Multi-Higgs production in EFT

Javier Martínez-Martín[1]

[1]In collaboration with Rafael L. Delgado, Raquel Gómez-Ambrosio, Alexandre Salas-Bernárdez and Juan J. Sanz-Cillero - 2311.04280v2 - JHEP 03 (2024) 037

8th RED LHC Workshop
29 May 2024

Outline

1. Introduction
   - Motivation
   - Framework
   - Higgs Effective Field Theory (HEFT)
   - Relation to SMEFT
2. Amplitudes and cross sections calculations
   - $\omega\omega \to 2h$
   - $\omega\omega \to 3h$
   - $\omega\omega \to 4h$
3. Cross section phenomenology
   - SMEFT-like model. Benchmark points
   - Non-SMEFT-like models. Benchmark points
   - BP study
4. Conclusions and next steps

Motivation

- Understanding the SM as the leading order of an EFT, we would like to find new terms.
- At the current energies, both SMEFT and HEFT are valid descriptions of the currently available LHC data.
- Multi-Higgs measurements at HL-LHC and beyond are crucial to check their validity range.

Framework

- We used the Equivalence theorem approximation where $W_L^a \sim \omega^a$.
- Assuming $m_h^2 \sim m_W^2 \ll s \ll \Lambda^2$.
- Using only derivative terms of the Lagrangian up to 2 derivatives (which scale with the energy).
- As a first approximation, only one SMEFT operator is needed (at each EFT dimenson).

Higgs Effective Field Theory

Canonical form

HEFT Lagrangian[1]

[Appelquist et al. - Phys. Rev. D 22 (1980) 200 , Longhitano et al. - Phys. Rev. D 22 (1980) 1166]

$$\mathcal{L}_{\rm HEFT} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\mathcal{F}(h)\,\partial_\mu \omega^a \partial^\mu \omega^a \; + \mathcal{O}(\omega^4)$$

Flare function[2]

[Gómez-Ambrosio, Llanes-Estrada, Salas-Bernárdez and Sanz-Cillero - 2204.01762]

$$\mathcal{F}(h) = 1 + a_1 \frac{h}{v} + a_2 \left(\frac{h}{v}\right)^2 + a_3 \left(\frac{h}{v}\right)^3 + a_4 \left(\frac{h}{v}\right)^4 + \mathcal{O}(h^5)$$

$$a \equiv \frac{a_1}{2}\,,\; a_2 \equiv b \quad \text{with} \quad a_{1,\rm SM} = 2\,,\; a_{2,\rm SM} = 1\,,\; a_{3,\rm SM} = 0\,,\; a_{4,\rm SM} = 0$$

[1] We only focus on the EW sector. In the conditions where, under the Goldstone equivalence theorem, longitudinal VBS is approximated by Goldstone scattering.

[2] Where a_n is the effective coupling of $\omega\omega$ with nh. This can be done similarly for the Yukawa sector through the study of $t\bar{t} \to n \times h$ processes, see:
Englert et al. - 2308.11722 , Gómez-Ambrosio et al. - 2207.09848

Higgs Effective Field Theory

Canonical form

HEFT Lagrangian[1]

[Appelquist et al. - Phys. Rev. D 22 (1980) 200 , Longhitano et al. - Phys. Rev. D 22 (1980) 1166]

$$\mathcal{L}_{\mathrm{HEFT}} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\mathcal{F}(h)\,\partial_\mu \omega^a \partial^\mu \omega^a \; + \mathcal{O}(\omega^4)$$

Flare function[2]

[Gómez-Ambrosio, Llanes-Estrada, Salas-Bernárdez and Sanz-Cillero - 2204.01762]

$$\mathcal{F}(h) = 1 + a_1 \frac{h}{v} + a_2 \left(\frac{h}{v}\right)^2 + a_3 \left(\frac{h}{v}\right)^3 + a_4 \left(\frac{h}{v}\right)^4 + \mathcal{O}(h^5)$$

$$a \equiv \frac{a_1}{2}\,,\; a_2 \equiv b \quad \text{with} \quad a_{1,\mathrm{SM}} = 2\,,\; a_{2,\mathrm{SM}} = 1\,,\; a_{3,\mathrm{SM}} = 0\,,\; a_{4,\mathrm{SM}} = 0$$

---

[1] We only focus on the EW sector. In the conditions where, under the Goldstone equivalence theorem, longitudinal VBS is approximated by Goldstone scattering.

[2] Where a_n is the effective coupling of $\omega\omega$ with nh. This can be done similarly for the Yukawa sector through the study of $t\bar{t} \to n \times h$ processes, see:
Englert et al. - 2308.11722 , Gómez-Ambrosio et al. - 2207.09848

Higgs Effective Field Theory

Check: Redefined form

Calculations have also been checked with:

Redefined HEFT Lagrangian

$$\mathcal{L}_{\text{HEFT}} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\hat{\mathcal{F}}(h)\,\partial_\mu \omega^a \partial^\mu \omega^a \; + \mathcal{O}(\omega^4)$$

Redefined Flare function[3]

$$\hat{\mathcal{F}}(h) = 1 + \hat{a}_2\left(\frac{h}{v}\right)^2 + \hat{a}_3\left(\frac{h}{v}\right)^3 + \hat{a}_4\left(\frac{h}{v}\right)^4 + \mathcal{O}(h^5)$$

$$\hat{a}_2 = b - a^2\,, \quad \hat{a}_3 = a_3 - \frac{4a}{3}\left(b - a^2\right)\,, \quad \hat{a}_4 = a_4 - \frac{3}{2}a\,a_3 + \frac{5}{3}a^2\left(b - a^2\right)$$

[3] This redefinition gives a more direct interpretation of the results, see: L. Delgado et al. 2311.04280v2

Higgs Effective Field Theory

Check: Redefined form

Calculations have also been checked with:

Redefined HEFT Lagrangian

$$\mathcal{L}_{\mathrm{HEFT}} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\hat{\mathcal{F}}(h)\,\partial_\mu \omega^a \partial^\mu \omega^a \; + \mathcal{O}(\omega^4)$$

Redefined Flare function[3]

$$\hat{\mathcal{F}}(h) \,=\, 1 + \hat{a}_2\left(\frac{h}{v}\right)^2 + \hat{a}_3\left(\frac{h}{v}\right)^3 + \hat{a}_4\left(\frac{h}{v}\right)^4 + \mathcal{O}(h^5)$$

$$\hat{a}_2 = b - a^2\,, \quad \hat{a}_3 = a_3 - \frac{4a}{3}\left(b - a^2\right)\,, \quad \hat{a}_4 = a_4 - \frac{3}{2}a\,a_3 + \frac{5}{3}a^2\left(b - a^2\right)$$

[3] This redefinition gives a more direct interpretation of the results, see: L. Delgado et al. 2311.04280v2

Higgs Effective Field Theory

Check: Redefined form

Calculations have also been checked with:

Redefined HEFT Lagrangian

$$\mathcal{L}_{\text{HEFT}} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\hat{\mathcal{F}}(h)\, \partial_\mu \omega^a \partial^\mu \omega^a \; + \mathcal{O}(\omega^4)$$

Redefined Flare function[3]

$$\hat{\mathcal{F}}(h) \,=\, 1 + \hat{a}_2 \left(\frac{h}{v}\right)^2 + \hat{a}_3 \left(\frac{h}{v}\right)^3 + \hat{a}_4 \left(\frac{h}{v}\right)^4 + \mathcal{O}(h^5)$$

$$\hat{a}_2 = b - a^2\,, \quad \hat{a}_3 = a_3 - \frac{4a}{3}\left(b - a^2\right)\,, \quad \hat{a}_4 = a_4 - \frac{3}{2} a\, a_3 + \frac{5}{3} a^2 \left(b - a^2\right)$$

[3] This redefinition gives a more direct interpretation of the results, see: L. Delgado et al. 2311.04280v2

Relation to SMEFT

SMEFT

SMEFT lagrangian

[Gómez-Ambrosio, Llanes-Estrada, Salas-Bernárdez and Sanz-Cillero - 2207.09848]

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{n=5}^{\infty} \sum_{i} \frac{c_i^{(n)}}{\Lambda^{n-4}} \mathcal{O}_i^{(n)}$$

$\mathcal{O}_{H\Box}$ operator

$$\mathcal{O}_{H\Box}^{(6)} = (H^\dagger H)\Box(H^\dagger H)\,, \quad \mathcal{O}_{H\Box}^{(8)} = (H^\dagger H)^2\Box(H^\dagger H)\,, \quad \partial^2 \equiv \Box$$

SMEFT parameters

$$d = \frac{2v^2 c_{H\Box}^{(6)}}{\Lambda^2} \quad , \quad \rho = \frac{c_{H\Box}^{(8)}}{2(c_{H\Box}^{(6)})^2}$$

Relation to SMEFT

SMEFT

SMEFT lagrangian

[Gómez-Ambrosio, Llanes-Estrada, Salas-Bernárdez and Sanz-Cillero - 2207.09848]

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{n=5}^{\infty} \sum_{i} \frac{c_i^{(n)}}{\Lambda^{n-4}} \mathcal{O}_i^{(n)}$$

$\mathcal{O}_{H\square}$ operator

$$\mathcal{O}_{H\square}^{(6)} = (H^\dagger H)\square(H^\dagger H)\,, \quad \mathcal{O}_{H\square}^{(8)} = (H^\dagger H)^2\square(H^\dagger H)\,, \quad \partial^2 \equiv \square$$

SMEFT parameters

$$d = \frac{2v^2 c_{H\square}^{(6)}}{\Lambda^2} \qquad , \qquad \rho = \frac{c_{H\square}^{(8)}}{2(c_{H\square}^{(6)})^2}$$

Relation to SMEFT

Relation between parameters

Relation with canonical parameters[4]

[Gómez-Ambrosio, Llanes-Estrada, Salas-Bernárdez and Sanz-Cillero - 2207.09848]

$$
\begin{aligned}
a_1/2 &= a = 1 + \frac{d}{2} + \frac{d^2}{2}\left(\frac{3}{4} + \rho\right) + \mathcal{O}\left(d^3\right) \\
a_2 &= b = 1 + 2d + 3d^2\left(1 + \rho\right) + \mathcal{O}\left(d^3\right) \\
a_3 &= \frac{4}{3}d + d^2\left(\frac{14}{3} + 4\rho\right) + \mathcal{O}\left(d^3\right) \\
a_4 &= \frac{1}{3}d + d^2\left(\frac{11}{3} + 3\rho\right) + \mathcal{O}\left(d^3\right)
\end{aligned}
$$

[4] a_5 and a_6 can be found in L. Delgado et al. 2311.04280v2.
a_n for $n \geq 7$ vanishes at order $1/\Lambda^4$.

$\omega\omega \to 2h$

The following results in this section are calculated in the massless limit.

Amplitude[5]

$$T_{\omega\omega\to 2h} \overset{\text{HEFT}}{=} -\frac{\hat{a}_2 s}{v^2} =$$
$$\overset{\text{SMEFT}}{=} -\frac{s}{v^2}\left[d + 2d^2\left(1+\rho\right)\right] + \mathcal{O}\left(d^3\right)$$

Cross section

$$\sigma_{\omega\omega\to 2h} \overset{\text{HEFT}}{=} \frac{8\pi^3\,\hat{a}_2^2}{s}\left(\frac{s}{16\pi^2 v^2}\right)^2 =$$
$$\overset{\text{SMEFT}}{=} \frac{8\pi^3}{s}\left[d^2 + 4d^3\left(1+\rho\right)\right]\left(\frac{s}{16\pi^2 v^2}\right)^2 + \mathcal{O}\left(d^4\right)$$

[5] Compatible with previous analysis in e.g. Arganda et al. - 1807.09763 , Dobado et al. - 1711.10310 .

$\omega\omega \to 3h$

Amplitude[6]

$$
\begin{aligned}
T_{\omega\omega\to 3h} &\overset{\text{HEFT}}{=} -\frac{3\hat{a}_3 s}{v^3} = \\
&\overset{\text{SMEFT}}{=} -\frac{4s}{v^3} d^2 (1+\rho) + \mathcal{O}\left(d^3\right)
\end{aligned}
$$

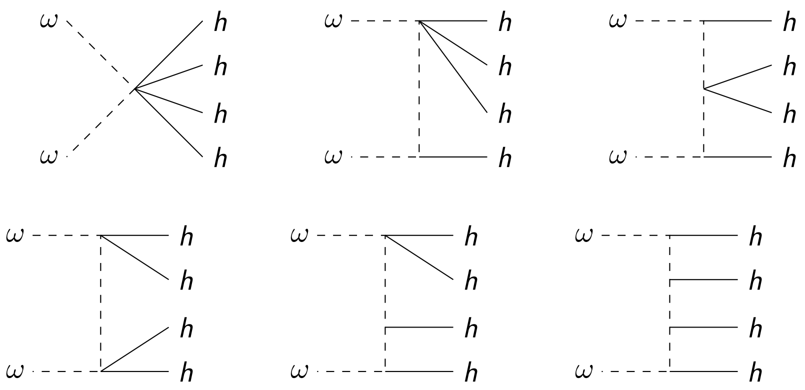
Cross section

$$
\begin{aligned}
\sigma_{\omega\omega\to 3h} &\overset{\text{HEFT}}{=} \frac{12\pi^3\,\hat{a}_3^2}{s}\left(\frac{s}{16\pi^2 v^2}\right)^3 = \\
&\overset{\text{SMEFT}}{=} \frac{64\pi^3}{3s} d^4 (1+\rho)^2 \left(\frac{s}{16\pi^2 v^2}\right)^3 + \mathcal{O}\left(d^5\right)
\end{aligned}
$$

[6] Previous analysis with modifications in e.g. Gonzalez-Lopez et al. - 2011.13915 , Chen et al. - 2105.11500 .

$\omega\omega \to 4h$

Contributing diagrams

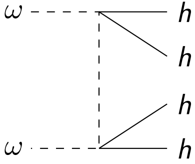


With permutations of external particles, there are a total of 75 diagrams

$\omega\omega \to 4h$

Amplitude

$$T_{\omega\omega\to 4h} \overset{\text{HEFT}}{=} -\frac{4s}{v^4}\left(3\hat{a}_4 + \hat{a}_2^2\,(B-1)\right) = \overset{\text{SMEFT}}{=} -\frac{4s}{v^4}d^2\,(1+\rho+B) + \mathcal{O}\left(d^3\right)$$



Cross section

$$\sigma_{\omega\omega\to 4h} \overset{\text{HEFT}}{=} \frac{8\pi^3}{9s}\left(\frac{s}{16\pi^2 v^2}\right)^4\left[\left(3\hat{a}_4 - \hat{a}_2^2\right)^2 + 2\left(3\hat{a}_4 - \hat{a}_2^2\right)\hat{a}_2^2\chi_1 + \hat{a}_2^4\chi_2\right] = \overset{\text{SMEFT}}{=} \frac{8\pi^3}{9s}\left(\frac{s}{16\pi^2 v^2}\right)^4 d^4\left[(1+\rho)^2 + 2(1+\rho)\chi_1 + \chi_2\right] + \mathcal{O}(d^5)$$

$\omega\omega \to 4h$

Parameters

Definitions[7]

$$B = f_1 f_2 f_3 f_4 \left(\mathcal{B}_{1234} + \mathcal{B}_{1324} + \mathcal{B}_{1423} + \mathcal{B}_{2314} + \mathcal{B}_{2413} + \mathcal{B}_{3412} \right)$$

$$\mathcal{B}_{ijk\ell} = \frac{z_{ij} z_{k\ell}}{2 f_i f_j z_{ij} - f_i z_i - f_j z_j}, \quad z_{ij} = z_{ji} = \frac{q^2 \left(p_i p_j\right)}{\left(q p_i\right)\left(q p_j\right)} \overset{\text{CM}}{=} 2 \sin^2(\theta_{ij}/2)$$

$$f_i = \frac{q p_i}{q^2} \overset{\text{CM}}{=} \|\vec{p}_i\|/\sqrt{s}, \quad z_i = \frac{2 k_1 p_i}{q p_i} \overset{\text{CM}}{=} 2 \sin^2(\theta_i/2)$$

$$q = k_1 + k_2 = p_1 + p_2 + p_3 + p_4$$

$$\chi_n = \mathcal{V}_4^{-1} \int \mathrm{d}\Pi_4 \, B^n, \quad \chi_1 = -0.124984\,(10), \quad \chi_2 = 0.0193760\,(16)$$

[7] Numerical values computed with a personal code, see: https://github.com/mamupaxs/mamupaxs

Cross section phenomenology. ATLAS and CMS data

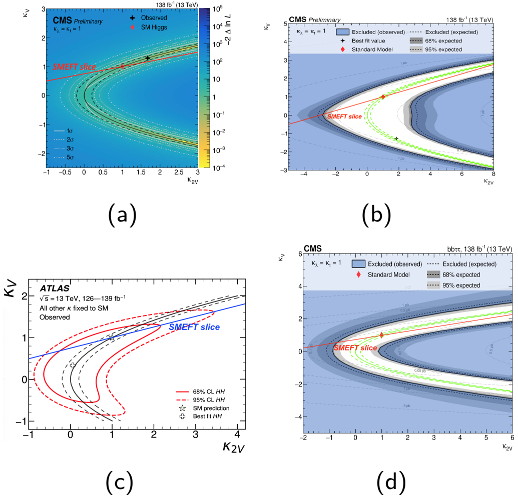


Figure: **(a)** Phys. Rev. Lett. 131 (2023) 041803 [2205.06667]
(b) CMS-PAS-HIG-21-005 **(c)** ATLAS-CONF-2022-050
(d) Phys. Lett. B 842 (2023) 137531 [2206.09401].

- a_1 is relatively well known from Higgs decays at LHC. Close to the SM, up to $\mathcal{O}(10\%)$.
- $\kappa_V = a = \frac{a_1}{2}$
  $\kappa_{2V} = a_2$
- Superimposed SMEFT correlation:
  $a_2 = 2a_1 - 3$
- Plotted parabolas:
  $\hat{a}_2 \equiv a_2 - \frac{a_1^2}{4} = 0$
  $\hat{a}_2 = \pm 0.2$

SMEFT-like model. Benchmark points

SMEFT$^{(D=6)}$ BP

$$d = 0.1$$

$$a = a_1/2 = 1.05\,, \quad b = a_2 = 1.20$$

$$a_3 = 0.1\widehat{3}\,, \quad a_4 = 0.0\widehat{3}$$

SMEFT$^{(D=8)}$ BP

$$d = 0.1\,, \quad \rho = 1$$

$$a = a_1/2 \approx 1.06\,, \quad b = a_2 = 1.26$$

$$a_3 = 0.22\,, \quad a_4 = 0.10$$

- d is compatible with the SM deviation range of ATLAS and CMS, $\Delta a = a - 1 \approx 0.05$ with $a \approx 1 + d/2$.
- d is crucial for the convergence of the expansion.
- ρ is not really relevant.
- $a_{1,\mathrm{SM}} = 2\,,\ a_{2,\mathrm{SM}} = 1$
  $a_{3,\mathrm{SM}} = 0\,,\ a_{4,\mathrm{SM}} = 0$

Non-SMEFT-like models[8]. Benchmark points

$\mathrm{BP1}^{(a_1)}$: input $a_1 = 2.1$

$$\mathcal{F}(h) = \exp\left\{a_1 \frac{h}{v}\right\}$$

$$a_2 = 2.205\,, a_3 \approx 1.54\,, a_4 \approx 0.81$$

$\mathrm{BP2}^{(a_1)}$: input $a_1 = 2.1$

$$\mathcal{F}(h) = \left(1 - \frac{a_1}{2}\frac{h}{v}\right)^{-2}$$

$$a_2 \approx 3.31\,, a_3 \approx 4.63\,, a_4 = 6.08$$

$\mathrm{BP1}^{(a_1,a_2)}$: input $a_1 = 2.1\,,\ a_2 = 1.2$

$$\mathcal{F}(h) = \exp\left\{a_1 \frac{h}{v} + \left(a_2 - \frac{a_1^2}{2}\right)\frac{h^2}{v^2}\right\}$$

$$a_3 \approx -0.57\,, \quad a_4 \approx -0.90$$

$\mathrm{BP2}^{(a_1,a_2)}$: input $a_1 = 2.1\,,\ a_2 = 1.2$

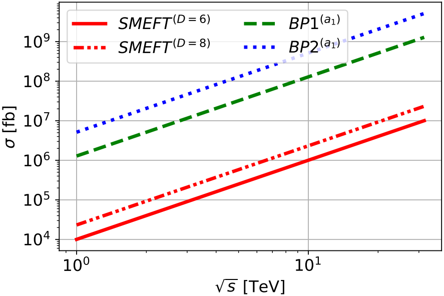
$$\mathcal{F}(h) = \left(1 - \frac{a_1}{2}\frac{h}{v} - \left(\frac{a_2}{2} - \frac{3a_1^2}{8}\right)\frac{h^2}{v^2}\right)^{-2}$$

$$a_3 \approx -2.01\,, a_4 \approx -4.53$$

[8]This flare functions have no real zeros [Cohen et al. - 2008.08597, Manohar et al. 1605.03602] but fulfil the postivity requirements in Gómez-Ambrosio et al. - 2204.01763

BP study

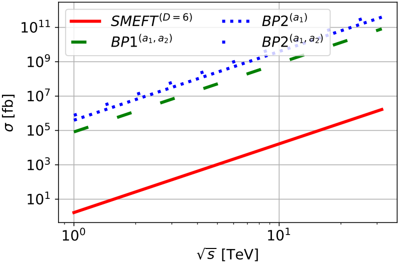
$\omega\omega \to 2h$

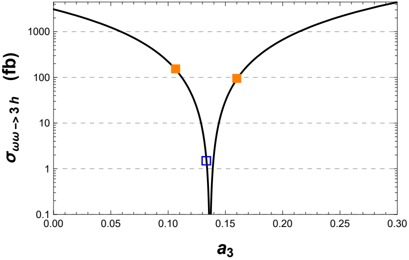


- SMEFT is suppressed by 2 orders of magnitude.
- $BP1^{(a_1,a_2)}$ and $BP2^{(a_1,a_2)}$ are identical to $SMEFT^{(D=6)}$.
- If a_1 and a_2 are set to the SM values the cross section vanishes.

BP study

$\omega\omega \to 3h$



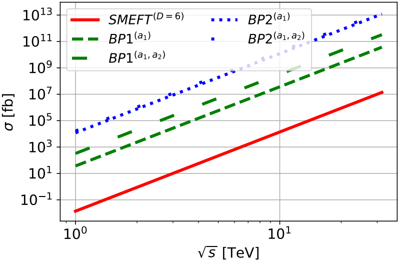


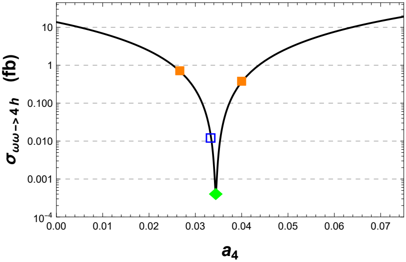
- SMEFT is suppressed by 5 orders of magnitude.
- BP1$^{(a_1)}$ XS accidentally vanishes for this parameters' values.
- Some very particular values could bring HEFT XS lower than SMEFT's (e.g. Dilaton model).

- a_1 and a_2 are set to their SMEFT$^{(D=6)}$ values. $\sqrt{s} = 1$ TeV.
- SMEFT is a particular value (blue).
- A 20% variation on a_3 varies the XS by 2 orders of magnitude.
- The XS vanishes for a particular value.

BP study

$\omega\omega \to 4h$





- SMEFT is suppressed by 4 orders of magnitude.
- Some very particular values could bring HEFT XS lower than SMEFT's (e.g. Dilaton model).

- a_1, a_2 and a_3 are set to their SMEFT$^{(D=6)}$ values. $\sqrt{s} = 1$ TeV.
- SMEFT is a particular value (blue).
- A 20% variation on a_4 varies the XS by 2 orders of magnitude.
- The XS doesn't vanish.

Conclusions and next steps

Conclusions

- We studied longitudinal VBS using the Goldstone equivalence theorem, massless approximation and $\mathcal{L} \sim \mathcal{O}(\partial^2)$.
- We computed analytic expressions for $2h$, $3h$ and $4h$ production at tree level for SMEFT and HEFT.
- Multi-Higgs production is very suppressed in SMEFT but not in general HEFT scenarios.

Next steps

- Full collider analysis: PDFs, mass corrections, NLO...

THANK YOU!

Outline

5. Back up
   - Back up 1. Redefinition of HEFT
   - Back up 2. HEFT-SMEFT parameters relation

Back up 1. Redefinition of HEFT

Fields redefinition

Fields redefinition

$$\omega^a \to \omega^a + g(h)\,\omega^a\,, \qquad h \to h + \mathcal{N}\,(1 + g(h))\,\frac{\omega^a\omega^a}{v}$$

Back up 1. Redefinition of HEFT

Fields redefinition

Fields redefinition

$$\omega^a \to \omega^a + g(h)\,\omega^a\,, \qquad h \to h + \mathcal{N}\,(1+g(h))\,\frac{\omega^a\omega^a}{v}$$

HEFT lagrangian

$$\mathcal{L}_{\mathrm{HEFT}} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\mathcal{F}(h)\,\partial_\mu\omega^a\partial^\mu\omega^a \;+\; \mathcal{O}(\omega^4)$$

Back up 1. Redefinition of HEFT

Fields redefinition

Fields redefinition

$$\omega^a \to \omega^a + g(h)\,\omega^a\,, \qquad h \to h + \mathcal{N}\,(1 + g(h))\,\frac{\omega^a\omega^a}{v}$$

Redefined HEFT lagrangian

$$\mathcal{L}_{\text{HEFT}} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\hat{\mathcal{F}}(h)\,\partial_\mu \omega^a \partial^\mu \omega^a + \mathcal{O}(\omega^4)$$

Back up 1. Redefinition of HEFT

Fields redefinition

Fields redefinition

$$\omega^a \to \omega^a + g(h)\,\omega^a\,, \qquad h \to h + \mathcal{N}\,(1 + g(h))\,\frac{\omega^a\omega^a}{v}$$

Redefined HEFT lagrangian

$$\mathcal{L}_{\text{HEFT}} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\hat{\mathcal{F}}(h)\,\partial_\mu \omega^a \partial^\mu \omega^a + \mathcal{O}(\omega^4)$$

Redefined flare function

$$\hat{\mathcal{F}}(h) \,=\, \mathcal{F}(h)\left(1 + g(h)\right)^2$$

Appendix 1. Redefinition of HEFT

Flare function redefinition

Normalization

$$\mathcal{N} = \frac{a}{2}$$

$$g(h) = -a\frac{h}{v} + a^2\frac{h^2}{v^2} + \frac{1}{3}a(b-4a^2)\frac{h^3}{v^3} + \frac{1}{4}a(a_3 - 4ab + 8a^3)\frac{h^4}{v^4} + \mathcal{O}(h^5)$$

Appendix 1. Redefinition of HEFT

Flare function redefinition

Normalization

$$\mathcal{N} = \frac{a}{2}$$

$$g(h) = -a\frac{h}{v} + a^2\frac{h^2}{v^2} + \frac{1}{3}a(b-4a^2)\frac{h^3}{v^3} + \frac{1}{4}a(a_3 - 4ab + 8a^3)\frac{h^4}{v^4} + \mathcal{O}(h^5)$$

Flare function

$$\mathcal{F}(h) = 1 + a_1\frac{h}{v} + a_2\left(\frac{h}{v}\right)^2 + a_3\left(\frac{h}{v}\right)^3 + a_4\left(\frac{h}{v}\right)^4 + \mathcal{O}(h^5)$$

Appendix 1. Redefinition of HEFT

Flare function redefinition

Normalization

$$\mathcal{N} = \frac{a}{2}$$

$$g(h) = -a\frac{h}{v} + a^2\frac{h^2}{v^2} + \frac{1}{3}a(b - 4a^2)\frac{h^3}{v^3} + \frac{1}{4}a(a_3 - 4ab + 8a^3)\frac{h^4}{v^4} + \mathcal{O}(h^5)$$

Redefined flare function

$$\hat{\mathcal{F}}(h) = 1 + \hat{a}_2\left(\frac{h}{v}\right)^2 + \hat{a}_3\left(\frac{h}{v}\right)^3 + \hat{a}_4\left(\frac{h}{v}\right)^4 + \mathcal{O}(h^5)$$

Back up 1. Redefinition of HEFT

Parameters redefinition

Redefined parameters

$$\begin{aligned}\hat{a}_2 &= b - a^2 \\ \hat{a}_3 &= a_3 - \frac{4a}{3}\left(b - a^2\right) \\ \hat{a}_4 &= a_4 - \frac{3}{2} a\, a_3 + \frac{5}{3} a^2 \left(b - a^2\right)\end{aligned}$$

Back up 2. HEFT-SMEFT parameters relation

Parameters

SMEFT parameters

$$d = \frac{2v^2 c^{(6)}_{H\Box}}{\Lambda^2} \qquad , \qquad \rho = \frac{c^{(8)}_{H\Box}}{2(c^{(6)}_{H\Box})^2}$$

Back up 2. HEFT-SMEFT parameters relation

Relation with canonical parameters

Relation with canonical parameters

$$
\begin{aligned}
a_1/2 &= a = 1 + \frac{d}{2} + \frac{d^2}{2}\left(\frac{3}{4} + \rho\right) + \mathcal{O}\left(d^3\right) \\
a_2 &= b = 1 + 2d + 3d^2\left(1 + \rho\right) + \mathcal{O}\left(d^3\right) \\
a_3 &= \frac{4}{3}d + d^2\left(\frac{14}{3} + 4\rho\right) + \mathcal{O}\left(d^3\right) \\
a_4 &= \frac{1}{3}d + d^2\left(\frac{11}{3} + 3\rho\right) + \mathcal{O}\left(d^3\right)
\end{aligned}
$$

Back up 2. HEFT-SMEFT parameters relation

Relation with redefined parameters

Relation with redefined parameters

$$
\begin{aligned}
\hat{a}_2 &= d + 2d^2(1+\rho) + \mathcal{O}(d^3) \\
\hat{a}_3 &= \frac{4}{3}d^2(1+\rho) + \mathcal{O}(d^3) \\
\hat{a}_4 &= \frac{1}{3}d^2(1+\rho) + \mathcal{O}(d^3)
\end{aligned}
$$