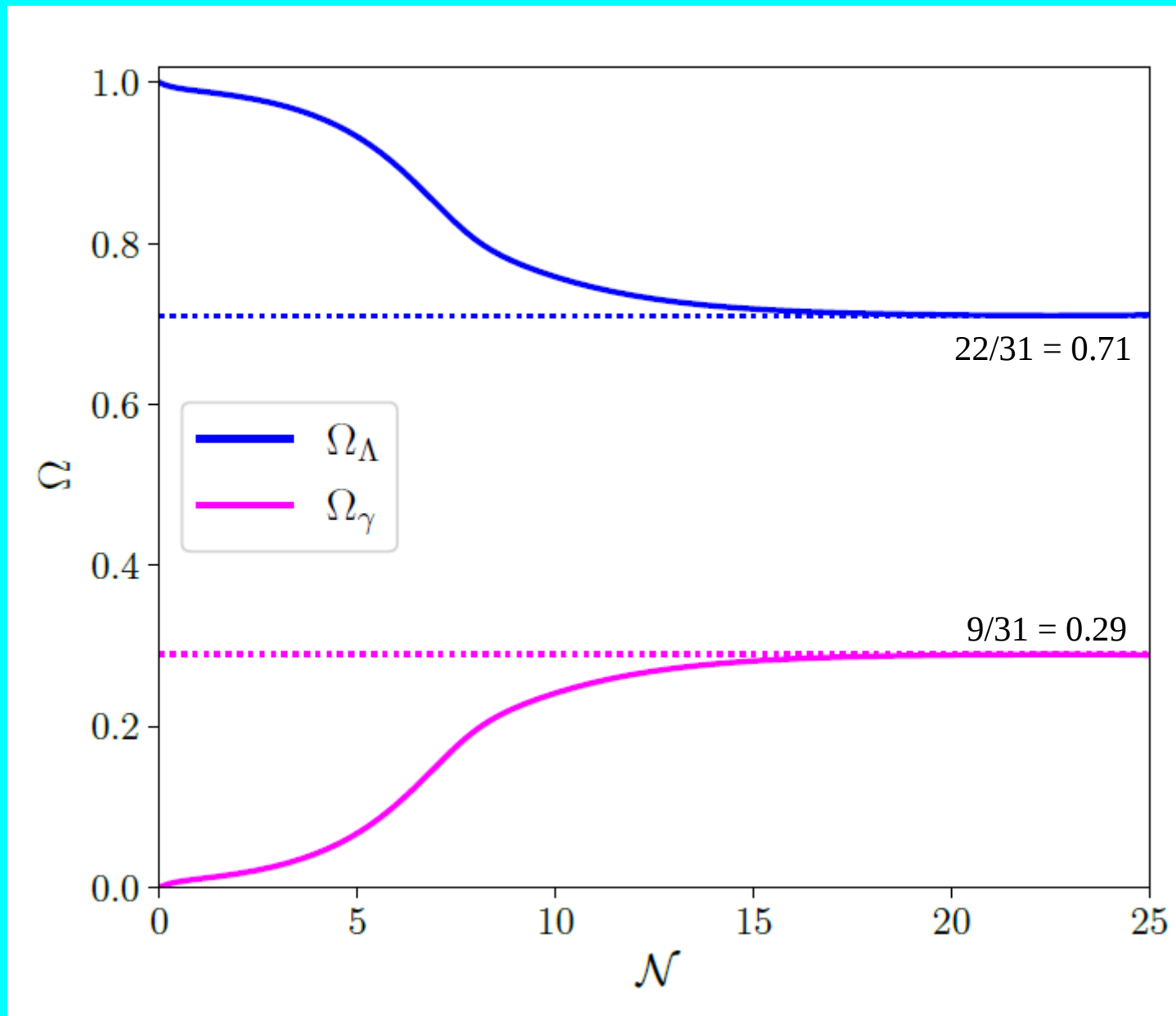
$$\bar{\Omega}_{\Lambda} = \frac{2(2X + 3w - 1)}{(1 - 3w)(X - 2)}$$

where

$$X \equiv (\alpha + 1/\delta)/\gamma$$

See paper for subtleties regarding decay of vacuum energy...

Example: Vacuum-energy/radiation stasis



$\alpha=0.5, \gamma=1, \delta=2, w=-0.7, H^{(0)}/\Gamma_{N-1}=20, \kappa=4/3$

Triple stasis

Thus far, we have shown the existence of three different kinds of pairwise stasis:

- Matter with radiation
- Vacuum energy with matter
- Vacuum energy with radiation

Each occurs in a universe consisting of only those two types of energy components.

Obvious next step:

Can we have a universe in which all three are in stasis with each other??

This is highly non-trivial!

Just because A comes into stasis with B in an A/B universe,
... and just because A comes into stasis with C in an A/C universe,
... and just because B comes into stasis with C in a B/C universe,
does *not* imply A, B, and C will all come into stasis in an A/B/C universe!

This is because each of our previous energy-transfer processes (decay and underdamping) would now need to operate in a universe which also contains a *third* energy component. This third component also affects Hubble and the overall expansion rates whose effects would need to cancel for a triple stasis.

In other words, both processes (decay and underdamping) must occur *simultaneously* while embedded in a **common cosmology**!

Triple stasis can arise only if these processes can co-exist with each other, potentially placing new constraints on each!

We can nevertheless proceed the same way as before, find

General algebraic structure of triple stasis

Let P_{ij} = “pump” that converts energy components i -type \rightarrow j -type

e.g., $P_{M\gamma}$ = particle decay

$P_{\Lambda M}$ = underdamping transition

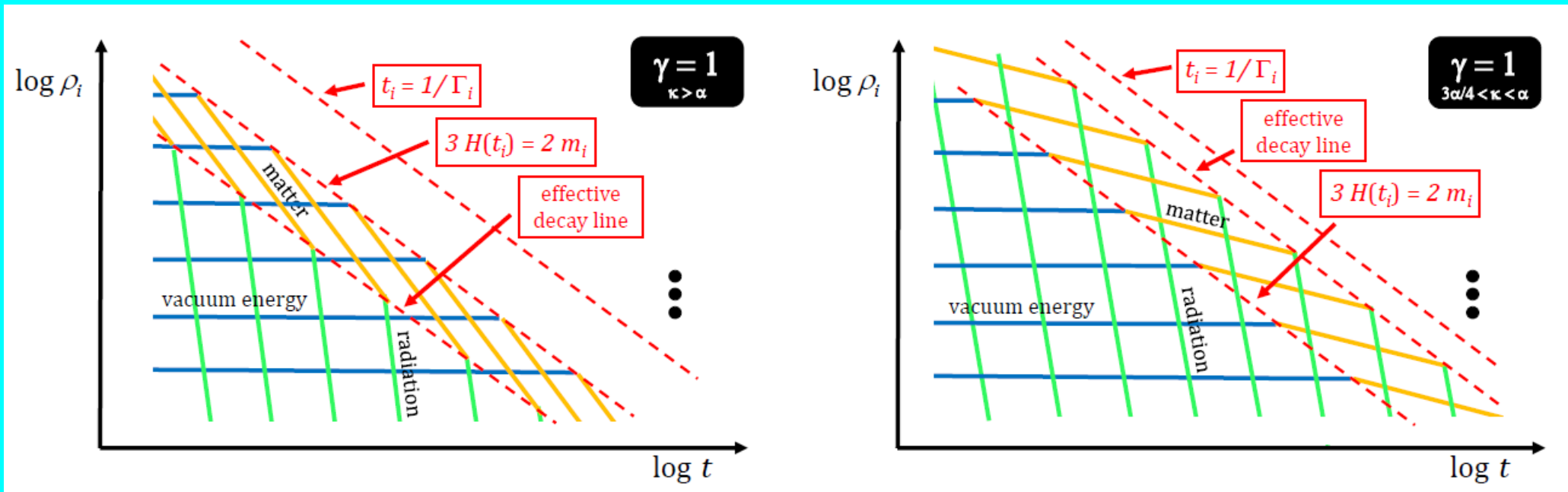
pump = abundance
conversion rate
(per unit time)

Then our stasis conditions now take the form

$$\begin{aligned} P_{\Lambda M} + P_{\Lambda\gamma} &= \left[2 - (1 + w)\bar{\kappa} \right] \bar{\Omega}_{\Lambda} \frac{1}{t} \\ -P_{\Lambda M} + P_{M\gamma} &= \left[2 - \bar{\kappa} \right] \bar{\Omega}_{M} \frac{1}{t} \\ -P_{\Lambda\gamma} - P_{M\gamma} &= \left[2 - \frac{4\bar{\kappa}}{3} \right] \bar{\Omega}_{\gamma} \frac{1}{t} . \end{aligned}$$

Pumping actions balanced against cosmological expansion
within a common cosmology (κ)

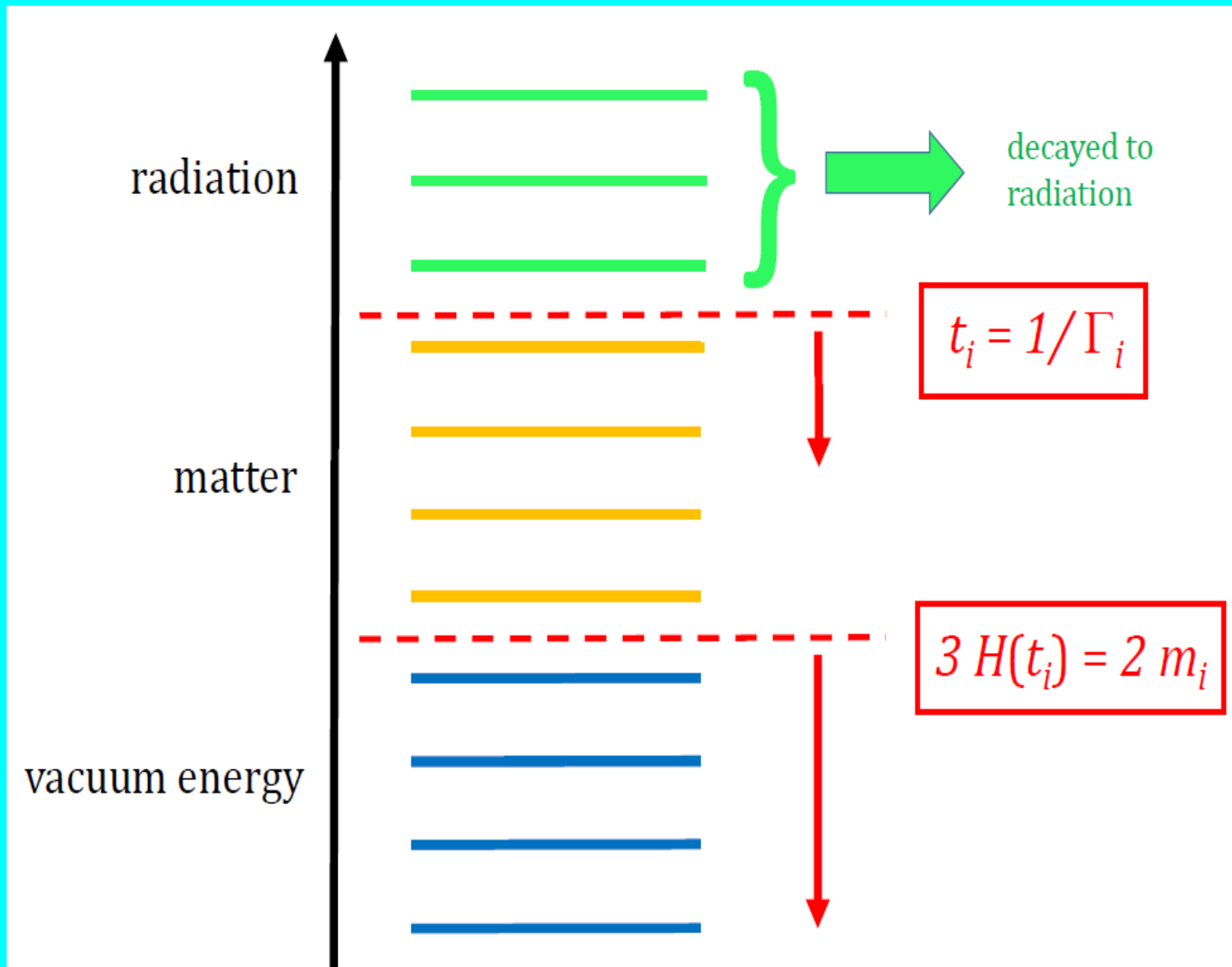
Such a triple stasis would have an even more complex dynamics.
 Depending on relevant parameters, there are two possible varieties...



Each case exhibits
 the interleaving
 characteristic of stasis.

Each state in the tower starts as **vacuum energy**
 ... then becomes underdamped, behaves as **matter**
 ... and then decays and becomes **radiation**.

Indeed, taking time-slices through the previous figures, we find that the dynamics takes the form



This is triple stasis in action!

Each phase transition proceeds down the tower, counterbalancing the effects of cosmic expansion.

But the fundamental question remains:

Can this system come into balance?? Is a true triple stasis possible?

Must satisfy the triple-stasis algebraic constraints, yielding relations for (α, γ, δ) .

$$\begin{aligned} P_{\Lambda M} + P_{\Lambda \gamma} &= \left[2 - (1 + w)\bar{\kappa} \right] \bar{\Omega}_{\Lambda} \frac{1}{t} \\ -P_{\Lambda M} + P_{M\gamma} &= \left[2 - \bar{\kappa} \right] \bar{\Omega}_M \frac{1}{t} \\ -P_{\Lambda \gamma} - P_{M\gamma} &= \left[2 - \frac{4\bar{\kappa}}{3} \right] \bar{\Omega}_{\gamma} \frac{1}{t}. \end{aligned}$$

Recall that this boils down to three kinds of constraints

1. Realize proper $1/t$ scaling
2. Avoid potential *logarithmic time dependence* in abundances
3. Match *prefactors* as well.

For the case of triple stasis, these constraints now give different parts of the puzzle.

- #1: gives relations for (α, γ, δ) , as before
- #2: is now an independent constraint, will rule out certain cases allowed by #1
- #3: allows us to determine the final stasis abundances for each component!

So what do we find?

#1: Now gives *two* constraints since there are now *two* pumps!

$$\alpha + \frac{1}{\delta} = 2 - (1 + w)\bar{\kappa} ,$$

$$\alpha + \frac{1}{\delta} = 2\gamma - (\gamma + w)\bar{\kappa} .$$

These separate constraints cannot be satisfied simultaneously unless

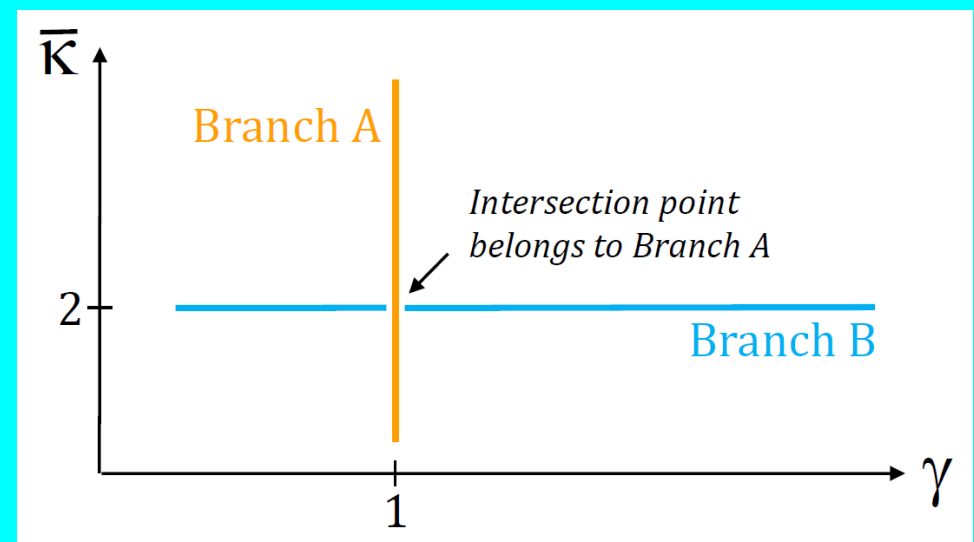
$$(2 - \bar{\kappa}) \left(1 - \frac{1}{\gamma}\right) = 0 .$$

This is the condition that correlates the two pumps, allowing them to co-exist!

Thus, our system splits into two disjoint branches:

Branch A : $\gamma = 1$, any $\bar{\kappa}$

Branch B : $\bar{\kappa} = 2$, any $\gamma \neq 1$



- #2: Avoiding spurious logarithmic time-evolution for abundances imposes an additional constraint: must have

$$\text{either } \gamma = 1 \text{ or } \bar{\kappa} \neq 2$$

This kills Branch B! (However, will play a role later ...)

Henceforth restrict to Branch A ($\gamma = 1$, κ unrestricted)

- #3: This relation imposes no further limitation and even allows us to solve for stasis abundances analytically!

Thus, once again, our triple-stasis conditions are satisfied for all (α, δ) . We have only one new constraint, namely $\gamma = 1$, and this is completely natural for fermions decaying to photons!

$$\bar{\Omega}_i = r_i / \sum_j r_j$$

where

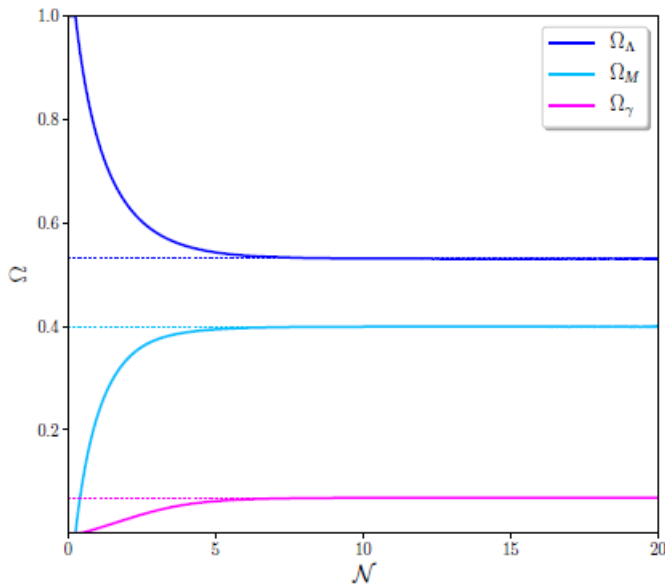
$$\begin{aligned} r_\Lambda &= (4\bar{\kappa} - 6)(X + \bar{\kappa} - 2) \\ r_M &= (4\bar{\kappa} - 6)\eta \\ r_\gamma &= 3\eta X . \end{aligned}$$

$$\eta = \alpha + 1/\delta$$

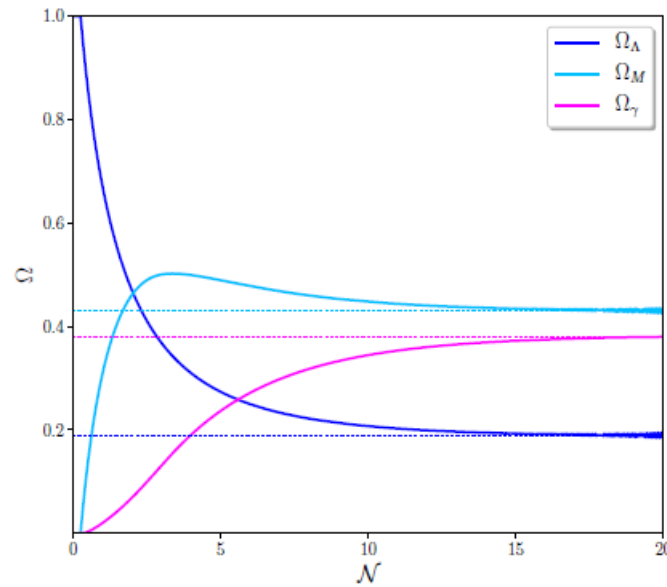


Like its pairwise cousins, triple stasis is also fairly generic!

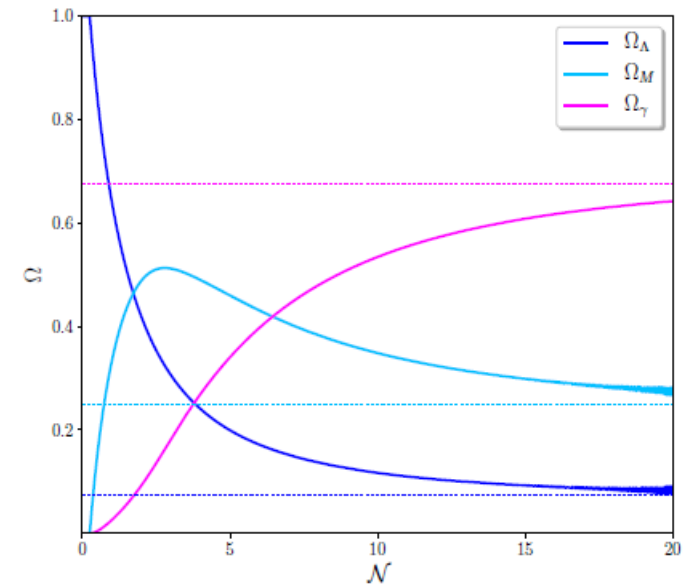
Triple stasis! Solutions for various underlying parameters ---



$\kappa > 2$



$\kappa = 2$

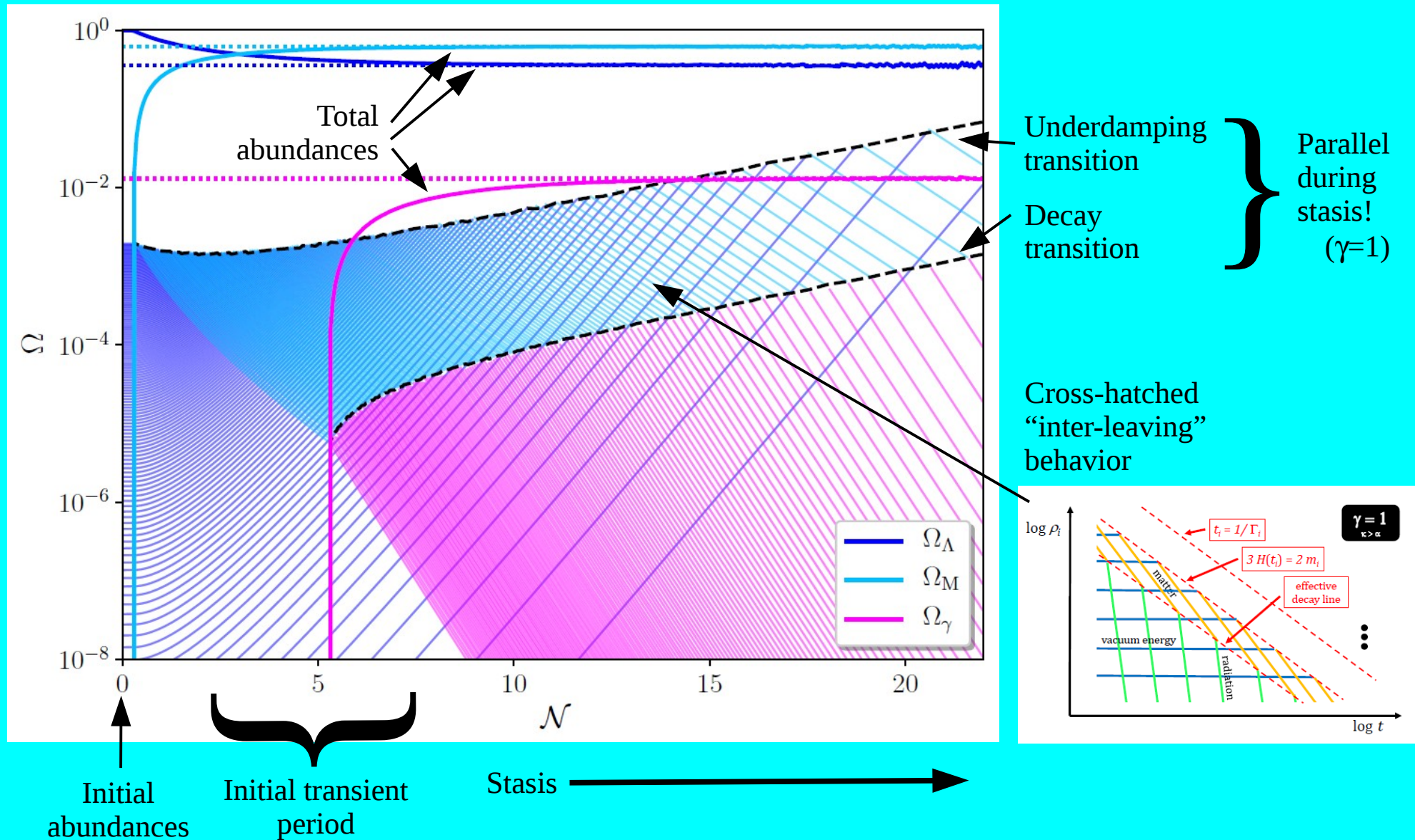


$\kappa < 2$

- In all cases shown, system approaches a triple-stasis configuration.
- As resulting κ increases, it takes less time to reach triple stasis.
- Triple stasis can have stasis abundances with various relative orderings of magnitudes.

How does triple stasis actually work at the level of the individual components in the tower?

Look “under the hood”, find



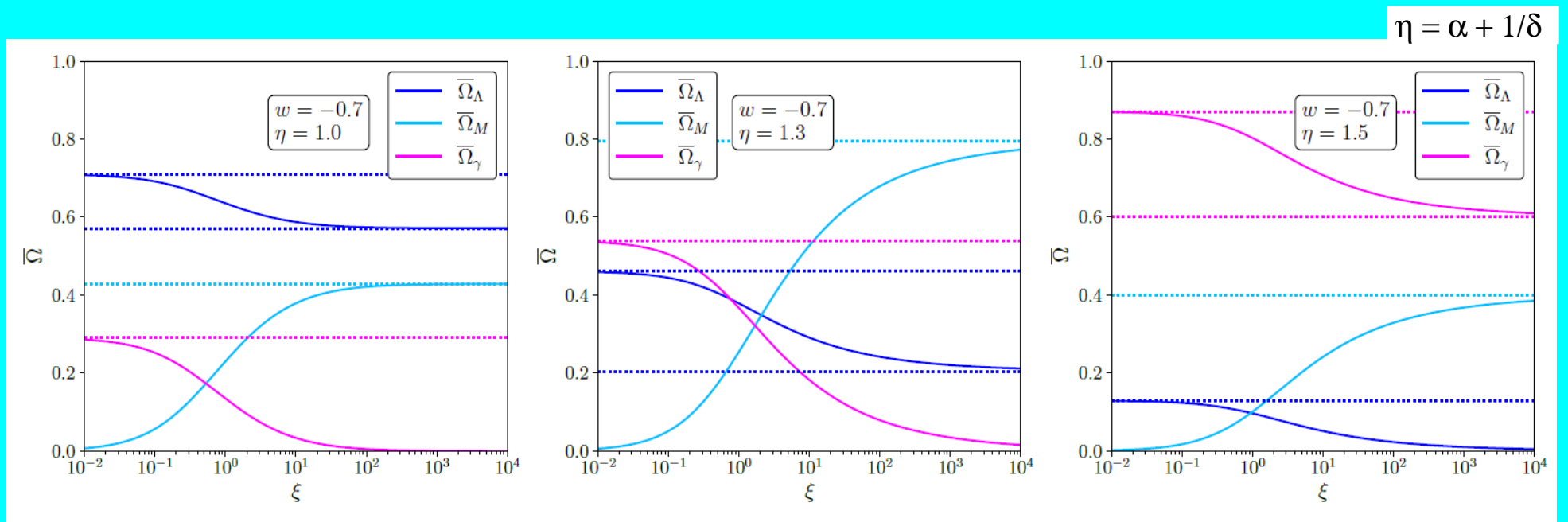
While important in its own right, triple stasis also *interpolates* between different pairwise stases!

Define parameter

$$\xi \equiv \frac{2}{\bar{\kappa}} \frac{m_0}{\Gamma_0}$$

Indicates duration of time interval between underdamping and decay (more ξ , more matter).

Plot final stasis abundances $\bar{\Omega}_i$ versus ξ for various values of $\alpha + 1/\delta$, obtain



$\Lambda\gamma$ ← triple stasis → ΛM

$\Lambda\gamma$ ← triple stasis → ΛM

$\Lambda\gamma$ ← triple stasis → $M\gamma$

Triple stasis connects different pairwise stases through parameter-space interiors.

Given the forms of the constraint equations, there is also a very “geometric” way of understanding the triple-stasis phenomenon.

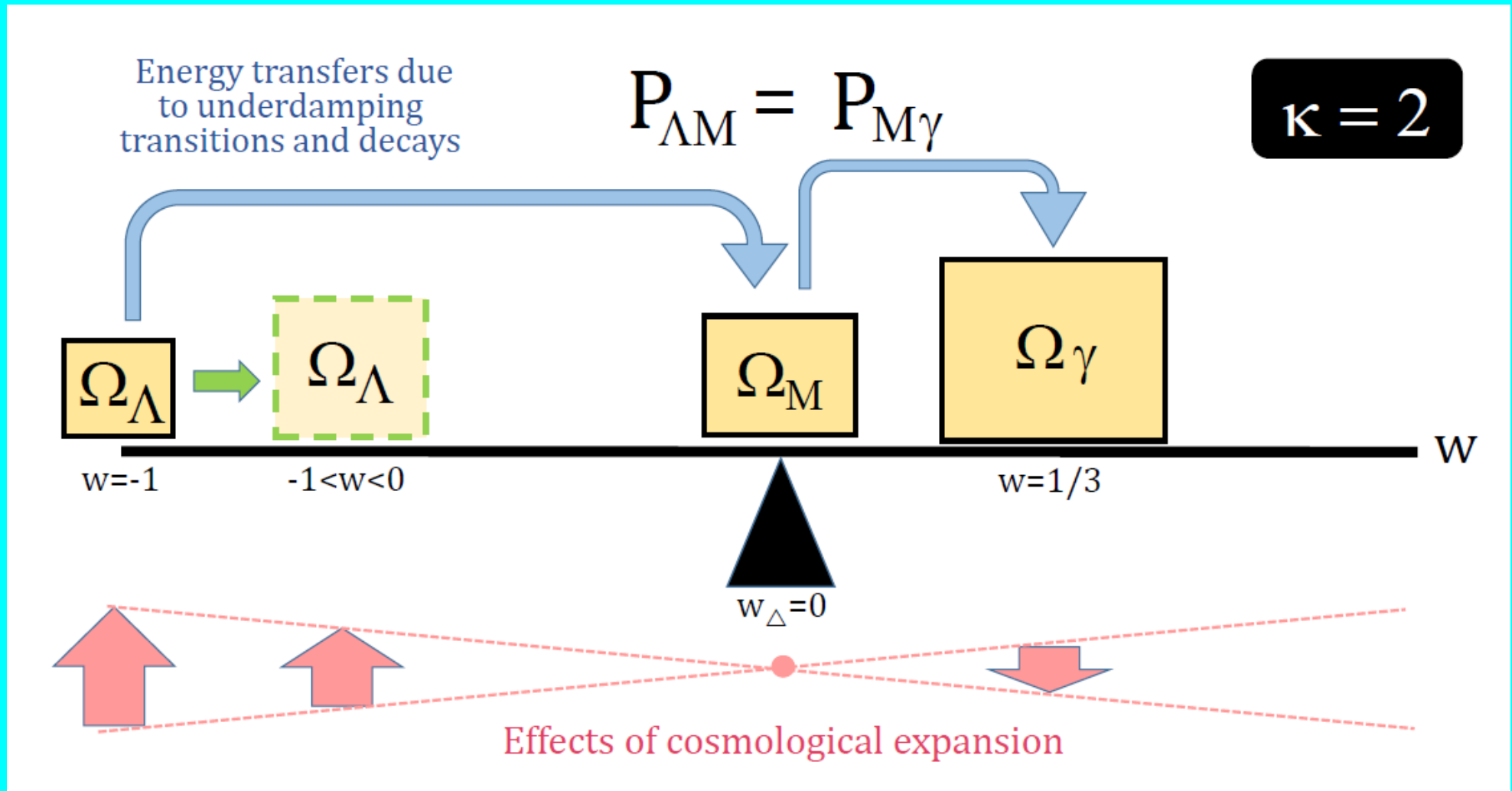
This depends on the values of κ that are being realized in each case.

Recall

$$\alpha + \frac{1}{\delta} = 2 - (1 + w)\bar{\kappa} ,$$

Let us first study the case with $\kappa = 2...$

The w-seesaw!



Triple stasis with $\kappa = 2$ is effectively matter-dominated.

This is the most “symmetric” situation,
with equal pumps and a fulcrum location at $w_{\Delta} = 0$.

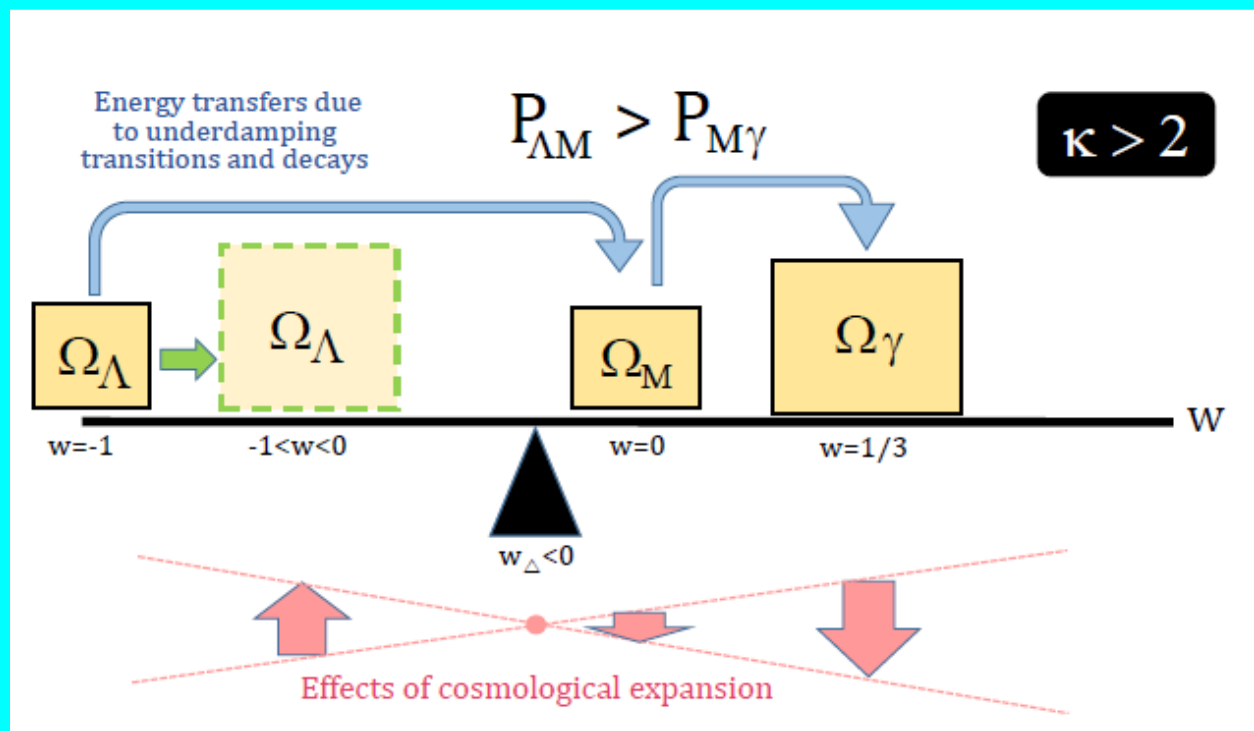
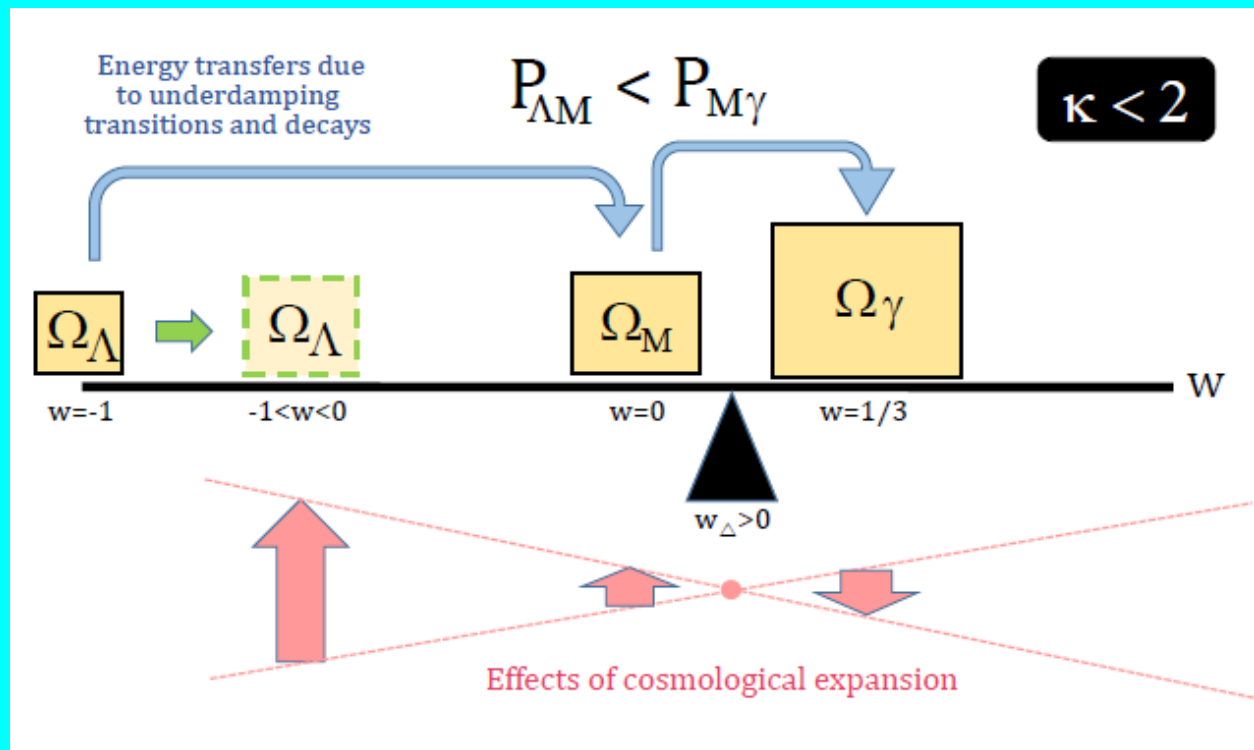
Shifting w does not destroy the stasis because the abundances compensate.

Other values of κ correspond to shifting the location of the fulcrum!

In general, we have

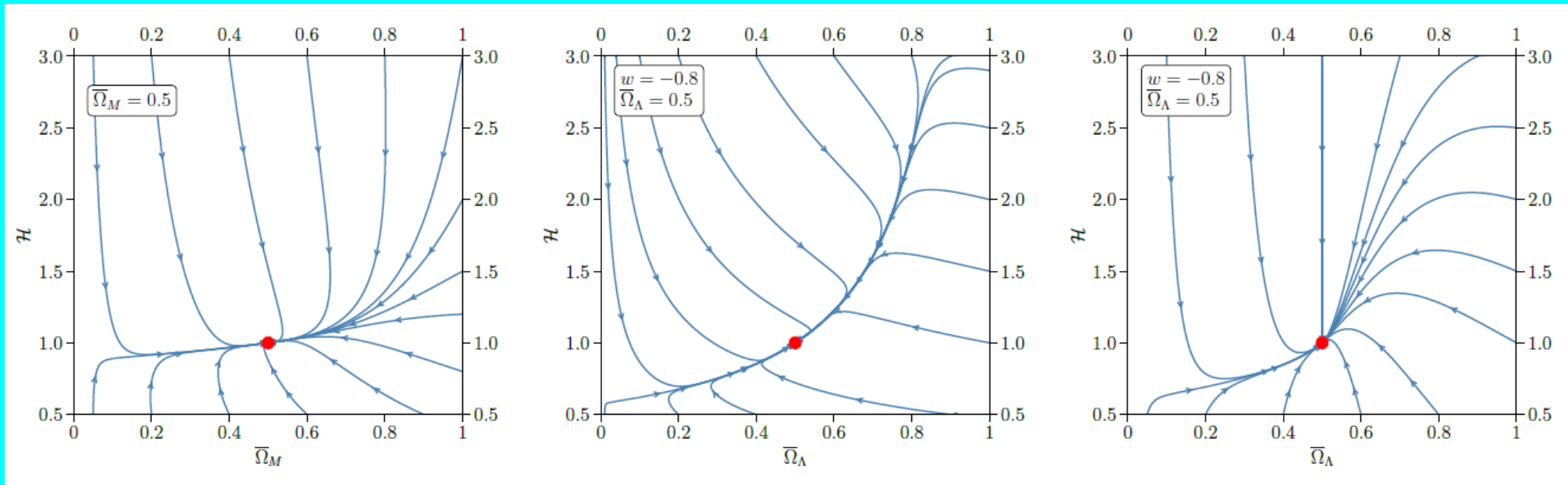
$$w_{\Delta} \equiv \frac{2}{\kappa} - 1$$

Pumps become unequal but stasis is always maintained!



All of these stases are global attractors.

This is true not only for the pairwise stases...



M/ γ stasis

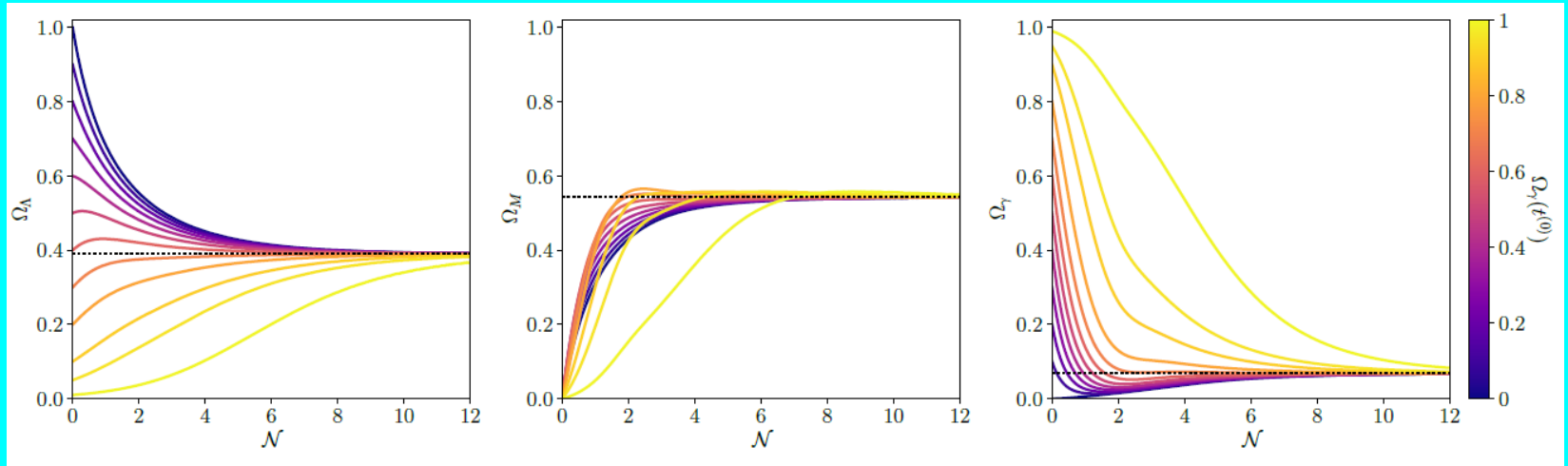
Λ /M stasis

Λ / γ stasis

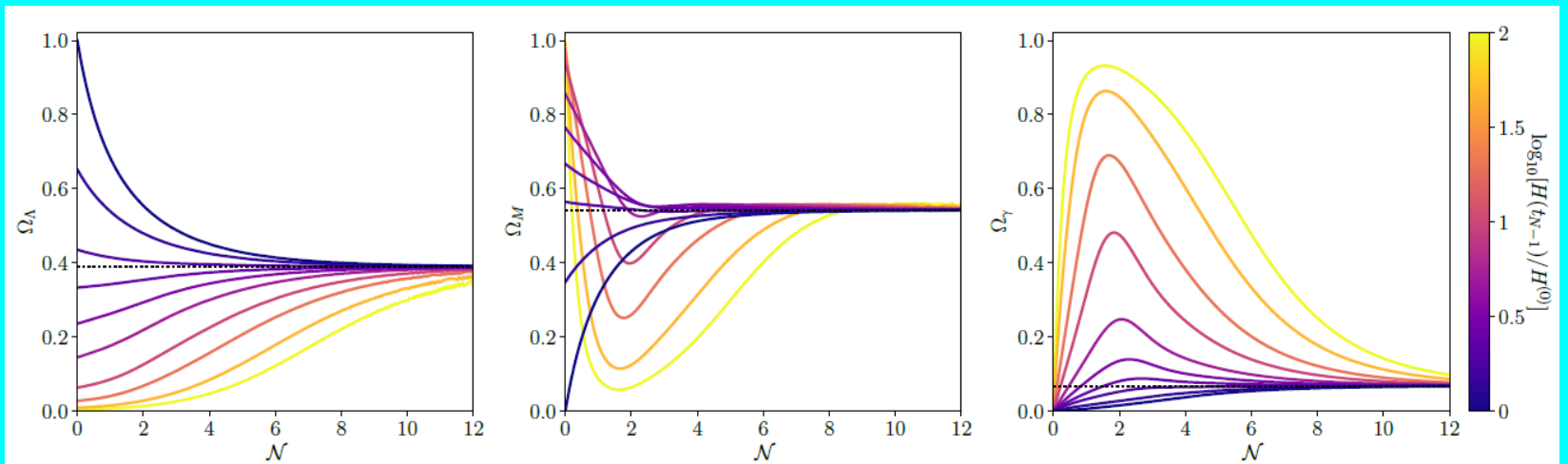
In each of these pairwise stases, the relevant abundances are drawn to their stasis values regardless of their initial values.

... but also for triple stasis!

Our three abundances $\{\Omega_\Lambda, \Omega_M, \Omega_\gamma\}$ all evolve toward their stasis values for all values of initial conditions...



...and also for different cosmological backgrounds (different Hubble parameters $H(t_{N-1})/H(t^{(0)})$)



A Phase Space for Stasis

All of the different stases we have discussed are deeply related to each other and populate different limiting regions of a common stasis “phase space”!

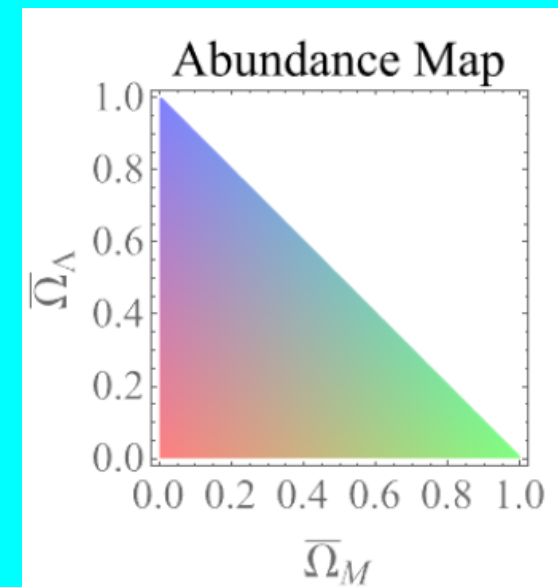
To see this, consider triple stasis. For each point within the plane

(w_Δ, ξ)

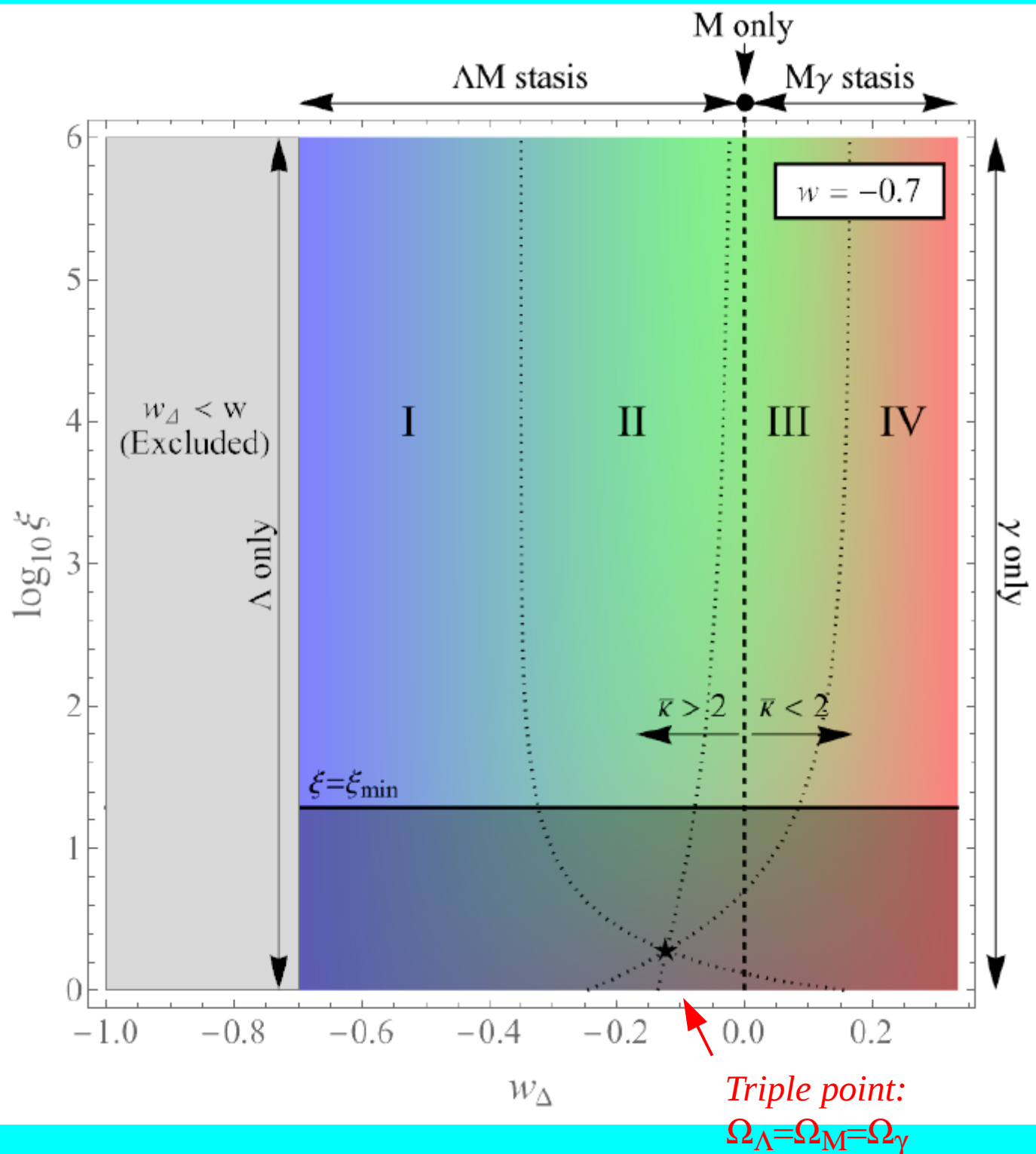
fulcrum
location

$$\xi \equiv \frac{2}{\bar{\kappa}} \frac{m_0}{\Gamma_0}$$

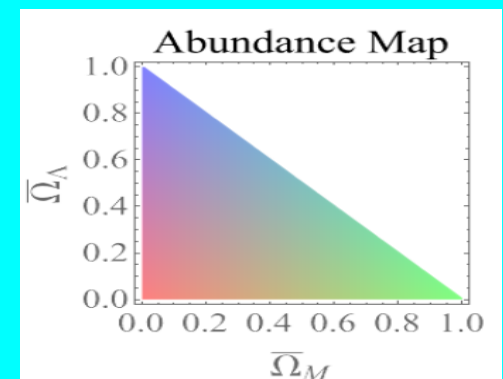
plot stasis abundances
according to the color map



find...



- Pairwise stases emerge for large ξ .
- Universes are effectively *matter-dominated* along vertical dashed line, with balanced Ω_{Λ} and Ω_{γ} in addition to Ω_M .
- As ξ increases, these other components die away, leaving only Ω_M .
- Nevertheless have same algebraic constraints and attractor behavior everywhere in plane.



Beyond stasis

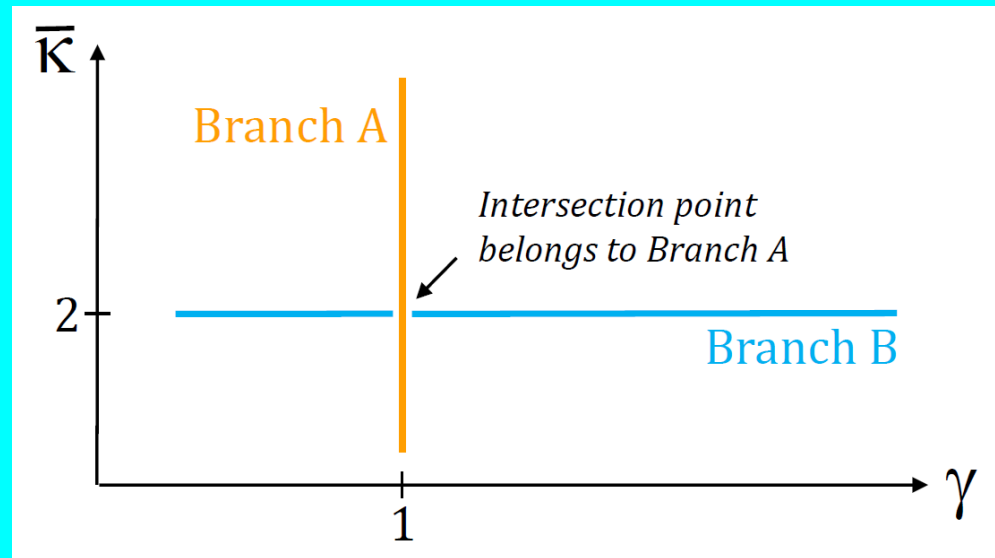
In addition to the stasis phenomenon, there are also *variants* of this phenomenon in which only some — but not all — of the features associated with stasis are retained.

Depending on which features are retained, we can obtain a variety of scenarios which may be exceedingly interesting in their own rights on both theoretical and phenomenological grounds.

1

Quasi-stasis

Recall that triple stasis requires that (γ, κ) lie along Branch A within the (γ, κ) plane. If (γ, κ) do not lie along Branch A, then we do not achieve triple stasis.

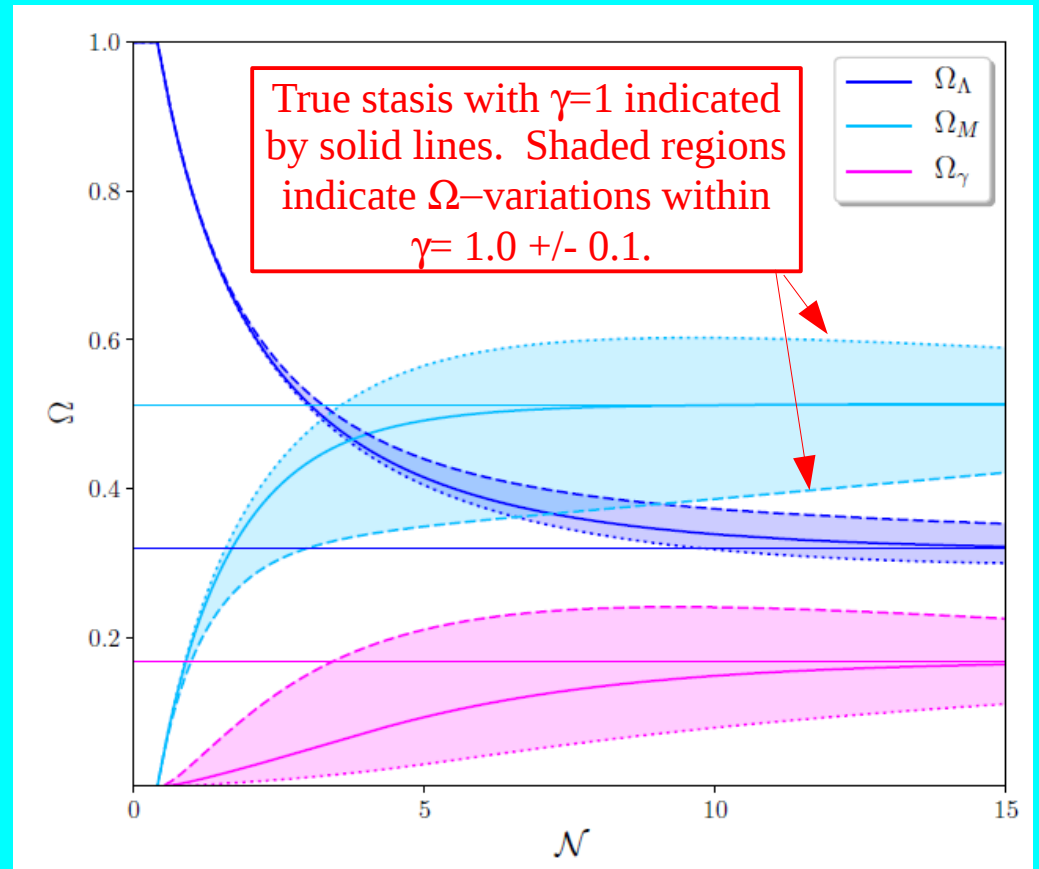


However, two other regions within this plane are of note:

- Points that lie along Branch B. These satisfy the overall scaling constraint but not the log-avoidance constraint.
- Points which lie near Branches A or B. These come *close* to satisfying the overall scaling constraints, but not completely.

Points within these regions correspond to situations in which our abundances do not remain constant, but evolve exceedingly slowly!

Within such scenarios, all of the leading power-law growth that would appear in the usual cosmology is absent but a weak, quasi-logarithmic time-dependence remains. Depending on the residual rate of change, such quasi-stasis solutions may effectively serve as (and be phenomenologically indistinguishable from) true stasis solutions over relevant cosmological timescales.



Stasis is a robust phenomenon. Even if real-world effects (e.g., radiative corrections) push our system beyond some of our precise input parameters, overall properties largely remain.

2

Oscillatory stasis

Another option is to consider what happens as Δm becomes large.

- Spacing between successive components of tower becomes significant.
- This increases the time intervals between successive underdamping and/or decay transitions.
- During these time intervals, cosmological expansion still occurs and has time to work its magic before being compensated by another “infusion” of abundance occurs from the next transition along the tower.

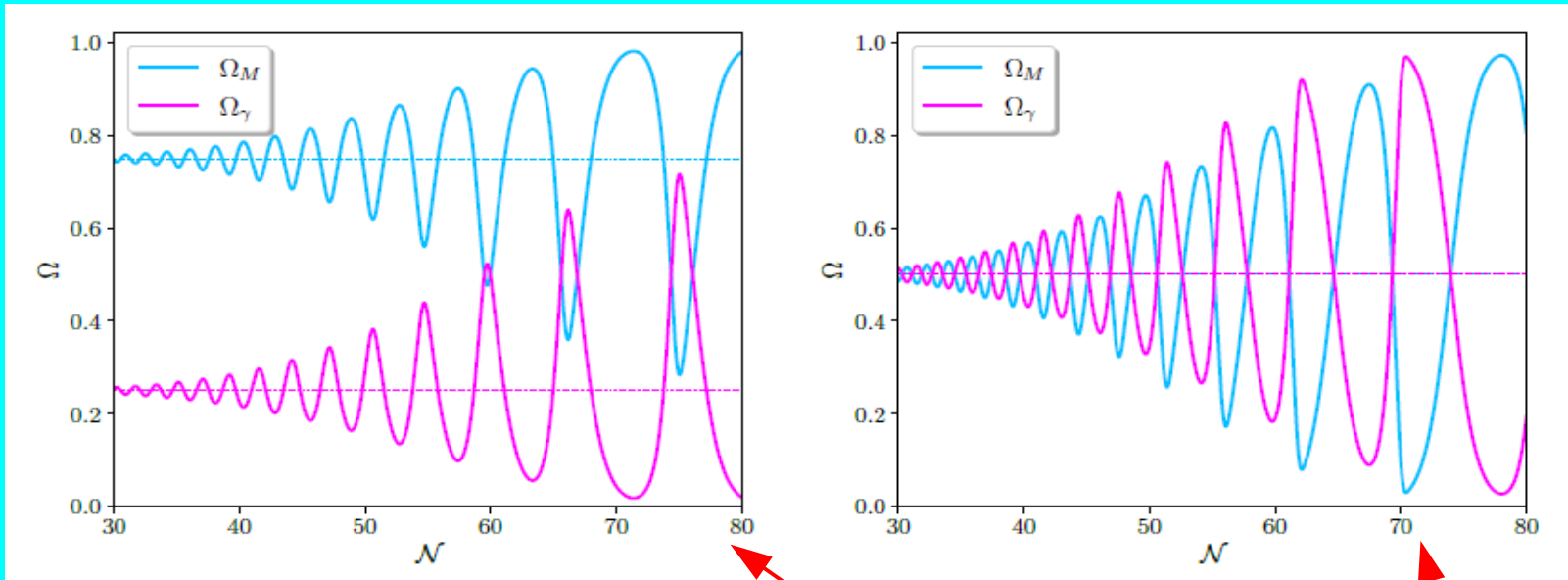
This then results in an “oscillatory” stasis!

This “discretization” effect is normally seen near the end of stasis, but can actually persist over many e -folds for large Δm .

It’s still a form of stasis because the abundance oscillations occur around *fixed* central values!

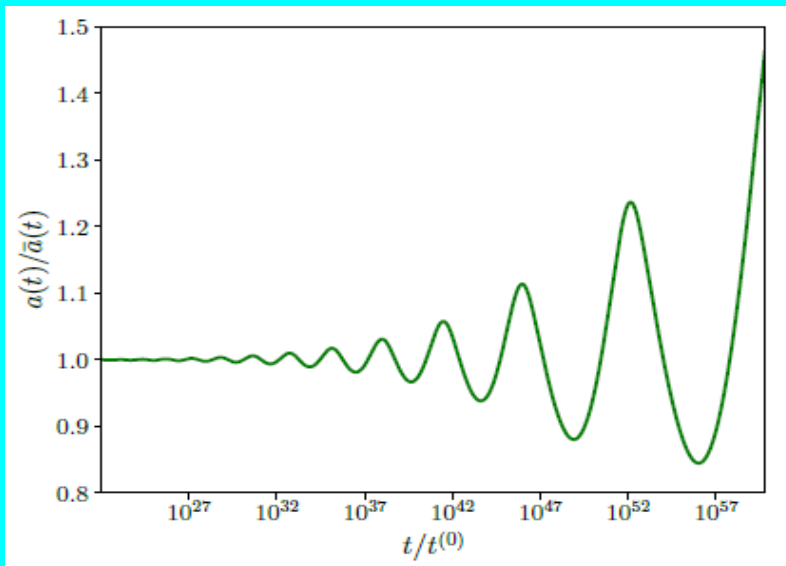
As such, the time-average of oscillatory stasis is ordinary stasis!

e.g., oscillatory $M\gamma$ stasis



Stasis abundances can be well separated

But when they are relatively close to each other, the abundances are “braided”.



The resulting universe continues to expand *but also vibrates or “breathes” as it expands!*

None of these behaviors are possible in traditional Λ CDM cosmologies!

3

Stasis unrealized

There is also another unique possibility.

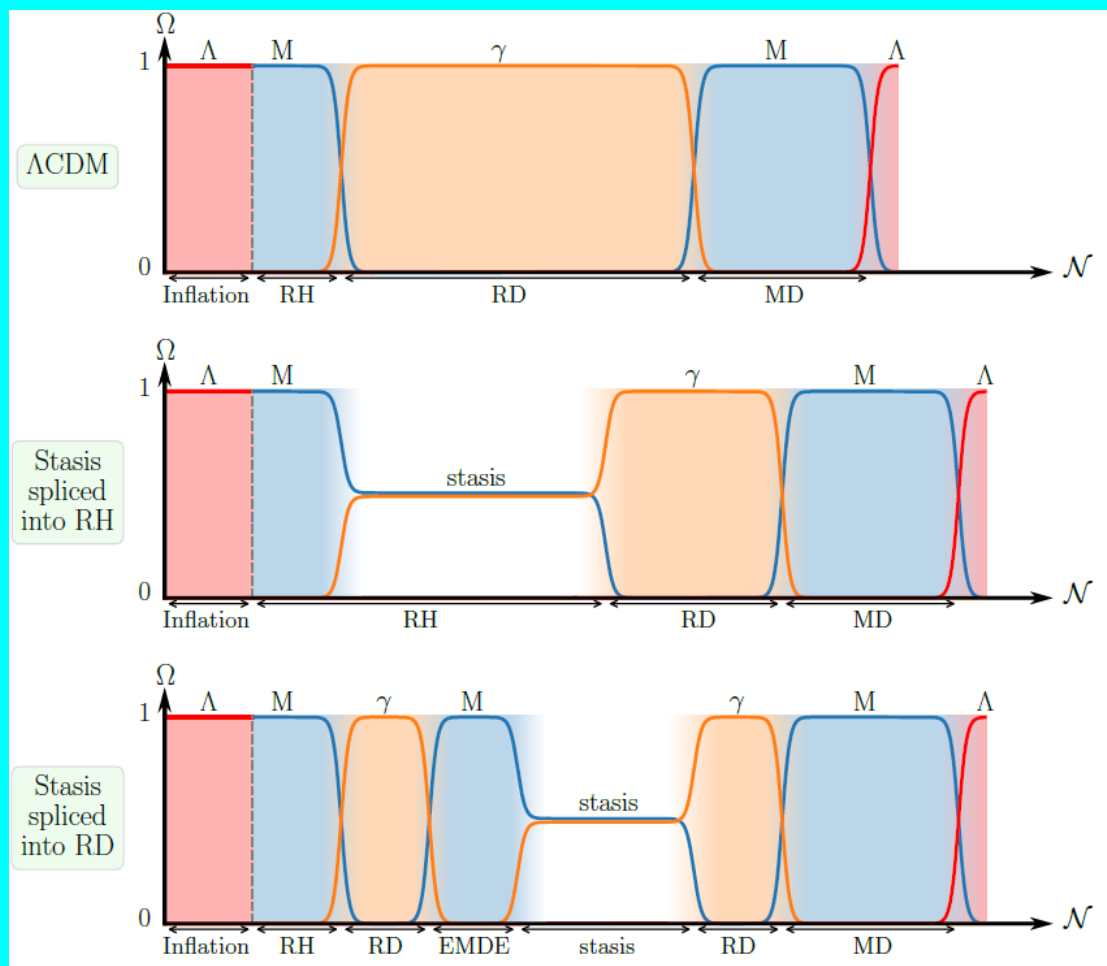
Depending on the underlying model parameters, it may happen that our system begins heading toward a stasis configuration as the result of the attractor behavior, *but never fully reaches this destination because our level-by-level transitions reach the bottom of the tower before the stasis is fully realized.*

This can then result in abundances whose time-evolution begins to slow over many e -folds (as appropriate for the approach to stasis), *but then grows again* as another post-stasis dynamics comes into play.

This phenomenon may also have important phenomenological implications, and represents one of the few ways in which a pairwise stasis may ultimately be avoided.

Conclusions

Stasis is nothing less than a new kind of cosmological epoch!
The implications are likely to be profound...



The existence of a stasis epoch within BSM cosmologies is likely to give rise to a host of new theoretical possibilities across the entire cosmological timeline, ranging from potential implications for primordial density perturbations, dark-matter production, and structure formation all the way to early reheating, early matter-dominated eras, and even the age of the universe. There may even be new alternatives to traditional inflation scenarios as well as reheating. The stasis phenomenon may also involve additional energy components such as kination, cosmic strings, and domain walls. **BSM cosmologies may therefore be much richer than previously imagined.**

We are only at the tip of the iceberg! Lots of implications to be explored!

Stasis epoch can find itself “spliced” into various points during the Λ CDM history.

Work in progress now...

- Can stasis be extended into the *thermal* domain, with *temperature* becoming part of the stasis phenomenon? (with J. Barber)
- How do density perturbations evolve during a stasis epoch? What kinds of new signatures are possible? (with D. Hoover and A. Paulsen)
- Is there an algebraic superstructure underlying stasis? Are all of the different kinds of stasis different manifestations of a single stasis phenomenon? (with J. Barber)
- What happens for a tower of actual realistic dynamical scalars for which the equation of motion is not a constant? Can one obtain stasis in such a situation? What new phenomenological possibilities might flow from this?

See Brooks' talk,
next!