



Is there an experimental way of directly showing that the neutrino is a Dirac particle?



Consider neutrino capture on Tritium

$$v_i + {}^3H \rightarrow {}^3He + e^-$$

$$Amp\sim 1-2h\sqrt{E^2-m^2}$$

Where h is the helicity of the incoming neutrino



 $Amp \sim 1 - 2h\sqrt{E^2 - m^2}$

If neutrinos are relativistic : $Amp \sim 1 - 2hE$

Right handed helical initial states ($h = \frac{1}{2}$) are suppressed

Dirac neutrinos : Antineutrinos are largely RH helical and will not interact due to lepton # conservation.

Majorana neutrinos : The right-handed helical state will be suppressed

$$\Gamma(Majorana) = \Gamma(Dirac)$$



 $Amp \sim 1 - 2h\sqrt{E^2 - m^2}$

If neutrinos are nonrelativistic : $Amp\!\sim\!1$

No helicity suppression

Dirac neutrinos : Antineutrinos will still not interact

Majorana neutrinos : No suppression – both LH and RH helical states can still interact.

$$\Gamma(Majorana) = 2\Gamma(Dirac)$$



Non-relativistic neutrinos can be used to distingush between Dirac and Majorana nature of neutrinos

But....

Only known source of non-relativistic neutrinos is the (yet to be observed) cosmic relic neutrino background

These have kinetic energies around 10⁻⁴ eV

Capture cross sections ~ 10^{-44} cm² (T2K : 10^{-38} cm²)

The Ptolemy experiment is an idea for an experiment designed to observe these neutrinos. Predicted capture rates for Ptolemy are

 $\Gamma(Dirac) \sim 5/year \qquad \Gamma(Majorana) \sim 10/year$

Lecture 3



The neutrino oscillation industry

Solar Neutrinos



SuperK : Solar neutrino-gram



•Light from the solar core takes a million years to reach the surface

- Fusion processes generate electron neutrinos which take
 2s to leave
- Solar neutrinos are a direct probe of the solar core
- Roughly 4.0 x 10^{10} solar v_e per cm² per second on earth

Solar Neutrino Flux





The Solar Neutrino Problem - Homestake





Homestake sensitive to ⁸B and ⁷Be *electron neutrinos*

 $E_v > 800 \text{ keV}$

Observe 1/3 of the expected number of solar neutrinos

1 SNU = 1 interaction per $10^{36} \text{ atoms per second}$



Experimental summary

Total Rates: Standard Model vs. Experiment Bahcall-Pinsonneault 2000



Atmospheric neutrinos



High energy cosmic rays interact in the upper atmosphere producing showers of mesons (mostly pions)



Neutrinos produced by

Expect $\frac{N(v_{\mu} + \overline{v_{\mu}})}{N(v_{e} + \overline{v_{e}})} \approx 2$

At higher energies, the muons can reach the ground before decaying so ratio increases



The Atmospheric Neutrino Anomaly



Neutrino Flavour Oscillations

MixingCKM
Mechanism
$$\begin{pmatrix} u \\ d' \end{pmatrix}_L$$
 $\begin{pmatrix} c \\ s' \end{pmatrix}_L$ $d' = d \cos \theta_c + s \sin \theta_c$
 $s' = -d \sin \theta_c + s \cos \theta_c$

In the quark sector, the flavour eigenstates (those states which couple to the W/Z) are not identical to the mass eigenstates (those states which are eigenstates of the Hamiltonian)

Weak
$$(d')_{s'} = \begin{pmatrix} 0.97 & 0.23 & 0.003 \\ 0.23 & 0.97 & 0.04 \\ 0.008 & 0.04 & 0.99 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} - Mass states$$

MixingImage: CKM
Mechanism
$$\begin{pmatrix} u \\ d' \end{pmatrix}_L$$
 $\begin{pmatrix} c \\ s' \end{pmatrix}_L$ $d' = d \cos \theta_c + s \sin \theta_c$
 $s' = -d \sin \theta_c + s \cos \theta_c$

In the quark sector, the flavour eigenstates (those states which couple to the W/Z) are not identical to the mass eigenstates (those states which are eigenstates of the Hamiltonian)





Neutrino Oscillations



If we don't know which mass state was created then the the amplitude involves a <u>coherent</u> superposition of v_i states

$$Prob(\nu_{\alpha} \rightarrow \nu_{\beta}) = \delta_{\alpha\beta} - 4\sum_{i>j} \Re (U_{\alpha i}^{*} U_{\beta i} U_{\alpha j} U_{\beta j}^{*}) \sin^{2}(\Delta m_{ij}^{2} \frac{L}{4E})$$
$$+ 2\sum_{i>j} \Im (U_{\alpha i}^{*} U_{\beta i} U_{\alpha j} U_{\beta j}^{*}) \sin(\Delta m_{ij}^{2} \frac{L}{2E})$$

> If $\Delta m_{ij}^2 = 0$ then neutrinos don't oscillate

- Oscillation depends on |Δm²| absolute masses cannot be determined
- > If there is no mixing (If $U_{ai} = 0$) neutrinos don't oscillate
- One can detect flavour change in 2 ways : start with v_α and look for v_β (appearance) or start with v_α and see if any disappears (disappearance)
- Flavour change oscillates with L/E. L and E are chosen by the experimenter to maximise sensitivity to a given Δm²
- Flavour change doesn't alter total neutrino flux it just redistributes it amongst different flavours (unitarity)

Two flavour oscillations



$$\begin{pmatrix} v_{\alpha} \\ v_{\beta} \end{pmatrix} = U \begin{pmatrix} v_{1} \\ v_{2} \end{pmatrix} \Rightarrow U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

$$P(v_{\alpha} \to v_{\beta}) = \delta_{\alpha\beta} - 4 \sum_{i>j} U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j} \sin^{2}(\Delta m_{ij}^{2} \frac{L}{4E})$$

 $P(v_{\alpha} \rightarrow v_{\beta})$: Appearance Probability $P(v_{\alpha} \rightarrow v_{\alpha})$: Survival Probability

$$P(\nu_{\alpha} \rightarrow \nu_{\beta}) = -4(U_{\alpha 1}U_{\beta 1}U_{\alpha 2}U_{\beta 2})\sin^{2}(\Delta m_{ij}^{2}\frac{L}{4E})$$

$$.=\sin^{2}(2\theta)\sin^{2}(1.27\Delta m^{2}(eV^{2})\frac{L(km)}{E(GeV)})$$

(changing to useful units)







Question : What would you observe if you were able to know which mass state propagated from source to detector?



Three Flavour Oscillation

The three flavour case is more complicated, but no different

$$\begin{pmatrix} \nu_{e} \\ \nu_{\mu} \\ \nu_{\mu} \\ \nu_{\tau} \end{pmatrix} = U \begin{pmatrix} \nu_{1} \\ \nu_{2} \\ \nu_{3} \end{pmatrix} \Leftrightarrow U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

U is the Pontecorvo-Maskawa-Nakayama-Sakata (PMNS) matrix

$$Prob(v_{\alpha} \rightarrow v_{\beta}) = \delta_{\alpha\beta} - 4\sum_{i>j} \Re (U_{\alpha i}^{*} U_{\beta i} U_{\alpha j} U_{\beta j}^{*}) \sin^{2}(\Delta m_{ij}^{2} \frac{L}{4E})$$
$$+ 2\sum_{i>j} \Im (U_{\alpha i}^{*} U_{\beta i} U_{\alpha j} U_{\beta j}^{*}) \sin(\Delta m_{ij}^{2} \frac{L}{2E})$$

$$\begin{array}{l}
\textbf{Oscillation parameters} \\
U = \begin{pmatrix} U_{el} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix} \\
\end{array}$$

$$\begin{array}{l}
\textbf{Prob}(v_{\alpha} \rightarrow v_{\beta}) = \delta_{\alpha\beta} - 4\sum_{i>j} \Re(U_{\alpha i}^{*}U_{\beta i}U_{\alpha j}U_{\beta j}^{*})\sin^{2}(\Delta m_{ij}^{2}\frac{L}{4E}) \\ + 2\sum_{i>j} \Im(U_{\alpha i}^{*}U_{\beta i}U_{\alpha j}U_{\beta j}^{*})\sin(\Delta m_{ij}^{2}\frac{L}{2E})
\end{array}$$

Oscillation parameters

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$
Three angles

$$Prob(v_{\alpha} \rightarrow v_{\beta}) = \delta_{\alpha\beta} - 4\sum_{i>j} \Re (U_{\alpha i}^{*} U_{\beta i} U_{\alpha j} U_{\beta j}^{*}) \sin^{2}(\Delta m_{ij}^{2} \frac{L}{4E})$$
$$+ 2\sum_{i>j} \Im (U_{\alpha i}^{*} U_{\beta i} U_{\alpha j} U_{\beta j}^{*}) \sin(\Delta m_{ij}^{2} \frac{L}{2E})$$



$$Prob(\nu_{\alpha} \rightarrow \nu_{\beta}) = \delta_{\alpha\beta} - 4\sum_{i>j} \Re (U_{\alpha i}^{*} U_{\beta i} U_{\alpha j} U_{\beta j}^{*}) \sin^{2} (\Delta m_{ij}^{2} \frac{L}{4E})$$
$$+ 2\sum_{i>j} \Im (U_{\alpha i}^{*} U_{\beta i} U_{\alpha j} U_{\beta j}^{*}) \sin (\Delta m_{ij}^{2} \frac{L}{2E})$$



The extra Majorana matrix does not affect flavour oscillation processes.....so is usually dropped. However it will affect the interpretation of neutrinoless double beta decay results



Explaining the solar data

Testing the oscillation hypothesis



Solar neutrino problem

 ν_e from sun would change to ν_μ or ν_τ . However these have too little energy to interact via the charged current, and all the detectors are only sensitive to charge current interactions.

Non- ν_e component would effectively disappear, reducing the apparent ν_e flux.

Proof : Neutral current event rate shouldn't change.

Sudbury Neutrino Observatory







1000 tonnes of D_20 6500 tons of H_20 Viewed by 10,000 PMTS In a salt mine 2km underground in Sudbury, Canada

SNO



$$v_e + d \rightarrow p + p + e$$

- Q = 1.445 MeV
- good measurement of v_e energy spectrum
- some directional info $\propto (1 1/3 \cos \theta)$
- Ve only

CC

NC
$$\nu_x + d \rightarrow p + n + \nu_x$$

-Q = 2.22 MeV

measures total ⁸B v flux from the Sun
 equal cross section for all v types

$$v_x + e^- \rightarrow v_x + e^-$$

- low statistics
- mainly sensitive to v_e , some v_{μ} and v_{τ}
- strong directional sensitivity

n captures on deuteron ²H(n, γ)³H Observe 6.25 MeV γ $\nu_e + \nu_{\mu} + \nu_{\tau}$

Produces Cherenkov Light Cone in D₂O

$$v_e + 0.15*(v_\mu + v_\tau)$$

SNO Results





Naively...



First instinct is to assume that neutrinos leave the sun as $v_{\rm e}$ and oscillate on their way to the earth. Assuming this

$$L \sim 10^8 \, km$$
, $E_v < 10 \, MeV \rightarrow \Delta m^2 \sim 3 \times 10^{-10} \, eV^2$

Naively...



First instinct is to assume that neutrinos leave the sun as $\nu_{\rm e}$ and oscillate on their way to the earth. Assuming this

$$L \sim 10^8 \, km$$
, $E_v < 10 \, MeV \rightarrow \Delta m^2 \sim 7 \times 10^{-5} \, eV^2$

Naively...



First instinct is to assume that neutrinos leave the sun as $v_{\rm e}$ and oscillate on their way to the earth. Assuming this

$$L \sim 10^8 \, km$$
, $E_v < 10 \, MeV \rightarrow \Delta m^2 \sim 7 \times 10^{-5} \, eV^2$

Oscillations come from phase difference between mass states. In a vacuum the phase diff comes from free particle Hamiltonian. In a material there are interaction potentials as well

$$-i\hbar\frac{\partial\psi}{\partial t} = E\psi = \frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} \rightarrow -i\hbar\frac{\partial\psi}{\partial t} = (E+V)\psi = \frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}$$
$$E_{vac}^2 - p^2 = m_{vac}^2 \rightarrow (E_{vac}+V)^2 - p^2 = m_{mat}^2 \rightarrow m_{mat} \approx \sqrt{m_{vac}^2 + 2E_{vac}V}$$

c.f. effective mass of an electron in a semiconductor or light in glass

Oscillations in Matter



Electrons exist in standard matter – μ/τ do not. Electron neutrinos travelling in matter can experience an extra charged current interaction that other flavours cannot.



Implications

$$\sin^{2} 2 \theta_{M} = \frac{\sin^{2} 2 \theta_{V}}{\sin^{2} 2 \theta_{V} + (\cos 2 \theta_{V} - \zeta)^{2}} \qquad \zeta = \frac{2 \sqrt{2} G_{F} N_{e} E}{\Delta m_{Vac}^{2}}$$

•If $\Delta m^2_{Vac} = 0$ or matter is very dense, $\zeta = \infty$ and $\theta_m = 0$ •Similarly, if $\theta_{vac}=0$, then $\theta_M = 0 \Rightarrow$ need mixing in vacuum •If there is no matter, then $\zeta = 0$ and we have vacuum mixing

•At a particular electron density, dependent on Δm^2 ,

$$\zeta = \frac{2\sqrt{2}G_F N_e E}{\Delta m^2} = \cos 2\theta \implies \sin^2 2\theta_M = 1$$

Even if the vacuum mixing angle is tiny, there is a density for which the matter mixing angle is maximal

Mass heirarchy

$$\sin^{2} 2 \theta_{M} = \frac{\sin^{2} 2 \theta}{\sin^{2} 2 \theta + (\cos 2 \theta - \zeta)^{2}} \qquad \zeta = \frac{2\sqrt{2} G_{F} N_{e} E}{\Delta m_{V}^{2}}$$

If mass of $v_1 < mass of v_2$, $\Delta m_V^2 = m_1^2 - m_2^2 < 0$

$$\zeta = -\frac{2\sqrt{2}G_F N_e E}{|\Delta m^2|} \rightarrow \sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta + |\zeta|)^2}$$

Positive definite – no resonance

If mass of $v_1 > mass of v_2$, $\Delta m^2 = m_1^2 - m_2^2 > 0$

$$\zeta = \frac{2\sqrt{2}G_F N_e E}{|\Delta m^2|} \rightarrow \sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - |\zeta|)^2}$$

Mass heirarchy

$$\sin^{2}2\theta_{M} = \frac{\sin^{2}2\theta}{\sin^{2}2\theta + (\cos 2\theta - \zeta)^{2}} \qquad \zeta = \pm \frac{2\sqrt{2}G_{F}N_{e}E}{|\Delta m_{V}^{2}|}$$

The effect of matter on neutrino oscillations can be used to measure the mass hierarchy.

This is about the only way we know how to do this.



Mixing matrix

$$U = \begin{pmatrix} U_{el} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

Solar oscillations
occur *in* the sun
via matter effects
$$\begin{cases} \text{Solar sector} \\ \theta_{e\mu} = 32.5^{\circ} \pm 2.4^{\circ} \\ \Delta m_{12}^{2} = +7.9 \times 10^{-5} eV^{2} \end{cases}$$



Explaining the atmospheric data

Cosmic Labs







Atmospheric results







Prediction for v_e rate agrees with data.
v_µ disappear at large baseline consistent with v_µ → v_τ
Don't detect v_τ as
below τ mass threshold
SuperK is awful at τ detection

$$\left|\Delta m_{atmos}^2\right| \approx 0.0025 \, eV^2$$

 $\sin^2(2\,\theta_{atmos}) \approx 1.0$



Accelerator Cross-check

$\Delta m_{atmos}^2 \approx 3 \times 10^{-3} eV^2 \rightarrow L/E \approx 400 \, km \, GeV^{-1}$

 $L=250 \, km \rightarrow E_{v} \approx 0.6 \, GeV$



Beam events tagged using GPS at both near and far detector sites





Use Near Detector to measure $\Phi_v(@ND)$

T2K and NOVA





> JPARC to Kamioka
> L = 295 km
> E_v ~ 0.6 GeV
> Far Det : 22.6 kton water Cerenkov detector

Fermilab to Ash River, MN
 L = 810 km
 E_v ~ 2.0 GeV
 Far Det : 14 kton of liquid scintillator (in bars)

detector at 295 km



T2K Disappearance







Mixing matrix

$$U = \begin{pmatrix} U_{el} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

Solar sector : $v_{\mu} \rightarrow v_{e}$
 $\theta_{e\mu} = 33.7^{\circ} \pm 1.1^{\circ}$
 $m_{12}^{2} = +(7.54 \pm 0.24) \times 10^{-5} eV^{2}$
$$Atmospheric sector$$

 $v_{\mu} \rightarrow v_{\tau}$
 $\theta_{\mu\tau} = 42^{\circ} \pm 3.0^{\circ}$
 $\Delta m_{23}^{2} = |(2.43 \pm 0.06) \times 10^{-3}| eV^{2}$

How do we measure θ_{13} ?



 $v_{\mu} \rightarrow v_{e}$ oscillations with atmospheric L/E

$$P(v_{\mu} \to v_{e}) = \sin^{2} 2 \theta_{13} \sin^{2} \theta_{23} \sin^{2} (1.27\Delta m_{23}^{2} \frac{L}{E})$$

 v_e appearance in a v_μ beam – ideal for *accelerator experiments*

 $\overline{v}_e \rightarrow \overline{v}_x$ disappearance oscillations with atmospheric L/E

$$p(\overline{\mathbf{v}_{e}} \rightarrow \overline{\mathbf{v}_{x}}) 1 - \sin^{2}(2\theta_{13}) \sin^{2}(1.27\Delta m_{23}^{2}\frac{L}{E})$$

 v_e disappearance – ideal for *reactor experiments* Probability only a function of θ_{13}



Example : Daya Bay



θ_{13} from reactors





 $\theta_{13} = (8.44(41) \pm 0.16)^{\circ} (NO(IO))$



Summary of Current Knowledge





$$U_{MNSP} \approx \begin{pmatrix} 0.82 & 0.54 & 0.14 \\ 0.35 & 0.56 & 0.68 \\ 0.35 & 0.55 & 0.69 \end{pmatrix}$$

Some elements only known to 10-30%

Very very different from the quark CKM matrix