**EFT intro part II**

# **Standard Model Effective Field Theory in more detail, practice**

Adam Martin [\(amarti41@nd.edu\)](mailto:amarti41@nd.edu)



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# **Standard Model Effective Field Theory (SMEFT)**

= SM extended by higher dimensional operators formed solely from SM field content and their covariant derivatives

$$
\mathcal{L} = \mathcal{L}_{SM} + \sum_{d} \sum_{i} \frac{c_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^{(d)}(Q, u_c, d_c, L, e_c, H, D_\mu, F_{\mu\nu} \cdots)
$$

 $\Lambda$  = scale of heavy new physics

 $c_i^{(d)}$  = `Wilson coefficients', encode info about interactions between heavy new physics and us *i*

$$
\text{Ex.}\n\begin{array}{ccc}\n\text{c} \ (HL)^2 & \text{c'} \ Q \ H \ \sigma_{\mu\nu} u_c \ B^{\mu\nu} & \text{c'} \ (Q^\dagger \bar{\sigma}^\mu Q) (i \ H^\dagger \overleftrightarrow{D}_\mu H) & \text{c'} \ (Q^\dagger \bar{\sigma}_\mu Q)^2 \\
\Lambda^2 & \Lambda^2 & \Lambda^2 & \end{array}
$$

EFT requirement: ratio of scales

$$
\frac{E}{\Lambda} = \frac{\text{Energy of process } (\sim \text{LHC } \sqrt{\hat{s}} \,)}{\text{Mass of some heavy stuff}}
$$

lowest dimension operators (= fewest  $\Lambda$ ) should be the most important effects

Top down: if we have ANY\* cool BSM particles but they are **too heavy** to produce on-shell, can map onto SMEFT ("integrate them out")

\*no other light particles (axions, DM), linearly realized EWSB

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*<sup>μ</sup>* <sup>−</sup> *<sup>M</sup>*<sup>2</sup> *Z*′ ℒ ⊃ *gZ*′*Q*† *σ*¯*μQ Z*′ ′*μ Z*′ *μZ* Ex.) 2 *g*2 *Z*′(*Q*† 2 *σ*¯*μQ*) ̂≪ *M* , expand <sup>2</sup> if *s Z*′ ̂− *M*<sup>2</sup> *s Z*′ −*g*<sup>2</sup> ) *ci* <sup>=</sup> <sup>−</sup> *<sup>g</sup>*<sup>2</sup> *Z*′ *Z*′ (*Q*† <sup>2</sup> + (*s*/̂*M*<sup>4</sup> *σ*¯*μQ*) *Z*′ *M*<sup>2</sup> *Z*′ Λ = *MZ*′ 

\*no other light particles (axions, DM), linearly realized EWSB

#### **Can also use as a bottom-up EFT: Turn on all operators**

Operators parameterize all possible deviations in SM inter-particle interactions. Bound these operators by precision measurements of how SM particles interact.

Use that information to inform about mass scale and properties of UV theories

**How many operators are there?** 

Dim 5: 12

Dim 6: 2499

Dim 7: 948

Dim 8: 36971

#### **But — can reduce the numbers by adding assumptions**

- Only keep B, L preserving operators. Removes all odd dimension operators
- Only keep CP preserving operators
- Assume some flavor symmetry

often `flavor universality' = effects are the same for all generations

$$
(Q_i^{\dagger} \bar{\sigma}^{\mu} Q_j)(i H^{\dagger} \overleftrightarrow{D}_{\mu} H) \longrightarrow (Q_i^{\dagger} \bar{\sigma}^{\mu} Q_i)(i H^{\dagger} \overleftrightarrow{D}_{\mu} H)
$$

Dim 6: 59 Dim 8: 993 **More manageable** 

# operators fixed, but exact form is not. Need to pick a basis to cover all effects

### **0-fermion and 2-fermion operators in "Warsaw basis"**



Table 2: Dimension-six operators other than the four-fermion ones. Here,  $\varphi = H; p, r, \cdots =$  flavor indices (so  $p = r$  -> flavor universal)

### **4-fermion operators in "Warsaw basis"**



**Implemented in MadGraph UFO models via SMEFTsim, SMEFT@NLO**

#### **What do these operators actually do?**

**SMEFT** is actually a double expansion, 
$$
\frac{v}{\Lambda}
$$
 and  $\frac{E}{\Lambda}$ 

Means higher dimensional operators can trickle down into lower dimension

Ex.)

\n
$$
\frac{c_{HQ}(Q_i^{\dagger} \bar{\sigma}^{\mu} Q_i)(i H^{\dagger} \overleftrightarrow{D}_{\mu} H)}{\Lambda^2} \qquad \varphi \qquad \
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$$
\n
$$
\Lambda^2 \quad \text{New } Q^2 h^2, Q^2 h Z_{\mu} \text{ vertices}
$$
\n
$$
(D_{\mu} H \supset Z_{\mu} H, H = (\nu + h) \quad \text{or}
$$
\n
$$
(\nu + h) \quad \text{or}
$$



but also changes how *Q* couples to 
$$
Z_{\mu}
$$
!  
\n
$$
\delta g_{ZQQ} \sim \frac{g c_{HQ} v^2}{2\Lambda^2}
$$

[quite constrained from LEP I!]





 $c_{HW} H^\dagger H W^I_{\mu\nu} W^{I,\mu\nu}$  $\overline{\Lambda^2}$  $h$ W W  $\bigotimes$ New  $h^2 W^2$  interaction **W**  $\mathsf{W}$ 

But, setting  $H \to \nu/\sqrt{2}$ , looks like a shift in the W kinetic term

$$
\mathcal{L} \supset -\frac{1}{4} \bigg( 1 - \frac{c_{HW} v^2}{2\Lambda^2} \bigg) W_{\mu\nu}^I W^{I,\mu\nu}
$$

We can redefine  $W_{\mu\nu} = \frac{R^2}{\sqrt{R^2-\mu^2}}$  = to get back usual  $\hat{W}_\mu^I$ ̂ *μν*  $1 - \frac{c_{HW}v^2}{2\Delta^2}$  $2\Lambda^2$ 

normalization, but we have to do that everywhere consistently

In particular, mass matrix:

$$
|D_{\mu}H|^2 \supset \frac{v^2}{2} (W_{3\mu} \quad B_{\mu}) \begin{bmatrix} g^2 & -gg' \\ -gg' & g^2 \end{bmatrix} {W_{3\mu} \choose B_{\mu}}
$$

Diagonalizing this gets us  $Z_\mu, A_\mu$  and coupling to  $Z_\mu, A_\mu$  is what we use to define electric couplings  $e, \, \sin \theta_W$ 

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But with the normalization change…

$$
\frac{v^2}{2} \left( \hat{W}_{3\mu} - B_{\mu} \right) \left[ \frac{\frac{g^2}{1 - c_{HW} v^2 / (2\Lambda^2)}}{\frac{gg'}{v^2}} - \frac{\frac{gg'}{\sqrt{1 - c_{HW} v^2 / (2\Lambda^2)}}}{g'^2} \right] \left( \hat{W}_{3\mu} \right)
$$

Mixing angles and couplings now depend on  $c_{HW}$ : sind

$$
\hat{\theta}_W = \frac{g'}{\sqrt{g^2 + g'^2}} + \mathcal{O}(c_{HW})
$$

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$$

Mixing angles and couplings now depend on  $c_{HW}$  :  $\sin \hat{\theta}_W =$ *g*′  $g^2 + g^2$ +  $\mathcal{O}(c_{HW})$ 

Net result: SMEFT operators enter in subtle ways and in multiple processes

### **SMEFT approach is a global approach**



**Operators impact multiple processes: Global approach needed**

Remember the goal: from pattern in deviations, **determine** Λ

# **SMEFT approach is a global approach**



[Ellis et al 2012.02779]

#### **SMEFT approach is a global approach** 3) Properly eigenvectors of constraint, not individual op limits - what are the spaces?



Composition

2012.02779 John Ellis, Maeve Madigan, Ken Mimasu, Veronica Sanze, and Tevong You



### **Looking for heavy new physics**





# **Looking for heavy new physics**



### **OK lets calculate something! Good idea to understand what we should get before diving in**

$$
A = A_{SM} + \frac{(A_{6,1}v^2 + A_{6,2}vE + A_{6,3}E^2)}{\Lambda^2} + \cdots
$$

 $A_{6,i}$  are functions of Wilson coefficients  $c_i$ . Formed by turning new operators into Feynman rules (ex. FeynRules), calculating away

$$
|A|^2 = |A_{SM}|^2 + 2 \operatorname{Re} \left( A_{SM}^* \frac{(A_{6,1}v^2 + A_{6,2}vE + A_{6,3}E^2)}{\Lambda^2} \right) \underset{\text{interference term}}{\longleftrightarrow} + \frac{1}{\Lambda^4} |(A_{6,1}v^2 + A_{6,2}vE + A_{6,3}E^2)|^2
$$
   
 
$$
\underbrace{\uparrow} \qquad \qquad \text{New physics}
$$

$$
2 \operatorname{Re} \left( A_{\text{SM}}^* \frac{(A_{6,1} v^2 + A_{6,2} v E + A_{6,3} E^2)}{\Lambda^2} \right)
$$

Know  $\Lambda \gg v, E$  . But  $v$  is fixed while  $E$  can vary (can be 'selected' by analysis cuts to focus on tails of distributions, etc.)

Means for fixed 
$$
\Lambda
$$
 (and  $c_i$ ) can have  $\left(\frac{E}{\Lambda}\right) \gg \left(\frac{v}{\Lambda}\right)$ , so we can be  
sensitive to smaller  $A_{6,3} \sim c_i$ 

This is **the main advantage** of SMEFT at LHC. Needs a combination of energy ( $E > v$ ) and precision!

$$
2 \operatorname{Re} \left( A_{\text{SM}}^* \frac{(A_{6,1} v^2 + A_{6,2} v E + A_{6,3} E^2)}{\Lambda^2} \right)
$$

Know  $\Lambda \gg v, E$  . But  $v$  is fixed while  $E$  can vary (can be 'selected' by analysis cuts to focus on tails of distributions, etc.)



### **But don't get carried away**

We're still at the mercy of perturbation theory expansion in

$$
c_i\left(\frac{v}{\Lambda}\right),\,c_i\left(\frac{E}{\Lambda}\right) < 1
$$

If >1, no sense that lowest order is adequate, expansion is invalid!

**Analysis with expansion > 1 is a straw man for some non-SM effect, but results don't contribute to SMEFT picture/goal**

[Running MC, **you** control this expansion:

- *E* is set by process and cuts
- $c_i$  and  $\Lambda$  are inputs set in parameter cards.

e.g.  $\bar{t}$ *th* production,  $E\gtrsim 500$  GeV...

Need <u>all three pieces</u> to know the expansion parameter! ]

### **But don't get carried away**

Even if  $c_i(\frac{1}{\Lambda}) < 1$ , larger expansion parameter (in kinematic tails) means higher order corrections are more important *E*  $\overline{\Lambda}$  ) < 1

MC already contain some higher order terms

$$
+\frac{1}{\Lambda^4} |(A_{6,1}v^2 + A_{6,2}vE + A_{6,3}E^2)|^2
$$

But these aren't the end of the story. Gotta do perturbation theory consistently, so if we need  $\mathcal{O}(1/\Lambda^4)$ , we need all terms with that 'power counting'

#### **SM** x **dimension-8 effects**

Even with our simplifying assumptions, 993 dim-8 operators…

**Can't we just use**  $\vert$  **dim**  $-6 \vert^2$ ?

Can suffice if no other choice, BUT

- $|\text{dim} 6|^2$  is positive definite, net  $\mathcal{O}(1/\Lambda^4)$  doesn't have to be (destructive interference)
- $|\text{dim} 6|^2$  is restricted to dim-6 operators, limited structure, some already bounded, small in some UV setups

Can lead to wildly inaccurate estimates of  $\mathcal{O}(1/\Lambda^4)$  ...

**Especially dangerous if**  $|\text{dim} - 6|^2 > SM \times (\text{dim} - 6)$ **without a good reason!!**

## **If**  $E/\Lambda$  vs.  $v/\Lambda$  is so important, how can I spot it in the operators **I choose for my analysis?**

• Check out the helicity/polarization/color structure. To interfere, need to match the SM structure. Non-Higgs SM interactions interactions involve fermions with the same helicity ( $Q^\dagger Q$ ,  $u_R^\dagger u_R^{\vphantom{\dagger}}$ , not  $Qu_R^{\vphantom{\dagger}},$  etc.)



**So SMEFT operators with opposite helicity can't interfere**

### **If**  $E/\Lambda$  vs.  $v/\Lambda$  is so important, how can I spot it in the operators **I choose for my analysis?**

• *E* comes from  $\partial_{\mu}$ . These hide in field strengths  $W_{\mu\nu}^{I} = \partial_{\mu}W_{\nu}^{I} - \partial_{\nu}W_{\mu}^{I} + \cdots$ 



### **If**  $E/\Lambda$  vs.  $v/\Lambda$  is so important, how can I spot it in the operators **I choose for my analysis?**

• Count the propagators,  $1/E^2$  for each



### **Takeaways**

• SMEFT is top-down or bottom-up EFT, SM + higher dimension operators formed from SM field content +  $D_{\mu}$ .

#### **Bottom up: determine** Λ **via global approach,** ≠ **resonance search**

- Lowest dimension operators (typically dim-6) are the most important, enter processes via interference with SM + new physics2.
- 'Energy enhanced effects' are both a blessing and a curse. Increase sensitivity for fixed  $c_i, \Lambda$ , but must be careful not to go too far to invalidate the EFT/introduce huge sensitivity to higher order effects
- Rules of thumb to help determine which SMEFT effects a particular analysis is most sensitive to (without huge  $c_i$ , small  $\Lambda$ )

### **Go forth and SMEFT!**

