EFT intro part II

Standard Model Effective Field Theory in more detail, practice

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Standard Model Effective Field Theory (SMEFT)

= SM extended by higher dimensional operators formed solely from SM field content and their covariant derivatives

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_{d} \sum_{i} \frac{c_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^{(d)}(Q, u_c, d_c, L, e_c, H, D_{\mu}, F_{\mu\nu} \cdots)$$

 Λ = scale of heavy new physics

 $c_i^{(d)}$ = `Wilson coefficients', encode info about interactions between heavy new physics and us

Ex.)
$$\frac{c (HL)^2}{\Lambda} = \frac{c' Q H \sigma_{\mu\nu} u_c B^{\mu\nu}}{\Lambda^2} = \frac{c (Q^{\dagger} \bar{\sigma}^{\mu} Q) (i H^{\dagger} \overleftrightarrow{D}_{\mu} H)}{\Lambda^2} = \frac{c (Q^{\dagger} \bar{\sigma}_{\mu} Q)^2}{\Lambda^2}$$

EFT requirement: ratio of scales

$$\frac{E}{\Lambda} = \frac{\text{Energy of process (~ LHC } \sqrt{\hat{s}} \text{)}}{\text{Mass of some heavy stuff}}$$

lowest dimension operators (= fewest Λ) should be the most important effects

Top down: if we have ANY* cool BSM particles but they are **too heavy** to produce on-shell, can map onto SMEFT ("integrate them out")

*no other light particles (axions, DM), linearly realized EWSB

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Can also use as a bottom-up EFT: Turn on all operators

Operators parameterize all possible deviations in SM inter-particle interactions. Bound these operators by precision measurements of how SM particles interact.

Use that information to inform about mass scale and properties of UV theories

How many operators are there?

Dim 5: 12

Dim 6: 2499

Dim 7: 948

Dim 8: 36971

But — can reduce the numbers by adding assumptions

- Only keep B, L preserving operators. Removes all odd dimension operators
- Only keep CP preserving operators
- Assume some flavor symmetry

often `flavor universality' = effects are the same for all generations

$$(Q_{i}^{\dagger}\bar{\sigma}^{\mu}Q_{j})(iH^{\dagger}\overleftrightarrow{D}_{\mu}H) \qquad \qquad (Q_{i}^{\dagger}\bar{\sigma}^{\mu}Q_{i})(iH^{\dagger}\overleftrightarrow{D}_{\mu}H)$$

Dim 6: 59 More manageable Dim 8: 993

operators fixed, but exact form is not. Need to pick a basis to cover all effects

0-fermion and 2-fermion operators in "Warsaw basis"

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2 arphi^3$	
Q_G	$f^{ABC}G^{A\nu}_{\mu}G^{B\rho}_{\nu}G^{C\mu}_{\rho}$	Q_{φ}	$(arphi^\dagger arphi)^3$	$Q_{e\varphi}$	$(arphi^{\dagger}arphi)(ar{l}_{p}e_{r}arphi)$
$Q_{\widetilde{G}}$	$f^{ABC} \widetilde{G}^{A\nu}_{\mu} G^{B\rho}_{\nu} G^{C\mu}_{\rho}$	$Q_{\varphi\Box}$	$(\varphi^{\dagger}\varphi)\Box(\varphi^{\dagger}\varphi)$	$Q_{u\varphi}$	$(\varphi^{\dagger}\varphi)(\bar{q}_{p}u_{r}\widetilde{\varphi})$
Q_W	$\varepsilon^{IJK} W^{I\nu}_{\mu} W^{J\rho}_{\nu} W^{K\mu}_{\rho}$	$Q_{\varphi D}$	$\left(\varphi^{\dagger}D^{\mu}\varphi\right)^{\star}\left(\varphi^{\dagger}D_{\mu}\varphi\right)$	$Q_{d\varphi}$	$(arphi^{\dagger}arphi)(ar{q}_p d_r arphi)$
$Q_{\widetilde{W}}$	$\varepsilon^{IJK}\widetilde{W}^{I\nu}_{\mu}W^{J\rho}_{\nu}W^{K\mu}_{\rho}$				
$X^2 \varphi^2$		$\psi^2 X \varphi$		$\psi^2 \varphi^2 D$	
$Q_{\varphi G}$	$\varphi^{\dagger}\varphiG^{A}_{\mu\nu}G^{A\mu\nu}$	Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I \varphi W^I_{\mu\nu}$	$Q_{\varphi l}^{(1)}$	$(\varphi^{\dagger}i\overleftrightarrow{D}_{\mu}\varphi)(\bar{l}_{p}\gamma^{\mu}l_{r})$
$Q_{arphi \widetilde{G}}$	$\varphi^{\dagger}\varphi\widetilde{G}^{A}_{\mu u}G^{A\mu u}$	Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r) \varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^{\dagger}i\overleftrightarrow{D}_{\mu}^{I}\varphi)(\overline{l}_{p}\tau^{I}\gamma^{\mu}l_{r})$
$Q_{\varphi W}$	$\varphi^{\dagger}\varphiW^{I}_{\mu\nu}W^{I\mu\nu}$	Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r) \widetilde{\varphi} G^A_{\mu\nu}$	$Q_{\varphi e}$	$(\varphi^{\dagger}i\overleftrightarrow{D}_{\mu}\varphi)(\bar{e}_{p}\gamma^{\mu}e_{r})$
$Q_{\varphi \widetilde{W}}$	$\varphi^{\dagger}\varphi\widetilde{W}^{I}_{\mu\nu}W^{I\mu\nu}$	Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \widetilde{\varphi} W^I_{\mu\nu}$	$Q^{(1)}_{\varphi q}$	$(\varphi^{\dagger}i\overleftrightarrow{D}_{\mu}\varphi)(\bar{q}_{p}\gamma^{\mu}q_{r})$
$Q_{\varphi B}$	$\varphi^{\dagger}\varphiB_{\mu u}B^{\mu u}$	Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \widetilde{\varphi} B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^{\dagger}i\overleftrightarrow{D}^{I}_{\mu}\varphi)(\bar{q}_{p}\tau^{I}\gamma^{\mu}q_{r})$
$Q_{arphi \widetilde{B}}$	$arphi^{\dagger}arphi\widetilde{B}_{\mu u}B^{\mu u}$	Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) \varphi G^A_{\mu\nu}$	$Q_{\varphi u}$	$(\varphi^{\dagger}i\overleftrightarrow{D}_{\mu}\varphi)(\bar{u}_{p}\gamma^{\mu}u_{r})$
$Q_{\varphi WB}$	$\varphi^{\dagger}\tau^{I}\varphiW^{I}_{\mu\nu}B^{\mu\nu}$	Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r) \tau^I \varphi W^I_{\mu\nu}$	$Q_{\varphi d}$	$(\varphi^{\dagger}i\overleftrightarrow{D}_{\mu}\varphi)(\bar{d}_{p}\gamma^{\mu}d_{r})$
$Q_{\varphi \widetilde{W}B}$	$\varphi^{\dagger}\tau^{I}\varphi\widetilde{W}^{I}_{\mu\nu}B^{\mu\nu}$	Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r) \varphi B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde{\varphi}^{\dagger}D_{\mu}\varphi)(\bar{u}_{p}\gamma^{\mu}d_{r})$

Here, $\varphi = H$; $p, r, \dots =$ flavor indices (so $p = r \rightarrow$ flavor universal)

4-fermion operators in "Warsaw basis"

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$		
Q_{ll}	$(\bar{l}_p \gamma_\mu l_r) (\bar{l}_s \gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p \gamma_\mu l_r) (\bar{e}_s \gamma^\mu e_t)$	
$Q_{qq}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r) (\bar{q}_s \gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p \gamma_\mu u_r)(\bar{u}_s \gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p \gamma_\mu l_r) (\bar{u}_s \gamma^\mu u_t)$	
$Q_{qq}^{(3)}$	$(\bar{q}_p \gamma_\mu \tau^I q_r) (\bar{q}_s \gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar{d}_p \gamma_\mu d_r) (\bar{d}_s \gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p \gamma_\mu l_r) (\bar{d}_s \gamma^\mu d_t)$	
$Q_{lq}^{(1)}$	$(\bar{l}_p \gamma_\mu l_r) (\bar{q}_s \gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$	
$Q_{lq}^{(3)}$	$(\bar{l}_p \gamma_\mu \tau^I l_r) (\bar{q}_s \gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar{e}_p \gamma_\mu e_r) (\bar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r) (\bar{u}_s \gamma^\mu u_t)$	
		$Q_{ud}^{(1)}$	$(\bar{u}_p \gamma_\mu u_r) (\bar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r) (\bar{u}_s \gamma^\mu T^A u_t)$	
		$Q_{ud}^{(8)}$	$(\bar{u}_p \gamma_\mu T^A u_r) (\bar{d}_s \gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r) (\bar{d}_s \gamma^\mu d_t)$	
				$Q_{qd}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r) (\bar{d}_s \gamma^\mu T^A d_t)$	
$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		<i>B</i> -violating				
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^{\alpha})^T C u_r^{\beta}\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$			
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r) \varepsilon_{jk} (\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^{\gamma})^T C e_t\right]$			
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r) \varepsilon_{jk} (\bar{q}_s^k T^A d_t)$	$Q_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\varepsilon_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$			
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r) \varepsilon_{jk} (\bar{q}_s^k u_t)$	$Q_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma}(\tau^{I}\varepsilon)_{jk}(\tau^{I}\varepsilon)_{mn}\left[(q_{p}^{\alpha j})^{T}Cq_{r}^{\beta k}\right]\left[(q_{s}^{\gamma m})^{T}Cl_{t}^{n}\right]$			
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r) \varepsilon_{jk} (\bar{q}_s^k \sigma^{\mu\nu} u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma} \left[(d_p^{\alpha})^T C u_r^{\beta} \right] \left[(u_s^{\gamma})^T C e_t \right]$			

Implemented in MadGraph UFO models via SMEFTsim, SMEFT@NLO

What do these operators actually do?

SMEFT is actually a double expansion, $\frac{v}{\Lambda}$ and $\frac{E}{\Lambda}$

Means higher dimensional operators can trickle down into lower dimension

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but also changes how
$$Q$$
 couples to Z_{μ} !
$$\delta g_{ZQQ} \sim \frac{g \, c_{HQ} \, v^2}{2 \Lambda^2}$$

[quite constrained from LEP I!]





But, setting $H \rightarrow v/\sqrt{2}$, looks like a shift in the W kinetic term

$$\mathcal{L} \supset -\frac{1}{4} \left(1 - \frac{c_{HW} v^2}{2\Lambda^2} \right) W^I_{\mu\nu} W^{I,\mu\nu}$$

We can redefine $W_{\mu\nu} = \frac{\hat{W}_{\mu\nu}^{I}}{\sqrt{1 - \frac{c_{HW}\nu^{2}}{2\Lambda^{2}}}} = \text{to get back usual}$

normalization, but we have to do that everywhere consistently

In particular, mass matrix:

$$|D_{\mu}H|^{2} \supset \frac{v^{2}}{2} \begin{pmatrix} W_{3\mu} & B_{\mu} \end{pmatrix} \begin{bmatrix} g^{2} & -gg' \\ -gg' & g'^{2} \end{bmatrix} \begin{pmatrix} W_{3\mu} \\ B_{\mu} \end{pmatrix}$$

Diagonalizing this gets us Z_{μ} , A_{μ} and coupling to Z_{μ} , A_{μ} is what we use to define electric couplings e, $\sin \theta_W$

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But with the normalization change...

$$\frac{v^2}{2} \left(\hat{W}_{3\mu} \quad B_{\mu} \right) \begin{bmatrix} \frac{g^2}{1 - c_{HW} v^2 / (2\Lambda^2)} & \frac{gg'}{\sqrt{1 - c_{HW} v^2 / (2\Lambda^2)}} \\ \frac{gg'}{\sqrt{1 - c_{HW} v^2 / (2\Lambda^2)}} & g'^2 \end{bmatrix} \begin{pmatrix} \hat{W}_{3\mu} \\ B_{\mu} \end{pmatrix}$$

Mixing angles and couplings now depend on c_{HW} : $\sin \hat{\theta}_{V}$

$$_{W} = \frac{g'}{\sqrt{g^2 + g'^2}} + \mathcal{O}(c_{HW})$$

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Net result: SMEFT operators enter in subtle ways and in multiple processes

SMEFT approach is a global approach



Operators impact multiple processes: Global approach needed

Remember the goal: from pattern in deviations, determine Λ

SMEFT approach is a global approach

8

4

-4

-6



[Ellis et al 2012.02779]

SMEFT approach is a global approach





Looking for heavy new physics





Looking for heavy new physics



OK lets calculate something! Good idea to understand what we should get before diving in

$$A = A_{SM} + \frac{(A_{6,1}v^2 + A_{6,2}vE + A_{6,3}E^2)}{\Lambda^2} + \cdots$$

 $A_{6,i}$ are functions of Wilson coefficients c_i . Formed by turning new operators into Feynman rules (ex. FeynRules), calculating away

$$|A|^{2} = |A_{SM}|^{2} + 2 \operatorname{Re} \left(A_{SM}^{*} \frac{(A_{6,1}v^{2} + A_{6,2}vE + A_{6,3}E^{2})}{\Lambda^{2}} \right) \xleftarrow{} \operatorname{interference term} + \frac{1}{\Lambda^{4}} |(A_{6,1}v^{2} + A_{6,2}vE + A_{6,3}E^{2})|^{2} \xrightarrow{} \operatorname{New physics} \overset{} \operatorname{squared}^{*} \operatorname{term}$$

$$2 \operatorname{Re} \left(A_{SM}^* \frac{(A_{6,1} v^2 + A_{6,2} v E + A_{6,3} E^2)}{\Lambda^2} \right)$$

Know $\Lambda \gg v, E$. But v is fixed while E can vary (can be 'selected' by analysis cuts to focus on tails of distributions, etc.)

Means for fixed
$$\Lambda$$
 (and c_i) can have $\left(\frac{E}{\Lambda}\right) \gg \left(\frac{v}{\Lambda}\right)$, so we can be sensitive to smaller $A_{6,3} \sim c_i$

This is **the main advantage** of SMEFT at LHC. Needs a combination of energy (E > v) and precision!

$$2\operatorname{Re}\left(A_{SM}^{*}\frac{(A_{6,1}v^{2} + A_{6,2}vE + A_{6,3}E^{2})}{\Lambda^{2}}\right)$$

Know $\Lambda \gg v, E$. But v is fixed while E can vary (can be 'selected' by analysis cuts to focus on tails of distributions, etc.)



But don't get carried away

We're still at the mercy of perturbation theory expansion in

$$c_i\left(\frac{v}{\Lambda}\right), \ c_i\left(\frac{E}{\Lambda}\right) < 1$$

If >1, no sense that lowest order is adequate, expansion is invalid!

Analysis with expansion > 1 is a straw man for some non-SM effect, but results don't contribute to SMEFT picture/goal

[Running MC, **you** control this expansion:

- *E* is set by process and cuts
- c_i and Λ are inputs set in parameter cards.

e.g. $\bar{t}th$ production, $E \gtrsim 500 \,\text{GeV}...$

Need <u>all three pieces</u> to know the expansion parameter!]

But don't get carried away

Even if $c_i \left(\frac{E}{\Lambda}\right) < 1$, larger expansion parameter (in kinematic tails) means higher order corrections are more important

MC already contain some higher order terms

$$+\frac{1}{\Lambda^4} |(A_{6,1}v^2 + A_{6,2}vE + A_{6,3}E^2)|^2$$

But these aren't the end of the story. Gotta do perturbation theory consistently, so if we need $\mathcal{O}(1/\Lambda^4)$, we need all terms with that 'power counting'

SM × dimension-8 effects

Even with our simplifying assumptions, 993 dim-8 operators...

Can't we just use $|\dim - 6|^2$?

Can suffice if no other choice, BUT

- $|\dim 6|^2$ is positive definite, net $\mathcal{O}(1/\Lambda^4)$ doesn't have to be (destructive interference)
- $|\dim 6|^2$ is restricted to dim-6 operators, limited structure, some already bounded, small in some UV setups

Can lead to wildly inaccurate estimates of $\mathcal{O}(1/\Lambda^4)$...

Especially dangerous if $|\dim - 6|^2 > SM \times (\dim - 6)$ without a good reason!!

If E/Λ vs. v/Λ is so important, how can I spot it in the operators I choose for my analysis?

• Check out the helicity/polarization/color structure. To interfere, need to match the SM structure. Non-Higgs SM interactions interactions involve fermions with the same helicity ($Q^{\dagger}Q$, $u_{R}^{\dagger}u_{R}$, not Qu_{R} , etc.)



So SMEFT operators with opposite helicity can't interfere

If E/Λ vs. v/Λ is so important, how can I spot it in the operators I choose for my analysis?

• E comes from ∂_{μ} . These hide in field strengths $W^{I}_{\mu\nu} = \partial_{\mu}W^{I}_{\nu} - \partial_{\nu}W^{I}_{\mu} + \cdots$



If E/Λ vs. v/Λ is so important, how can I spot it in the operators I choose for my analysis?

• Count the propagators, $1/E^2$ for each



Takeaways

• SMEFT is top-down or bottom-up EFT, SM + higher dimension operators formed from SM field content + D_{μ} .

Bottom up: determine Λ via global approach, \neq resonance search

- Lowest dimension operators (typically dim-6) are the most important, enter processes via interference with SM + new physics².
- 'Energy enhanced effects' are both a blessing and a curse. Increase sensitivity for fixed c_i , Λ , but must be careful not to go too far to invalidate the EFT/introduce huge sensitivity to higher order effects
- Rules of thumb to help determine which SMEFT effects a particular analysis is most sensitive to (without huge c_i , small Λ)

Go forth and SMEFT!

