Making sense of Initial conditions for hydro in small systems

G.Torrieri

In collaboration with Gabriel Rabelo-Soares,Gojko Vujanovic,Jun Takahashi

(2004) Matter in heavy ion collisions seems to behave as a perfect fluid, characterized by a very rapid thermalization

RHIC Scientists Serve Up 'Perfect' Liquid

New state of matter more remarkable than predicted — raising many new questions

April 18, 2005

TAMPA, FL — The four detector groups conducting research at the Relativistic Heavy Ion Collider (RHIC) — a giant atom "smasher" located at the U.S. Department of Energy's Brookhaven National Laboratory — say they've created a new state of hot, dense matter out of the quarks and gluons that are the basic particles of atomic nuclei, but it is a state quite different and even more remarkable than had been predicted. In peer-reviewed papers summarizing the first three years of RHIC findings, the scientists say that instead of behaving like a gas of free quarks and gluons, as was expected, the matter created in RHIC's heavy ion collisions appears to be more like a *liquid*.

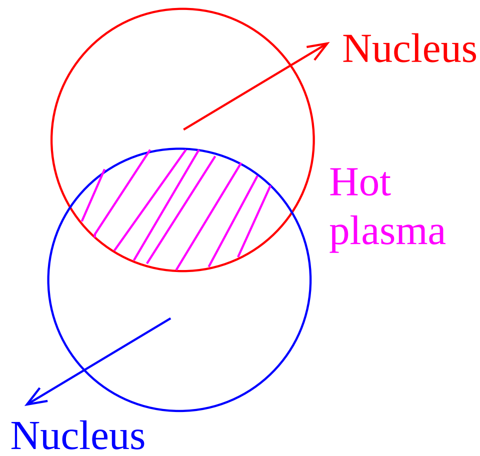
"Once again, the physics research sponsored by the Department of Energy is producing historic results," said Secretary of Energy Samuel Bodman, a trained chemical engineer. "The DOE is the principal federal funder of basic research in the physical sciences, including nuclear and high-energy physics. With today's announcement we see that investment paying off."

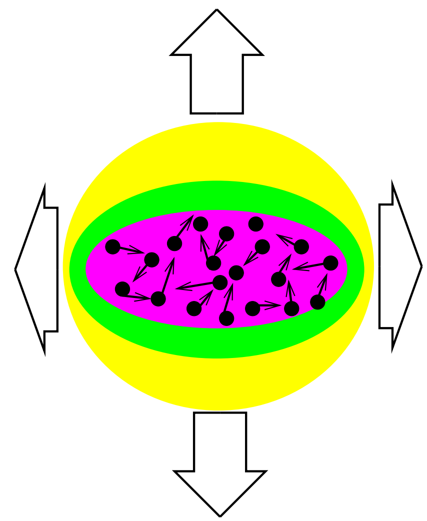
"The truly stunning finding at RHIC that the new state of matter created in the collisions of gold ions is more like a liquid than a gas gives us a profound insight into the earliest moments of the universe," said Dr. Raymond L. Orbach, Director of the DOE Office of Science.

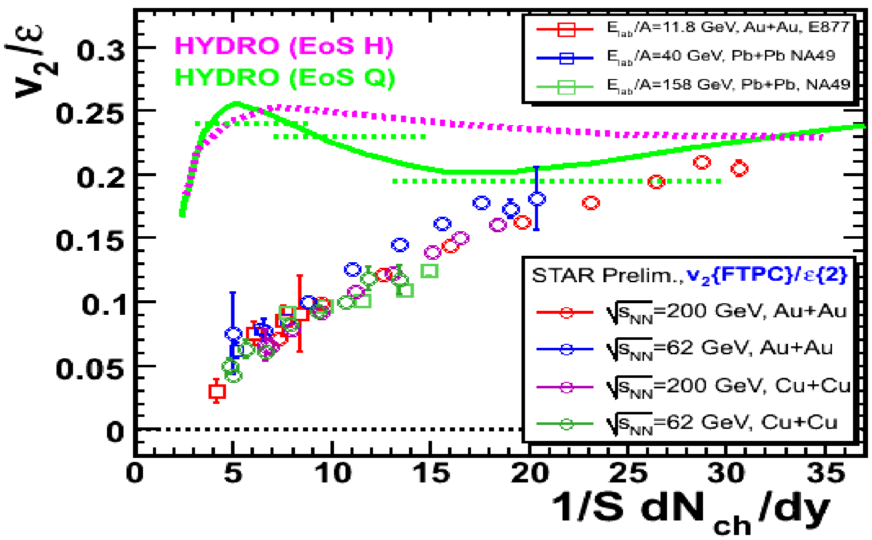
Initial azimuthal density gradients $\to$ pressure gradients $\to$ p_T gradient

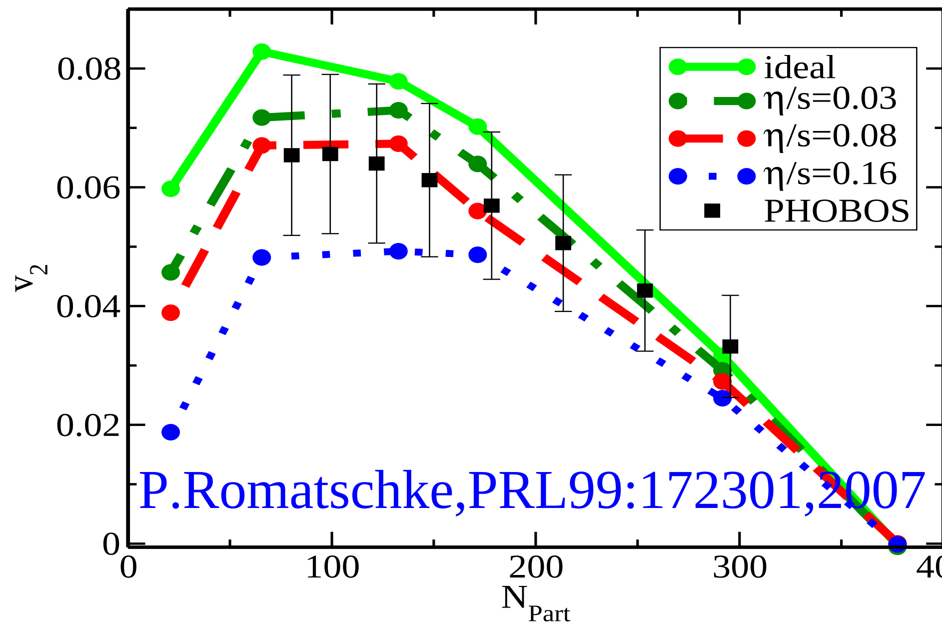
$$\frac{dN}{p_T dp_T dy d\phi} = \frac{dN}{p_T dp_T dy}\left[1 + 2v_n(p_T, y)\cos\left(n\left(\phi - \phi_0\left(n, p_T, y\right)\right)\right)\right]$$

"trivial" effects ($\vec{p}$ conservation) also give you a v_n . "Collectivity": Same v_n from $\forall$ n-particle correlations , $\left\langle \frac{dN}{d\phi_1}\frac{dN}{d\phi_2}... \right\rangle$, reaction plane dependence









$$v_n \sim A\mathcal{O}\left(\epsilon_n\right) + B\mathcal{O}\left(\langle T_{ini}\rangle \times R\right)^{\beta} \sum_m \mathcal{O}\left(\epsilon_n \epsilon_m\right) + C\mathcal{O}\left(\frac{\epsilon_n \eta}{sTR}\right) + \ldots$$

Need hydrodynamic code to get A,B,C . Term in A 80-100%

What is (ideal) hydrodynamics?

Conservation of momentum and Charge always gives us 5 Equations:

$$\underbrace{\partial_\mu \langle T^{\mu\nu} \rangle = 0}_{4} \quad , \quad \underbrace{\partial_\mu \langle j^\mu \rangle = 0}_{1}$$

Local equilibrium/isotropy, in some frame (at rest with u^μ), reduces these 10+4 independent components

$$\langle T^{\mu\nu} \rangle = \underbrace{(e+p)u^\mu u^\nu - p g^{\mu\nu}}_{5 \quad e,p,u_{x,y,z}} \quad , \quad j^\mu = \underbrace{\langle \rho \rangle}_{1} u^\mu$$

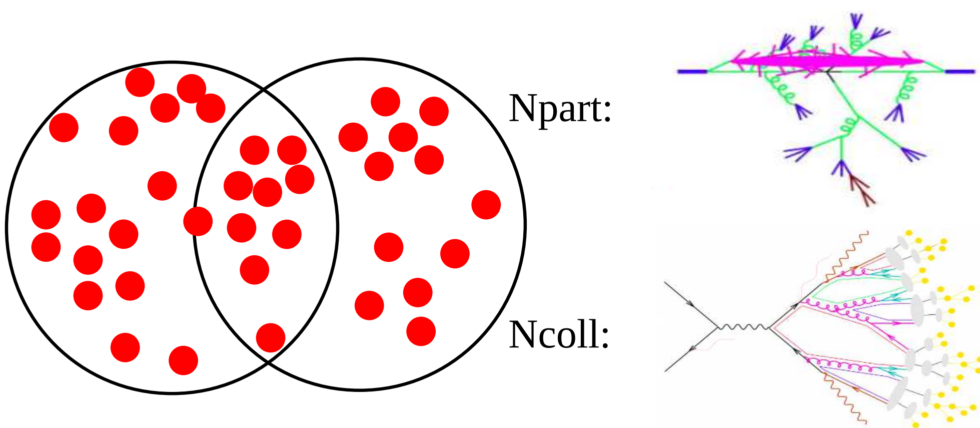
Together with the equation of state, system closed Viscosity gives more corrections equations, but still closed

$$p(e,\rho) \equiv \frac{\partial S}{\partial V} = T \ln \mathcal{Z} \quad , \quad e = -\frac{\partial \ln \mathcal{Z}}{\partial 1/T} \quad , \quad \rho = T \frac{\partial \ln \mathcal{Z}}{\partial \mu}$$

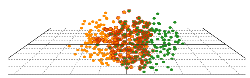
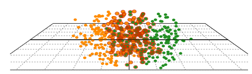
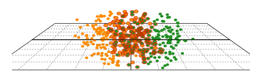
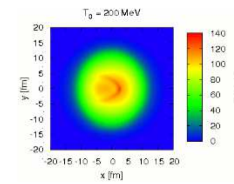
However,to solve we need <u>non-</u>hydrodynamic ingredients!

“easy” EoS at low chemical potential it is known from the lattice. Extending into μ is part of ongoing research

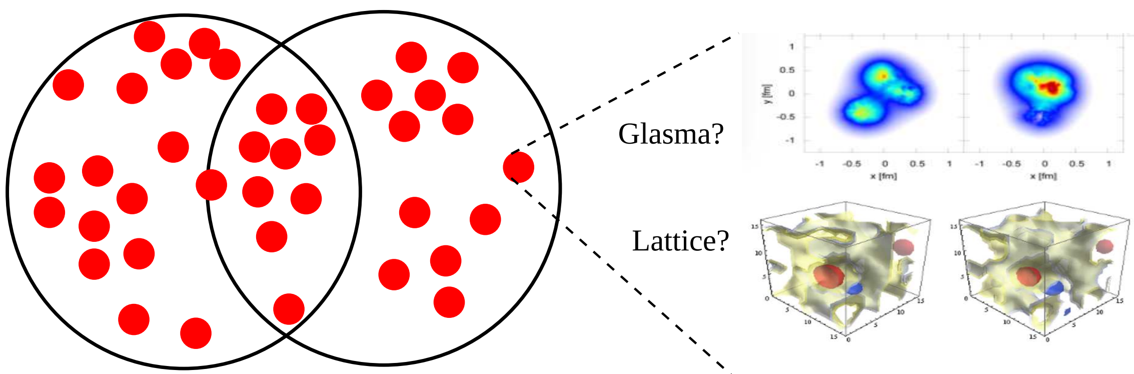
“Hard” Initial conditions (Energy/entropy density) reasonable baseline: Incoherent superposition of nucleon-nucleon collision (Glauber model)+smearing. works as $R_{nucleus} \gg 1/Q^2 \gg 1/\sqrt{s}$



Complication: e-by-e correlations,subnucleonic structure

At most $<$  $+$,  $+$,  $+ .. >=$,....$\neq$ 

e-by-e fluctuations and correlations complicated). For subnucleonic structe have estimates from color glass/saturation, but Need $\hat{T}^{\mu\nu}_{event} \simeq \left\langle \hat{T}^{\mu\nu} \right\rangle_{event}$
Hydro is a classical theory,need classical input all info in $\langle ... \rangle$



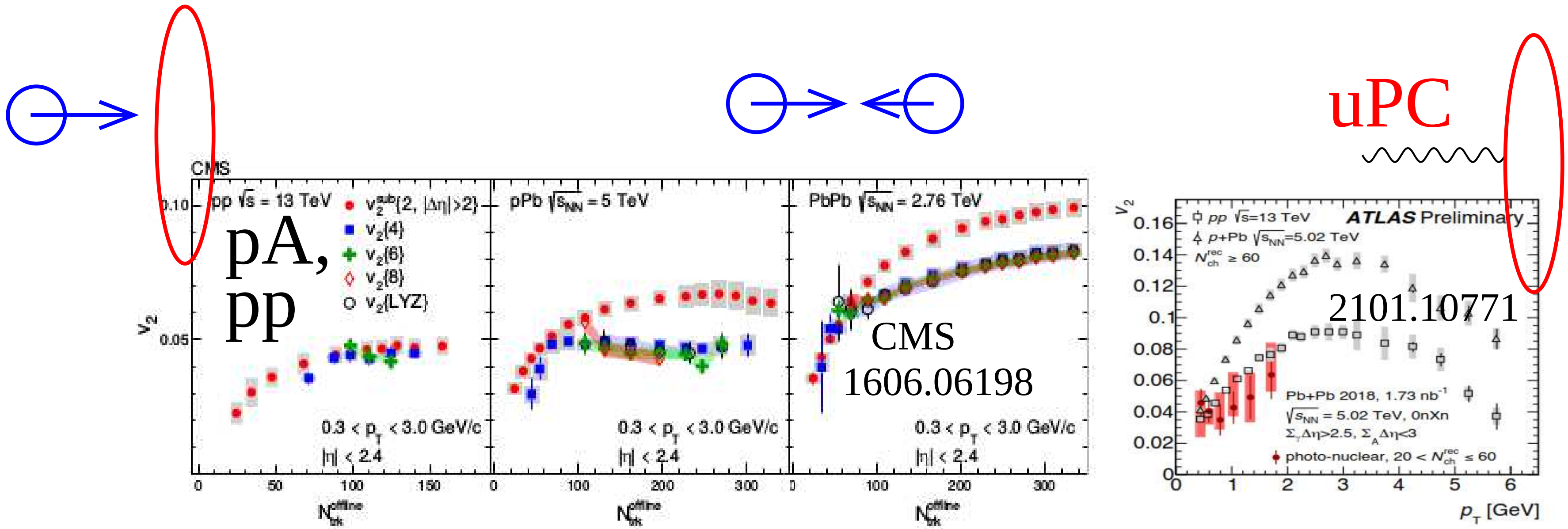
However the LHC turned on and...

SCIENCE

The LHC Might Have Created The Smallest Drop Of Liquid Ever

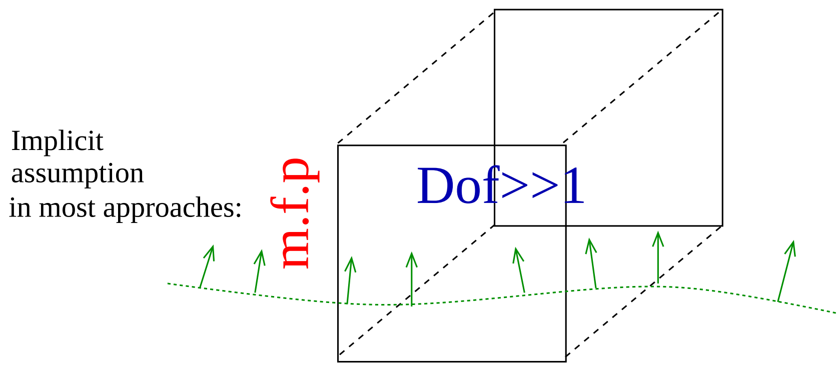
A tiny drop could have big implications for our understanding of particle collisions.

By Shaunacy Ferro *May 8, 2013*



1606.06198 (CMS) : When you consider geometry differences, hydro with $\mathcal{O}(20)$ particles "just as collective" as for 1000. Thermalization scale. 2101.10771 (ATLAS) also UPCs $\gamma^* A - \rho A$! It is clear that Subnucleonic degrees of freedom crucial

This raises conceptual problems

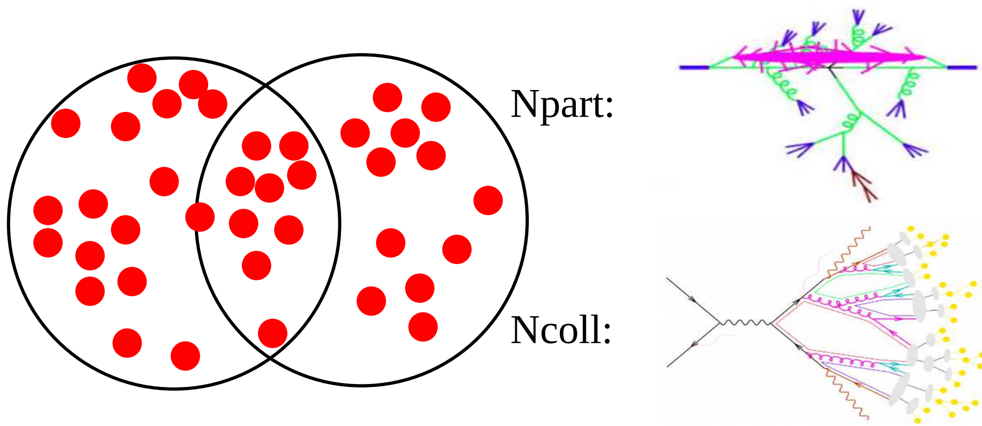


Hydrodynamics implicitly assumes a "thermodynamic limit in every cell", so $\left\langle e^2 \right\rangle - \left\langle e \right\rangle^2 \ll \left\langle e^2 \right\rangle$,thermodynamic fluctuations do not propagate. Need to study better connection to statistical mechanics . I work on this 2307.07021 , 2309.05154 2007.09224 ,2109.06389 ,But for this talk I will focus on an immediate "semi-technical" isse

Qualitatively system seem equally collective as $v_n\left\{N\right\}$ independent of N everywhere But what is the initial eccentricity of a nucleon-nucleon collision? no quantitative recipe that makes sense!

Deterministic motion of averages (hydro) vs wavefunctions (initial state)

$\langle O \rangle \neq \hat{O}$ But nuclei are big, $R_{nucleus} \gg p_{fermi}^{-1}$

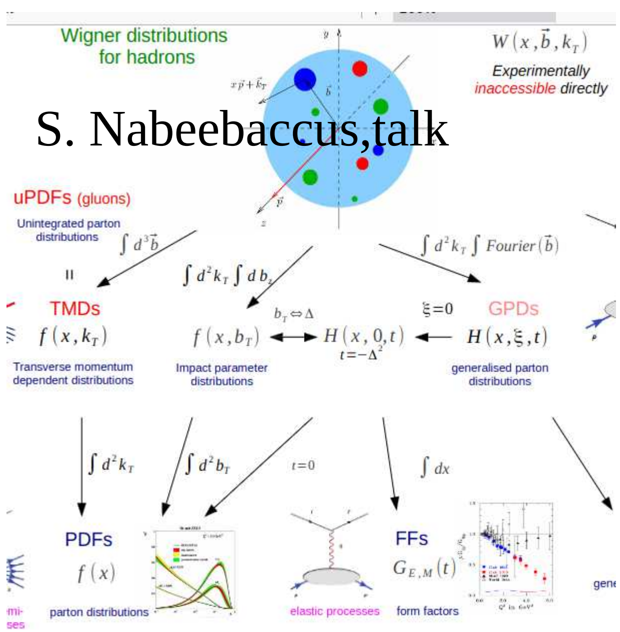


$$\langle O \rangle = \int \psi_A^* \psi_A O dx \simeq \sum_{incoherent} \langle O \rangle_N$$

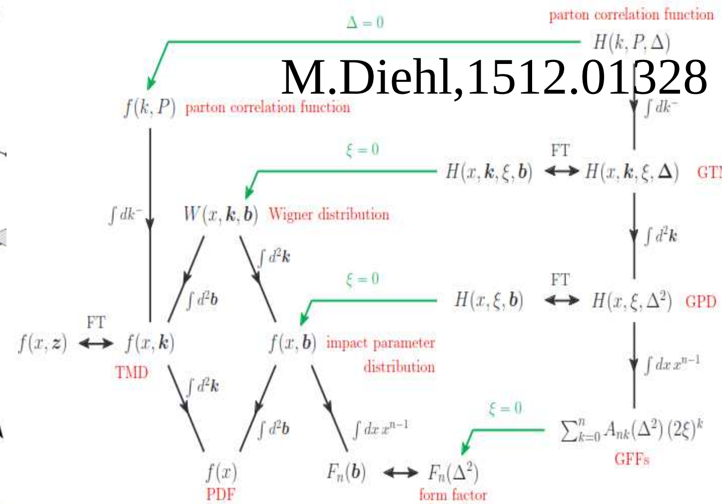
And this is true in any basis, ie for both gluon density and $T_{\mu\nu}$.

Hydro is a classical theory initial conditions are either energy-momentum tensor $T_{\mu\nu}(x)$ or entropy density $s(x)$ Either works because one goes to the other via the EoS

You'd get those from the 3D wave-function of the nucleon . but ...



S. Nabeebaccus,talk



M.Diehl,1512.01328

Big issue 1 Shape distributions are defineable (Wigner functions) and calculable (lattice), but relation to experimentally measureable processes goes via transformations as well as limits and projections (non-invertible)

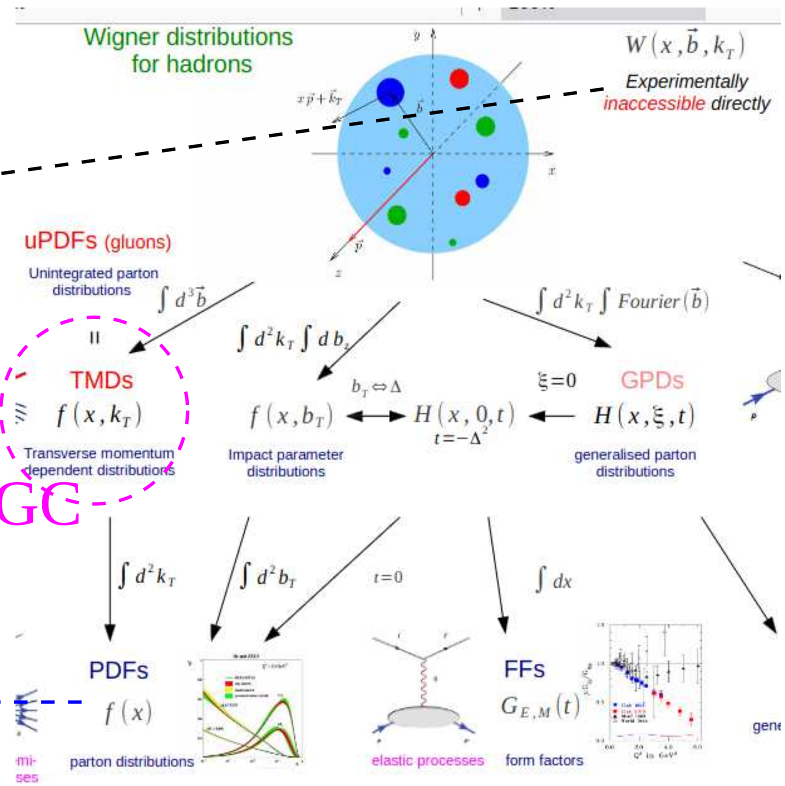
S. Nabeebaccus,talk

QCD
(including lattice)

???
Small systems?

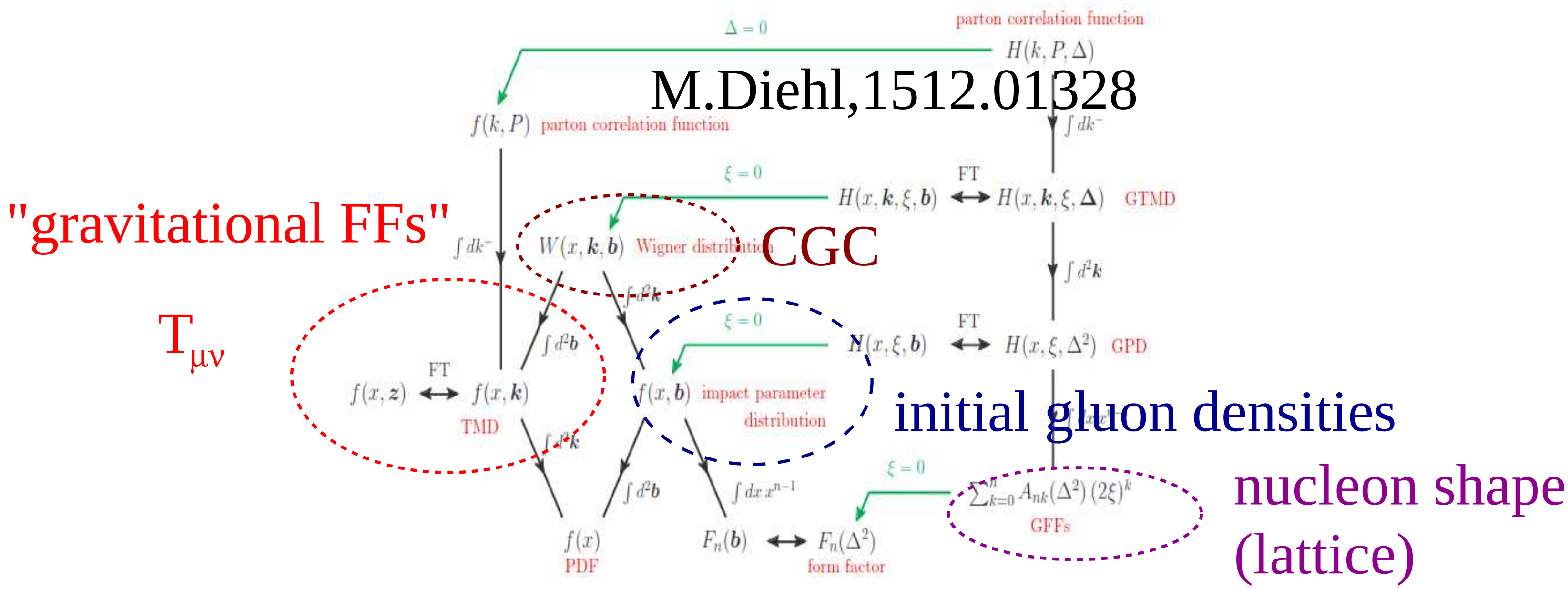
CGC

Heavy ion
initial conditions



Big issue 2 (A huge one!)

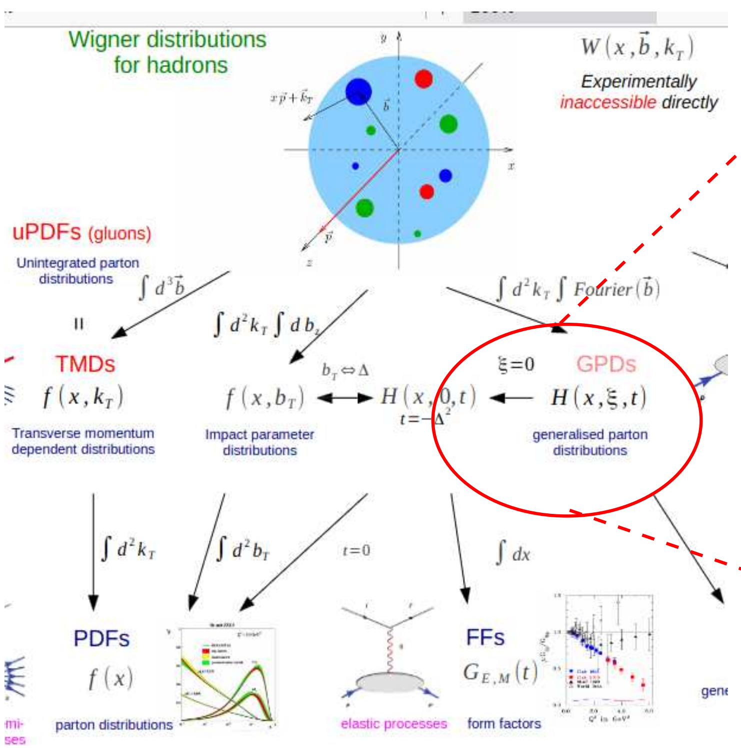
$T_{\mu\nu}(b_\perp, x_{bj})$ and $W_{partons}(b_\perp, x_{bj})$ characterized respectively by TMDs and GPDs . These are not transforms of each other, contain different information. What kind of "initial state" is it? CGC: TMD,Gaussians,thermalization not clear! (Kompost,free-flow etc)



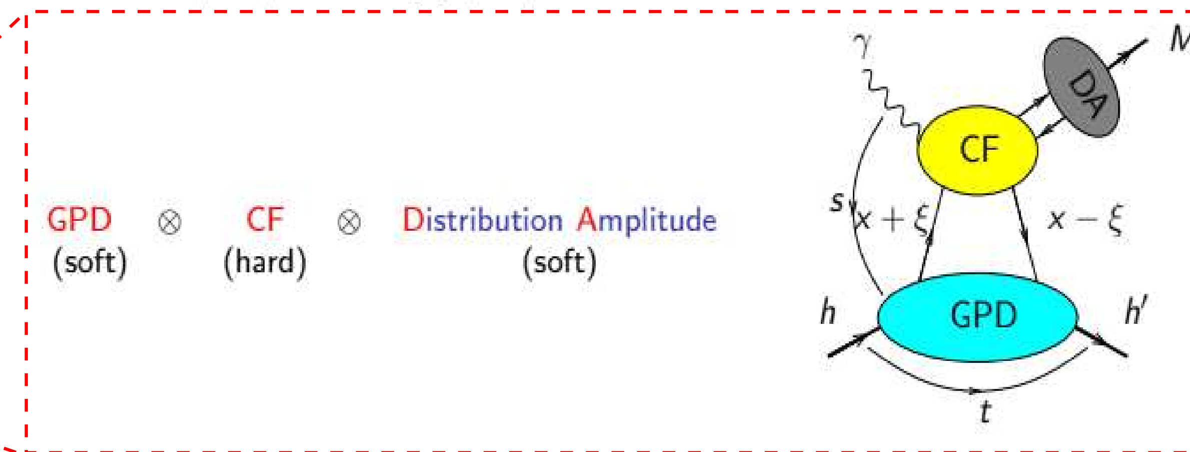
Beyond DiS structure process dependent! ,factorization less useful

transverse structure/motion of partons $\Rightarrow \underbrace{x_{structure}}_{z_{parton}/t_{parton} \sim p_z/E} \neq \underbrace{x_{kinematic}}_{p_z^{process}/E}$

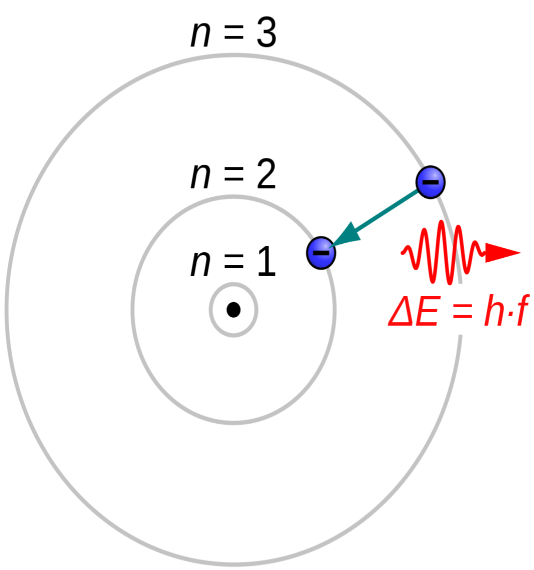
. How does one square this with <u>universality of thermalization</u>?



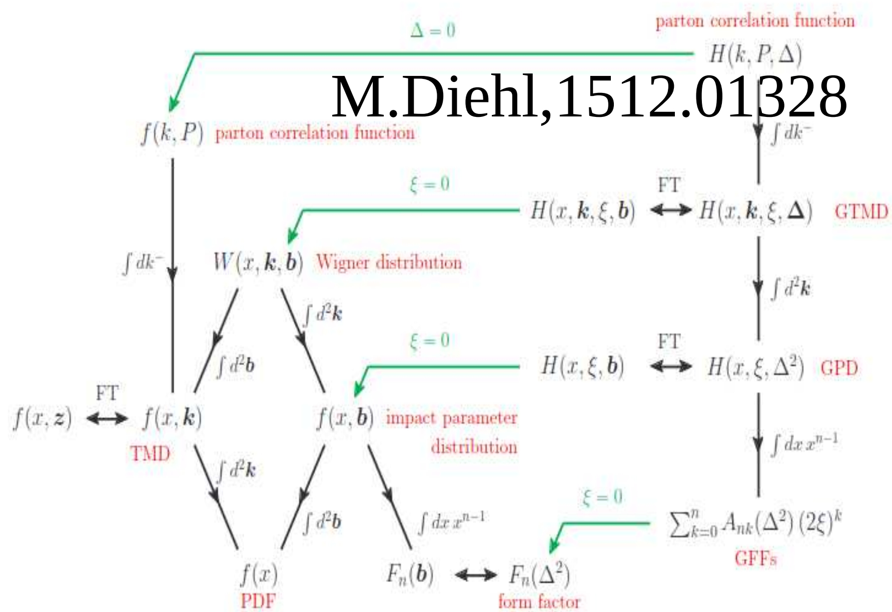
S. Nabeebaccus,talk



If you understand quantum mechanics this is not surprising We calculate wavefunctions... $\langle\psi|$ but measure process-dependent matrix elements $\left|\langle\psi_1|\,\hat{O}\,|\psi_2\rangle\right|^2$. Bohr picture works as $m_{electron} \gg r_{bohr}^{-1}$ so $\langle\psi_1|\,\hat{O}\,|\psi_2\rangle \simeq \int d^3x \,\langle\psi_{free}(x)|\,V(r)\,|\psi_{free}(x)\rangle$ but $r_{had}^{-1} \sim m_{had} \sim \Lambda_{QCD}$

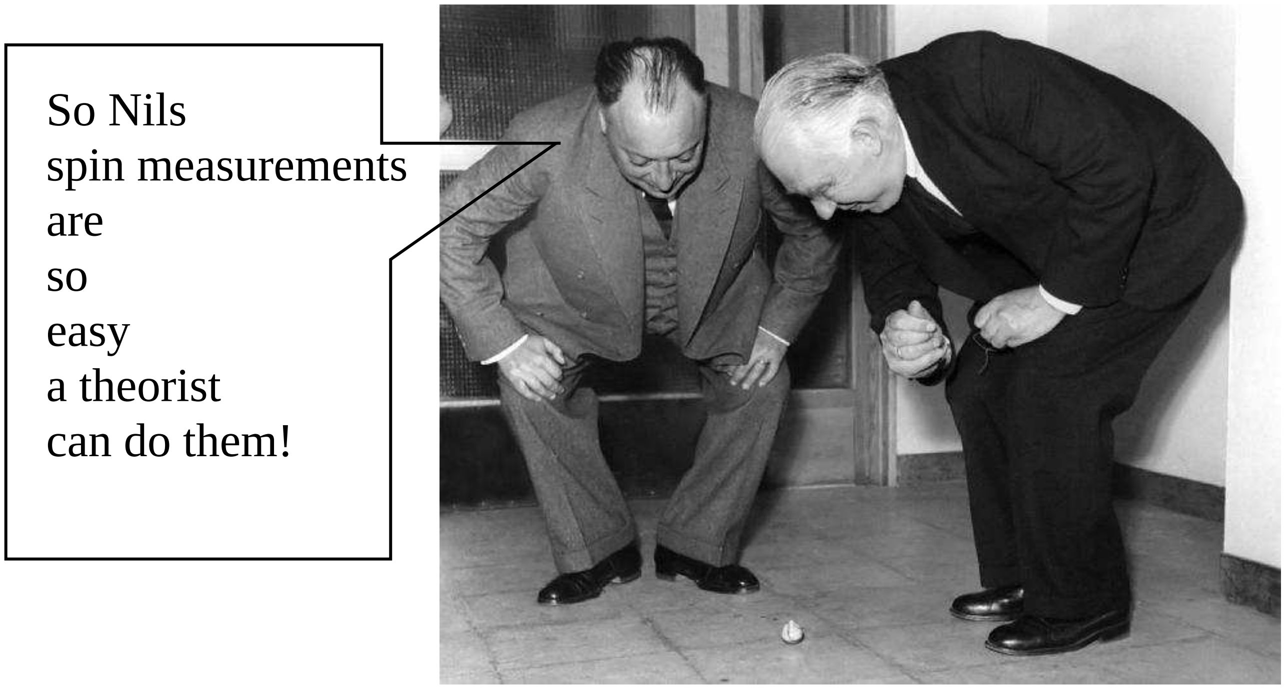


≠



M.Diehl,1512.01328

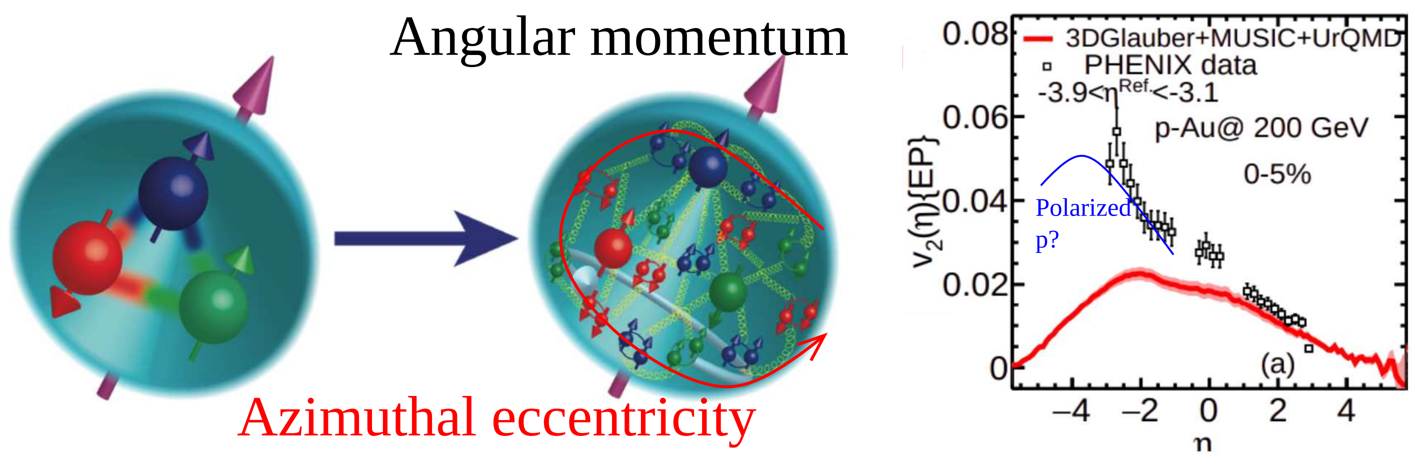
But how seriously can one take “initial states” in small systems as a “quantitative science”? What’s the hydro-relevant shape of a nucleon?



A concrete question: if Hydro “classicalizes” initial conditions. It means Hydro in small systems could lead to “classical spin measurement”

Remember v_n sensitivity to eccentricity

$$v_n \sim A\mathcal{O}\left(\epsilon_n\right) + B\mathcal{O}\left(\langle T_{ini}\rangle \times R\right)^{\beta} \sum_m \mathcal{O}\left(\epsilon_n \epsilon_m\right) + C\mathcal{O}\left(\frac{\epsilon_n \eta}{sTR}\right) + \ldots$$



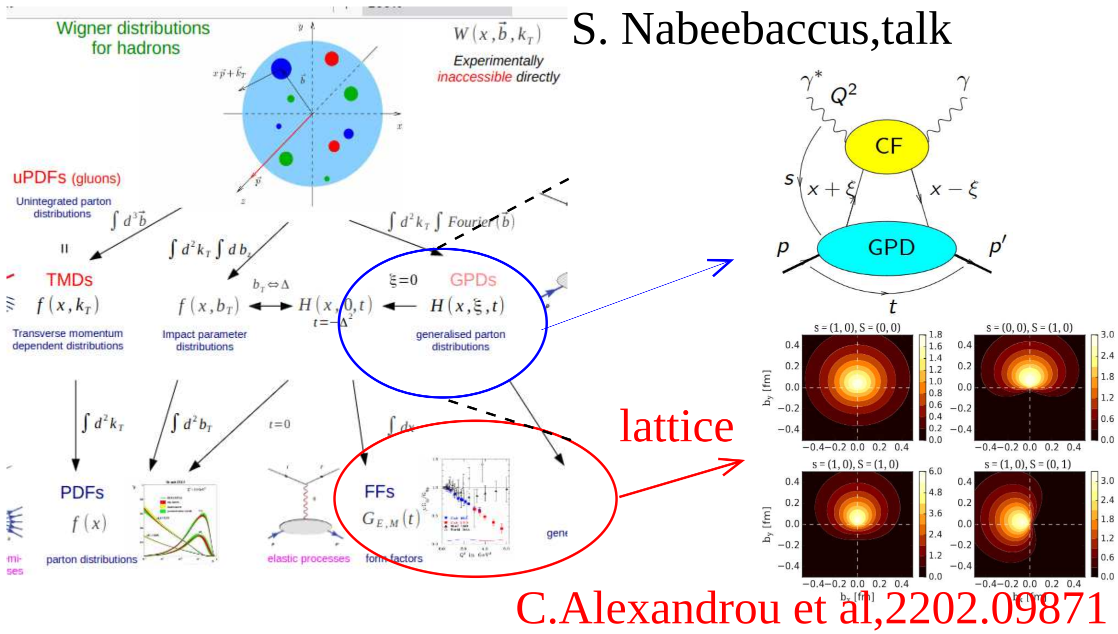
Spin dependent nucleon shape changes v_2 in polarized pA collisions. ultimate small system hydrodynamics? Experimentally feasible (data is there), But how does one get a theoretical estimate given the issues before?

What distribution to measure: A guess

TMDs integrate out all configuration information of the Wigner function. Can give $\langle T_{\mu\nu} \rangle$

GPDs integrate out all momentum information of the of the Wigner function, get S^μ

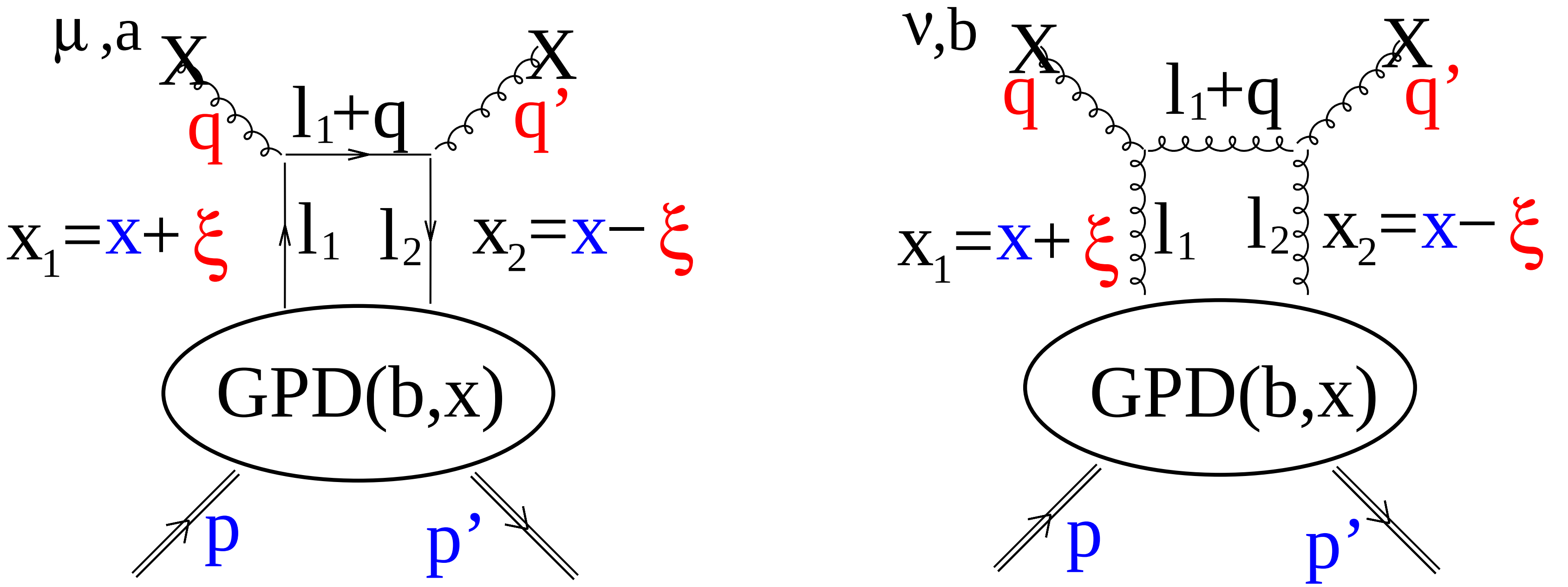
Therefore GPDs are more appropriate for "real" hydrodynamics, TMDs for "hydrodynamization"/free streaming. But in both cases, "real" distribution hidden by process dependence.



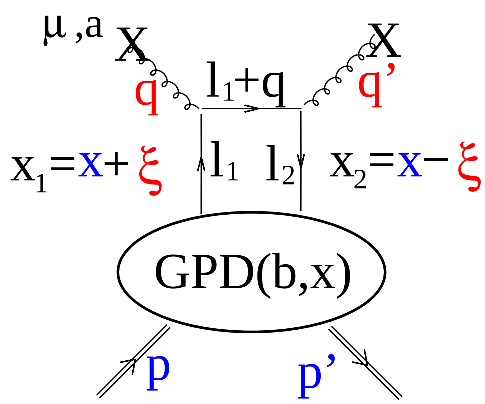
Lattice can measure azimuthally dependent Mellin moments

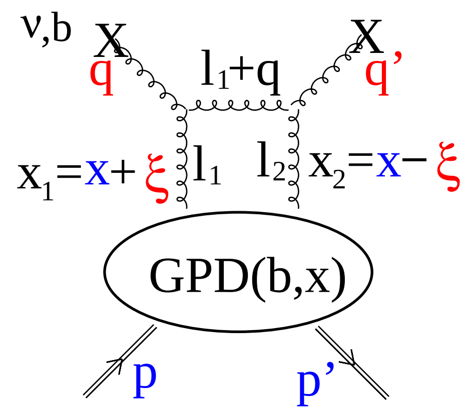
$$\int dx x^{n=0,1} \rho(b_x, b_y, x) \quad , \quad \rho \simeq g_0 + g_1 \cos(2\phi)$$

DVCS experimentally rare but conntects to $\lim_{\xi\to 0} \rho(b_x, b_y, x \pm \xi)$



Let us replace the virtual photon by an on-shell gluon from a thermal bath!
Same diagram up to $\alpha_s, tr[\lambda_{Gell-mann}]$. A GPD talking to a heat-bath!





Simple but consistent: Instant thermalization+wounded nucleon picture

Instant thermalization The quanta coming in and out, photons in DVCS but gluons for hadronic collisions, are "in detailed balance with GPD"

The Wounded nucleon picture The longitudinal structure of the Nucleon is unchanged before and after, any energy transferred is transverse.

Detailed balance gives expression for $T(b_x, b_y, x_{bj})$ in terms of the GPD

$$\int d^3q d^3q' d^4 l_{1,2} \mathcal{H}\left(b_x, b_y, q, q', p, p', T(b_x, b_y, x)\right) = p', q' \leftrightarrow p, q$$

$$\mathcal{H}\left(...\right) = \rho_2\left(b_x, b_y, \zeta_1(q, q', p, p'), \zeta_2(q, q', p, p')\right) \left|M(p, p', q, q')\right|^2 \times$$

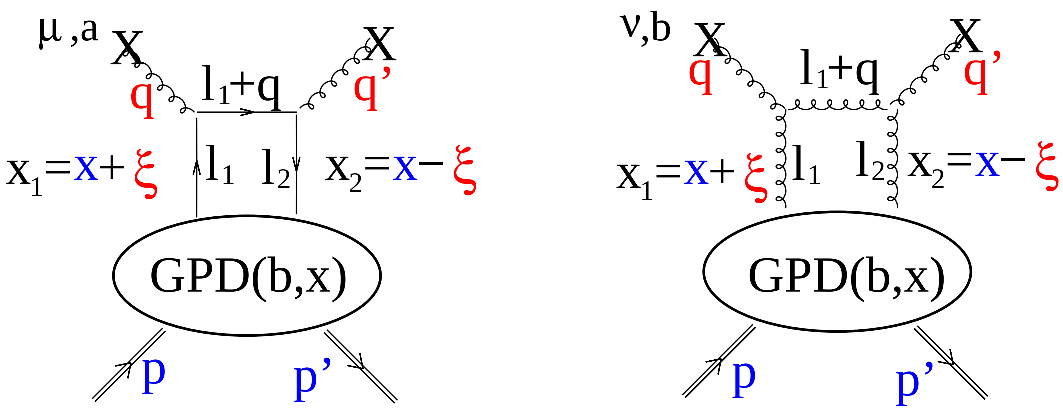
$$\times e^{-q_\mu \beta^\mu(b_x, b_y, x)} \left(1 + e^{-q'_\mu \beta^\mu(b_x, b_y, x)}\right) \delta^4\left(p + q - p' - q'\right)$$

$$\mathcal{H}\left(...\right) = \rho_2\left(b_x, b_y, x - \xi, x + \xi\right) \left|M(p, p', q, q')\right|^2 \times$$

$$\times e^{-q_\mu \beta^\mu(b_x, b_y, x)} \left(1 + e^{-q'_\mu \beta^\mu(b_x, b_y, x)}\right) \delta^4\left(p + q - p' - q'\right)$$

GPD enters via $\zeta_{1,2} = x_{bj} \pm \xi \; x_{bj}$ (observable),ξ (integrated over)

$$\rho_2\left(b_x, b_y, \zeta_1(q, q', p, p'), \zeta_2(q, q', p, p')\right) = \rho(b_x, b_y, \zeta_1) \times \rho(b_x, b_y, \zeta_2)$$

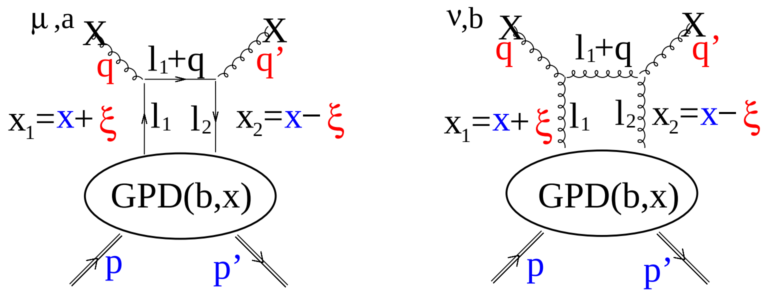


Partons co-move with $x \sim p_\perp e^{\pm y}$,in lightcone frame

$$p = \left(\frac{m}{2}\left(\cosh y \pm \sinh y\right), \mathbf{0}_\perp\right) \qquad , \qquad p' = \left(\frac{m_\perp}{2}\left(\cosh y \pm \sinh y\right), \mathbf{p}'_\perp \hat{\phi}_p\right)$$

q, q' is thermal bath (integrated over),so

$$\int d^8 l_{1,2} 1 d^3 q d^3 q' \delta^4 \left(l_1 - l_2 + q - q'\right) \rightarrow \int d\xi d^2 \mathbf{\Delta}_\perp d\phi_{\mathbf{k}_\perp} |\mathbf{k}_\perp|^2 \frac{dq^+ d^2 \mathbf{q}_\perp}{2q^+} \frac{dq^{+'} d^2 \mathbf{q}'_\perp}{2q^{+'}}$$

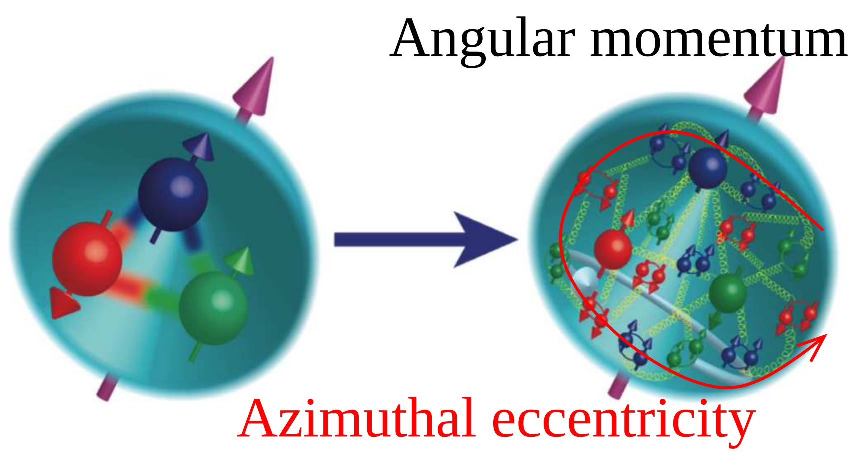


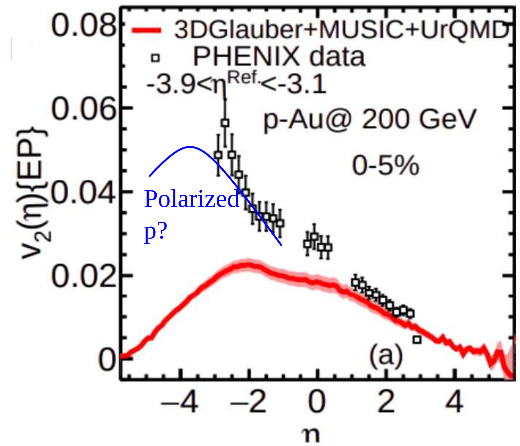
$$l_{2,1} = \left(P^{+}(x \pm \xi), \pm \frac{1}{8} \frac{\xi \mathbf{\Delta}_{\perp}^{2} + 4\xi m^{2}}{(1-\xi^{2})P^{+}}, \left[k_{\perp} \pm \frac{1}{2} \mathbf{\Delta}_{\perp} \right] \hat{\phi} \right)$$

problem: GPD has no $k_{\perp}$ Diffractive limit fixes kinematics but is unrealistic ("all exchange in energy transverse", "wounded nucleon" has same mass as before impact $\mathbf{k}_{\perp} = -\frac{2x\xi p^{+2}}{\mathbf{\Delta}_{\perp}}$ Easy to go beyond this with saturation

$$\mathbf{k}_{\perp} = \sqrt{\left(\frac{2x\xi p^{+2}}{\mathbf{\Delta}_{\perp}}\right)^{2} + Q^{2}} \quad , \quad f(Q)dQ \sim \exp\left[-\left(\frac{Q}{Q_s}\right)^{2} \right]$$

Now we are getting somewhere!

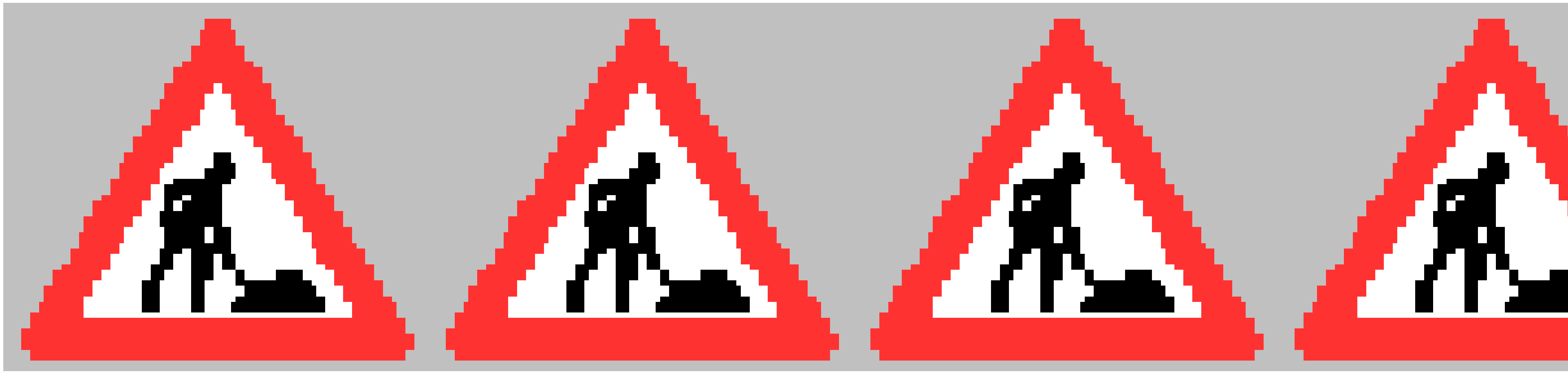




Our approach is simplified but consistent, and gets a transverse spin-dependent $T(b_x, b_y, x_{bj})$ out of lattice data, including spin! $\epsilon_n(y) = \frac{c_n(y)}{c_0(y)}$

$$c_n(y) = \int db_x db_y \cos(n\phi) T^m(b_x, b_y, x) \delta\left(y + \ln\left(\frac{1}{x}\right)\right) \delta\left(\phi - \tan^{-1}\left(\frac{b_y}{b_x}\right)\right)$$

An estimate...



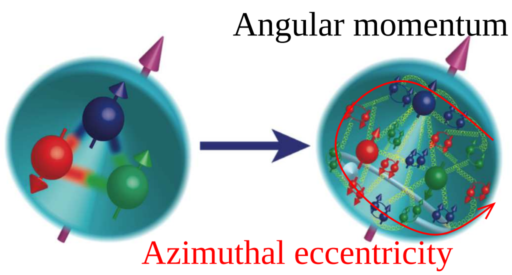
Unfortunately we didnt quite produce one in time for this workshop...

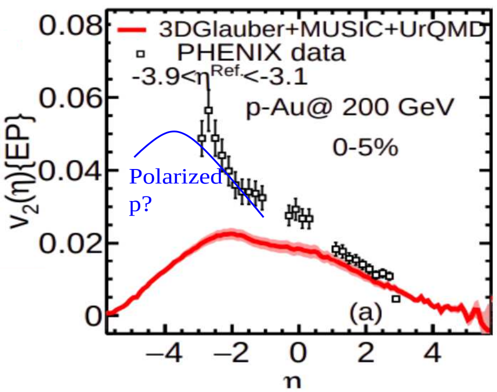
Conclusions

- The observation of fluids in small systems throws a bunch of conceptual problems at us!

- Not clear how a classical fluid initial condition emerges out of deeply quantum configuration of a nucleon in small systems.

- Fast thermalization (detailed balance) and wounded nucleons (transverse energy exchange) could be a way forward,exciting prospect of linking lattice to hydro initial conditions





- EiC is coming and RHIC,LHC data is here , the symbiosis needs to be used to its fullest! Final frontier for quantitative hydro in small systems concurrent v_2 for $pA, p^{\Uparrow}A, \gamma^* A$ together with EiC 3D tomography!

SPARE SLIDES