

# **Practical introduction to EFT**

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### Data rich era spanning multiple scales



### **Motivation**

### EFT in a nutshell

A QFT which describes a low energy limit of a 'more fundamental' theory (can also be an EFT)

Allows calculation of experimental quantities with expansion to finite order in small parameter

 $EFT \equiv fields + symmetries at energy scale of interest constructed as a self-consistent theory$ 

### **Benefits**

Simplifies calculations by dealing with one scale at a time

Make symmetries in limits of full theory manifest

Only deal with relevant interactions at scale of interest

Can resum large logs with RG systematically



## Philosophy

**Idea:** Appropriate theory for each scale - you don't need SUSY to catch a ball (at all!)

Near ( $L \sim R$ ): needs position of each charge to describe E-field in her proximity

Far ( $r \gg R$ ): E-field described by multipole expansion,  $V(r) = \frac{q}{r} + \mathcal{O}(R/r^2)$ 



Lagrangian-level: Take QFT with large energy scale  $\Lambda$  and we are interested in momenta  $p\ll\Lambda$ 

EFT is then same QFT in an expansion in  $p/\Lambda \Rightarrow$  fields of mass  $\Lambda$  are `integrated out'

### Formalism

**EFT Lagrangian** written generally as expansion in inverse powers of large scale M

$$\mathscr{L}_{\mathsf{EFT}} = \mathscr{L}_{\mathsf{full}}^{\mathsf{rem}} + \frac{c_1}{M} \mathscr{O}_1 + \frac{c_2}{M^2} \mathscr{O}_2 + \dots$$

with invariant operators  $\mathcal{O}_i$  (forming a complete basis), matching coefficients  $c_i$  and  $1/M \sim$  Compton wavelength of particles of mass  $\sim M$ 

**Note:** EFT is a theory like any QFT and Feynman rules are obtainable to study processes as in FT

Accuracy improved by going to higher powers in p/M as well as higher powers in couplings (as in FT)

## **Matching and running**

**Matching:** define  $c_i$  to ensure that at each perturbative order, EFT and FT predictions identical  $\leftrightarrow$  Match scattering amplitudes for corresponding operators

Toy example: Scalar field theory with light field  $\varphi$  and heavy field H

At scales  $p^2 \ll m_H^2$  propagation of H indistinguishable from a contact interaction

Processes probing energy (distance) scales  $m_H$  ( $1/m_H$ ) cannot resolve H propagation

**Running:** beyond tree-level  $c_i = c_i(\mu)$  and one matches EFT to FT at UV scale  $\mu = M \Rightarrow$  can use RGE's to run coefficients down to scale of interest



## **Classes of EFT**

**Top-down:** Full theory known - e.g. UV BSM model, SM, QCD/QED - interested in physics at specific lower energies, higher mass scales integrated out

**Bottom-up:** Full theory at high energy scale unknown, but must match to SM at measured energies

### **Key examples:**

Pre-QFT: Hydrogen atom, multipole expansion, nuclear physics

Precision/low-energy EFTs: HQET/(p)NRQCD, ChPT, NR DM, SCET

High energy EFTs: SMEFT, HEFT

Many more: WEFT, ALP-SMEFT, DM EFTs, NR gravity etc.

Let's examine a few now...

# HQET and (p)NRQCD

Heavy quarks and their bound states call for **multi-scale** treatment

Expansion at the Lagrangian level in  $m_Q \gg v \sim \alpha_s \Rightarrow$  **Top-down perturbative EFT** description



**HQET/NR QCD(QED):** Operators and matching known to D = 8, NNLO and above for specific applications [Gerlach et al. '19; Gunawardana et al. '17; BA, Soto, Kniehl '20]

**pNRQCD(QED):** Static 1/r potentials known to  $N^3LO$  and finite mass + spin-dependent terms to  $m^2NNLO$  [Review: Pineda '11]

**Applications:** *t*, *b*, *c*, *B*,  $\eta_{b,c}$ ,  $\Upsilon(1S)$ ,  $J/\psi$ , ... mass, decay, splitting, cross-sections etc.

[Bodwin, Braaten, Brambilla, Caswell, Eichten, Georgi, Kniehl, LePage, Manohar, Paz, Pineda, Soto, Wise,...;1987-2024]

# HQET and (p)NRQCD

**HQET:** describes heavy quarks from QCD with momentum separated

$$p^{\mu} = mv^{\mu} + k^{\mu} : k \sim \Lambda_{QCD}, \quad v \cdot v = 1, \quad p \cdot p = m^2$$

light dof's (light quarks and gluons)  $\Rightarrow$  LO Lagrangian

$$\mathscr{L}_{\text{HQET}} = \bar{h}_{\nu} i\nu \cdot Dh_{\nu} + \sum_{i} \bar{q} i\gamma^{\mu} D_{\mu} q - \frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \mathcal{O}(1/m)$$



(p)NRQCD: Describes quark/anti-quark pair (quarkonium) ↔ 3 well separated scales

hard: *m* soft: mv ultrasoft:  $mv^2$ 



# **Composite EFTs**

**Very light quarks:** Predictions from ChPT sensitive to poorly constrained lowenergy constants (LECs) and choice of power counting

Berengut et al. PRD 87 (2013), Beane and Savage, Nucl. Phys. A717 (2003)

**Medium-mass quarks:** Chiral EFT poorly describe quark-mass dependence of the nucleon mass for  $m_{\pi} > m_{\pi}^{\text{phys}} \Rightarrow$  lattice QCD needed to provide a first principles description of the quark-mass dependence of hadron interactions (hard/inefficient)



BA and Wagman: PRD 108 (2023), 2311.01498

# **Dark Hadrons**

**Composite dark matter** attractive candidate since implicit stability due to global flavour symmetry

$$\mathscr{L}_{dQCD} = -\frac{1}{2} \text{Tr}G_{\mu\nu}^2 + \bar{q}iDq + m_d\bar{q}q$$

New  $SU(N_d)$  gauge sector **confines** at

$$\Lambda_{\rm dQCD} \sim \exp\left(-\frac{2\pi}{\beta_0 \alpha_d}\right)$$



DM stability is maintained beyond Gyrs  $\sim$  proton stability in SM

SM-DM interactions **suppressed** in the EFT above confinement

## Dark quark masses

Standard model fermions exist over a wide range of masses

Dark matter candidates span an even wider range of scales



Can we understand what nuclear interactions look like for dark nuclei comprised of quarks across this mass range?

## **Chiral perturbation theory**

**Low-energy regime:** of QCD dof.'s are hadrons due to **confinement**, parity-invariant theory of Goldstone bosons

$$U(x) = \exp\left\{\frac{i}{F} \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}\right\}$$

 $F = \Lambda_{\chi}/4\pi = 93 \text{ MeV} \leftrightarrow \pi$ -decay constant,  $\Lambda_{\chi} \sim 1 \text{ GeV} \leftrightarrow \text{CSB}$  scale

For SU(2) theory the Lagrangian

$$\mathscr{L}_{2} = g_{2} \mathrm{tr} \left( \partial_{\mu} U \partial^{\mu} U^{\dagger} \right) + g_{4}^{(1)} \mathrm{tr} \left( \partial_{\mu} U \partial^{\mu} U^{\dagger} \right)^{2} + g_{4}^{(2)} \mathrm{tr} \left( \partial_{\mu} U \partial^{\nu} U^{\dagger} \right) \mathrm{tr} \left( \partial_{\nu} U \partial^{\mu} U^{\dagger} \right) + \dots$$

# LECs $g_i$ encode high mass states integrated out

**N.B.** can describe Kaons as well with U(x) as SU(3) matrix



## SCET

LHC-Like scales: COM energy/jet momentum orders of magnitude above mass of heaviest SM particles

Processes involves physics from large scale, Q, down to low scale  $\sim$  proton mass



SCET describes interactions of S & C dof's in presence of hard scale,  $Q \gg p_{\rm soft}$  and  $Q \gg \Lambda_{\rm QCD}$  (pert) or  $Q \sim \Lambda_{\rm QCD}$  (non-pert).

#### **Power counting**

*n*-collinear:  $Q(\lambda^2, 1, \lambda) \Rightarrow p^2 = Q^2 \lambda^2$   $\bar{n}$ -collinear:  $Q(1, \lambda^2, \lambda) \Rightarrow p^2 = Q^2 \lambda^2$ soft:  $Q\lambda(1, 1, 1) \Rightarrow p^2 = Q^2 \lambda^2$  and ultrasoft  $p_{\text{US}} \sim Q\lambda^2(1, 1, 1) \ll p_{\text{soft}}$ 

## **Soft-Collinear EFT (SCET)**

Derive S & C SCET Lagrangian from FT, e.g. QCD:

$$\mathscr{L} = \bar{\psi} i \gamma_{\mu} D^{\mu} \psi - \frac{1}{4} G^{\mu\nu} G_{\mu\nu}$$

by splitting quarks and gluons of different modes:

$$\psi \to \psi_c + \psi_s \quad A^\mu \to A^\mu_c + A^\mu_s$$

**Factorization** at Lagrangian-level in SCET, splitting collinear into large and small components (as in HQET)  $\psi_c = \eta + \xi$  gives

$$\mathcal{L}_{\text{SCET}} = \bar{\psi}_{s} i \not\!\!\!\!D_{s} \psi_{s} + \bar{\xi} \frac{\vec{n}}{2} \left[ in \cdot D + i \not\!\!\!D_{c\perp} \frac{1}{i\bar{n} \cdot D_{c}} i \not\!\!\!D_{c\perp} \right] \xi - \frac{1}{4} \left( F^{s,a}_{\mu\nu} \right)^{2} - \frac{1}{4} \left( F^{c,a}_{\mu\nu} \right)^{2}$$

Second collinear sector obtained by taking  $n \rightarrow \bar{n}$ 



**Goal:** EFT that systematically classifies "all" BSM physics without requiring knowledge of UV theory

Assumptions: new nearly physics decoupled  $\Rightarrow \Lambda \sim$  few TeV  $\gg v$ and at the accessible scale only SM fields + symmetries





[Henning et al. 1512.03433]

Number of operators grows quickly with increasing mass dimension

### And what do these operators do?

[Martin '22]



For 2- and 3-point interactions # of contributing SMEFT operators is **small** and **constant** with operator dimension  $\Rightarrow$  pheno can be done with small set of operators

### Warsaw basis at d = 6

Grzadkowski, Iskrzynski, Misiak, Rosiek 1008.4884

$X^3$		$\varphi^6$ and $\varphi^4 D^2$		$\psi^2 arphi^3$	
$Q_G$	$f^{ABC}G^{A\nu}_{\mu}G^{B\rho}_{\nu}G^{C\mu}_{\rho}$	$Q_{arphi}$	$(arphi^\dagger arphi)^3$	$Q_{e\varphi}$	$(\varphi^{\dagger}\varphi)(\bar{l}_{p}e_{r}\varphi)$
$Q_{\widetilde{G}}$	$f^{ABC} \widetilde{G}^{A\nu}_{\mu} G^{B\rho}_{\nu} G^{C\mu}_{\rho}$	$Q_{\varphi \Box}$	$(\varphi^{\dagger}\varphi)\Box(\varphi^{\dagger}\varphi)$	$Q_{uarphi}$	$(\varphi^{\dagger}\varphi)(\bar{q}_{p}u_{r}\widetilde{\varphi})$
$Q_W$	$\varepsilon^{IJK} W^{I\nu}_{\mu} W^{J\rho}_{\nu} W^{K\mu}_{\rho}$	$Q_{\varphi D}$	$\left( \varphi^{\dagger} D^{\mu} \varphi \right)^{\star} \left( \varphi^{\dagger} D_{\mu} \varphi \right)$	$Q_{darphi}$	$(\varphi^{\dagger}\varphi)(\bar{q}_{p}d_{r}\varphi)$
$Q_{\widetilde{W}}$	$\varepsilon^{IJK}\widetilde{W}^{I\nu}_{\mu}W^{J\rho}_{\nu}W^{K\mu}_{\rho}$				
$X^2 \varphi^2$		$\psi^2 X \varphi$		$\psi^2 arphi^2 D$	
$Q_{arphi G}$	$\varphi^{\dagger}\varphiG^{A}_{\mu u}G^{A\mu u}$	$Q_{eW}$	$(\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I \varphi W^I_{\mu\nu}$	$Q^{(1)}_{arphi l}$	$(\varphi^{\dagger}i\overleftrightarrow{D}_{\mu}\varphi)(\bar{l}_{p}\gamma^{\mu}l_{r})$
$Q_{arphi \widetilde{G}}$	$\varphi^{\dagger} \varphi  \widetilde{G}^{A}_{\mu u} G^{A\mu u}$	$Q_{eB}$	$(\bar{l}_p \sigma^{\mu\nu} e_r) \varphi B_{\mu\nu}$	$Q^{(3)}_{arphi l}$	$(\varphi^{\dagger}i\overleftrightarrow{D}^{I}_{\mu}\varphi)(\bar{l}_{p}\tau^{I}\gamma^{\mu}l_{r})$
$Q_{arphi W}$	$arphi^{\dagger} arphi W^{I}_{\mu u} W^{I\mu u}$	$Q_{uG}$	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r) \widetilde{\varphi}  G^A_{\mu\nu}$	$Q_{arphi e}$	$(\varphi^{\dagger}i \overleftrightarrow{D}_{\mu} \varphi)(\bar{e}_{p} \gamma^{\mu} e_{r})$
$Q_{arphi \widetilde{W}}$	$arphi^{\dagger} arphi  \widetilde{W}^{I}_{\mu u} W^{I\mu u}$	$Q_{uW}$	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \widetilde{\varphi} W^I_{\mu\nu}$	$Q^{(1)}_{arphi q}$	$(\varphi^{\dagger}i\overleftrightarrow{D}_{\mu}\varphi)(\bar{q}_{p}\gamma^{\mu}q_{r})$
$Q_{arphi B}$	$arphi^\dagger arphi  B_{\mu u} B^{\mu u}$	$Q_{uB}$	$(\bar{q}_p \sigma^{\mu u} u_r) \widetilde{\varphi}  B_{\mu u}$	$Q^{(3)}_{arphi q}$	$\left(\varphi^{\dagger}i\overleftrightarrow{D}_{\mu}^{I}\varphi)(\bar{q}_{p}\tau^{I}\gamma^{\mu}q_{r})\right)$
$Q_{arphi \widetilde{B}}$	$arphi^{\dagger}arphi\widetilde{B}_{\mu u}B^{\mu u}$	$Q_{dG}$	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) \varphi  G^A_{\mu\nu}$	$Q_{arphi u}$	$(\varphi^{\dagger}i\overleftrightarrow{D}_{\mu}\varphi)(\bar{u}_{p}\gamma^{\mu}u_{r})$
$Q_{\varphi WB}$	$arphi^\dagger  au^I arphi  W^I_{\mu u} B^{\mu u}$	$Q_{dW}$	$(\bar{q}_p \sigma^{\mu\nu} d_r) \tau^I \varphi W^I_{\mu\nu}$	$Q_{arphi d}$	$(\varphi^{\dagger}i\overleftrightarrow{D}_{\mu}\varphi)(\bar{d}_{p}\gamma^{\mu}d_{r})$
$Q_{\varphi \widetilde{W}B}$	$arphi^\dagger  au^I arphi  \widetilde{W}^I_{\mu u} B^{\mu u}$	$Q_{dB}$	$(\bar{q}_p \sigma^{\mu u} d_r) \varphi  B_{\mu u}$	$Q_{arphi u d}$	$i(\widetilde{\varphi}^{\dagger}D_{\mu}\varphi)(\bar{u}_{p}\gamma^{\mu}d_{r})$

### **Warsaw basis at** d = 6

Grzadkowski, Iskrzynski, Misiak, Rosiek 1008.4884

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$		
$Q_{ll}$	$(ar{l}_p \gamma_\mu l_r) (ar{l}_s \gamma^\mu l_t)$	Qee	$(\bar{e}_p \gamma_\mu e_r) (\bar{e}_s \gamma^\mu e_t)$	$Q_{le}$	$(ar{l}_p \gamma_\mu l_r) (ar{e}_s \gamma^\mu e_t)$	
$Q_{qq}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r) (\bar{q}_s \gamma^\mu q_t)$	$Q_{uu}$	$(\bar{u}_p \gamma_\mu u_r) (\bar{u}_s \gamma^\mu u_t)$	$Q_{lu}$	$(ar{l}_p \gamma_\mu l_r) (ar{u}_s \gamma^\mu u_t)$	
$Q_{qq}^{(3)}$	$(\bar{q}_p \gamma_\mu \tau^I q_r) (\bar{q}_s \gamma^\mu \tau^I q_t)$	$Q_{dd}$	$(ar{d}_p \gamma_\mu d_r) (ar{d}_s \gamma^\mu d_t)$	$Q_{ld}$	$(ar{l}_p \gamma_\mu l_r) (ar{d}_s \gamma^\mu d_t)$	
$Q_{lq}^{(1)}$	$(ar{l}_p \gamma_\mu l_r) (ar{q}_s \gamma^\mu q_t)$	$Q_{eu}$	$(\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$	$Q_{qe}$	$(\bar{q}_p \gamma_\mu q_r) (\bar{e}_s \gamma^\mu e_t)$	
$Q_{lq}^{(3)}$	$(\bar{l}_p \gamma_\mu \tau^I l_r) (\bar{q}_s \gamma^\mu \tau^I q_t)$	$Q_{ed}$	$(\bar{e}_p \gamma_\mu e_r) (\bar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r) (\bar{u}_s \gamma^\mu u_t)$	
		$Q_{ud}^{(1)}$	$(ar{u}_p \gamma_\mu u_r) (ar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$\left  (\bar{q}_p \gamma_\mu T^A q_r) (\bar{u}_s \gamma^\mu T^A u_t) \right $	
		$Q_{ud}^{(8)}$	$\left( (\bar{u}_p \gamma_\mu T^A u_r) (\bar{d}_s \gamma^\mu T^A d_t) \right)$	$Q_{qd}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r) (\bar{d}_s \gamma^\mu d_t)$	
				$Q_{qd}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r) (\bar{d}_s \gamma^\mu T^A d_t)$	
$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		<i>B</i> -violating				
$Q_{ledq}$	$(ar{l}_p^j e_r) (ar{d}_s q_t^j)$	$Q_{duq}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_{p}^{\alpha})^{T}Cu_{r}^{\beta}\right]\left[(q_{s}^{\gamma j})^{T}Cl_{t}^{k}\right]$			
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r) \varepsilon_{jk} (\bar{q}_s^k d_t)$	$Q_{qqu}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^{\gamma})^T C e_t\right]$			
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r) \varepsilon_{jk} (\bar{q}_s^k T^A d_t)$	$Q_{qqq}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\varepsilon_{mn}\left[(q_{p}^{\alpha j})^{T}Cq_{r}^{\beta k}\right]\left[(q_{s}^{\gamma m})^{T}Cl_{t}^{n}\right]$			
$Q_{lequ}^{(1)}$	$(\bar{l}_{p}^{j}e_{r})arepsilon_{jk}(\bar{q}_{s}^{k}u_{t})$	$Q_{duu}$	$arepsilon^{lphaeta\gamma}\left[(d_p^{lpha})^T C u_r^{eta} ight]\left[(u_s^{\gamma})^T C e_t ight]$			
$Q_{lequ}^{(3)}$	$(\bar{l}_{p}^{j}\sigma_{\mu u}e_{r})\varepsilon_{jk}(\bar{q}_{s}^{k}\sigma^{\mu u}u_{t})$					

 $\mathscr{L}_6$  has 2499 parameters in the most general case  $\mathcal{O}(100)$  with flavor symmetries and CP

typically each process is corrected by  $\mathcal{O}(10)$  parameters: constrains a direction in param. space

each parameter enters multiple processes



<sup>[</sup>Brivio '20]

Analyses combining several measurements necessary to avoid bias in interpretation

Large # of operators  $\Rightarrow$  many operators can contribute to same observable

**Ideal:** global SMEFT fit to very precise measurement, all  $C_i$  free parameters

**Reality:** only partial fits are feasible since too many operators to constrain

**Aim:** come up with set of **observables** sensitive to a close manageable set of operators

**Dominant effect:** the tree-level interference e.g.  $|\mathscr{A}_{\rm SM}\mathscr{A}_{d=6}^*| \sim C_i/\Lambda^2$ 





### **Extensive studies** done for $\mathscr{L}_6$

- 1) Complete RGEs and various 1-loop results
- 2) Tools for matching and numerical analysis
- 3) Many tree-level calculations of EW, Higgs, & flavour observables

Similarly but to lesser extent for  $\mathscr{L}_{7.8}$  (RGEs and tree-level)

### What more is needed for successful program?

- 1) Improving EFT uncertainties: H.O. in loops and EFT, MC simulation
- 2) Understanding corrections beyond ME: PDF, PS, acceptances [Carrazza et al 1905.05215, Greljo et al. 2104.02723, Iranipour, Ubiali 2201.07240 Goldouzian et al 2012.06872]
- 3) Handling & understanding large-dimensional likelihoods
- 4) More refined process treatment: differential info, target CPV, flavor...

## **SMEFT-Adjacent EFTs to account for**

**HEFT**  $\supset$  **SMEFT**  $\leftrightarrow$  fusion of ChPT in scalar sector and SMEFT in gauge/ fermion sector = SMEFT + no assumptions about Higgs being scalar doublet

**ALP-SMEFT** includes dynamics of light/heavy axion or axion-like particles which are not present in SMEFT or HEFT

**Below EW scale** low energy effective theory (LEFT) with quark and lepton fields with only QCD and QED gauge fields

**Combining EFTs:** If scales widely separated can match and run between EFTs systematically



## Summary

Explored the main EFTs, current status and associated challenges

Introduced the process of matching and running in various contexts

Enormous improvements made but many technical challenges remain

SMEFT alternatives are also important candidates for BSM interpretation

### Some references

(p)NRQCD/QED reviews: Bodwin, Brambilla, Georgi, LePage, Manohar, Pineda, Soto, Wise. tools: FeynOnium,...

ChPT reviews: Pich, Scherer, Schindler. tools: CHIRON,...

**SCET reviews:** Becher, Beringer, Cohen, Manohar, Neumann, Rothstein, Stewart. **tools:** SoftSERVE, MCFM-CuTe,...

**SMEFT/HEFT, LEFT reviews:** Bauer, Brivio, Cohen, Dawson, Jenkins, Manohar, Neubert, Trott. **tools:** SMEFTsim, SMEFTfit, WCxf, flavio, matchete,... [hep-ph/1910.11003]