Flavour Physics

What is flavour physics (FP)?

- Study of properties of quarks + leptons that each come is 6 flavours in 3 generations in SM.
  - we will focus on weak decays of heavy (b) quarks

- In SM this is the study of

$$\overbrace{\underbrace{\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}}_{\text{Quarks}} \qquad \underbrace{\begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}}_{\text{leptons}}}^{\text{fermions}}$$

$$\underbrace{W^\pm, Z^0}_{\text{bosons}}$$

-

- BUT we know SM is incomplete
  - 95% of Universe unexplained in SM
  - Matter antimatter asym.
  - Gravity?...

- The level of CP violation allowed for in SM is 10^4 too small to explain our matter dominated Universe
  - → Our existance necessitates BSM physics and now players in the flavour family.

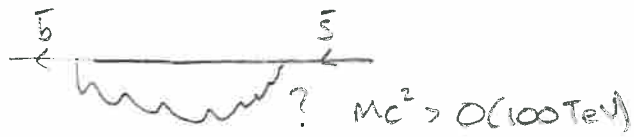
Direct vs. Indirect BSM searches

Direct

"Energy frontier"

- Produce + detect new particles from collision
- Sensitive to particles with $mc^2 \leq E$ (or $\leq \frac{E}{2}$ if pair production)

Indirect



"Precision frontier"

- Make precision measurements of known processes that when compared to SM predictions infer NP.
- Probes quantum nature of particles so sensitive to energy scales far beyond collision energies of current or next-gen colliders

- Indirect searches are proven discovery tool.
  - First evidence of new particles from indirect searches that precede direct evidence

    - Charm quark discovery
    - 3^{rd} flavour generation

    ] we will look more at these

- If BSM physics exists at > 14 TeV indirect searches are the only method by which it will be seen for ≥ 5 decades!

Flavour physics experiments (slides 2-9)

Belle
- 1999 - 2010
- Collisions from asym. $e^- - e^+$ collider KEKB
- Most data collected at $\Upsilon(4S)$ ($b\bar{b}$ quarkonia) $\Rightarrow$ 771 $\underset{(\pm,0)}{B\bar{B}}$ pairs
- Some data at $\Upsilon(5S)$ for B_s studies
- Highlights - 1^{st} observation of $X(3872)$ (tetraquark)
- $\phi_3(\gamma)$ measurements using Dalitz method
- 1^{st} obs. of FCNC $B \to K^{(*)} \ell\ell$

BaBar

- 1999-2008
- Collisions from asym e^+e^- collider PEP-II at SLAC
- Data collected at $\Upsilon(4S) \Rightarrow$ 384M $B\bar{B}$ pairs
- Highlights - 1st obs. of bottonium ground state $\eta_b(1S)$
- Obs. of $B \to D^{(*)}\tau\nu$ exceeding SM rate predictions ($R(D^{(*)})$) (in 2012 AFTER data taking)

Belle + BaBar are collectively "B factories"

CLEO

- 1979-2008
- Collisions from symmetric e^+e^- collider CESR at Cornell Uni
- Ran at 3.5-12 GeV for b and c physics (cleo-c)
- Highlights - Longest running particle physics exp (29 yrs!)
- First obs of FCNC $b \to s\gamma$
- Discovered 13 Charm baryons
- Discovered $\Upsilon(4S)$ and confirmed $\Upsilon(3S)$
- key charm inputs for γ measurements

All 3 are hermetic (4π) detectors - cover as large an area around interaction point to observe all decay products. Roughly cylindrical with sub-detectors in concentric layers.

LHCb (00)

- ~2011-2018
- Collisions from sym. p-p collider LHC
- Ran at 7-13.6 TeV
- Highlights - discoverd 64/72 of new hadrons discovered at LHC
- (re-) discovery of Pentaquarks
- 1st obs. of CPV in Charm
- γ precision of 4°

LHCb is not hermetic, it is a single-arm forward spectrometer
- LHC is essentially g-g collider and $b\bar{b}$ production is highest when there is larger asym. in collision

D e^+-e^- vs. p-p collisions

p-p

- Higher energy for same radius
  - large mass ⇒ small synchroton radiation
- Discovery machines at "energy frontier"
- Huge bkg from colliding strongly interaction <u>composite</u> particles
- Collision energy of constituents unknown

e^+-e^-

- Lower energy
  - High synchrotron radiation
- Precision machines at "precision frontier"
- Clean collisions from fundamental particles
- Collision energy known

- We will now look at flavour in SM formulation
  - this will <u>NOT</u> be rigourous, it is designed to give an overview (good for a thesis theory chapter :))

The Standard Model

Action and Lagrangians

field

$$\underset{\text{action for field } \phi}{S} = \underbrace{\int dt \int d^3x}_{\text{integral over spacetime}} \; \mathcal{L}(\phi, \partial_\mu \phi)$$

lagrangian density

- From "principle of least action" that says that systems follow path to minimise action we get the Euler-Lagrange eqn for fields (using calculus of variations)

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial \phi)} \right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0$$

- Consider Dirac lagrangian for free, relativistic, spin-½ particle

$$\mathcal{L}_{Dirac} = \bar{\Psi} (i \gamma^\mu \partial_\mu - m) \psi \qquad [\bar{\Psi} = \psi^\dagger \gamma^0]$$

Dirac field for fermions

- Can use E-L eqn to find (use $\bar{\Psi}$ version)

$$i\gamma^\mu \partial_\mu \psi - m\psi = 0 \implies \text{Dirac eqn.}$$

Symmetries in SM

- SM is QFT with a lagrangian density, $\mathcal{L}$, that has
  - global Poincaré sym.
  - local gauge sym.

Poincaré sym.

- By Noether's theorem "every global, continuous sym of a system gives conservation law"

Translations in spacetime → Conservation of energy and momentum

Rotations → Cons of angular momentum

Lorentz transformations → Invariance to inertial reference frame

Local gauge symmetries

- Impose local (non-)abelian gauge invariance on $\mathcal{L}$.

eg Impose U(1) abelian gauge invariance of $\mathcal{L}_{Dirac}$

$$\mathcal{L}_{Dirac} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi \xrightarrow[\text{under}]{\text{Invariant}} \psi \to \psi' = \underbrace{e^{iq\overbrace{\theta(x)}^{\text{local}}}}_{\text{U(1) generator}}\psi$$

- For this to be true we need to introduce spin-1 4-vector field A_μ transforming as

$$A_\mu \to A_\mu' = A_\mu + \partial_\mu \theta(x)$$

with covariant derivative

$$D_\mu = \overbrace{\partial_\mu}^{\text{free fermion}} - \underbrace{iqA_\mu}_{\text{interaction term}}$$

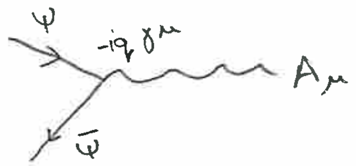
$\Longrightarrow$ Condition for local U(1) invariance is interaction with field A_μ with coupling q.

$$\mathcal{L}_{QED} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \overbrace{\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}}^{\text{gauge inv. kinetic term - free propagation of } A_\mu.}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

- Note that mass terms are quadratic in the field (eg. $m\bar{\psi}\psi$). A mass term for A_μ would not be U(1) invariant so quantisation of A_μ field - the photon - is massless

- Note we can also write



$$\mathcal{L}_{QED} = \mathcal{L}_{free} - \overbrace{J^\mu A_\mu}^{\text{Interaction term}} - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

where $J^\mu_{EM} = q\bar{\psi}\gamma^\mu\psi$ is EM 4-vector current

- All fundamental interactions result from local gauge invariance

- $\mathcal{L}_{SM}$ is invariant under

$$\underbrace{U(1)}_{\substack{\text{QED-like}\\ \text{photon}}} \otimes \underbrace{SU(2)}_{\substack{\text{weak}\\ W^\pm, Z^0}} \otimes \underbrace{SU(3)}_{\substack{\text{strong}\\ 8 \text{ gluons}}} \Longrightarrow 12 \text{ SM spin-1 gauge bosons}$$

Flavour in the SM

- SM lagrangian can be written as

$$\mathcal{L}_{SM} = \mathcal{L}_{gauge}(\underbrace{A_a}_{\text{gauge fields}}, \overbrace{\Psi_i}^{\text{massless fermion fields}}) + \mathcal{L}_{Higgs}(\underbrace{\Phi}_{\text{Higgs field}}, A_a, \Psi_i)$$

$$\mathcal{L}_{gauge} = \sum_{1\leq i\leq 3}^{\text{fermions}} \bar{\Psi}_i (i\gamma^\mu D_\mu)\Psi_i - \sum_{a}^{\text{forces}} \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu,a}$$

- Need to find D_μ for $U(1) \otimes SU(2) \otimes SU(3)$
- We know LH fermions and RH fermions behave differently in SM → RH fermions do not take part in weak interaction

LH doublets

$$Q_{Li} = \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix}$$

$$L_{Li} = \begin{pmatrix} \nu_{Li} \\ e_{Li} \end{pmatrix}$$

Need $U(1)\otimes SU(2) \otimes SU(3)$ invariance

RH singlets

$$U_{Ri} = (u_{Ri}) \quad D_{Ri} = (d_{Ri}) \quad e_{Ri} = (e_{Ri})$$

(no RH neutrinos)

$1 \leq i \leq 3$

Need $U(1) \otimes SU(3)$ invariance

- Recall projection operator $\hat{P}_{L/R} = \frac{1}{2}(1 \mp \gamma^5)$; $\Psi = (\hat{P}_L + \hat{P}_R)\Psi$

- Imposing the required invariance on doublets and singlets

$$D_{Q_L,\mu} = \partial_\mu + ig_s \underbrace{\lambda_\alpha}_{\text{Gell-Mann matrices}} G^\alpha_\mu + ig \overbrace{\sigma_j}^{\text{Pauli matrices - SU(2) generators}} W^j_\mu + i Y_{Q_L} g' B_\mu \qquad \text{(LH doublets)}$$

$$D_{U/D,\mu} = \partial_\mu + ig_s \overbrace{\lambda_\alpha}^{\text{SU(3) generators}} G^\alpha_\mu + i Y_{U/D} g' B_\mu \qquad \text{(RH singlets)}$$

- $SU(2)$ generators $\underline{\sigma}$ give weak isospin ⇒ $\underline{T} = \frac{\underline{\sigma}}{2}$. $T^3 = \pm\frac{1}{2}$ for doublets and $T^3 = 0$ for singlets. W^μ_j are $\underline{\underline{3}}$ vector fields
- Y generator from $U(1)$ is hypercharge. Values fixed so $Q = T^3 + Y$. B_μ is $\underline{\underline{1}}$ $(\Phi, \underline{B})$ vector field
- λ_α generators from $SU(3)$ are colour charge. G^α_μ are $\underline{\underline{8}}$ vector fields

- Mass terms remain forbidden for gauge bosons <u>AND</u> now fermions

$$(\bar{\psi}_L + \bar{\psi}_R)(\psi_L + \psi_R) = \cancel{\bar{\psi}_L \psi_L}^{\,0} + \bar{\psi}_L \psi_R + \psi_R \psi_L + \cancel{\bar{\psi}_R \psi_R}^{\,0}$$

(use $\hat{P}$ and $\gamma^5\gamma^\mu = -\gamma^\mu\gamma^5$)

- Mixes RH fermions that are singlets and LH fermions that are doublets $\Longrightarrow$ Cannot be invariant under $SU(2)$

- At this point SM is flavour ($1 \le i \le 3$) universal
  - 3 gen. have same couplings
  - same (zero) mass

DEGENERACY

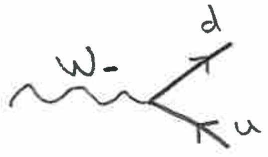
<u>BUT</u> we do already have our weak charged current with up and down "interaction" states of same generation.

$$\mathcal{L}_{CC} \supset ig\,(\bar{u}_{Li}\ \bar{d}_{Li})\,\gamma^\mu\,[\sigma_j W_\mu^j]\begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix}$$

$$ig\,(\bar{u}_{Li}\ \bar{d}_{Li})\,\gamma^\mu \begin{pmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & W_\mu^3 \end{pmatrix}\begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix}$$

$$\overset{\text{terms}}{\underset{\text{like}}{\Longrightarrow}} \bar{u}_{Li}\gamma^\mu W_\mu^+ d_{Li} + \bar{d}_{Li}\gamma^\mu W_\mu^- u_{Li}$$

where $W_\mu^\pm = (W_\mu^1 \mp W_\mu^2)$



<u>Higgs Mechanism</u> (may miss factors of 2...) (ignore SU(3) atm)

- Now look at $\mathcal{L}_{Higgs}$

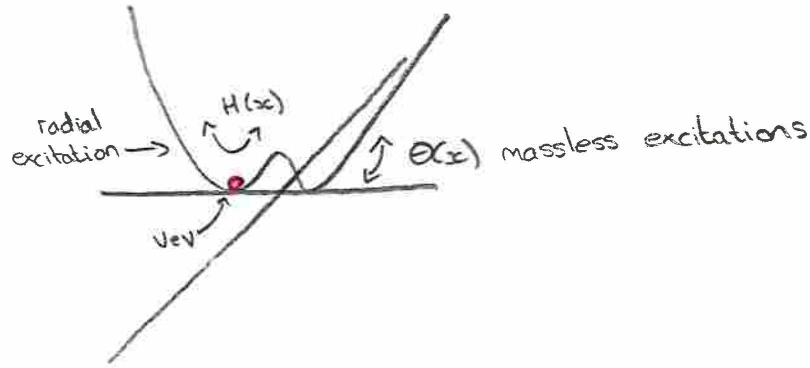
$$\mathcal{L}_{Higgs} = |D_\mu \Phi|^2 - V(\Phi) + \mathcal{L}_{YUKAWA}$$

- Φ is doublet of complex, scalar fields $\Phi = \begin{pmatrix} \Phi^+ \\ \Phi^c \end{pmatrix} = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$ (4 dof)
- V is potential $V(\Phi) = -\mu^2(\Phi^\dagger\Phi) + \lambda(\Phi^\dagger\Phi)^2$

  $\lambda > 0$ for V to be bound.

$\mu^2 < 0$

- Definite ground state



$\mu^2 > 0$

- Degenerate ground state $(\Phi^+ \Phi) = \frac{\mu^2}{2\lambda} = \frac{v^2}{2}$
- System itself is invariant under $SU(2) \otimes U(1)$ but ground state is not
  $\Longrightarrow$ SSB
- Broken 3/4 generators (directions)
- $T^3 + Y = Q$ remains unbroken

$$SU(2) \otimes U(1) \xrightarrow{\text{SSB}} U(1)_{EM}$$

- Choose a ground state to expand around

$$\Phi_0 = e^{i \underline{\Theta}(x) \cdot \underline{T}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

- Remember $\mathcal{L}_{SM}$ is still invariant under $SU(2)$

$$\Phi_0 \xrightarrow[\text{excitations using } SU(2)]{\text{Gauge away 3 massless}} \Phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

- With $|D_\mu \Phi_0|^2 - V(\Phi_0)$ we get terms quadratic in the fields including

$$-\mu^2 H^2 + v^2 \left[g^2 W_\mu^{+2} + g^2 W_\mu^{-2} + (g^2 + g'^2) Z_\mu^2 + 0 \cdot A_\mu^2 \right.$$

$\uparrow$ spin-0 Higgs particle

where $W^\pm_\mu = W_1^\mu \mp W_2^\mu$

$$Z_\mu = \sin\Theta_W (g W_\mu^3 - g' B_\mu)$$
$$A_\mu = \cos\Theta_W (g W_\mu^3 - g' B_\mu)$$

$$\sin\Theta_W = \frac{g'}{\sqrt{g^2 + g'^2}} \qquad \cos\Theta_W = \frac{g}{\sqrt{g^2 + g'^2}}$$

Θ_W is weak mixing angle

- Massless excitations reappear as extra dof to make $W^\pm$ Z^0 massive

Before SSB	dof	After SSB	dof	
Complex Higgs doublet	4	Real Higgs field	1	
Massless B_μ	2	3 massive $W^\pm, Z^0$	9	[+8 gluons]
3 massless W_μ	6	Massless γ	2	
	12		12	

Yukawa Couplings

- Higgs doublet allows fermion mass terms invariant under $U(1) \otimes SU(2) \otimes SU(3)$
- Back to $\mathcal{L}_{Yuk}$ (focusing on quarks)

$$\mathcal{L}_{Yuk} = \underbrace{Y^d_{ij} (\bar{u}_{Li} \; \bar{d}_{Li}) \Phi \, d_{Rj}}_{\text{for down-type}} + \underbrace{Y^u_{ij} (\bar{u}_{Li} \; \bar{d}_{Li}) \tilde{\Phi}_c \, u_{Rj}}_{\text{for up-type}} + h.c$$

- where $\tilde{\Phi}_c = i\sigma_2 \Phi^*$

- After SSB $\Phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+H \end{pmatrix}$ and $\tilde{\Phi}_{0c} = \frac{1}{2} \begin{pmatrix} v+H \\ 0 \end{pmatrix}$

$$\mathcal{L}_{Yuk} = Y^d_{ij} \bar{d}_{Li} \frac{v}{\sqrt{2}} d_{Rj} + Y^u_{ij} \bar{u}_{Li} \frac{v}{\sqrt{2}} u_{Rj} + h.c \; + \text{Higgs int. terms}$$

- we have fermion mass terms $Y^d_{11} \bar{d}_L \frac{v}{\sqrt{2}} d_R + h.c \; ((\bar{d}_L + \bar{d}_R)(d_L + d_R))$

BUT also mixed mass terms $Y^d_{12} \bar{d}_L \frac{v}{\sqrt{2}} s_R + h.c$:(

- To get physical (mass e-state) quarks we need to "remove" mixed mass terms

$$\mathcal{L}_{Yuk} = \underline{\bar{d}_{Li} M^d_{ij} d_{Rj}} + \bar{u}_{Li} M^u_{ij} u_{Rj} + \ldots$$

$$\xrightarrow{\text{diagonalise } M} \underbrace{\bar{d}_{Li} V^{d\dagger}_L}_{\bar{d}^M_{Li}} \underbrace{V^d_L M^d_{ij} V^{d\dagger}_R}_{M^d_{diag}} \underbrace{V^d_R d_{Rj}}_{d^M_{Rj}} \qquad (V^{d\dagger}_L V^d_L = I)$$

- Mass estates are superposition of interaction estates

$$d^M_{Li} = (V^d_L)_{ij} d^I_{Lj} \qquad u^M_{Li} = (V^u_L)_{ij} u^I_{Lj}$$

- Returning to $\mathcal{L}_{cc}$ (LH now implied)

$$\mathcal{L}_{cc} \supset \bar{u}_i \gamma^\mu W_\mu^+ d_i + \bar{d}_i \gamma^\mu W_\mu^- u_i$$

$$\xrightarrow[\text{estates}]{\text{in mass}} \bar{u}_j \underbrace{(V^u V^{d\dagger})_{jk}}_{V_{CKM}} \gamma^\mu W_\mu^+ d_k + \bar{d}_k \underbrace{(V^d V^{u\dagger})_{kj}}_{V_{CKM}^\dagger} \gamma^\mu W_\mu^- u_j$$

- By convention $u_{Li}^I = u_{Li}^M$ and $d_{Li}^I = V_{CKM}\, d_{Li}^M$

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix}^I = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}^M$$

- Applying $\hat{C}\hat{P}$ on $\mathcal{L}_{cc}$

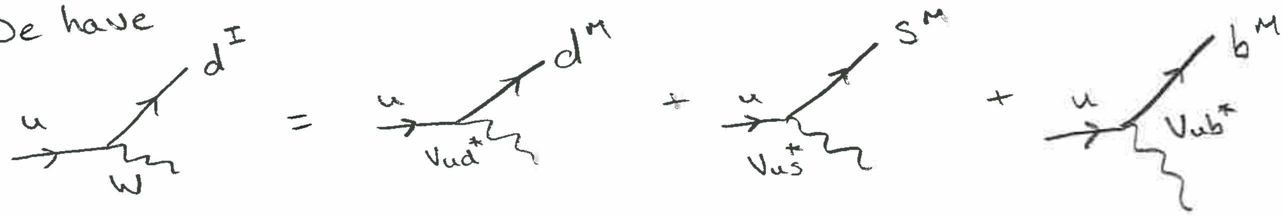
$$\bar{u}_j \gamma^\mu W_\mu^+ V_{jk}^{CKM} d_k + \bar{d}_k \gamma^\mu W_\mu^- V_{jk}^{CKM*} u_j$$

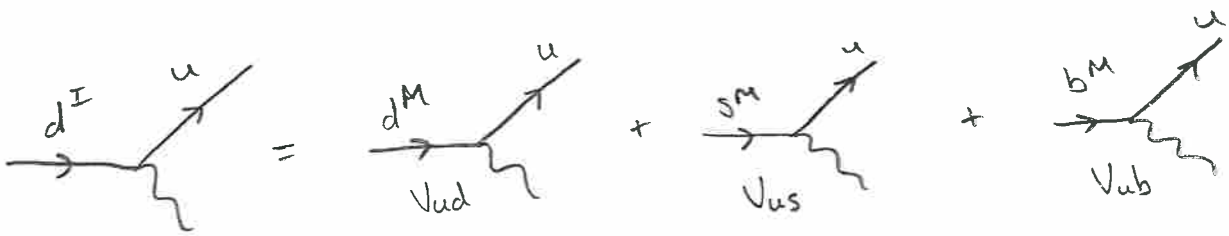
$$\Big\downarrow \hat{C}\hat{P}$$

$$\bar{d}_k \gamma^\mu W_\mu^- V_{jk}^{CKM} u_j + \bar{u}_j \gamma^\mu W_\mu^+ V_{jk}^{CKM*} d_k$$

$\therefore$ SM is only $\hat{C}\hat{P}$ invariant if $V_{CKM} = V_{CKM}^*$!

- We have





SM parameters and puzzles

- There are 18 SM parameters that must be experimentally determined

  - 3 gauge couplings g, g', g_s (alternatively g_s, α, θ_W)
  - Higgs coupling and mass μ, λ (alt. vev and μ/λ)
  - 9 Yukawa couplings for 9 fermion masses
  - 3 quark mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$ and δ_{13} of CKM

  15/18 are related to Higgs mech and flavour.

  [If massive neutrinos + 3 masses, + 3 mixing angles + 1 phase]

- Flavour provides only source of CPV in SM.

Open questions in flavour (slide 10)

- Quark mass hierarchy ie. Yukawa couplings
  - Why?
  - Fine tuning suggest SM is part of larger theory with naturalness

  eg of fine tuning: $M_d \gtrsim M_u \Rightarrow M_n \gtrsim M_p$ else p decays to n and no stable atoms form

- 3 generations
  - Why?
  - Need ≥ 3 for CPV but why not 4+ ($\Rightarrow$ 3 CPV phases)

- Now we will go back and see the discovery of the SM formulation and beyond.