# **Nuclear Structure Theory (lecture III)**



### Gianluca Colò Dep. of Physics and INFN, Milano



2024 STFC Nuclear Physics Summer School

## **"Collective" spectroscopy**



## Nuclear spectroscopy

In Magda's lectures: ample discussions about the **relationships between observable properties of the low-lying spectra and nuclear shapes**.

In lecture II we have also introduced the paradigmatic case of vibrational and rotational nuclei





### Transition between spherical and well-deformed nuclei



When the potential is not very stiff, it is quite natural to assume that the **density is not the one corresponding to a single value of β or < Q >**.

The density can be seen as a superposition of the densities associated with different quadrupole deformations β.

$$
|\Psi\rangle = \int dq f(q) |\Phi(q)\rangle \qquad E =
$$

 $\langle \Psi | H | \Psi \rangle$  $\langle \Psi | \Psi \rangle$ 

- The energy should be minimized (again, variational principle).
- Generator coordinate method or **multi-reference DFT**.
- Angular momentum projection needed!



### Examples of multi-reference DFT calculations nonadiabatic limit, to provide more reliable prediction and nne i



2024 STFC Nuclear Physics Summer School  $5$ 

deformation, and also from the theoretical side, e.g., with the

NOVEL TRIAXIAL STRUCTURE IN LOW-LYING STATES . . . PHYSICAL REVIEW C **93**, 054324 (2016)

# Changing the quadrupole moment or?

### The shapes of nuclei

G.F. Bertsch<sup>∗</sup>

Department of Physics and Institute for Nuclear Theory, University of Washington, Seattle, Washington 98915, USA



 $f1G. 3$ : The spe FIG. 3: The spectrum of <sup>40</sup>Ca, showing the first three levels of the deformed band and the gamma transitions that establish the deformation of the band [11]



-336 -334 **particles that are promoted** n. -328 **constraint on the number of** 5 l G Instead of Q one can put a  $\sim$  10  $\sim$  100  $\sim$  100  $\sim$  1100  $\sim$  r<br>C q20 (fm2 ) **to the next shell.**

FIG. 5: Density distribution of the <sup>40</sup>Ca configurations. Left: ground state; center: GCM configuration at  $q = 108$  fm<sup>2</sup>; right: 4p-4h  $K^{\pi}$ -constrained configuration.



### Nuclear spectra above the particle threshold

At "high" energy, above the particle emission threshold, there is a continuum of states – and yet some prominent states show up: **the Giant Resonances.**

 $\tau =$ 

 $\bar{h}$ 

 $\Gamma$ 

**They lie around 10-30 MeV, and they have large widths.**

**They can undergo particle emission.**

**They are "giant" because they have large cross sections.**



## Dipole oscillations: isovector

- GDR (Giant Dipole Resonance): excited by a photon beam impinging on a nucleus.  $\hbar c \approx 197$  MeV fm
- $E \approx 10{\text -}20$  MeV implies that  $\lambda_{el\,\text{field}} = \text{hc}/E \gg R$  (nuclear dimension).

• Consequently, **the e.m. field in the nuclear region can be considered constant (dipole approximation).**



- Analogy with excitations in molecules or solids (plasmons)
- In nuclei there have been intensive searches of "pygmy" dipole states.





# "Pygmy" dipole resonance (?)

#### Progress in Particle and Nuclear Physics 70 (2013) 210–245





Review

Experimental studies of the Pygmy Dipole Resonance





#### Progress in Particle and Nuclear Physics 129 (2023) 104006

**Contents lists available at ScienceDirect** 



article in the control of t

Progress in Particle and Nuclear Physics and the last decade. This review gives a The electric dipole response of atomic nuclei has attracted a lot of attracted a lot of  $\mathcal{L}(\mathcal{L})$ status of experimental approaches to study low-lying electric dipole strength (or  $\frac{1}{2}$ noted as Pygmy Dipole Resonance) in stable and radioactive atomic nuclei and discusses the

journal homepage: www.elsevier.com/locate/ppnp © 2013 Elsevier B.V. All rights reserved. Description and the served of the served. Description and the served

Review **Contents** Theoretical studies of Pygmy Resonances  $\begin{array}{|c|c|}\hline \textbf{0} & \textbf{0} & \textbf{0}\ \hline \end{array}$ E.G. Lanza  $a,b,*$ , L. Pellegri  $c,d$ , A. Vitturi  $e,f$ , M.V. Andrés  $g$ Lanza $\mathbb{R}^n$ , L. Penegri $\mathbb{S}^n$ , A. Vitturi $\mathbb{S}^n$ , M.V. Andres $\mathbb{S}^n$ 

D. Savran<sup>a,b,\*</sup>, T. Aumann<sup>c,d</sup>, A. Zilges<sup>e</sup>



### **There have been strong discussions if it really exists.**



### **"Resonance" or SPS?**

**Fig. 9. Isovector (upper frame) dipole response obtained in a self-consistent HF + RPA calculation for 208Pb.** The strength  $\sim$ function were calculated by convoluting the corresponding reduced transition probability with a Lorenzian of 1 MeV width. Impact for  $(n,y)$ : cf. Ann-



**INFN** New 2024 STFC Nuclear Physics Summer School <sub>9</sub>

#### constant, and the beam waist is at zero. gauge has the advantage of being readily and being readily and being readily and being readily amenable in all coordinate systems and leads in this case to considerable systems and leads in this case to considerable systems  $\sim$ symmetry in the symmetry in the results and y directions are  $\overline{a}$ We have so far considered linearly polarized light;  $h$ nd on the vector position  $h$ Vortex photons

*kz*

 $\mathcal{K}% _{0}$ 

 $\theta_{k}$ 

 $\hat{z}$ 

Use of photons with orbital angular istene with sister a 2 momentum can open new avenues for the physics of collective states. for <sup>a</sup> beam of unit amplitude, where <sup>z</sup> is the unit vector in

are best expressed in cylindrical control in cylindrical control in cylindrical control in cylindrical control

Within

where zz is the Rayleigh range, w(z) is the radius of the beam, L' is the associated Laguerre polynomial, <sup>C</sup> is <sup>a</sup>

coordinates.

this description we have shown that the time

$$
M_f - M_i = m_\gamma = m_\ell + m_s
$$



## Types of giant resonances

 $\frac{1}{n}$ 



 $IS = Iso-Scalar$  $IV = Iso-Vector$  $S = Spin$  $G =$ Giant  $M = Monopole$  $D = Dipole$  $Q = Quadrupole$  $O = Octupole$ 

Image courtesy of M. Harakeh

tituto Nazionale





 $202$  Stephendiers Summer Summer Summer Summer Summer Summer School 11 **They are excited by photons or by particle inelastic scattering**

#### Strength function and operators  $\alpha$  duce the years to describe  $\alpha$  is described states, and not only the IVGDR, and not only the IVGDR, and  $\alpha$ starting from the seminal work of Ref. (Brown and Bolsterli (1959)). *F*IVGDR = *eN A Z* **<u>≀perators</u>** and van der Woude (2001)). In the non spin-flip case (electric GRs), the operators *F* associated with different values of the transferred angular momentum  $\mathbf{a}$  reading  $\mathbf{a}$  reading The nuclear translation of the motion of  $\mathbf{u}$ at  $z$  at zero excitation energy in an exact calculation, and should be decoupled from  $\mathcal{L}$

*i*=1

*<sup>F</sup>*ISGDR <sup>=</sup> ∑

∑

*<sup>i</sup> Y*1*M*(*r*ˆ*i*)*.* (35)

*riY*1*M*(*r*ˆ*i*)*.* (38)

*i*=1

whose height is larger by the factor  $N$  is larger by the factor  $N$  introduced between  $N$  introduced been widely intro-

When this term becomes meaningless, one of the next term in the next term in the next term in the next term in the Taylor and the Ta

di Fisica Nucleare. *rYM<sup>2</sup>* 

expansion of the Bessel function, that is *rL*+2. For instance, in the monopole case,

$$
S(E) = \sum_{n} |\langle n|F|0\rangle|^2 \delta(E - E_n)
$$
  

$$
S(E) = \sum_{n} |\langle n|F|0\rangle|^2 \frac{\Gamma_n}{(E - E_n)^2 + \frac{\Gamma_n^2}{4}}
$$
  

$$
F_{\text{IS}} = \sum_{i} r_i^L Y_{LM}(\hat{r}_i), \qquad F_{\text{IS}} = \sum_{i} r_i^L [Y_{LM}(\hat{r}_i) \otimes \sigma(i)]_J,
$$

$$
F_{\text{IS}} = \sum_{i} r_{i}^{T} I_{LM}(r_{i}), \qquad F_{\text{IS}} = \sum_{i} r_{i}^{T} [Y_{LM}(r_{i}) \otimes \sigma(i)]_{J},
$$
\n
$$
F_{\text{IV}} = \sum_{i} r_{i}^{L} Y_{LM}(\hat{r}_{i}) \tau_{z}(i).
$$
\n
$$
F_{\text{IV}} = \sum_{i} r_{i}^{L} [Y_{LM}(\hat{r}_{i}) \otimes \sigma(i)]_{J} \tau_{z}(i).
$$
\n
$$
F_{\text{ISGMR}} = \sum_{i} r_{i}^{2}, \qquad F_{\text{IVGDR}} = \frac{eN}{A} \sum_{i=1}^{Z} r_{i} Y_{1M}(\hat{r}_{i}) - \frac{eZ}{A} \sum_{i=1}^{N} r_{i} Y_{1M}(\hat{r}_{i}).
$$
\n
$$
F_{\text{IVGDR}} = \frac{eN}{A} \sum_{i=1}^{Z} r_{i} Y_{1M}(\hat{r}_{i}) - \frac{eZ}{A} \sum_{i=1}^{N} r_{i} Y_{1M}(\hat{r}_{i}).
$$
\n2024 STFC Nuclear Physics Summer School

# The Giant Monopole Resonance

Breathing mode: in this case its energy is correlated with the compressibility of nuclear matter.



 $\chi^{-1} = \rho^3 \frac{d^2}{d\rho}$ 

PHYSICAL REVIEW LETTERS 129, 032701 (2022)

 $K_{\infty} = 9 \rho_0^2 \frac{d^2}{d\rho^2} \left(\frac{E}{A}\right)$ 

Probing the Incompressibility of Nuclear Matter at Ultrahigh Density through the Prompt Collapse of Asymmetric Neutron Star Binaries

Albino Perego<sup>o,1,2,\*</sup> Domenico Logoteta<sup>o,3,4</sup> David Radice<sup>o,5,6,7</sup> Sebastiano Bernuzzi<sup>o</sup>,<sup>8</sup> Rahul Kashyap<sup>o,5,6</sup> Abhishek Das  $\mathbb{D}^{5,6}$  Surendra Padamata  $\mathbb{D}^{5,6}$  and Aviral Prakash  $\mathbb{D}^{5,6}$ 

 $\begin{array}{|l|}\n\hline\n\textbf{U. Garg, GC, PPNP 101 (2018) 55}\n\hline\n\end{array}$ **Department of Astrophysics, U. Garg, GC, PPNP 101 (2018) 55 U. Bark, Park, Park,** 

 $(\mathcal{R}_\mathcal{D})$  and  $\mathcal{R}_\mathcal{D}$  accepted 6 July 2022; published 13 July 2022; published 13 July 2022; published 13 July 2022

### Charge-exchange transitions and β-decay



They are induced by reactions, like (p,n) or  $(3He,t)$ .

Some transitions may be inside the allowed β-decay window.





Z N







2024 STFC Nuclear Physics Summer School  $14$ 

**Giant Resonances and highly excited states Spectroscopy of heavy and superheavy nuclei Reactions (transfer, fusion, fission) Neutron stars**





 $E_X$ 

# Time-dependent DFT and RPA

$$
\boxed{h\phi_i = \varepsilon_i\phi_i}
$$

In the time-dependent case, one can solve the evolution equation for the density directly:

$$
h(t) = h + f(t) \qquad [h(t), \rho(t)] = i\hbar \dot{\rho}(t)
$$

$$
\rho(t=0) \neq \rho_{\rm g.s.}
$$

$$
\begin{array}{c}\n 5 \times 10^{-3} \\
 \underline{\overline{E}} \\
 0 \\
 \underline{\overline{E}} \\
$$

From: P. Stevenson (U. Surrey)

$$
\rho(t = \Delta t) = U(t = 0, t = \Delta t)\rho(t = 0) \qquad U = e^{-i\frac{\Delta t}{\hbar}}
$$

If the equation for the density is **linearized** and solved on a basis: **Random Phase Approximation or RPA**.

$$
U = e^{-i\frac{\Delta t}{\hbar} \cdot h}
$$

$$
\begin{pmatrix} A & B \ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \hbar \omega \begin{pmatrix} X \\ Y \end{pmatrix}
$$

G.C. *et al.*, Computer Physics Commun. 184, 142 (2013).

 $\frac{20}{10}$  The superpositions  $\frac{16}{16}$ In RPA the excited states are 1p-1h superpositions

### The external field excites, at lowest order, a one particle-one hole state (1p-1h)



in TDA. The single-particle orbitals up to the Fermi energy ε*<sup>F</sup>* are fully occupied by *A* particles from the bottom of the potential *V (r)* in order in the HF vacuum for TDA. (Lower left) RPA ground state. Figure taken from: A. Obertelli and H. Sagawa, *Modern Nuclear Physics*, Springer (2021)





 $2029$  moder spaces (in strict terms,  $10$   $17$ One can extend RPA to second RPA (2p-2h) Advantage of Shell Model: automatically, many p-many h included. Advantage of RPA: possibility to go to large model spaces (in other terms, "no core".

## Random Phase Approximation (RPA)

$$
\begin{pmatrix}\n\varepsilon_{\rm ph} + v_{\rm ph,ph} & v_{\rm ph,p'h'} & \dots \\
v_{\rm p'h',ph} & \varepsilon_{\rm p'h'} + v_{\rm p'h',p'h'} & \dots \\
\dots & \dots & \dots\n\end{pmatrix}
$$

In RPA (or TDA) the excited states are **superpositions of particle-hole excitations**, that interfere to form a state that we can view as an excitation of the nucleus as a whole. Cf. the picture below and the toy model in the next slides.



PHYSICAL REVIEW LETTERS 121, 252501 (2018)

### Enhanced Quadrupole and Octupole Strength in Doubly Magic <sup>132</sup>Sn

D. Rosiak,<sup>1</sup> M. Seidlitz,<sup>1,\*</sup> P. Reiter,<sup>1</sup> H. Naïdja,<sup>2,3,4</sup> Y. Tsunoda,<sup>5</sup> T. Togashi,<sup>5</sup> F. Nowacki,<sup>2,3</sup> T. Otsuka,<sup>6,5,7,8,9</sup> G. Colò,<sup>10,11</sup> K. Arnswald,<sup>1</sup> T. Berry,<sup>12</sup> A. Blazhev,<sup>1</sup> M. J. G. Borge,<sup>13,†</sup> J. Cederkäll,<sup>14</sup> D. M. Cox,<sup>15,16</sup> H. De Witte,<sup>8</sup> L. P. Gaffney,<sup>13</sup> C. Henrich,<sup>17</sup> R. Hirsch,<sup>1</sup> M. Huyse,<sup>8</sup> A. Illana,<sup>8</sup> K. Johnston,<sup>13</sup> L. Kaya,<sup>1</sup> Th. Kröll,<sup>17</sup> M. L. Lozano Benito,<sup>13</sup> J. Ojala,<sup>15,16</sup> J. Pakarinen,<sup>15,16</sup> M. Queiser,<sup>1</sup> G. Rainovski,<sup>18</sup> J. A. Rodriguez,<sup>13</sup> B. Siebeck,<sup>1</sup> E. Siesling,<sup>13</sup> J. Snäll,<sup>14</sup> P. Van Duppen,<sup>8</sup> A. Vogt,<sup>1</sup> M. von Schmid,<sup>17</sup> N. Warr,<sup>1</sup> F. Wenander,<sup>13</sup> and K. O. Zell<sup>1</sup>  $I_2^{2,3,4}$  V Teunoda <sup>5</sup> T Togschi <sup>5</sup> E Nowacki <sup>2,3</sup> T Oteuka <sup>6,5,7,8,9</sup> G Colo<sup>10,10,1</sup> G. Borge,  $^{13, \dagger}$  J. Cederkäll,  $^{14}$  D. M. Cox,  $^{13, 16}$  H. De Witte,  $^{8}$  L. P. Gaffney,  $^{13}$  $10^{13}$  J. Ojala,  $\frac{15,18}{8}$  $\cdot$  J. A. Rodriguez,  $\cdot$  B. Siebeck, E. Siesling, J. Snall, P. Van Duppen, chmid  $^{17}$  N. Warr  $^{1}$  F. Wenander  $^{13}$  and K. O. Zell<sup>1</sup>  $56$   $58$   $73$   $65780$   $1011$  $n_{\text{min}}$  number of 0.1 (1.28) protons and 0.23 (0.23) neutrons are  $n_{\text{min}}$  of 0.23 (0.22) neutrons are  $n_{\text{min}}$  in  $n_{\text{min}}$  and 0.233 except and D.M. Cox, The Let White, E.T. Galiney,  $^{13}$  L. Kaya,<sup>1</sup> Th. Kröll,<sup>17</sup> M. L. Lozano Benito,<sup>13</sup> J. Ojala,<sup>15,16</sup> odriguez,  $^{13}$  B. Siebeck, E. Siesling,  $^{13}$  J. Snäll,  $^{14}$  P. Van Duppen,  $^{8}$ 

(MINIBALL and HIE-ISOLDE Collaborations) HIE-ISOLDE COLLECTRONS

<sup>18</sup>Department of Atomic Physics, University of Sofia, 5 James Bourchier Boulevard, BG-1164 Sofia, Bulgaria

yield BðE3; 0<sup>þ</sup>



values derived from LSSM, MCSM, and RRPA calculations compare well with the new experimental value with the error bars  $\mathbb{R}^n$  and  $\mathbb{R}^n$  approaches  $\mathbb{R}^n$  approaches  $\mathbb{R}^n$ 

respectively, in agreement with experimental data

theory  $[11.12]$ . Because of the computational limits of the  $\mu$  and  $\mu$  1.1,12]. Because of the computational finites of the valence space, the SM approaches do not provide information on the  $3<sub>1</sub><sup>-</sup>$  state. The RPA and  $R_{\text{Hubble 1}}$  on the  $\sigma_1$ -state. The K mation on the  $3<sub>1</sub><sup>-</sup>$  state. The RPA and RRPA calculations between performance as well as well as mean-field calculations utilizations uti theory [11,12]. Because of the computational limits of the <sup>g</sup>:s: → 3<sup>−</sup>

 $\kappa$ ise(s): try vourself comparison between RPA  $\kappa$  $T$  and  $3-$  states of the doubly magic nucleus 132Sn are populated via safe Coulomb excitation  $\alpha$ quadrupole particle-hole excitations in <sup>132</sup>Sn. Details on the investigation of <sup>132</sup>Sn requires np − nh excitations from the Exercise(s): try yourself comparison between RPA and SM.

 $\mathbf{v}$  mpinged on a 206Pb target. Deep the low-lying excited states of the target and the target an projectile are recorded in coincidence with scattered particles. The reduced transition strengths are ns4exp.mi.infn.it determined for the transitions 0<sup>þ</sup>

sphericity and double magicity of <sup>132</sup>Sn.

ations: RPA only for spherical systems, Hamiltonian was diagonalized employing the ANTOINE nly for spherical systems, SM no corroborate the local ly enhanced quadrupole strength in the local ly enhanced strength in the local ly enhanced strength in the local Limitations: RPA only for spherical systems, SM not suitable for large model spaces. LSSM yield 0.028, 0.100, and 0.027 e<sup>2</sup>b<sup>2</sup> for <sup>130</sup>;132;<sup>134</sup>Sn,

<sup>1</sup> Þ values in agreement with the

A novel MCSM calculation was performed recently in a



<sup>1</sup> state is well reproduced

<sup>g</sup>:s: → 2<sup>þ</sup>

### RPA and collectivity: schematic model (I)

### **Schematic 2 x 2 case**

$$
\left(\begin{array}{cc} \varepsilon+v & v \\ v & \varepsilon+v \end{array}\right)
$$

$$
\hbar\omega_1 = \varepsilon, \qquad X^{(1)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix},
$$

$$
\hbar\omega_2 = \varepsilon + v \qquad X^{(2)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}.
$$

### **Magnetic spin-flip states (M1)**

<sup>208</sup>Pb : h<sub>11/2</sub> 
$$
\rightarrow
$$
 h<sub>9/2</sub> (proton)  
 $i_{13/2}$   $\rightarrow$  i<sub>11/2</sub> (neutron)





### RPA and collectivity: schematic model (II)

**Schematic** *N* **x** *N* **case**

There is one "coherent state":

$$
\frac{1}{\sqrt{N}}\left(\begin{array}{c}1\\1\\ \cdots\\1\end{array}\right)
$$



Its transition amplitude is enhanced:

$$
\langle n|F|0\rangle = \sum_{ph} X_{ph} \langle p|F|h\rangle \approx N \frac{1}{\sqrt{N}} M = \sqrt{N} M
$$

G. C., *Theoretical Methods for Giant Resonances*, in: Handbook of Nuclear Physics, edited by I. Tanihata, H. Toki and T. Kajino (Springer, 2022).



# **Single-particle spectroscopy**



#### Department ofMathematical Physics, Lund Institute of Technology, Lund, Sweden Center for Mathematical Sciences, University ofAizu, Aizu Wa-kamatsu, Fukushima 965, Japan  $\Box$ ngrtic | Single-particle spectroscopy: neutron-rich  $\begin{array}{l} \hline \end{array}$  calculation decays  $\begin{array}{l} \hline \end{array}$ and neutron-deficient nuclei such as 'zo and neutron-deficient nuclei is a beta decay to the GT giant resonance state. The amplitude of  $T = 1$ 3.1. Spherical case



ground state in N  $=$  nuclei is also calculated and discussed.

 $\frac{1}{\sqrt{N}}$  is a line nucleus, 6C is a line  $\mathbb{R}$ shown, which is calculated by using the BKN interaction interaction  $\mathbb{R}$  $\mathbb{Z}$  in  $\mathbb{Z}$ . All other shown at respectively.

 $\sim$ 

0

I I <sup>I</sup> I ] <sup>I</sup> I <sup>I</sup> I

 $\mathbb{R}\mathbb{N}$  int

If the neutron number increases, neutrons occupies higher levels – protons become more bound due to the dominance of the p-n interaction.

#### Dancoff approximation (TDA) as well as using a I. Hamamoto and H. Sagawa, Phys. Rev. C48, R960 (1993) i. Hamamolo and H. Sayawa, Filys. Nev. 040, N90







2024 STFC Nuclear Physics Summer School 23 lation is performed with the Coulomb potential, there is a small

 $\overline{1}$ 



 $^{213}Rn_{86}$  Changes of orbitals (either upward or downward) are LARGER for HIGHER angular momenta.

> In fact, orbitals with smaller values of angular momenta are less constrained by the centrifugal barrier and overlap less with the other states.

> In particular, the trend of s- and porbitals becomes almost flat when their energies approach zero.

stability valley  $24$ We expect changes in the shell structure when going far from the

$$
U_q(\vec{r}) = \int d^3r' \sum_{q'} v_{qq'}(\vec{r}, \vec{r}') \rho_{q'}(r')
$$

**Tensor force** is a further element that (a) affects the evolution of shell structure and (b) is in common between KS/DFT and SM.

> FIG. 32. Evolution of the proton  $h_{11/2}$ - $g_{7/2}$  gap in the Sb isotopes with and without the tensor term. (Upper left panel)  $\pi + \rho$  meson-exchange tensor force on top of the usual Woods-Saxon potential. From Otsuka et al., 2005. (Upper right panel) A Gogny-type calculation with the tensor force (GT2) and without it (D1S). From Otsuka, Matsuo, and Abe, 2006. (Lower panel) A zero-range tensor force calculation added to the SLy5 force. From Colò et al., 2007.

KS/DFT language | Shell-model language  $\blacksquare$ 

Mean field **Music algebra et al.: Evolution of Shell Structure interaction** Monopole interaction

$$
\hat{v}_{nn}^{m}(j,j') = \begin{cases} V_{nn}^{m}(j,j) \frac{1}{2} \hat{n}_{j} (\hat{n}_{j} - 1) & \text{for } j = j' \\ V_{nn}^{m}(j,j') \hat{n}_{j} \hat{n}_{j'} & \text{for } j \neq j' \end{cases}
$$





2024 STFC Nuclear Physics Summer School  $25$ isotopes with and without the tensor term. (Upper left panel)  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ elements of the zero-range tensor force of  $\mathsf{z}$ 

### **Bonus: neutron stars**



The existence of a netron star can be, at our level, understood by means of a simple exercise based on the semi-empirical mass formula,

$$
BE(A, Z) = a_V A - a_S A^{2/3} - a_A \frac{(N - Z)^2}{A} + \text{pairing term.}
$$

A system with only neutrons has no Coulomb, and is expected to be very large if the binding energy of the latter formula, which is negative, must be counterbalanced by the gravitational energy (9.7), that we can take with  $k = 3/5$  assuming that the density is uniform. Being the system very large, we can neglect the surface energy and obviously the pairing contribution. We arrive at the following balance between nuclear and gravitational energies, namely

$$
a_V A - a_A A + \frac{3}{5} \frac{Gm^2}{r_0} A^{5/3} = 0,
$$
\n(9.26)

where  $m$  is the nucleon mass and the radius is taken as usual from Eq.  $(2.1)$ .

If we replace reasonable values for the parameters of the mass formula, like  $a_V = 15.85$  MeV and  $a_A = 23.21$  MeV, we obtain

$$
A \approx 5 \ 10^{55}.\tag{9.27}
$$

The corresponding mass and radius are

$$
M = mA \approx 10^{29} \text{ kg} \approx 0.05 \ M_{\odot} \qquad R = r_0 A^{1/3} \approx 5 \text{ km.}
$$
 (9.28)



 $\sqrt{PN}$  2024 STFC Nuclear Physics Summer School 27



### *Credit: B. Giacomazzo*



## Measurements of masses and radii

Up to some time ago, measurements of radii have been plagued by large uncertainties.

Moreover, radii and masses were determined for different systems.

Recently, NASA's NICER mission has measured, for the first time, mass and radius of the same star.

The results will be shown here below.

The observation of gravitational waves has also set new constraints ("chirp mass", see below).



# The TOV equation



General relativity corrections (TOV)

$$
\frac{dP(r)}{dr} = -\frac{Gm(r)\varepsilon(r)}{r^2c^2} \frac{\left(1 + \frac{P(r)}{\varepsilon(r)}\right)\left(1 + \frac{4\pi r^3 P(r)}{\varepsilon(r)}\right)}{1 - \frac{2Gm(r)}{rc^2}}
$$



$$
\frac{dP(r)}{dr} = -\frac{Gm(r)\varepsilon(r)}{r^2c^2}
$$

$$
\begin{array}{|c|} \hline \\[-1.5mm] \hline \\[-1.5mm] \hline \end{array}
$$



### 2024 STFC Nuclear Physics Summer School 30

### EoS and the solution of the TOV equation





masses of the millisecond pulsar PSR J0740 + 6620 [12] and of J0384-0432 [11] are also shown, as well

as constraints inferred from the analysis of the GW170817 event and observations reported by the

Figure 3. Pressure vs. baryon density for the symmetric case (left panels), and the beta-stable case (right panels). The upper (lower) panels display results for microscopic (phenomenological) EOSs. Constraints derived from HIC data are displayed in the left panels as orange (KaoS experiment) and grey (flow data) bands. Limits deduced by the GW170817 event are labelled by blue bands in the right panels. See text for details.

### F. Burgio *et al.*, Symmetry 13, 400 (2021)



NICER mission [18,19]. See text for details.

### Merging of neutron star AND the nuclear EoS

- 2015: first observation of Gravitational Waves (GW), awarded with Nobel prize in 2017.
- 17/8/2017 "multi-messenger" observation of NS merging (GW, GRB, X-rays...).



### GW170817 Press Release

**LIGO and Virgo make first** detection of gravitational waves produced by colliding neutron stars

B.P. Abbott et al., Astrophysical J. Lett. 848:L12 (2017)

**In simulations tidal effects are relevant. A more compact star is harder to distort. But this depends on S!**

$$
\Lambda = \frac{2}{3} k_2 \left(\frac{c^2 R}{GM}\right)^2
$$



*Mass polarizability: ratio quadrupole moment / gravitational field*



THE ASTROPHYSICAL JOURNAL LETTERS, 848:L12 (59pp), 2017 October 20 @ 2017 The American Astronomical Society, All rights reserved **OPEN ACCESS** 

#### https://doi.org/10.3847/2041-8213/aa91c9



#### Multi-messenger Observations of a Binary Neutron Star Merger'

LIGO Scientific Collaboration and Virgo Collaboration, Fermi GBM, INTEGRAL, IceCube Collaboration, AstroSat Cadmium Zinc Telluride Imager Team, IPN Collaboration, The Insight-HXMT Collaboration, ANTARES Collaboration, The Swift Collaboration, AGILE Team, The 1M2H Team, The Dark Energy Camera GW-EM Collaboration and the DES Collaboration, The DLT40 Collaboration, GRAWITA: GRAvitational Wave Inaf TeAm, The Fermi Large Area Telescope Collaboration, ATCA: Australia Telescope Compact Array, ASKAP: Australian SKA Pathfinder, Las Cumbres Observatory Group, OzGrav, DWF (Deeper, Wider, Faster Program), AST3, and CAASTRO Collaborations. The VINROUGE Collaboration, MASTER Collaboration, J-GEM, GROWTH, JAGWAR, Caltech-NRAO, TTU-NRAO, and NuSTAR Collaborations, Pan-STARRS, The MAXI Team, TZAC Consortium, KU Collaboration, Nortic Optical Telescope, ePESSTO, GROND, Texas Tech University, SALT Group, TOROS: Transient Robotic Observatory of the South Collaboration, The BOOTES Collaboration, MWA: Murchison Widefield Array, The CALET Collaboration, IKI-GW Follow-up Collaboration, H.E.S.S. Collaboration, LOFAR Collaboration, LWA: Long Wavelength Array, HAWC Collaboration, The Pierre Auger Collaboration, ALMA Collaboration, Euro VLBI Team, Pi of the Sky Collaboration. The Chandra Team at McGill University, DFN: Desert Fireball Network, ATLAS, High Time Resolution Universe Survey, RIMAS and RATIR, and SKA South Africa/MeerKAT (See the end matter for the full list of authors.)

Received 2017 October 3; revised 2017 October 6; accepted 2017 October 6; published 2017 October 16

#### **A** hstract

On 2017 August 17 a binary neutron star coalescence candidate (later designated GW170817) with merger time 12:41:04 UTC was observed through gravitational waves by the Advanced LIGO and Advanced Virgo detectors. The Fermi Gamma-ray Burst Monitor independently detected a gamma-ray burst (GRB 170817A) with a time delay of  $\sim$  1.7 s with respect to the merger time. From the gravitational-wave signal, the source was initially localized to a sky region of 31 deg<sup>2</sup> at a luminosity distance of  $40^{+8}_{-8}$  Mpc and with component masses consistent with neutron stars. The component masses were later measured to be in the range 0.86 to 2.26  $M_{\odot}$ . An extensive observing campaign was launched across the electromagnetic spectrum leading to the discovery of a bright optical transient (SSS17a, now with the IAU identification of AT 2017gfo) in NGC 4993 (at  $\sim$  40 Mpc) less than 11 hours after the merger by the One-Meter, Two Hemisphere (1M2H) team using the 1 m Swope Telescope. The optical transient was independently detected by multiple teams within an hour. Subsequent observations targeted the object and its environment. Early ultraviolet observations revealed a blue transient that faded within 48 hours. Optical and infrared observations showed a redward evolution over  $\sim$  10 days. Following early non-detections, X-ray and radio emission were discovered at the transient's position  $\sim$ 9 and  $\sim$  16 days, respectively, after the merger. Both the X-ray and radio emission likely arise from a physical process that is distinct from the one that generates the UV/optical/near-infrared emission. No ultra-high-energy gamma-rays and no neutrino candidates consistent with the source were found in follow-up searches. These observations support the hypothesis that GW170817 was produced by the merger of two neutron stars in NGC 4993 followed by a short gamma-ray burst (GRB 170817A) and a kilonova/macronova powered by the radioactive decay of r-process nuclei synthesized in the ejecta.

 $Key words: gravitational waves - stars: neutron$ 



### **GW from NS-NS merging provide information on the EoS**

This is a very active domain in which more progress is expected





### 2024 STFC Nuclear Physics Summer School  $_{33}$ (Recepted 20 December 2010)<br>STFC Nuclear Physics Summer School STFC Nuclear Physics Summer School<br>
STFC Nuclear Physics Summer School **STFC Nuclear Physics Summer School** STFC Nuclear Physics Summer School<br>
either from binaryies with two different mass of products with two different mass with two different mass of the m SIFC NUCIEAR PHYSICS SUMMER SCHOOL

## Structure of neutron stars



Outer core: nuclei and free neutrons coalesce. This defines the so-called outer core. Elongated shapes vs. uniform matter? Balance between surface energy and Coulomb energy.

Inner core: completely uncertain is the composition of neutron stars at even higher densities. Leptons,  $\Sigma$ ,  $\Lambda$ ,  $\Xi$  hyperons? Quark matter??

