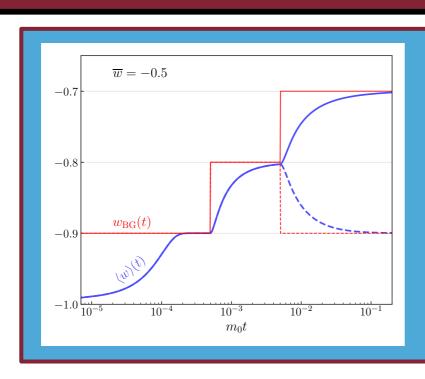
# Cosmological Stasis from Dynamical Scalars



#### **Brooks Thomas**

LAFAYETTE COLLEGE



#### Based on work done in collaboration with:

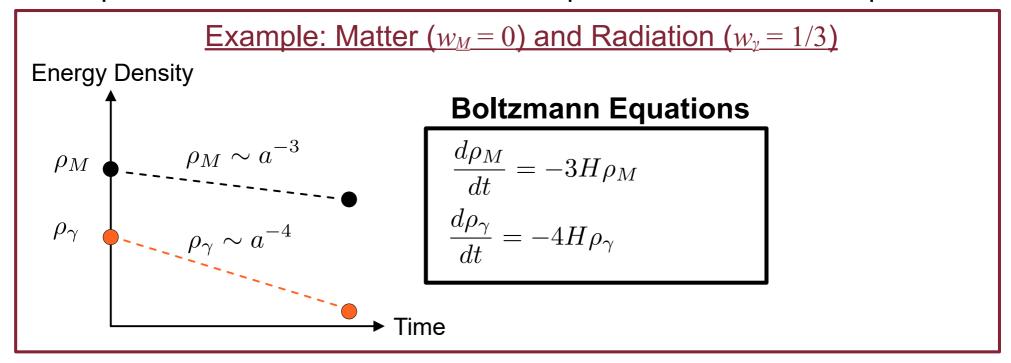
Keith R. Dienes, Fei Huang, Lucien Heurtier, and Timothy M. P. Tait [arXiv:2405.xxxxx]

Mitchell Conference, Texas A&M University, May 24th, 2024

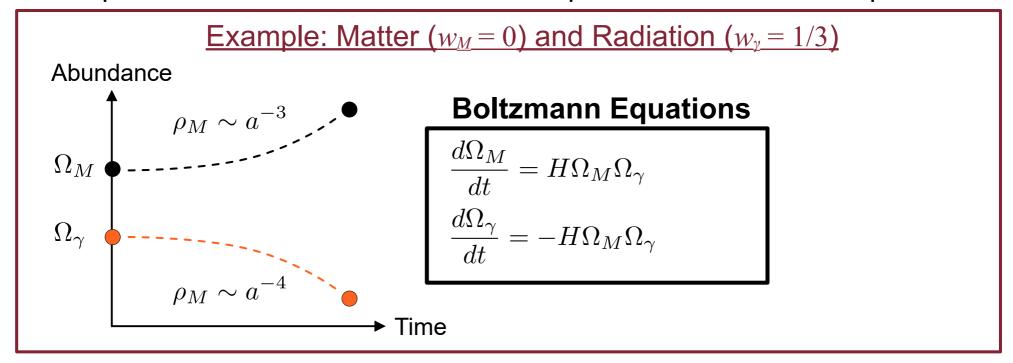
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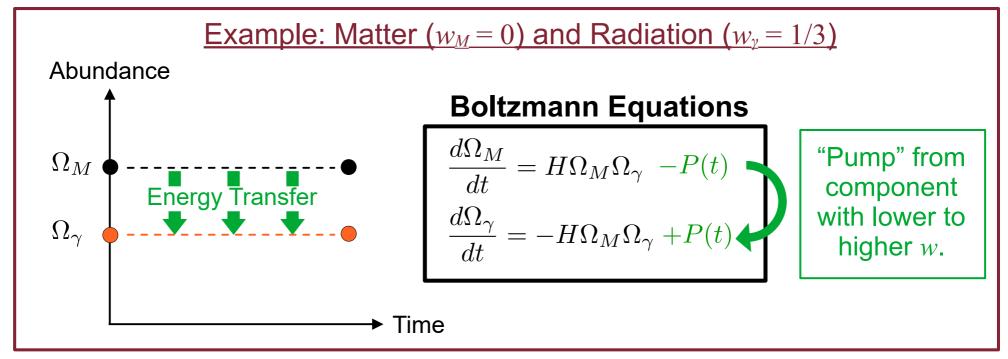
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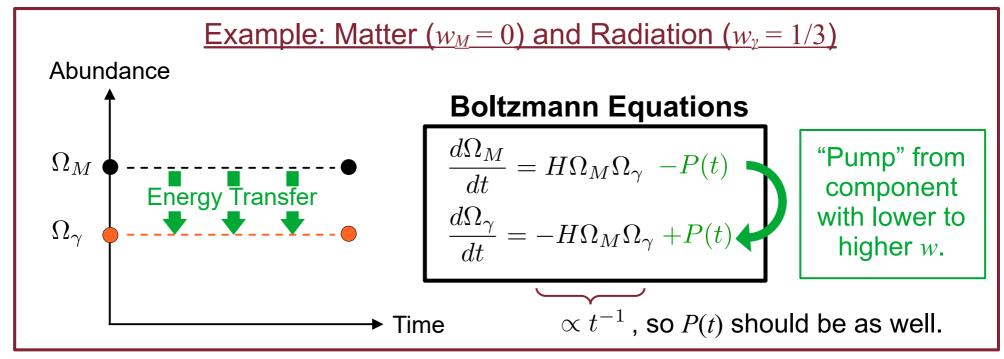
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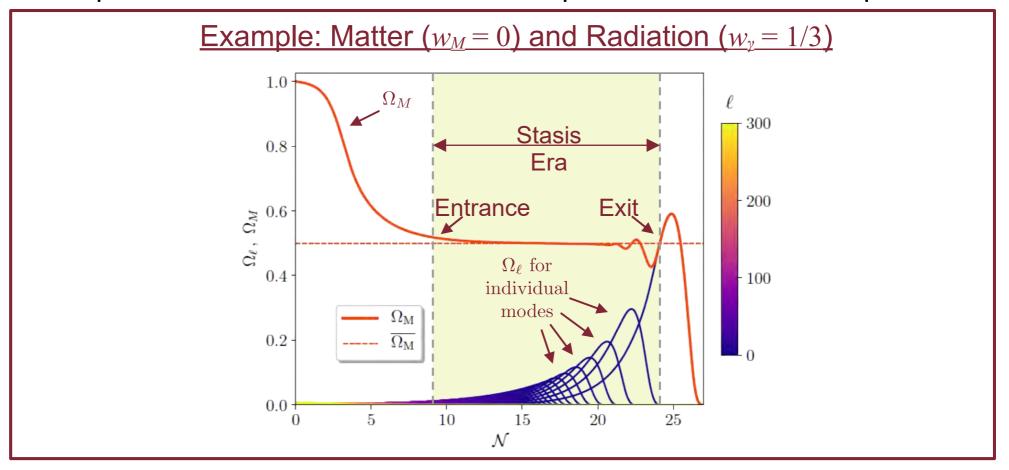
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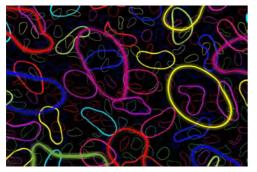


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• Pump terms with the right time-dependence for stasis emerge naturally in scenarios involving <u>towers of states</u> with broad spectra of masses, cosmological abundances, lifetimes, etc.

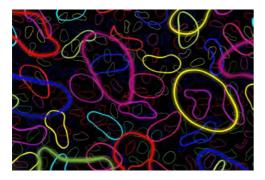
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- Such towers are a facet of numerous BSM-physics scenarios including...
  - String theory (string moduli, axions, etc.)
  - Theories with extra spacetime dimensions (KK towers)
  - Scenarios which lead to the production of primordial black holes with an extended mass spectrum (the black holes themselves)







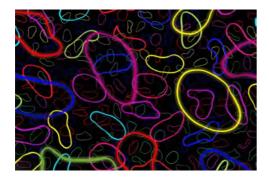
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- When they do emerge, stasis is typically a **global attractor**: the universe will evolve toward stasis regardless of initial conditions.



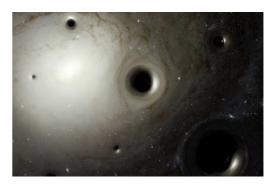




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- When they do emerge, stasis is typically a **global attractor**: the universe will evolve toward stasis regardless of initial conditions.
- The modified cosmological histories associated with stasis can affect the evolution of <u>scalar and tensor perturbations</u>.







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• Such a stasis, as we'll see, would be characterized by an effective equation-of-state parameter between that of vacuum energy  $(w_{\Lambda} = -1)$  and matter  $(w_{\rm M} = 0)$ .

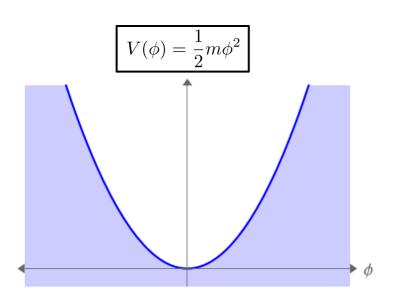
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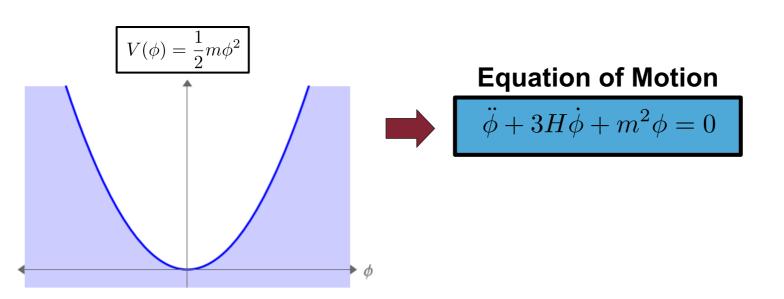
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- Such a stasis, as we'll see, would be characterized by an effective equation-of-state parameter between that of vacuum energy  $(w_{\Lambda} = -1)$  and matter  $(w_{\rm M} = 0)$ .
- Moreover, stases involving dynamical scalars give rise to some
   <u>phenomena not seen in other realizations of stasis</u> which could potentially useful for addressing fundamenal questions in cosmology.

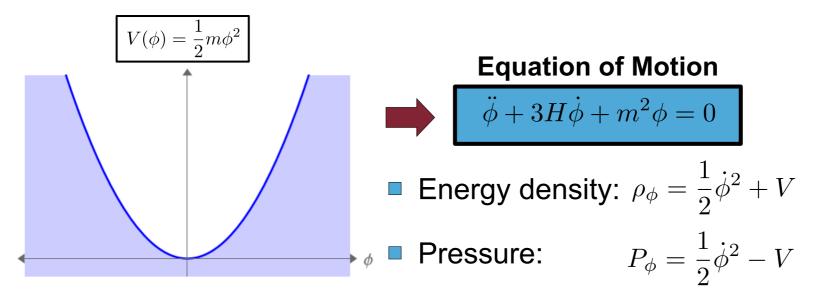
• To set the stage, let's recall how the homogeneous zero-mode of a **single scalar field**  $\phi$  of mass m with a quadratic potential  $V(\phi)$  evolves in a flat FRW universe.



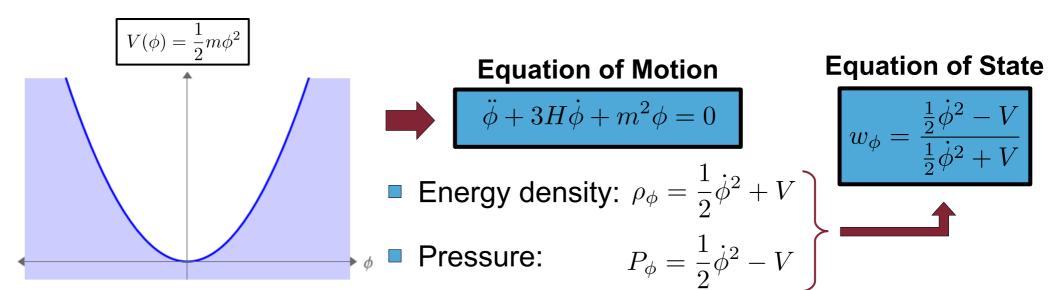
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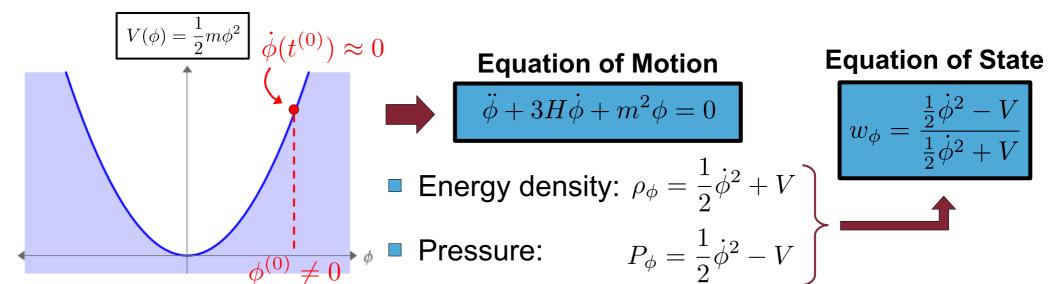
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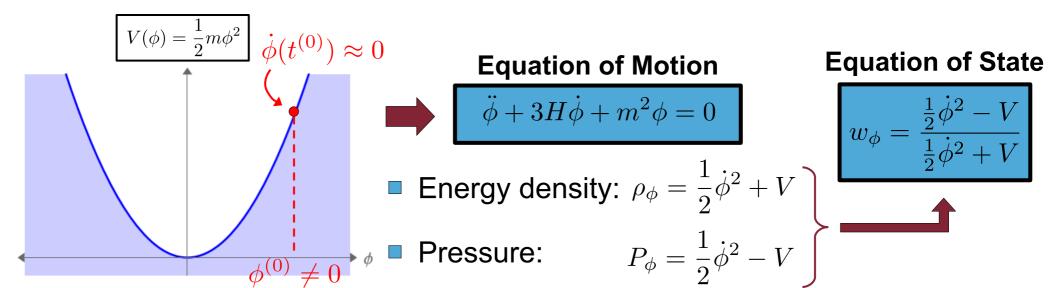
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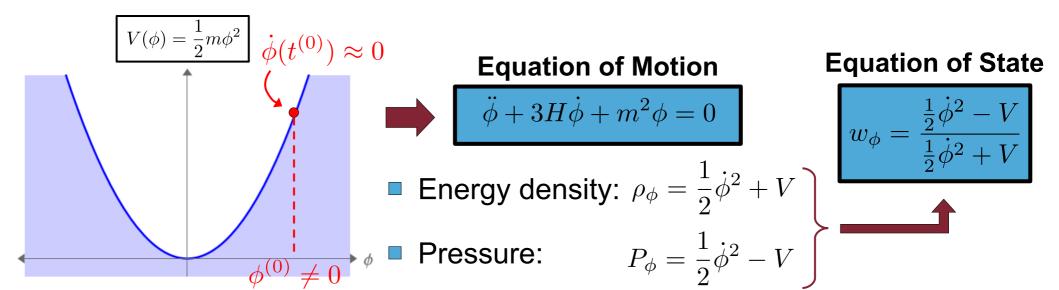
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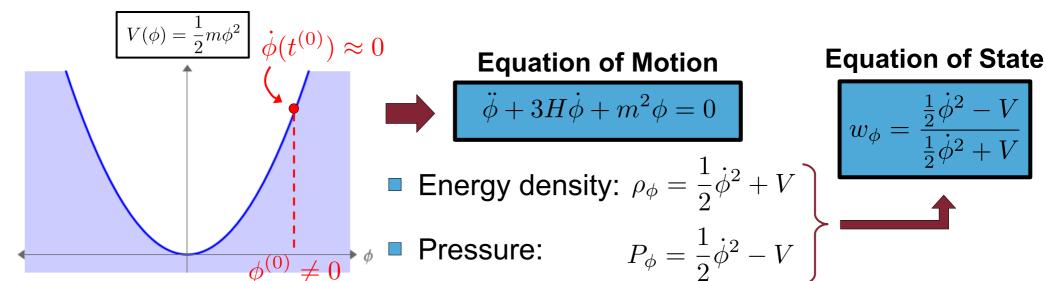
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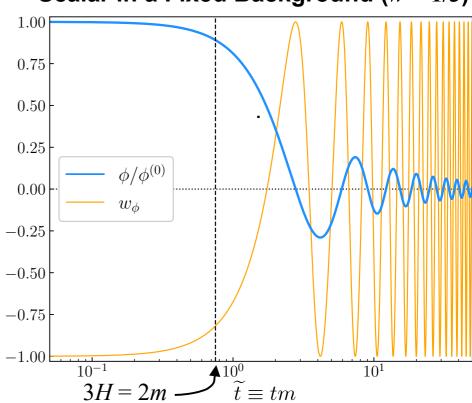
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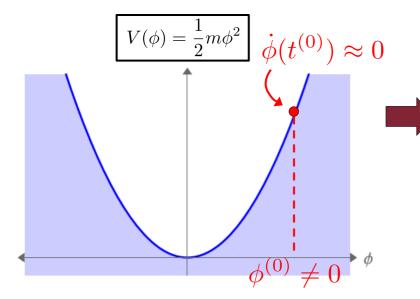


 $Hpprox rac{\kappa}{3t}$  , where  $\kappa \equiv rac{2}{1+w}$ 



#### Scalar in a Fixed Background (w = 1/3)





#### **Equation of Motion**

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0$$

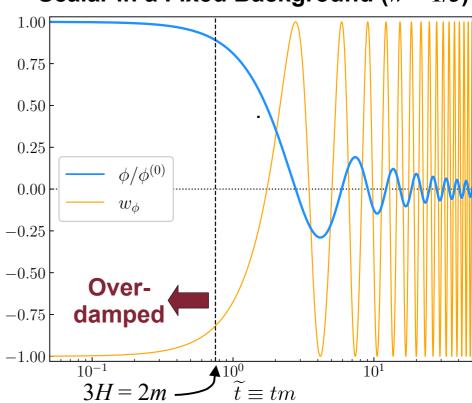
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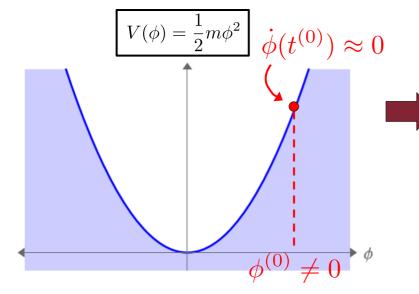
$$\phi(\tilde{t}) \approx c_J \, \tilde{t}^{(1-\kappa)/2} J_{(\kappa-1)/2}(\tilde{t})$$

$$w_{\phi} = \frac{\frac{1}{2}\dot{\phi}^2 - V}{\frac{1}{2}\dot{\phi}^2 + V}$$

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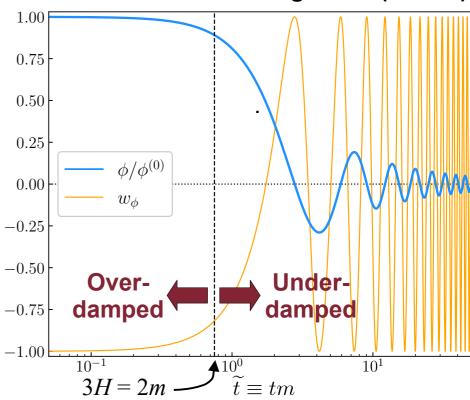
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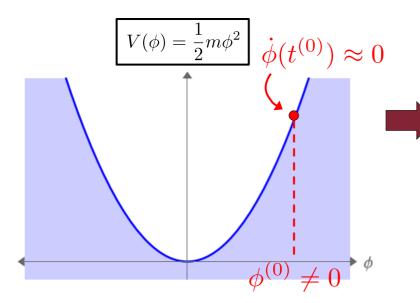
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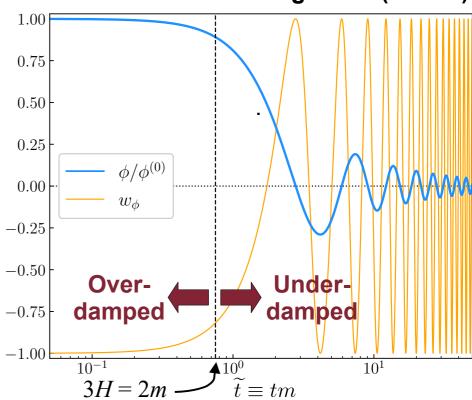
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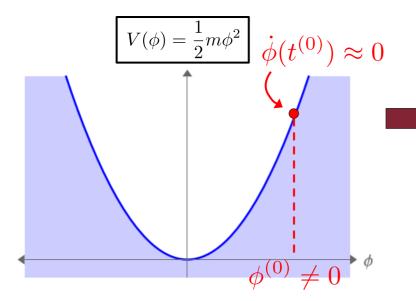
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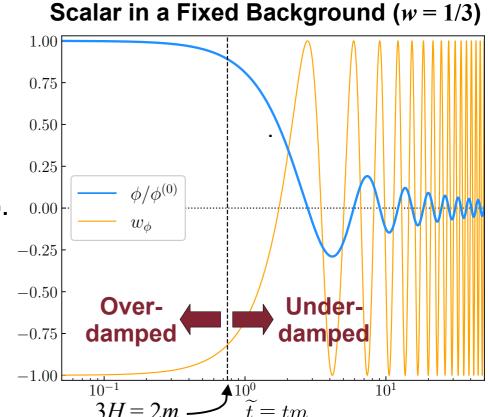
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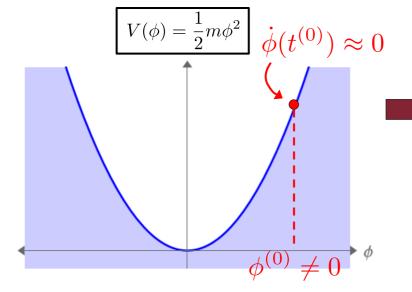
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  - **➡** Behaves like massive matter





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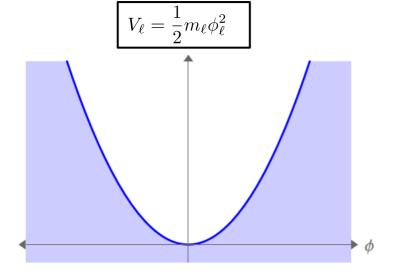
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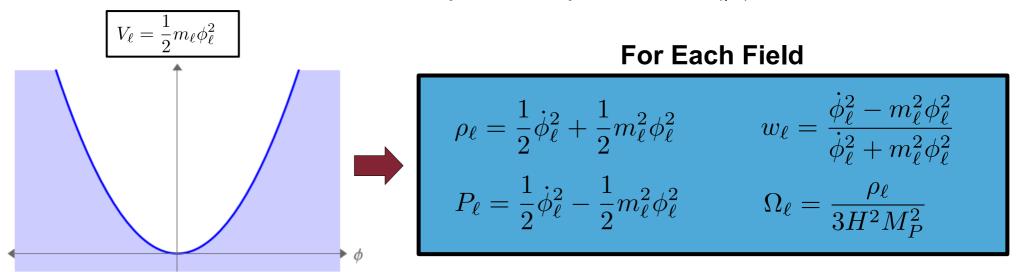
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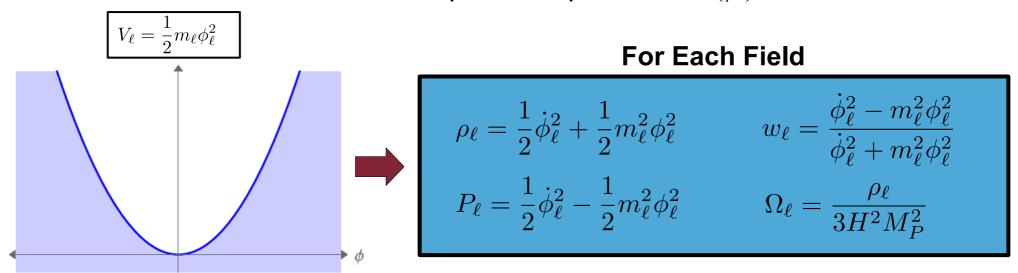
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• We'll also assume (for the moment) that there's **no background energy component**: the collective energy density of the  $\phi_{\ell}$  dominates the universe.

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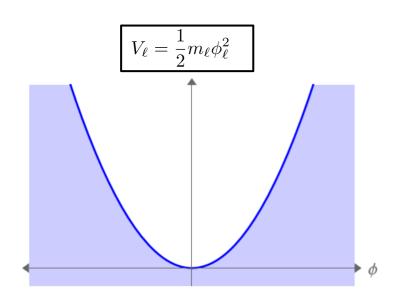
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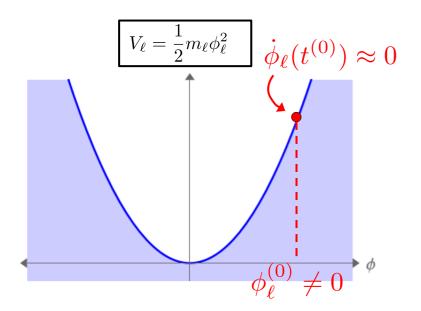
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Let's see what the cosmology of such a tower of scalar-field zero modes looks like!

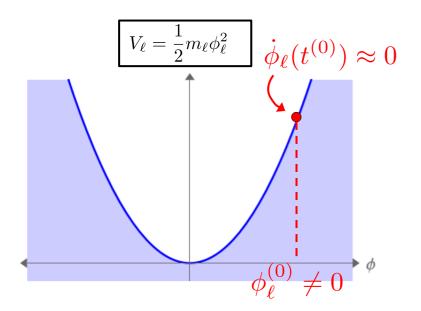
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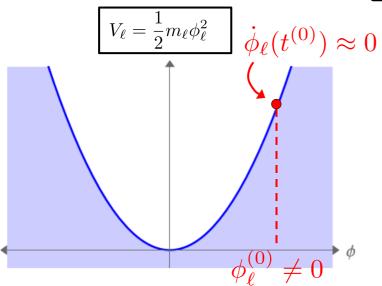


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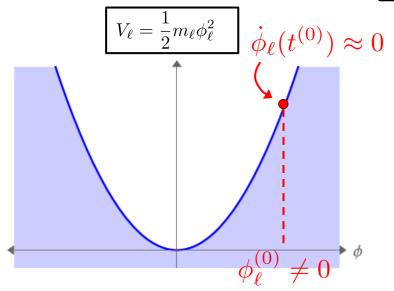
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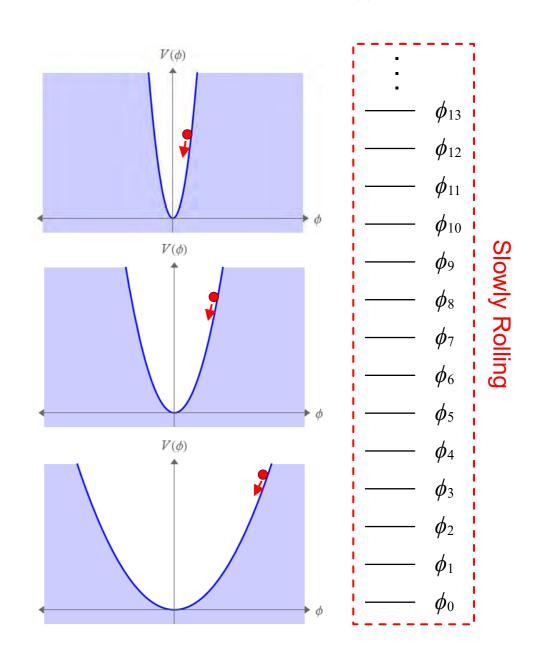
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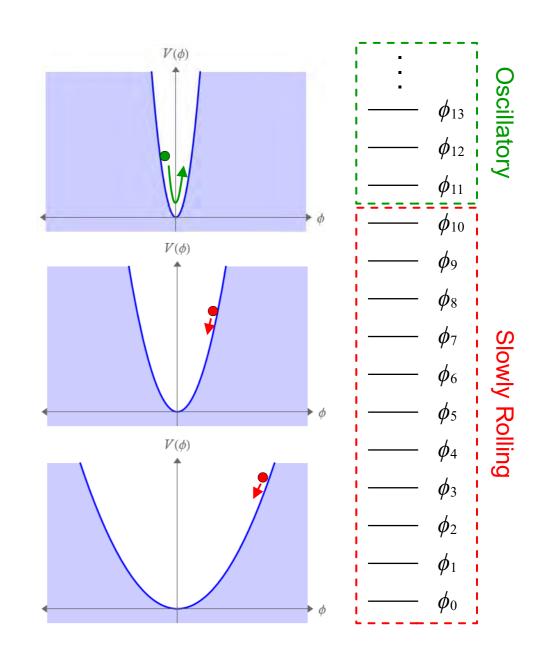
• For a given mass spectrum, the overall scale of the abundances can be parameterized by the ratio  $\phi_0^{(0)}/M_P$ , or, equivalently, by the ratio  $H^{(0)}/m_{N-1}$ .

• Each  $\phi_{\ell}$  transitions to the underdamped phase when when  $3H(t) = 2m_{\ell}$ .

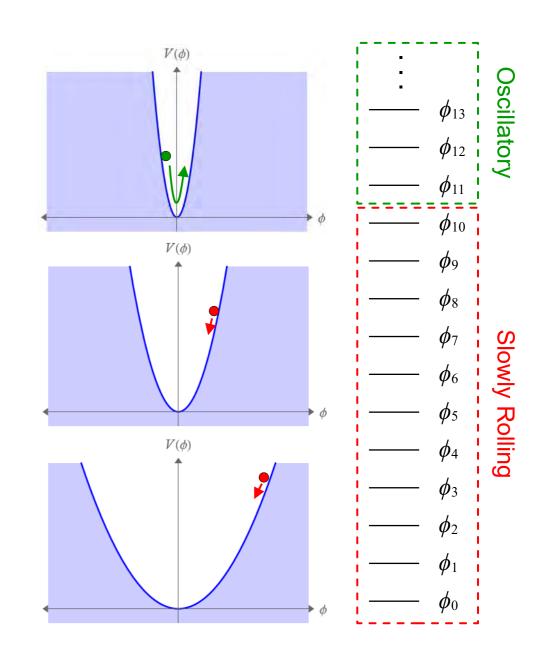
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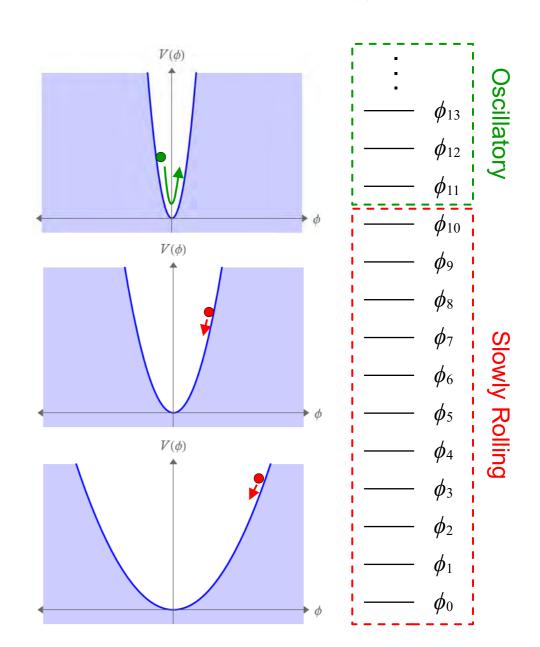
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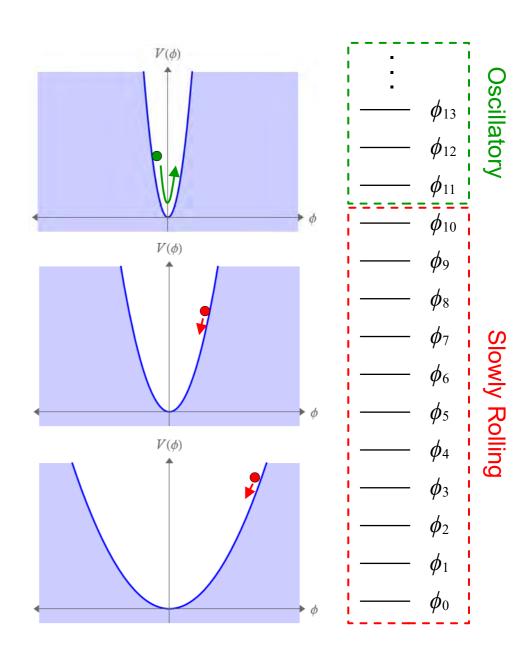


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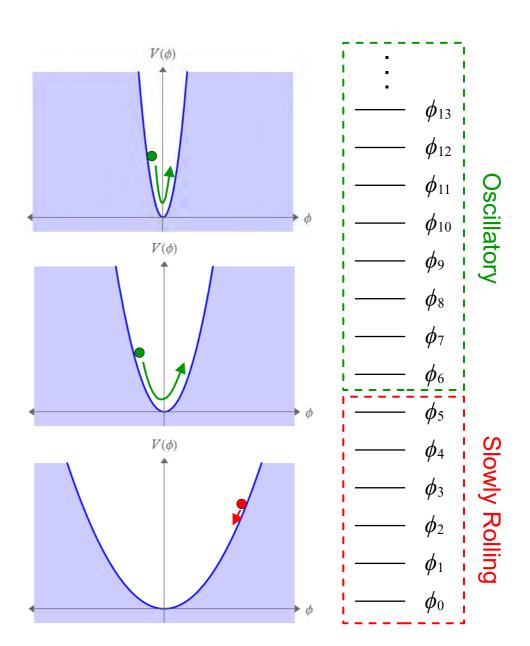


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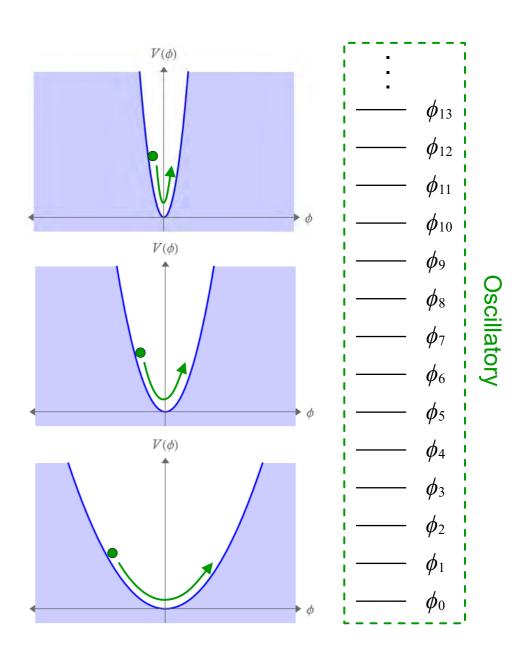


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# **Oscillatory**

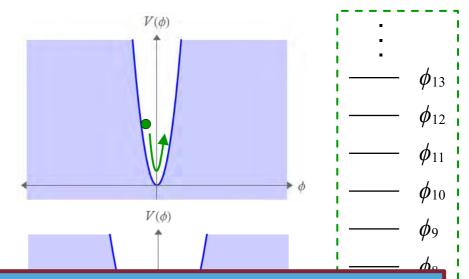
### **Dynamical Evolution**

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#### The Question:

Can we achieve a stasis
between these slow-roll and
oscillatory cosmological
energy components, which act
like vacuum energy and
matter, respectively?

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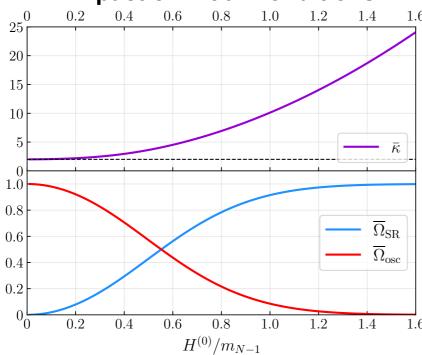
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Towers which satisfy this relation give rise to stasis. For  $\delta = 1$ , this corresponds to

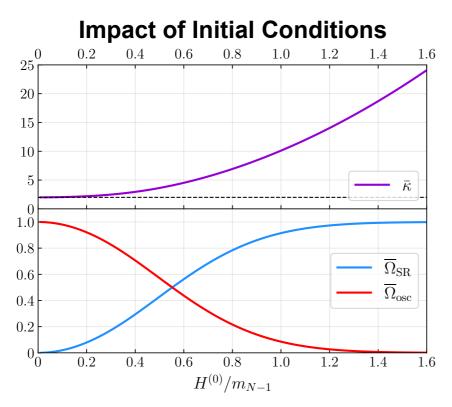
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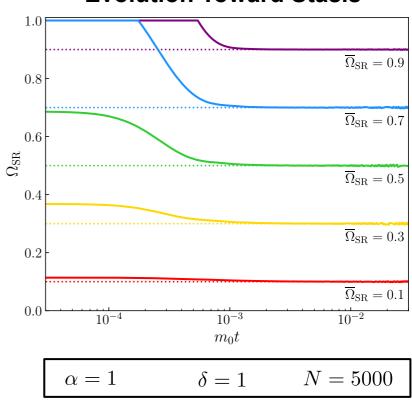
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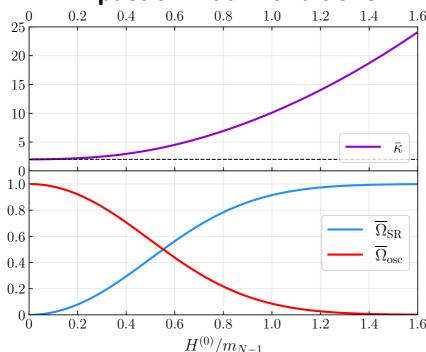


#### **Evolution Toward Stasis**



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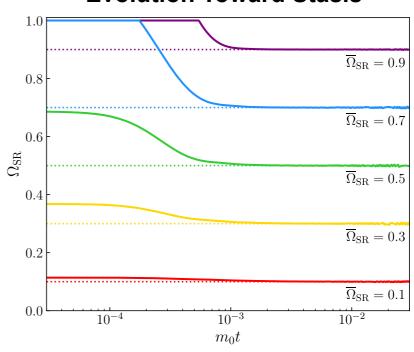
#### Impact of Initial Conditions



#### **Duration of Stasis**

## $\mathcal{N}_s \approx \frac{\overline{\kappa}}{3} \left[ \delta \log N + \log \left( \frac{\Delta m}{m_0} \right) + \log \left( \frac{3H^{(0)}}{2m_{N-1}} \right) \right]$

#### **Evolution Toward Stasis**

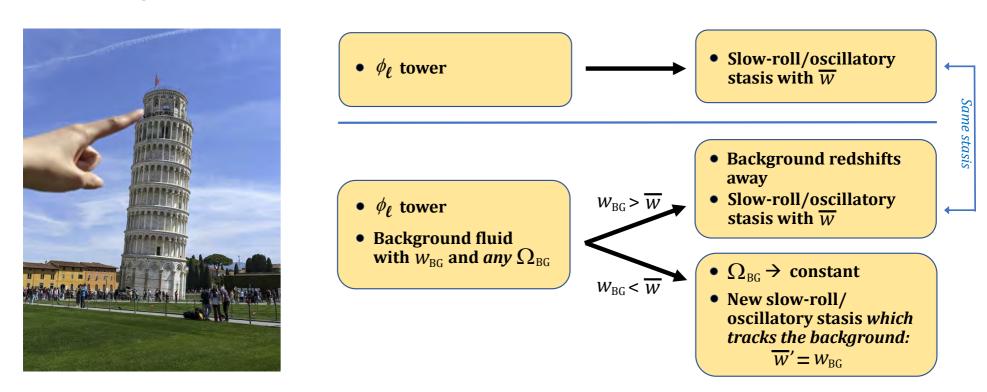


$$\alpha = 1$$
  $\delta = 1$   $N = 5000$ 

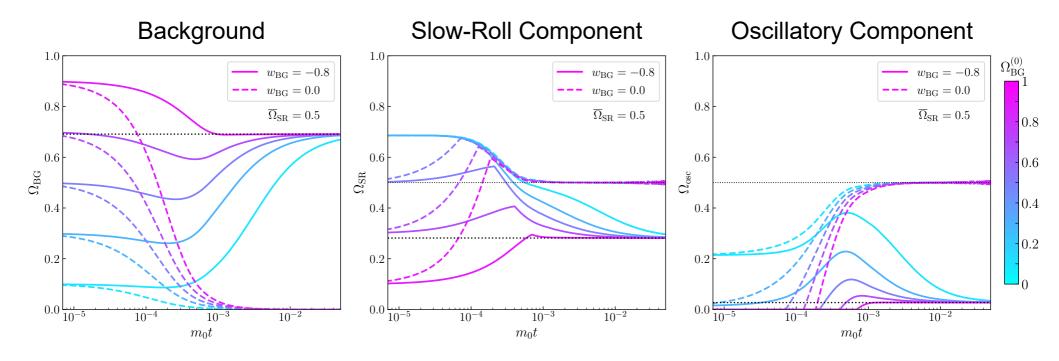
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- It turns out that in the presence of such an energy component, the universe still evolves toward stasis (or something like it).
- However, the outcome depends on the relationship between  $w_{BG}$  and the equation-of-state parameter  $\overline{w}$  the tower would have had during stasis if the background component weren't present.

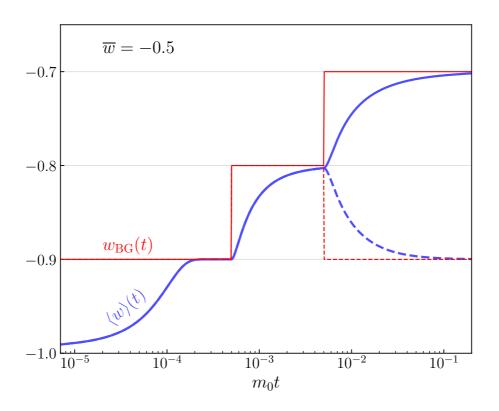


 The <u>tracking phenomenon</u> which arises in wBG < w has not been observed in other realizations of stasis.

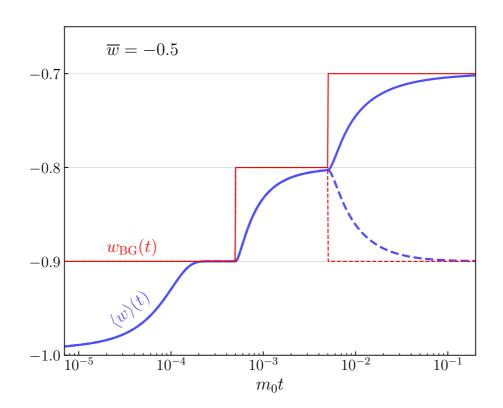


• These results provide insight about how the universe might **enter into** – or exit from – an stasis epoch involving dynamical scalars.

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- Indeed, as long as  $w_{BG}$  remains below  $\overline{w}$ , the tower's equation-of-state parameter  $\langle w \rangle$  continues to evolve toward the new value of  $w_{BG}$  after the shift, regardless of whether this shift is positive or negative.



#### Stasis-Induced Inflation?

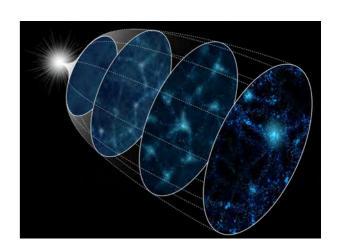
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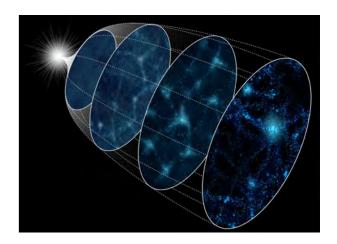


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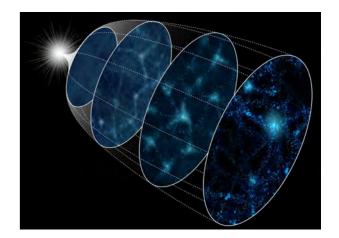
Can such a stasis furnish a framework for cosmic inflation?

This is an intriguing possibility for a number of reasons.



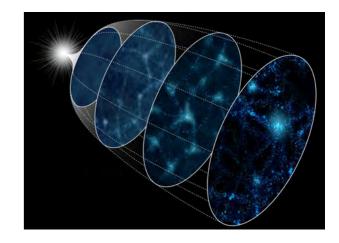
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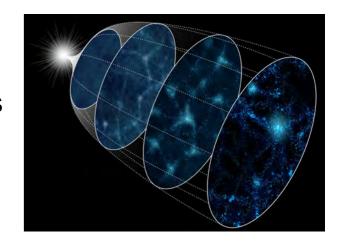
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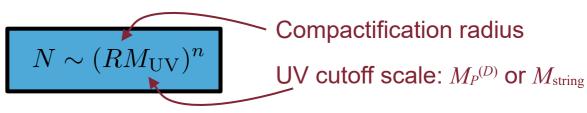
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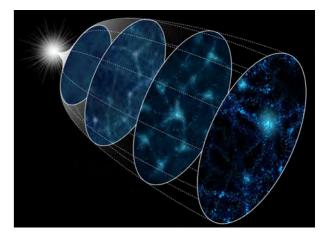
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- A "graceful exit" from inflation is built into this scenario. It ends with the  $\phi_{\ell}$  behaving like massive matter. Reheating can be achieved in principle via their subsequent decays.



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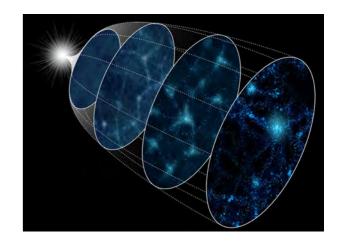
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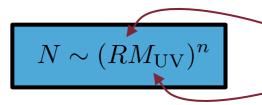
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The abundance of matter (i.e.,  $\Omega_{\rm osc}$ ) remains non-zero throughout the inflationary epoch (and yet nevertheless unwanted relics are inflated away).



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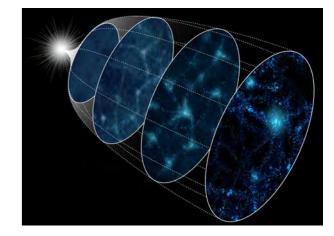
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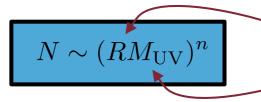
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  - ...give rise to a (nearly scale-invariant) spectrum of density perturbations consistent with CMB data, etc.



The number of e-folds of inflation is often related in a deep way to <u>hierarchies among fundamental scales</u> – e.g., in KK theories...

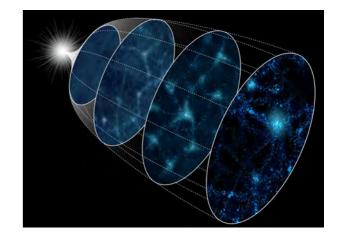
 $\mathcal{N}_e \sim \log N$  , where



Compactification radius

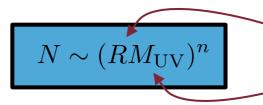
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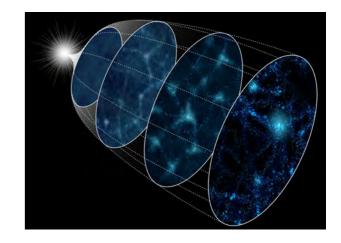
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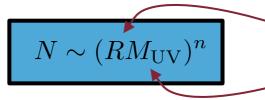
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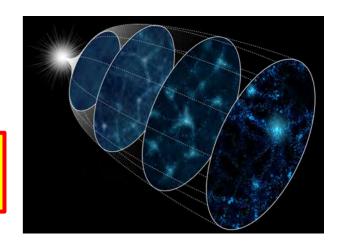


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This is an intriguing possibility – and one that warrants further exploration!



# **Summary**

- <u>Stable, mixed-component cosmological eras</u> i.e. <u>stasis eras</u> are indeed a viable cosmological possibility and one that can arise naturally in many extensions of the Standard Model.
- A tower of scalar fields which undergo a transition from overdamped to underdamped evolution can give rise to stasis.
- Stasis itself is an <u>attractor</u> in these systems, but several fundamental characteristics of the stasis epoch toward which the universe evolves depend on the initial conditions.
- In the presence of an additional background component with equation-of-state parameter  $w_{BG}$ , the tower exhibits a <u>tracking behavior</u> in which its own equation-of-state parameter evolves toward  $w_{BG}$ .

