Simulation of Beam-Loading effects in linacs

CLIC Project Meeting #45
19th of March 2024

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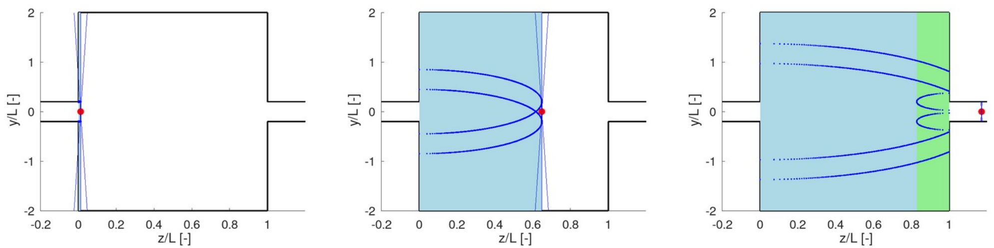
Outline

- **PART I**: Introduction and BL power diffusive model
- **PART II**: Beam Loading simulations in RF-Track
  - Benchmark for CLIC AS and CLIC PETS
- **PART III:** Experimental results @ CLEAR facility
- **PART IV**: BL Compensation simulations
- **PART V**: Conclusions

I. Beam Cavity Interaction

- Beam excites EM modes, described by
  - Resonating frequency
  - Quality factor $$Q = \omega_{\mathrm{RF}} \frac{w}{p_{\mathrm{diss}}}$$
  - Shunt impedance $$r_e = \frac{G_{\mathrm{eff}}^2}{p_{\mathrm{diss}}} \; [\Omega/\mathrm{m}]$$



> Particle flying through a cavity and leaving an EM field behind it (excited mode).

- Ideally, cavities exhibit low Q_{HOM} and high Q_{FM}

- The **TM_{01} induced excitation lasts for a long time**:
  - Long range effect
  - Accumulated from bunch to bunch

Beam Loading Effect:
Reduction of available accelerating gradient
Due to beam – cavity interaction

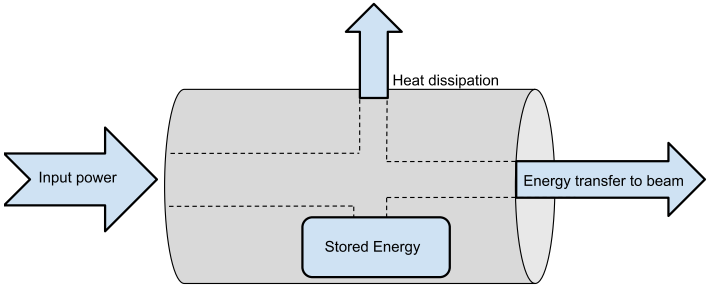
[1] Chao A. *Physics of collective beam instabilities in high energy particle accelerators*. New York, US: John Wiley and Sons (1993).

I. Power-Diffusion PDE

- From Poynting Theorem: Gradient reduction PDE:

TW $$-\frac{\partial G}{\partial t} = v_g \frac{\partial G}{\partial z} + \left(-\frac{v_g}{r/Q}\frac{\partial (r/Q)}{\partial z} + \frac{\omega}{Q} + \frac{\partial v_g}{\partial z}\right)\frac{G}{2} + \underbrace{\frac{\omega r \tilde{I}}{2Q}}_{\text{Beam Loading term}}$$

SW $$-\frac{\partial G}{\partial t} = +\frac{\omega}{Q}\frac{G}{2} - \frac{\omega}{2Q}\frac{G^2_{\text{unloaded}}}{G}\left(1 - \exp\frac{\omega t}{2Q}\right) + \underbrace{\frac{\omega r \tilde{I}}{2Q}}_{\text{Beam Loading term}}$$



> Energy balance schematics for an accelerating structure

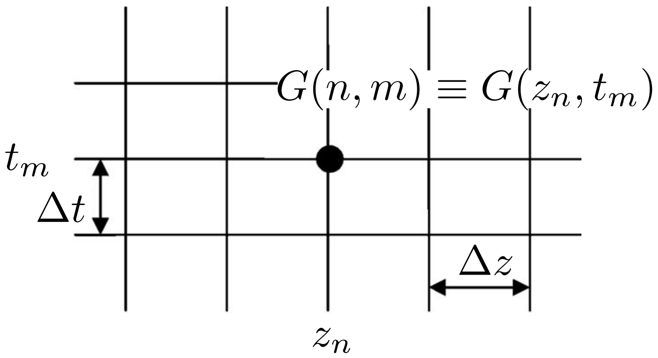
Common features:

- **Beam Loading term**: **Decelerating** gradient dependent on **Intensity (i.e charge)**.
- **Quasi-static** approximation - Admitted temporal dependency of phasors → G depends on t

Expansion of the expressions found in:
[3] Leiss JE. *Beam Loading and Transient behavior in travelling wave electron linear accelerators*. In: PM Lapostolle, editor. Linear accelerators. Amsterdam, Holland: A.L. North Holland Publishing Company (1970). p. 147–72.
[4] Lunin A, Yakovlev V, Grudiev A. *Analytical solutions for transient and steady-state beam loading in arbitrary travelling wave accelerating structures*. Phys Rev Spec Top Accel Beams (2011) 14:05.

II. BL in RF-Track

- Particle tracking code developed at CERN
  - Allows integration of motion of arbitrary charged particles in user-provided 3D fieldmaps
  - C++; GSL & FFTW libraries; Octave and Python interface
- **Beam Loading Module**:
  - Based on numerical resolution of Power-Diffusion PDEs (Finite-Difference Method)



$$B(n) = \frac{1}{2}\frac{\mathrm{d}v_g}{\mathrm{d}z}\bigg|_{z_n} + \frac{\mathrm{d}r/Q}{\mathrm{d}z}\bigg|_{z_n}\frac{v_g(z_n)}{2r/Q(z_n)} - \frac{\omega}{2Q(z_n)} - \frac{v_g(z_n)}{\Delta z} + \frac{1}{\Delta t}$$

$$C(n,m) = \frac{\omega \frac{r}{Q}(z_n)\tilde{I}(z_n,t_m)}{2}$$

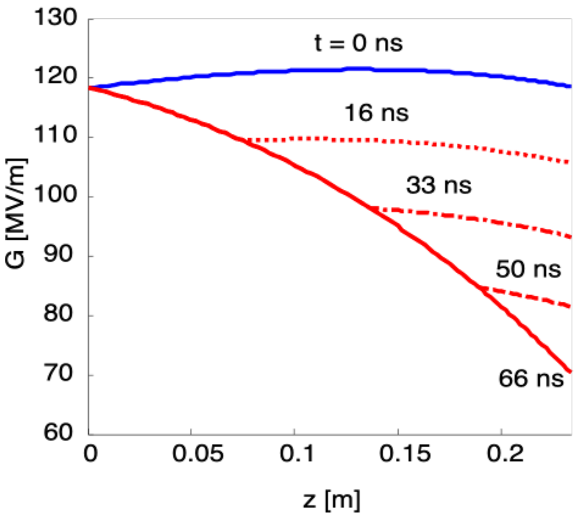
TW	$G(n,m+1) = G(n,m)\Delta t B(n) + \Delta t \frac{v_g(z_n)}{\Delta z} G(n-1,m) - C(n,m)$
SW	$G(n,m+1) = G(n,m)\left(1 - \frac{\omega \Delta t}{2Q(n)}\right) - C(n,m) + \frac{P_{\text{input}}}{L}$

[5] A. Latina. *RF-Track Reference Manual*. CERN, Geneva, Switzerland, June 2020 DOI: 10.5281/zenodo.3887085

II. Transient Gradient Reduction – CLIC AS

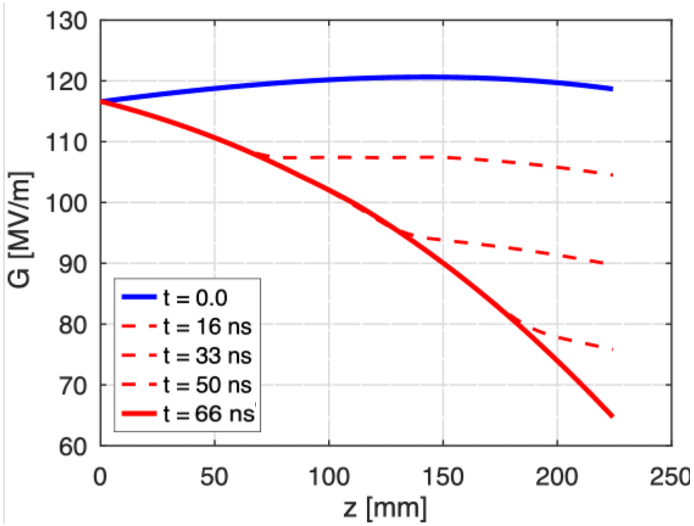
- Reliable **gradient reduction** calculation

(A)



(A) CLIC main AS gradient reduction for a train of I = 1.02A and length of 152 ns. Theoretical calculation. Fig. from [4]

(B)



(B) CLIC main Accelerating structure gradient reduction for a train of I = 1.02A and length of 152 ns

Magnitude	Units	Value
r/Q_{average}	Ω/m	16178
Q_{average}	-	5636
$v_{g\,\text{average}}$	$c/100$	1.21
f_0	GHz	12.00
f_0/f_b	-	6
N_{bunches}	-	312
σ	mm/c	0.3
$\langle I \rangle$	A	1.20
t_{train}	ns	152.0

> CLIC main Accelerating structure gradient details [4,6]

[4] Lunin A, Yakovlev V, Grudiev A. *Analytical solutions for transient and steady-state beam loading in arbitrary travelling wave accelerating structures*. Phys Rev Spec Top Accel Beams (2011) 14:05.
[6] Aicheler M, Burrows P, Draper M, Garvey T, Lebrun P, Peach K, et al. A Multi-TeV linear collider based on CLIC technology: CLIC Conceptual Design Report. Menlo Park, CA, United States: SLAC National Accelerator Lab (2014).

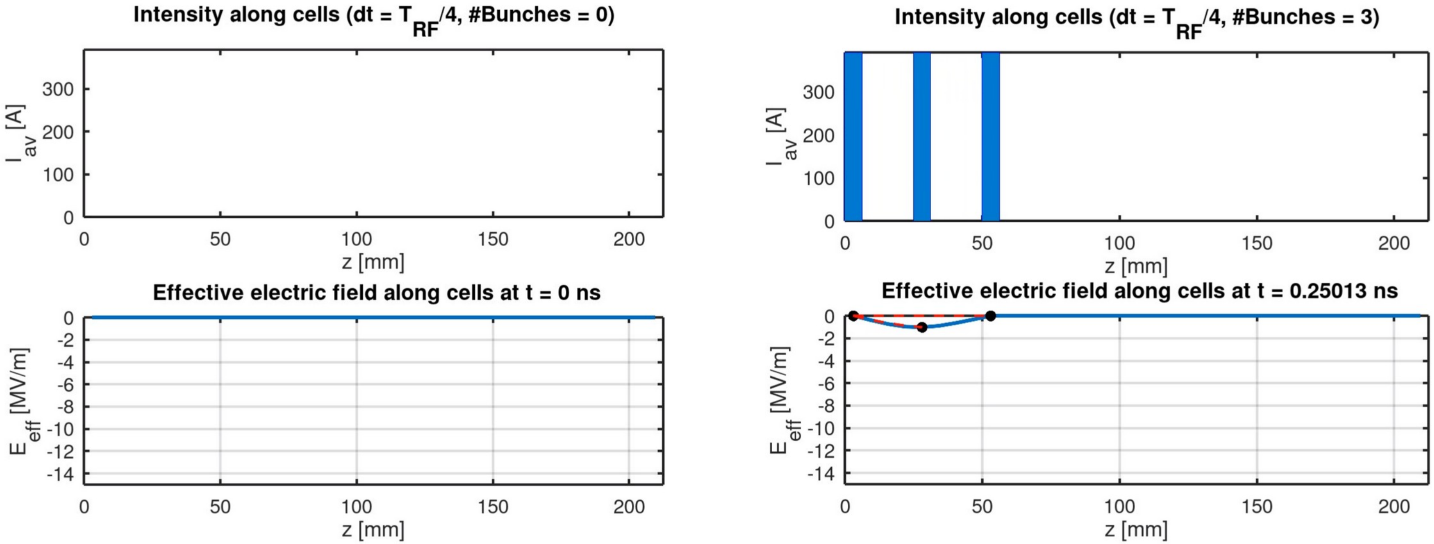
II. Tracking algorithm with BL

- **2 different strategies** depending on particle v
  - **Ultrarelativistic** bunches:
    - Solve G prior to tracking (Finite differences)
    - $E_z(z,t) = \mathrm{Re}\left[G(z,t)\mathrm{e}^{j(k(z-z_{\mathrm{mean}})-\omega t)}\right]$
    - Apply $F_z = qE_z(z_{\mathrm{part}}, t_{\mathrm{part}})$ (Cubic interpolation)
  - **Relativistic** bunches:
    - Intensity depends on particle's velocity: $\tilde{I}(z,t) = \beta_r(z,t)\frac{q_{\mathrm{bunch}}}{T_{\mathrm{RF}}}$
    - Therefore, gradient reduction depends on instantaneous bunch info
    - G is solved on the fly

[7] Olivares Herrador J, Latina A, Aksoy A, Fuster Martínez N, Gimeno B and Esperante D (2024), Implementation of the beam-loading effect in the tracking code RF-track based on a power-diffusive model. Front. Phys. 12:1348042. doi: 10.3389/fphy.2024.1348042

II. Bunch-to-bunch effect – CLIC PETS

- **Transient** tracking
  - **PETS**: Power Extraction and Transfer Structures. → **Deceleration** due to BL



Magnitude	Units	Value
r/Q_{average}	Ω/m	2294
Q_{average}	-	7200
$v_{g\,\text{average}}$	$c/100$	45.3
f_0	GHz	12.00
f_0/f_b	-	1
N_{bunches}	-	2928
σ	mm/c	1.0
$\langle I \rangle$	A	101
E_0	MeV	2400

> PETS gradient details [6]

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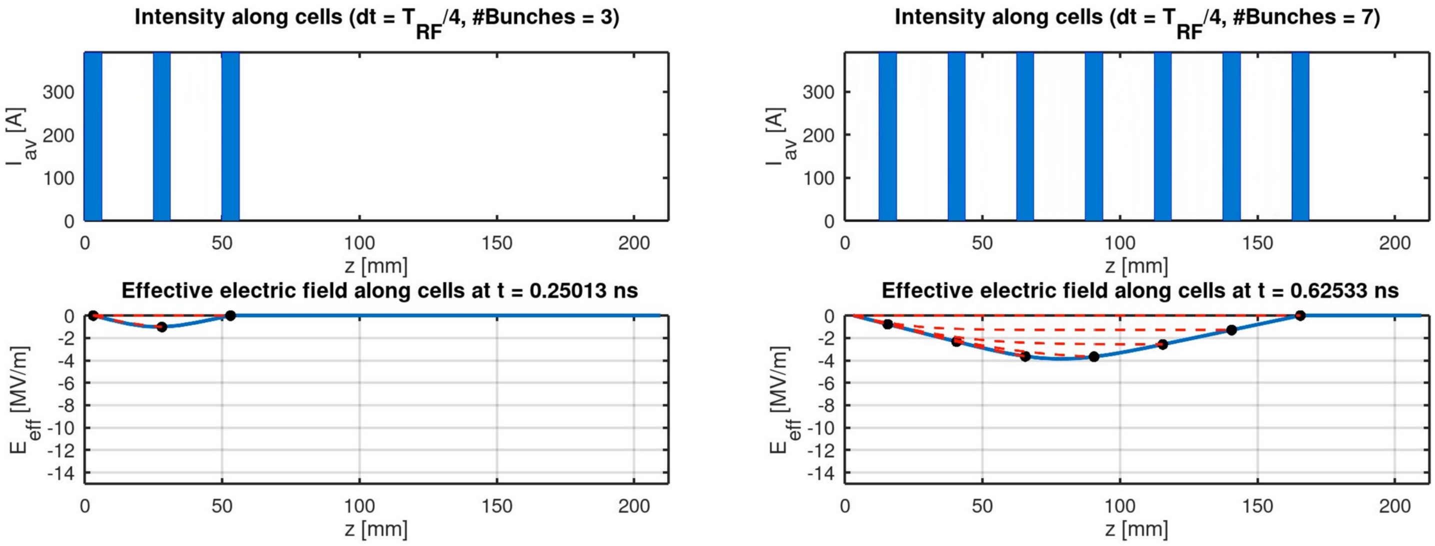

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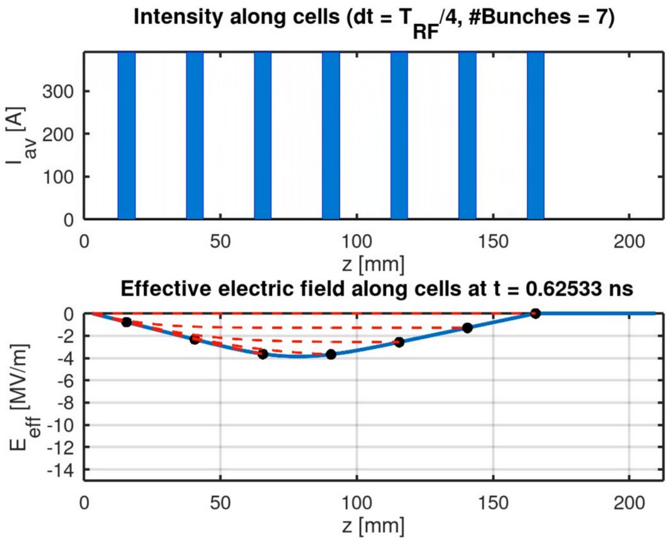

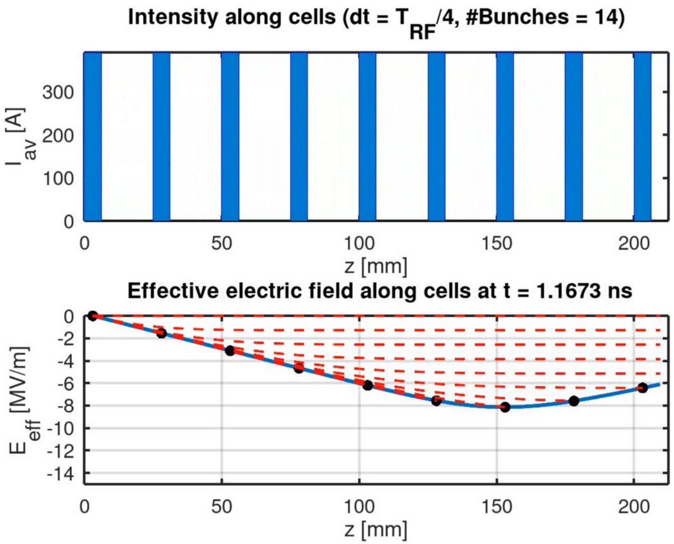

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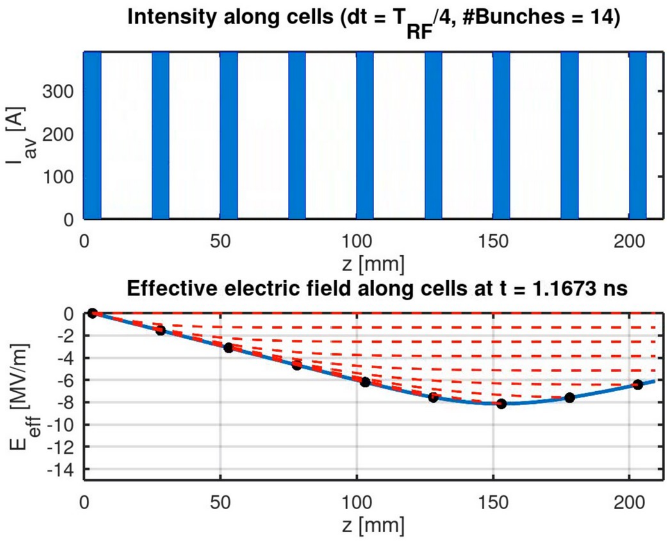

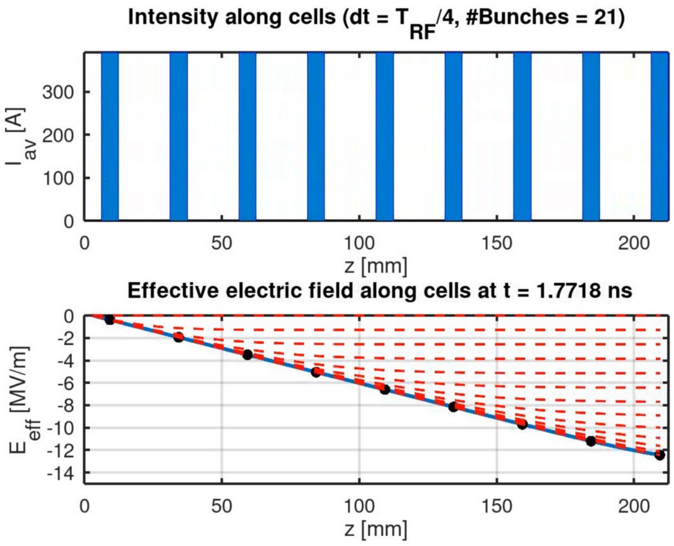

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Q_{average}	-	7200
$v_{g\text{average}}$	$c/100$	45.3
f_0	GHz	12.00
f_0/f_b	-	1
N_{bunches}	-	2928
σ	mm/c	1.0
$\langle I \rangle$	A	101
E_0	MeV	2400

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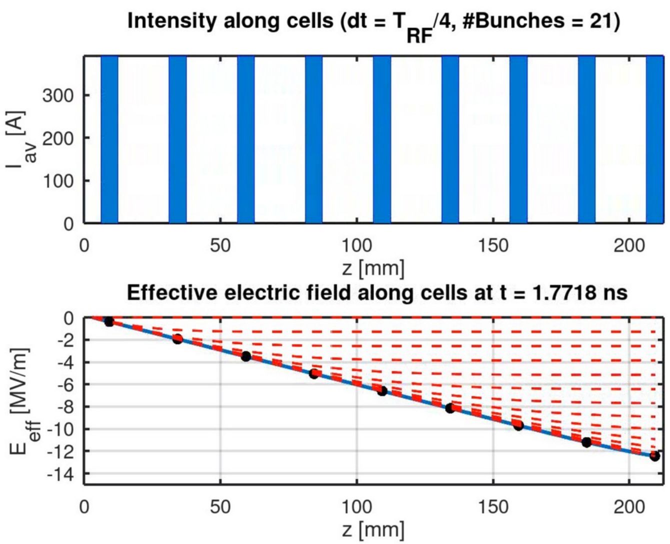

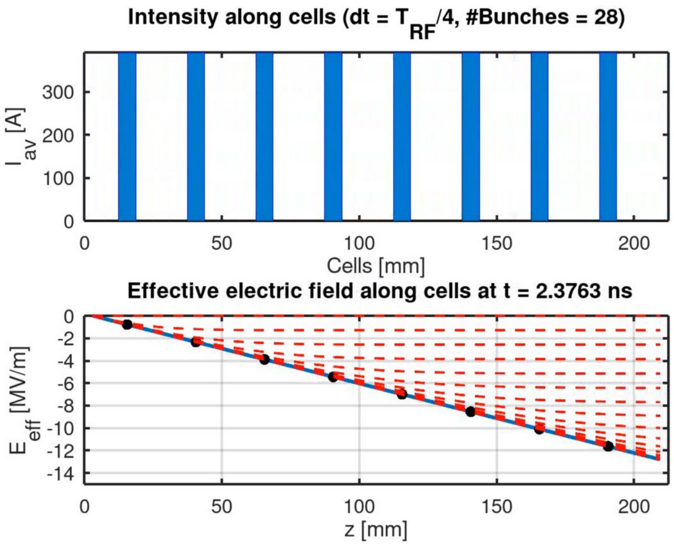

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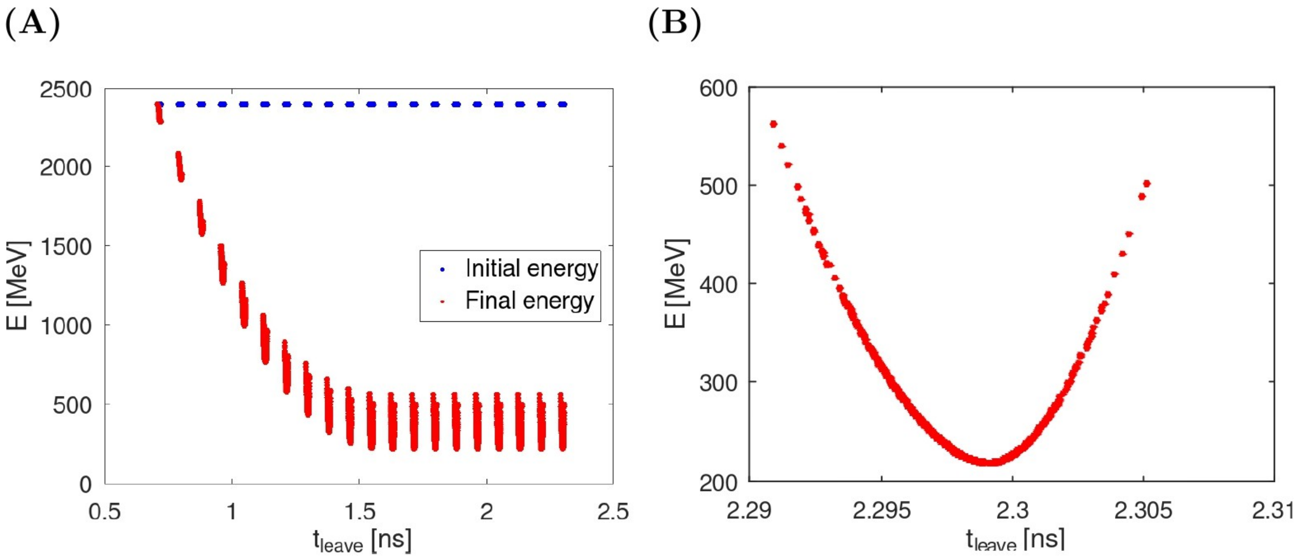


(A) Beam Energy after 20 bunches go through 1492 PETS.

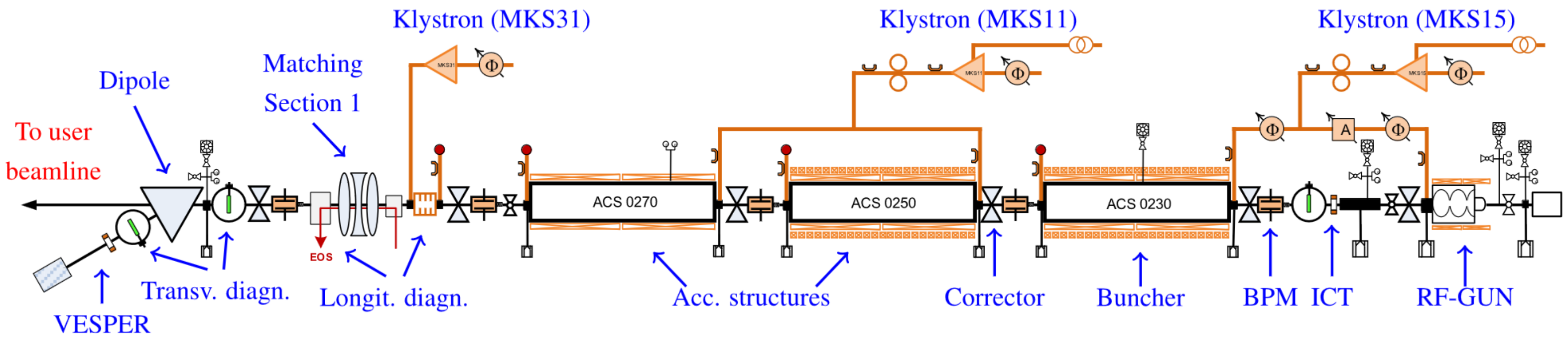
(B) Beam Energy of the 20th bunch after 1492 PETS.

$$\eta = \frac{E_0 - E_{\min}}{E_0}$$

$\boldsymbol{E_0}$	[GeV]	2.40
$\boldsymbol{E_{\min}}$	[MeV]	241.6
$\boldsymbol{\eta_{\text{PLACET}}}$	[%]	90.0
$\boldsymbol{\eta_{\%}}$	[%]	89.7
$\boldsymbol{\delta}$	[%]	0.67

[8] Erik Adli (2009). *A Study of the Beam Physics in the CLIC Drive Beam Decelerator.* PhD Thesis.

III. BL measurements at CLEAR



Magnitude	Units	Value
r/Q_{average}	Ω/m	4400
Q_{average}	-	15000
f_0	GHz	3.00
f_0/f_b	-	2
N_{bunches}	-	150
σ	mm/c	1.0

>CLEAR TW structures information

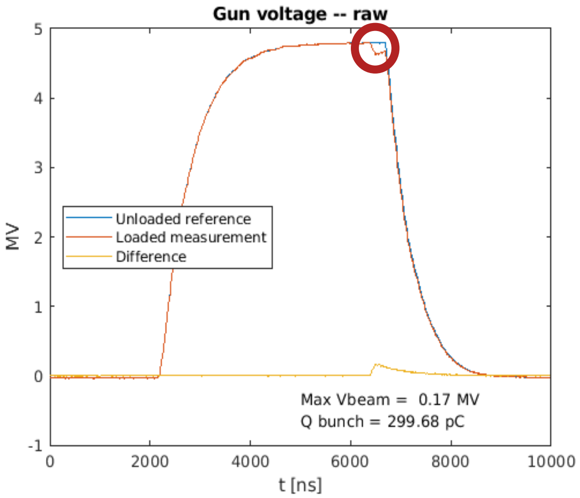
Magnitude	Units	Value
r/Q_{average}	Ω/m	3765
Q_{average}	-	5920
f_0	GHz	3.00
f_0/f_b	-	2
N_{bunches}	-	150
σ	mm/c	1.0

> CLEAR SW photoinjector information

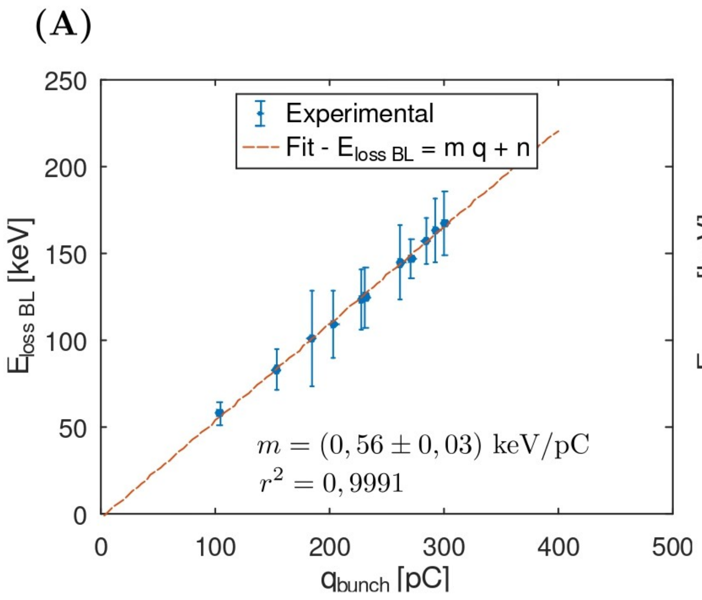
[9] CLEAR Official Website [Accessed October 2023]. https://clear.cern/content/beam-line-description

III. BL in CLEAR photo-injector

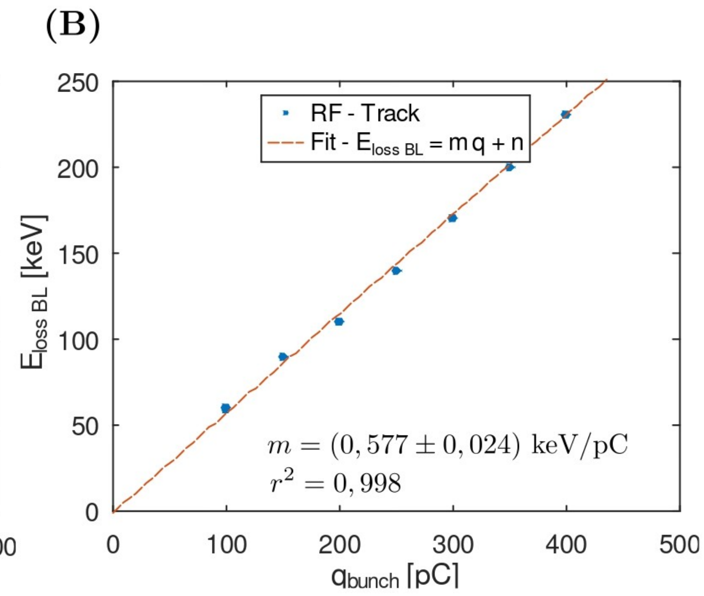
- Train of **150 bunches** with **variable charge (Q_{bunch})** per bunch; $f_b = f_{RF}/2$



> Voltage measurement at the gun loop in CLEAR photo-injector.



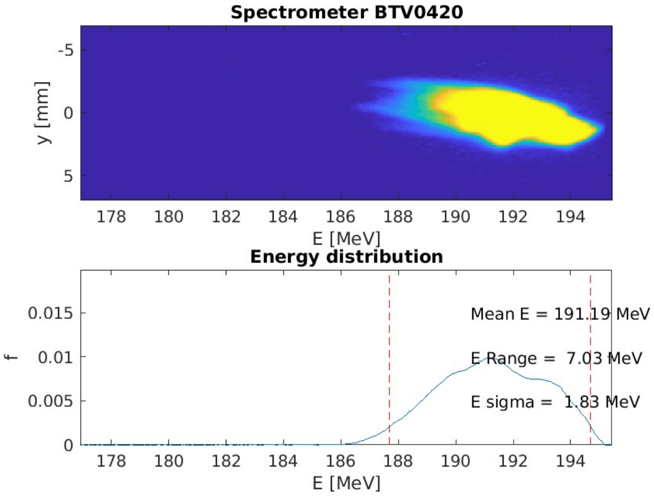
(A) Beam Loading Energy Spread induced in a train of 150 bunches as a function of charge (measurement at CLEAR)



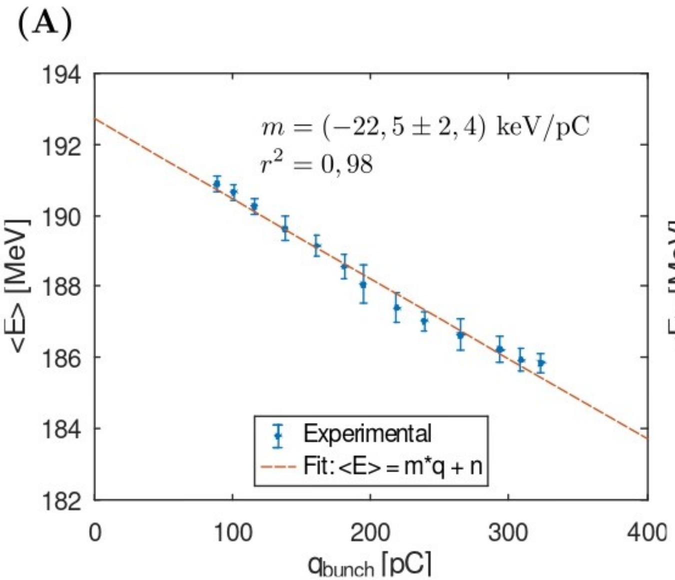
(B) Beam Loading Energy Spread induced in a train of 150 bunches as a function of charge (RF-Track)

III. BL measurements in VESPER (after TW + SW)

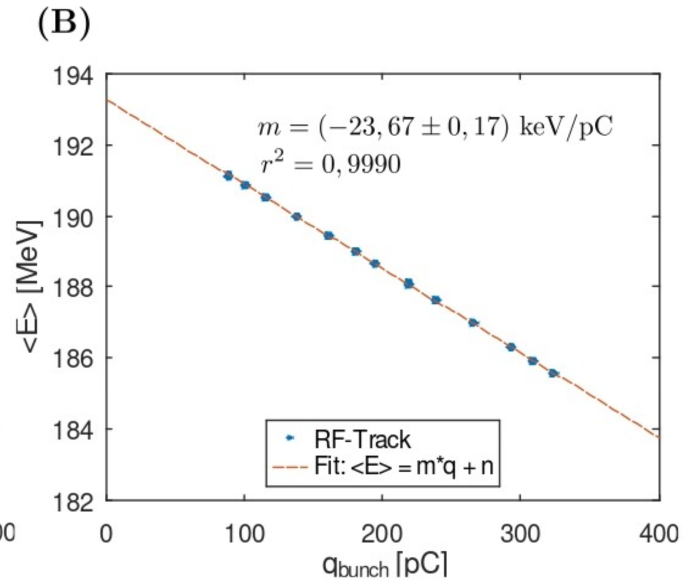
- Train of **50 bunches** with **variable charge (Q_{bunch})** per bunch; $f_b = f_{RF}/2$



> Spectrometer image and energy distribution for a **train of 13 bunches @ VESPER**



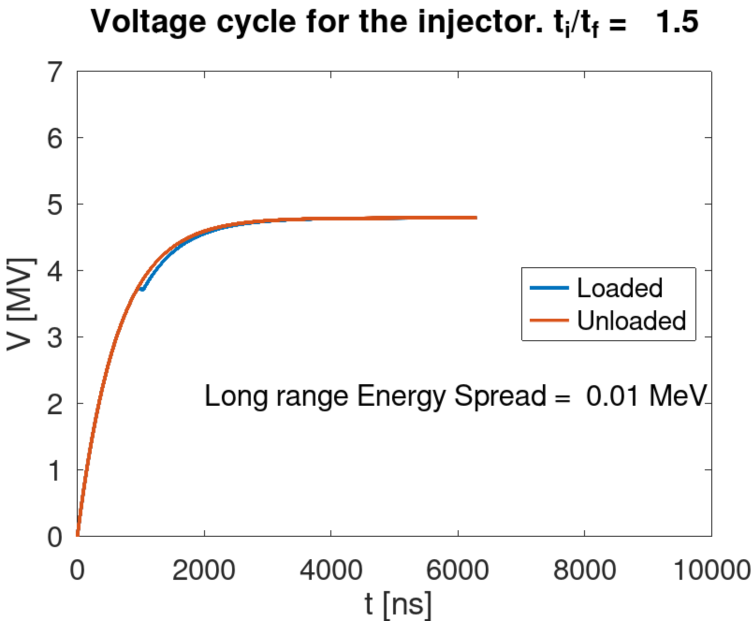
(A) Beam Loading Energy Spread induced in a train of 150 bunches as a function of charge



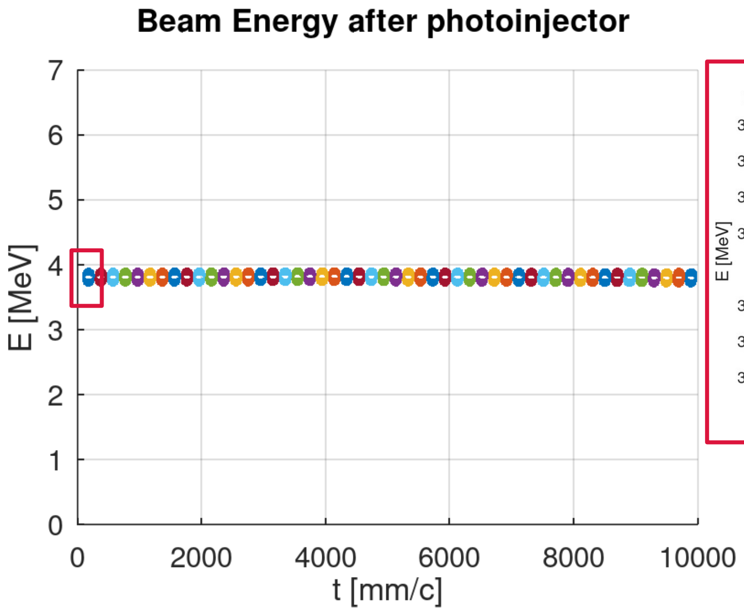
(B) Beam Loading Energy Spread induced in a train of 150 bunches as a function of charge (RF-Track)

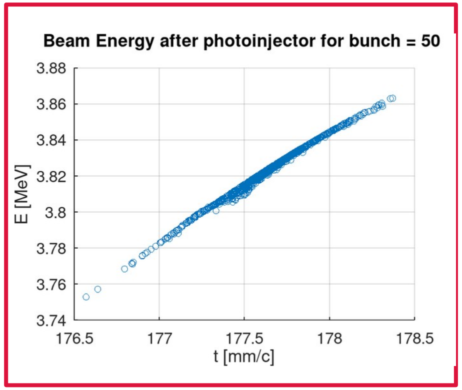
IV. BL compensation - Photocathode

- BL can be compensated with **early injection** of the particles
  - **RF-Track** allows the simulation of this scenario



> Voltage along an RF-cycle for CLEAR SW photo-injector.

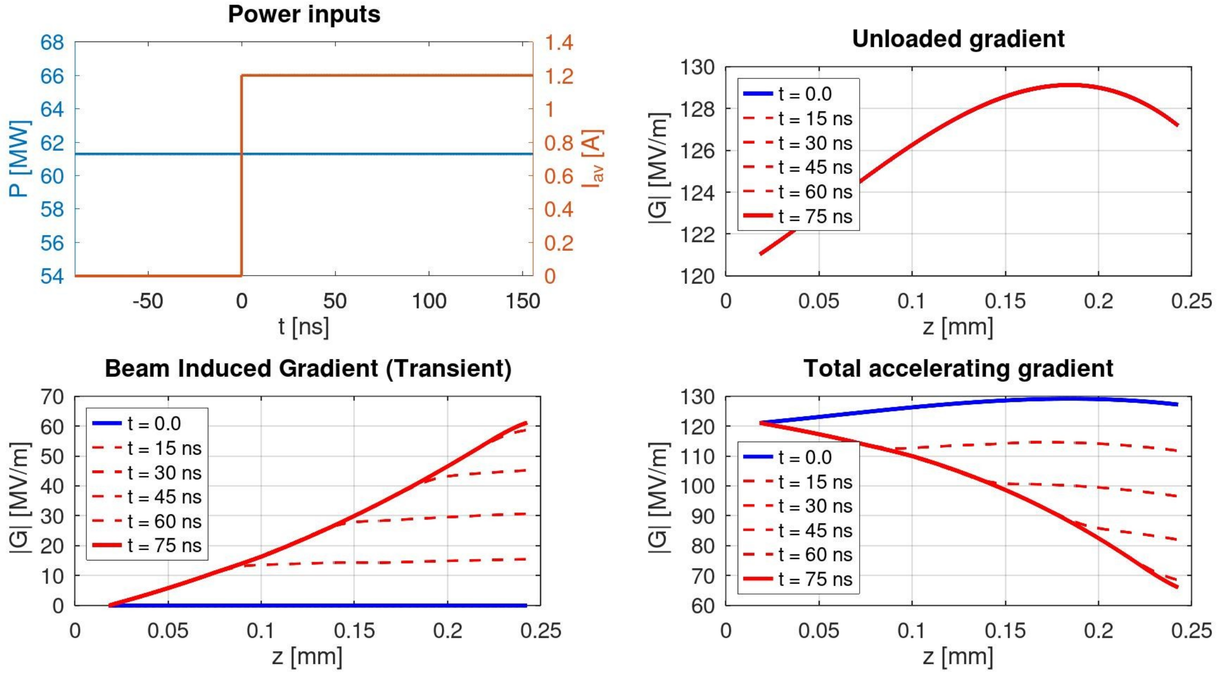


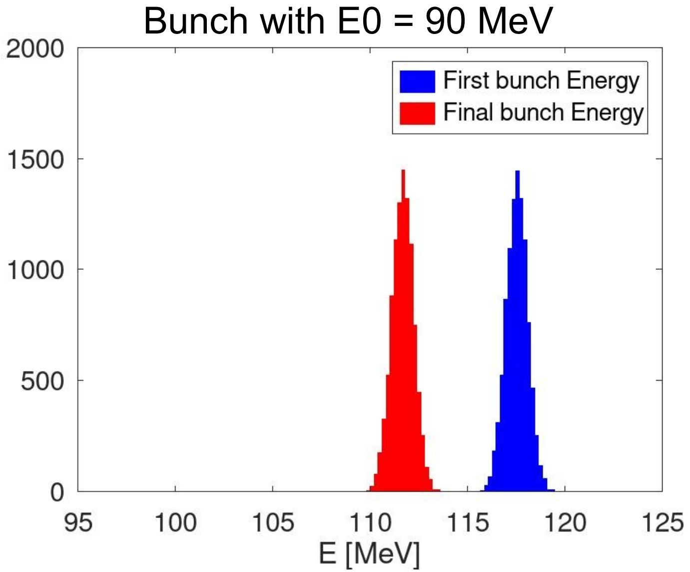


> Energy gain of a train of 50 bunches with a charge per bunch of 300 pC.

IV. BL compensation in TW structures

- Latest feature: possibility to tune input power → BL compensation






```
Elapsed time is 0.32189 seconds.
BL Energy spread from tracking =  5.87 MeV
BL Energy spread from fieldmap =  6.00 MeV
```


[4] Lunin A, Yakovlev V, Grudiev A. *Analytical solutions for transient and steady-state beam loading in arbitrary travelling wave accelerating structures*. Phys Rev Spec Top Accel Beams (2011) 14:05.

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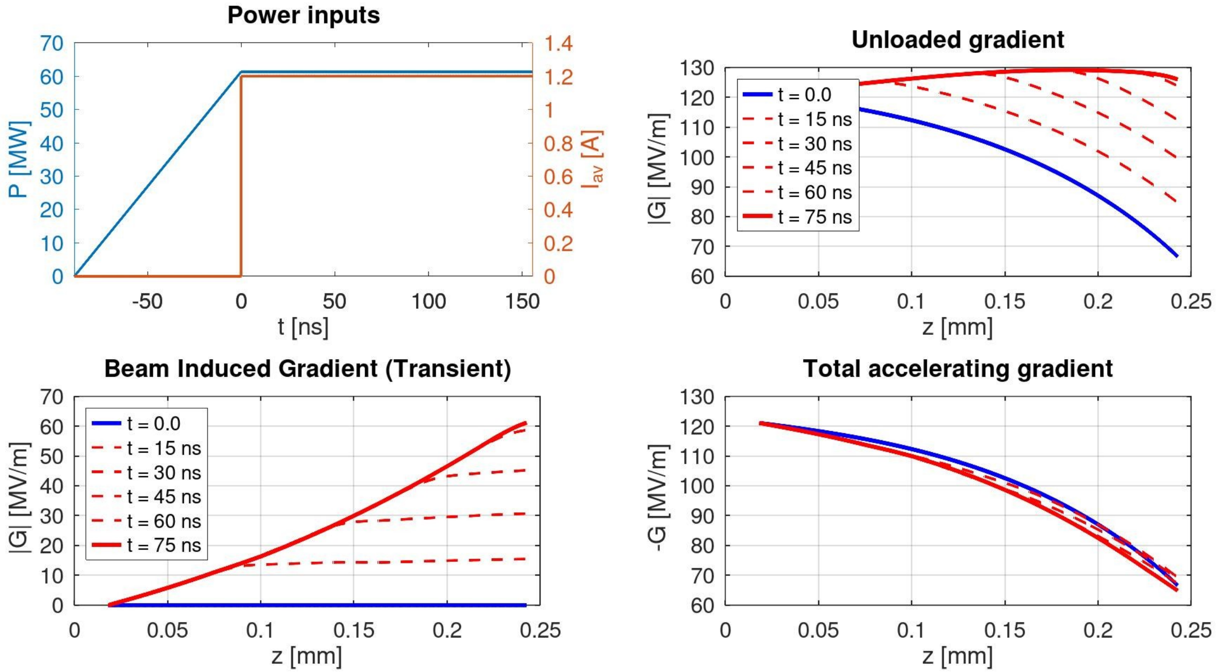
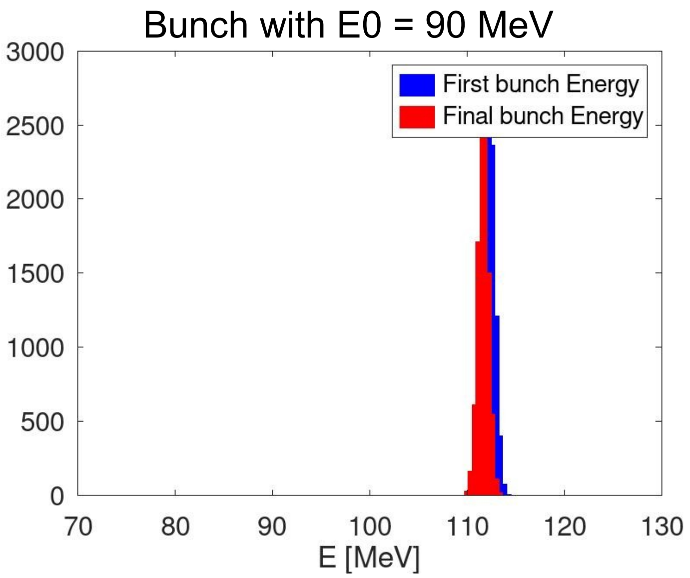




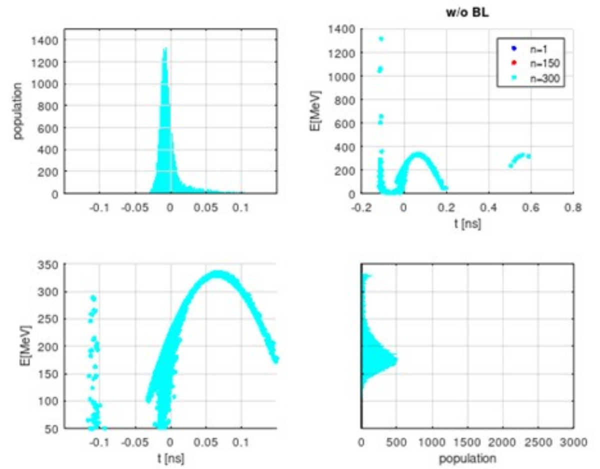

```
BL Energy spread from tracking =  0.71 MeV
BL Energy spread from fieldmap =  0.64 MeV
```


[4] Lunin A, Yakovlev V, Grudiev A. *Analytical solutions for transient and steady-state beam loading in arbitrary travelling wave accelerating structures*. Phys Rev Spec Top Accel Beams (2011) 14:05.

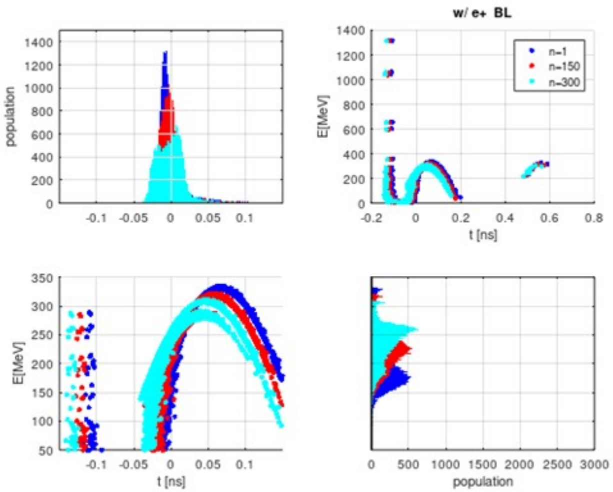
V. CLIC Positron Source

Courtesy of Nafiseh Mesbah

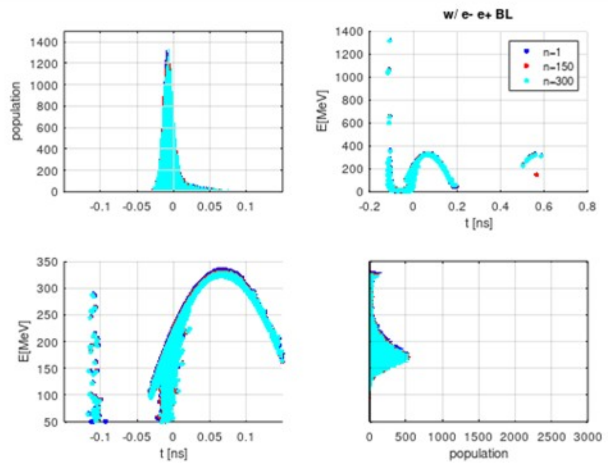
- Both e- and e+ travel together through 11 TW structures (See Nafiseh's talk in CLIC Mini Week)
  - Both BL contribution seem to cancel each other → Bunch-to-bunch E, t spread is minimized



> Positron bunches longitudinal phase-space for optimized phases by Yongke Zhao.



> Positron bunches longitudinal phase-space for optimized phases by Yongke Zhao with BLe+



> Positron bunches longitudinal phase-space for optimized phases by Yongke Zhao with both BL

Contact: nafiseh.mesbah@cern.ch

V. Conclusions

- Basic principles → Understanding of power-diffusive model for BL
- Implementation in RF-Track with fast computational times
  - Orders of magnitude below EM solvers
- Consistent with experimental measurements @ CLEAR
- Bridges beam dynamics simulations with RF-BL compensation
- Potential:
  - Compact high-intensity linac for medical and industrial applications – Neutron sources
  - Multi-species structures such as CLIC positron source

Thanks for your attention

Contact: Javier.olivares.herrador@cern.ch

BACK UP SLIDES

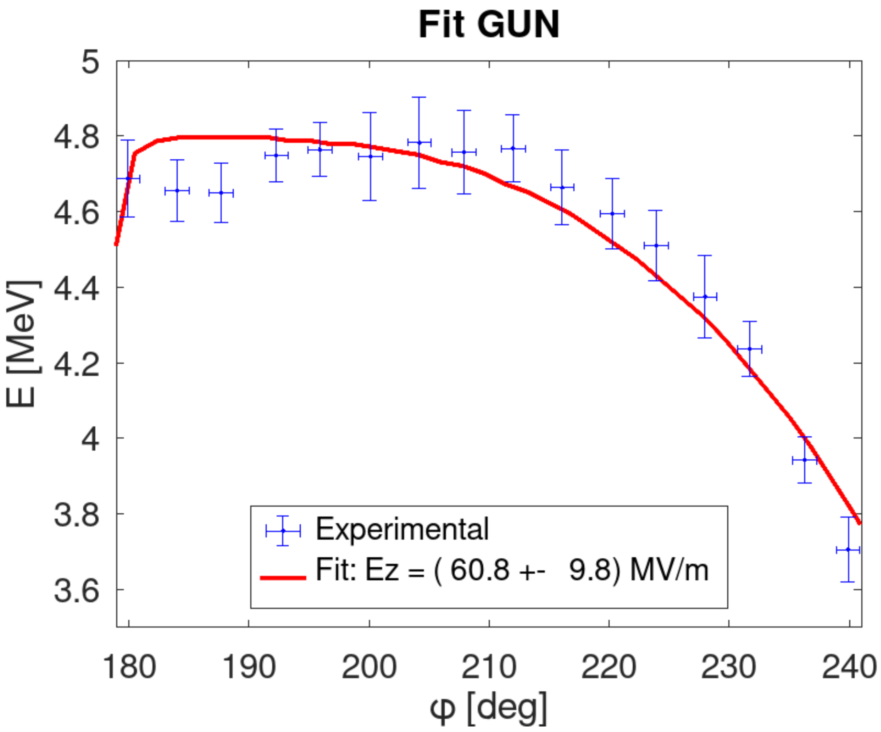
GUN: (Ez, φ) Calibration

With Avni Aksoy

- For a 50 pC bunch:
  - Collect (E_k, φ_k) measurements
  - Fit then to target function F(E_z, φ)
    - F: RF-Track calculation of E after gun.

Magnitude	Units	Value
E_z^{max}	MV/m	60.8 ± 9.8
r^2		0.94

> Results of the minimum square fitting with a test function computed with RF-Track.



> Energy gain after the gun as a function of the phase. In red, RF-Track prediction. In blue, experimental results

GUN: Beam Loading Measurements

- 2) Divergent slope
  - Looking again at the BL equation …

$$-\frac{\partial G}{\partial t} = \frac{\omega}{Q}\frac{G}{2} + \frac{\omega r_{\text{eff}} I}{2Q} + \frac{G_{\text{init}}\omega}{2Q}$$

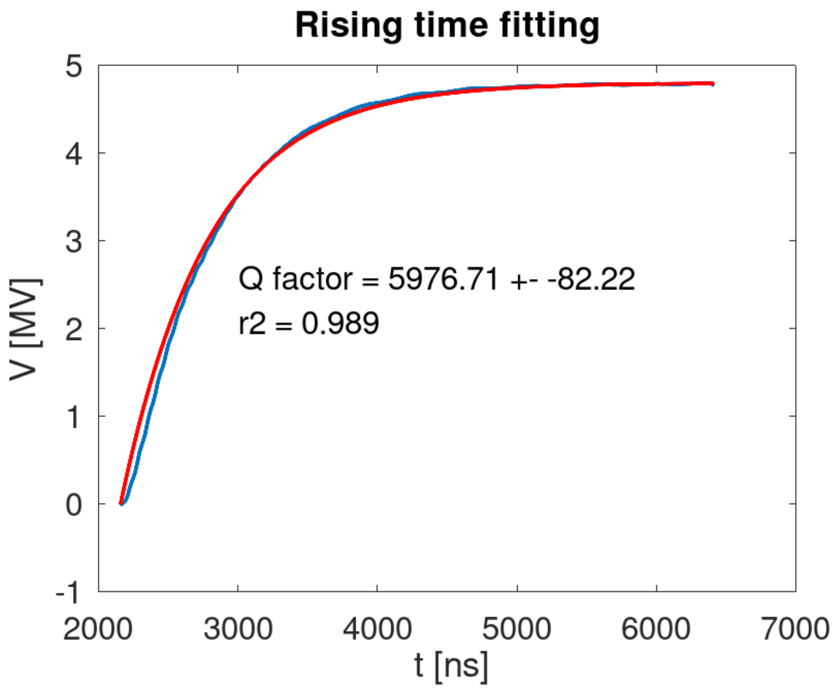
… the slope of the plot E vs Q depends on **r/Q and Q**

  - From design report: r/Q = ; Q0 = 14530;
  - However, we learn that the Q governing the dynamics is

$$Q = (598 \pm 8) \cdot 10$$

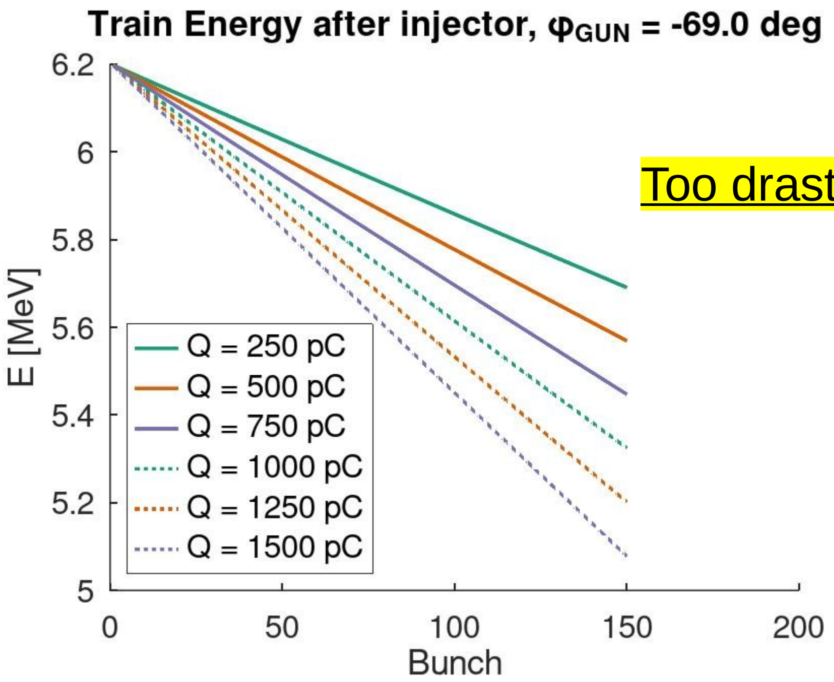
  - This is the loaded quality factor!

$$Q_l = \frac{Q_0}{1+\beta} = (598 \pm 8) \cdot 10 \implies \beta = 1.5$$



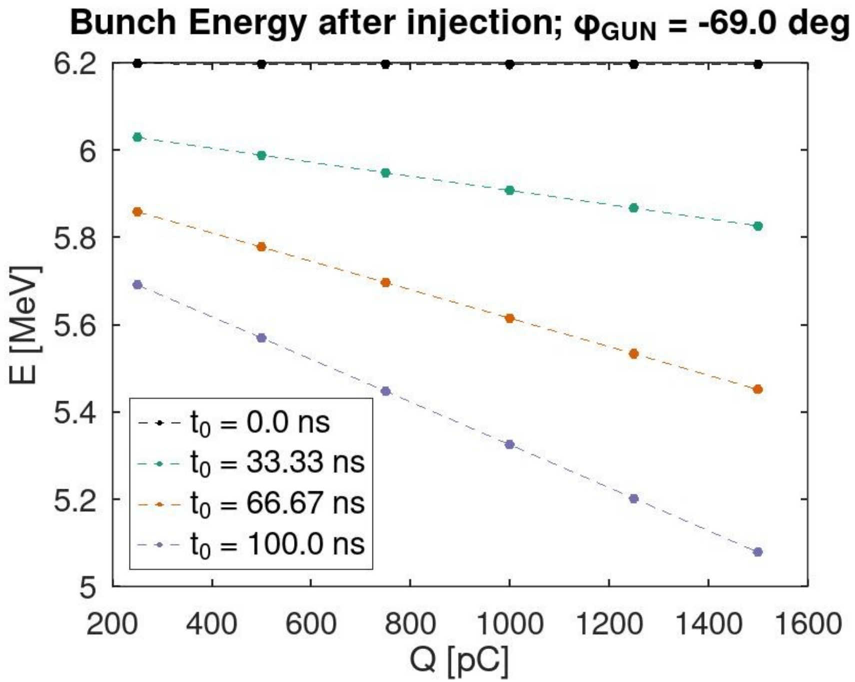
CLEAR – Injector BL

- Train: 150 bunches; $f = f_{RF}/2$; Q_{bunch} = 250 pC – 1500 pC



> Energy profile for bunches with different charges after having travelled along the photoinjector

Too drastic!



> Energy profile for different bunches depending on their charge

CLEAR – Injector BL

- First consequence: Energy loss depending on Q
- Another consequence: Arrival time to TWS1
  - If all particles travel with same β, then the arrival time to TWS1 would be equally spaced. Ideally, it would be perfectly synchronized so that

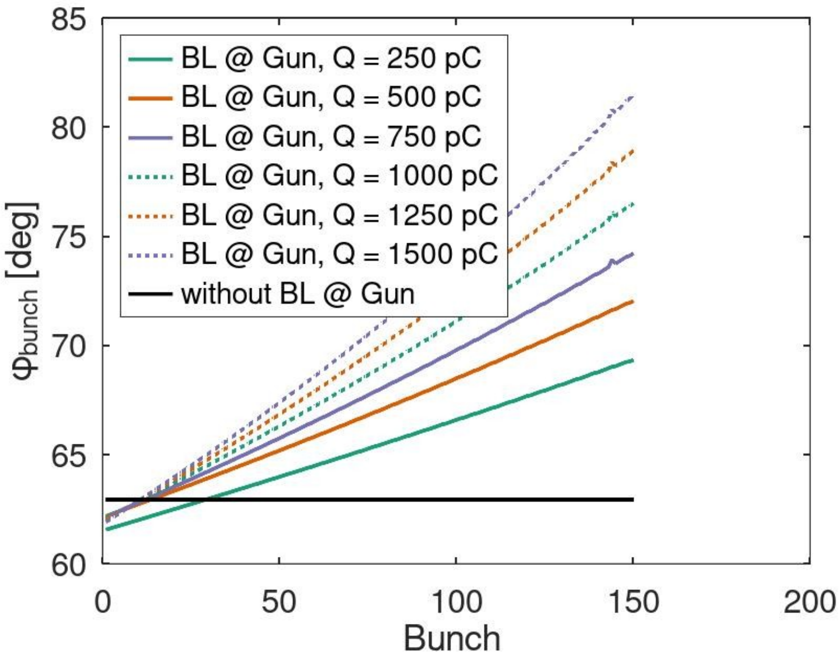
$$t_k = \underbrace{\frac{4\pi k}{\omega}}_{\text{Injection time}} + \underbrace{\int_0^L \frac{\mathrm{d}z}{\beta(z)c}}_{\text{Flight time along gun}}; \; k = 0, .., N-1 \implies \Delta t_k = \frac{4\pi}{\omega}$$

  - However, particles have different β because of **Gradient reduction** $\implies$ **Different spacing!**

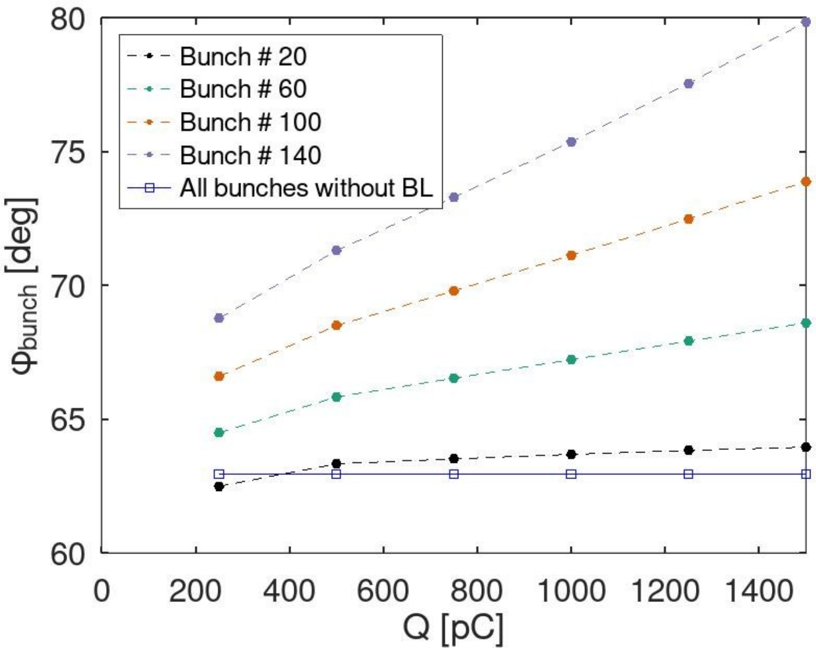
- Definition: Bunch phase $\varphi_{\mathrm{bunch},k} = \omega t_k \pmod{2\pi}$

CLEAR – Injector BL

- Bunch phase distortion depends on Q



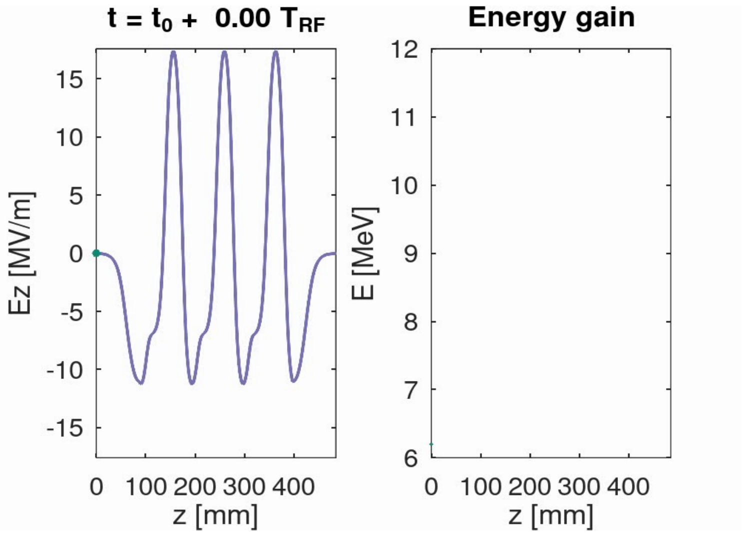
> Bunch phase for differently charged trains.



> Comparison of the bunch phase for different bunches and its dependency on charge

CLEAR – TW structures

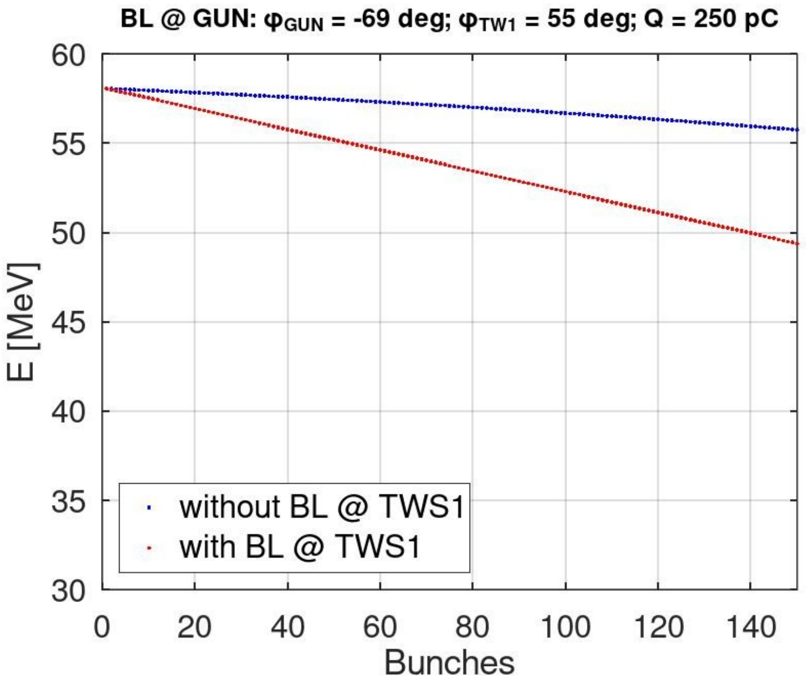
- Inhomogeneous $\varphi_{\text{bunch}} \implies$ Spoils train E homogeneity
  - t_k no longer synchronized with RF.

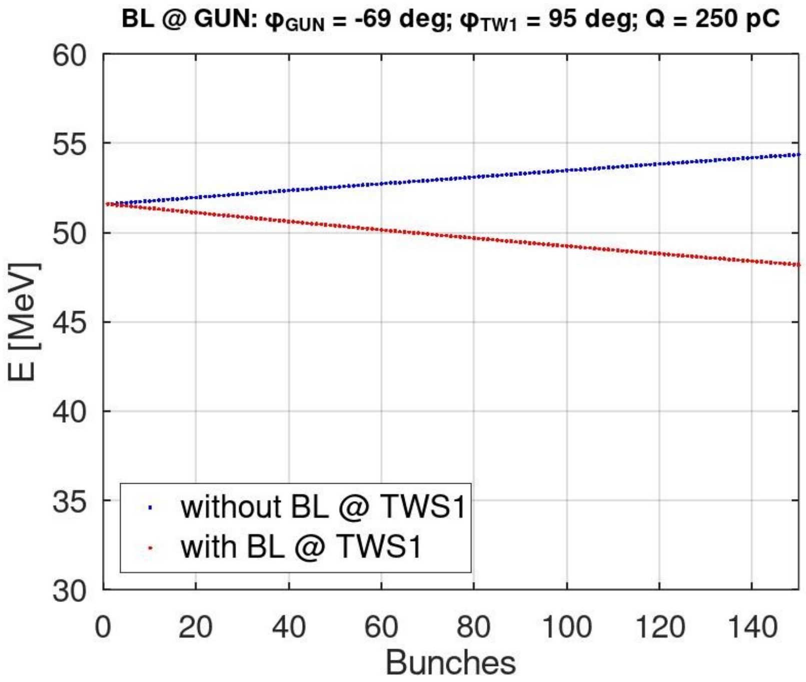


$$F_z \propto E_z \propto \cos\left(\varphi_{\text{bunch}} + \phi_{\text{RF}}\right)$$

CLEAR – TWS1 BL

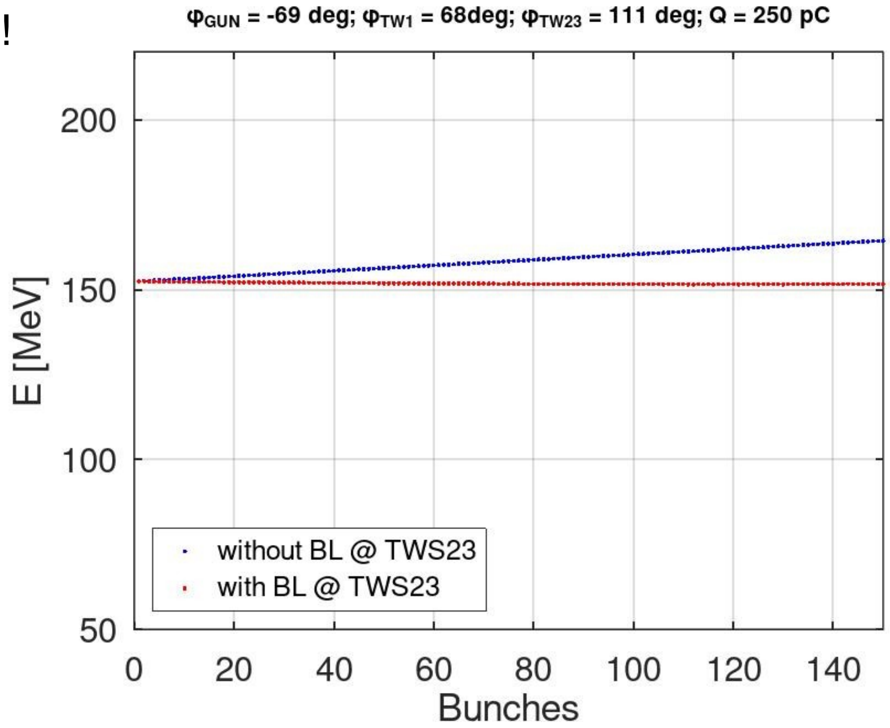
- BL @ GUN $\Longrightarrow \phi_{\mathrm{RF}}$ affects Energy profile!





CLEAR – TW structure

- **Beam Loading at GUN helps compensating overall Beam Loading**
  - Correct phase choice!



> Beam Loading compensated train.