Which orders?

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In SMEFT framework

$$|A|^2 = |A_{SM}|^2 + \frac{2Re(A^*_{SM}A_6)}{\Lambda^2} + \frac{1}{\Lambda^4}\Big(|A_6|^2 + 2Re(A^*_{SM}A_8)\Big) + \cdots$$

interference piece,
usually largest effect.
State of the art
SMEFT

'Higher order'
$\mathcal{O}(1/\Lambda^4)$
corrections

Dual expansion: gotta match dimensions, so numerator ~ powers of

$$v, \partial_\mu \sim E$$

At high energy $\left(\frac{E^n}{\Lambda^n}\right) > \left(\frac{v^n}{\Lambda^n}\right)$: main advantage of SMEFT at LHC

In SMEFT framework

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'Higher order'
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corrections

larger expansion parameter = more sensitive to higher orders!

- To know error on $1/\Lambda^2$ piece, we should know next order!
- Additionally, there are circumstances where interference is suppressed. Then $1/\Lambda^4$ is the leading SMEFT piece

OK, so we'd like to include $\mathcal{O}(1/\Lambda^4)$ effects

BUT!

SMEFT Warsaw basis: $\mathcal{O}(60)$ operators at dim-6

(flavor universal, CP) $\mathcal{O}(1000)$ operators at dim-8

Can't we just do $|\text{dim} - 6|^2$?

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can be okay if nothing else, but **lots** of pitfalls

- $|\text{dim}-6|^2$ is positive definite, total $\mathcal{O}(1/\Lambda^4)$ need not be
- $|\text{dim}-6|^2$ limited to $\text{dim}-6$ operators…
  limited structure, some already bounded, small in some UV setups

Can lead to wildly inaccurate estimates of $\mathcal{O}(1/\Lambda^4)$ …

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Can lead to wildly inaccurate estimates of $\mathcal{O}(1/\Lambda^4)$ …

Especially dangerous if $|\text{dim} - 6|^2 > SM \times (\text{dim} - 6)$
without a good reason!!

geoSMEFT-ist perspective

[2001.01453 Helset, AM, Trott]

geoSMEFT = re-organization of SMEFT that makes many key processes (for LHC SMEFT global fit) calculable $\mathcal{O}(1/\Lambda^4)$ without needing 1000 operators. Clarifies E vs. v counting

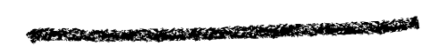
Calculate away, forming a library of process to use as a laboratory to study 'truncation error'.

geoSMEFT

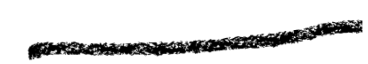
[2001.01453 Helset, AM, Trott]

Organize operators by the smallest vertex (# of particles that enter) they can impact at tree level: 2, 3,4, etc. Minimize the # of operators affecting 2, 3-particle vertices by strategically placing derivatives (IBP)

- $(H^\dagger D_\mu H)^*(H^\dagger D^\mu H) \supset v^2 (\partial_\mu h)^2$ contributes to 2-particle vertex



$(\psi^\dagger \psi)^2$ contributes to 4-particle vertex

- $\square (H^\dagger H) \square (H^\dagger H) \supset v^4 (\partial_\mu h)^2$ would contribute to 

but can use IBP to manipulate to

$(D_\mu H^\dagger D^\mu H D_\nu H^\dagger D^\nu H)$ which only affects 4+ particle vertices

At dimension-6, assuming B,L, flavor universal (59 total)

Min vertex:

Operator type: [X = field strength, D = deriv]	H^6 H^4D^2 H^2X^2 ψ^2H^3	X^3 ψ^2HX ψ^2H^2D	ψ^4	
Number:	**14**	**20**	**25**	**0**

At dimension-8, assuming B,L, flavor universal (993 total)

Min vertex:				
Operator type: [X = field strength, D = deriv]	H^8 $H^6 D^2$ $H^4 X^2$ $\psi^2 H^5$	$H^2 X^3$ $\psi^2 H^3 X$ $\psi^2 H^4 D$ $H^4 D^2 X$	X^4 $H^4 D^4$ $X^2 H^2 D^2$ ...	$\psi^4 X$
Number:	**19**	**47**	**927!**	

At dimension-8, assuming B,L, flavor universal (993 total)

Min vertex:

Operator type: [X = field strength, D = deriv]	H^8 H^6D^2 H^4X^2 ψ^2H^5	H^2X^3 ψ^2H^3X ψ^2H^4D H^4D^2X	X^4 H^4D^4 $X^2H^2D^2$ ...	ψ^4X
Number:	**19**	**47**	**927!**	

If we also impose CP, U(3)⁵ (remember, must interfere to enter $1/\Lambda^4$)

	8	**22**

[trend continues to dim > 8 too!]

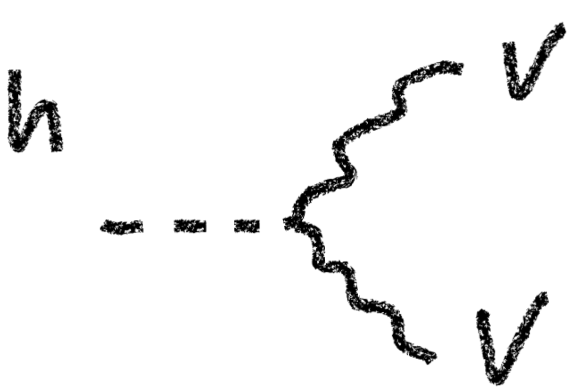
Why is this a good idea?

- "Universal" corrections related to inputs ~ O(10) new operators. Simplest building block vertices ~ O(20) ops

- Bulk of operators pushed to more process-specific, 4^+-particle interactions

- 2-, 3- particle interactions: going from dim-6 to dim-8 doesn't change kinematics — just added additional H^2! Additional derivatives aren't possible, as all momentum products reduce to masses = constants. So the energy/vev scaling of these terms is set by whatever happens at dim-6

Ex.)



$$\sim \frac{E^2 v}{\Lambda^2} \text{ at dim-6} \qquad \sim \frac{E^2 v}{\Lambda^2}\left(\frac{v^2}{\Lambda^2}\right) \text{ at dim-8}$$

$$H^\dagger H X_{\mu\nu} X^{\mu\nu} \qquad (H^\dagger H)^2 X_{\mu\nu} X^{\mu\nu}$$

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  So the energy/vev scaling of these terms is set by whatever happens at dim-6

So, if we're hunting for energy enhanced effects —> energy enhanced dim-6 3-particle + dim-6, dim-8 contact

Ok, what do I do with this?

1.) Simplest LHC processes: resonances, 2 -> 2 can be done 'fully' to $\mathcal{O}(1/\Lambda^4)$ without an order of magnitude increase in operators

$gg \to h \to \gamma\gamma, \gamma Z$ Z-pole, Drell-Yan $pp \to V(\ell\ell)\,h$

$pp \to W(\ell\nu)\,\gamma$

[Kim, AM 2203.11976] [Boughezal et al 2106.05337, 2207.01703

[Corbett, AM, Trott 2107.07470] [AM, Trott 2305.05879] [Hays, Helset, AM, Trott 2007.00565]

For these, can use $\mathcal{O}(1/\Lambda^4)$ as an uncertainty on extraction of dim-6 operators [how to do this systematically?]

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For these, can use $\mathcal{O}(1/\Lambda^4)$ as an uncertainty on extraction of dim-6 operators [how to do this systematically?]

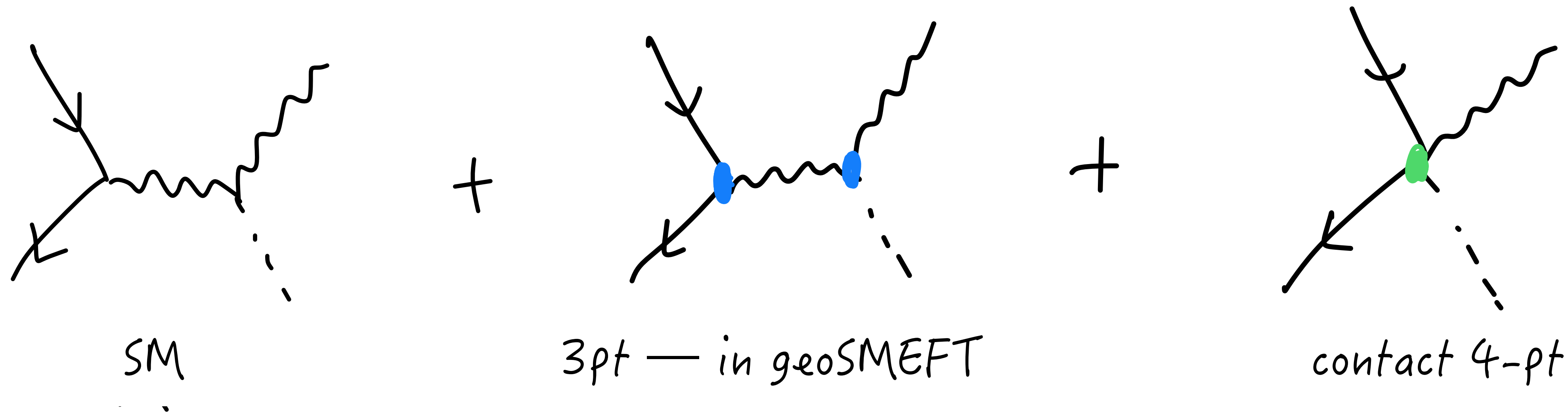
2.) Initial step: focus on terms that grow with energy (fully, to $\mathcal{O}(1/\Lambda^4)$). Assuming all WC are same size, these effects will be largest

$pp \to W^+W^-, W^\pm Z$ $\qquad$ $VBF\, pp \to hjj$

[2303.10493 Degrande] [Assi,AM in prep]

Do I gain something vs. using |dim-6|²

Example: VH



Energy enhanced effects

dim-6: vertex $H^\dagger H\, W_{\mu\nu} W^{\mu\nu}$

contact $(Q^\dagger \bar{\sigma}^\mu \tau^I Q)\, H^\dagger \overleftrightarrow{D}_I H$

dim-8: contact $\psi^2 H^2 D^3 \supset (Q^\dagger \sigma^\mu D^\nu Q)(D^\mu H^\dagger D_\nu H)$

$\psi^2 H^2 X\, D \supset (Q^\dagger \bar{\sigma}^\mu Q) D^\nu (H^\dagger H) B_{\mu\nu}$

[2306.00053 Corbett, AM]

Do I gain something vs. using |dim-6|^2

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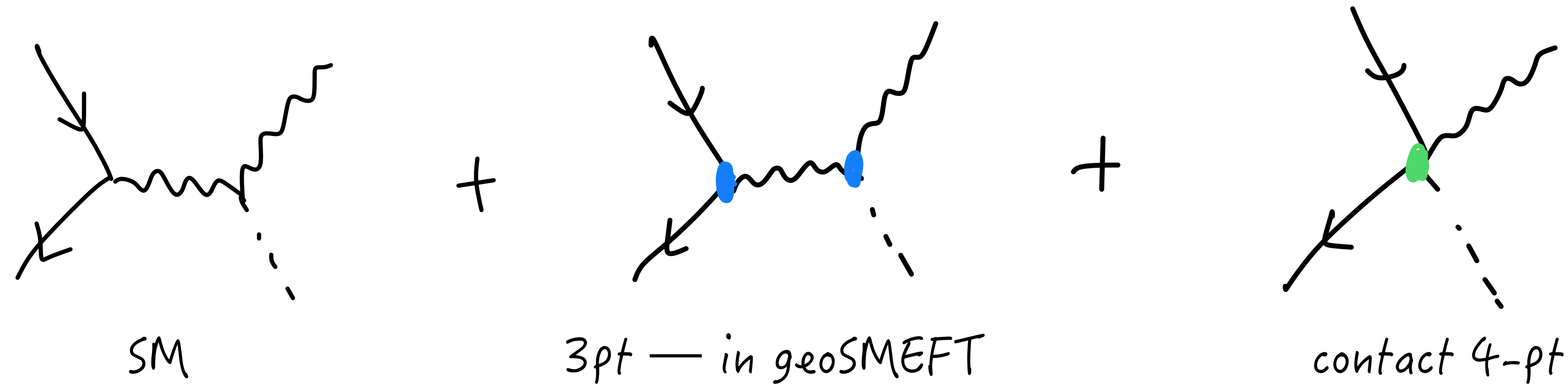

Energy enhanced effects

dim-6: vertex ~~$H^\dagger H\, W_{\mu\nu} W^{\mu\nu}$~~

contact $(Q^\dagger \bar{\sigma}^\mu \tau^I Q)\, H^\dagger \overleftrightarrow{D}_I H$

SM dominantly V_L, won't interfere with these!

dim-8: contact $\psi^2 H^2 D^3 \supset (Q^\dagger \sigma^\mu D^\nu Q)(D^\mu H^\dagger D_\nu H)$

~~$\psi^2 H^2 X\, D \supset (Q^\dagger \bar{\sigma}^\mu Q) D^\nu (H^\dagger H) B_{\mu\nu}$~~

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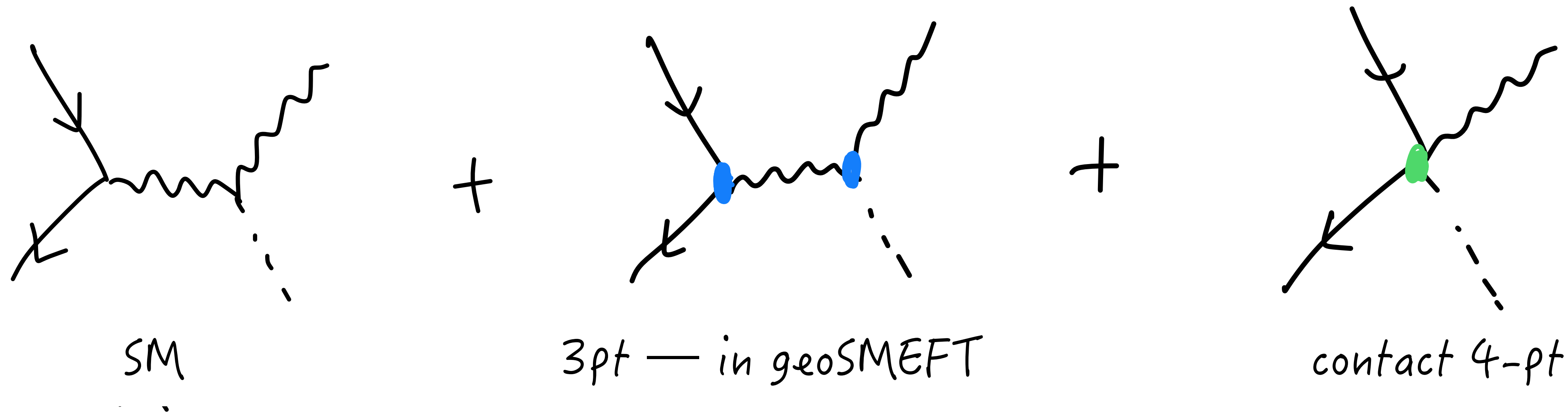


Energy enhanced effects

dim-6: vertex ~~$H^\dagger H\, W_{\mu\nu} W^{\mu\nu}$~~

contact $(Q^\dagger \bar{\sigma}^\mu \tau^I Q)\, H^\dagger \overleftrightarrow{D}_I H$

interference $\sim g_{SM}^2 \dfrac{\hat{s}}{\Lambda^2}$

squared $\sim \dfrac{\hat{s}^2}{\Lambda^4}$

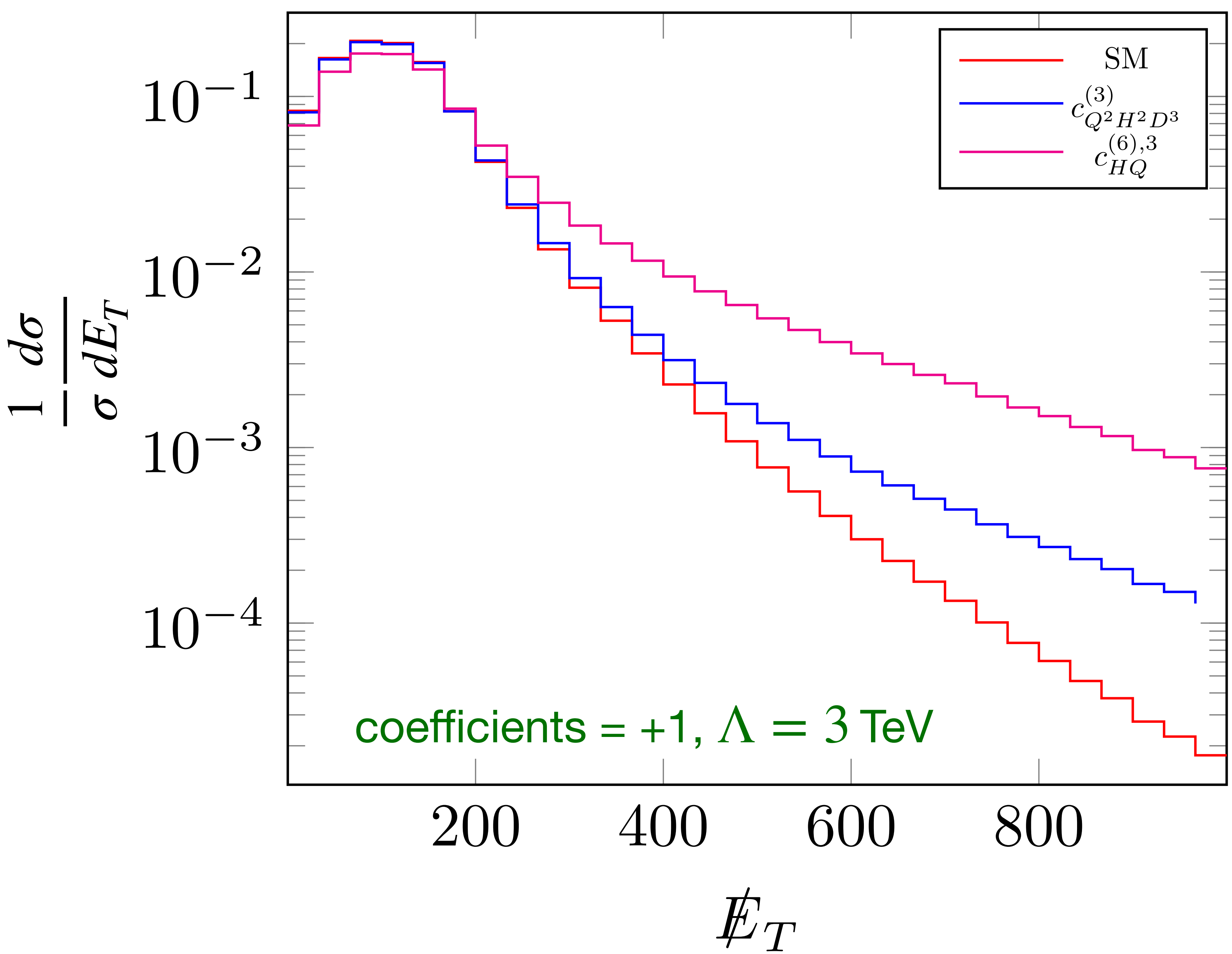
dim-8: contact $\psi^2 H^2 D^3 \supset (Q^\dagger \sigma^\mu D^\nu Q)(D^\mu H^\dagger D_\nu H)$

~~$\psi^2 H^2 X D \supset (Q^\dagger \bar{\sigma}^\mu Q) D^\nu (H^\dagger H) B_{\mu\nu}$~~

interference $g_{SM}^2 \dfrac{\hat{s}^2}{\Lambda^4}$

Do I gain something vs. using |dim-6|²

Effects at large $\hat{s}$ controlled by:

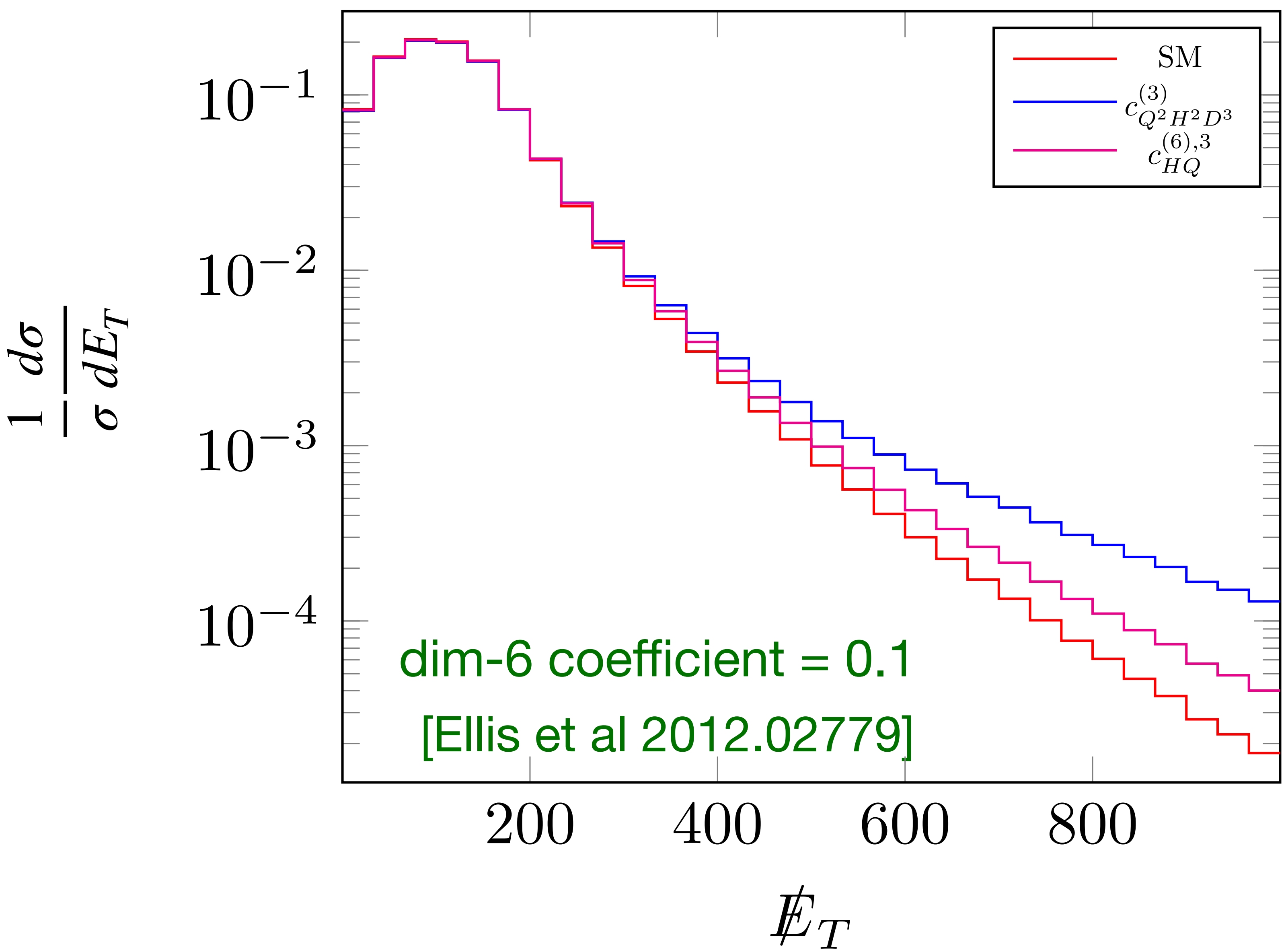


$$(Q^\dagger \bar{\sigma}^\mu \tau^I Q)\, H^\dagger \overleftrightarrow{D}_\mu \tau^I H$$

$$Q^\dagger \bar{\sigma}^\mu \tau^I D_\nu Q\, D^\mu H^\dagger \tau_I D_\nu H$$

Do I gain something vs. using |dim-6|^2

But, $Q^\dagger \bar{\sigma}^\mu \tau^I Q H^\dagger \overleftrightarrow{D}_I H$ etc. $\supset Q^\dagger \bar{\sigma}^\mu Q\, Z_\mu$ are constrained by LEP, while $Q^\dagger \bar{\sigma}^\mu \tau^I D_\nu Q\, D^\mu H^\dagger \tau_I D_\nu H$ are not ($\not\supset Q^\dagger \bar{\sigma}^\mu Q\, Z_\mu$)



complying with those constraints, dim-8 terms dominate in large $\hat{s}$ regime

Do I gain something vs. using |dim-6|²

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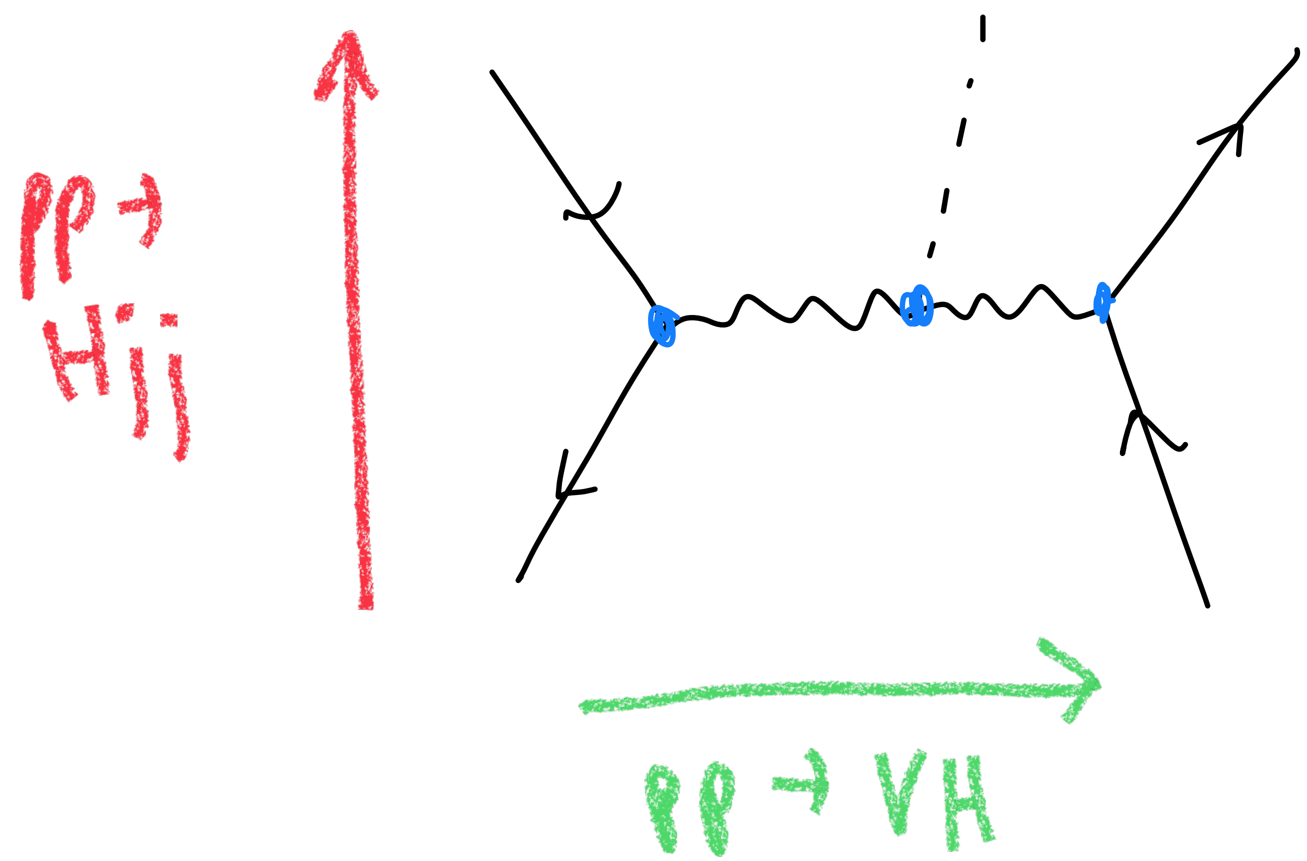
$Q^\dagger \bar{\sigma}^\mu \tau^I D_\nu Q \, D^\mu H^\dagger \tau_I D_\nu H$ are not

$\bar{q}q \to V(\bar{q}q)H$ + crossing symmetry gets us VBF

Therefore, expect similar operators to dominate, though kinematics and cuts are slightly different

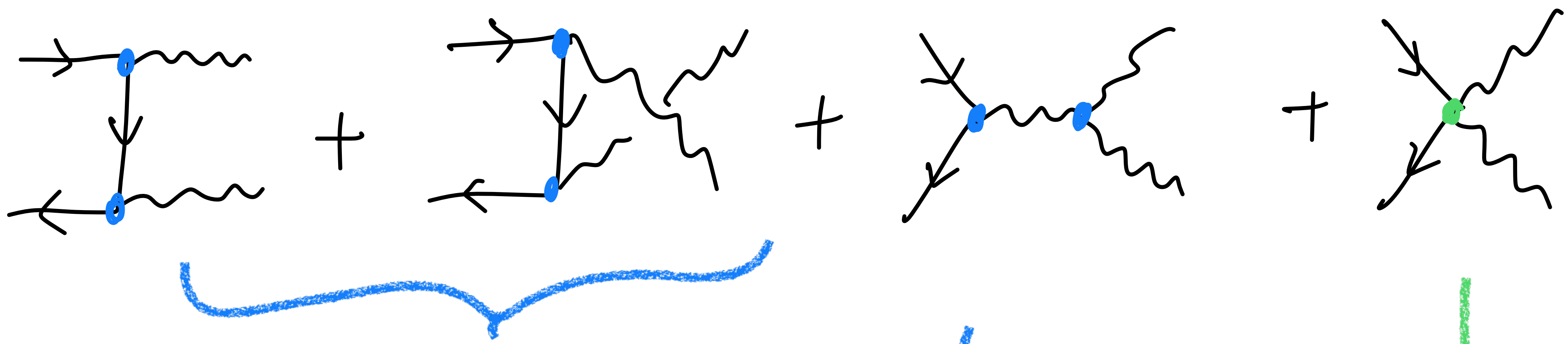
i.e.
$\psi^4 H^2$



[Assi, AM in progress]

Do I gain something vs. using |dim-6|²

Diboson



Assuming CP, $U(3)^5$, no energy enhancements

VVV is energy enhanced ($C_W W^3$). Important in global fit program, as first place triple gauge operators as appear.

Contact terms only show up at dim-8, ex. class $\psi^2 X^2 D$

Example: $\gamma W^{\pm}$, organize calculation by the polarizations of the W, γ

Do I gain something vs. using |dim-6|²

Energy scaling of different polarization amplitudes

$\epsilon_\gamma \epsilon_W$	SM	dim-6 C_W
$++$	$\frac{v^2}{s}$	$\frac{s}{\Lambda^2}$
$+-$	1	0
$+0$	$\frac{v}{\sqrt{s}}$	$\frac{v\sqrt{s}}{\Lambda^2}$

$\hat{s} \gg m_W^2$

$$|A_{SM}|^2 + \frac{2Re(A_{SM}^* A_6)}{\Lambda^2} + \frac{1}{\Lambda^4}|A_6|^2$$

with dim-6 alone, largest energy enhancement (to $\mathcal{O}(1/\Lambda^4)$) comes from from

$$|\textbf{dim-6 } C_W|^2 \sim \frac{s^2}{\Lambda^4}$$

Do I gain something vs. using |dim-6|^2

[AM, 2312.09867]

$\epsilon_\gamma \epsilon_W$	SM	dim-6 C_W (W^3)	dim-8 contact ($\psi^2 W^2 D$)
$++$	$\frac{v^2}{s}$	$\frac{s}{\Lambda^2}$	$\frac{s^2}{\Lambda^4}$
$+-$	1	0	$\frac{s^2}{\Lambda^4}$
$+0$	$\frac{v}{\sqrt{s}}$	$\frac{v\sqrt{s}}{\Lambda^2}$	$\frac{v s^{3/2}}{\Lambda^4}$

$\hat{s} \gg m_W^2$

$$|A_{SM}|^2 + \frac{2Re(A_{SM}^* A_6)}{\Lambda^2} + \frac{1}{\Lambda^4}|A_6|^2$$

$$+\frac{2Re(A_{SM}^* A_8)}{\Lambda^4}$$

But: dim 8

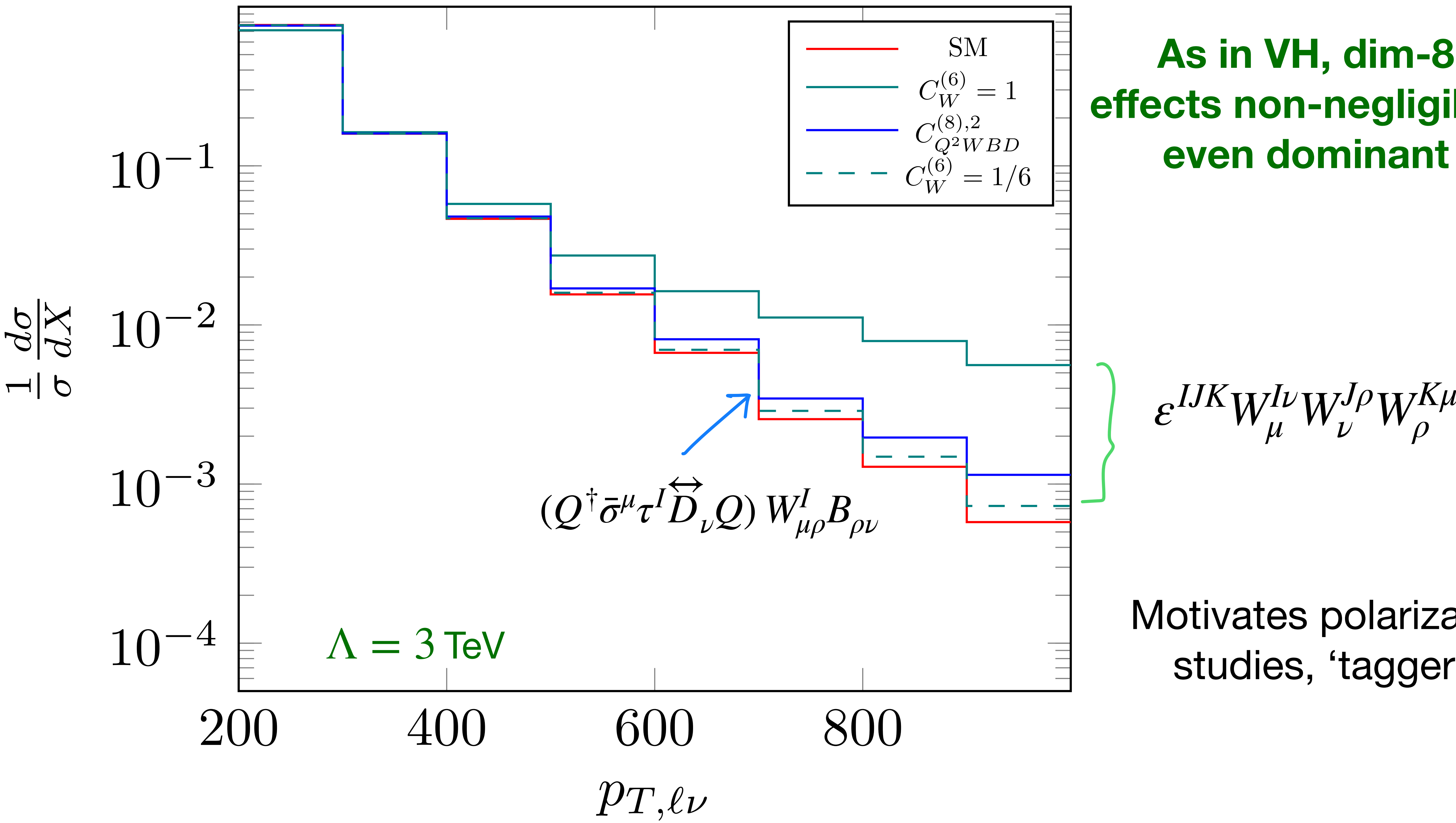
$$(Q^\dagger \bar{\sigma}^\mu \tau^I \overleftrightarrow{D}_\nu Q)\, W^I_{\mu\rho} B_{\rho\nu}$$

can interfere with dominant SM polarization

$$SM \times \textbf{dim-8} \sim \frac{s^2}{\Lambda^4}$$

See also Degrande 2303.10493 (for WW, WZ)

Do I gain something vs. using |dim-6|²



As in VH, dim-8 effects non-negligible, even dominant

Motivates polarization studies, ‘taggers’

Takeaways

To take advantage of 'energy frontier' at LHC, need to know next order SMEFT corrections.

$|\text{dim-6}|^2$ is an unreliable estimate at best! (And $|\text{dim-6}|^2$ > dim-6 x SM without good reason I don't trust at all)

geoSMEFT organization: minimizes operators that enter smallest (& most universal) vertices. Pushes new energy-enhanced effects to process-specific 4+ particle vertices

Facilitates full $\mathcal{O}(1/\Lambda^4)$ calculations. Several key processes relevant for global SMEFT program worked out.

Easy energy vs. vev counting: as first step, focus on energy enhanced terms to $\mathcal{O}(1/\Lambda^4)$. Assuming all WC are the same size, these will dominate kinematic tails

Takeaways

Facilitates full $\mathcal{O}(1/\Lambda^4)$ calculations. Several key processes relevant for global SMEFT program worked out.

Easy energy vs. vev counting: as first step, focus on energy enhanced terms. Assuming all WC are the same size, these will dominate kinematic tails

In either scenario, how do you actually include $\mathcal{O}(1/\Lambda^4)$ as uncertainty? At least from examples worked out so far, impact of $\mathcal{O}(1/\Lambda^4)$ strongly depends on process and kinematic regime…

What's the best format for $\mathcal{O}(1/\Lambda^4)$ theory calculations?