# Lepton Flavour Universality tests at LHCb: Simultaneous $R(D^0)$ and $R(D^{*0})$ measurement with hadronic $\tau$ decays

8<sup>th</sup> RED LHC workshop Universidad Complutense de Madrid (UCM)

Arnau Brossa Gonzalo Universidade de Santiago de Compostela













Fondos Europeos



Cofinanciado pola Unión Europea

### Outline

- Lepton Flavour Universality (LFU) tests: what are they and why are they important?
- LFU at the LHCb: Hadronic vs Muonic
- Simultaneous R(D<sup>0</sup>) and R(D<sup>\*0</sup>) hadronic measurement at LHCb
- Future prospects

0.4R(D\*) 68% CL contours HFLAV Belle<sup>a</sup> BaBar Moriond 2024 0.35 LHCb<sup>c</sup> BelleII 0.3 LHCb<sup>t</sup> **LHC**b<sup>a</sup> 0.25 Belle 0.2  $R(D) = 0.342 \pm 0.026_{total}$ HFLAV SM Prediction  $R(D^*) = 0.287 \pm 0.012_{total}$  $R(D) = 0.298 \pm 0.004$  $\rho = -0.39$  $R(D^*) = 0.254 \pm 0.005$  $P(\chi^2) = 35\%$ 0.2 0.3 0.4 0.5 R(D)

HFLAV Moriond 2024

Conclusions

#### LFU: what is that and why do we care?



No mixing between generationsSame coupling for all generations

Differences in amplitudes of similar processes only depend on the mass of the leptons

Theory predictions can be computed with high precision:



Advantages of computing  $R(X_C)$  ratios also are:

• Remove dependence on  $|V_{cb}|$ 

 $W^{-}$ 

 $\sim$ 

- Partial cancellation of most experimental uncertainties
- Partial cancellation of theoretical uncertainties due to hadronic form-factors uncertainties

### LFU: The big picture



HFLAV Moriond 2024

4

#### LFU: The COMBINED big picture



3.31 $\sigma$  from SM prediction

HFLAV Moriond 2024

5

#### PDG 2023

 $\mathcal{B}(\%)$ 

### LFU in LHCb: hadronic vs muonic <sub>Channel</sub>

$$R = \frac{BR(B \to X_c \tau^+ \nu_{\tau})}{BR(B \to X_c l^+ \nu_l)}$$
$$R(D^0)_{SM} = 0.298 \pm 0.004,$$
$$R(D^{*0})_{SM} = 0.258 \pm 0.005.$$

In LHCb we distinguish between two main approaches:

*R* hadronic:  $\tau^- \rightarrow \pi^- \pi^+ \pi^- (\pi^0) \nu_{\tau}$ 

- 'Only' two neutrinos in the final state
- Three charged tracks enable tau vertex reconstruction
- Lower yields due to smaller BF

$\mathcal{D}(70)$
$17.39 \pm 0.04$
$17.82\pm0.04$
$25.49\pm0.09$
$10.82\pm0.05$

 $\begin{aligned} \tau^- &\to \pi^- \pi^+ \pi^- \nu_\tau & 9.02 \pm 0.05 \\ \tau^- &\to \pi^- \pi^+ \pi^- \pi^0 \nu_\tau & 4.49 \pm 0.05 \end{aligned}$ 

*R* muonic:  $\tau^- \rightarrow \mu^- \overline{\nu}_\mu \nu_\tau$ 

- Same final state as normalization for the ratio
- Larger yields

 $\tau^{-}$ 

 Challenging to distinguish μ from τ (3 neutrinos in the final state!)

#### Muonic analysis: in a nutshell

Signal has 3 neutrinos that cannot be detected, while normalization channel has only one!



Consider the system:

B

Good observables to distinguish contributions are:

- Muon energy in the *B* rest frame:  $E_{I}$ 
  - Missing mass squared:  $\mathbf{M}_{miss}^2 = \left(p_B - p_{X_c} - p_{\mu}\right)^2$
- Squared momenta transfer:  $q^2 = (p_B - p_D)^2$

		PDG 2023
LFU in LHCb: hadronic vs muonic	Channel	${\cal B}~(\%)$
$P = \frac{BR(B \to X_c \tau^+ \nu_{\tau})}{BR(B \to X_c \tau^+ \nu_{\tau})}$	$\tau^-  o \mu^- \overline{\nu}_\mu \nu_\tau$	$17.39 \pm 0.04$
$R = BR(B \to X_c l^+ \nu_l)$	$\tau^- \to e^- \overline{\nu}_e \nu_\tau$	$17.82 \pm 0.04$
$R(D^0)_{SM} = 0.298 \pm 0.004,$	$\tau^- \to \pi^- \pi^0 \nu_\tau$	$25.49 \pm 0.09$
$R(D^{*0})_{SM} = 0.258 \pm 0.005.$	$\tau^- \to \pi^- \nu_{\tau}$	$10.82\pm0.05$
	$\tau^- \to \pi^- \pi^+ \pi^- \nu_\tau$	$9.02 \pm 0.05$
In LHCb distinguish between two main approaches:	$\tau^- \to \pi^- \pi^+ \pi^- \pi^0 \nu_\tau$	$4.49 \pm 0.05$
<i>R</i> hadronic: $\tau^- \to \pi^- \pi^+ \pi^- (\pi^0) \nu_{\tau}$ <i>R</i> • 'Only' two neutrinos in the final state       •	<b>R</b> muonic: $\tau^- \rightarrow \mu^- \overline{\nu}_{\mu}$ Same final state as r for the ratio	TALK
Three charged tracks enable tau vertex reconstruction	Larger IN THE	iguish $\mu$ from $ au$
Lower yields due to smaller BF	atrinos in the fi	nal state!)

### Hadronic analysis

$$R(D^{(*)0}) = \frac{BR(B^+ \to D^{(*)0}\tau^+\nu_{\tau})}{BR(B^+ \to D^{(*)0}l^+\nu_{l})}$$



In practice, as usual, we compute this as a dual ratio using a control channel

Channel	${\cal B}~(\%)$
$\tau^-  ightarrow \mu^- \overline{\nu}_\mu \nu_\tau$	$17.39\pm0.04$
$ au^-  ightarrow e^- \overline{ u}_e  u_ au$	$17.82\pm0.04$
$ au^-  ightarrow \pi^- \pi^0  u_ au$	$25.49\pm0.09$
$ au^-  ightarrow \pi^-  u_ au$	$10.82\pm0.05$
$ au^-  ightarrow \pi^- \pi^+ \pi^-  u_ au$	$9.02\pm0.05$
$\tau^-  ightarrow \pi^- \pi^+ \pi^- \pi^0 \nu_{ au}$	$4.49\pm0.05$

Using 3-prong hadronic decays:

- Despite having lower BF, it offers an easier reconstruction.
- Similar topology to normalization channel, which allow us to reduce systematics

 $R(D^{(*)0}) = \frac{BR(B^+ \to D^{(*)0}\tau^+\nu_{\tau})}{BR(B^+ \to D^{(*)0}l^+\nu_{l})} = \frac{BR(B^+ \to D^{(*)0}\tau^+\nu_{\tau})}{BR(B^+ \to D^0D_s^+)} \frac{BR(B^+ \to D^0D_s^+)}{BR(B^+ \to D^{(*)0}l^+\nu_{l})}$ Normalization channel

#### **Analysis strategy**

 $R(D^{(*)0}) = \frac{BR(B^+ \to D^{(*)0}\tau^+\nu_{\tau})}{BR(B^+ \to D^{(*)0}l^+\nu_{l})} = \frac{BR(B^+ \to D^{(*)0}\tau^+\nu_{\tau})}{BR(B^+ \to D^0D_s^+)} + \frac{BR(B^+ \to D^0D_s^+)}{BR(B^+ \to D^{(*)0}l^+\nu_{l})}$ 



#### Invariant mass fit:

Control channel much cleaner with almost no background from other sources

Run II analysis (2016-2018 data)

**3D simultaneous signal fit** to properly identify all components:

**External Inputs** 

- *τ* decay time: sensitivity to discriminate against prompt and other charm decays
- $q^2 = (p_B p_{D^0})^2$  momenta transfer: discriminates between the  $D^0$  and  $D^{*0}$  signal
- BDT ouput: trained against B → D<sup>(\*)</sup>D<sup>+</sup><sub>s</sub>(X) decays to discriminate most relevant background

#### The BDT

 $q^{2} = (p_{B} - p_{D^{0}})^{2}$  $\tau \to \pi^{+} \pi_{1}^{-} \pi_{2}^{-}$ 

BDT trained to **reject dominant**  $B \to D^{(*)}D_s^+(X)$  backgrounds

Variables chosen to be uncorrelated to  $q^2$  and  $t_{\tau}$  (nominal fit variables)

Soft cut on BDT output selected, as it is also used as a third variable in the nominal fit

BDT variables:

- $\min[m_{\pi_1^-\pi^+}, m_{\pi^+\pi_2^-}]$
- $\max[m_{\pi_1^-\pi^+}, m_{\pi^+\pi_2^-}]$
- $m_{\pi} \pi^{-}_{p_t(\gamma)}$
- $p_t(3\pi) + p_t(\gamma)$
- $N_{\gamma}(\Delta R < 0.4)$ , where  $\Delta R$  is the radius of the cone around  $\gamma$

Best cut selected maximizing

$$FoM = \frac{S}{\sqrt{S+B}} \epsilon_S$$

TMVA overtraining check for classifier: BDTG



### **Background studies. Control samples:**

Despite all the selection steps, an important portion of the data sample is still dominated by background events. These are modelled using templates extracted from simulated events.

Data control samples are produced to improve background modelling. The backgrounds components considered are:



### **Background studies. Control samples:**

Despite all the selection steps, an important portion of the data sample is still dominated by background events. These are modelled using templates extracted from simulated events.

Data control samples are produced to improve background modelling. The backgrounds components considered are:



# Control samples: $B \to \overline{D}^0 D_S^+(X)$ events

Control sample for  $\overline{D}^0 D_s^+$  selected by studying the events with  $m(3\pi) = \pm 20 \text{ MeV/c}^2$  around the  $D_s^+$  mass.

The control sample is split in two:

- $\overline{D}^0 D_s^+$  sample by adding a  $D^{*-}$  veto
- $D^{*-}D_s^+$  sample by studying events with  $m\left(\overline{D}^0\pi^-\right) m\left(\overline{D}^0\right) = [143,148] \text{MeV/c}^2$  (anti  $D^{*-}$  veto)
- Simultaneous fit of  $m(\overline{D}^0 D_s^+)$  and  $m(D^{*-}D_s^+)$  to extract ratios between all components:

Component	Yield
$B^+ \to \overline{D}{}^0 D^+_s$	$ N_{D_{s}^{+}}^{(\overline{D}^{0}D_{s}^{+})} \times \varepsilon_{B^{+} \to \overline{D}^{0}D_{s}^{+}} \times r_{B^{+} \to \overline{D}^{0}D_{s}^{+}}^{D_{s}^{+}} / \sum_{i} r_{i}^{D_{s}^{+}} $
$B^+ \to \overline{D}^{*0} D_s^+$	$N_{D_{s}^{+}}^{(\overline{D}^{0}D_{s}^{+})} \times \varepsilon_{B^{+} \to \overline{D}^{*0}D_{s}^{+}} \times r_{B^{+} \to \overline{D}^{*0}D_{s}^{+}}^{D_{s}^{+}} / \sum_{i} r_{i}^{D_{s}^{+}}$
$B^+  ightarrow \overline{D}{}^0 D_s^{*+}$	$ N_{D_{s}^{+}}^{(\overline{D}{}^{0}D_{s}^{+})} \times \varepsilon_{B^{+} \to \overline{D}{}^{0}D_{s}^{*+}} \times r_{B^{+} \to \overline{D}{}^{0}D_{s}^{*+}}^{D_{s}^{+}} / \sum_{i} r_{i}^{D_{s}^{+}} $
$B^+ \to \overline{D}^{*0} D_s^{*+}$	$N_{D_{s}^{+}}^{(\bar{D}^{0}D_{s}^{+})} \times \varepsilon_{B^{+} \to \bar{D}^{*0}D_{s}^{*+}} \times r_{B^{+} \to \bar{D}^{*0}D_{s}^{*+}}^{D_{s}^{+}} / \sum_{i} r_{i}^{D_{s}^{+}}$
$B^+ \to \overline{D}{}^0 D_{s1}(2460)^+$	$ N_{D_{s}^{+}}^{(\overline{D}^{0}D_{s}^{+})} \times \varepsilon_{B^{+} \to \overline{D}^{0}D_{s1}(2460)^{+}} \times r_{B^{+} \to \overline{D}^{0}D_{s1}(2460)^{+}}^{D_{s}^{+}} / \sum_{i} r_{i}^{D_{s}^{+}} $
$B^+ \to \overline{D}^{*0} D_{s1}(2460)^+$	$N_{D_{s}^{+}}^{(\overline{D}^{0}D_{s}^{+})} \times \varepsilon_{B^{+} \to \overline{D}^{*0}D_{s1}(2460)^{+}} \times r_{B^{+} \to \overline{D}^{*0}D_{s1}(2460)^{+}}^{D_{s}^{+}} / \sum_{i} r_{i}^{D_{s}^{+}}$
$B^+ B^0_s\to \overline{D}{}^0D^+_sX$	$\Big  N_{D_{s}^{+}}^{(\overline{D}^{0}D_{s}^{+})} \times \varepsilon_{B^{+} B_{s}^{0} \to \overline{D}^{0}D_{s}^{+}X} \times r_{B^{+} B_{s}^{0} \to \overline{D}^{0}D_{s}^{+}X}^{D_{s}^{+}} / \sum_{i} r_{i}^{D_{s}^{+}}$
$B^0 \to \overline{D}{}^0 D^+_s X$	$ N_{D_s^+}^{(\overline{D}{}^0D_s^+)} \times \varepsilon_{B^0 \to \overline{D}{}^0D_s^+X} \times r_{B^0 \to \overline{D}{}^0D_s^+X}^{D_s^+} / \sum_i r_i^{D_s^+} $
$B^0 \to D^{*-} D^+_s$	$N_{D_{s}^{+}}^{(\bar{D}^{0}D_{s}^{+})} \times \varepsilon_{B^{0} \to D^{*-}D_{s}^{+}} \times r_{B^{0} \to D^{*-}D_{s}^{+}}^{D_{s}^{+}} / \sum_{i} r_{i}^{D_{s}^{+}}$
$B^0 \to D^{*-} D^{*+}_s$	$ N_{D_{s}^{+}}^{(\bar{D}^{0}D_{s}^{+})} \times \varepsilon_{B^{0} \to D^{*-}D_{s}^{*+}} \times r_{B^{0} \to D^{*-}D_{s}^{+}}^{D_{s}^{+}} \times r_{B^{0} \to D^{*-}D_{s}^{*+}}^{*D_{s}^{+}} / \sum_{i} r_{i}^{D_{s}^{+}} $
$B^0 \to D^{*-} D^*_{s0}(2317)^+$	$ \left  \begin{array}{c} N_{D_s^+}^{(\bar{D}^0 D_s^+)} \times \varepsilon_{B^0 \to D^{*-} D_{s0}^*(2317)^+} \times r_{B^0 \to D^{*-} D_s^+}^{D_s^+} \times r_{B^0 \to D^{*-} D_{s0}^*(2317)^+}^{*D_s^+} / \sum_i r_i^{D_s^+} \right  \\ \end{array} \right  \\$
$B^0 \to D^{*-} D_{s1}(2460)^+$	$ \left  \begin{array}{c} N_{D_{s}^{+}}^{(\bar{D}^{0}D_{s}^{+})} \times \varepsilon_{B^{0} \to D^{*-}D_{s1}(2460)^{+}}^{D_{s}^{+}} \times r_{B^{0} \to D^{*-}D_{s}^{+}}^{b_{s}^{+}} \times r_{B^{0} \to D^{*-}D_{s1}(2460)^{+}}^{*D_{s}^{+}} / \sum_{i} r_{i}^{D_{s}^{+}} \right  $

## Control samples: $B \to \overline{D}^0 D_S^+(X)$ events

Simultaneous fit produces ratios between components that are used as weights in the nominal fit



These ratios enter as weights in the MC to generate background teamplates in the signal fit.

Some of these components enter in the fit gaussianly constrained using their uncertainty

#### **Signal fit: Results**





Signal components as well as the  $D_S^+$  background randomly scaled to blind the results.

Parameter	Fit result	Constraint
$N(B^+ \to \overline{D}{}^0 \tau^+ \nu_{\tau})$	$xxx \pm 1572$	
$N(B^+ \to \overline{D}^{*0} \tau^+ \nu_{\tau})$	$xxx \pm 956$	
$N_{D_{+}^{+}}$	$xxx \pm 982$	
$N_{\rm sv}(B \to \overline{D}{}^0 D^0(X))$	$2742 \pm 392$	$2844 \pm 427$
$N_{\rm dv}(B \to \overline{D}{}^0 D^0(X))$	$8008 \pm 621$	
$N(B \to \overline{D}{}^0D^+(X))$	$8743\pm508$	
$N(B \to \overline{D}{}^0 3\pi(X))$	$7260\pm265$	
$N_{{ m fake}-\overline{D}{}^0}$	10902	10902
$N_{ m combi}$	3247	3247
$r^{D^+_s}_{B^+ \to \overline{D}^{*0} D^+_s}$	$0.960 \pm 0.036$	$0.894 \pm 0.056$
$r_{B^+ \rightarrow \overline{D}^0 D^{*+}}^{D^+_s}$	$0.894 \pm 0.042$	$0.949 \pm 0.057$
$r_{B+\rightarrow\overline{D}^{*0}D^{*+}}^{D_s}$	$1.829 \pm 0.042$	$1.828\pm0.048$
$r_{B+\rightarrow\overline{D}^0D}^{D^*}D_{s}^{(2460)+}$	$0.337 \pm 0.033$	$0.338 \pm 0.039$
$r_{B^+ \to \bar{D}^{*0} D^{-1}}^{D_{s1}}$ (2460)+	$0.682 \pm 0.030$	$0.706 \pm 0.042$
$r_{B+ B^0 \rightarrow \overline{D}^0 D^+ X}^{D_s^+}$	$0.845 \pm 0.038$	$0.836 \pm 0.042$
$r_{P_0}^{D_s^+}$	0.118	0.118
$r_{B^0 \rightarrow D^* - D^+ Y}^{D_s}$	0.088	0.088
$r_{P0}^{*D_s^*}$	1.726	1.726
$r_{B^0 \rightarrow D^{*-} D^{*}}^{*D_s^+} $	0.137	0.137
$r_{B^0 \to D^{*-} D^{*-$	0.414	0.414
$f_{2\pi}^{D^0}$	0.835	0.835
$f_{2-}^{D^{*0}}$	0.842	0.842
$f_{2-}^{D^{*-}}$	0.857	0.857
f <sub>D**</sub> / <sub>D*0</sub>	0.044	0.044
$f_{D^{*-}/D^{*0}}$	0.138	0.138
w <sub>BDTDelite</sub>	$0.073 \pm 0.033$	

Table 38: Results of the nominal 3D fit.

#### **Normalization fit: Results**

Normalization yield extracted from a fit on the deconvoluted mass:  $m(\overline{D}^0 D_s^+) \equiv M(\overline{D}^0 D_s^+) - M(\overline{D}^0) - M(D_s^+) + m_{\text{PDG}}(\overline{D}^0) + m_{\text{PDG}}(D_s^+),$ 

 Crystall Ball (Normalization) + Exponential (Background) model used to describe the data. With CB tail parameters fixed from fit in simualtion



Parameter	Fit result (MC)	Fit result (data)
$N_{norm}$	$162650\pm400$	$3047\pm56$
$m_{mean}$	$5279.455 \pm 0.026$	$5278.87 \pm 0.17$
$\sigma$	$8.288 \pm 0.030$	$8.66\pm0.14$
$lpha_R$	$-1.674 \pm 0.031$	-1.674 (fixed)
$lpha_L$	$1.510\pm0.023$	1.510 (fixed)
$n_R$	$44 \pm 17$	44 (fixed)
$n_L$	$15.7\pm2.0$	15.7 (fixed)
$N_{bkg}$	-	$27 \pm 10$
$r_{bkg}$	-	$-0.0043 \pm 0.0068$





### Systematic uncertainties

LHCb Preliminary Unofficial

Source	$N_{ m sig}^{D^0}$	$N_{\mathrm{sig}}^{D^{*0}}$	$\rho(N_{\rm sig})$	$\mathcal{B}^{D^0}_{ ext{sig}}(\%)$	$\mathcal{B}^{D^{st 0}}_{\mathrm{sig}}(\%)$	$ ho(\mathcal{B}_{\mathrm{sig}})$	$R(D^0)$	$R(D^{*0})$	$\rho(R)$
Templates statistics	1163	696	-0.93	0.140	0.099	-0.93	0.061	0.018	-0.93
Form-factors (templates shape)	676	491	-0.99	0.076	0.065	-0.99	0.033	0.012	-0.99
$D_s^+$ decay model	260	136	-1	0.031	0.019	-1	0.014	0.003	-1
Simulation Statistics (efficiency)		—		0.003	0.006	+0.52	0.001	0.001	+0.52
Form-factors (efficiency)		—		0.001	0.022	0	0.0004	0.004	0
Normalisation yield (model shape)				0.0004	0.0009	+1	0.0002	0.0002	+1
Total Systematic	1348	844	-0.94	0.162	0.122	-0.93	0.070	0.022	-0.93
Stat. Signal Fit	1572	956	-0.92	0.189	0.136	-0.92	0.082	0.024	-0.92
Stat. Norm. Fit				0.013	0.026	+1	0.005	0.005	+1
Total Statistical	1572	956	-0.92	0.189	0.138	-0.89	0.082	0.025	-0.89
External				0.073	0.154	+1	0.034	0.029	+0.88
Total	2071	1275	-0.93	0.259	0.240	-0.49	0.113	0.044	-0.47

Last step (ongoing!) is the complete assessment of systematic uncertainties. Dominant contributions have been studied, only minor ones remaining:

- PID efficiency
- Background shapes

- ...

#### **Blinded Results**



20

#### **Future Prospects: Beyond this analysis**



#### *RevModPhys*.94.015003

Still a lot of work to do with the available data!

- Reduction of systematics due to many factors (better understanding of backgrounds, improvement in simulation, new theory inputs...)
- Future LHCb measurements include: *R*(*D<sub>s</sub>*), *R*(*D\*\**), *Hadronic R*(*D*) and *R*(*D\**), *R*(Λ<sup>\*</sup><sub>c</sub>) and more!
- And even more data coming from the new LHC era!

#### Conclusions

- Lepton Flavour Universality tests can be a window to NP
- Still some tension can be seen in a few observables with respect SM predictions, we need to keep improving to find a clear picture!
- LHCb has a important role in uncovering the final picture, with many LFU test results in the last few years and more to come
- Simultaneous  $R(D^0)$  and  $R(D^{*0})$  measurement with hadronic  $\tau$  soon to be added to the global combination.



Thank you for your attention.

#### Efficiencies

Last remaining ingredient for the computation of the LFU ratios is the signal and normalization efficiencies, which are extracted directly from simulation

In addition, for the computation of  $R(D^0)$  and  $R(D^{*0})$  some other external parameters are required

Parameter	Value
$f^{D^0}_{3\pi}$	$0.835 \pm 0.002$
$f_{3\pi}^{D^{*0}}$	$0.842\pm0.002$
$f_{3\pi}^{D^{*+}}$	$0.857 \pm 0.004$
$f_{D^{*+}/D^{*0}}$	$0.138 \pm 0.002$

Decay	$\varepsilon_{\rm gen}$ Up/Down	$\varepsilon_{\mathrm{filtering}}$	$\varepsilon_{ m Stripping}$ (×10 <sup>-2</sup> )	$\varepsilon_{\mathrm{offline}}$	$arepsilon_{ ext{Total}} ( imes 10^{-5})$
$ \begin{array}{c} B^+ \to \overline{D}{}^0 \tau^+ \nu_\tau \\ \tau^+ \to \pi^+ \pi^- \pi^+ \overline{\nu}_\tau \end{array} \end{array} $	$\frac{0.04178 \pm 0.00011}{0.04186 \pm 0.00011} /$	0.0501	0.116	0.089	$10.36\pm0.04$
$ \begin{array}{c} B^+ \rightarrow \overline{D}{}^0 \tau^+ \nu_{\tau}, \\ \tau^+ \rightarrow \pi^+ \pi^- \pi^+ \pi^0 \overline{\nu}_{\tau} \end{array} \end{array} $	$\frac{0.037156 \pm 0.000099}{0.037107 \pm 0.000097}$	0.0494	0.096	0.043	$4.11\pm0.02$
$ \begin{array}{c} B^+ \to \overline{D}{}^{*0} \tau^+ \nu_\tau, \\ \tau^+ \to \pi^+ \pi^- \pi^+ \overline{\nu}_\tau \end{array} \end{array} $	$\begin{array}{r} 0.04004 \pm 0.00010 \\ 0.04032 \pm 0.00011 \end{array}$	0.0493	0.111	0.080	$8.82\pm0.03$
$ \begin{array}{c} B^+ \to \overline{D}^{*0} \tau^+ \nu_\tau, \\ \tau^+ \to \pi^+ \pi^- \pi^+ \pi^0 \overline{\nu}_\tau \end{array} \end{array} $	$\begin{array}{c} 0.036045 \pm 0.000096 \\ 0.036071 \pm 0.000096 \end{array}$	0.0488	0.093	0.036	$3.32\pm0.02$
$ \begin{array}{c} B^0 \to D^{*-} \tau^+ \nu_\tau, \\ \tau^+ \to \pi^+ \pi^- \pi^+ \overline{\nu}_\tau \end{array} $	$\frac{0.1601 \pm 0.0022}{0.1592 \pm 0.0022} /$	0.0227	0.149	0.012	$1.84\pm0.03$
$ \begin{array}{c} B^0 \rightarrow D^{*-} \tau^+ \nu_{\tau}, \\ \tau^+ \rightarrow \pi^+ \pi^- \pi^+ \pi^0 \overline{\nu}_{\tau} \end{array} \end{array} $	$\frac{0.1559 \pm 0.0021}{0.1578 \pm 0.0022} /$	1.0	0.108	0.0057	$0.61\pm0.02$
$ \begin{array}{c} B^- \rightarrow D^0 D^s, \\ D^s \rightarrow \pi^- \pi^+ \pi^- \end{array} $	$\begin{array}{c} 0.05980 \pm 0.00015 \\ 0.05992 \pm 0.00015 \end{array}$	0.0614	0.241	0.174	$42.10\pm0.13$

 $f_{3\pi}^X$ : ratio of  $B \to X\tau\nu_{\tau}, \tau \to 3\pi\nu$  with respect total number of  $B \to X\tau\nu_{\tau}$ 

 $f_{D^{*+}/D^{*0}}$ : ratio of  $B^0 \to D^{*-}\tau^+\nu$  with respect  $B^+ \to D^{*0}\tau^+\nu$