# Tutorial on Effective Field Theories and SMEFT

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LPC EFT Workshop
April 2024

Local quantum field theory is formulated on a continuum. It includes quantum fluctuations will all momenta and energies, up to $\infty$ . So it is no surprise that, when we compute, we find infinite results.

In your QFT course, you learned that this problem is solved by renormalization. You consider only “renormalizable QFTs”, that is QFTs with only operators of dimension $\leq 4$ in the Lagrangian. There are a finite number of coefficients, and these can be adjusted to absorb the infinite terms. Then it can be shown that all further predictions of the theory are finite.

Eventually, the infinities should be resolved by effects of new physics that comes in at higher mass scales. In this paradigm, those mass scales are viewed as being very far above the energy scales of interest.

[Note: in this lecture, I consider only UV infinities. IR infinities have a different origin and are cancelled in a different way.]

At the LHC, we have a different problem, and we need to address it in a different way. We have the Standard Model, which is well tested at energies up to 200 GeV. We ask: Is there new physics, in addition to the Standard Model, due to particles and forces of higher mass?

We do not need any new tool to discover effects of physics beyond the Standard Model. All we need to do is to take the most precise Standard Model prediction and compare it to experiment. That is quite enough for a discovery.

But if we want to get more out of the data than yes or no, we need a tool. If we do not discover a deviation, we would like to quantify the strength of the constraint that we have achieved. If we do discover a discrepancy, we would like to express its form in a precise way that can be compared to theory.

In particular, if we have not explicitly discovered new particles, we would like to test for effects of particles with mass M larger than our bounds. We do not know the nature of these (still hypothetical) particles, so our parametrization of their effects should be as general as possible.

For this, we should write the most general quantum field theory that extends the Standard Model, including systematically all possible terms of order $1/M^2$, order $1/M^4$, etc. due to unknown particles with mass M.

This is the question addressed by Effective Field Theory (EFT). To describe corrections to the Standard Model from higher energy physics, we will need a description that includes not only renormalizable operators but also “nonrenormalizable operators”, operator with dimension $> 4$ . We will need to “renormalize” these operators, that is to compute with them to achieve finite predictions in the context of local quantum field theory.

The role of nonrenormalizable operators in QFT was explained by Ken Wilson in the 1960’s. He considered the formulation of QFT as a functional integral and solved this integral by integrating out the degrees of freedom, starting systematically from the highest momenta and energies. Non-renormalizable operators naturally arise in this process. You can find references to Wilson’s work in my review, arXiv:1405.7086.

Wilson’s method was not so elegant. In 1979, Steven Weinberg showed that EFT calculations could be done very straightforwardly using dimensional regularization: Weinberg, Physica A96, 1 (1979). This is the method used in all current LHC analyses.

Now I can answer: What is an EFT ?

An Effective Field Theory is a local quantum field theory that

1. Is applicable to a particular momentum and energy scale
2. Is formulated using only fields associated with particles visible at that energy scale
3. Contains nonrenormalizable operators representing the (integrated-out) effects of physics at higher energy scales.

To guide the analysis of an EFT, we define a small parameter and work to a fixed order in this small parameter. Typically, the parameter is (p/M), where p is a relevant momentum and M is a heavy mass of a particle that is integrated out.

If the more complete theory underlying the EFT has a symmetry, the EFT must also have that symmetry. This is a strong restriction on the EFT.

For some problems, one uses a different ordering variable than energy or particle mass. For example, in Soft and Collinear Effective Theory (SCET), the ordering variable is transverse momentum. (Also, SCET is actually a collection of EFT's, linked by matching conditions.) In this lecture, I will stick to (p/M) as the ordering variable.

Our goal in this lecture is to understand Standard Model Effective Theory (SMEFT). But I would like to start by reviewing an earlier EFT that illustrates some of these general principles. This is the Chiral Effective Field Theory of QCD.

First, some background. The Lagrangian of QCD with 2 flavors (u,d) of massless quarks is

$$\mathcal{L} = -\frac{1}{4} G^A_{\mu\nu} G^{A\mu\nu} + \overline{Q}_{aL} i\overline{\sigma} \cdot D \; Q_{aL} + \overline{Q}_{aR} i\sigma \cdot Q_{aR}$$

where Q = (u,d). Remember that the kinetic term in the Dirac Lagrangian does not mix L and R components. The u and d quark masses are small on the hadronic mass scale of GeV, and so this Lagrangian is very close to real QCD.

This model manifestly has a large global symmetry

$$Q_{La} \to V_{Lab} Q_{Lb} \qquad Q_{Ra} \to V_{Rab} Q_{Rb}$$

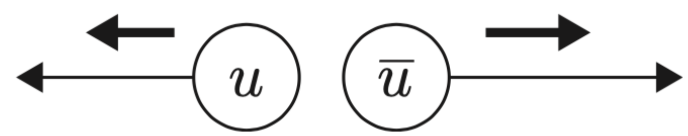
where $V_L, \; V_R$ are independent 2x2 unitary matrices. Thus the theory has an

$$SU(2)_L \times SU(2)_R$$

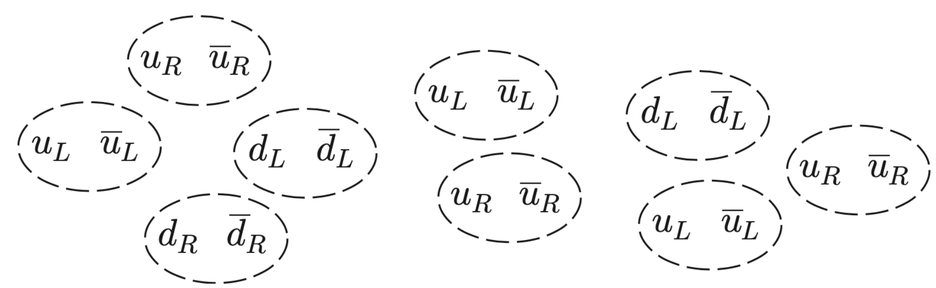
global symmetry.

A nonzero mass term would connect the L and R quarks and give them small masses. But the mass of quarks inside hadrons is about 300 MeV. A a larger effect must be at work.

In the massless theory, it costs almost noting to make a $q\bar{q}$ pair. Since these pairs are strongly bound at low momenta, we can lower the energy of the vacuum by creating a large number of pairs condensed into the vacuum structure. The physics is very similar to the physics that creates the superconducting ground state of a metal from electron pairing, except that in 3-dimensions, a strong coupling is needed. To form a Lorentz-invariant vacuum state, the condensing $q\bar{q}$ pairs should have zero spin




The QCD vacuum then has the structure:



Notice that a right-handed antiquark is the antiparticle of a left-handed quark; thus, this structure links the two SU(2)'s. This is spontaneous breaking of

$$SU(2)_L \times SU(2)_R \to SU(2)$$

Spontaneously breaking 3 global symmetries leads to 3 Goldstone bosons, forming an iso-triplet. These are just the pions. In this theory, the pions are massless. If we turn on the quark masses, the pions acquire small masses, with $m_\pi^2 \sim (m_u + m_d)$ .
In real QCD,

$$m_\pi^2 / m_\rho^2 = 0.03$$

In the massless theory, there are actually many ways to pair the L and R quarks. Let

$$\Delta_{ab} = \langle\Omega|\, Q_{La}\overline{Q}_{Rb}\, |\Omega\rangle$$

If u pairs with u and d with d,

$$\Delta_{ab} = \Delta \cdot \mathbf{1}_{ab}$$

Under SU(2)xSU(2), $\Delta_{ab}$ transforms as

$$\Delta_{ab} \to V_{Lac}\Delta_{cd}V^{\dagger}_{Rdb}$$

If SU(2)xSU(2) is a global symmetry, then every vacuum with

$$\Delta_{ab} = \Delta \cdot U_{ab}$$

where U is an SU(2) unitary matrix, has the same energy. If we let U depend on x, then the fluctuations in U(x) quantize to massless particles. These are just the pion Goldstone bosons. Adding back the quark mass term breaks this degeneracy, but only weakly, giving a small preference for the vacuum state with U = 1.

For future reference, U transforms under SU(2)xSU(2) as

$$U \to V_L U V_R^{\dagger}$$

Now we have the ingredients to formulate an EFT for the low-energy pion-pion dynamics of QCD. I propose — following Weinberg's papers of the 1960's — to write this Lagrangian in terms of the field U(x) or π(x):

$$U(x) = \exp[-i\pi^j(x) \cdot \sigma^j / f_\pi]$$

The Lagrangian will be applicable at energies

$$E \sim m_\pi \ll \mathrm{GeV}$$

but, in that energy range, it will be general enough to represent QCD as precisely as we would wish.

I will write this EFT as an expansion in momenta p (or, derivatives of π). It turns out that, at each order in momenta, there are only a finite number of terms that are allowed by SU(2)xSU(2) symmetry. With 2 derivatives, there is only 1 term:

$$\mathcal{L} = \frac{f_\pi^2}{4} tr[\partial_\mu U^\dagger \partial^\mu U]$$

There are no fully symmetric terms with zero derivatives, since $U^\dagger U = 1$ .
It can be shown that the parameter I call $f_\pi$ actually is equal to the pion decay constant, $f_\pi = 93\ \mathrm{MeV}$ (to leading order).

This effective Lagrangian already makes predictions for pion-pion scattering. Let's work these out, for simplicity, in the massless model. Notice that the pion-pion interaction is zero at zero momentum, so this interaction is always weak within our approximation scheme. Then expand in pion fields:

$$U = 1 - i\frac{\pi^j \sigma^j}{f_\pi} - \frac{1}{2}\frac{\pi^2}{f_\pi^2} + i\frac{\pi^2 \pi^j \sigma^j}{6 f_\pi^3} + \cdots$$

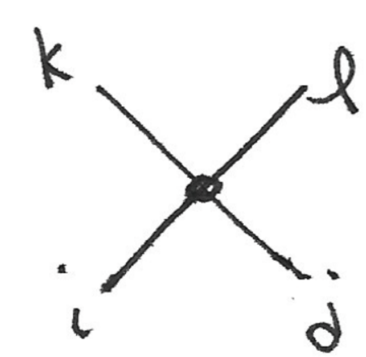
$$\partial_\mu U = -i\frac{\partial_\mu \pi^j \sigma^j}{f_\pi} - \frac{\pi^j \partial_\mu \pi^j}{f_\pi^2} + i\frac{2\pi^j \partial_\mu \pi^j \pi^k \sigma^k + \pi^2 \partial_\mu \pi^j \sigma^j}{6 f_\pi^3} + \cdots$$

$$\frac{f_\pi^2}{4}\mathrm{tr}[\partial_\mu U^\dagger \partial^\mu U] = \frac{1}{2}(\partial_\mu \pi^j)^2 + \frac{1}{2}\frac{(\pi^j \partial_\mu \pi^j)^2}{f_\pi^2} + \cdots$$

The Lagrangian is nonlinear in π, and in fact a specific interaction is predicted. According to the structure, the scattering amplitude is of order $p^2$:

$$i\mathcal{M} = -i\frac{2}{f_\pi^2}\Big(s(\delta^{ik}\delta^{jl} + \delta^{i\ell}\delta^{jk}) + t(\delta^{ij}\delta^{k\ell} + \delta^{i\ell}\delta^{jk}) + u(\delta^{ij}\delta^{k\ell} + \delta^{ik}\delta^{j\ell})\Big)$$

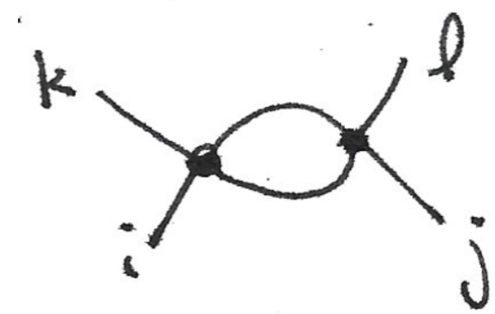
Projected onto partial waves, this predicts:




strong attraction in I = 0 J= 0
weak attraction in I = 1 J = 1
weak repulsion in I = 2 J = 0

"Weinberg scattering lengths"

Notice that the Lagrangian is already non-renormalizable; the interaction term is dimension 6. This will cause a problem if we go to 1-loop order, e.g.




This diagram generates a log-divergent term with 4 derivatives — dimension 8. To control this divergence, we need to add dimension 8 operators with arbitrary coefficients. These coefficients will need to be determined by experiment. The Lagrangian of the massless theory becomes

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{tr}[\partial_\mu U^\dagger \partial^\mu U] + \ell_1\ (\mathrm{tr}[\partial_\mu U^\dagger \partial^\mu U])^2 + \ell_2\ \mathrm{tr}[\partial_\mu U^\dagger \partial^\nu U \partial_\mu U^\dagger \partial_\nu U]$$

Actually, there are 3 structures. The third one can be generated from the two that are given by use of the equations of motion for U. It can be shown that terms in the Lagrangian that vanish by the equations of motion do not affect the values of scattering amplitudes. So this is the most general Lagrangian consistent with the symmetries.

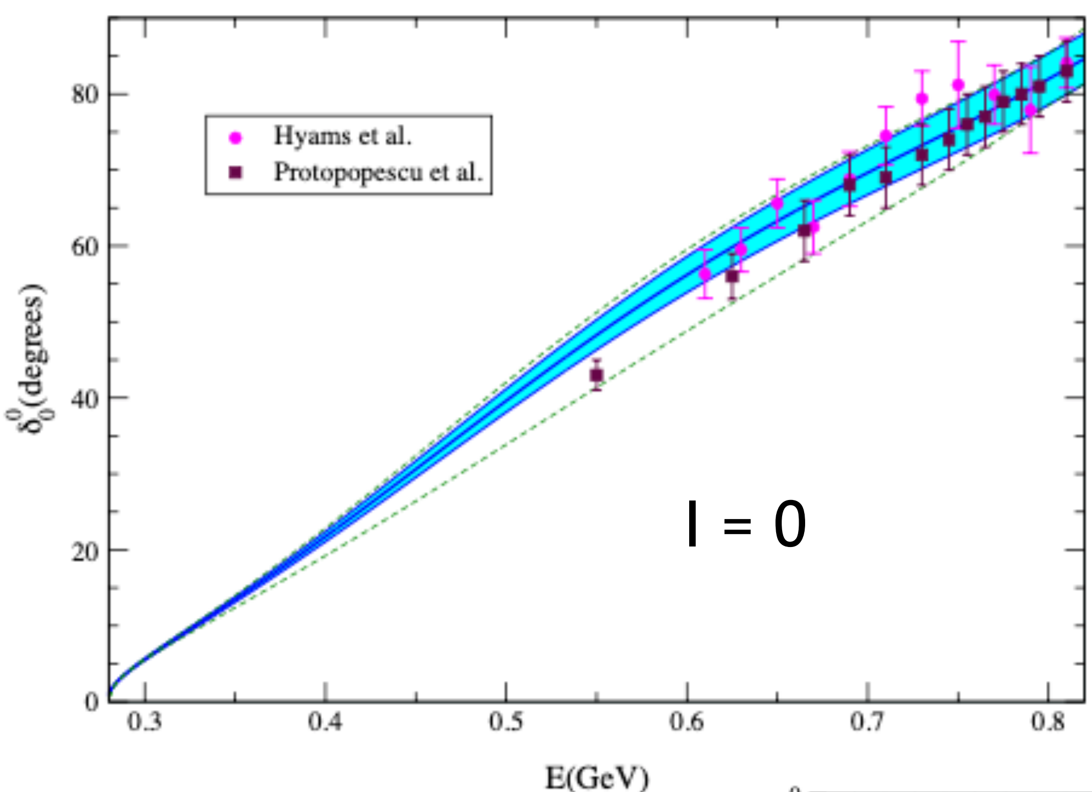
The new terms generate changes in the order $p^4$ term in the scattering amplitude.

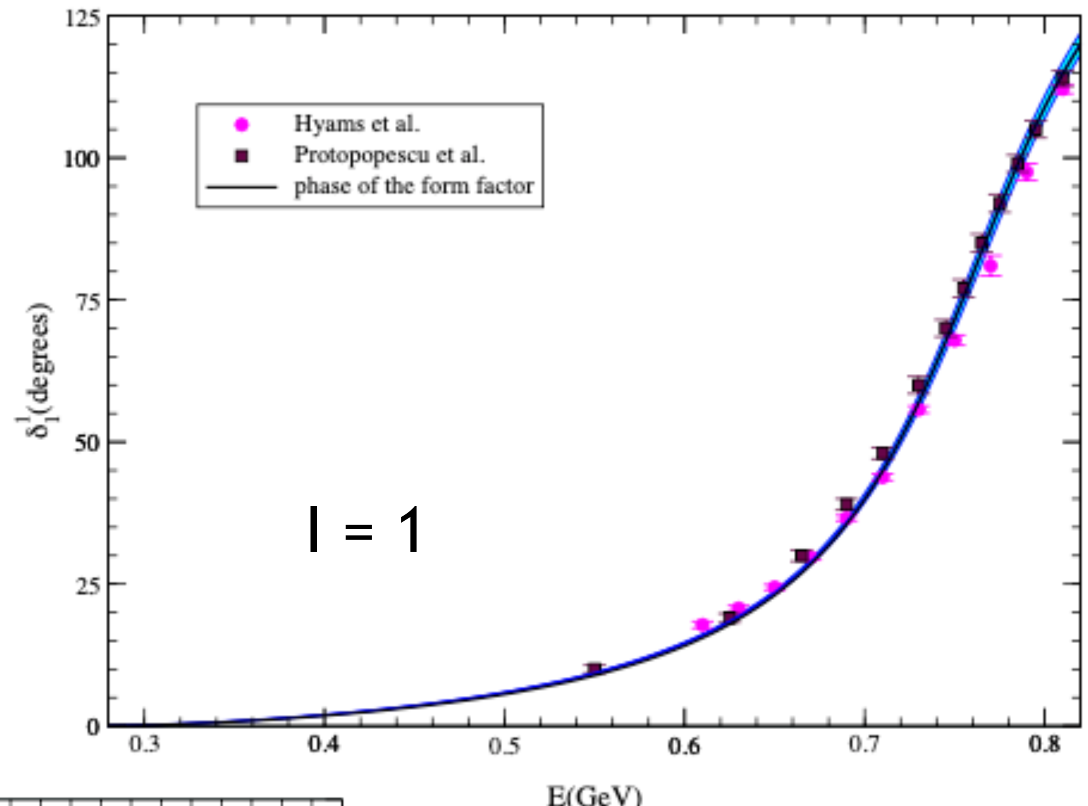
Now we can compute the complete $\pi - \pi$ scattering amplitude, up to order $p^4$, in terms of 3 parameters!

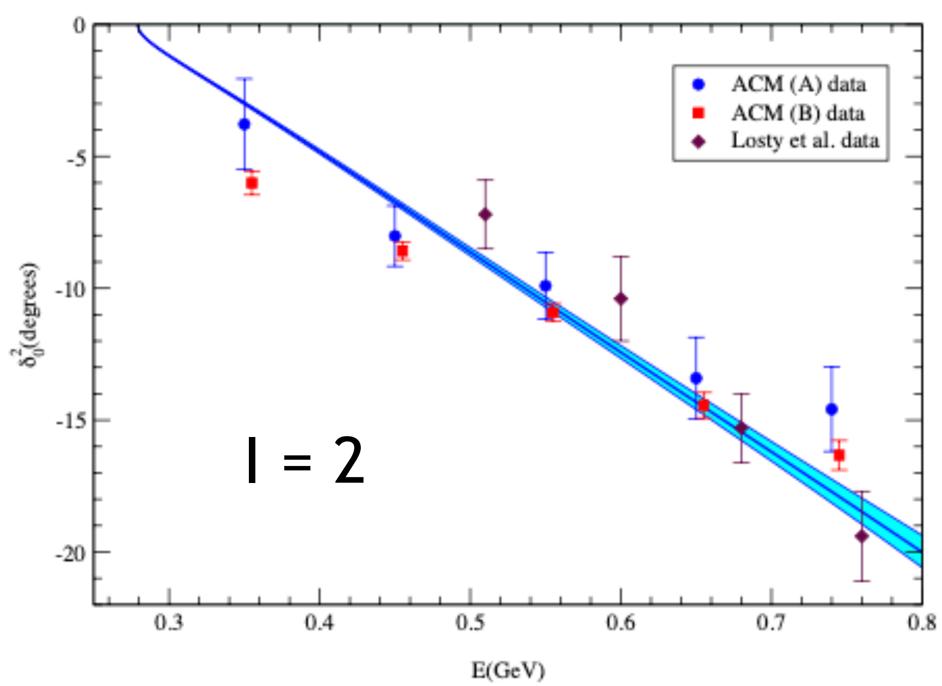
This program was carried out explicitly by (including finite quark mass effects)

1-loop order: Gasser and Leutwyler, Ann. Phys. 158, 142 (1984)
2-loop order: Colangelo, Gasser, and Leutwyler, arXiv:hep-ph/0103088 (2001)

Here are the resulting fits to the three partial wave amplitudes:



The partial wave amplitude for I = 1, J= 1 gets an especially large positive correction due to a large value of $\ell_2$ . Actually, we could measure $\ell_2$ even at 500-600 MeV.

Of course, the ρ meson was well known before any of this theory was developed. But what if we had been in the situation that, for reasons of technology, we could not reach CM energies greater than 600 MeV in pion-pion scattering? Using this precision theory, we could observe that the major resonance in pion-pion scattering is present in the I = 1, J = 1 channel. Then we could recognize that QCD has as its low-lying resonances

$$pion: \qquad 0^{-+} \qquad\qquad rho: \qquad 1^{--}$$

These are the lowest bound states of a system of spin-1/2 fermions and antifermions. Then we would be very close to discovering the quark model!

We would like to do something similar to this to discover physics beyond the Standard Model from LHC precision measurements.

Now we are ready to define SMEFT.

SMEFT is the EFT with the gauge symmetries of the Standard Model and the particle content that we observe in nature today.

That is:

SMEFT has the gauge symmetry SU(3)xSU(2)xU(1)
SMEFT is built from these gauge fields, plus 3 generations of quarks and leptons, plus 1 Higgs boson SU(2) doublet
SMEFT describes additional new physics present at higher energies by adding to its Lagrangian all possible operators consistent with the above.

Because SMEFT is constrained by its symmetries, any underlying theory with additional particles at masses above 1 TeV, as long as this theory has the full gauge symmetry, will be described by SMEFT at energies below 1 TeV.

SMEFT is easier than the Chiral Effective Field Theory because the symmetries act linearly on the usual fields; there is no need for a nonlinear variable like U(x). However, there are compensating complications, as we will see.

When we analyze using SMEFT as the Lagrangian, we can organize the calculation in terms powers of (1/M), where M is the mass scale of new physics. By dimensional analysis, an operator of dimension d has in front of it a coefficient

$$c_i \sim 1/M^{d-2}$$

The general SMEFT Lagrangian can be written as an expansion in the operator dimension:

$$\mathcal{L} = (d=2) + (d=4) + (d=5) + (d=6) + \cdots$$

There is one possible dimension 2 operator, the Higgs field mass term

$$\mathcal{L}_2 = \mu^2 \Phi^\dagger \Phi$$

The dimension 4 operators are the SM kinetic terms, Yukawa couplings, and Higgs quartic interaction.

$$\mathcal{L}_4 = -\frac{1}{4} F^A_{\mu\nu} F^{A\mu\nu} + \overline{f}(i\not{D})f + |D_\mu \Phi|^2 + \lambda |\Phi^\dagger \Phi|^2 \\ + y_e^{ij} \overline{L}^i \cdot \Phi e_R^j + y_d^{ij} \overline{Q}^i \cdot \Phi d_R^j + y_u^{ij} \overline{Q}^i_a \epsilon_{ab} \Phi^*_b u_R^j + h.c.$$

It can be shown that the Yukawa coefficients $y_e^{ij}$ etc. can be diagonalized by a change of variables, with the only price being that the charged current weak interaction acquires a 3x3 unitary matrix (the CKM matrix).

Thus, the dimension 2 and 4 terms are nothing else but the Standard Model with its most general parameters. The effect of new physics on these terms is to modify the parameters of the Standard Model, but these are in any event fit to experiment.

There is one possible dimension 5 operator, the Weinberg operator for generating neutrino mass:
$$\mathcal{L}_5 = -\mu_{ij}(\epsilon_{ab}L^i_a\Phi_b)(\epsilon_{cd}L^j_c\Phi_d)$$

This is unimportant for LHC experiments. In fact, all operators of odd dimension violate either Lepton or Baryon Number, so I will ignore all of these from here on.

Then the SMEFT Lagrangian takes the form
$$\mathcal{L} = \mathcal{L}_{SM} + \sum_j \frac{\bar{c}_j}{M^2}\mathcal{O}^{(6)}_j + \sum_k \frac{\bar{d}_k}{M^4}\mathcal{O}^{(8)}_k + \cdots$$
Alternatively, since we do not know where the new physics scale is, we can write the Lagrangian as
$$\mathcal{L} = \mathcal{L}_{SM} + \sum_j \frac{c_j}{v^2}\mathcal{O}^{(6)}_j + \sum_k \frac{d_k}{v^4}\mathcal{O}^{(8)}_k + \cdots$$
where $v = 246\ \mathrm{GeV}$ is the expectation value of the Higgs field. The coefficients, in whatever normalization, are called “Wilson coefficients”, suggesting a relation to operator product expansions.

To calculate with this Lagrangian and obtain finite results, we must include all operators obeying the symmetries of the Standard Model, including dimension 6 for calculations up to $1/M^2$, dimension 8 for calculations up to $1/M^4$, etc.

In this context, SMEFT is the most general description of the Standard Model and its extensions.

When we integrate out the effect of heavy particles, there is no visible effect on the dimension 2, 4 operators. Any corrections can be absorbed by changing the SM input parameters. Thus, this effect can only be seen in the coefficients of dimension 6, dimension 8, etc. operators.

If the heavy particles affect low-energy physics at the tree level (e.g. heavy Higgs bosons mixing with the 125 GeV Higgs boson, heavy Z' appearing in fermion-fermion scattering), the effect on $\overline{c}_j, \overline{d}_k$ will be of order 1. If the heavy particles affect low-energy physics only at the loop level, the effect on $\overline{c}_j, \overline{d}_k$ will be of order $g^2/(4\pi)^2$. We might be lucky and find large dimensionless factors, but this is the general expectation.

Thus, according to SMEFT, all visible effects of new physics on low-energy observables are naturally at the level of $v^2/M^2$ ~ a few percent.

An issue with SMEFT is that there is a very large number of dimension 6 operators satisfying the requirements to be included in the SMEFT Lagrangian, and this number increases exponentially with dimension. For dimension 6, counting an operator and its conjugate as separate, one finds

| | |
|---|---|
| 1 fermion generation: | 84 operators |
| 3 fermion generations: | 3045 operators |

(A detailed counting scheme is given in Henning, Lu, Melia, and Murayama, 1512.03433). Any fit to the SMEFT Lagrangian must reduce the set of free coefficients using assumptions that may or may not be justified.

The full set of dimension 6 operators was first worked out by Grzadkowski, Iskrzynski, Misiak, and Rosiek, 1008.4888 . Their particular set of operators is called the "Warsaw basis". Adam Martin gave you a list of these operators in his talk on Monday (pp. 7-8).

As I noted above, operators that are equivalent to others using the equations of motion are redundant. The Warsaw basis is a set inequivalent operators, so it involves some conventions. For example, operators like

$$\Phi^\dagger t^a \overleftrightarrow{D}_\mu \, \Phi \; D_\nu W^{a\mu\nu}$$

are eliminated in favor of others.

Many dimension 6 operators do not play the roles that appear at first sight. For example, consider the product of a fermion current and a Higgs current,

$$\Phi^\dagger \overleftrightarrow{D}_\mu \Phi = \Phi^\dagger D_\mu \Phi - D_\mu \Phi^\dagger \Phi$$

Thus, for the electron, we have

$$\Phi^\dagger \overleftrightarrow{D}_\mu \Phi \; L^\dagger \overline{\sigma}^\mu L \;, \quad \Phi^\dagger t^a \overleftrightarrow{D}_\mu \Phi \; L^\dagger \overline{t}^a \sigma^\mu L \;, \quad \Phi^\dagger \overleftrightarrow{D}_\mu \Phi \; e_R^\dagger \sigma^\mu e_R$$

In general, each pair of quarks yields 4 such operators. Since, in the electroweak vacuum,

$$\Phi^\dagger \overleftrightarrow{D}_\mu \Phi = -i \frac{g}{c_w} v^2 Z_\mu + \cdots$$

these operators correct the strength of the Z-fermion vertices in a flavor-dependent way. The precision electroweak constraints are quite strong, so the corresponding Wilson coefficients are constrained at the $c < 0.001$ level.

Similarly, consider

$$(\Phi^\dagger \Phi) \partial^2 (\Phi^\dagger \Phi) = -|\partial_\mu (\Phi^\dagger \Phi)|^2 = v^2 (\partial_\mu H)^2 + \cdots$$

This operator corrects the field strength of the Higgs field and thus gives a uniform rescaling of all Higgs boson couplings.

An important aspect of SMEFT is that it relates the BSM corrections to processes with different numbers of HIggs bosons. For example,

$$\Phi^\dagger \Phi W^a_{\mu\nu} W^{a\mu\nu} = \frac{1}{2}(v^2 + 2vh + h^2)(W^a_{\mu\nu})^2$$

The first term here corrects the W field strength, the second is a correction to the form factors in H-> WW decay, the third is a correction to H pair production. At the level of $1/M^2$ corrections (dimension 6 operators), these all have the same coefficient.

There are also vertices that receive no correction from dimension 6 operators. Thus, SMEFT, despite its large number of parameters, has nontrivial predictions. For example, the HHWW vertex with no derivatives arises only from the Higgs kinetic term

$$(D_\mu \phi)^\dagger (D_\mu \Phi)$$

and does not receive any correction from dimension 6 operators (except from the Higgs rescaling correction mentioned earlier). In the current LHC Higgs pair production analysis, this vertex is assigned a correction parameter called $\kappa_{2V}$. It is a prediction of dimension 6 SMEFT that $\kappa_{2V} = 1$ , and this holds experimentally to about 20% accuracy.

What are the conditions for the applicability of SMEFT ?

So far, I have been very conservative, quoting SMEFT predictions only up to linear order in the dimension 6 Wilson coefficients. If this suffices to explain the experimental data, we are on safe ground and we can use SMEFT as a quantitative tool. Beyond this,

(1) If the data does not fit the Standard Model expectation, that is a discovery. SMEFT is not needed for this. SMEFT is only useful to interpret a discrepancy or a constraint.

(2) SMEFT is a general parametrization of new physics effects only if all relevant operators are included with adjustable Wilson coefficients. Since there are 59 or 84 dimension 6 operators at least, this is difficult to achieve. Typically we would put a subset of these operators into our theory, with arguments that these are the leading effects. For many operators, this is well justified. (e.g. u and d Yukawa couplings are expected to be very small, so their SMEFT corrections are probably also small on an absolute scale.) But, formally, dropping any dimension 6 operator is a model-dependent assumption.

(3) Since SMEFT is an expansion in $1/M^2$ , where $M$ is the mass of heavy new particles, expressions proportional to the square of dimension 6 Wilson coefficients $c_j^2$ are of the same order as terms linear in dimension 8 Wilson coefficients $d_k$ . So going beyond linear order in the $c_j$ invites the addition of many more variables to the fit. There are many, many dimension 8 operators. So, IMHO, inclusion of quadratic terms in the $c_j$ is not meaningful. If the fit does not close, so be it.

(4) One reason that a $c_j$ can be large is that the new particles that generate this term are light (e.g. 200 GeV extended Higgs bosons or superpartners not yet excluded by LHC). In that case, SMEFT cannot be exact.

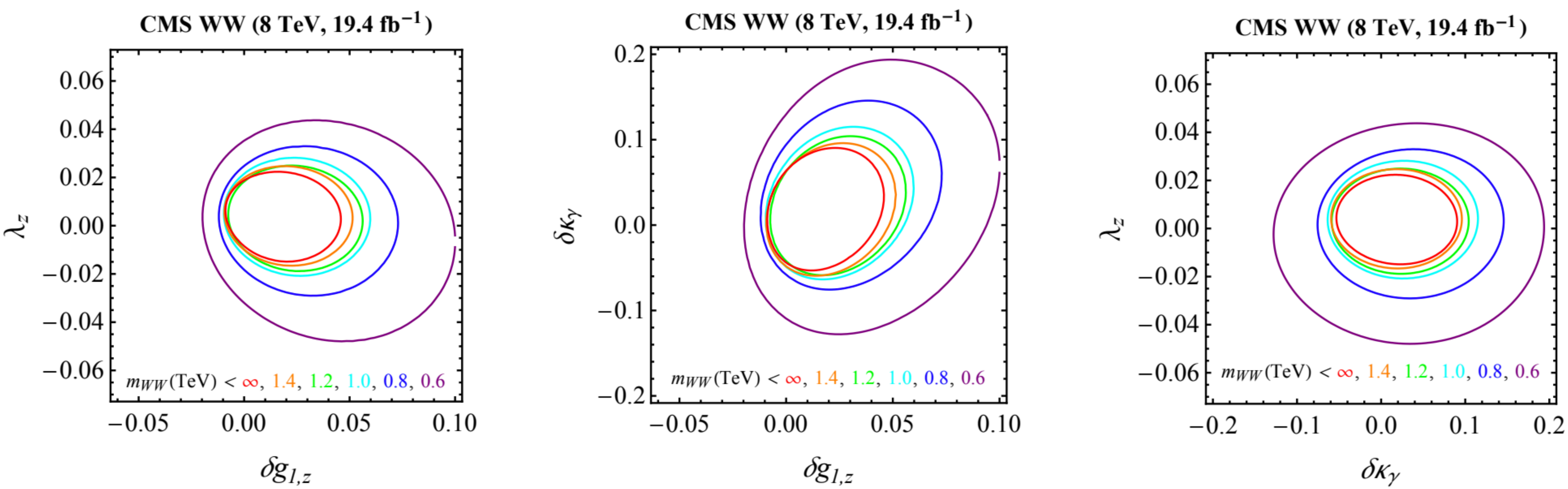
But it may still be useful to get qualitative information from a linear SMEFT analysis. In precision electroweak, analyses linear in S and T give reasonable estimates of the top quark and Higgs boson masses.

(5) In reactions with high energy and large momentum transfer, amplitudes using dimension 6 operators will typically be enhanced over amplitudes from dimension 4 operators only by a factor $E_{CM}^2$. Then the effect of the dimension 6 operators will be enhanced for higher parton CM energy. The relevant expansion parameter for dimension 6 SMEFT operators is then

$$\frac{E_{CM}^2}{v^2}\, c_j \sim \frac{E_{CM}^2}{M^2}$$

So "energy helps accuracy" (arXiv:1609.08157). But, this can be a devil's bargain. If you optimize for maximum sensitivity, you push the analysis into the region where SMEFT is not valid.

In arXiv:1609.06312, Falkowski et al. discuss the extraction of the SMEFT parameters appearing in the triple gauge boson coupling at the LHC, going into these issues in more detail. In general, one must choose where to cut off an analysis of W pair production at a hadron collider as $m(W^+W^-)$ increases in order not to exceed the region of validity of SMEFT.




m(WW) < 0.6, 0.8, 1.2, 1.4, ∞ TeV

It is interesting to ask whether the estimation $c_j \sim v^2/M^2$ is generally valid, or whether there can be hierarchies within the $c_j$ .

My personal attitude is that if new physics is associated with the mystery of electroweak symmetry breaking, then those Wilson coefficients closest to the Higgs boson should be the largest. Then we can hope for

$$c_f, c_H, c_{WW} \sim \text{few \%}$$

while Wilson coefficients contributing only to precision electroweak are smaller, maybe by a factor of $\alpha_w/4\pi$ . If there are light scalar bosons that mix with the Higgs field it is possible that $c_6$ could be of order 1. (It had better be so large if we expect to see a deviation in $c_6$ from foreseen accelerators.)

On the other hand, it could be that all $c_j$ are roughly of the same order of magnitude. Then it would be better to pursue further precision electroweak experiments, which will allow stronger absolute bounds on the $c_j$ .

What do you think ?

Some examples of effects that generate especially large contributions to the Wilson coefficients $c_f, c_H, c_{WW}$ affecting the Higgs vertices are given in arXiv:2209.03303 .

In general, operators in quantum field theory are not scale-invariant. In addition to dimensional analysis scaling, loop corrections produce additional, logarithmic, scalings. For a single operator, one can compute the “anomalous dimension” in perturbation theory. For QCD renormalization, this anomalous dimension has the form

$$\gamma_j = -a_j \, \frac{g_s^2}{(4\pi)^2}$$

In general, the operator will be normalized by the value of an expectation value at a particular momentum scale $M$. Then, operators normalized at the scales $M_1$ and $M_2$ are related by

$$[\mathcal{O}_j]_{M_1} = Z_j(M_1, M_2) \; [\mathcal{O}_j]_{M_2}$$

where, for QCD above the top quark mass scale, to leading order,

$$Z_j(M_1, M_2) = \left( \frac{\log(M_1^2/\Lambda^2)}{\log(M_2^2/\Lambda^2)} \right)^{-a_j/2b_0}$$

where $b_0 = 7$ is the first β function coefficient. In general, $Z_j(M_1, M_2)$ is found by solving a renormalization group equation.

In a theory with many operators of the same dimension, the anomalous dimension will be a matrix, corresponding to mixing of the operators on a logarithmic scale. In the simplest case of QCD corrections above the top mass scale,

$$\gamma_{jk} = -\mathbf{a}_{jk} \frac{g_s^2}{(4\pi)^2}$$

and

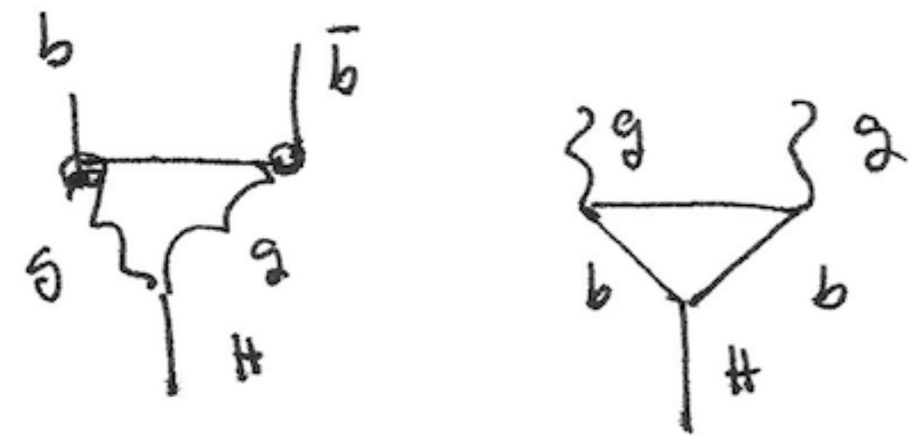
$$[\mathcal{O}_j]_{M_1} = \left[ \left( \frac{\log(M_1^2/\Lambda^2)}{\log(M_2^2/\Lambda^2)} \right)^{-\mathbf{a}/2b_0} \right]_{jk} [\mathcal{O}_k]_{M_2}$$

The complete anomalous dimension matrix was worked out for the Warsaw basis by Jenkins, Manohar, Trott, and Alonso, arXiv:1308.2627, 1310.4838, 1312.2014.

This operator rescaling and mixing is relevant for two reasons.

First, integrating out new particles of mass M generates SMEFT operators at the scale M. These need to be carried down to the scale of v or $m_h$ using the renormalization group. This may induce additional operators not present in the expression at the scale M.

Second, operator mixing is one part of a general phenomenon that operators can produce different physical outcomes as the result of radiative corrections. For example, the diagrams below allow the operator $\Phi^\dagger \Phi G_{\mu\nu} G^{\mu\nu}$ to produce b-tagged final states and the operator $y_b \Phi^\dagger \Phi\, Q^\dagger \cdot \Phi b_R$ to produce non-b-tagged final states.




The upshot is that SMEFT gives a well-defined way to define the Higgs couplings, and the deviations from the Standard Model more generally, at one-loop and also at higher orders in perturbation theory. Alternative methods such as the κ parameters and pseudo-observables eventually become ambiguous. To fully specify SMEFT, we do need to choose a reference renormalization scale for all dimension 6 (and higher dimension) operators.

This completes my short tutorial on SMEFT.   I have given you the basic definitions of SMEFT and some theoretical background to supplement the practical advice you received earlier in the week.

I have also pointed out many subtleties of SMEFT that you will meet as you work in this subject, and given you my personal opinions on how to deal with them. Tomorrow, I think, you will see many of these points debated, with different solutions offered.  Eventually, with more experience, you can find your own point of view.

Good luck, and happy SMEFTing !

General references, in addition to those given in the text:

An excellent general review of SMEFT:

Brivio and Trott, arXiv:1706.08945   Phys. Repts.  793, 1 (2019)

Discussion of SMEFT for Higgs and the Higgs self-coupling:

Henning, Murayama, Lu, arXiv:1412.1837    JHEP 2016, 023 (2016)
De Florian et al.  “Handbook of LHC Higgs Cross Sections - 4”
arXiv:1610.07922 ,   Section II
DiMicco et al.,  arXiv:1910.00012    Rev. Phys. 5, 100045 (2020)