Addressing the Axion Quality Problem

K.S. Babu

Oklahoma State University

Mitchell Conference on CD&NP
Texas A&M University, College Station
May 23-26, 2024

Credits

Talk based on work done in collaboration with:
Bhaskar Dutta and Rabi Mohapatra (paper forthcoming)

Discussions started at Mitchell Conference 2023

Supported by the US Department of Energy

The Strong CP Problem

- Strong interactions appear to conserve Parity (P) and Time Reversal (T) symmetries, and therefore also CP symmetry. However, QCD Lagrangian admits a source of P and T violation:

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \theta_{QCD} \frac{g_s^2}{32\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu} + \overline{q} \left(i\gamma^\mu D_\mu - m_q e^{i\theta_q \gamma_5} \right) q$$

- A chiral rotation on the quark field, $q \to e^{i\alpha\gamma_5/2} q$, can remove the phase of the quark mass as $\theta_q \to \theta_q - \alpha$. Due to the anomalous nature of this rotation, θ_{QCD} also changes to $\theta_{QCD} \to \theta_{QCD} + \alpha$

- The parameter

$$\overline{\theta} = \theta_{QCD} + \theta_q$$

  is invariant, and is physical

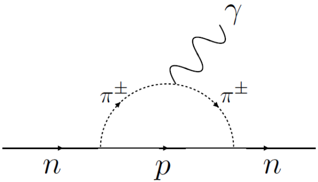
- With multiple flavors of quarks, the invariant physical parameter is

$$\overline{\theta} = \theta_{QCD} + \mathrm{ArgDet}(M_Q)$$

- $\overline{\theta}$ contributes to neutron Electric dipole moment (EDM)

Neutron EDM from $\overline{\theta}$

- In presence of $\overline{\theta}$ neutron will develop an EDM:



$$d_n \simeq \frac{e\,\overline{\theta}\,g_A\,c_+\,\mu}{8\pi^2 f_\pi^2}\mathrm{Log}\left(\frac{\Lambda^2}{m_\pi^2}\right) \simeq 3\times 10^{-16}\ \overline{\theta}\,\mathrm{e\ cm}$$

Here $\mu = \frac{m_u m_d}{m_u+m_d}$, $g_A \simeq 1.27$, $c_+ \simeq 1.6$, $\Lambda = 4\pi f_\pi$

- From $d_n < 1.8\times 10^{-26}$ e cm, one obtains $\Rightarrow \overline{\theta} < 10^{-10}$
- A lack of understanding of the extreme smallness of $\overline{\theta}$ is the strong CP problem
- Setting $\overline{\theta}$ to zero is unnatural, since weak interactions require $\mathcal{O}(1)$ CP violation in that sector

Axion Solution to Strong CP Problem

- The Peccei-Quinn (PQ) mechanism, which leads to a light pseudoscalar particle, the axion (a), is an elegant solution to the strong CP problem

- The PQ mechanism assumes a global $U(1)_{PQ}$ symmetry that has a QCD anomaly. This $U(1)$ is spontaneously broken by a Higgs scalar, and also explictly by the QCD anomaly term

- Axion is the pseudo-Goldstone boson associated with the $U(1)_{PQ}$ symmetry breaking. The QCD anomaly induces a coupling of axion to the gluon field so that

$$\mathcal{L} = \frac{g_s^2}{32\pi^2}\left(\overline{\theta} + \frac{a}{f_a}\right) G_{\mu\nu}\tilde{G}^{\mu\nu}$$

- QCD also induces a potential for the axion field, which upon minimization dynamically sets $\overline{\theta} + \frac{a}{f_a} = 0$

Axion and the PQ Mechanism

- The Lagrangian for a PQ symmetric theory contains the terms

$$\mathcal{L} = \left(\frac{g_s^2}{32\pi^2}\right)\frac{a}{f_a}G_{\mu\nu}\tilde{G}^{\mu\nu} - \lambda(|\Phi|^2 - f_a^2)^2 - \Lambda^4\cos\left(\frac{Na}{f_a}\right),$$

- Note that $\overline{\theta}$ has been absorbed into the dynamical axion field a which is the phase of Φ: $\Phi = \frac{(\rho+f_a)}{\sqrt{2}}e^{i\frac{a}{f_a}}$
- Here Λ is the QCD scale, N is an integer ≥ 1 related to the QCD anomaly coefficient, and f_a is the axion decay constant.
- Minimizing the potentail leads to $a = 0$, which thus solves the strong CP problem

Axion Quality Problem

- Quantum gravity is expected to break all global symmetries, including $U(1)_{PQ}$. This gives rise to the axion quality problem
- For example, a gravity-induced term in the Higgs potential,

$$V_{gravity} = \frac{\kappa}{M_{\rm Pl}} |\Phi|^4 (e^{i\delta}\Phi + h.c.)$$

  would shift the vacuum value $a = 0$
- Minimizing the potential in presence of the quantum gravity correction one has

$$\bar{\theta} \simeq \frac{\kappa \sin\delta}{2\sqrt{2}} \frac{f_a^5}{\Lambda^4 M_{\rm Pl} N^2}$$

- For currently favored values of $f_a = (10^9 - 10^{12})$ GeV, with $\Lambda \simeq 200$ MeV and $M_{\rm Pl} = 1.22 \times 10^{19}$ GeV, one finds the limits

$$\kappa \sin\delta \leq \{10^{-38} - 10^{-53}\}$$

- This is rather severe, much worse than the strong CP problem itself!
  Holman et. al. (1992); Kamionkowski, March-Russell (1992); Barr, Seckel (1992); Ghigna, Lusignoli, Roncadelli (1992)

Attempts to Solve the Quality Problem

- Attempts to solve the axion quality problem have used gauge symmetries with an accidental global $U(1)$, composite axion, and discrete gauge symmetries

  Barr, Seckel (1992); Randall (1992); Babu, Gogoladze, Wang (2003),...

- Realizing accidental PQ symmetry from a gauge symmetry is nontrivial, since PQ symmetry should have a QCD anomaly, but the original gauge symmetry has no anomaly

- In the rest of the talk I shall present our attempts to construct such models and discuss briefly phenomenology of successful models

Accidental PQ From Gauged U(1)

- Standard Model is extended with a gauged $U(1)_a$. All SM particles are neutral under this $U(1)_a$. Vectorlike fermions are added which carry axial $U(1)_a$ charges:

New fermion	$SU(3)_c \times SU(2)_L \times U(1)_Y$	$U(1)_a$ **charge**
$(Q_L)_{1,2,3}$	$(3, 1, y)$	$(1, 1, -2)$
$(Q_R)_{1,2,3}$	$(3, 1, y)$	$-(1, 1, -2)$
$(N_R)_{1,2,3}$	$(1, 1, 0)$	$(1, 3, -4)$
New scalar	$SU(3)_c \times SU(2)_L \times U(1)_Y$	$U(1)_a$ **charge**
T	$(1, 1, 0)$	4
S	$(1, 1, 0)$	$2/n$

- All anomalies cancel, including $[U(1)_a]^3$ via the singlet fermions
- Axial nature of $U(1)_a$ charges implies that $U(1)_Y \times [U(1)_a]^2$ anomaly cancels for any hypercharge y
- n is an integer with $n \geq 5$. This choice guarantees quality of the axion

High Quality Axion

- Model has two global $U(1)$s, one acting on (Q_3, T) and another acting on $(Q_{1,2}, S)$. Each has a QCD anomaly
- Thus there are two Goldstone bosons, one eaten up by the $U(1)_a$ gauge boson, with the other identified as the axion
- Axion is orthogonal to Goldstone:

$$G = \frac{\left(\frac{2}{n} f_S \eta_S + 4 f_T \eta_T\right)}{\sqrt{(4f_T)^2 + (\frac{2}{n} f_S)^2}}, \quad a = \frac{\left(4 f_T \eta_S - \frac{2}{n} f_S \eta_T\right)}{\sqrt{(4f_T)^2 + (\frac{2}{n} f_S)^2}}$$

Here $S = \frac{(\rho_S + f_S)}{\sqrt{2}} e^{i\eta_S/f_S}$ and $T = \frac{(\rho_T + f_T)}{\sqrt{2}} e^{i\eta_T/f_T}$ are used

- Yukawa interactions of S and T generate vectorlike quark masses:

$$-\mathcal{L}_{\rm Yuk} = Y_3\, \overline{Q}_{3L} Q_{3R}\, T^* + Y_{1,2}\, \overline{Q}_{1,2,L} Q_{1,2,R}\, \frac{S^n}{(M_*)^{(n-1)}} + h.c.$$

- Axion decay constant f_a can be worked out to be

$$f_a = \frac{f_T f_S}{\sqrt{4 f_T^2 n^2 + f_S^2}}$$

High Quality Axion - cont.

- This model is a KSVZ type axion model, but with high quality axion
- The leading quantum gravity correction that can destabilize the axion potential is

$$V_{\mathrm{gravity}} = \frac{\kappa e^{i\delta} S^{2n} T^*}{M_{\mathrm{Pl}}^{2n-3}} + h.c.$$

- The induced $\overline{\theta}$ is

$$\overline{\theta} \simeq \frac{\kappa \sin\delta}{(\sqrt{2})^{(2n-1)}} \frac{f_S^{2n} f_T}{\Lambda^4 M_{\mathrm{Pl}}^{2n-3}}$$

- For $f_S = f_T = f_a\sqrt{1+4n^2}$, and with $n = 5$, $f_a = 10^{10}$ GeV, one has $\overline{\theta} \sim 7 \times 10^{-12}$. If $f_a = 10^{12}$ GeV is used, $n = 7$ gives $\overline{\theta} \sim 10^{-12}$, showing the high quality, all for $\kappa \sin\delta = 1$
- Since y of vectorlike quarks is free, one can choose $y = 2/3$ or $y = -1/3$ in which case these quarks mix with the usual quarks and there won't be stable heavy quarks

Generalization to a Family of Models

- A family of models can be found as extensions of the model. $m+1$ vectorlike quarks are used:

New fermion	$SU(3)_c \times SU(2)_L \times U(1)_Y$	$U(1)_a$ **charge**
$(Q_L)_{1,2,3,\ldots,m,m+1}$	$(3,1,y)$	$(1,1,1,..1,-m)$
$(Q_R)_{1,2,3,\ldots,m,m+1}$	$(3,1,y)$	$-(1,1,1,...1,-m)$
$(N_R)_{1,2,3}$	$(1,1,0)$	$(-1+m,1+m,-2m)$
New scalar	$SU(3)_c \times SU(2)_L \times U(1)_Y$	$U(1)_a$ **charge**
T	$(1,1,0)$	$2m$
S	$(1,1,0)$	$2/n$

- All anomalies cancel, with the previous model being $m=2$ case
- Axion field is given as

$$a = \frac{\left(2mf_T\eta_S - \frac{2}{n}f_S\eta_T\right)}{\sqrt{(2mf_T)^2 + (\frac{2}{n}f_S)^2}}, \quad f_a = \frac{f_T f_S}{\sqrt{f_T^2 m^2 n^2 + f_S^2}}$$

- This leads to a high quality KSVZ type axion. For $m \geq 9$, $n=1$ can be chosen, whence all quarks acquire mass at renormalizable level
- Couplings of axion to nucleon and electron are same as KSVZ

Domain Wall Number

- In KSVZ model, with one flavor of quarks, the number of domain walls in $N_{DW} = 1$, which is not cosmologically stable and thus harmless Sikivie (1982)
- In the present model there are 3 or more flavors, but $N_{DW} = 1$ nevertheless.
- When axion field is a mixed combination of fields in presence of a gauge symmetry, N_{DW} needs careful consideration:

$$N_{\rm DW} = \text{minimum integer} \left\{ \frac{1}{f_a} \sum_i n_i\, c_i\, v_i \;, \quad n_i \in \mathcal{Z} \right\}, \quad a = \sum_i c_i a_i$$

Ernst, Ringwald, Tamarit (2018)

SO(10) Model with Gauged U(1) and Axion

- Unified $SO(10) \times U(1)$ can lead to high quality axion
- The attractive features of $SO(10)$ GUT are preserved, including coupling unification and predictive fermion spectrum
- The fermion content and transformation under $SO(10) \times U(1)$:

$$\{3 \times 16_1 + 1 \times 10_{-6} + 1 \times 1_{12}\} + \{2 \times 1_{-4} + 1 \times 1_8\}$$

- All gauge anomalies cancel. Some resemblance to E_6 charges
- Higgs sector contains the usual $SO(10)$ fields and two singlets:

$$\{10_H(-2) + \overline{126}_H(-2) + 45_H(0) \text{ or } 54_H(0) + 10'_H(0) + T(1_{+1}) + S(1_{12})\}$$

- New features are the two singlet scalars T, S and a real $10'_H$. The $10'_H$ is needed to avoid weak scale axion

High Quality SO(10) Axion

- Yukawa couplings:

$$\begin{aligned} \mathcal{L}_{\rm Yuk} &= Y_{10} 16\, 16\, 10_H + Y_{126} 16\, 16\, \overline{126}_H + y_{10} 10_{-6} 10_{-6} S_{12} \\ &+ y'_{10} 10_{-6} 1_8\, 10_H + 1_{12} 1_{12} \frac{(S^*)^2}{M_{\rm Pl}} + ...h.c. \end{aligned}$$

- Realistic fermion masses are induced, including exotics
- Model has two decoupled sectors, one with 16-fermions, and one with 10-fermion. This results in accidental PQ symmetry
- Leading correction to PQ symmetry from gravity is

$$V \supset \frac{T^{12} S^*}{M_{\rm Pl}^9}$$

- Resulting shift in $\overline{\theta}$ is highly suppressed. $f_a < 2 \times 10^{11}$ GeV is required for quality. Domain wall number
- $N_{DW} = 1$ in the model

High Quality SO(10) Axion - cont.

- Axion field is orthogonal to pseudoscalars and Goldstones

$$
\begin{aligned}
a &\simeq 1/\sqrt{1+\frac{144 v_S^2 v^2}{X}}\left(\eta_S - 12 v_T v_S v^2 \eta_T / X\right) + \ldots \\
f_a &= v_S/\sqrt{1+\frac{144 v_S^2 v^2}{X}} \\
X &= v_T^2 v^2 + 4\tilde{v}^2 (v_u^{02} + v_d^{02}) + 16 v_u^{02} v_d^{02}
\end{aligned}
$$

- Axion coupling to fermions are modified compared to DFSZ or KSVZ models:

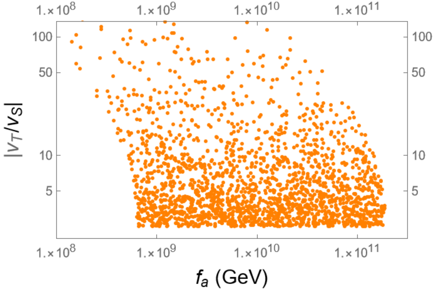
$$
\begin{aligned}
\mathcal{L}(f,a) &= i\frac{m_u}{f_a}\left[\frac{24 v_S^2 (2 v_d^{0\,2} + \tilde{v}^2)}{X + 144 v_S^2 v^2}\right]\bar{u}\gamma_5 u a + \; i\frac{m_d}{f_a}\left[\frac{24 v_S^2 (2 v_u^{0\,2} + \tilde{v}^2)}{X + 144 v_S^2 v^2}\right]\bar{d}\gamma_5 d a \\
&+ i\frac{m_e}{f_a}\left[\frac{24 v_S^2 (2 v_u^{0\,2} + \tilde{v}^2)}{X + 144 v_S^2 v^2}\right]\bar{e}\gamma_5 e a
\end{aligned}
$$

An upper limit on f_a in SO(10) model

- The induced $\bar{\theta}$ from quantum gravity is

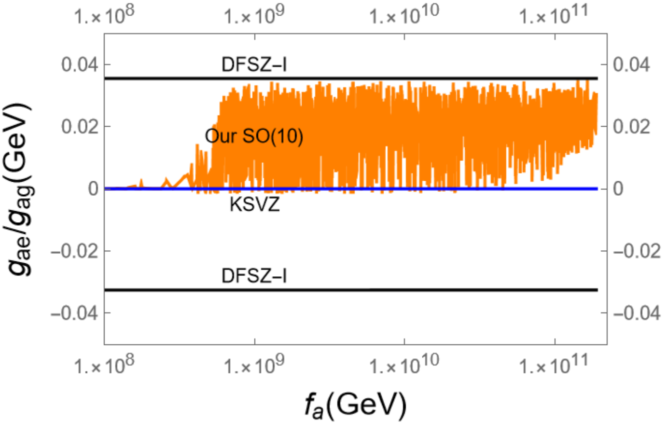
$$\bar{\theta} \simeq \frac{\kappa \sin\delta}{2^{(11/2)}} \sqrt{1 + \frac{144 v_S^2}{v_T^2}} \frac{v_T^{12} f_a}{\Lambda^4 M_{\rm Pl}^9}$$

- This sets a limit $f_a < 2 \times 10^{11}$ GeV

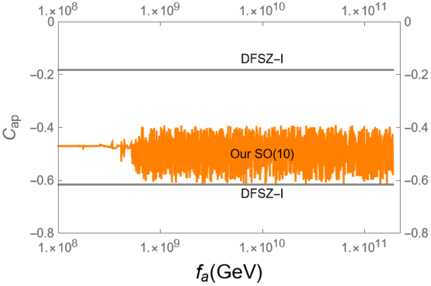


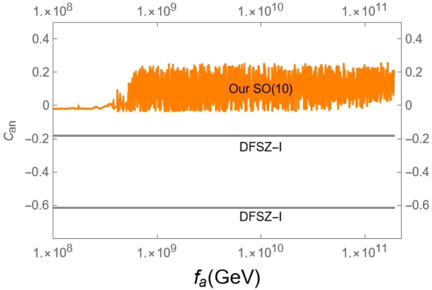
Axion couplings to electron in SO(10) model

Ratio of electron coupling of axion versus gluon coupling:



Axion couplings to proton and neutron in SO(10)





Conclusions

- Two classes of models presented which have an accidental PQ symmetry
- In one class, SM $\times U(1)$ resulted in high quality axion which is similar to KSVZ model
- $N_{DW} = 1$ in these models for domain wall number, causing no cosmological issues
- A second class in within the framework of $SO(10)$ unification. It leads to a hybrid KSVZ-DFSZ axion with high quality
- $N_{DW} = 1$ in these models without cosmological problems
- Axion couplings to fermions can potentially distinguish these models from standard benchmarks
- In all cases there is room for the axion to be the entire dark matter content of the universe