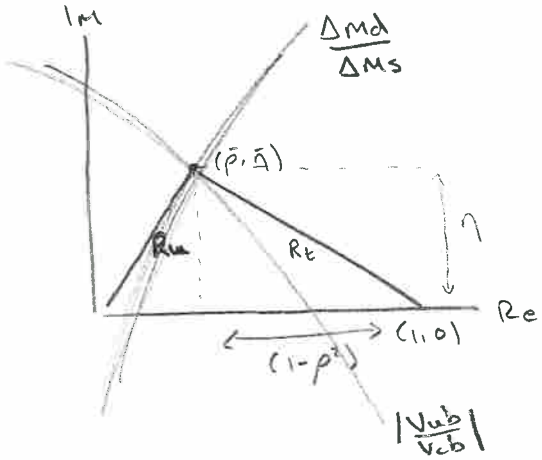
Measuring the CKM

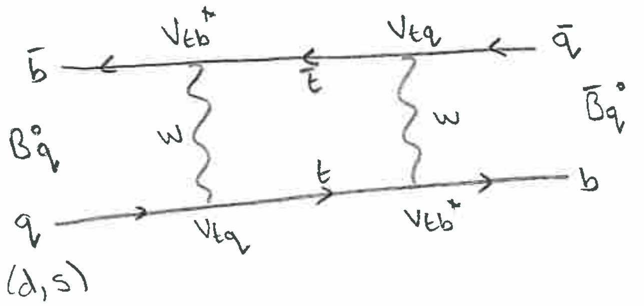
- Take the B^0 UT - there are 4 measurements that provide strongest constraints

$$|V_{ub}|,\ \Delta M,\ \sin 2\beta \text{ and } \varepsilon_K$$



- R_u side constrained by $\left|\frac{V_{ub}}{V_{cb}}\right|$ as $(\bar\rho, \bar\eta) = (1-\frac{\lambda^2}{2})(\rho, \eta)$ and $\left|\frac{V_{ub}}{V_{cb}}\right|^2 \propto (\rho^2 + \eta^2)$

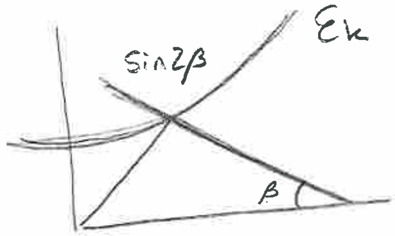
- R_t side constrained by $\frac{\Delta M_d}{\Delta M_s}$ as $\left|\frac{V_{td}}{V_{ts}}\right|^2 \propto (1-\rho)^2 + \eta^2$



- t quark contribution dominates as $m_t \gg m_c$

- Although strong constraints, on their own, these do not give CPV as η can be $0 \Rightarrow J = 0$

- Two other constraints are



- $|\varepsilon_K|$ related to kaon mixing

Measuring phases

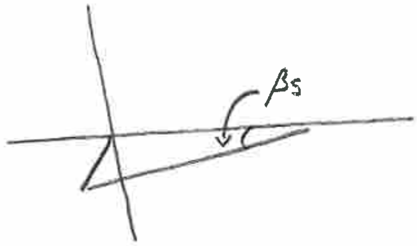
- Recall

$$V_{CKM} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{-i\beta_s} & |V_{tb}| \end{pmatrix} + O(\lambda^5)$$

- Access γ through V_{ub} in interference btw $b \to u$ and $b \to c$
- Access β through V_{td} in int. btw B^0 mixing and decay
- Access β_s through V_{ts} in int btw B_s^0 mixing and decay
- Access α through int. btw different $b \to u$ transitions

Measuring β_s

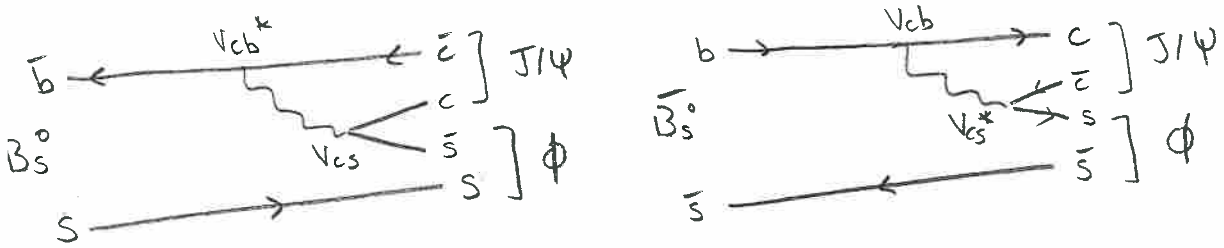
- Recall B_s UT
- Access β_s through V_{ts}



- β_s is small!

- Golden mode is $\overset{0^-}{B_s^0} \rightarrow \overset{1^{--}}{J/\psi}\ \overset{1^{--}}{\phi}$ with 3 amps

$A_{\parallel}\,(\uparrow\uparrow)$ $L=2$
$A_{\perp}\,(\uparrow\rightarrow)$ $L=1$
$A_0\,(\uparrow\downarrow)$ $L=0$

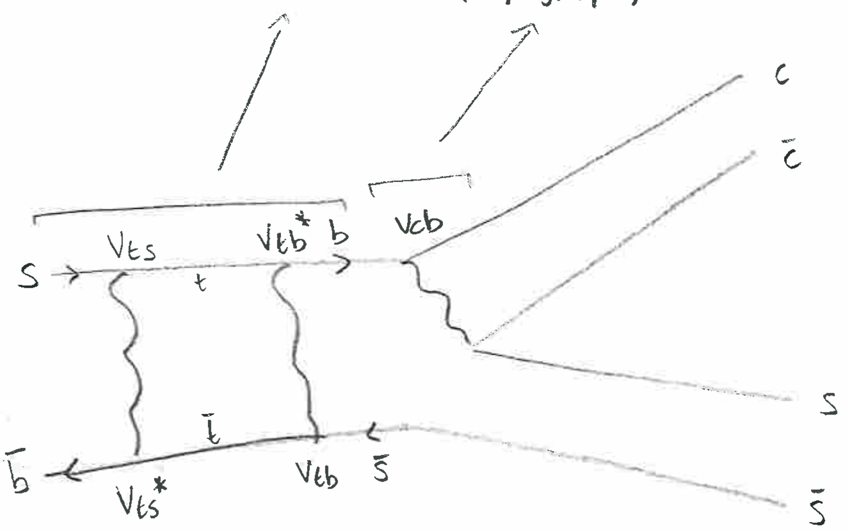


$$|A_f| \sim |\bar{A}_f| \therefore |\lambda_f| \sim 1$$

- Look closer at λ_f

$$\lambda_{J/\psi\phi} = \underbrace{\left(\frac{q}{p}\right)_{B_s}}_{\text{mix}} \underbrace{\left(\frac{\bar{A}_{J/\psi\phi}}{A_{J/\psi\phi}}\right)}_{\text{decay}} = \left(\frac{\cancel{V_{tb}^*} V_{ts}}{\cancel{V_{tb}}\ V_{ts}^*}\right)\left(\frac{\cancel{V_{cb}} \cancel{V_{cs}^*}}{\cancel{V_{cb}^*} \cancel{V_{cs}}}\right)$$

no phase



$$\lambda = \frac{-|V_{ts}|e^{-i\beta_s}}{-|V_{ts}|e^{i\beta_s}}$$

$$\lambda = e^{-2i\beta_s}$$

- Returning to A_{CP} where $|p| \sim |q|$ as for B_s^0

$$A_{CP}(t) = \frac{C_f \cos(\Delta m t) - S_f \sin(\Delta m t)}{\cosh\left(\frac{\Delta\Gamma t}{2}\right) + D_f \sinh\left(\frac{\Delta\Gamma t}{2}\right)} \xrightarrow{|\lambda_f| \sim 1} \frac{-\mathrm{Im}(\lambda_f)\sin(\Delta m t)}{\cosh\left(\frac{\Delta\Gamma}{2}t\right) + \mathrm{Re}(\lambda_f)\sinh\left(\frac{\Delta\Gamma}{2}t\right)}$$

- Therefore

$$A_{CP}(t) = \frac{-\eta \sin(2\beta_s)\sin(\Delta m t)}{\cosh\left(\frac{\Delta\Gamma}{2}t\right) + \eta\cos(2\beta_s)\sinh\left(\frac{\Delta\Gamma}{2}t\right)}$$

(3 amps to consider)

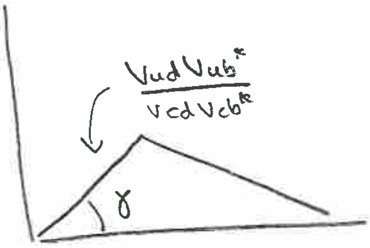
$$\hat{C}\hat{P}|J/\psi\phi\rangle = (-1)^L |J/\psi\phi\rangle \Rightarrow \eta = (-1)^L$$

- β is measured similarly using $B^0 \to J/\Psi K_{S(L)}$ with following differences

  - Sensitive to V_{td} $(e^{-i\beta})$ not V_{ts} $(-e^{-i\beta_s})$
  - Actual decay product is K^0 so have to account for kaon mixing in λ_f
  - Only one amplitude so $l=0$
  - $\Delta T \sim 0$

$$\Rightarrow A_{CP}(t) = C_f \cos(\Delta m t) - S_f \sin(\Delta m t)$$

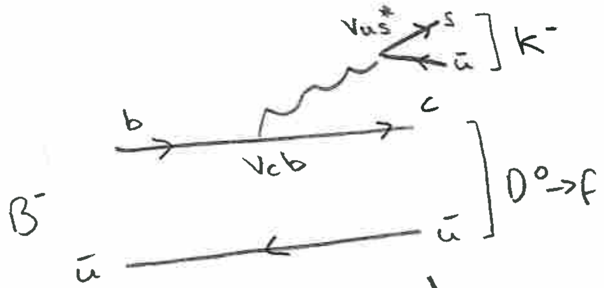
Measuring γ



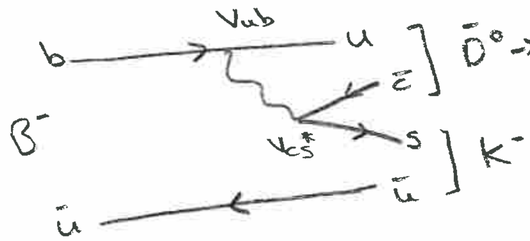
- Recall B^0 UT
- γ is phase btw $V_{ud}V_{ub}^*$ and $V_{cb}^*V_{cd}$

$\Rightarrow$ need int btw $b \to c$ and $b \to u$

- No dependence on top quarks $\to$ do not need loops, γ can be measured at tree-level called SM "standard candle"
- Most familiar case is



Colour favoured
$V_{us}^* V_{cb} = |V_{us}||V_{cb}|$



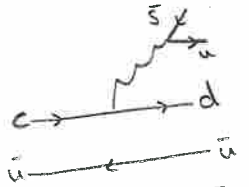
Colour suppressed
$V_{ub}V_{cs}^* = |V_{ub}|e^{-i\gamma}|V_{cs}|$

- Interference when D^0 and $\bar{D}^0$ decay to same f. lots of f to choose from but all have <u>same</u> weak phase γ
- There are lots of ways to measure γ categorised depending on f.

ADS Atwood, Dunietz + Soni

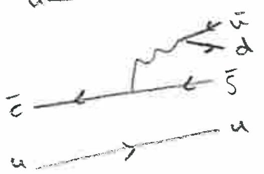
- $\overset{(-)}{D}{}^0$ 2-body decay

DCS $D^0 \to \pi^- K^+$



$V_{us}V_{cd} \sim \lambda^2$

CF $\bar{D}^0 \to \pi^- K^+$



$V_{cs}V_{ud} \sim 1$

- Look at the favoured B decay followed by suppressed D decay and v.v. $\Rightarrow$ similar overall amps $\Rightarrow$ large interference

$$\begin{tikzcd}
 & Dh^- \arrow[dr, "r_D e^{i\delta_D}", "DCS"'] & \\
B^- \arrow[ur, "CF"'] \arrow[dr, "r_B e^{i(\delta-\gamma)}", "CS"'] & & (h^+h^{-'})h^{-''} \\
 & \bar{D}h^- \arrow[ur, "CF"'] &
\end{tikzcd}$$

$$\underbrace{r_B = \frac{|S|}{|F|}, \quad \delta_B = \delta_F - \delta_S}_{\text{depend on B decay}}, \quad \underbrace{\gamma = \phi_F - \phi_S}_{\text{Universal}}$$

$$\underbrace{r_D = \frac{|DCS|}{|CF|}, \quad \delta_D = \delta_{CF} - \delta_{DCS}}_{\text{Depend on D decay}} \qquad \text{(neglect CPV in D system)}$$

- Need "hadronic" parameters for D decays from CLEO, BESIII "Charm factories"

$$A_{CP}^{ADS} = \frac{2 r_D r_B \sin(\delta_B + \delta_D) \sin(\gamma)}{r_D^2 + r_B^2 + 2 r_B r_D \cos(\delta_B + \delta_D) \cos(\gamma)}$$

- Can extend ADS to 3 body modes like $D \to k_S \pi\pi$. Here we will have interfering resonances in final state which "dilute" (K_D) A_{CP} when we integrate over PS.

BPGGSZ

- $\overset{(-)}{D}{}^0$ 3 body decay
  - interfering resonances
  - r_D, δ_D vary over PS (on DP)

- If we actually know $A_{D\to f}$ as a func^n of phasespace we can use this to measure γ in bins of PS optimising sensitivity to γ.

$\rightarrow$ CLEO provided best DP binning to give greatest γ sensitivity

(Slide 25)

(Slide 26-29)

EFTs

(largely copied from S. Renner) $\frac{1}{4}$

Idea of EFTs

- To understand system at length/energy scale we do not need to describe it at a completely different scale
  eg. Understand Mars' orbit without knowing position of all rocks on Mars

- In natural units ($c = \hbar = 1$) $\frac{1}{\text{length}}$ = mass = energy

mass	momentum	$\frac{1}{\text{length}}$	all have same unit
energy	frequency	$\frac{1}{\text{time}}$	

$\Rightarrow$ All dimensionful quantities can be expressed in powers of mass

$\Rightarrow$ $\uparrow$ mass = $\uparrow$ energy = $\downarrow$ length = $\downarrow$ time

- Often we do not need to know exact physics at higher energies/masses

Action

- In natural units, S, is dimensionless

$[x]$ = dim of x

$$S = \int \underbrace{d^4x}\ \mathcal{L}$$

$$[d^4x] = \text{mass}^{-4} \Rightarrow [\mathcal{L}] = \text{mass}^4$$

- Use this to find field dim

$$\mathcal{L}^{kin}_{Higgs} = \partial_\mu \phi\, \partial^\mu \phi \qquad [\partial_\mu] = 1 \Rightarrow [\phi] = 1$$

(in mass dim)

$$\mathcal{L}^{kin}_{fermion} = i\bar{\Psi}\gamma\partial_\mu \Psi \qquad [\partial_\mu] = 1 \Rightarrow [\Psi] = 3/2$$

- Looking at mass term $m\Psi\bar{\Psi}$, $[m] = 4 - 2\times\frac{3}{2} = 1$ ✓ ☺

Operators

- Counting dimensions helps understand importance of interactions at different energies
  - Consider dim = 4 int. like QED

    $$\mathcal{L}_{Dirac} = \bar{\Psi}\left(i\gamma^\mu(\partial_\mu + ieA_\mu)\right)\Psi \text{, int. term is } \bar{\Psi}\gamma^\mu A_\mu \Psi$$

    $[A_\mu] = 1$, $[\Psi] = 3/2$ so dim 4 $\quad A^{(4)} \sim 1$

    "Marginal" → int. term is dimensionless and constant with energy

  - Consider dim=3 term like $\bar{\Psi}\Psi$ (mass term)

    $$A^{(3)} \sim \frac{1}{E}$$

    "Relevant" → more important at low energies. eg. $[\Psi\bar{\Psi}] = 3$ has more effect at low energies (mass & fermion matters more)

  - Consider dim = 5 term $\quad A^{(5)} \sim E$

    "Irrelevant"/"Non-renormalisable" → less important at low energies

- Irrelevant operators can be important if they provide the leading effect for a process (ie. process is forbidden by relevant + marginal operators)

- Write $\mathcal{L}_{EFT}$ as, $[\Lambda] = 1$

  wilson coeffs

  $$\mathcal{L}_{EFT} = \underbrace{\mathcal{L}^{d\leq 4}}_{\text{M+R ops}} + \underbrace{\overbrace{\frac{C_i^{(d=5)}}{\Lambda}}^{\sim E} \underbrace{O_i^{(d=5)}}_{\text{operators}} + \overbrace{\frac{C_i^{(d=6)}}{\Lambda^2}}^{\sim E^2} O_i^{(d=6)} + \dots}_{\text{Important at high energies}}$$

  - as long as $\Lambda \gg E$ operators are less important as dim increases

- Two restrictions
  - operators must respect gauge + global sym that exist in the theory at scale E
  - Operators must be bilt from fields that exist in theory at scale E ($M \leq E$)

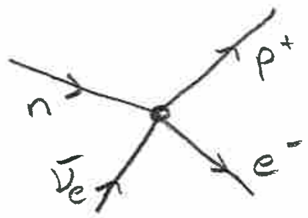
Fermi theory

- Fermi theory is an EFT ($\mathcal{L}_{EFT}$) below EW scale
- Below EW scale, SM is just $SU(3) \otimes U(1)$ with no W, Z, H, t particles and neutrons are stable
  - neutrons decay through 4-fermion point interaction (do not resolve W - it doesnt exist in the EFT), with the leading irrelevant operator for $d \leq 4$ interactions below EW scale

  $\Rightarrow$ Flavour is preserved

- Fermi described β-decay as 4-point int. and proposed a matrix element analogous to EM int.

$$M = G_F \cdot g_{\mu\nu} [\bar{U}_p \gamma^\mu U_n][\bar{u}_e \gamma^\nu u_{\bar{\nu}_e}]$$



in $\mathcal{L}$ form this is

$$\mathcal{L}_{\beta\text{-decay}} = \frac{C_{\beta\text{-decay}}}{\Lambda^2} \underbrace{(\bar{u}_p \gamma^\mu u_n)(\bar{u}_e \gamma^\nu u_{\bar{\nu}_e})}_{dim=6}$$ where $\Lambda \sim$ EW scale

- In "full" theory we have $W^\pm$ vertices and $W^\pm$ propogators that describe $d \to u e^- \bar{\nu}_e$

- We can "match" $\mathcal{L}_{EFT}$ to "full" theory at $q^2 \ll \Lambda$
- W fermion vertex is dim = 4, $\mathcal{L}_{cc} \supset [\bar{u}\gamma^\mu d_L] W^+$, in full theory

"Full"
$$M \supset (g_w)^2 \underbrace{\left[\bar{u}_u \gamma^\mu u_d\right]}_{W^\pm \text{ fermion vertex}} \underbrace{\left[\frac{g_{\mu\nu} - q_\mu q_\nu / M_W^2}{q^2 - M_W^2}\right]}_{W \text{ propagator}} \left[\bar{u}_e \gamma^\mu u_{\bar{\nu}_e}\right]$$

(easier to "match" with matrix elements but use $\mathcal{L}$ for dim)

Fermi
$$M \supset \underbrace{G_F \cdot g_{\mu\nu}}_{\frac{C_{\beta\text{-decay}}}{\Lambda^2}} \underbrace{\left[\bar{u}_p \gamma^\mu u_n\right]\left[\bar{u}_e \gamma^\mu u_\nu\right]}_{\Rightarrow \text{dim} = 6 \text{ operator}}$$

$$W \text{ propagator} \xrightarrow{q^2 \ll M_W^2} \frac{g_w^2 \, g_{\mu\nu}}{M_W^2}$$

$$\Rightarrow G_F = \frac{g_w^2}{M_W^2} = \frac{C_{\beta\text{-decay}}}{\Lambda^2}$$ CONSTANT COUPLING GF AT LOW ENERGY

- Fermi theory is low energy limit of weak int. where $M_W^2 \gg q^2$

- A $\mathcal{L}_{EFT}$ respecting $SU(3) \otimes U(1)$ with operators up to dim = 6 can provide good descriptions of SM below EW scale
  $\Rightarrow$ known as the <u>LEFT</u>

(slide 30)